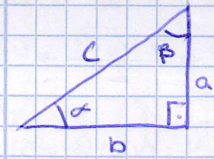
Trigonometria:

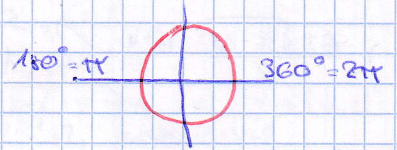
Arragoi trigonometrikoak



$\sin\alpha = \dfrac{\text{aurkako katetoa}}{\text{hipotenusa}} = \dfrac{a}{c} \rightarrow \operatorname{cosec}\alpha = \dfrac{1}{\sin\alpha} = \dfrac{\text{hip}}{\text{aurkakoa}} = \dfrac{c}{a}$

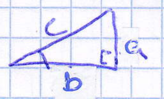
$\cos\alpha = \dfrac{\text{alboko katetoa}}{\text{hipotenusa}} = \dfrac{b}{c} \rightarrow \sec\alpha = \dfrac{1}{\cos\alpha} = \dfrac{\text{hip}}{\text{albokoa}} = \dfrac{c}{b}$

$\tan\alpha = \dfrac{\sin\alpha}{\cos\alpha} = \dfrac{a}{b} \rightarrow \operatorname{cotan}\alpha = \dfrac{1}{\tan\alpha} = \dfrac{\text{albokoa}}{\text{aurkakoa}} = \dfrac{b}{a}$



Arragoi trigonometrikoen arteko erlazioak:

$\boxed{\tan\alpha = \dfrac{\sin\alpha}{\cos\alpha}}$ $\boxed{\cot\alpha = \dfrac{\cos\alpha}{\sin\alpha}}$



Pitagoras: $a^2 + b^2 = c^2$

$(c \cdot \sin\alpha)^2 + (c \cdot \cos\alpha)^2 = c^2$

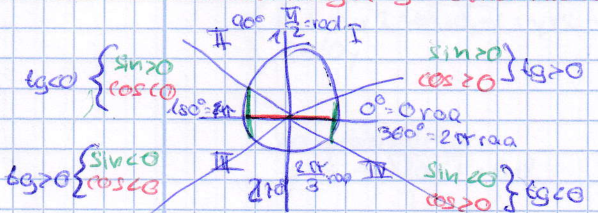
$c^2 \sin\alpha^2 + c^2 \cdot \cos\alpha^2 = c^2$

$c^2(\sin\alpha^2 + \cos\alpha^2) = c^2$

$\boxed{\sin^2\alpha + \cos^2\alpha = 1}$ $\rightarrow$ $\dfrac{\sin^2\alpha}{\cos^2\alpha} + \dfrac{\cos^2\alpha}{\cos^2\alpha} = \dfrac{1}{\cos^2\alpha}$

$\boxed{\tan^2\alpha + 1 = \dfrac{1}{\cos^2\alpha}}$

Zirkunferentzia goniometrikoa:



$\cos\alpha = \dfrac{1}{2} \rightarrow \sec\alpha = -2$, $\alpha \in$ IV. koadrantea

$270° \leq \alpha \leq 360°$

$\dfrac{3\pi}{2} \leq \alpha \leq 2\pi$

rad $\cdot \dfrac{180°}{\pi \text{ rad}}$ ° ; $\cdot \dfrac{\pi \text{ rad}}{180°}$

$\sin^2\alpha + \cos^2\alpha = 1$

$\sin^2\alpha + \left(\dfrac{1}{2}\right)^2 = 1$

$\sin^2\alpha = 1 - \dfrac{1}{4}$ $\tan\alpha = \dfrac{\sin\alpha}{\cos\alpha} = \dfrac{-\sqrt{3}/2}{1/2} = -\sqrt{3}$

$\sin^2\alpha = \dfrac{3}{4}$ $\cot\alpha = -\dfrac{1}{\sqrt{3}} \cdot \dfrac{\sqrt{3}}{\sqrt{3}} = -\dfrac{\sqrt{3}}{3}$

