

1. GAIA

SARRERA ETA OINARRIZKO KONTZEPTUAK

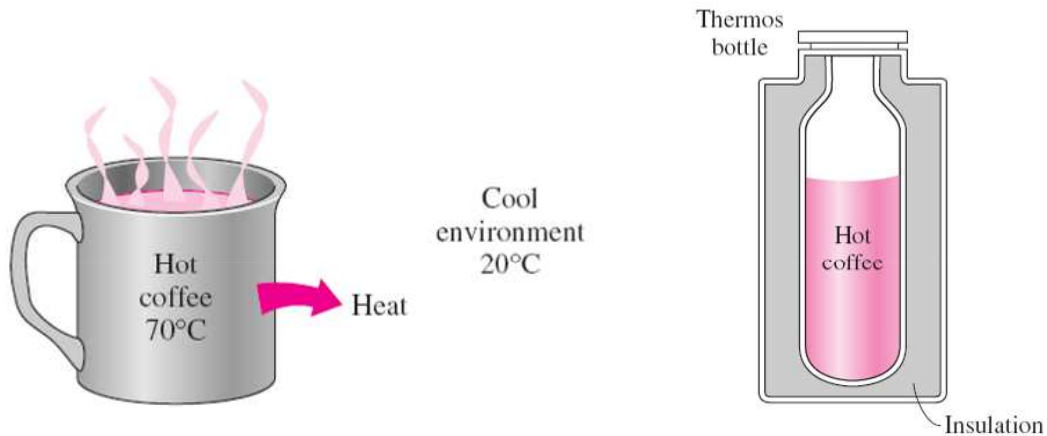
1.0 - HELBURUAK

2/33

- **Termodinamikaren** eta **bero-transferentziaren** arteko harremana zein den ulertu
- Energia termikoa bereizi **beste energia mota** batzuetatik, eta bero-transferentzia beste energia-transferentzia batzuetatik
- **Energia-balantze** orokorrak eta gainazalen energia-balantzeak egin
- Bero-transferentziaren oinarriko mekanismoak, hots, **eroapena**, **konbekzioa** eta **erradiazioa**, ulertu
- Praktikan aldi berean gertatzen diren bero-transferentziako mekanismoak identifikatu
- Bero-galerek eragiten duten **kostuaz** jabetu
- Praktikan aurkitzen diren bero-transferentziako zenbait **problema** ebatzi

TERMODINAMIKA:

Irudiko bi kasuetarako, zein da kafearen oreka-tenperatura?



BERO (ETA MASA) TRANSFERENTZIA:

Oreka-tenperatura heldu arte, zenbat denbora igaroko da ?

BERO-TRANSFERENTZIAREN APLIKAZIO-EREMUAK



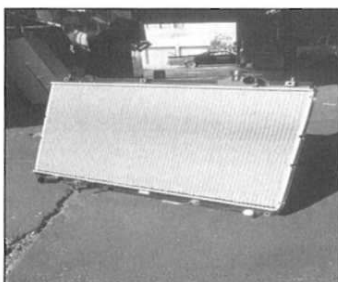
The human body



Air conditioning systems



Airplanes



Car radiators

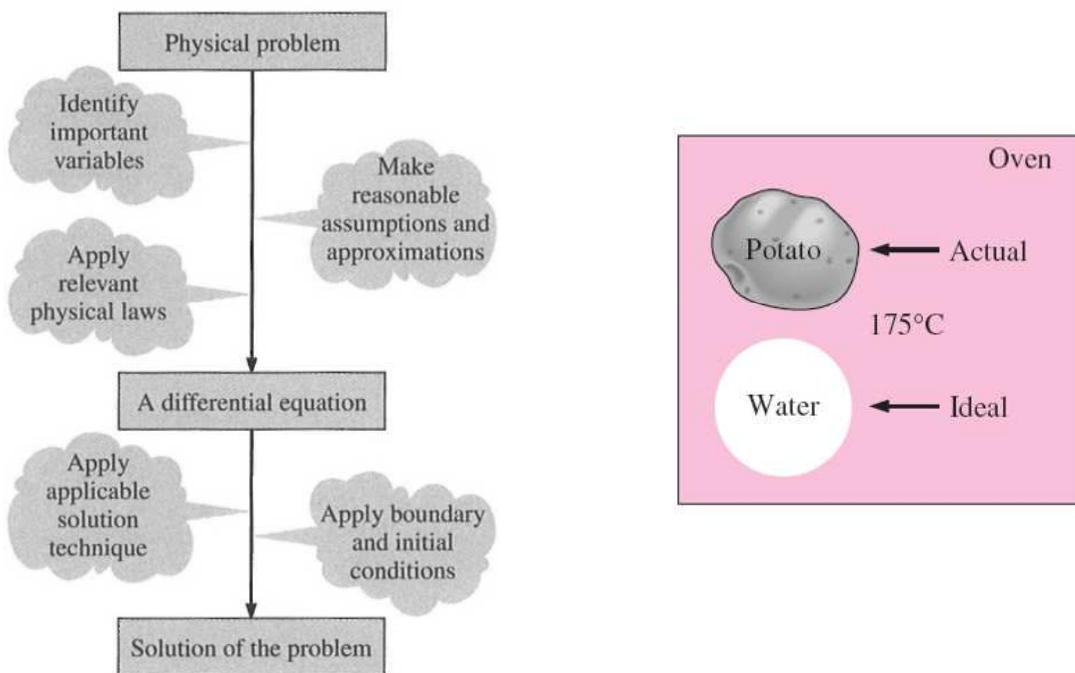


Power plants

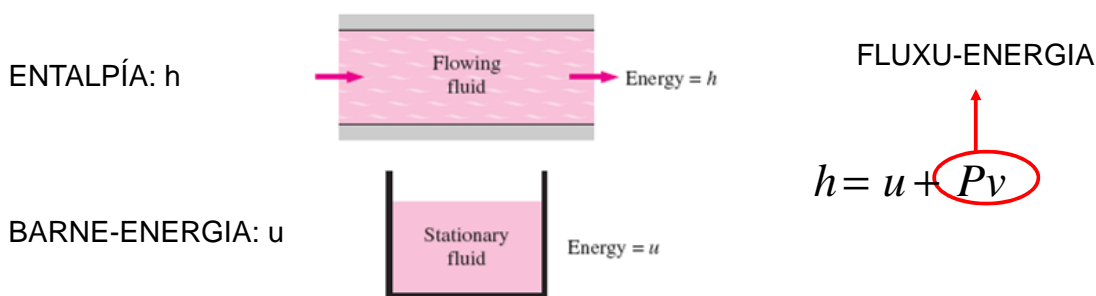


Refrigeration systems

EREDUGINTZA BERO TRANSFERENTZIAN

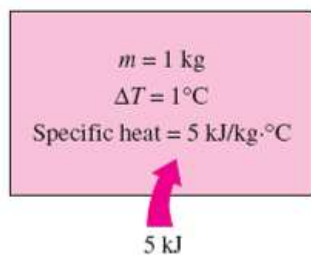


1.3 - BEROA ETA BESTE ENERGIA MOTA BATZUK



GASEN, LIKIDOEN ETA SOLIDOEN BERO ESPEZIFIKOAK

BERO-ESPEZIFIKOAK: PRESIO KONSTANTEKO BERO-ESPEZIKIFOA: C_p



BOLUMEN KONSTANTEKO BERO-ESPEZIKIFOA:

• GAS IDEALAK: $Pv = RT$ or $P = \rho RT$

$$C_p = C_v + R$$

• SUBSTANTZIA KONPRIMAETZINAK (solido eta likidoak):

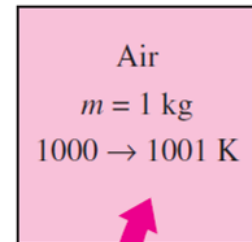
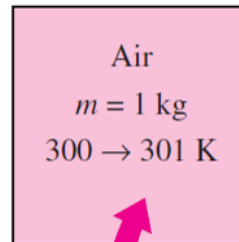
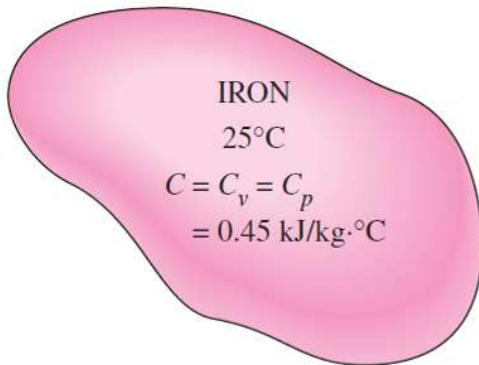
$$C_p \cong C_v \cong C$$

GASEN, LIKIDOEN ETA SOLIDOEN BERO ESPEZIFIKOAK

$1 \text{ kJ/kg} \cdot ^\circ\text{C} \equiv 1 \text{ kJ/kg} \cdot \text{K}$??????????????

$\Delta u = C_{v,ave} \Delta T$ eta $\Delta h = C_{p,ave} \Delta T$

$\Delta U = m C_{v,ave} \Delta T$ eta $\Delta H = m C_{p,ave} \Delta T$



¿URA?

TERMOTEKNIA

ENERGIA-TRANSFERENTZIA (ENERGI-KALITATEA)

	MEKANIKA	ELEKTRIKA	BEROA	NUKLEARRA, KIMIKOA,...
INDARRA edo POTENTZIALA	INDARRA [N]	POTENTZIALA [V]	TENPERATURA DIFERENTZIA [°C]	
ENERGIA, LANA edo BEROA	POTENTZIALA: $m \cdot g \cdot H$ ZINETIKOA: $\frac{1}{2} \cdot m \cdot V^2$ [J] = [N·m]	ENERGIA ELEKTRIKOA: $E = P \cdot t$ [J] = [N·m]	BEROA edo TRUKATUTAKO ENERGIA: $Q = \dot{Q} \cdot t$ [J] = [N·m]	
POTENTZIA edo BERO TRANSFERENTZIA ABIADURA	POTENTZIA MEKANIKOA $P = F \cdot V$ $P = M \cdot w$ [W] = [J/s]	POTENTZIA ELEKTRIKOA $P = V \cdot I \cdot \cos(\varphi)$ [W] = [J/s]	BERO TRANSFERENTZIA ABIADURA $\dot{Q} = f(\Delta T)$ [W] = [J/s]	

¿Zein kalitate haundien duen energia? ¿Nola neur daiteke energiaren kalitatea?

ENERGIA-TRANSFERENTZIA

BERO-TRANSFERENTZIA [J]
(ELKARTRUKATUTAKO BEROA)

BERO-TRANSFERENTZIA
ABIADURA [W]

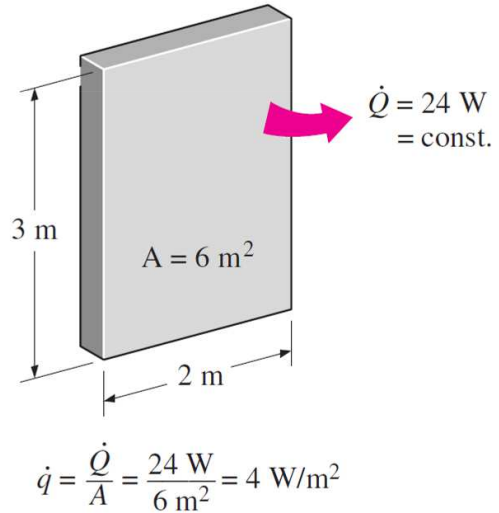
KONSTANTEA [W]

$$Q = \dot{Q} \Delta t \quad (\text{J})$$

$$Q = \int_0^{\Delta t} \dot{Q} dt \quad (\text{J})$$

BERO-FLUXUA [W/m²]

$$\dot{q} = \frac{\dot{Q}}{A} \quad (\text{W/m}^2)$$



1.4 - TERMODINAMIKAREN LEHEN LEGEA

$$\left(\begin{matrix} \text{Total energy} \\ \text{entering the} \\ \text{system} \end{matrix} \right) - \left(\begin{matrix} \text{Total energy} \\ \text{leaving the} \\ \text{system} \end{matrix} \right) = \left(\begin{matrix} \text{Change in the} \\ \text{total energy of} \\ \text{the system} \end{matrix} \right)$$

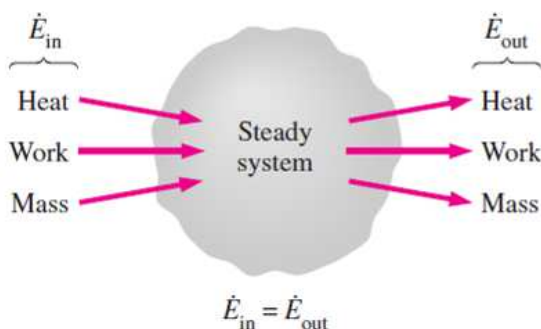
ENERGIAREN KONTSERBAZIO-PRINTZIOA

$$\underbrace{E_{in} - E_{out}}_{\text{Net energy transfer by heat, work, and mass}} = \underbrace{\Delta E_{system}}_{\text{Change in integral, kinetic, potential, etc., energy}}$$

ENERGIA-BALANTZEA (DENBORA TARTEA)

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\text{Rate of net energy transfer by heat, work, and mass}} = \underbrace{dE_{system} / dt}_{\text{Rate of change in integral, kinetic, potential, etc., energy}}$$

POTENTZI-BALANTZEA (ALDIUNEKOA)



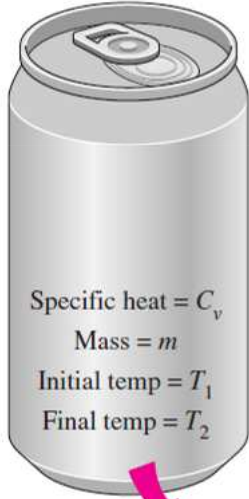
$$\left(\frac{dE_{system}}{dt} = 0 \right)$$

$$\underbrace{\dot{E}_{in}}_{\text{Rate of net energy transfer in by heat, work, and mass}} = \underbrace{\dot{E}_{out}}_{\text{Rate of net energy transfer out by heat, work, and mass}}$$

$$\underbrace{Q_{in} - Q_{out}}_{\text{Net heat transfer}} + \underbrace{E_{gen}}_{\text{Heat generation}} = \underbrace{\Delta E_{thermal, system}}_{\text{Change in thermal energy of the system}}$$

BERO-BALANTZEA

ENERGIA-BALANTZEA SISTEMA ITXIETAN



GELDIRIK DAGOEN SISTEMA ITXIA:

$$E_{in} - E_{out} = \Delta U = mC_v \Delta T \quad (J)$$

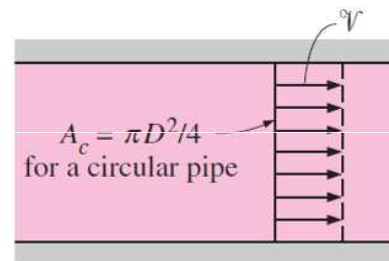
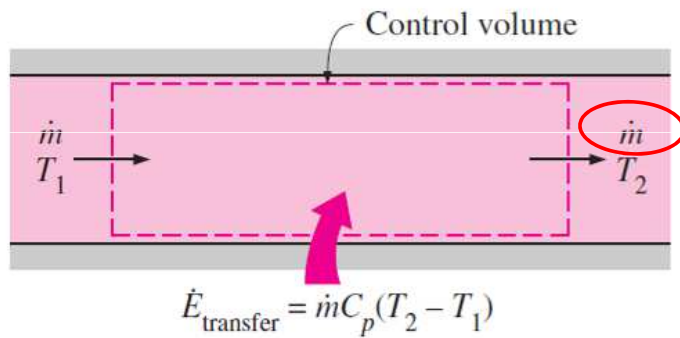
GELDIRIK DAGOEN SISTEMA ITXIA, LANIK EZ:

$$Q = mC_v \Delta T \quad (J)$$

$$\underbrace{Q_{in} - Q_{out}}_{\text{Net heat transfer}} + \underbrace{E_{gen}}_{\text{Heat generation}} = \underbrace{\Delta E_{thermal, system}}_{\text{Change in thermal energy of the system}}$$

BERO-BALANTZEA

ENERGIA-BALANTZEA FLUXU GELDIKORREKO SISTEMETAN



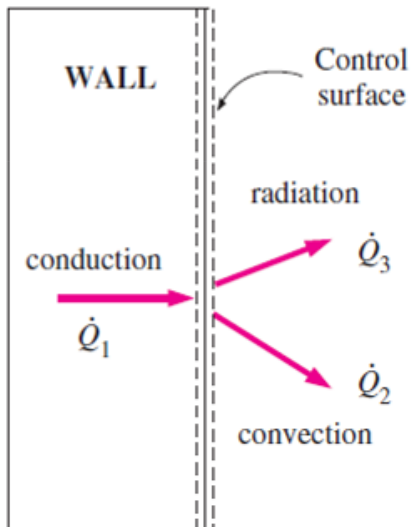
$$\dot{Q} = \dot{m} \Delta h = \dot{m} C_p \Delta T \quad (\text{kJ/s})$$

$$\dot{m} = \rho v A_c \quad (\text{kg/s})$$

$$\underbrace{Q_{in} - Q_{out}}_{\text{Net heat transfer}} + \underbrace{E_{gen}}_{\text{Heat generation}} = \underbrace{\Delta E_{thermal, system}}_{\text{Change in thermal energy of the system}}$$

BERO-BALANTZEA

GAINAZALEKO ENERGIA-BALANTZEA



$$\dot{Q}_1 = \dot{Q}_2 + \dot{Q}_3$$

1.5 – BERO TRANSFERENTZIAKO MEKANISMOAK

EROAPENA



KONBEKZIOA



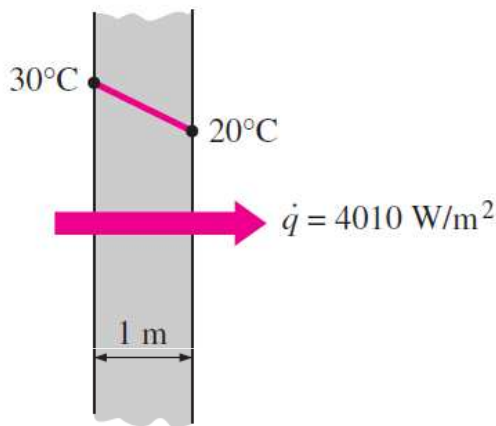
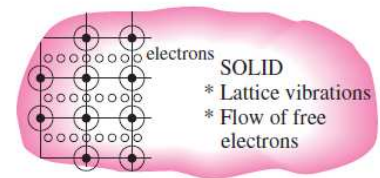
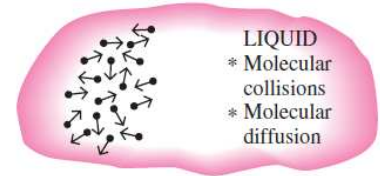
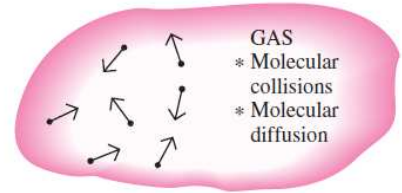
ERRADIAZIOA



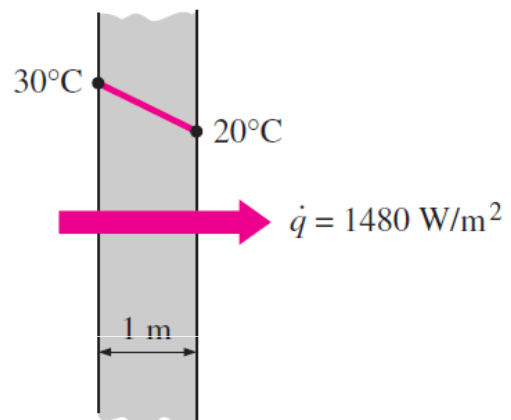
Partikulen arteko elkarrekintzen ondorioz substantzia bateko energia handiagoko partikuletatik energia txikiagoko inguruko partikuletara gertatzen den energia-transferentzia, **eroapena** da.

