Zenbaki Konplexuak:

$\mathbb{N} < \mathbb{Z} < \mathbb{Q} < \mathbb{R} < \mathbb{C}$

$0,1,2,\dots$ $0 \pm 1, \pm 2 \dots$ $\frac{a}{b} \neq 0$ $a,b = \mathbb{Z}$

$x^2+1=0$
$x^2=-1$
$x=\pm\sqrt{-1}$
$\sqrt{-1}=i \rightarrow 2i \quad -i \quad 1+i \quad -3+\frac{i}{2}$ → $a=-3$, $b=\frac{1}{2}$

$a=0, b=2$ $a=0, b=-1$ $a=1, b=1$

Adierazpen binomikoa

$\boxed{z = a + bi}$ $z \in \mathbb{C}$ $a,b \in \mathbb{R}$

bi → zati irudikaria
a → zati erreala

$a=0$ bada → $z=bi$ → zenbaki irudikaria soila
$b=0$ bada → $z=a \in \mathbb{R}$ → zenbaki erreala

Ariketak:

1) $2x^2+8=0 \rightarrow x=\sqrt{-4}=\sqrt{4}\cdot\sqrt{-1}=\pm 2i$

2) $z^2+z+1=0$

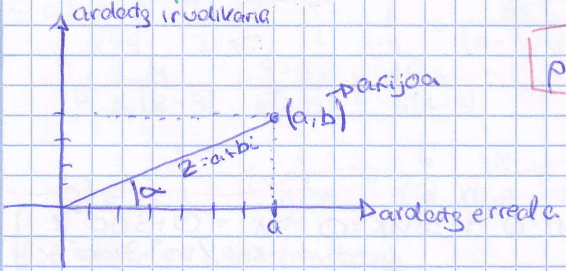
$$z=\frac{-1\pm\sqrt{1-4}}{2}=\frac{-1\pm\sqrt{-3}}{2}$$

$\frac{-1+\sqrt{-3}}{2} \rightarrow \frac{-1+\sqrt{3}i}{2}$ → $a=-1/2$, $b=\sqrt{3}/2$

$\frac{-1-\sqrt{-3}}{2} \rightarrow \frac{-1-\sqrt{3}i}{2}$ → $a=-1/2$, $b=-\sqrt{3}/2$

$\boxed{\begin{array}{l} i=\\ i^2=-1\\ i^3=-i\\ i^4=(-1)^2=1\end{array}}$

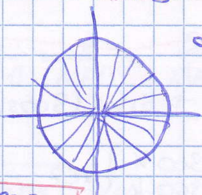
Plano Komplexua:



ρ · Tamainoa/moduloa $|z|=\sqrt{a^2+b^2}$ ← pitagoras

$|z|=2$ → Argumentua $=\tan=\frac{b}{a}$

Era polarra: $z=\rho_\alpha$



∞ soluzio, norabidea eta noranzkoa jakin behar dira.

$\tan\alpha=\frac{b}{a}$ $\cos\alpha=\frac{a}{|z|}=\frac{a}{\rho} \rightarrow \boxed{a=\rho\cdot\cos\alpha}$

$\sin\alpha=\frac{b}{|z|}=\frac{b}{\rho} \rightarrow \boxed{b=\rho\cdot\sin\alpha}$

$\rightarrow z=a+bi \rightarrow z=\rho\cdot\cos\alpha+(\rho\cdot\sin)i$

$\boxed{z=\rho(\cos\alpha+i\sin\alpha)}$
Era trigonometrikoa

$z=2_{90°}=2\cdot(\underbrace{\cos 90}_{0}+i\underbrace{\sin 90°}_{1})=2i$

$z=1+\sqrt{3}i \rightarrow a=1$, $b=\sqrt{3}$

$\rho=|z|=\sqrt{a^2+b^2}=\sqrt{1+3}=\sqrt{4}=2$

$\tan\alpha=\frac{\sqrt{3}}{1}=\sqrt{3}$

→ B. zenbaki konplexu **berdinak** dira zati erreala eta zati irudikoa berdinak dituztenean;
$a+bi = c+di \begin{cases} a=c \\ b=d \end{cases}$

Adb:
Kalkulatu x eta y $z_1 = 4x-2yi$, $z_2 = 2-xi$ berdinak izateko

$z_1 = 4x-2yi$ $\quad 4x-2yi = 2-xi$
$z_2 = 2-xi$

• $-2y = -x \to -2y = -1/2 \to y = \frac{1}{4}$
• $4x = 2 \to x = 1/2$

→ Zenbaki konplexu baten **aurkakoa**:

