

TERMOTERKINIA

1. GAIA : OINARRIZKO KONZEPТУAK

- [DEF]: BERO TRANSFERENTZIA Q (J)
- BERO TRANSFERENTZIA ABIADURA \dot{Q} (W)
- BERO-FLUXUA \dot{q} (W/m²)

- TERMODINAMIKAREN 1^o LEGEA:
$$Q_{in} - Q_{out} + E_{gen} = E_{MET}$$

- BERO TRANSFERENTZIARAKO MEKANISMOAK:

1) EROAPENA:
$$\dot{Q} = -kA \frac{dT}{dx} = -kA \frac{\Delta T}{\Delta x} \quad (W)$$

FOURIER

- [DEF]: EROANKORTASUN TERMIKOA K (W/m°C)

- [DEF]: DIFUSIBILITATE TERMIKOA
$$\alpha = \frac{K}{\rho C_p} \quad (m^2/s)$$

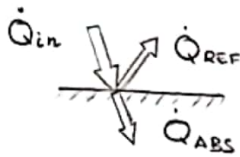
2) KONBEKZIOA:
$$\dot{Q} = h A_s (T_s - T_\infty) \quad (W)$$

NEWTON

- [DEF]: KONBEKZIO KOFIZIENTEA h (W/m²°C)

3) ERRADIAZIOA:
$$\dot{Q} = \epsilon \sigma A_s (T_s^4 - T_{SUR}^4) \quad (W)$$

KIRCHOFF



$$\dot{Q}_{REF} = (1 - \alpha) \dot{Q}_{in}$$
$$\dot{Q}_{ABS} = \alpha \dot{Q}_{in}$$

2. GAIA : BERO EROAPENAREN EKUAZIOA

- [DEF]: GRADIENTE TERMIKOA
$$\vec{\nabla} T = \frac{\partial T}{\partial x} \vec{i} + \frac{\partial T}{\partial y} \vec{j} + \frac{\partial T}{\partial z} \vec{k} \quad (K/m)$$

• $\dot{Q} \Rightarrow \vec{\nabla} T$ • $\dot{Q} \perp$ ISOTERMA

• [DEF] : BERO EROAPENAREN EKVAZIO OROKORRA (3D)

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{e}_{gen}}{k} = \frac{1}{\alpha} \frac{\partial T}{\partial t}$$

• [DEF] : BERO EROAPENAREN EKVAZIO UNIDIMENSIONALA (1D)

$$\frac{1}{r^n} \frac{d}{dr} \left(r^n \frac{dT}{dr} \right) + \frac{\dot{e}_{gen}}{k} = \frac{1}{\alpha} \frac{\partial T}{\partial t}$$

$n=0$ NORMA LAUA
 $n=1$ NORMA ZILINDRIKOA
 $n=2$ NORMA ESFERIKOA

• HUSTUGAILU GABE $\dot{e}_{gen} = 0$ • 2 MB

• EGOERA EGONKORRA $\frac{\partial T}{\partial t} = 0$

• [PROZ] : EK. DIF. \rightarrow INTEGRATU \rightarrow M.B. $\rightarrow T = T(x, y, z, t)$

(GOMENDAGARRIA : EGONKORRA + $\dot{e}_{gen} \neq 0$ DENEAN)

• MUGALDE BALDINTZAK : ① $T(x=\zeta) = T^{\circ}C$ ② $Q_i = Q_j$

3 GAIA : BERO EROAPEN GELDIKORRA

• [DEF] : ANALOGIA TERMOELEKTRIKOA $I = \frac{\Delta V}{R} \Leftrightarrow \dot{Q} = \frac{T_1 - T_2}{R}$

• [PROZ] : $\frac{dT}{dt} = 0 \wedge \dot{e}_{gen} = 0$

- EROAPENEN ERRESISTENTZIA :