FOURIERREN BERO-EROAPENAREN LEGEA:

$$\dot{Q}_{cond} = -kA \frac{dT}{dx} \quad (W)$$



(a) Copper ($k = 401 \text{ W/m}\cdot\text{°C}$)

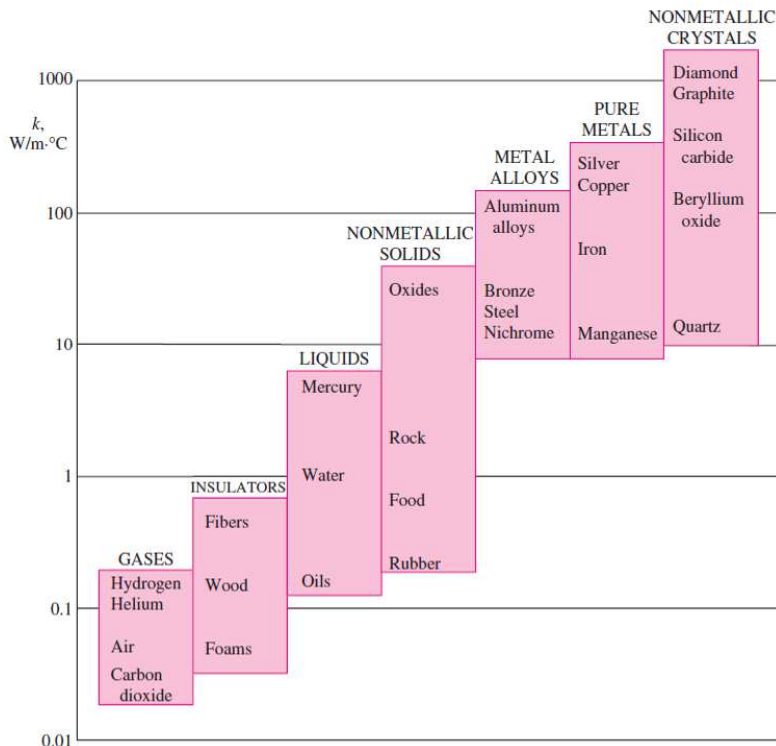


(b) Silicon ($k = 148 \text{ W/m}\cdot\text{°C}$)

$$\dot{Q}_{cond} = -kA \frac{dT}{dx} \quad \longrightarrow \quad \dot{Q}_{cond} = kA \frac{T_1 - T_2}{\Delta x} = -kA \frac{\Delta T}{\Delta x} \quad (W)$$

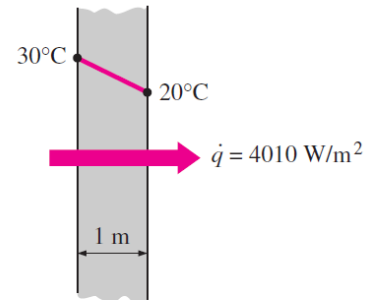
¿UNITATEAK?

EROANKORTASUN TERMIKOA



$$\dot{Q}_{cond} = -kA \frac{dT}{dx} \text{ (W)}$$

$$\dot{Q}_{cond} = kA \frac{T_1 - T_2}{\Delta x} \text{ (W)}$$



(a) Copper ($k = 401 \text{ W/m}\cdot\text{°C}$)

EROANKORTASUN TERMIKOA

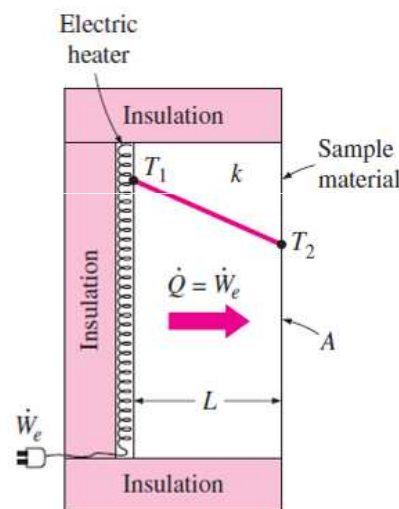
The thermal conductivities of some materials at room temperature

Material	$k, \text{ W/m} \cdot \text{°C}^*$
Diamond	2300
Silver	429
Copper	401
Gold	317
Aluminum	237
Iron	80.2
Mercury (l)	8.54
Glass	0.78
Brick	0.72
Water (l)	0.613
Human skin	0.37
Wood (oak)	0.17
Helium (g)	0.152
Soft rubber	0.13
Glass fiber	0.043
Air (g)	0.026
Urethane, rigid foam	0.026

*Multiply by 0.5778 to convert to Btu/h · ft · °F.

$$\dot{Q}_{cond} = -kA \frac{dT}{dx}$$

$$\dot{Q}_{cond} = kA \frac{T_1 - T_2}{\Delta x}$$

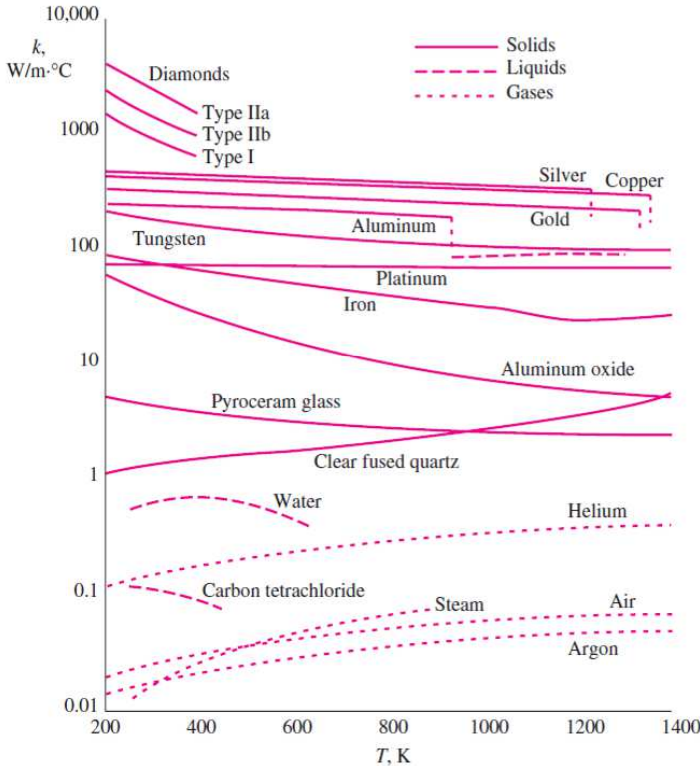


$$k = \frac{L}{A(T_1 - T_2)} \dot{Q}$$

EROANKORTASUN TERMIKOA

$$\dot{Q}_{cond} = -kA \frac{dT}{dx}$$

Thermal conductivities of materials vary with temperature



T, K	Copper	Aluminum
100	482	302
200	413	237
300	401	237
400	393	240
600	379	231
800	366	218

The thermal conductivity of an alloy is usually much lower than the thermal conductivity of either metal of which it is composed

Pure metal or alloy	k, W/m · °C, at 300 K
Copper	401
Nickel	91
Constantan (55% Cu, 45% Ni)	23
Copper	401
Aluminum	237
Commercial bronze (90% Cu, 10% Al)	52

DIFUSIBITATE TERMIKOA

$$\alpha = \frac{\text{Heat conducted}}{\text{Heat stored}} = \frac{k}{\rho C_p} \quad (\text{m}^2/\text{s})$$

BERO-DIFUSIOA MATERIALETAN ZENBATEKO ABIADURAZ GERTATZEN DEN ADIERAZTEN DU

The thermal diffusivities of some materials at room temperature

Material	α , m ² /s*
Silver	149 × 10 ⁻⁶
Gold	127 × 10 ⁻⁶
Copper	113 × 10 ⁻⁶
Aluminum	97.5 × 10 ⁻⁶
Iron	22.8 × 10 ⁻⁶
Mercury (l)	4.7 × 10 ⁻⁶
Marble	1.2 × 10 ⁻⁶

Ice	1.2 × 10 ⁻⁶
Concrete	0.75 × 10 ⁻⁶
Brick	0.52 × 10 ⁻⁶
Heavy soil (dry)	0.52 × 10 ⁻⁶
Glass	0.34 × 10 ⁻⁶
Glass wool	0.23 × 10 ⁻⁶
Water (l)	0.14 × 10 ⁻⁶
Beef	0.14 × 10 ⁻⁶
Wood (oak)	0.13 × 10 ⁻⁶

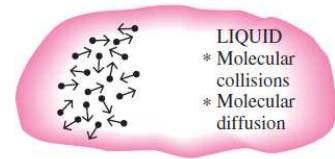
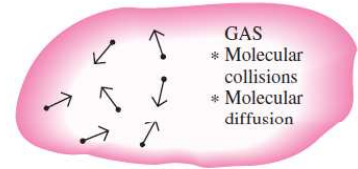
*Multiply by 10.76 to convert to ft²/s.

Gainazal solido baten eta haren inguruan mugimenduan dagoen likido edo gasaren artean energia transferitzeko modua, eta eroapenaren eta fluido-mugimenduaren efektuen konbinazioa da, **konbekzioa** da.

NEWTONEN HOZTE-LEGEA

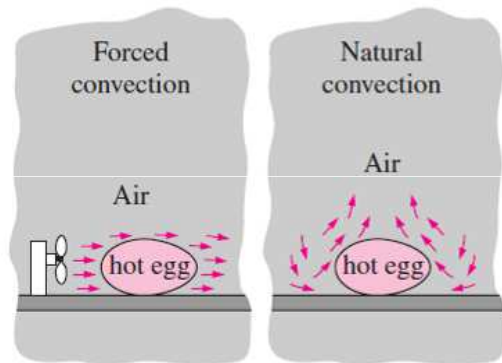
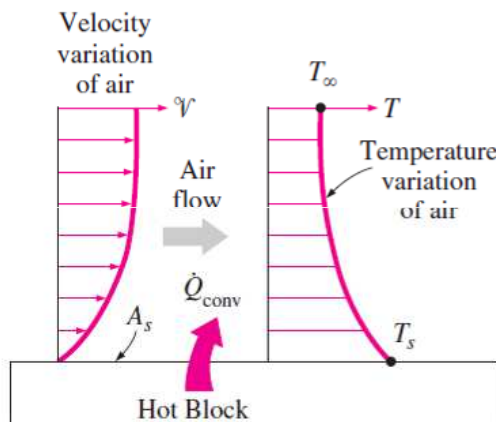
$$\dot{Q}_{conv} = hA_s (T_s - T_\infty) \quad (W)$$

Fluidoaren mugimendu globalik gabe, gainazal solidoaren eta inguruko fluidoaren arteko bero-transferentzia eroapen hutsekoa da.



$$\dot{Q}_{conv} = hA_s (T_s - T_\infty) \quad (W)$$

KONBEKZIO KOEFIZIENTEA



$$\dot{Q}_{conv} = hA_s (T_s - T_\infty) \quad (W)$$

KONBEKZIO KOEFIZIENTEA

Typical values of convection heat transfer coefficient

Type of convection	$h, W/m^2 \cdot ^\circ C^*$
Free convection of gases	2–25
Free convection of liquids	10–1000
Forced convection of gases	25–250
Forced convection of liquids	50–20,000
Boiling and condensation	2500–100,000

ZEREN MENPEKOA DA ?

- LIKIDO vs GAS
- BEHARTUA vs NATURALA
- SOLIDOAREN ITXURA
- SOLIDOAREN POSIZIOA
- ...

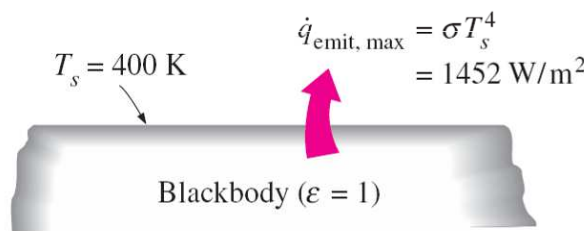
*Multiply by 0.176 to convert to Btu/h · ft² · °F.

1.8 - ERRADIAZIOA

Materiak, atomoen edo molekulen konfigurazio elektronikoaren aldaketen ondorioz, uhin elektromagnetiko (edo fotoi) moduan igortzen duen energia, **erradiazioa** da.

STEFAN-BOLTZMANNEN LEGEA:

$$\dot{Q}_{emit, max} = \sigma A_s T_s^4 \quad (W)$$



Eroapenak eta konbektzioak ez bezala, erradiazio bidezko bero-transferentziak ez du behar bitartekorik. Izatez, erradiazio bidezko bero-transferentzia lasterragoa da (argiaren abiadura du), eta ez da moteltzen hutsean.

$$\dot{Q}_{emit, max} = \sigma A_s T_s^4 \rightarrow \dot{Q}_{emit} = \epsilon \sigma A_s T_s^4 \quad (W)$$

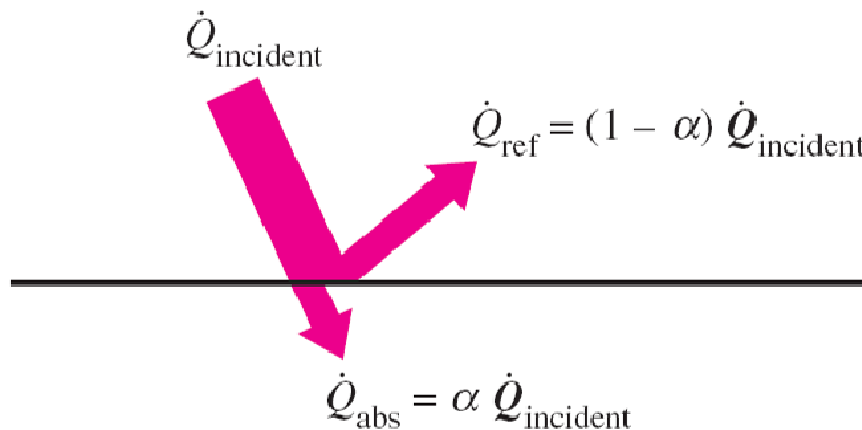
Emissivities of some materials at 300 K

Material	Emissivity
Aluminum foil	0.07
Anodized aluminum	0.82
Polished copper	0.03
Polished gold	0.03
Polished silver	0.02
Polished stainless steel	0.17
Black paint	0.98
White paint	0.90
White paper	0.92–0.97
Asphalt pavement	0.85–0.93
Red brick	0.93–0.96
Human skin	0.95
Wood	0.82–0.92
Soil	0.93–0.96
Water	0.96
Vegetation	0.92–0.96

EMISIBITATEA

$$\dot{Q}_{absorbed} = \alpha \dot{Q}_{incident} \quad (W)$$

ABSORTIBITATEA, ISLADAPENA ETA (TRANSMISIBITATEA)



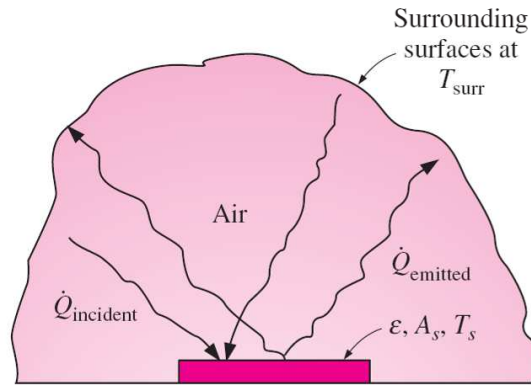
ERRADIAZIO BIDEZKO BERO-TRANSFERENTZIA:

Xurgatu eta emititu arteko diferentzia

$$\dot{Q}_{emit} = \epsilon\sigma A_s T_s^4 \quad (W) \qquad \dot{Q}_{absorbed} = \alpha\dot{Q}_{incident} \quad (W)$$

KIRCHOFFEN LEGEA

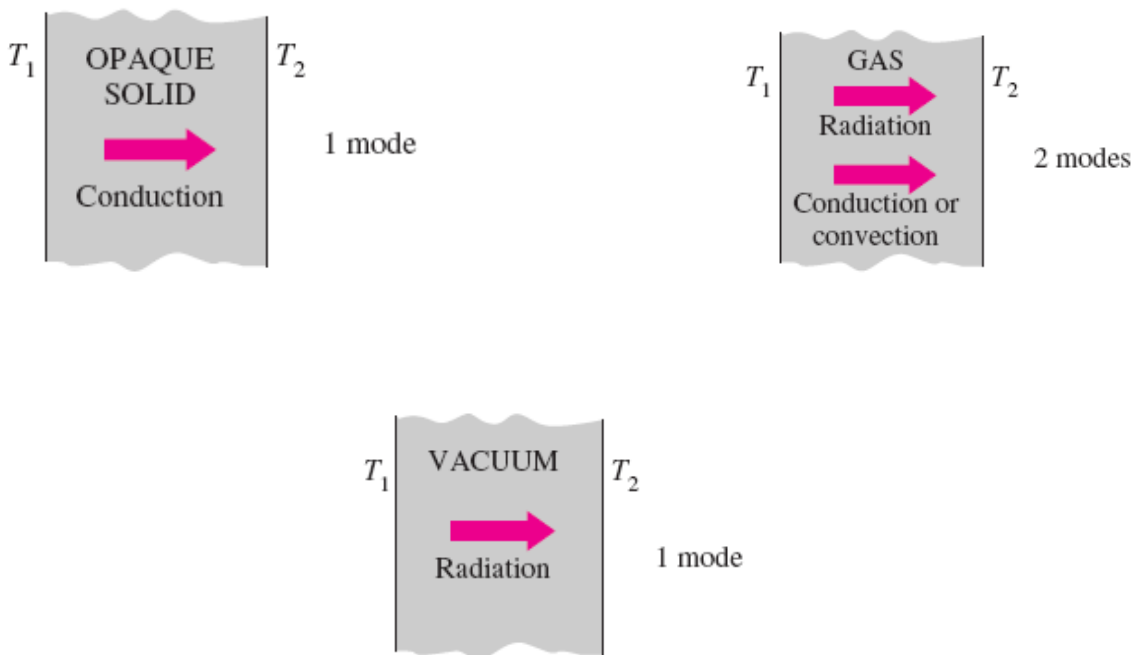
$$\dot{Q}_{rad} = \epsilon\sigma A_s (T_s^4 - T_{surr}^4) \quad (W)$$



$$\dot{Q}_{rad} = \epsilon\sigma A_s (T_s^4 - T_{surr}^4)$$

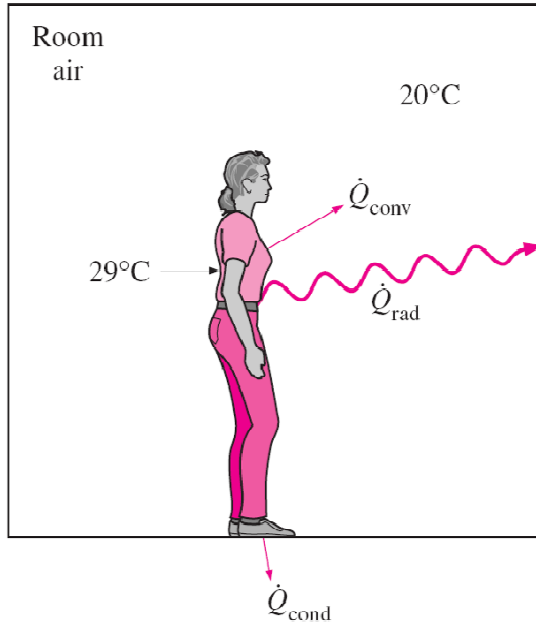
TERMOTEKNIA

1.9 - ALDIBEREKO BERO-TRANSFERENTZIAKO MEKANISMOAK



TERMOTEKNIA

1. ADIBIDEA – Pertsona baten bero-galerak



$$\epsilon = 0,95$$

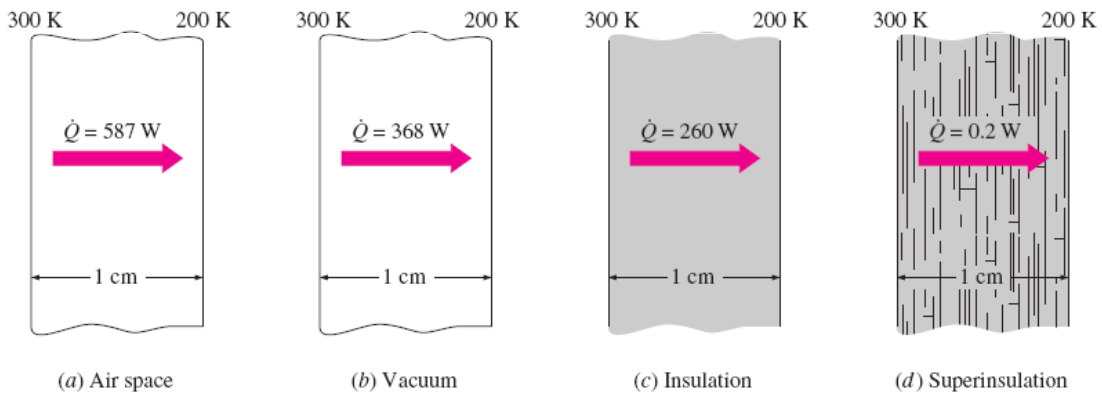
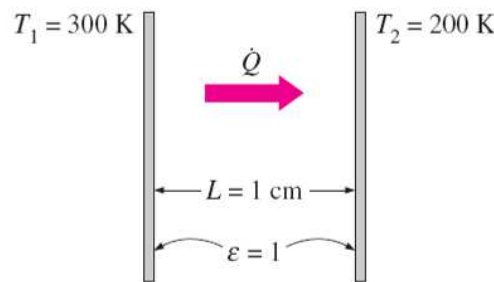
$$\begin{aligned} \dot{Q}_{conv} &= hA_s(T_s - T_\infty) \\ &= (6 \text{ W / m}^2 \cdot \text{°C})(1,6 \text{ m}^2)(29 - 20) \text{ °C} \\ &= 86,4 \text{ W} \end{aligned}$$

$$\begin{aligned} \dot{Q}_{rad} &= \epsilon \sigma A_s (T_s^4 - T_{surr}^4) \\ &= (0,95)(5,67 \times 10^{-8} \text{ W / m}^2 \cdot \text{K}^4)(1,6 \text{ m}^2) \\ &\quad \times [(29 + 273)^4 - (20 + 273)^4] \text{ K}^4 \\ &= 81,7 \text{ W} \end{aligned}$$