$z = a+bi \to -z = -a-bi \quad || \quad z = r_\alpha \to -z = r_{\alpha \pm 180}$

Adb: $z_1 = 3-2i \to -3i = -3+2i$
$z_2 = -3+3i \to -z_2 = 3-3i$

→ Zenbaki konplexu baten **konjokatua**:

$z = a+bi \to \bar{z} = a-bi$ → beti irudikarraren zeinua aldatzen da

$z_1 = 3-2i \to \bar{z}_1 = 3+2i$ $\quad || \quad z = r_\alpha \to \bar{z} = r_{-\alpha}$
$z_2 = -3+3i \to \bar{z}_2 = -3-3i$

Adb:
$z = \sqrt{3}+i \to -z = \sqrt{3}-i$
$a = \sqrt{3}$ $\quad \bar{z} = \sqrt{3}-i$
$b = 1$

$r = \sqrt{(\sqrt{3})^2+1} = \sqrt{4} = 2 > 0$

$\tan\alpha = \frac{b}{a} = \frac{1}{\sqrt{3}} \to 30°$

$2_{30°}$ → $z = 2 \to -z = 2_{210}$ (30 ± 180) → $a = r.\cos\alpha$, $b = r.\sin\alpha$
$2_{30°}$ → $\bar{z} = r_{-30} = r_{330}$

Zenbaki konplexuen arteko $\boxed{\text{batuketak}}$ eta $\boxed{\text{kenketak}}$ (beti era binomikoan):

$\left.\begin{matrix} z_1 = a+bi \\ z_2 = c+di \end{matrix}\right\}$ $z_1+z_2 = a+bi+c+di = a+c+bi+di = \overbrace{(a+c)}^{\mathbb{R}} + \overbrace{(b+d)}^{\mathbb{R}}i$
$z_1-z_2 = a+bi-(c+di) = a+bi-c-di = a-c+(b-d)i$

Adb:
$\left.\begin{matrix} z_1 = \sqrt{3}+i \\ z_2 = \sqrt{3}-i \end{matrix}\right\}$ $\sqrt{3}+\sqrt{3}+\cancel{1-1}i = 2\sqrt{3}$
$\cancel{\sqrt{3}-\sqrt{3}}+(1+1) = 2i$

$z_1 = a+2i$
$z_2 = -2+bi$ $\quad z_1+z_2 = a+2i+(-2+bi) = a-2+(2+b)i = 1+i$ $\quad \boxed{\begin{matrix} a-2=1 \to a=3 \\ 2+b=1 \to b=-1 \end{matrix}}$

a) a, b? $z_1+z_2 = 1+i$
b) a, b? z_1-z_2 zenbaki erreala izateko → $z_1-z_2 = a+2i-(-2+bi) = a+2+2i-bi$ →
$a+2+\underbrace{(2-b)i}_{=0} \in \mathbb{R}$
→ $b = +2$, $a \in \mathbb{R}$
$a =$
c) a, b? z_1-z_2 zenbaki purua izateko

Biderketa:
Eragiketa binomikoan: $z_1 = a+bi$, $z_2 = c+di$ $\quad z_1 \cdot z_2 = (a+bi)\cdot(c+di) = ac+adi+cbi+bdi^2 = ac-bd+(ad+cb)i$ ($i^2 = -1$)

Era polarrean: $\left.\begin{matrix} z_1 = r_\alpha \\ z_2 = r'_\beta \end{matrix}\right\}$ $z_1 \cdot z_2 = (r.r')_{\alpha+\beta}$

Zatiketa:

Era binomikoa: $\frac{z_1}{z_2}=\frac{a+bi}{c+di}=\frac{(a+bi)(c-di)}{(c+di)(c-di)}=\frac{ac-adi+cbi-bdi^2}{c^2-(di)^2}=\frac{ac-adi+bci+bd}{c^2-(-d)}=\frac{ac+bd+(bc-ad)i}{c^2+d}$

Era polarrean: $\frac{z_1}{z_2}=\frac{r_\alpha}{r'_\beta}=\left(\frac{r}{r'}\right)_{\alpha-\beta}$

Adb:

$z_1=1+\sqrt{3}$
$z_2=-1+\sqrt{3}$ $\rightarrow \frac{1\cdot(-1)+\sqrt{3}\cdot\sqrt{3}+[(\sqrt{3}\cdot(-1))-(1\cdot\sqrt{3})]i}{(\sqrt{3})^2+\sqrt{3}} \rightarrow \frac{-1+3+(-\sqrt{3}-\sqrt{3})i}{3+\sqrt{3}}=\frac{-3+(-2\sqrt{3})i}{3+\sqrt{3}}=$