1) NORMA LAUA : $R = \frac{L}{k \cdot A} \text{ (K/W)}$

2) ZILINDROA : $R = \frac{\ln(r_2/r_1)}{2\pi L \cdot k} \text{ (K/W)}$

3) ESFERA : $R = \frac{r_2 - r_1}{4\pi \cdot r_1 \cdot r_2 \cdot k} \text{ (K/W)}$

- KONBEKZIO BIDEZKO ERRESISTENTZIA : $R = \frac{1}{h \cdot A} \text{ (K/W)}$

- ERRADIAZIO BIDEZKO ERRESISTENTZIA : $R = \frac{1}{A \cdot \epsilon \cdot \sigma \cdot (T_S^2 + T_{SURR}^2) \cdot (T_S + T_{SURR})}$

• ERRESISTENTZIA KONBINAZIOAK : • PARALELO : $\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2}$

• SERIE : $R_T = R_1 + R_2$

• ERRADIO KRITIKOA:

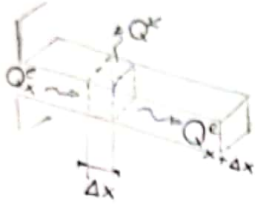
$$r_{21L} = \frac{K_{150}}{h}$$

ZILINDROA

$$r_{ESP} = \frac{2 \cdot K_{150}}{h} \quad (v < 1 \text{ cm})$$

ESFERA

• GAINAZAL HEGALDUNAK:



- HEGAL EKUAZIOAK:

- 1) $\dot{Q}_{e,x} = \dot{Q}_{e,x+\Delta x} + \dot{Q}_k$

- 2) $\dot{Q}_k = h_p \Delta x (T_b - T_\infty)$

- 3) $\dot{Q}_{e,x} = -k A_c \frac{dT(x)}{dx}$

- 4) $\frac{d}{dx} \left(k A_c \frac{dT(x)}{dx} \right) - h_p (T_b - T_\infty) = 0$

- 5) $\frac{d^2\theta}{dx^2} - m^2 \theta = 0 \quad [E.D.A.]$

- 6) $\theta(x) = C_1 e^{m \cdot x} + C_2 e^{-m \cdot x}$

$$\theta(x) = T(x) - T_\infty$$

$$m^2 = \frac{h_p}{K A_c}$$

- ZERO TRANSFERENTZIA MAXIMOA:

$$\dot{Q}_{FIN, MAX} = h A_{FIN} (T_b - T_\infty)$$

- ERRENDIMENDUA:

$$\eta = \frac{\dot{Q}_{FIN}}{\dot{Q}_{FIN, MAX}}$$

$$A_{FIN} = p \cdot L_c$$

$$L_c = L + \frac{A_c}{p}$$

$$Q_{FIN} = N \left(-k A_c \frac{dT(x)}{dx} \right)_{x=0}$$

$$Q_{NOFIN} = h \cdot A_{NOFIN} (T_b - T_\infty)$$

$$Q_{BARE} = h A_{BARE} (T_b - T_\infty)$$

- ERAGINKORTASUNA:

$$\epsilon = \frac{N Q_{FIN} + Q_{NOFIN}}{Q_{BARE}}$$

- $\epsilon < 1$ ISOLATZAILA
- $\epsilon = 1$ ERAGINIK EZ
- $\epsilon > 1$ GALERA HANDITU

4. GAIA: BERO EROAPEN IRAGANKORRA

• PARAMETRO KONZENTRATUAK:

$$T(r, z) \sim T(z)$$

$$Bi = \frac{h L_c}{K}$$

BIOT

$$L_c = \frac{V}{A_s}$$

- LUZERA CHARACTERISTICO

- BALDINIZA: $Bi \leq 0,1$

$$\frac{T(z) - T_\infty}{T_i - T_\infty} = \exp\left(-\frac{h A_s}{\rho V c_p} z\right)$$

- EKUAZIOAK:

$$Q = m c_p (T(z) - T_i)$$

BT. TOT

$$Q = m c_p (T_\infty - T_i)$$

BT. MAX

• 1D-KO PROBLEMAK DIMENTSIOGABETUAK (IRAGANKORRA)