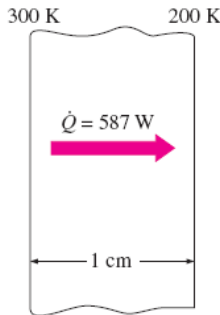
$$\dot{Q}_{total} = \dot{Q}_{conv} + \dot{Q}_{rad} = (86,4 + 81,7) \text{ W} = 168,1 \text{ W}$$

1.9 - ALDIBEREKO BERO-TRANSFERENTZIAKO MEKANISMOAK

2. ADIBIDEA – Bi xafra isotermikoren arteko bero-transferentzia



2. ADIBIDEA – Bi xafla isotermikoren arteko bero-transferentzia

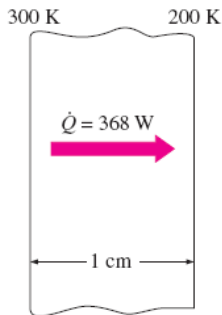


(a) Air space

$$\dot{Q}_{cond} = kA \frac{T_1 - T_2}{L} = (0,0219 \text{ W} / \text{m} \cdot \text{°C})(1 \text{ m}^2) \frac{(300 - 200)}{0,01 \text{ m}} = 219 \text{ W}$$

$$\begin{aligned} \dot{Q}_{rad} &= \epsilon \sigma A (T_1^4 - T_2^4) \\ &= (1)(5,67 \times 10^{-8} \text{ W} / \text{m}^2 \cdot \text{K}^4)(1 \text{ m}^2) [(300 \text{ K})^4 - (200 \text{ K})^4] = 368 \text{ W} \end{aligned}$$

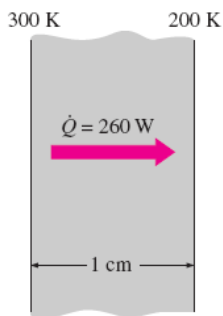
$$\dot{Q}_{total} = \dot{Q}_{cond} + \dot{Q}_{rad} = 219 + 368 = 587 \text{ W}$$



(b) Vacuum

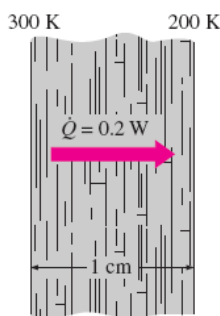
$$\dot{Q}_{total} = \dot{Q}_{rad} = 368 \text{ W}$$

2. ADIBIDEA – Bi xafla isotermikoren arteko bero-transferentzia



(c) Insulation

$$\dot{Q}_{total} = \dot{Q}_{cond} = kA \frac{T_1 - T_2}{L} = (0,026 \text{ W} / \text{m} \cdot \text{°C})(1 \text{ m}^2) \frac{(300 - 200) \text{°C}}{0,01 \text{ m}} = 260 \text{ W}$$



(d) Superinsulation

$$\dot{Q}_{total} = kA \frac{T_1 - T_2}{L} = (0,00002 \text{ W} / \text{m} \cdot \text{°C})(1 \text{ m}^2) \frac{(300 - 200) \text{°C}}{0,01 \text{ m}} = 0,2 \text{ W}$$

- 1.1eko azpiatala: AURREKARI HISTORIKOAK

2. GAIA

BERO EROAPENAREN EKUAZIOA

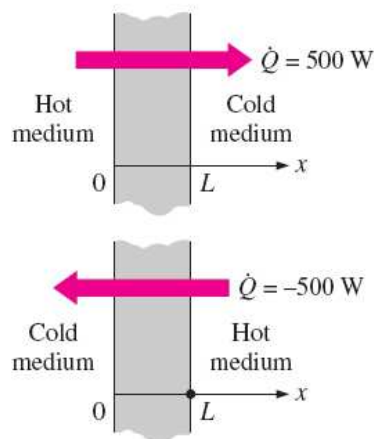
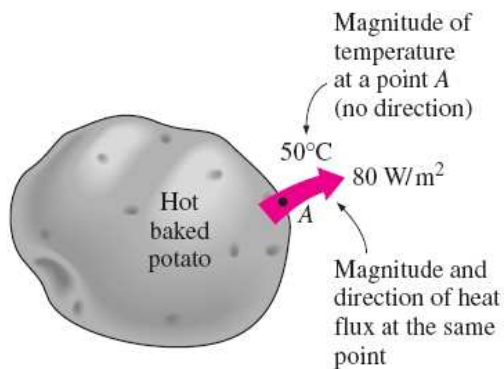
2.0 - HELBURUAK

2/28

- Bero-transferentziaren **multidimentsionaltasuna** eta denborarekiko menpekotasuna ulertu, eta orobat zer balditzen pean trata daitezkeen bero-transferentziaren problemak dimentsio bakarrekoak balira bezala.
- **Bero-eroapenaren ekuazio diferentzialak** zenbait koordinatu-sistematan lortu, eta dimentsio bakarreko kasu geldikorretarako sinplifikatu.
- Gainazalen baldintza termikoak identifikatu, eta matematikoki adierazi, **mugalde (edo inguruko) baldintza eta hasierako baldintza** gisa.
- Dimentsio bakarreko bero-eroapenaren problemak ebatzi, eta ingurune bateko **temperatura-banaketa** eta **bero-fluxua** kalkulatu.
- **Beroa sorrera** duten solidoetako dimentsio bakarreko bero-eroapena aztertu.

TENPERATURA ETA BERO TRANSFERENTZIA

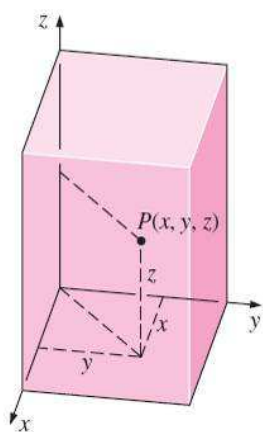
- Eskalar magnitudea vs. Bektorial magnitudea



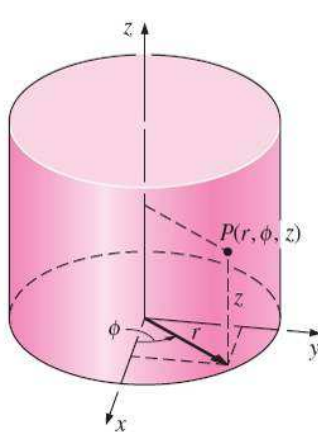
BERO FLUXUAREN NORANTZA

TENPERATUREN BANAKETA

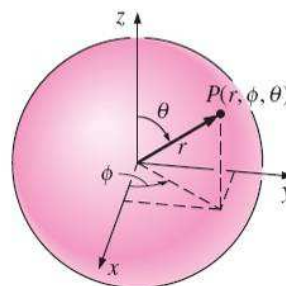
Koordenatu-sistema → angeluzuzenak, zilindrikoak, esferikoak.



$$T = T(x, y, z, t)$$



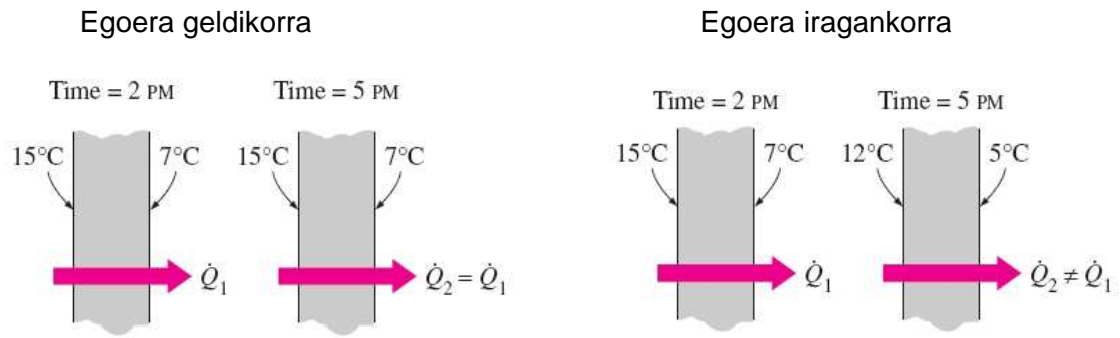
$$T = T(r, \phi, z, t)$$



$$T = T(r, \phi, \theta, t)$$

Kasu berezia: $T = T(x)$ → **Dimentsio bakar eta Geldikorra**

BERO-TRANSFERENTZIA GELDIKORRA VS. IRAGANKORRA



* Kasu berezia: **Parametro kontzentratuen sistemak.**

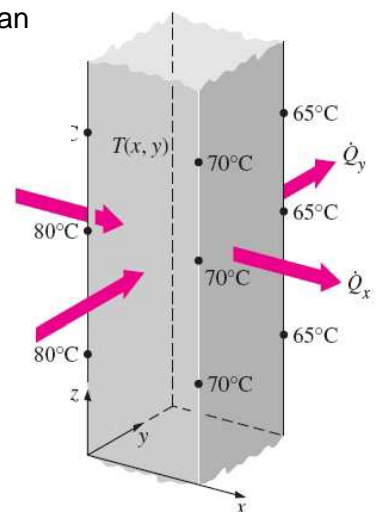
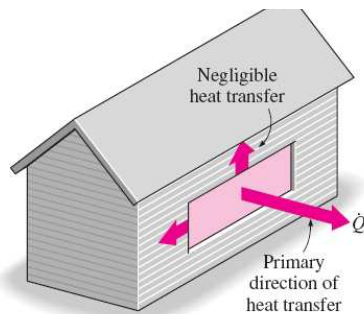


Naturan ematen diren prozesuak iragankorrak badira, **nola aztertzen dira?**

BERO-TRANSFERENTZIA MULTIDIMENSIONALA

Dimentsio bakarreko, bi dimentsioko edo hiru dimentsiokoa izan daiteke.

Temperatura-aldakuntzaren araberakoa izango da.



Zer motako transferentzia gertatzen da...
... ur beroa daraman hodi batean?

eta lapiko baten barnean ura irakiten dagoen arraultz batean?

BERO-TRANSFERENTZIA MULTIDIMENSIONALA



ZER DA EROAPEN PROBLEMA BAT EBAZTEA?

Helburua:

- Punto jakin bateko tenperatura $T = T(x, y, z, t)$ [°C]

- Bero fluxua $\dot{Q}_n = -k \cdot A \cdot \overrightarrow{\text{grad}}(T)$ [W]

2.1 – SARRERA

BERO-TRANSFERENTZIA-MULTIDIMENSIONALA

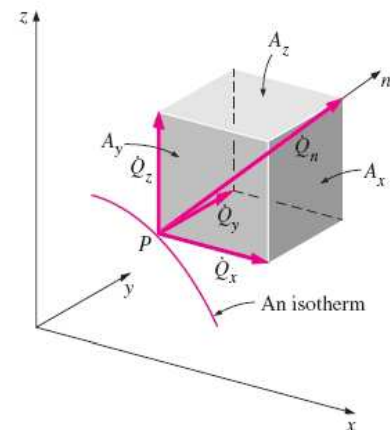
Fourier legearen adierazpen orokorra:

$$\dot{Q}_n = -k \cdot A \cdot \frac{\partial T}{\partial n} = -k \cdot A \cdot \overrightarrow{\text{grad}}(T) \quad [\text{W}]$$

$$\vec{\dot{Q}}_n = \dot{Q}_x \cdot \vec{i} + \dot{Q}_y \cdot \vec{j} + \dot{Q}_z \cdot \vec{k}$$

$$\dot{Q}_x = -k \cdot A_x \cdot \frac{\partial T}{\partial x} \quad \dot{Q}_y = -k \cdot A_y \cdot \frac{\partial T}{\partial y} \quad \dot{Q}_z = -k \cdot A_z \cdot \frac{\partial T}{\partial z}$$

Material isotropoak – anisotropoak



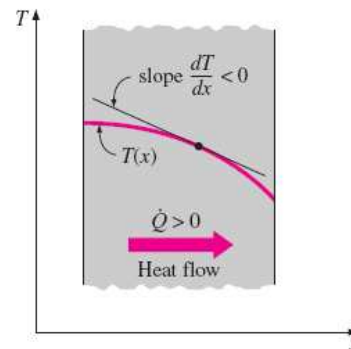
BERO-TRANSFERENTZIA MULTIDIMENTSIONALA

$$\text{FOURIERREN LEGEA } \dot{Q}_{cond} = -k \cdot A \cdot \frac{dT}{dx} \quad (\text{W})$$

Bero-eroapenaren abiadura ingurunean zeharreko temperatura-diferentziarekiko eta bero-transferentziaren norabidearekiko elkarzuta den azalerarekiko proportzionala da, baina norabide horretako distantziarekiko alderantziz proportzionala.

k eroankortasun termikoa da.

$\frac{dT}{dx}$ tenperaturaren gradientea da.



2.1 – SARRERA

BERO-SORRERA

Beste energia mota bat (mekanikoa, elektrikoa, nuklearra, kimikoa) bero energian bihurtzean sortzen da.

Adibidez: Zirkuito elektrikoak, erregai nuklearra, eguzkia, etc.

Fenomeno bolumetrikoa da.

Bero-sorrera abiadura: \dot{e}_{gen} [W/m³] o [Btu/h·ft³]

Orokorrean posizio eta denborarekin aldatzen da: $\dot{E}_{gen} = \int_V \dot{e}_{gen} \cdot dV$

Bero-sorrera konstantea bada: $E_{gen} = \dot{e}_{gen} \cdot V$

Bero-transferentzian norabide nagusi bat dagoenenan erabiltzen da, beste bi norabideak mesprezagarriak izanik.

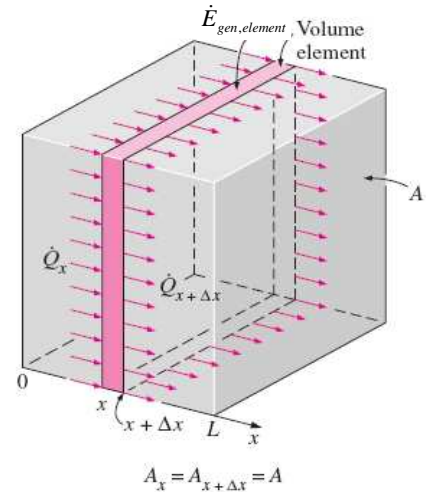
HORMA LAU HANDI BATEKO BERO-EROAPENAREN EKUAZIOA

Energia-balatzea aplikatuz:

$$\dot{Q}_x - \dot{Q}_{x+\Delta x} + \dot{E}_{gen,element} = \frac{\Delta E_{element}}{\Delta t} \quad [\text{W}]$$

$$\dot{E}_{gen,element} = \dot{e}_{gen} \cdot V_{element} = \dot{e}_{gen} \cdot A \cdot \Delta x$$

$$\Delta E_{element} = E_{t+\Delta t} - E_t = m \cdot c \cdot (T_{t+\Delta t} - T_t) = \rho \cdot c \cdot A \cdot \Delta x \cdot (T_{t+\Delta t} - T_t)$$



2.2 – DIMENTSIO BAKARREKO BERO-EROAPENAREN EKUAZIOA

HORMA LAU HANDI BATEKO BERO-EROAPENAREN EKUAZIOA

Ordezkatuz, $\Delta t \rightarrow 0$ y $\Delta x \rightarrow 0$ limitea hartuz eta Fourierren legea aplikatuz:

Eroankortasun aldakorra:

$$\frac{\partial}{\partial x} \left(k \frac{\partial T}{\partial x} \right) + \dot{e}_{gen} = \rho \cdot c \frac{\partial T}{\partial t}$$

Eroankortasun konstantea:

Difusibitate termikoa: $\alpha = \frac{k}{\rho \cdot c}$

$$\frac{\partial^2 T}{\partial x^2} + \frac{\dot{e}_{gen}}{k} = \frac{1}{\alpha} \frac{\partial T}{\partial t}$$

Kasu bereziak:

- Egoera egonkorra $\frac{d^2 T}{dx^2} + \frac{\dot{e}_{gen}}{k} = 0$

- Bero-sorrera gabeko egoera iragankorra $\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha} \frac{\partial T}{\partial t}$

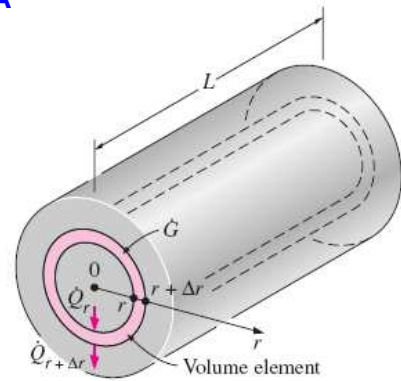
- Bero-sorrera gabeko egoera egonkorra $\frac{d^2 T}{dx^2} = 0$

ZILINDRO LUZE BATEKO BERO-EROAPENAREN EKUAZIOA

Energia-balantzea aplikatuz:

$$\dot{Q}_r - \dot{Q}_{r+\Delta r} + \dot{E}_{gen,element} = \frac{\Delta E_{element}}{\Delta t} \quad [W]$$

$$\dot{E}_{gen,element} = \dot{e}_{gen} \cdot V_{element} = \dot{e}_{gen} \cdot A \cdot \Delta r$$



$$\Delta E_{element} = E_{t+\Delta t} - E_t = m \cdot c \cdot (T_{t+\Delta t} - T_t) = \rho \cdot c \cdot A \cdot \Delta r \cdot (T_{t+\Delta t} - T_t)$$

Ordezkatuz, $\Delta t \rightarrow 0$ y $\Delta x \rightarrow 0$ limitea hartuz eta Fourierren legea aplikatuz:

Eroankortasun aldakorra:

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \cdot k \frac{\partial T}{\partial r} \right) + \dot{e}_{gen} = \rho \cdot c \frac{\partial T}{\partial t}$$

ZILINDRO LUZE BATEKO BERO-EROAPENAREN EKUAZIOA

Eroankortasun konstantea

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) + \frac{\dot{e}_{gen}}{k} = \frac{1}{\alpha} \frac{\partial T}{\partial t}$$

Kasu bereziak:

- Egoera egonkorra

$$\frac{1}{r} \frac{d}{dr} \left(r \frac{dT}{dr} \right) + \frac{\dot{e}_{gen}}{k} = 0$$

- Bero-sorrera gabeko egoera iragankorra

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) = \frac{1}{\alpha} \frac{\partial T}{\partial t}$$

- Bero-sorrera gabeko egoera egonkorra

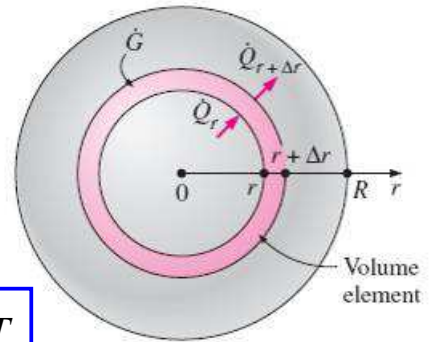
$$\frac{d}{dr} \left(r \frac{dT}{dr} \right) = 0$$

ESFERA BATEKO BERO-EROAPENAREN EKUAZIOA

Beste geometriekiko parekotasuna eginez:

Eroankortasun aldakorra:

$$\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \cdot k \frac{\partial T}{\partial r} \right) + \dot{e}_{gen} = \rho \cdot c \frac{\partial T}{\partial t}$$



