ZENBAKI KONPLEXUAK

ERA BINOMIKOA: $z = a + i \cdot b \;/\; a, b \in \mathbb{R} \wedge i = \sqrt{-1}$

$a = \mathrm{Re}\{z\}$
$b = \mathrm{Im}\{z\}$

Batuketa: $z_1 = a_1 + i \cdot b_1$, $z_2 = a_2 + i \cdot b_2$ $\quad z_1 + z_2 = (a_1 + a_2) + i \cdot (b_1 + b_2)$

Biderketa: $z_1 \cdot z_2 = (a_1 a_2 - b_1 b_2) + i(a_1 b_2 + b_1 a_2)$

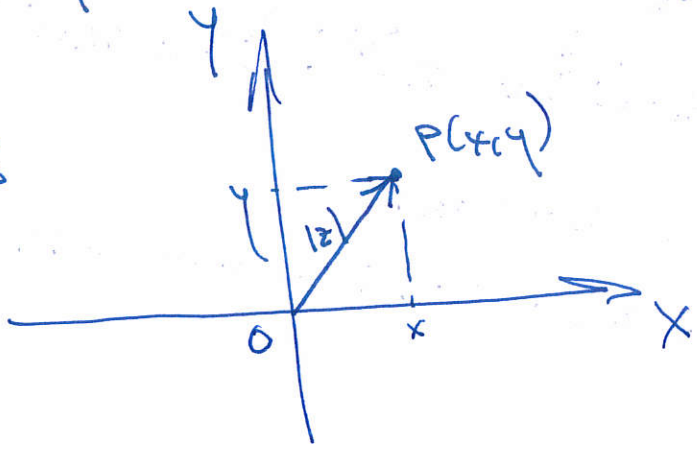
Konjokatua: $z = a + i \cdot b \Rightarrow \bar{z} = a - i \cdot b$

Zatiketa: $\frac{z_1}{z_2} = \frac{z_1 \cdot \bar{z}_2}{\bar{z}_2}$

Alderantzizkoa: $z^{-1} = \frac{1}{z} = \frac{\bar{z}}{z \cdot \bar{z}} = \frac{a - i \cdot b}{(a + i \cdot b)(a - i \cdot b)} = \frac{a - i \cdot b}{a^2 + b^2}$

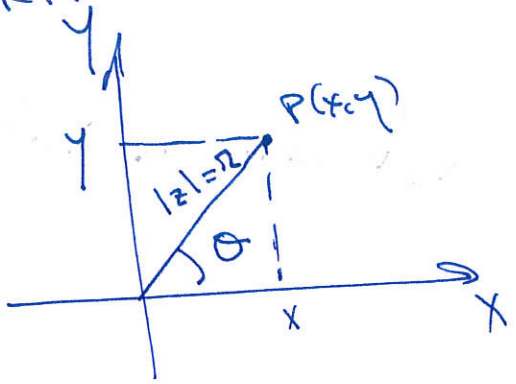
Modulua: $z = x + i \cdot y \Rightarrow |z| = \sqrt{x^2 + y^2}$

Plano konplexua:



Afixua

ERA TRIGONOMETRIKOA:



$z = x + iy$

$x = r\cos\theta$

$y = r\sin\theta$

$r = \sqrt{x^2+y^2}$

$\Rightarrow z = r(\cos\theta + i\sin\theta)$

$r: |z|$

$\theta: \arg(z)$ periodikoa

ERA ESPONENTZIALA:

Euler formula $e^{i\theta} = \cos\theta + i\sin\theta$

$z = re^{i\theta}$ edo $re^{i(\theta+2k\pi)}$ θ periodikoa dela

ERROTAZIOAK:

$z_1 \in \mathbb{C}$ / $|z_1|, \arg(z_1)$

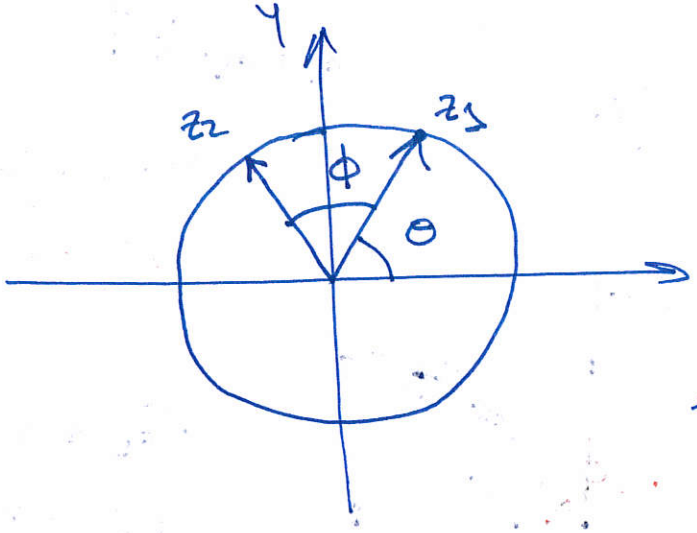
$z_1 \cdot e^{i\phi} = z_2 \in \mathbb{C}$ / $|z_2| = |z_1|$ eta $\arg(z_2) = \arg(z_1) + \phi$