- T(°C) DIMENTSIOGABEA: $\theta(x, z) = \frac{T(x, z) - T_m}{T_i - T_m} = \frac{Q(x, z)}{Q_{max}}$
- DISTANTZIA DIMENTSIOGABEA: $\bar{x} = x/L$
- BIOT: $Bi = \frac{hL}{k}$ (B.T. KOEFIZIENTEA)
- FOURIER (TAU): $z = \frac{\alpha \cdot t}{r_0^2}$ ($z = \frac{Q_{LOND}}{Q_{MET.}}$)

5 GAIA: ZENBAKIZKO METODOAK BERO-EROAPENEAN

• EKUAZIO DIFERENZIALEN DIFERENZIA FINITUKO FORMULAZIOA:

- 1D: $\frac{T_{m-1} - 2T_m + T_{m+1}}{\Delta x^2} + \frac{\dot{e}_m}{k} = 0$

- 2D: $\frac{T_{m+1, n} - 2T_{m, n} + T_{m-1, n}}{\Delta x^2} + \frac{T_{m, n+1} - 2T_{m, n} + T_{m, n-1}}{\Delta y^2} + \frac{\dot{e}_{m, n}}{k} = 0$

• ADI! BEI ERREXAGO



• EGOERA IRAGANKORRA:

- METODO ESPLIZITUA: $\sum_{\text{all sides}} \dot{Q}^i + \dot{E}_{gen, element} = \rho V_{element} \cdot c_p \frac{T_m^{i+1} - T_m^i}{\Delta t}$
(CERRAZA, Δt MUGATUA)

- METODO INPLIZITUA: $\sum_{\text{all sides}} \dot{Q}^{i+1} + \dot{E}_{gen, element} = \rho V_{element} \cdot c_p \frac{T_m^{i+1} - T_m^i}{\Delta t}$
(ZAILA, Δt EZ-MUGATUA)

————— NODO GUTIAK BATERA EBATEI —————

• EGONKORTASUN IRIZPIDEA:

- 1D: $\tau = \frac{\alpha \Delta t}{\Delta x^2} \leq \frac{1}{2}$

- 2D: $\tau = \frac{\alpha \Delta t}{l^2} \leq \frac{1}{4}$

6. GAIA KONBEKZIOAREN OINARRIAK

• BAZE BESTERO KONBEKZIO KOEFIZIENTEA:
$$h = \frac{-k_{\text{fluid}} \cdot (dT/dy)_{y=0}}{T_s - T_{\infty}}$$

• NUSELTEN ZENBAKIA:
$$Nu = \frac{h \cdot L_c}{k} \quad (Nu = \frac{q_{\text{konB}}}{q_{\text{konB}}})$$

• GAINAZALEKO ERABAKIDURA-TENISIOA:
$$Z_c = C_f \cdot \rho \frac{V^2}{2} \quad [N/m^2]$$

• BISKOSITATE ZINEMATIKOA:
$$\nu = \frac{\mu}{\rho} \quad [m^2/s] \quad (\mu [kg/m \cdot s])$$

• PRANDTLEN ZBK:
$$Pr = \frac{\nu}{\alpha} = \frac{\mu C_p}{k} \quad \left(\frac{\text{MOMENTUAREN DIFUSIBILITATE MOLEKULARRA}}{\text{BEROAREN DIFUSIBILITATE MOLEKULARRA}} \right)$$

• REYNOLDS ZBK:
$$Re = \frac{V \cdot L_c}{\nu} = \frac{\rho V \cdot L_c}{\mu}$$

• STANTONEN ZBK:
$$St = \frac{h}{\rho C_p V} = \frac{Nu}{Re \cdot Pr}$$

• CHILTON-COLBURNEN ZBK:
$$\frac{C_{f,x}}{2} = \frac{h_x}{\rho C_p V} \cdot Pr^{2/3} \equiv j_H$$