Eroankortasun konstantea:

$$\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial T}{\partial r} \right) + \frac{\dot{e}_{gen}}{k} = \frac{1}{\alpha} \frac{\partial T}{\partial t}$$

Kasu bereziak:

- Egoera egonkorra $\frac{1}{r^2} \frac{d}{dr} \left(r^2 \frac{dT}{dr} \right) + \frac{\dot{e}_{gen}}{k} = 0$

- Bero-sorrera gabeko egoera iragankorra $\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial T}{\partial r} \right) = \frac{1}{\alpha} \frac{\partial T}{\partial t}$

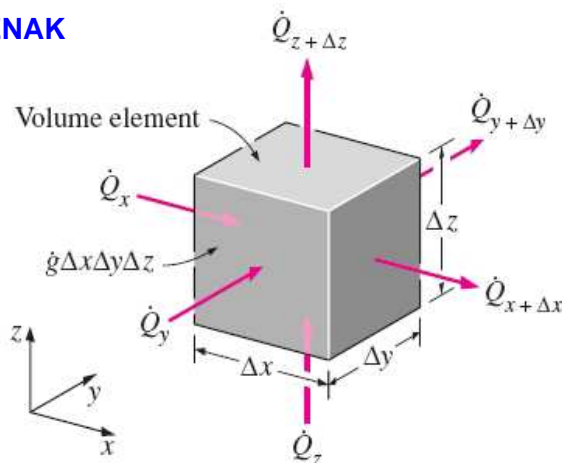
- Bero-sorrera gabeko egoera egonkorra $\frac{d}{dr} \left(r^2 \frac{dT}{dr} \right) = 0$

DIMENTSIO BAKARREKO BERO-EROAPENAREN EKUAZIO KONBINATUA

Hiru geometrientzako baliogarria den adierazpen trinkoa da:

$\frac{1}{r^n} \frac{\partial}{\partial r} \left(r^n \frac{\partial T}{\partial r} \right) + \frac{\dot{e}_{gen}}{k} = \frac{1}{\alpha} \frac{\partial T}{\partial t}$	<p>n = 0 Forma laua.</p> <p>n = 1 Forma zilindrikoa.</p> <p>n = 2 Forma esferikoa.</p>
---	---

KOORDENATU ANGELUZUZENAK



$$\dot{Q}_x + \dot{Q}_y + \dot{Q}_z - \dot{Q}_{x+\Delta x} - \dot{Q}_{y+\Delta y} - \dot{Q}_{z+\Delta z} + \dot{E}_{gen,element} = \frac{\Delta E_{element}}{\Delta t} \quad [\text{W}]$$

$$\dot{E}_{gen,element} = \dot{e}_{gen} V_{element} = \dot{e}_{gen} \Delta x \Delta y \Delta z$$

$$\Delta E_{element} = E_{t+\Delta t} - E_t = m \cdot c \cdot (T_{t+\Delta t} - T_t) = \rho \cdot c \cdot \Delta x \Delta y \Delta z (T_{t+\Delta t} - T_t)$$

KOORDENATU ANGELUZUZENAK

$$\frac{\partial}{\partial x} \left(k \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left(k \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z} \left(k \frac{\partial T}{\partial z} \right) + \dot{e}_{gen} = \rho \cdot c \frac{\partial T}{\partial t}$$

Eroankortasun konstantea:

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{e}_{gen}}{k} = \frac{1}{\alpha} \frac{\partial T}{\partial t}$$

Kasu bereziak:

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{e}_{gen}}{k} = 0$$

Egoera egonkorra.

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = \frac{1}{\alpha} \frac{\partial T}{\partial t}$$

Bero-sorrera gabeko egoera iragankorra.

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = 0$$

Bero-sorrera gabeko egoera egonkorra.

MUGALDE-BALDINTZAK

Gorputzaren mugaldean adierazpen matematikoa osatzen duten baldintza termikoak dira.

The differential equation:

$$\frac{d^2T}{dx^2} = 0$$

General solution:

$$T(x) = C_1x + C_2$$

Arbitrary constants

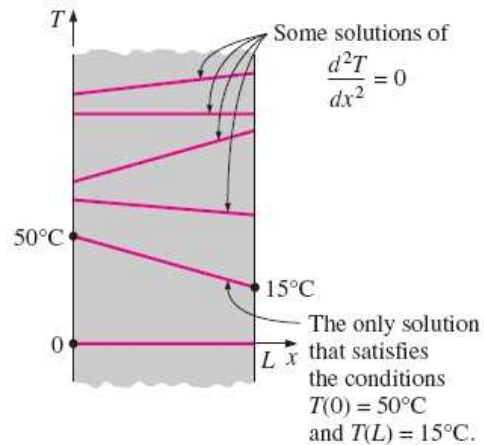
Some specific solutions:

$$T(x) = 2x + 5$$

$$T(x) = -x + 12$$

$$T(x) = -3$$

$$T(x) = 6.2x$$

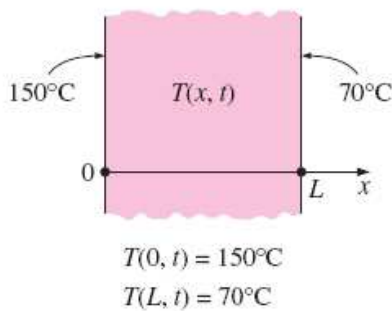
$$\vdots$$


HASIERAKO BALDINTZAK

Aldiune zehatz batean adierazpen matematikoa osatzen duten baldintza termikoak dira. Orokorrean $t = 0$ aldiuneari buruzko informazioa da.

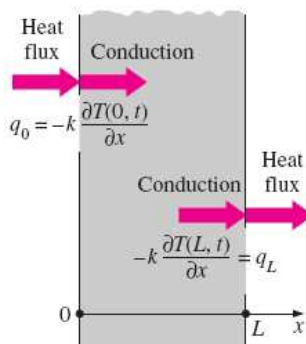
$$T(x, y, z, 0) = f(x, y, z)$$

1- TEMPERATURA ZEHAZTUAREN MUGALDE-BALDINTZA



$$\begin{cases} T(0, t) = T_1 \\ T(L, t) = T_2 \end{cases}$$

2- BERO-FLUXU ZEHAZTUAREN MUGALDE-BALDINTZA



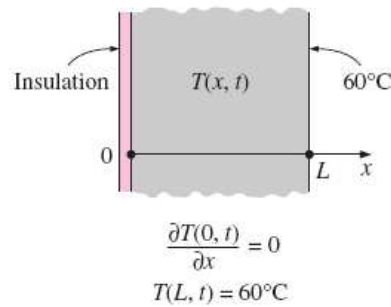
$$\begin{cases} \dot{q} = -k \frac{\partial T(0, t)}{\partial x} & [\text{W/m}^2] \\ \dot{q} = -k \frac{\partial T(L, t)}{\partial x} & [\text{W/m}^2] \end{cases}$$

Bero fluxua x norabide positiboan

2- BERO-FLUXU ZEHAZTUAREN MUGALDE-BALDINTZA

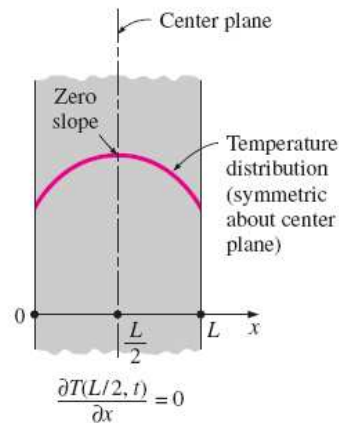
Mugalde isolatua

$$k \frac{\partial T}{\partial x} = 0 \rightarrow \frac{\partial T}{\partial x} = 0$$



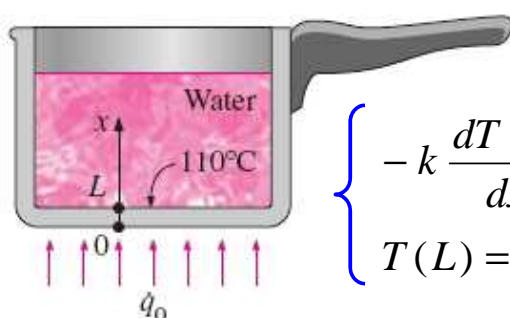
Simetri termikoa

$$\frac{\partial T(L/2, t)}{\partial x} = 0$$



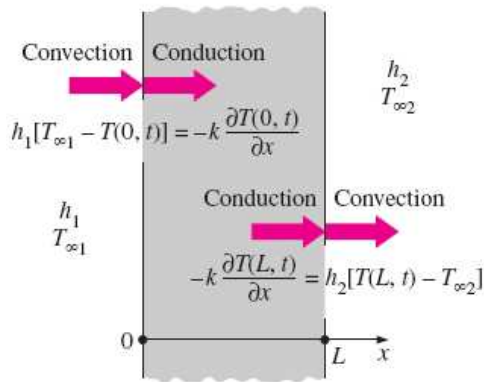
1. ADIBIDEA

Jar dezagun aluminiozko zartagin bat, haragia erregosteko erabiltzen dena, su elektrikoan. Zartagin-ipurdia $L = 0,3$ cm lodi da, eta $D = 20$ cm-ko diametroa du. Berogailu elektrikoak 800 W kontsumitzen du bete-betean ari denean haragia erregosten, eta berotze-elementuak sortzen duen beroaren ehuneko 90 transferitzen dio zartaginari. Operazio geldikorrean neurtu da zartaginaren barne-gainazalaren temperatura 110°C dela. Adierazi zartagin-ipurdiaren mugalde-baldintzak haragia erregosten ari denean.



$$\left\{ \begin{array}{l} -k \frac{dT(0)}{dx} = \dot{q}_0 = \frac{0,720 \text{ kW}}{\pi(0,1 \text{ m})^2} = 22,9 \text{ kW/m}^2 \\ T(L) = 110^\circ\text{C} \end{array} \right.$$

3- KONBEKZIOAREN MUGALDE-BALDINTZA



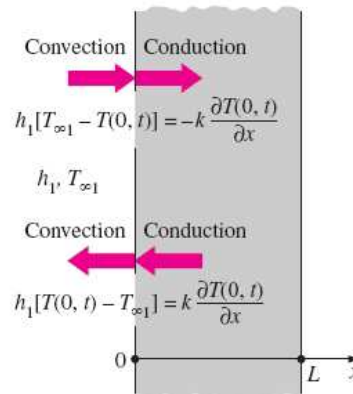
$$\begin{cases} -k \frac{\partial T(0,t)}{\partial x} = h_1 [T_{\infty,1} - T(0,t)] \\ -k \frac{\partial T(L,t)}{\partial x} = h_2 [T(L,t) - T_{\infty,2}] \end{cases}$$

Iruzkinek:

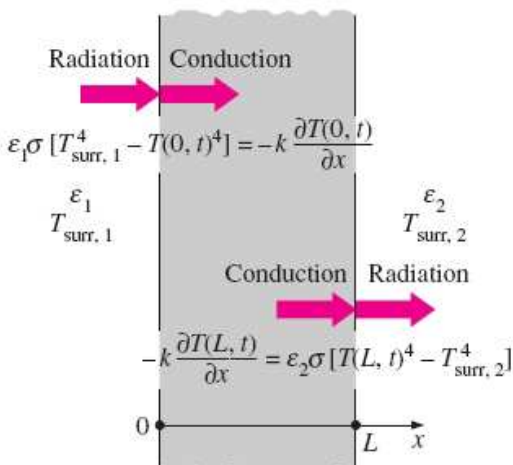
Zeinuak

Gainazaleko balantzearen esanahi fisikoa

Gainazaleko tenperatura ezezagunak



4- ERRADIAZIOAREN MUGALDE-BADINTZA



$$\begin{cases} -k \frac{\partial T(0,t)}{\partial x} = \epsilon_1 \sigma [T_{surr,1}^4 - T(0,t)^4] \\ -k \frac{\partial T(L,t)}{\partial x} = \epsilon_2 \sigma [T(L,t)^4 - T_{surr,2}^4] \end{cases}$$

Stefan-Boltzmannen

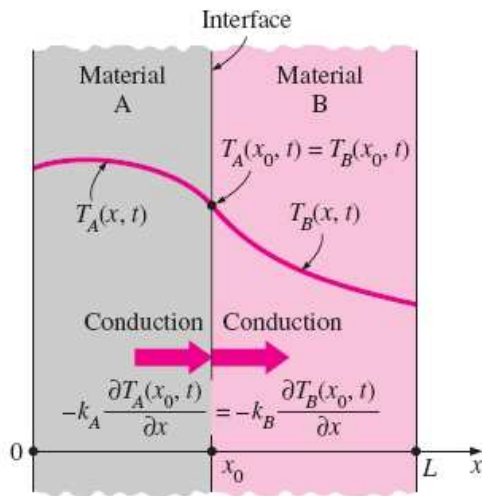
$\sigma = 5,67 \cdot 10^{-8} \text{ [W/m}^2 \cdot \text{K}^4]$

konstantea

Iruzkinek:

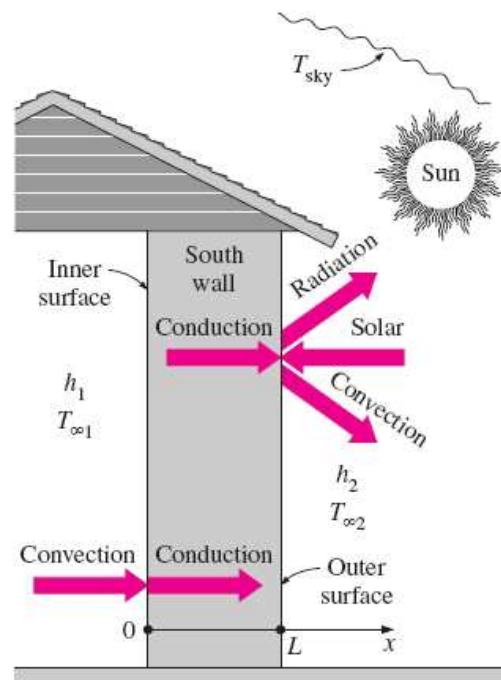
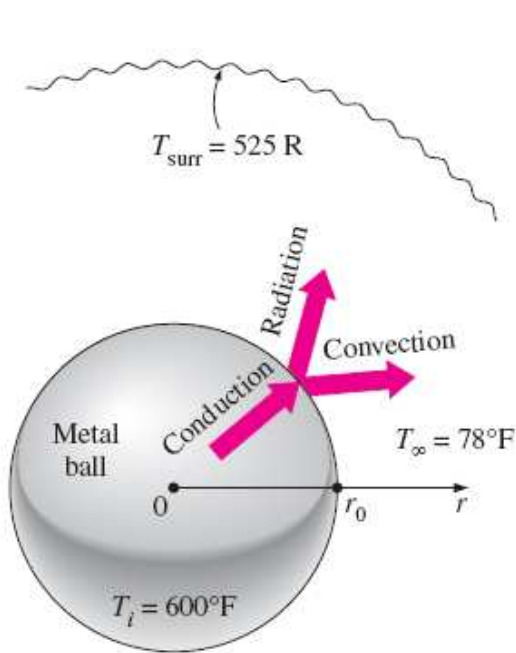
- Tenperaturak
- Linealtasuna

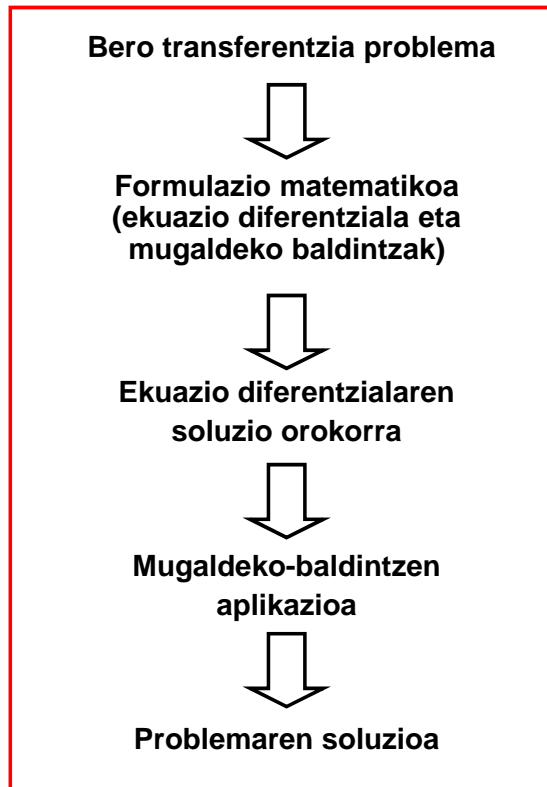
5- FASEARTEKO MUGALDE BALDINTZAK



$$\left\{ \begin{aligned} T_A(x_0, t) &= T_B(x_0, t) \\ -k_A \frac{\partial T_A(x_0, t)}{\partial x} &= -k_B \frac{\partial T_B(x_0, t)}{\partial x} \end{aligned} \right.$$

6- MUGALDE-BALDINTZA OROKORRAK





$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{e}_{gen}}{k} = \frac{1}{\alpha} \frac{\partial T}{\partial t}$$

$$T = T(x, y, z, t) \quad [^{\circ}\text{C}]$$

$$\dot{Q}_n = -k \cdot A \cdot \left| \overrightarrow{\text{grad}}(T) \right| \quad [\text{W}]$$

2.6 – IRAKATSIKO EZ DIREN ATALAK

- 2.3ko azpiatala: KOORDENATU ZILINDRIKOAK
- 2.3ko azpiatala : KOORDENATU ESFERIKOAK
- 2.6 atala: BERO-SORRERA SOLIDOETAN
- 2.7 atala: EROANKORTASUN TERMIKO ALDAKORRA $k(T)$.

3. GAIA

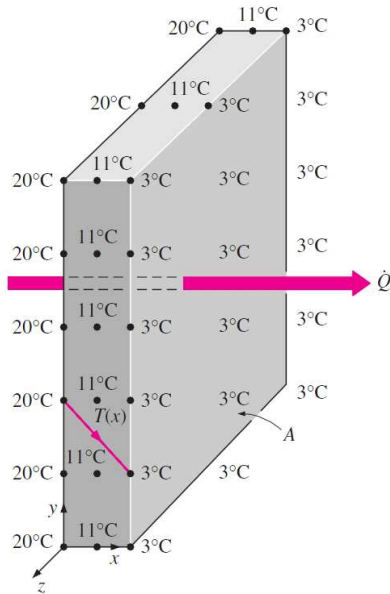
BERO-EROAPEN GELDIKORRA

3.0 - HELBURUAK

2/25

- **Erresistentzia termikoaren** kontzeptua eta haren mugak ulertu, eta erresistentzia termikoen sareak garatu bero-eroapeneko problema praktikoetarako.
- Geruza anitzeko geometria angeluzuzen, zilindriko edo esferikoen **eroapen geldikorreko** problemak ebatzi.
- **Kontaktu-erresistentzia** termikoa eta hori zer egoeratan izan daitekeen garrantzitsua era intuitiboan ulertzeko gaitasuna landu.
- Isolatuz gero bero-transferentzia handitu dezaketen aplikazioak identifikatu (**erradio kritikoa**).
- **Gainazal hegaldunak** aztertu, eta kalkulatu zenbateko errendimendua eta eraginkortasuna duen hegalk berotransferentzia handitzeko.