Froga: $z_1 = re^{i\theta}$

$z_2 = z_1 \cdot e^{i\phi} = re^{i\theta} \cdot e^{i\phi} = re^{i(\theta+\phi)} \in \mathbb{C}$

$|z_2| = r = |z_1|$

$\arg(z_2) = \theta + \phi = \arg(z_1) + \phi$



$\vec{Oz_1}$ bektoreari ϕ angelueran errotazio bat eginez, $\vec{Oz_2}$ lortzeko

x diogu

* Ilusi grafikoa !

Berretura: $z = re^{i\theta} \Rightarrow z^n = r^n e^{in\theta}$

Erroa: $\sqrt[n]{z}$ $\quad w \in \mathbb{C} \;/\; w^n = z$

$z = re^{i(\theta + 2\pi k)} \Rightarrow z^{1/n} = r^{1/n} \cdot e^{i\frac{(\theta+2\pi k)}{n}}$ $\quad k = 0, 1, \dots, n-1$ n erro!

$k = 0 \quad w_1 = r^{1/n} e^{i\theta/n}$

$k = 1 \quad w_2 = r^{1/n} e^{i(\theta/n + \frac{2\pi}{n})}$

$\vdots$

$k = n-1 \quad w_n = r^{1/n} e^{i(\theta/n + \frac{2\pi(n-1)}{n})}$

$\arg(w_2) - \arg(w_1) = \frac{2\pi}{n}$

$\arg(w_3) - \arg(w_2) = \frac{2\pi}{n}$

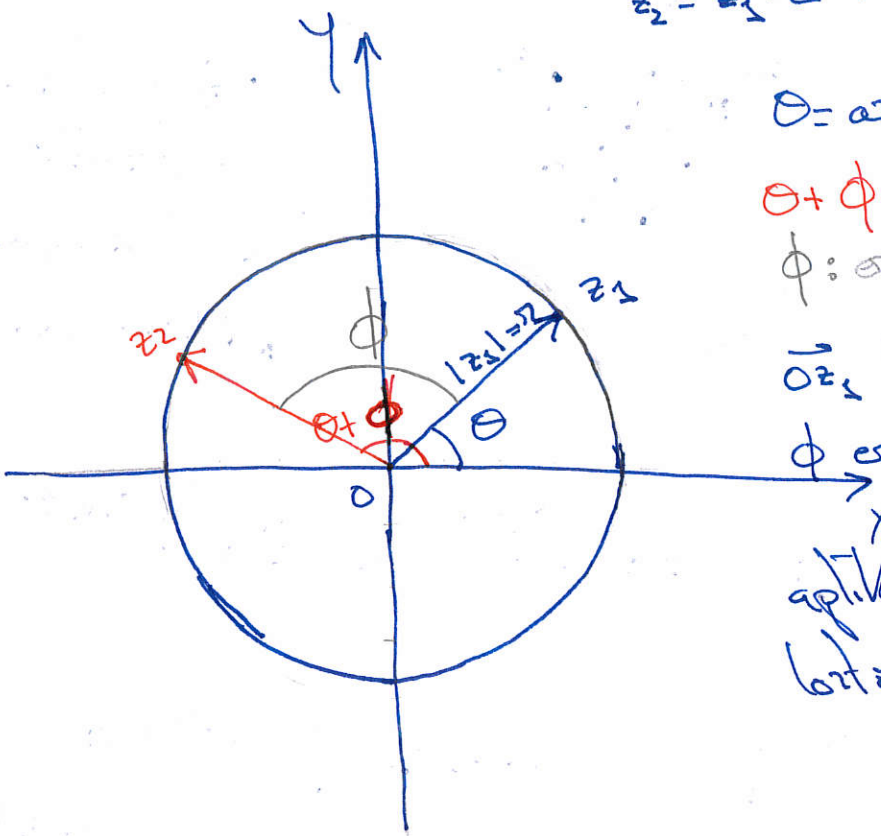
$\vdots$

$|w_1| = |w_2| = \dots = |w_n| = |z|^{1/n}$

** Ilusi grafikoa ?

Grafika 1:

$z_2 = z_1 \cdot e^{i\phi}$



$\theta = \arg(z_1)$

$\theta + \phi = \arg(z_2)$

ϕ: errotazioa

$\vec{Oz_1}$ bektoreari ϕ errotazio bat aplikatzen $\vec{Oz_2}$ lortzen dugu.