$\sin\alpha = \pm\sqrt{\dfrac{3}{4}}$

$\sin\alpha = -\dfrac{\sqrt{3}}{2} \rightarrow \operatorname{cosec}\alpha = -\dfrac{2}{\sqrt{3}} \cdot \dfrac{\sqrt{3}}{\sqrt{3}} = -\dfrac{2\sqrt{3}}{3}$

$\operatorname{tg}\alpha = \dfrac{\sin\alpha}{\cos\alpha}$

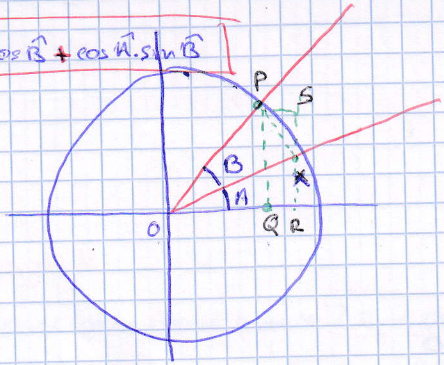
$\sin^2\alpha + \cos^2\alpha = 1$

$1 + \tan^2\alpha = \dfrac{1}{\cos^2\alpha}$

Angeluen batura eta kenduren arrazoi trigonometrikoak:

Batura:

$\rightarrow \sin(\hat{A}+\hat{B}) = \boxed{\sin\hat{A}\cdot\cos\hat{B} + \cos\hat{A}\cdot\sin\hat{B}}$



$\sin(\hat{A}+\hat{B}) = \frac{aurk.}{h} = \frac{\overline{PQ}}{1} = \overline{PQ} = \overline{RS} = \overline{RT}+\overline{TS}$

$\sin\hat{B} = \frac{\overline{PT}}{1} \rightarrow \overline{PT} = \sin\hat{B}$

$\cos\hat{B} = \frac{\overline{OT}}{1} \rightarrow \overline{OT} = \cos\hat{B}$

$\sin\hat{A} = \frac{\overline{RT}}{\overline{OT}} \rightarrow \overline{RT} = \sin\hat{A}\cdot\overline{OT} = \sin\hat{A}\cdot\cos\hat{B}$

$\left(\cos\hat{A} = \frac{\overline{OR}}{\overline{OT}} \rightarrow\right) \rightarrow \cos\hat{A} = \frac{\overline{TS}}{\overline{PT}} \rightarrow$

$\rightarrow \overline{TS} = \cos\hat{A}\cdot\overline{PT} = \cos\hat{A}\cdot\sin\hat{B}$

$\rightarrow \sin(\hat{A}-\hat{B}) = \sin(\hat{A}+(-\hat{B})) = \sin\hat{A}\cdot\cos(-\hat{B}) + \cos\hat{A}\cdot\sin(-\hat{B}) = \boxed{\sin\hat{A}\cdot\cos\hat{B} - \cos\hat{A}\cdot\sin\hat{B}}$

$\rightarrow \cos(\hat{A}+\hat{B}) = \sin\left[90^\circ - (\hat{A}+\hat{B})\right] = \sin\left[(90-\hat{A})-\hat{B}\right] = \underbrace{\sin(90^\circ-\hat{A})}_{\cos\hat{A}}\cdot\cos\hat{B} - \underbrace{\cos(90^\circ-\hat{A})}_{\sin\hat{A}}\cdot\sin\hat{B}$

$\cos(\hat{A}+\hat{B}) = \boxed{\cos\hat{A}\cdot\cos\hat{B} - \sin\hat{A}\cdot\sin\hat{B}}$

$\rightarrow \cos(\hat{A}-\hat{B}) = \cos(\hat{A}+(-\hat{B})) = \cos\hat{A}\cdot\underbrace{\cos(-\hat{B})}_{\cos\hat{B}} - \sin\hat{A}\cdot\underbrace{\sin(-\hat{B})}_{-\sin\hat{B}} = \boxed{\cos\hat{A}\cdot\cos\hat{B} + \sin\hat{A}\cdot\sin\hat{B}}$