- Etxe baten horma neguan



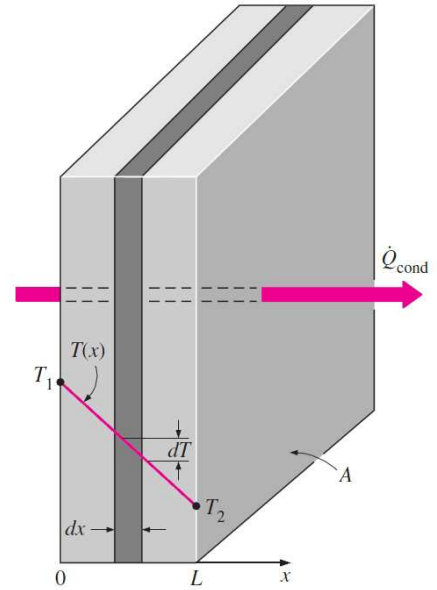
$$\dot{Q}_{in} - \dot{Q}_{out} = \frac{dE_{wall}}{dt} = 0$$



$$\dot{Q}_{cond,wall} = -kA \frac{dT}{dx}$$



$$\dot{Q}_{cond,wall} = kA \frac{T_1 - T_2}{L}$$

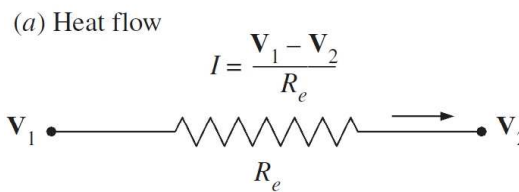
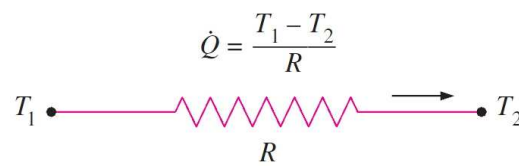


ERRESISTENTZIA TERMIKOAREN KONTZEPTUA

Analogia termoelektrikoa

Baldintzak:

- Egoera egonkorra
- Bero-sorrera gabe

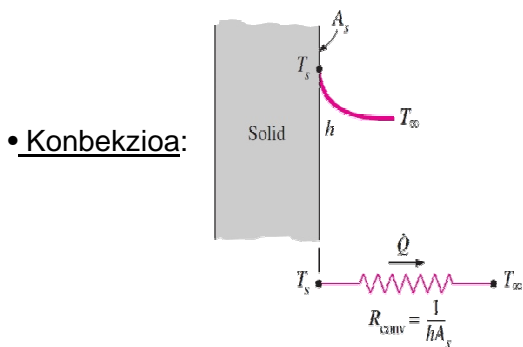


(b) Electric current flow

- Eroapena:

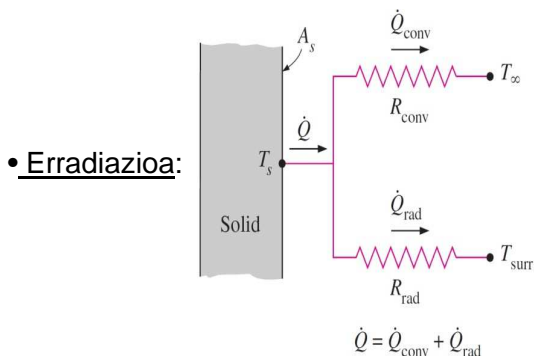
$$\dot{Q}_{cond,wall} = kA \frac{T_1 - T_2}{L} = \frac{T_1 - T_2}{R_{wall}} \Rightarrow R_{wall} = \frac{L}{kA}$$

ERRESISTENTZIA TERMIKOAREN KONTZEPTUA



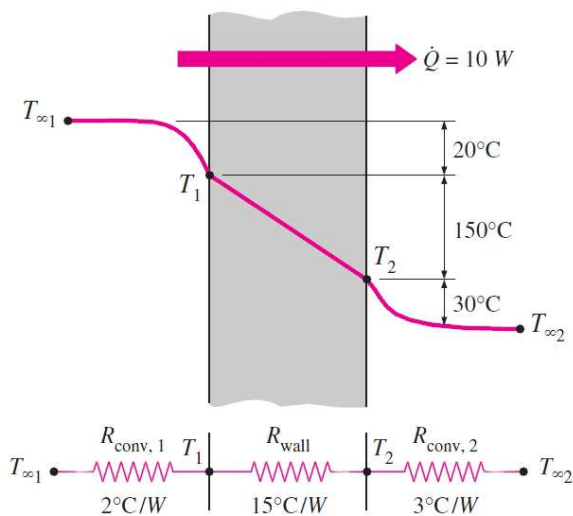
$$\dot{Q}_{conv} = hA(T_s - T_{\infty}) \Rightarrow \boxed{R_{conv} = \frac{1}{hA_s}}$$

$$\Rightarrow \dot{Q}_{conv} = \frac{T_s - T_{\infty}}{R_{conv}}$$

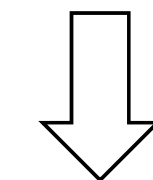


$$\dot{Q}_{rad} = \epsilon \cdot \sigma \cdot A_s (T_s^4 - T_{surr}^4) = h_{rad} \cdot A_s (T_s - T_{surr}) = \frac{T_s - T_{surr}}{R_{rad}} \Rightarrow \boxed{R_{rad} = \frac{1}{h_{rad} A_s}}$$

ERRESISTENTZIA TERMIKOEN SAREA



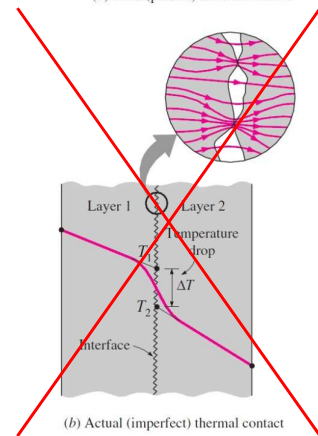
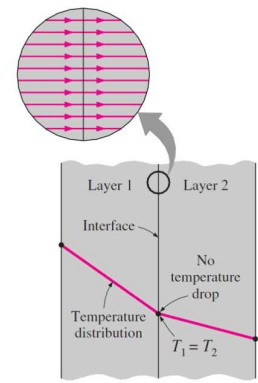
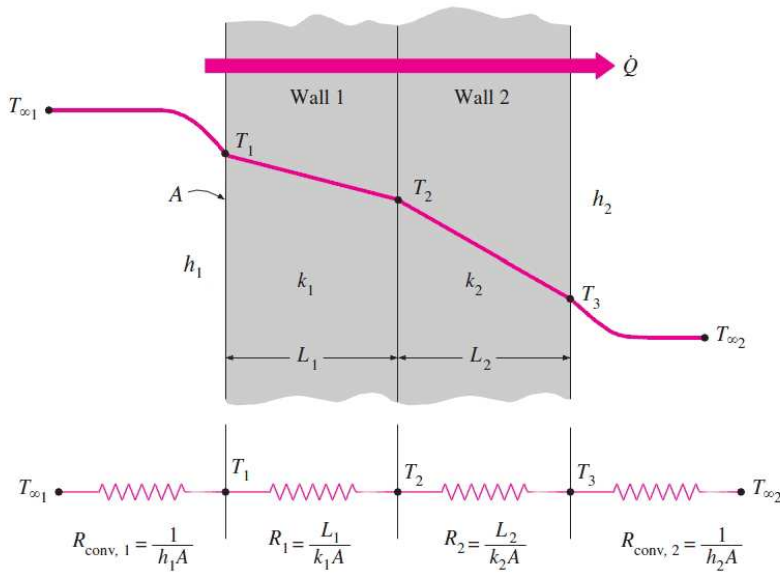
$$\dot{Q} = \frac{\Delta T}{R} = UA\Delta T$$



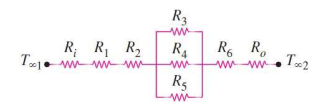
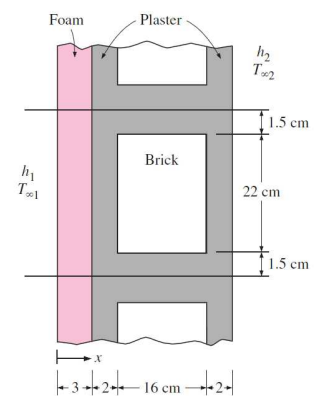
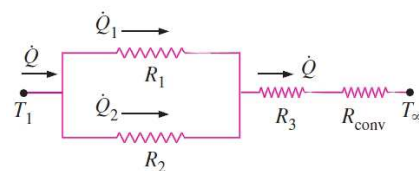
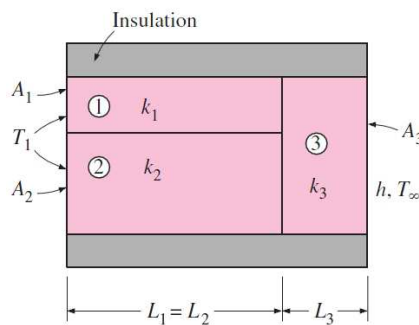
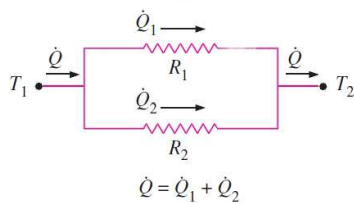
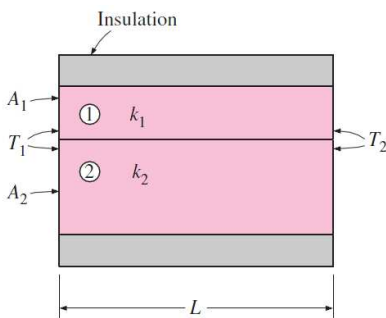
$$\boxed{UA = \frac{1}{R_{total}}}$$

$$\dot{Q} = \frac{T_{\infty 1} - T_1}{R_{conv,1}} = \frac{T_1 - T_2}{R_{cond}} = \frac{T_2 - T_{\infty 2}}{R_{conv,2}} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}}$$

GERUZA ANITZEKO HORMA LAUAK

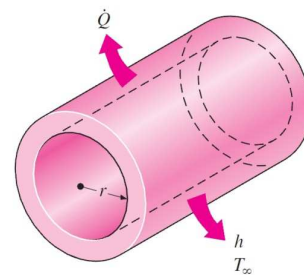


3.3 – ERRESISTENTZIA TERMIKOEN SARE OROKORTUAK

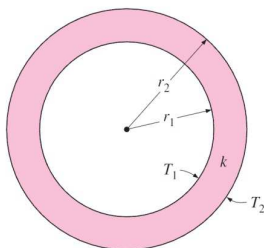


Helburua →

$$\dot{Q}_{cond} = -kA \frac{dT}{dr} \Rightarrow \dot{Q}_{cond} = \frac{\Delta T}{R}$$



• Zilindroak:



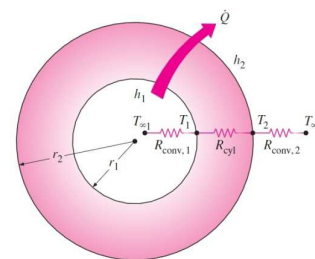
$$\int_{r_1}^{r_2} \frac{\dot{Q}_{cond}}{A} dr = -\int_{T_1}^{T_2} k dT \Rightarrow \dot{Q}_{cond} = 2\pi L k \frac{T_1 - T_2}{\ln\left(\frac{r_2}{r_1}\right)}$$



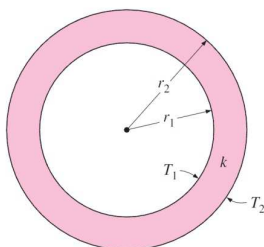
$$R_{cyl} = \frac{\ln\left(\frac{r_2}{r_1}\right)}{2\pi L k}$$

Helburua →

$$\dot{Q}_{cond} = -kA \frac{dT}{dr} \Rightarrow \dot{Q}_{cond} = \frac{\Delta T}{R}$$



• Esferak:



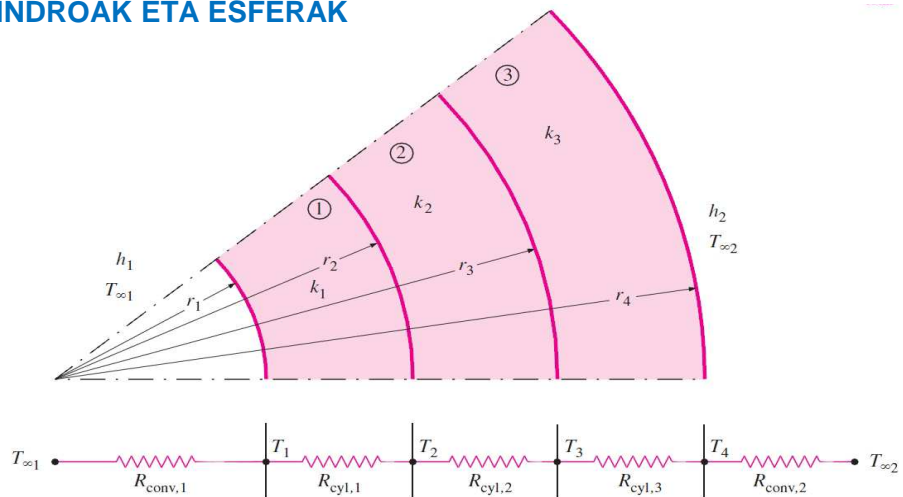
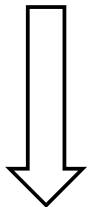
$$\int_{r_1}^{r_2} \frac{\dot{Q}_{cond}}{A} dr = -\int_{T_1}^{T_2} k dT \Rightarrow \dot{Q}_{cond} = 4\pi \cdot r_1 r_2 k \frac{T_1 - T_2}{r_2 - r_1}$$



$$R_{sph} = \frac{r_2 - r_1}{4\pi \cdot r_1 r_2 k}$$

GERUZA ANITZEKO ZILINDROAK ETA ESFERAK

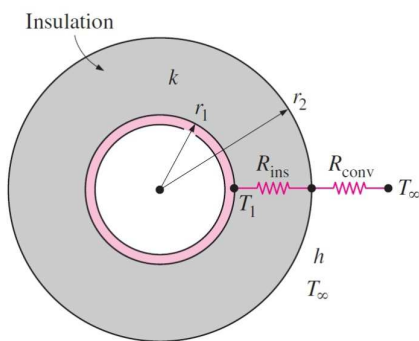
$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}}$$



$$R_{total} = R_{conv,1} + R_{cyl,1} + R_{cyl,2} + R_{cyl,3} + R_{conv,2} \Rightarrow$$

$$\Rightarrow R_{total} = \frac{1}{h_1 A_1} + \frac{\ln\left(\frac{r_2}{r_1}\right)}{2\pi L k_1} + \frac{\ln\left(\frac{r_3}{r_2}\right)}{2\pi L k_2} + \frac{\ln\left(\frac{r_4}{r_3}\right)}{2\pi L k_3} + \frac{1}{h_2 A_4}$$

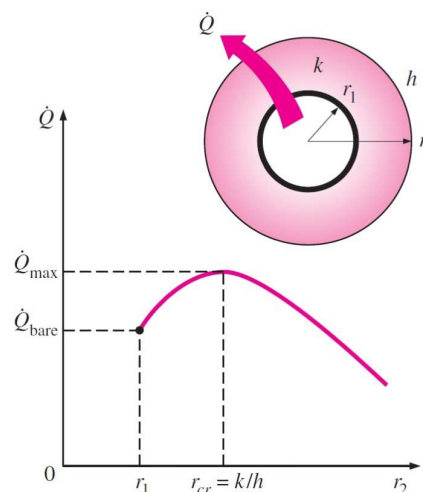
3.5 – ISOLAMENDU-ERRADIO KRITIKOA



$$\dot{Q} = \frac{T_1 - T_{\infty}}{R_{ins} + R_{conv}} = \frac{T_1 - T_{\infty}}{\frac{\ln\left(\frac{r_2}{r_1}\right)}{2\pi L k} + \frac{1}{h 2\pi r_2 L}}$$

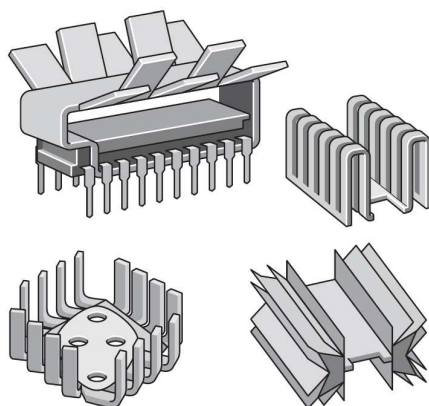
Baldin $r_2 \uparrow$ $\begin{cases} R_{ins} \uparrow \\ R_{conv} \downarrow \end{cases}$

$$\frac{d\dot{Q}}{dr_2} = 0 \Rightarrow \begin{cases} r_{cr,cyl} = k/h \\ r_{cr,sph} = 2k/h \end{cases}$$

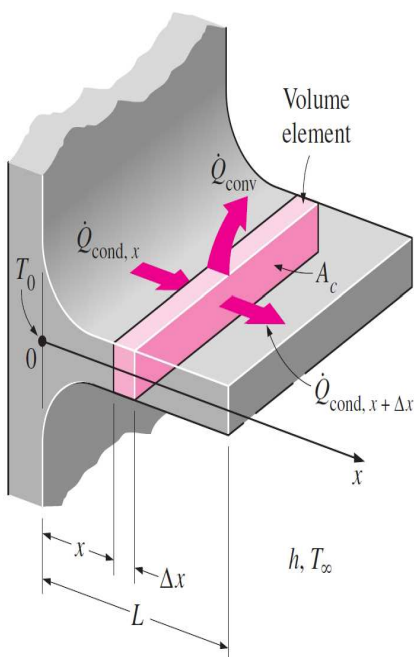




$$\dot{Q}_{conv} = hA_s(T_s - T_\infty)$$



HEGAL-EKUAZIOA



$$\left. \begin{aligned} \dot{Q}_{cond,x} &= \dot{Q}_{cond,x+\Delta x} + \dot{Q}_{conv} \\ \dot{Q}_{conv} &= h(p\Delta x) \cdot (T - T_\infty) \end{aligned} \right\} \Rightarrow$$

$$\Rightarrow \frac{\dot{Q}_{cond,x+\Delta x} - \dot{Q}_{cond,x}}{\Delta x} + hp(T - T_\infty) = 0 \xrightarrow{\Delta x \rightarrow 0} \frac{d\dot{Q}_{cond,x}}{dx} + hp(T - T_\infty) = 0$$

$$\left. \begin{aligned} \dot{Q}_{cond,x} &= -kA_c \frac{dT}{dx} \end{aligned} \right\} \Rightarrow$$

$$\Rightarrow \frac{d}{dx} \left(kA_c \frac{dT}{dx} \right) - hp(T - T_\infty) = 0$$

HEGAL-EKUAZIOA

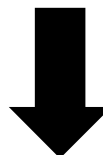
Suposatuz x-ekiko ez direla aldatzen:

- Eroankortasun termikoa
- Zeharkako azalera
- Perimetroa

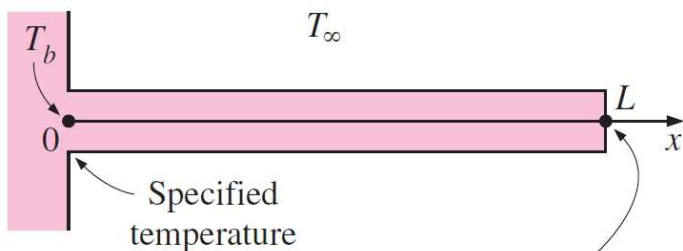


$$\frac{d^2\theta}{dx^2} - m^2 \theta = 0 \rightarrow m^2 = \frac{hp}{kA_c}$$

$$\theta(x) = T(x) - T_\infty$$



$$\theta(x) = C_1 e^{mx} + C_2 e^{-mx}$$



- (a) Specified temperature
- (b) Negligible heat loss
- (c) Convection
- (d) Convection and radiation

HEGAL-EKUAZIOA

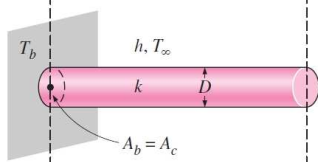
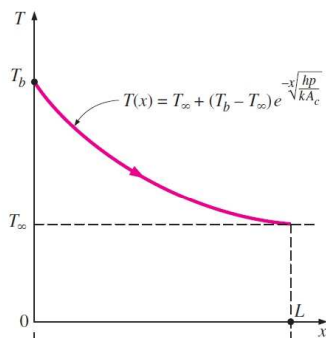
1. KASUA: Hegal luzera infinitukoa



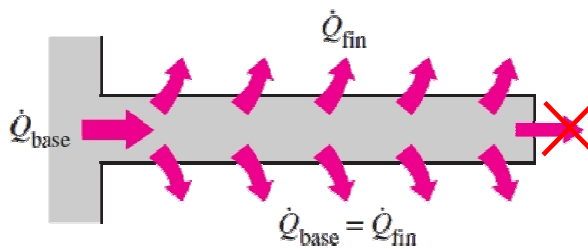
$$\theta(x) = C_1 e^{mx} + C_2 e^{-mx}$$



$$\frac{T(x) - T_\infty}{T_b - T_\infty} = e^{-mx} = e^{-x\sqrt{hp/kA_c}}$$



($p = \pi D, A_c = \pi D^2/4$ for a cylindrical fin)



$$\dot{Q}_{long\ fin} = -kA_c \left. \frac{dT}{dx} \right|_{x=0} = \sqrt{hp k A_c} (T_b - T_\infty)$$

HEGAL-EKUAZIOA

Hegalak ez dira izaten beren muturreko temperatura inguruneko temperaturara hurbiltzeko adina luzeak $\rightarrow T(L) = T_\infty$