Grafika 2:

$\sqrt[4]{-16}$:

$z = -16 = 16e^{i(\pi+2k\pi)}$

$w = z^{1/4} = \sqrt[4]{16}\, e^{i\left(\frac{\pi+2k\pi}{4}\right)}$

$k = 0, 1, 2, 3$

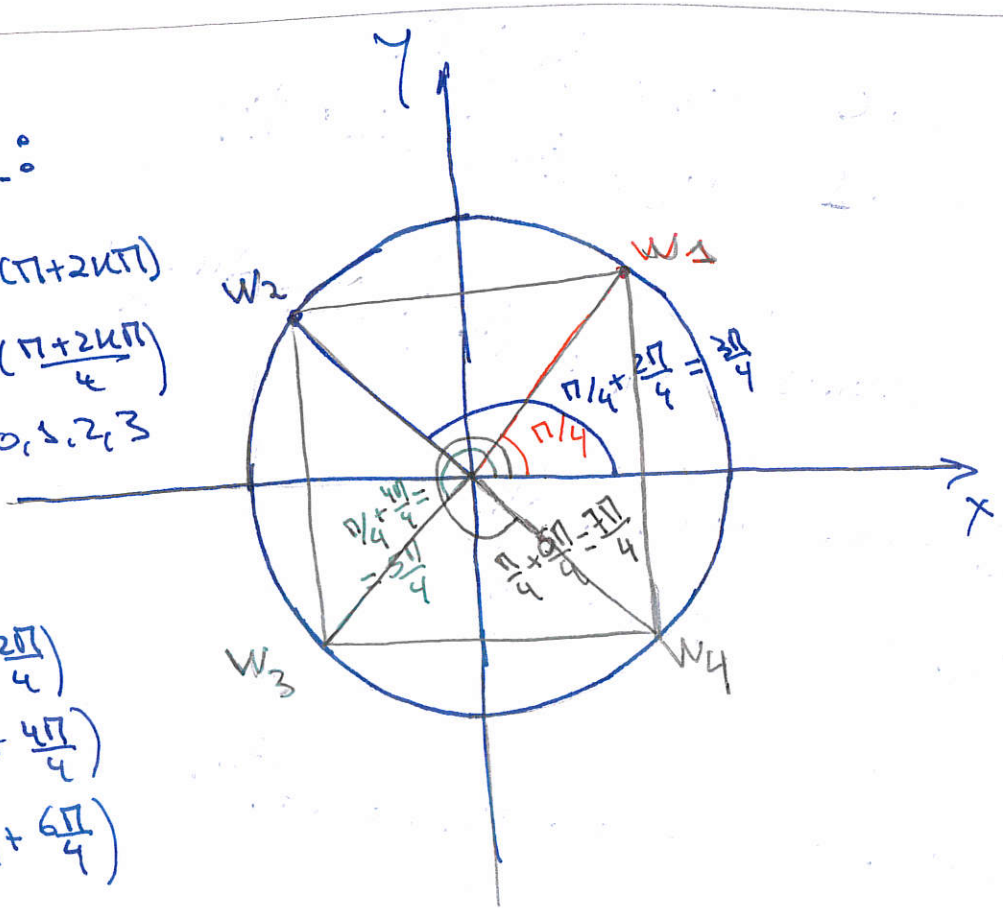
$\Downarrow$

$w_1 = 2e^{i\pi/4}$

$w_2 = 2e^{i\left(\pi/4 + \frac{2\pi}{4}\right)}$

$w_3 = 2e^{i\left(\pi/4 + \frac{4\pi}{4}\right)}$

$w_4 = 2e^{i\left(\pi/4 + \frac{6\pi}{4}\right)}$



$|w_1| = |w_2| = |w_3| = |w_4| \Rightarrow$

$\arg(w_2) - \arg(w_1) = \frac{2\pi}{4}$