— BALIOGARRIA —
 $0,6 < Pr < 60$
 • FLUXU TURBULENTU PRESIO ∇
 BARRAUPERE
 • FLUXU LAMINARRA BALDIN
 $dP/dx = 0$

1) EKUAZIO DIMENTSIOGABEAK:

JARRAITASUNA:
$$\frac{du}{dx} + \frac{dv}{dy} = 0$$

MOMENTUA:
$$u \frac{du}{dx} + v \frac{du}{dy} = \frac{1}{Re_L} \frac{d^2 u}{dy^2} - \frac{dP}{dx}$$

ENERGIA:
$$u \frac{dT}{dx} + v \frac{dT}{dy} = \frac{1}{Re_L \cdot Pr} \left(\frac{d^2 T}{dy^2} \right)$$

7 GAIA : KANPO KONBEZIO BEHARTUA

PROZEDURA :

1) E.B. PLANIEATU

2) h ? (T_s ?...)

2.1) [SUPOSATU T_s] : $T_f = \frac{T_s + T_\infty}{2}$ T_f KARAKTERISTIKOA

2.2) ITALETAN : FLUIDOAREN PROP.

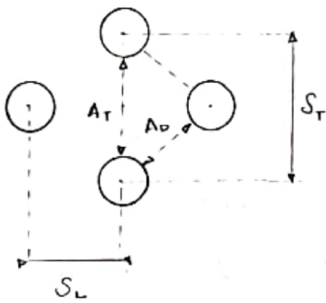
2.3) Re KALKULATU

2.4) Nu KORRELAZIO BAT AUKERATU ETA KALKULATU

2.5) h KALKULATU

3) BERO TRANSFERENTZIA KALKULATU.

MODU MULTZOETAN ZEHARREKO FLUXUA :



$$Re = \frac{V_{max} \cdot D}{\nu}$$

$$S_D = \sqrt{S_L^2 + (S_T/2)^2}$$

$$2A_D > A_T$$

$$V_{max} = \frac{S_T}{S_T - D} \cdot V$$

$$2A_D < A_T$$

$$V_{max} = \frac{S_T}{2(S_D - D)} \cdot V$$

FLUXUA: $\dot{Q} = h A_s \Delta T_{lm} = \dot{m} c_p (T_e - T_i)$

$$\Delta T_{lm} = \frac{(T_s - T_e) - (T_s - T_i)}{\ln[(T_s - T_e)/(T_s - T_i)]}$$

$$T_e = T_s - (T_s - T_i) \cdot e^{-\frac{A_s h}{\dot{m} c_p}}$$

8 GAIA : BARNEKO KONBEZIO BEHARTUA

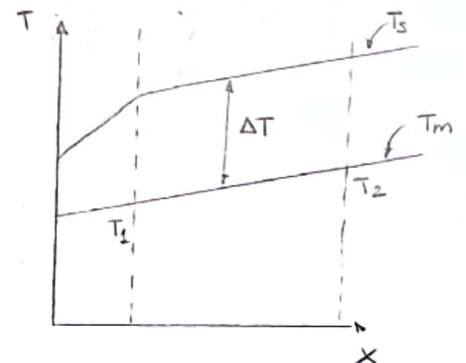
ANALISI TERMICO OROKORRA :

1) \dot{q}_s KONSTANTE :

$$\left[\begin{array}{l} \dot{Q} = \dot{q}_s A_s = \dot{m} c_p (T_2 - T_1) \\ \dot{q}_s = h (T_s - T_m) \end{array} \right]$$