$\rightarrow \tan(\hat{A}+\hat{B}) = \frac{\sin(\hat{A}+\hat{B})}{\cos(\hat{A}+\hat{B})} = \frac{\sin\hat{A}\cdot\cos\hat{B} + \cos\hat{A}\cdot\sin\hat{B}}{\cos\hat{A}\cdot\cos\hat{B} + \sin\hat{A}\cdot\sin\hat{B}} = \frac{\frac{\sin\hat{A}\cdot\cancel{\cos\hat{B}}}{\cos\hat{A}\cdot\cancel{\cos\hat{B}}} + \frac{\cancel{\cos\hat{A}}\cdot\sin\hat{B}}{\cancel{\cos\hat{A}}\cdot\cos\hat{B}}}{\cancel{\frac{\cos\hat{A}\cdot\cos\hat{B}}{\cos\hat{A}\cos\hat{B}}} + \frac{\sin\hat{A}\cdot\sin\hat{B}}{\cos\hat{A}\cdot\cos\hat{B}}} =$

$= \boxed{\frac{\tan\hat{A} + \tan\hat{B}}{1 - \tan\hat{A}\cdot\tan\hat{B}}}$

$\rightarrow \tan(\hat{A}-\hat{B}) = \frac{\tan(\hat{A}+(-\hat{B}))}{1-\tan\hat{A}\tan\hat{B}} \rightarrow \frac{\tan\hat{A} + \tan(-\hat{B})}{1-\tan\hat{A}\cdot\tan(-\hat{B})} = \boxed{\frac{\tan\hat{A} - \tan\hat{B}}{1+\tan\hat{A}\cdot\tan\hat{B}}}$

Angelu bikoitzaren arrazoiak:

$\sin(2\alpha) \to \sin(\alpha+\alpha) \to \sin\alpha \cdot \cos\alpha + \cos\alpha \cdot \sin\alpha \to \boxed{2 \cdot \sin\alpha \cdot \cos\alpha}$

$\cos(2\alpha) \to \cos(\alpha+\alpha) \to \cos\alpha \cdot \cos\alpha - \sin\alpha \cdot \sin\alpha \to \boxed{\cos^2\alpha - \sin^2\alpha}$

$tg(2\alpha) \to tg(\alpha+\alpha) \to \frac{\tan\alpha + \tan\alpha}{1-\tan\alpha \cdot \tan\alpha} \to \boxed{\frac{2\tan\alpha}{1-\tan^2\alpha}}$

$\cos\alpha = \cos\left(2\frac{\alpha}{2}\right) = \cos^2\left(\frac{\alpha}{2}\right) - \sin^2\left(\frac{\alpha}{2}\right)$

$+\ 1 = \sin^2\left(\frac{\alpha}{2}\right) + \cos^2\left(\frac{\alpha}{2}\right)$

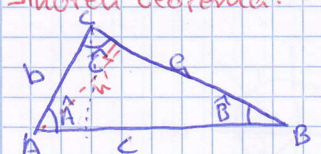
$1+\cos\alpha = 2\cos^2\left(\frac{\alpha}{2}\right)$

$\frac{1+\cos\alpha}{2} = \cos^2\left(\frac{\alpha}{2}\right) \to \boxed{\cos\left(\frac{\alpha}{2}\right) = \pm\sqrt{\frac{1+\cos\alpha}{2}}}$

$\cos\alpha - 1 = -2\sin^2\left(\frac{\alpha}{2}\right) \to \frac{1-\cos\alpha}{2} = \sin^2\left(\frac{\alpha}{2}\right) \to \boxed{\sin\left(\frac{\alpha}{2}\right) = \pm\sqrt{\frac{1-\cos\alpha}{2}}}$