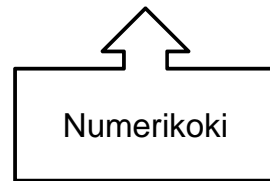
2. KASUA: Hegalaren muturreko bero-galera baztergarria



$$\frac{T(x) - T_\infty}{T_b - T_\infty} = \frac{\cosh m(L - x)}{\cosh mL}$$

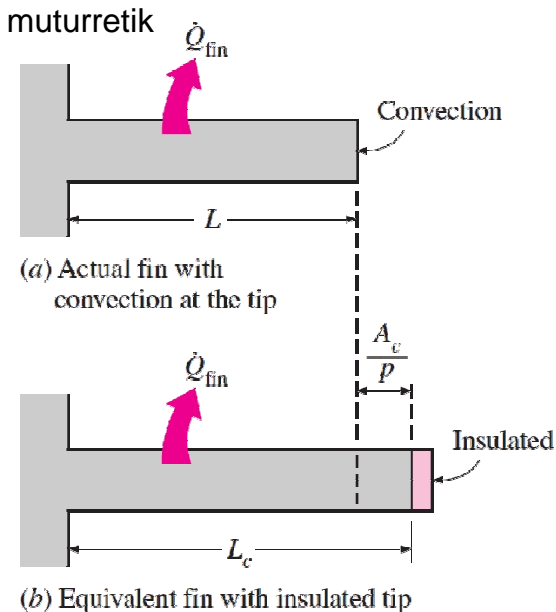
$$T(x) - T_\infty = C_1 e^{mx} + C_2 e^{-mx}$$

$$\dot{Q}_{insulated\ tip} = -kA_c \left. \frac{dT}{dx} \right|_{x=0} = \sqrt{hpkA_c} (T_b - T_\infty) \tanh mL$$



HEGAL-EKUAZIOA

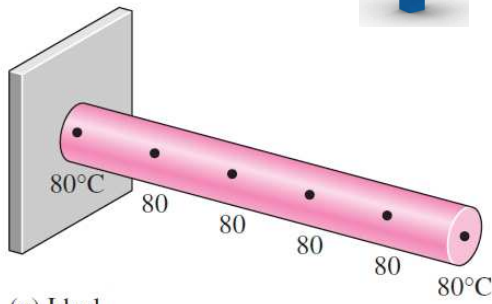
3. KASUA: Konbekzioa (edo konbekzio eta erradiazio konbinatuak) hegal-muturretik



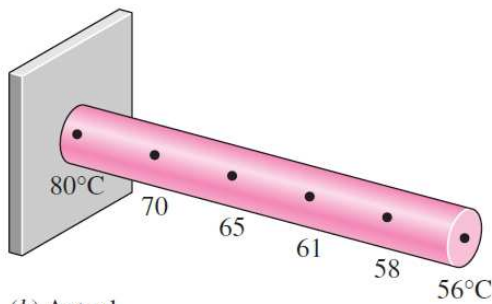
Hegal-mutur adiabatikoaren kasua luzera zuzenduz

$$L_c = L + \frac{A_c}{p}$$

HEGAL ERRENDIMENDUA



(a) Ideal



(b) Actual

Hegalaren bero-transferentzia maximoa

$$\dot{Q}_{fin,max} = hA_{fin} (T_b - T_\infty)$$

Hegalaren errendimendua

$$\eta_{fin} = \frac{\dot{Q}_{fin}}{\dot{Q}_{fin,max}} \Rightarrow \dot{Q}_{fin} = \eta_{fin} \cdot \dot{Q}_{fin,max}$$

f(geom,m)

HEGAL ERRENDIMENDUA

1. KASUA: Hegal luzera infinitua

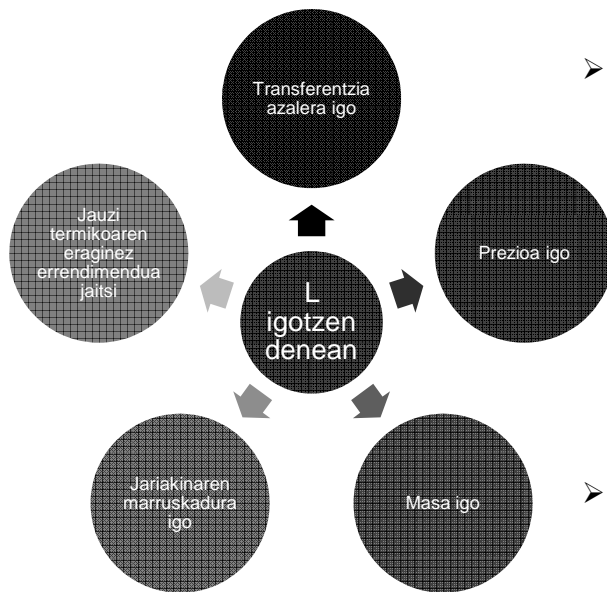
$$\eta_{long\ fin} = \frac{\dot{Q}_{fin}}{\dot{Q}_{fin,max}} = \frac{\sqrt{hp}kA_c (T_b - T_\infty)}{hA_{fin} (T_b - T_\infty)} = \frac{1}{L} \sqrt{\frac{kA_c}{hp}} = \frac{1}{mL}$$

2. KASUA: Hegalaren muturreko bero-galera baztergarria

$$\eta_{insulated\ tip} = \frac{\dot{Q}_{fin}}{\dot{Q}_{fin,max}} = \frac{\sqrt{hp}kA_c (T_b - T_\infty) \tanh mL}{hA_{fin} (T_b - T_\infty)} = \frac{\tanh mL}{mL}$$

HEGAL ERRENDIMENDUA

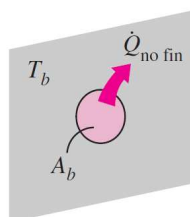
Hegalaren luzerari buruzko iruzkinak:



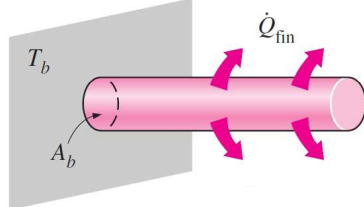
➤ Hegal-errendimendua % 60tik jaitsarazten duten luzerak ezin dira justifikatu ekonomiaren ikuspegitik, normalean

➤ Praktikan erabiltzen diren hegal gehienen errendimendua % 90etik gorakoa da.

HEGAL-ERAGINKORTASUNA



$$\epsilon_{fin} = \frac{\dot{Q}_{fin}}{\dot{Q}_{no\ fin}}$$



- $\epsilon < 1$** • Hegalak isolatzaile moduan jokatzen du
- $\epsilon = 1$** • Ez du bero-transferentzian eraginik
- $\epsilon > 1$** • Bero-transferentzia handitzen da
• Errentagarria da?

$$\epsilon_{fin} = \frac{\dot{Q}_{fin}}{\dot{Q}_{no\ fin}} = \frac{\eta_{fin} h A_{fin} (T_b - T_{\infty})}{h A_b (T_b - T_{\infty})} = \frac{A_{fin}}{A_b} \eta_{fin}$$

HEGAL-ERAGINKORTASUNA

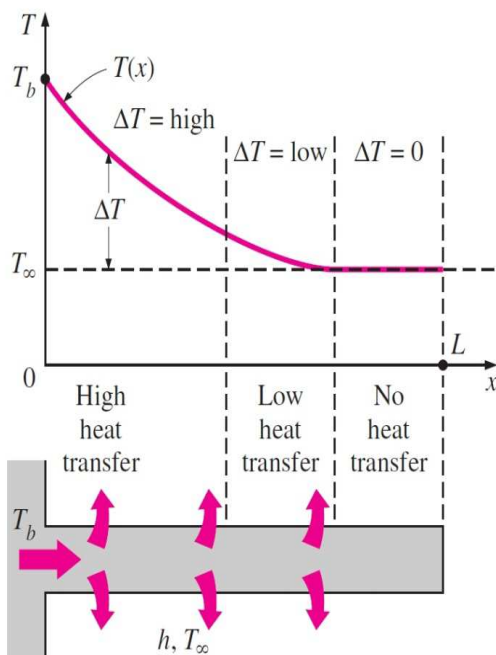
1. KASUA: Hegal luzera infinitua

$$\epsilon_{long\ fin} = \frac{\dot{Q}_{fin}}{\dot{Q}_{no\ fin}} = \frac{\sqrt{hp k A_c} (T_b - T_\infty)}{h A_b (T_b - T_\infty)} = \sqrt{\frac{kp}{h A_c}}$$

Ondorioak:

- ✓ h txikia → Eraginkortasun handiagoa konbektzio naturalean eta gasetan
- ✓ k altua (metalak)
- ✓ p/A_c erlazio altua (xafla meheko hegalak)

HEGALAREN LUZERA EGOKIA



$$\frac{\dot{Q}_{fin}}{\dot{Q}_{long\ fin}} = \frac{\sqrt{hp k A_c} (T_b - T_\infty) \tanh mL}{\sqrt{hp k A_c} (T_b - T_\infty)} = \tanh mL$$

mL	$\frac{\dot{Q}_{fin}}{\dot{Q}_{long\ fin}} = \tanh mL$
0.1	0.100
0.2	0.197
0.5	0.462
1.0	0.762
1.5	0.905
2.0	0.964
2.5	0.987
3.0	0.995
4.0	0.999
5.0	1.000

- 3.2 atala: KONTAKTU-ERRESISTENTZIA TERMIKOA
- 3.7 atala: BERO-TRANSFERENTZIA KONFIGURAZIO ARRUNTETAN

4. GAIA

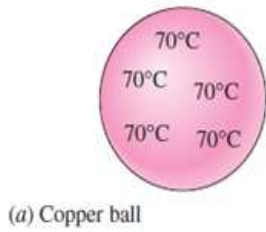
BERO-EROAPEN IRAGANKORRA

4.0 - HELBURUAK

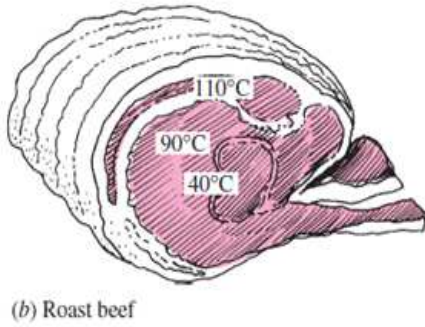
2/30

- Baloratu noiz den baztergarria temperaturaren aldaketa espaziala eta noiz aldatzen den temperatura ia era uniformean denborarekin, parametro **kontzentratuen sistemen** analisi sinplifikatuak aplikagarri eginez.
- Ebazpide analitikoak lortu **dimentsio bakarreko eroapen-problema iragankorretarako** geometria angeluzuzen, zilindriko eta esferikoetan, aldagai-bereizkuntzaren metodoa erabilia, eta ulertu zergatik den hurbiltze onargarria, gehienetan, gai bakarreko soluzioa.
- **Ingurune handietako eroapen iragankorreko** problemak ebatzi, antzekotasun-aldagaia erabilia, eta temperaturaren aldaketa aurrean, denboraren eta gainazaletiko distantziaren arabera.

SARRERA

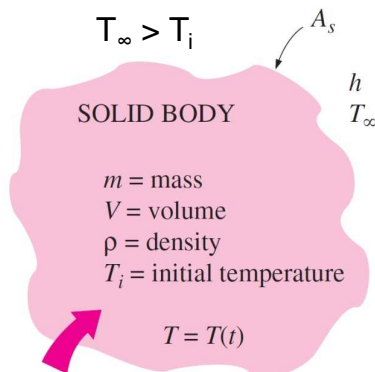


Barne erresistentzia= 0
 T = kte espazioan
 T ≠ kte denbora
 $T(x,y,z,t) \rightarrow T(t)$



T ≠ kte espazioan
 T ≠ kte denbora
 $T(x,y,z,t)$

4.1 – PARAMETRO KONTZENTRATUEN SISTEMEN ANALISIA



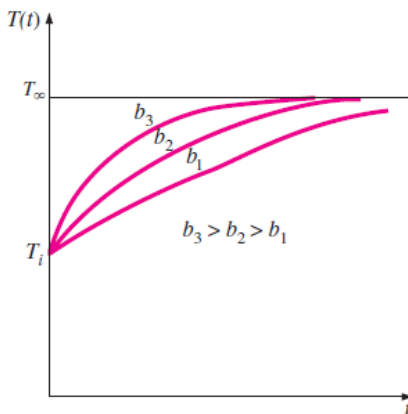
$$\left(\begin{array}{l} \text{Bero-} \\ \text{transferezentzia} \\ \text{gorputzera, dt-n} \end{array} \right) = \left(\begin{array}{l} \text{Gorputzaren} \\ \text{energia gehitzea} \\ \text{dt-n} \end{array} \right)$$

$$hA_s(T_\infty - T(t))dt = mc_p d(T(t))$$

$$\frac{d(T(t) - T_\infty)}{T(t) - T_\infty} = -\frac{hA_s}{\rho V c_p} dt$$

t=0-tik integratuz non T=T_i
 t berdin T=T(t)-rarte →

$$\ln \frac{T(t) - T_\infty}{T_i - T_\infty} = -\frac{hA_s}{\rho V c_p} t$$



$$\frac{T(t) - T_\infty}{T_i - T_\infty} = e^{-bt}$$

$$b = \frac{hA_s}{\rho V c_p} \quad (1/s)$$

Newtonen hozte-legea

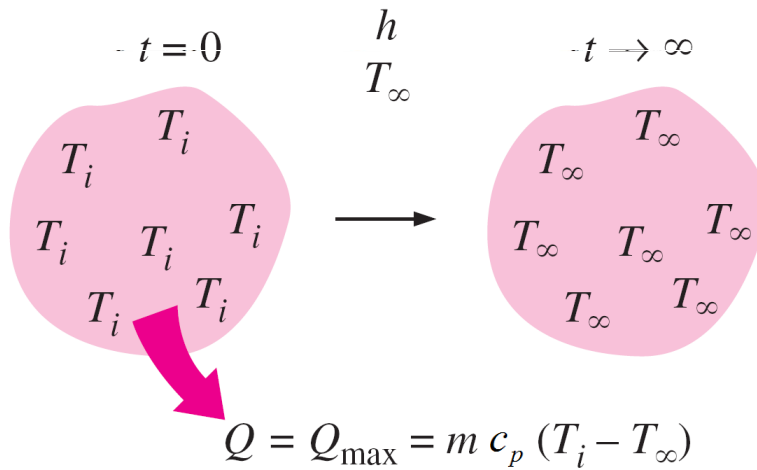
$$\dot{Q}(t) = hA_s [T(t) - T_\infty] \quad (W)$$

Gorputzaren eta haren ingurunearen arteko bero-transferentzia kantitate totala

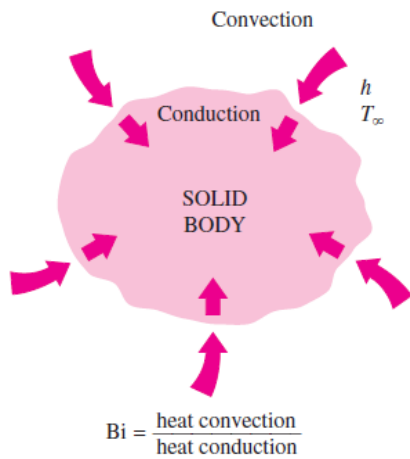
$$Q = mc_p [T(t) - T_i] \quad (J)$$

Gorputzaren eta haren ingurunearen arteko bero-transferentzia maximoa

$$Q_{max} = mc_p [T_\infty - T_i] \quad (J)$$



PARAMETRO KONTZENTRATUEN SISTEMEN ANALISIRAKO IRIZPIDEAK



$$L_c = \frac{V}{A_s}$$

$$Bi = \frac{hL_c}{k}$$

$$Bi = \frac{h}{k/L_c} \frac{\Delta T}{\Delta T}$$

$$Bi = \frac{L_c/k}{1/h}$$



Zer adierazten du Biot zenbaki txikiak?

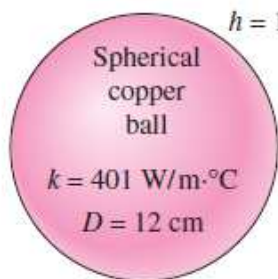
PARAMETRO KONTZENTRATUAK APLIKATZEKO IRIZPIDEAK **Bi = 0**



$$Bi \leq 0,1 \rightarrow T - T_\infty \text{ errorea } \pm 5 \%$$

PARAMETRO KONTZENTRATUEN SISTEMEN ANALISIRAKO IRIZPIDEAK

1. ADIBIDEA:



$$L_c = \frac{V}{A_s} = \frac{\frac{1}{6} \pi D^3}{\pi D^2} = \frac{1}{6} D = 0,02 \text{ m}$$

$$Bi = \frac{hL_c}{k} = \frac{15 \times 0,02}{401} = 0,00075 < 0,1$$

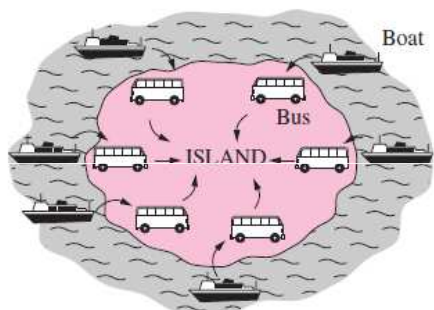
$$\frac{T(t) - T_\infty}{T_i - T_\infty} = e^{-bt}$$

$$b = \frac{hA_s}{\rho V C_p}$$

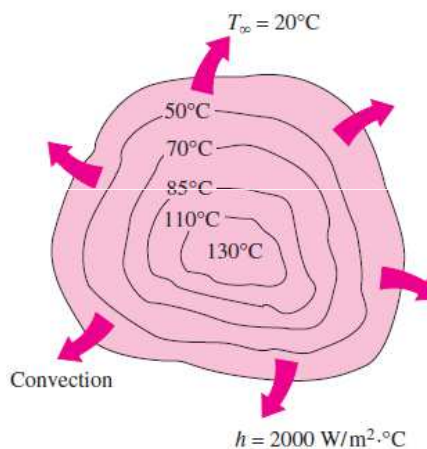
Eroankortasun termiko handiko eta konbektzio-koefiziente txikiko gorputz txikiak dira parametro kontzentratuen sistemen analisisen irizpideak betetzeko aukera gehien dituztenak

OHAR BATZUK PARAMETRO KONTZENTRATUEN SISTEMETAKO BERO-TRANSFERENTZIAZ

Solido baterako bero-transferentzia



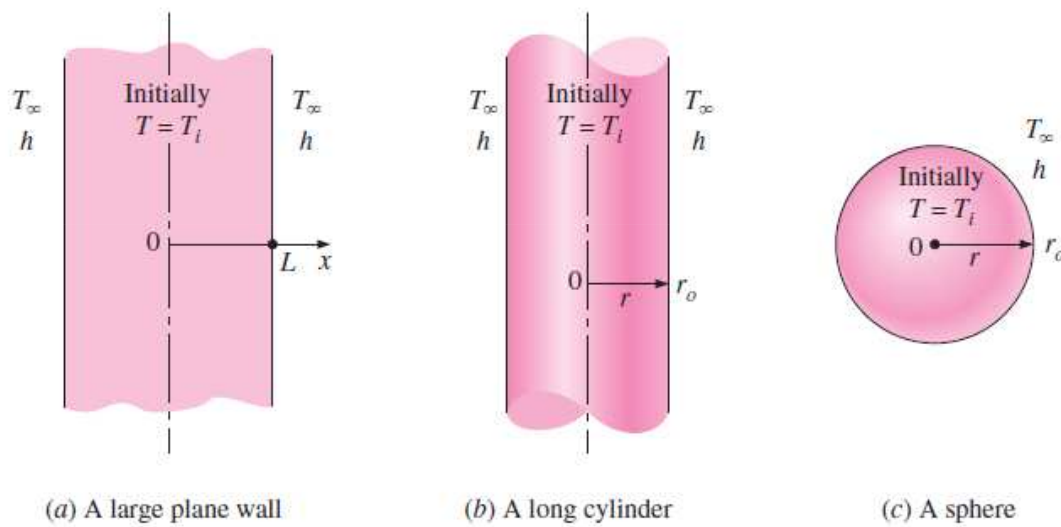
Gorputz baten hozte prozesua



4.2 – BERO-EROAPEN IRAGANKORRA HORMA LAU HANDIETAN, ZILINDRO LUZEETAN ETA ESFERETAN, EFEKTU ESPAZIALAK KONTUAN HARTUZ

9/30

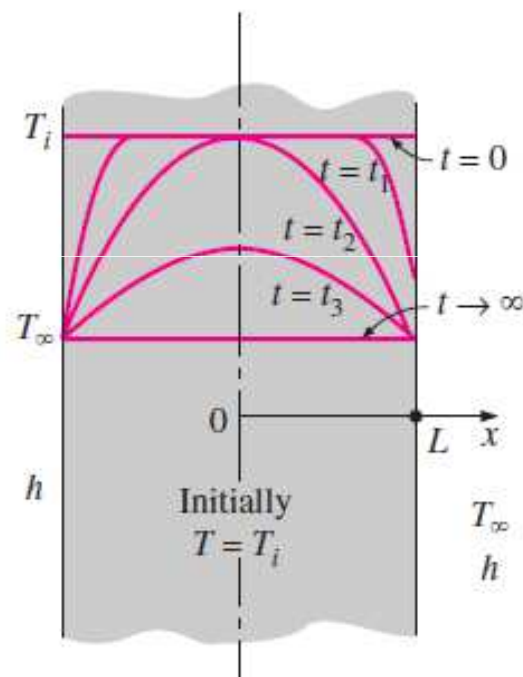
Bero-transferentzia dimentsio bakarrekoa duten geometria sinpleen eskema



4.2 – BERO-EROAPEN IRAGANKORRA HORMA LAU HANDIETAN, ZILINDRO LUZEETAN ETA ESFERETAN, EFEKTU ESPAZIALAK KONTUAN HARTUZ

10/30

Tenperatura-profil iragankorra gainazaletiko konbektziopean dagoen horma lau batean, $T_i > T_\infty$ -rako.