$=\frac{-3-2\sqrt{3}i}{3+\sqrt{3}}\cdot\frac{3-\sqrt{3}}{3-\sqrt{3}}=\frac{(-3-2\sqrt{3}i)\cdot(3-\sqrt{3})}{6}$

$z_1=\sqrt{1+3}=2$
$z_2=\sqrt{1+3}=2$

$\tan\alpha=\frac{\sqrt{3}}{1}=\sqrt{3}=60^\circ \rightarrow 2_{60^\circ}$

$\tan\alpha=\frac{\sqrt{3}}{-1}=180^\circ-60^\circ=120^\circ \rightarrow 2_{120^\circ}$

$\left(\frac{2}{2}\right)_{60^\circ-120^\circ}=1_{-60^\circ}=1_{300^\circ}$

$\frac{z_1}{z_2}=\frac{1+\sqrt{3}i}{-1+\sqrt{3}i}\cdot\frac{-1-\sqrt{3}i}{-1-\sqrt{3}i}=\frac{-1-\sqrt{3}i-\sqrt{3}i-(\sqrt{3}i)^2}{1+3}=\frac{-1-2\sqrt{3}i+3}{4}=\frac{2-2\sqrt{3}i}{4}=\frac{1-\sqrt{3}i}{2}=\frac{1}{2}-\frac{\sqrt{3}}{2}i$

Era binomikoa:

$\boxed{z=a+bi}$ $a,b\in\mathbb{R}$ → beti egin horrela!!!

Era polarra

$z=r_\alpha$ r=modulua ≥ 0
α=argumentua

$z=r(\cos\alpha+i\sin\alpha)$

Berreketak:

Era binomikoan:

$$\binom{n}{m} = \frac{n!}{m!(n-m)!}$$

$$z^n = (a+bi)^n$$

n^1 1 1

n^2 1 2 1

n^3 1 3 3 1

n^4 1 4 6 4 1

n^5 1 5 10 10 5 1

Newtonen binomioak:

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(a+b)^4 = a^4 + 2a^3 \cdot b + 6a^2 \cdot b^2 + 4ab^3 + b^4$$

$$(a+b)^n = \underbrace{\binom{n}{0}}_{1} a^n + \binom{n}{1} a^{n-1} \cdot b + \binom{n}{2} a^{n-2} \cdot b^2 + \dots + \underbrace{\binom{n}{n}}_{1} b^n$$

Adib:

$$z = \sqrt{2} + \sqrt{2}i$$

$$z^4 = (\sqrt{2}+\sqrt{2}i)^4 = \underbrace{\binom{4}{0}}_{1} \cdot \underbrace{\sqrt{2}^4}_{2^2} + \underbrace{\binom{4}{1}}_{4} \underbrace{\sqrt{2}^3}_{2\sqrt{2}} \cdot \sqrt{2}i + \underbrace{\binom{4}{2}}_{6} \underbrace{\sqrt{2}^2}_{2} \cdot \underbrace{(\sqrt{2}i)^2}_{-1} + \underbrace{\binom{4}{3}}_{4} \cdot \sqrt{2} \cdot \underbrace{(\sqrt{2}i)^3}_{2\sqrt{2}i^3} + \underbrace{\binom{4}{4}}_{1} \underbrace{(\sqrt{2}i)^4}_{2^2 \cdot i^4}$$

$$4 + \underbrace{8\sqrt{2}\cdot\sqrt{2}i}_{16i} - 6\cdot 2 \cdot 2 \underbrace{- 4\sqrt{2}\cdot 2\sqrt{2}i}_{-16i} + 4 = -16$$

Era polarrean

$$z = r_\alpha \rightarrow z^n = r^n_{\alpha \cdot n}$$

Adib $\quad z^4 = (\sqrt{2}+\sqrt{2}i)^4$

$$r = \sqrt{(\sqrt{2})^2 + (\sqrt{2})^2} = 2$$

$$\tan\alpha = \frac{\sqrt{2}}{\sqrt{2}} = 1 = 45°$$

$$z = 2_{45°} \rightarrow z^4 = 2^4_{45° \cdot 4} = \boxed{16_{180°}}$$

$$16(\underbrace{\cos 180}_{-1} + i\underbrace{\sin 180}_{0}) = -16$$

$$z^n = [r(\cos\alpha + i\sin\alpha)]^n = r^n(\cos\alpha + i\sin\alpha)^n = r^n(\cos(\alpha n) + i\sin(\alpha n))$$