$\tan\left(\frac{\alpha}{2}\right) = \frac{\sin\left(\frac{\alpha}{2}\right)}{\cos\left(\frac{\alpha}{2}\right)} = \frac{\pm\sqrt{\frac{1-\cos\alpha}{2}}}{\pm\sqrt{\frac{1+\cos}{2}}} = \boxed{\pm\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}}}$

Sinuen teorema:



$\sin\hat{A} = \frac{h}{b} \to h = b \cdot \sin\hat{A}$

$\sin\hat{B} = \frac{h}{a} \to h = a \cdot \sin\hat{B}$

$a \cdot \sin\hat{B} = b \cdot \sin\hat{A}$

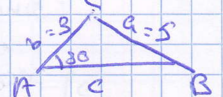
$\frac{a}{\sin\hat{A}} = \frac{b}{\sin\hat{B}}$

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$\sin\hat{C} = \frac{h'}{b}$

$\sin\hat{B} = \frac{h'}{c}$

$\frac{b}{\sin\hat{B}} = \frac{c}{\sin\hat{C}}$



$\frac{a}{\sin\hat{A}} = \frac{b}{\sin\hat{B}} \to \frac{5}{\sin 30} = \frac{3}{\sin\hat{B}} =$

$= 0'5\,6088\ldots$

$\hat{B} = \arcsin\,(0'5\,6088\ldots) =$

$= 36'22^\circ$

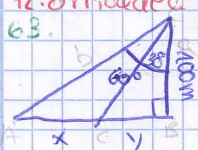
$\hat{C} = 180 - A - B = 63'78^\circ$

$\frac{c}{\sin\hat{C}} = \frac{a}{\sin\hat{A}} =$

$= c = 4'55\,cm$

72. orrialdea

63.

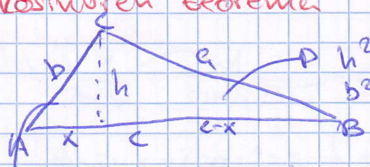


$tg 30^\circ = \frac{y}{100} \to 100 \cdot tg 30^\circ = y \to y = 57'7\,m$

$tg 60^\circ = \frac{x+y}{100} \to tg 60 \cdot 100 = 57'7 + x \to x = 115'5\,m$

$\frac{y}{\sin 30^\circ} = \frac{100}{\sin 60^\circ} \to y = \frac{100 \cdot \sin 30}{\sin 60} = 57'7\,m$

Kosinuen teorema



$h^2 + (c-x)^2 = a^2$

$b^2 - x^2 + (c-x)^2 = a^2 \to b^2 - x^2 + c^2 - 2cx = a^2$

$b^2 + c^2 - 2cb\cos\hat{A} = a^2$

Pitagoras

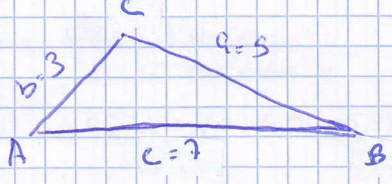
$x^2 + h^2 = b^2$

$h^2 = b^2 - x^2$

$a^2 = b^2 + c^2 - 2bc \cdot \cos\hat{A}$

$b^2 = a^2 + c^2 - 2ac \cdot \cos\hat{B}$

$c^2 = a^2 + b^2 - 2ab \cdot \cos\hat{C}$



$a^2 = b^2 + c^2 - 2bc \cdot \cos\hat{A}$

$25 = 9 + 49 - 2 \cdot 3 \cdot 7 \cdot \cos\hat{A}$

$25 - 58 = -42\cos\hat{A}$

$\frac{33}{42} = \cos\hat{A} \to \cos\hat{A} = 0'80489\ldots = 36'4^\circ$

$\frac{5}{\sin 36'4} = \frac{3}{\sin\hat{B}} \to \hat{B} = 20'85^\circ$

$C = 180 - 36'4 - 20'85 = 15'55^\circ$