4.2 – BERO-EROAPEN IRAGANKORRA HORMA LAU HANDIETAN, ZILINDRO LUZEETAN ETA ESFERETAN, EFEKTU ESPAZIALAK KONTUAN HARTUZ 11/30

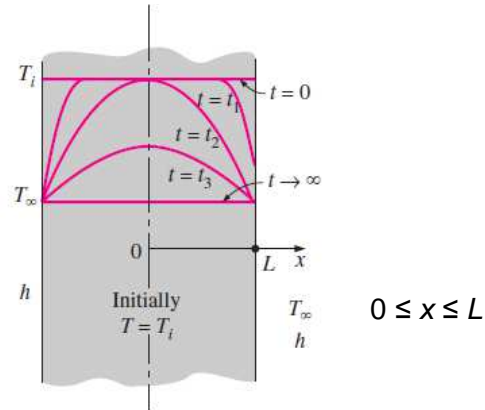
JATORRIZKO BERO-EROAPENENKO PROBLEMA:

- Ekuazio diferentziala $\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha} \frac{\partial T}{\partial t}$ $\alpha = k/\rho c_p$ difusibitate termikoa

- Mugalde-baldintzak $\frac{\partial T(0,t)}{\partial x} = 0$ $-k \frac{\partial T(L,t)}{\partial x} = h[T(L,t) - T_\infty]$

- Hasierako baldintzak $T(x,0) = T_i$

- Emaita $T = F(x, t, L, k, \alpha, h, T_i)$
aldagaia parametroa



4.2 – BERO-EROAPEN IRAGANKORRA HORMA LAU HANDIETAN, ZILINDRO LUZEETAN ETA ESFERETAN, EFEKTU ESPAZIALAK KONTUAN HARTUZ 12/30

DIMENTSIO BAKARREKO EROAPEN IRAGANKORREKO PROBLEMA DIMENTSIOGABETUA

Aldagai dimentsiogabeak : $X = x/L$

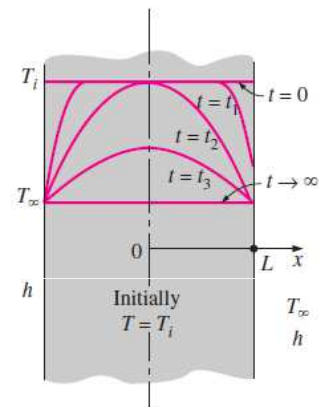
$$\theta(x,t) = [T(x,t) - T_\infty] / [T_i - T_\infty]$$

- Ekuazio diferentziala $\frac{\partial^2 \theta}{\partial X^2} = \frac{\partial \theta}{\partial \tau}$

- Mugalde-baldintzak $\frac{\partial \theta(0,\tau)}{\partial X} = 0$ $\frac{\partial \theta(1,\tau)}{\partial X} = -Bi \theta(1,\tau)$

- Hasierako baldintzak $\theta(X,0) = 1$

- Emaita $\theta = f(X, Bi, Fo)$



4.2 – BERO-EROAPEN IRAGANKORRA HORMA LAU HANDIETAN, ZILINDRO LUZEETAN ETA ESFERETAN, EFEKTU ESPAZIALAK KONTUAN HARTUZ 13/30

DIMENTSIO BAKARREKO EROAPEN IRAGANKORREKO PROBLEMA DIMENTSIOGABETUA

Temperatura dimentsiogabea $\theta(X, \tau) = \frac{T(x, t) - T_i}{T_\infty - T_i}$

Distantzia dimentsiogabea, zentrotik

$$X = x/L$$

Bero-transferentziaren koefiziente dimentsiogabea

$$Bi = \frac{hL}{k}$$

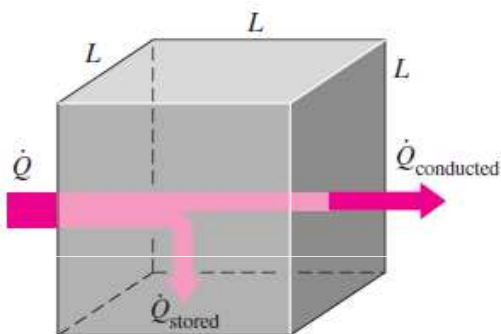
Bioten zenbakia

Denbora dimentsiogabea

$$\tau = \frac{\alpha \cdot t}{L^2} = Fo$$

Fourierren zenbakia

4.2 – BERO-EROAPEN IRAGANKORRA HORMA LAU HANDIETAN, ZILINDRO LUZEETAN ETA ESFERETAN, EFEKTU ESPAZIALAK KONTUAN HARTUZ 14/30



$$\text{Fourierren zenbakia} = \frac{\text{Eroandako Beroa}}{\text{Metatutako Beroa}}$$

$$\tau = \frac{\alpha t}{L^2} = \frac{kL^2 (1/L) \Delta T}{\rho C_p L^3 / t \Delta T}$$

$$\text{Fourier number: } \tau = \frac{\alpha t}{L^2} = \frac{\dot{Q}_{\text{conducted}}}{\dot{Q}_{\text{stored}}}$$

4.2 – BERO-EROAPEN IRAGANKORRA HORMA LAU HANDIETAN, 15/30 ZILINDRO LUZEETAN ETA ESFERETAN, EFEKTU ESPAZIALAK KONTUAN HARTUZ

DIMENTSIO BAKARREKO EROAPEN IRAGANKORREKO PROBLEMEN SOLUZIO ZEHATZA

Aldagai-bereizkuntzaren metodoa

$$\theta(X, \tau) = F(X)G(\tau) \xrightarrow{\text{Ek Diferentziala}} \frac{1}{F} \frac{d^2 F}{dX^2} = \frac{1}{G} \frac{dG}{d\tau} = \text{KTE} = (-\lambda^2)$$

$$\frac{d^2 F}{dX^2} + \lambda^2 F = 0 \quad \frac{dG}{d\tau} + \lambda^2 G = 0$$

APLIKAGARRITASUNA

- Geometria sinplea eta finitua bada
- Ekuazio diferentziala eta mugalde-baldintzak nahiz hasierakoak, linealak badira
- Gai ez-homogeneo bakarra badute

Soluzioak $\theta = \sum_{n=1}^{\infty} A_n e^{-\lambda_n^2 \tau} \cos(\lambda_n X)$

Honako ingurune baldintzapean (Ekuazio karakteristikoa) $\lambda_n \tan \lambda_n = Bi$

Eta honako hasierako baldintza $A_n = \frac{4 \sin \lambda_n}{2\lambda_n + \sin(2\lambda_n)}$

4.2 – BERO-EROAPEN IRAGANKORRA HORMA LAU HANDIETAN, 16/30 ZILINDRO LUZEETAN ETA ESFERETAN, EFEKTU ESPAZIALAK KONTUAN HARTUZ

DIMENTSIO BAKARREKO EROAPEN IRAGANKORREKO PROBLEMEN SOLUZIO ZEHATZA

Geometria **Ebazpidea** **λ_n -ak hauen erroak dira**

Horma Laua $\theta = \sum_{n=1}^{\infty} \frac{4 \operatorname{sen} \lambda_n}{2\lambda_n + \operatorname{sen}(2\lambda_n)} e^{-\lambda_n^2 \tau} \cos(\lambda_n x / L)$ $\lambda_n \tan \lambda_n = Bi$

Zilindroa $\theta = \sum_{n=1}^{\infty} \frac{2}{\lambda_n} \frac{J_1(\lambda_n)}{J_0^2(\lambda_n) + J_1^2(\lambda_n)} e^{-\lambda_n^2 \tau} J_0(\lambda_n r / r_0)$ $\lambda_n \frac{J_1(\lambda_n)}{J_0(\lambda_n)} = Bi$

Esfera $\theta = \sum_{n=1}^{\infty} \frac{4(\operatorname{sen} \lambda_n - \lambda_n \cos \lambda_n)}{2\lambda_n - \operatorname{sen}(2\lambda_n)} e^{-\lambda_n^2 \tau} \frac{\operatorname{sen}(\lambda_n r / L)}{\lambda_n r / L}$ $1 - \lambda_n \cot \lambda_n = Bi$

Ebazpideak: Serie infinituak eta ekuazio implizitua

4.2 – BERO-EROAPEN IRAGANKORRA HORMA LAU HANDIETAN, ZILINDRO LUZEETAN ETA ESFERETAN, EFEKTU ESPAZIALAK KONTUAN HARTUZ 17/30

HURBILKETA BIDEZKO EBAZPIDE : ANALITIKO ETA GRAFIKOAK

Aplikagarritasuna $\tau > 0,2$

a.EBAZPIDE ANALITIKOA

Horma lauetan
$$\theta_{wall} = \frac{T(x,t) - T_{\infty}}{T_i - T_{\infty}} = A_1 e^{-\lambda_1^2 \tau} \cos(\lambda_1 x/L), \quad \tau > 0,2$$

Zilindroetan
$$\theta_{cyl} = \frac{T(r,t) - T_{\infty}}{T_i - T_{\infty}} = A_1 e^{-\lambda_1^2 \tau} J_0(\lambda_1 r/r_o), \quad \tau > 0,2$$

Esferetan
$$\theta_{sph} = \frac{T(r,t) - T_{\infty}}{T_i - T_{\infty}} = A_1 e^{-\lambda_1^2 \tau} \frac{\sin(\lambda_1 r/r_o)}{\lambda_1 r/r_o}, \quad \tau > 0,2$$

Non A_1 eta λ_1 konstanteak Bi zenbakiaren funtzioak diren (4-2 Taula)
 Non J_0 Bessel-en funtzioa den (4-3 Taula)

4.2 – BERO-EROAPEN IRAGANKORRA HORMA LAU HANDIETAN, ZILINDRO LUZEETAN ETA ESFERETAN, EFEKTU ESPAZIALAK KONTUAN HARTUZ 18/30

HURBILKETA BIDEZKO EBAZPIDE : ANALITIKO ETA GRAFIKOAK

Aplikagarritasuna $\tau > 0,2$

a.EBAZPIDE ANALITIKOA

Horma lauetan
$$\left(\frac{Q}{Q_{max}} \right)_{wall} = 1 - \theta_{0,wall} \frac{\sin \lambda_1}{\lambda_1}$$

Zilindroetan
$$\left(\frac{Q}{Q_{max}} \right)_{cyl} = 1 - 2\theta_{0,cyl} \frac{J_1(\lambda_1)}{\lambda_1}$$

Esferetan
$$\left(\frac{Q}{Q_{max}} \right)_{sph} = 1 - 3\theta_{0,sph} \frac{\sin \lambda_1 - \lambda_1 \cos \lambda_1}{\lambda_1^3}$$

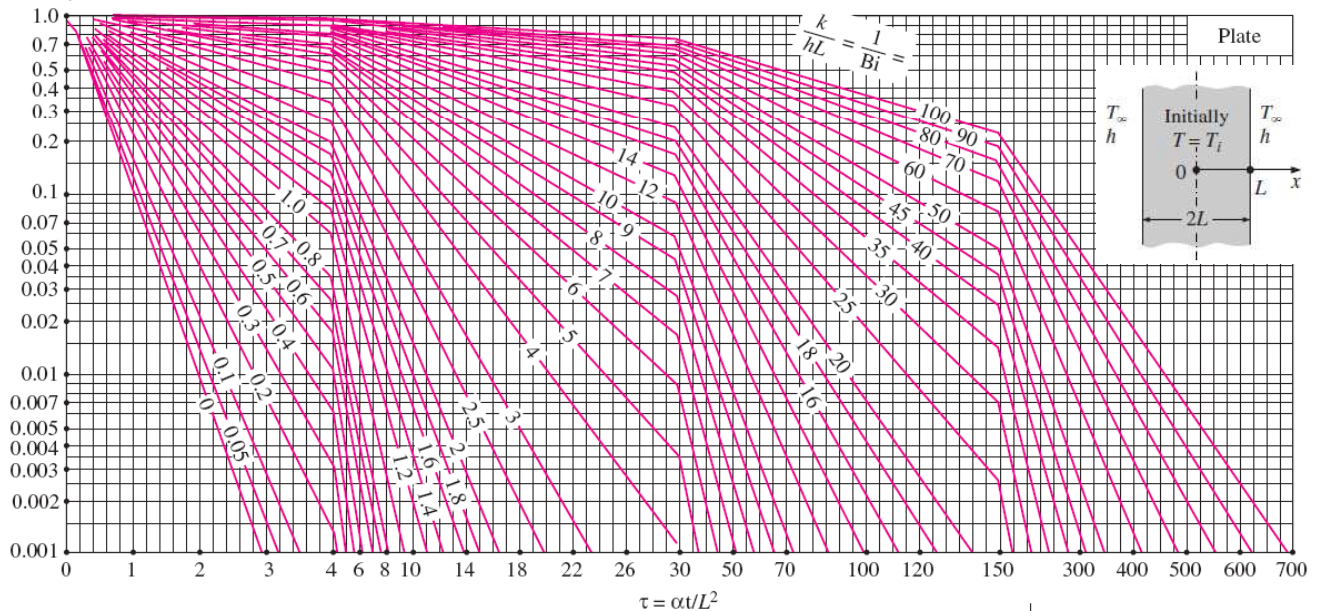
4.2 – BERO-EROAPEN IRAGANKORRA HORMA LAU HANDIETAN, ZILINDRO LUZEETAN ETA ESFERETAN, EFEKTU ESPAZIALAK KONTUAN HARTUZ 19/30

HURBILKETA BIDEZKO EBAZPIDE : ANALITIKO ETA GRAFIKOAK Aplikagarritasuna $\tau > 0,2$

b.EBAZPIDE GRAFIKOA. HEISLER-EN GRAFIKOAK (3) Geometria bakoitzarekin lotuta.

b.1. Geometriaren zentroan eta t denbora jakin batean T_0 **temperatura** kalkulatzeko balio du

$$\theta_o = \frac{T_o - T_\infty}{T_i - T_\infty}$$



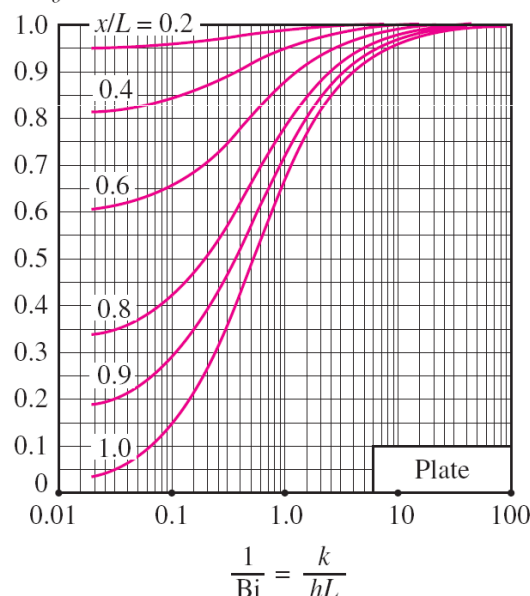
4.2 – BERO-EROAPEN IRAGANKORRA HORMA LAU HANDIETAN, ZILINDRO LUZEETAN ETA ESFERETAN, EFEKTU ESPAZIALAK KONTUAN HARTUZ 20/30

HURBILKETA BIDEZKO EBAZPIDE : ANALITIKO ETA GRAFIKOAK Aplikagarritasuna $\tau > 0,2$

b.EBAZPIDE GRAFIKOA. HEISLER-EN GRAFIKOAK (3) Geometria bakoitzarekin lotuta

b.2. **Beste kokapenetako** eta aldiune bereko temperatura T_0 -ren arabera kalkulatzeko

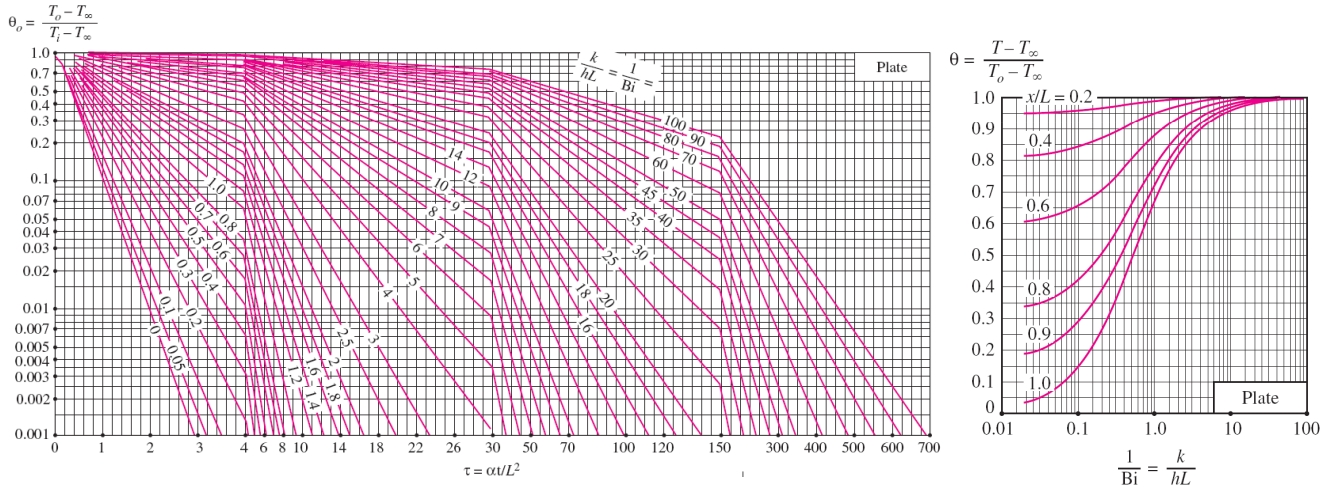
$$\theta = \frac{T - T_\infty}{T_o - T_\infty}$$



4.2 – BERO-EROAPEN IRAGANKORRA HORMA LAU HANDIETAN, 21/30 ZILINDRO LUZEETAN ETA ESFERETAN, EFEKTU ESPAZIALAK KONTUAN HARTUZ