[LAMINAR] : $Nu = 4,36$ // $\dot{q}_s = k_l e \Delta T = k_l e h = k_l e$

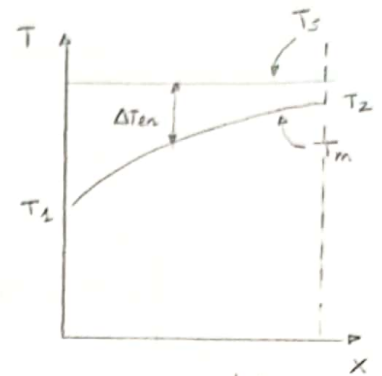
$$\hookrightarrow T_s - T_m = k_l e$$



2) T_s KONSTANTE :

$$\left[\begin{aligned} \dot{Q} &= h A_s \Delta T_{\text{en}} = \dot{m} c_p (T_2 - T_1) \\ \Delta T_{\text{en}} &= \frac{(T_s - T_2) - (T_s - T_1)}{\ln[(T_s - T_2)/(T_s - T_1)]} \end{aligned} \right]$$

[LAMINAR] : $Nu = 3,66$



$$\frac{T_2 - T_s}{T_1 - T_s} = e^{-\frac{h A_s}{\dot{m} c_p}}$$

• PRESIO GALERAK :

- FLUXU LAMINARRA : $\Delta P = \frac{32 \mu L V_{\text{AVG}}}{D^2} = f \frac{L}{D} \rho \frac{V_{\text{AVG}}^2}{2}$

$$\left[f = \frac{64}{Re} = \frac{8 z_w}{\rho V_{\text{AVG}}^2} \right]$$

• KARGA GALERAK :

- FLUXU LAMINARRA : $h_L = f \frac{L}{D} \frac{V_{\text{AVG}}^2}{2g}$ $W = \dot{V} \Delta P = \dot{m} h_L$

• KORRELAZIOAK :

- LAMINAR : $L_h = 0,05 Re D$ // $L_z = Pr L_h$

- TURBULENTIO : $L_h \approx L_z \approx 10D$

9. GAIA : KONBEKZIO NATURALA

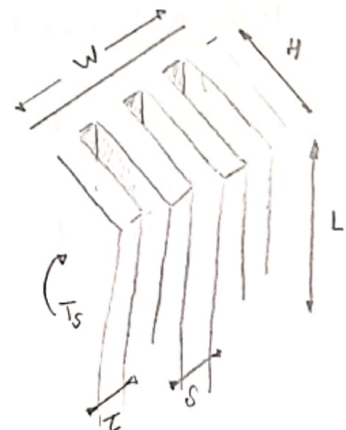
• KONBEKZIO BEHARTUA (Pr, Re) \longrightarrow KONBEKZIO NATURALA (Gr, Pr)

$$\boxed{Ra = Gr \cdot Pr = \frac{g \beta (T_s - T_{\infty}) L_c^3}{\nu^2} Pr}$$

• GAINAZAL HEGALDUNAK ($T_s = KHe$) :

$$S_{\text{OPT}} = 2,714 \frac{L}{Ra^{0,25}}$$

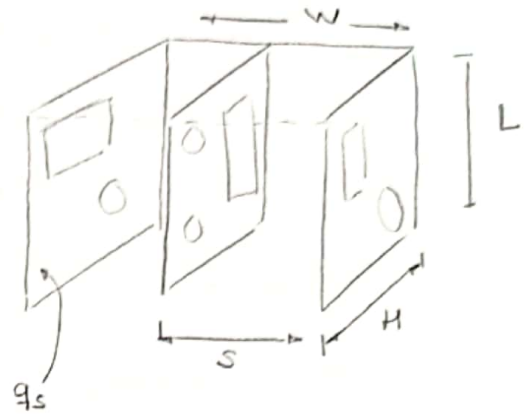
$$\left[\begin{aligned} \dot{Q} &= h (2n L H) (T_s - T_{\infty}) \\ n &= \frac{W}{S} \end{aligned} \right]$$



• ZIRKUITU INPRIMATU BERTKALETAN ($q_s = kL$)

$$So_{Pr} = 2,12 \left(\frac{5^4 L}{Ra_s} \right)^{0,2}$$

$$\left[\begin{aligned} \dot{Q} &= \dot{q}_s A_s = \dot{q}_s (2nLH) \\ n &= \frac{W}{S} \quad // \quad \dot{q}_s = h(T_L - T_\infty) \end{aligned} \right]$$



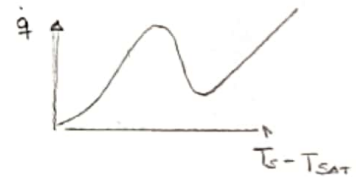
• EROANKORTASUN TERMICO ERAGINKORRA:

$$\dot{Q} = h A_s (T_1 - T_2) = k N u A_s \frac{(T_1 - T_2)}{L_c}$$

$$\boxed{K_{eff} = k N u}$$

10. GAIA: IRAKITEA ETA KONDENTSAZIOA

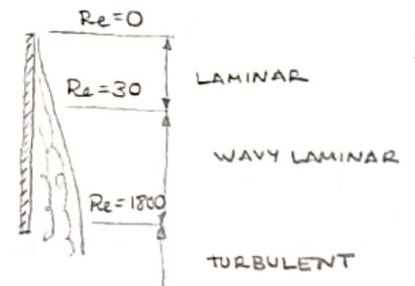
1) IRAKITEA: $\boxed{\dot{q}_{BOIL} = h (T_s - T_{SAT})}$



2) GERUZA ERAKO KONDENTSAZIOA:

$$\boxed{h_{fg}^* = h_{fg} + 0,68 C_{pL} (T_{SAT} - T_s)}$$

$$\boxed{\dot{Q}_{COND} = h A_s (T_{SAT} - T_s) = \dot{m} h_{fg}^*}$$



• TANTA ERAKO KONDENTSAZIOA

• MODI HORIZONTALAK

11. GAIA: BERO TRUKAGAILUAK

• BERO-TRANSFERENTZIAREN KOEFIZIENTE OROKORRA:

$$\boxed{\dot{Q} = U A_s \Delta T_{en} \quad U = \frac{1}{\frac{1}{h_o} + \frac{1}{h_i}}}$$

METODOS LOGARITMIKOA :

1) $\dot{Q} = m c_p (T_2 - T_1)$

$\dot{Q} = U A F \Delta T_{lm}$

2) $\Delta T_{lm} : \Delta T_{lm} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1 / \Delta T_2)}$

$\left\{ \begin{array}{l} \Delta T_1 = T_{h,in} - T_{c,out} \\ \Delta T_2 = T_{h,out} - T_{c,in} \end{array} \right.$

$A = \pi D \cdot L \cdot n_{TUB} \cdot n_{FASES}$

3) F : ZUZENTZEA (TAULAK)

FASE TUBO 1 $\rightarrow F = 1$

ϵ - NTU METODOA :

1) $\dot{Q} : \dot{Q} = m c_p (T_2 - T_1)$

$\dot{Q}_{max} = C_{min} (T_{h,in} - T_1)$

2) $\epsilon : \epsilon = \frac{\dot{Q}}{\dot{Q}_{max}}$

3). KORRELAZIOA $\epsilon \propto NTU = \frac{U A}{C_{min}}$

12. GAIA : ERRADIAZIO TERMIKOEN OINARRIAK

$\lambda T \xrightarrow{\text{TAULAK}} f_{\lambda}$

$\epsilon(T) = f \epsilon_1 + (1-f) \epsilon_2$

$\alpha + \rho + z = 1$

MB : $\dot{q}_{in} = \dot{q}_{out} \quad G_s = G_{sD} \cdot \cos \alpha + G_{sdif}$

KIRCHOFF $\epsilon = \alpha$

$F_{i-j} = \frac{\sum \text{HARI. GUR.} - \sum \text{HARI. EZ. GUR.}}{2 \cdot L_i}$

$E_g = E_{g1} + E_{g2} - \Delta E$

$Q_{TRASM} = q_{SOLAR} \cdot \alpha \cdot A (f_{\lambda 2} - f_{\lambda 1})$