HURBILKETA BIDEZKO EBAZPIDE : ANALITIKO ETA GRAFIKOAK Aplikagarritasuna $\tau > 0,2$

b.EBAZPIDE GRAFIKOA. HEISLER-EN GRAFIKOAK (3) Geometria bakoitzarekin lotuta



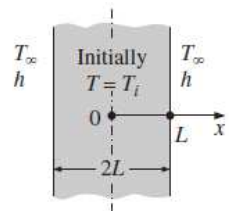
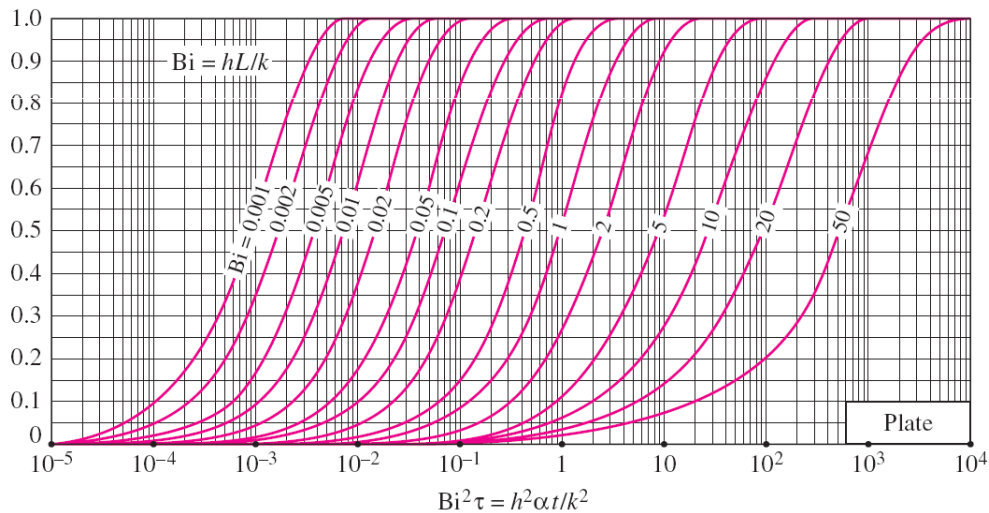
4.2 – BERO-EROAPEN IRAGANKORRA HORMA LAU HANDIETAN, 22/30 ZILINDRO LUZEETAN ETA ESFERETAN, EFEKTU ESPAZIALAK KONTUAN HARTUZ

HURBILKETA BIDEZKO EBAZPIDE : ANALITIKO ETA GRAFIKOAK Aplikagarritasuna $\tau > 0,2$

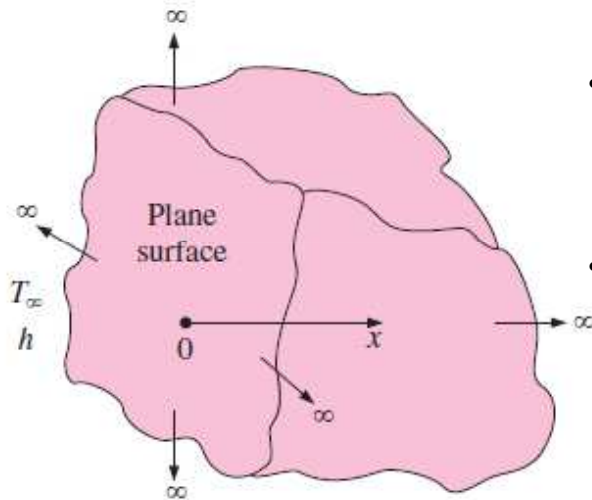
b.EBAZPIDE GRAFIKOA. HEISLER-EN GRAFIKOAK (3) Geometria bakoitzarekin lotuta

b.3. t aldiunera arteko **bero-transferentziaren** kantitate totala kalkulatzeko

$$\frac{Q}{Q_{\max}} = Q_{\max} = mc_p (T_{\infty} - T_i) = \rho V C_p (T_{\infty} - T_i) \quad (\text{kJ})$$



GORPUTZ ERDIINFINITU BATEN ESKEMA



• Ekuazio diferentziala $\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha} \frac{\partial T}{\partial t}$

$T(0, t) = T_s$

• Mugalde-baldintzak

$T(x \rightarrow \infty, t) = T_i$

• Hasierako baldintzak $T(x, 0) = T_i$

BERO-EROAPEN IRAGANKORRA SOLIDO ERDIINFINITUETAKO EBAZPIDEA

Antzekotasun-Aldagai Metodoa

x eta t aldagai independenteak η aldagai bakarrean konbinatzen ditu

$$\eta = \frac{x}{\sqrt{4 \cdot \alpha \cdot t}}$$

▪ $T = T(\eta)$ suposatuz eta y kate-erregela erabiliz:

$$\alpha = k/\rho c_p$$

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha} \frac{\partial T}{\partial t} \quad \text{eta} \quad \eta = \frac{x}{\sqrt{4\alpha t}}$$

$$\frac{\partial T}{\partial t} = \frac{dT}{d\eta} \frac{\partial \eta}{\partial t} = \frac{x}{2t\sqrt{4\alpha t}} \frac{dT}{d\eta}$$

$$\frac{\partial T}{\partial x} = \frac{dT}{d\eta} \frac{\partial \eta}{\partial x} = \frac{1}{\sqrt{4\alpha t}} \frac{dT}{d\eta}$$

$$\frac{\partial^2 T}{\partial x^2} = \frac{d}{d\eta} \left(\frac{\partial T}{\partial x} \right) \frac{\partial \eta}{\partial x} = \frac{1}{4\alpha t} \frac{d^2 T}{d\eta^2}$$

• Ekuazio Diferentziala $\frac{d^2 T}{d\eta^2} = -2\eta \frac{dT}{d\eta}$

$T(0) = T_s$

• Mugalde-baldintzak

$T(\eta \rightarrow \infty) = T_i$

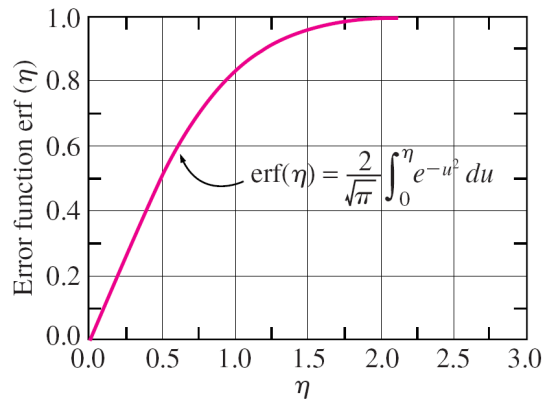
Ekuazio eraldatua nahiz mugalde-baldintzak, η -ren menpekoak baino ez dira, eta x-rekiko nahiz t-rekiko independenteak dira.

BERO-EROAPEN IRAGANKORRA SOLIDO ERDIINFINITUETAKO EBAZPIDEA

Temperatura-ren aldaketa

$$\frac{T - T_s}{T_i - T_s} = \frac{2}{\sqrt{\pi}} \int_0^\eta e^{-u^2} du = erf(\eta) = 1 - erfc(\eta)$$

Gauss-en ekuazioa Errore funtzioa: numerikoki ebaluatzen da



$$\eta = \frac{x}{\sqrt{4\alpha t}}$$

BERO-EROAPEN IRAGANKORRA SOLIDO ERDIINFINITUETAKO EBAZPIDEA

Temperatura-banaketa jakinda, gainazaleko bero-fluxua Fourierren legearekin

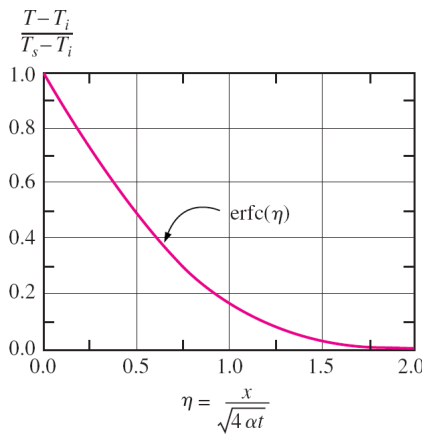
$$\dot{q}_s(t) = -k \frac{\partial T}{\partial x} \Big|_{x=0} = -k \frac{dT}{d\eta} \frac{\partial \eta}{\partial x} \Big|_{\eta=0} = -k \cdot C_1 \cdot e^{-\eta^2} \frac{1}{\sqrt{4 \cdot \alpha \cdot t}} \Big|_{\eta=0} = \frac{k(T_s - T_i)}{\sqrt{\pi \cdot \alpha \cdot t}}$$

BERO-EROAPEN IRAGANKORRA SOLIDO ERDIINFINITUETAKO EBAZPIDEA BESTE MUGALDE BALDINTZETARAKO

1. **kasua** Gainazal-temperatura zehaztua, $T_s = \text{konstantea}$

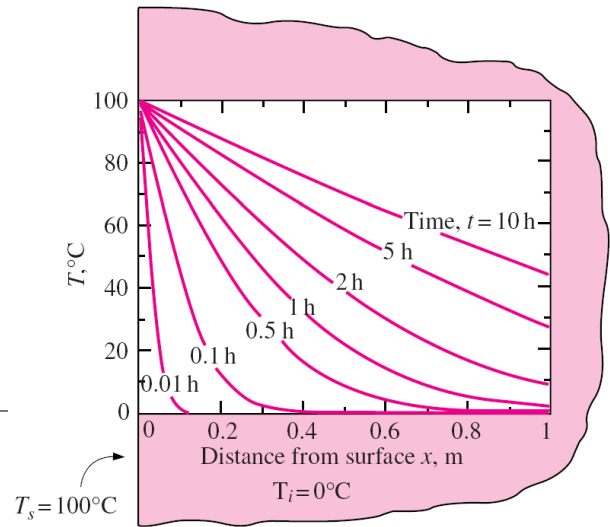
$$\frac{T(x,t) - T_i}{T_s - T_i} = \text{erfc}\left(\frac{x}{2\sqrt{\alpha t}}\right)$$

$$\dot{q}_s(t) = \frac{k(T_s - T_i)}{\sqrt{\pi\alpha t}}$$



4-4 Taula

η	$\text{erfc}(\eta)$
0.00	1.00000
0.02	0.9774
0.04	0.9549
0.06	0.9324
0.08	0.9100



(a) Specified surface temperature, $T_s = \text{constant}$.

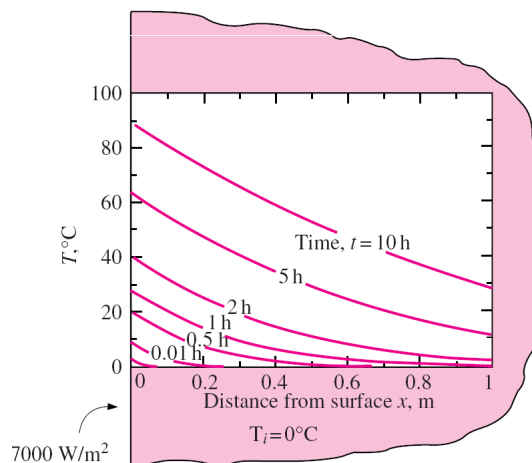
TERMOTEKNIA



BERO-EROAPEN IRAGANKORRA SOLIDO ERDIINFINITUETAKO EBAZPIDEA BESTE MUGALDE BALDINTZETARAKO

2. **Kasua** Gainazaleko bero-fluxu zehaztua $q = \text{cte}$

$$T(x,t) - T_i = \frac{\dot{q}_s}{k} \left[\sqrt{\frac{4\alpha t}{\pi}} \exp\left(-\frac{x^2}{4\alpha t}\right) - x \text{erfc}\left(\frac{x}{2\sqrt{\alpha t}}\right) \right]$$



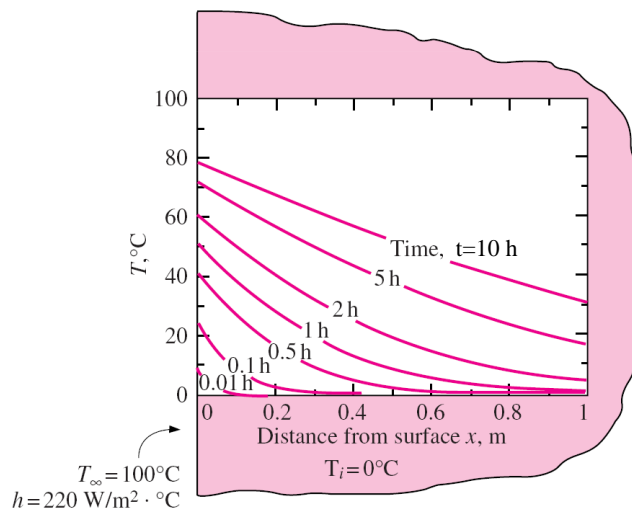
(b) Specified surface heat flux, $\dot{q}_s = \text{constant}$.

BERO-EROAPEN IRAGANKORRA SOLIDO ERDIINFINITUETAKO EBAZPIDEA BESTE MUGALDE BALDINTZETARAKO

3. Kasua Konbekzioa gainazalean

$$\dot{q}_s(t) = h[T_\infty - T(0, t)]$$

$$\frac{T(x, t) - T_i}{T_\infty - T_i} = \operatorname{erfc}\left(\frac{x}{2\sqrt{\alpha t}}\right) - \exp\left(\frac{hx}{k} + \frac{h^2 \alpha t}{k^2}\right) \operatorname{erfc}\left(\frac{x}{2\sqrt{\alpha t}} + \frac{h\sqrt{\alpha t}}{k}\right)$$

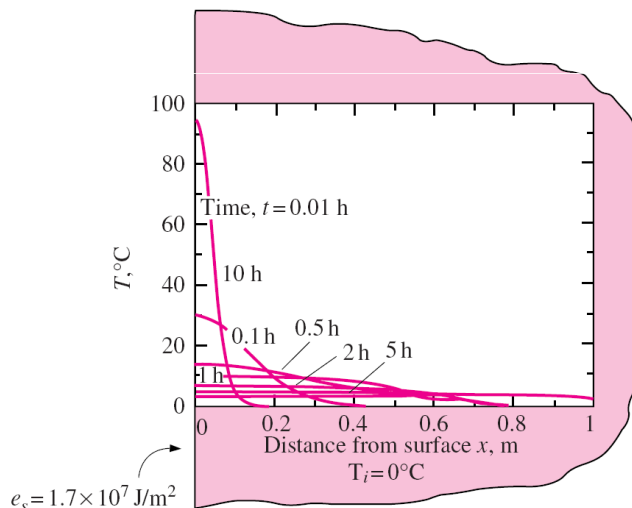


(c) Convection at the surface

BERO-EROAPEN IRAGANKORRA SOLIDO ERDIINFINITUETAKO EBAZPIDEA BESTE MUGALDE BALDINTZETARAKO

4. Kasua Energia-pultsua gainazalean, $e_s = kte$

$$T(x, t) - T_i = \frac{e_s}{k\sqrt{\pi t / \alpha}} \exp\left(-\frac{x^2}{4\alpha t}\right)$$



(d) Energy pulse at the surface, $e_s = \text{constant}$

- 4.3ko azpiatala: BI SOLIDO ERDIINFITUREN ARTEKO KONTAKTUA
- 4.4 atala: BERO-EROAPEN IRAGANKORRA SISTEMA MULTIDIMENTSIONALETAN

5. GAIA

ZENBAKIZKO METODOAK BERO-EROAPENEAN

5.0 - HELBURUAK

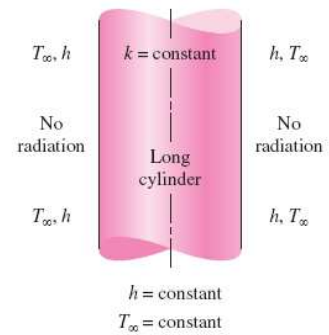
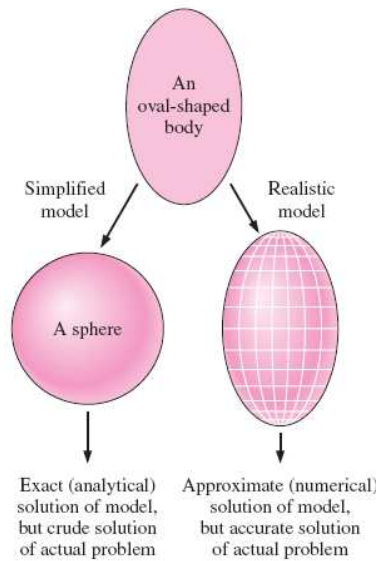
- Eroapen-problemen ebazpen analitikoan mugak eta ordenagailu bidezko **zenbakizko metodo** intentsiboen **beharra** ulertu.
- Deribatuak diferentzia gisa adierazi eta **diferentzia finituko formulazioak** lortu.
- Dimentsio bakarreko edo biko eroapen geldikorreko **problema numerikoki ebaztea** diferentzia finituen metodoa baliatuta.
- Dimentsio bakarreko edo biko eroapen **iragankorreko problema** ebaztea diferentzia finituen metodoa baliatuta.

Ekuazio diferentziala
+
Mugaldeko baldintzak



SOLUZIO ANALITIKOA

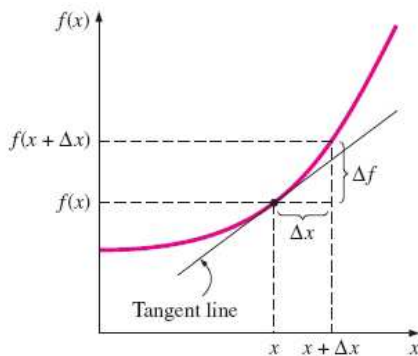
- Mugak
- Eredu hobeak egitea
- Malgutasuna
- Arazoak
- Giza izaera



5.2 – EKUAZIO DIFERENTZIALEN DIFERENTZIA FINITUKO FORMULAZIOA

Ekuazio diferentziala → Ekuazio algebraikoa

Deribatuak → Diferentziak



$$\frac{df(x)}{dx} = \lim_{\Delta x \rightarrow 0} \frac{\Delta f}{\Delta x} = \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$\frac{df(x)}{dx} \cong \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

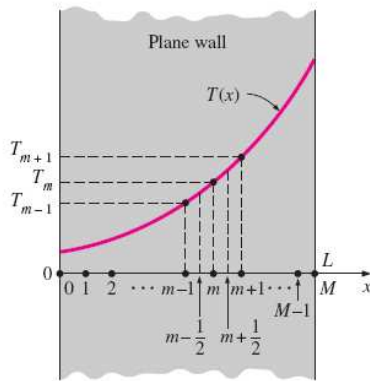
Aukera: f(x) funtzioaren Taylorren garapena

~~$$f(x + \Delta x) = f(x) + \Delta x \cdot \frac{df(x)}{dx} + \frac{1}{2} \Delta x^2 \cdot \frac{d^2 f(x)}{dx^2} + \dots$$~~

Errorea Δx-ekiko proportzional

5.2 – EKUAZIO DIFERENTZIALEN DIFERENTZIA FINITUKO FORMULAZIOA

5/22



$$\left. \frac{dT}{dx} \right|_{m-\frac{1}{2}} \cong \frac{T_m - T_{m-1}}{\Delta x}$$

$$\left. \frac{dT}{dx} \right|_{m+\frac{1}{2}} \cong \frac{T_{m+1} - T_m}{\Delta x}$$

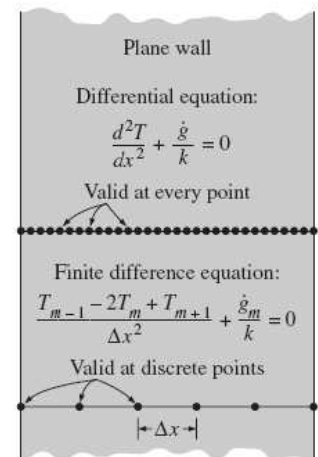
$$\left. \frac{d^2T}{dx^2} \right|_m \cong \frac{T_{m-1} - 2T_m + T_{m+1}}{\Delta x^2}$$

$$\frac{d^2T}{dx^2} + \frac{\dot{e}}{k} = 0 \quad \Rightarrow \quad \frac{T_{m-1} - 2T_m + T_{m+1}}{\Delta x^2} + \frac{\dot{e}_m}{k} = 0$$

$$m = 1, 2, 3, \dots, M-1$$

T_0 eta T_M ezagunak: $M - 1$ ekuazio $M - 1$ ezezagun

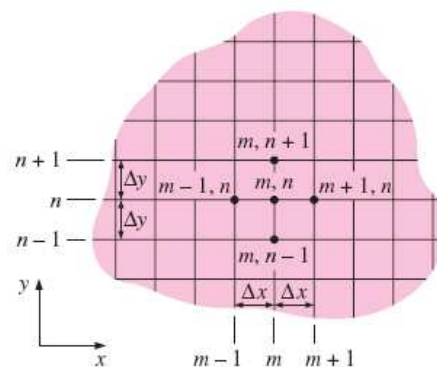
T_0 eta T_M ezezagunak: $\left\{ \begin{array}{l} M - 1 \text{ ekuazio} \\ + \\ 2 \text{ mugaldeko baldintza} \end{array} \right\}$ $M + 1$ ezezagun



5.2 – EKUAZIO DIFERENTZIALEN DIFERENTZIA FINITUKO FORMULAZIOA

6/22

Bi dimentsiotan:



$$\frac{T_{m+1,n} - 2T_{m,n} + T_{m-1,n}}{\Delta x^2} + \frac{T_{m,n+1} - 2T_{m,n} + T_{m,n-1}}{\Delta y^2} + \frac{\dot{e}_{m,n}}{k} = 0$$

$$m = 1, 2, 3, \dots, M - 1$$

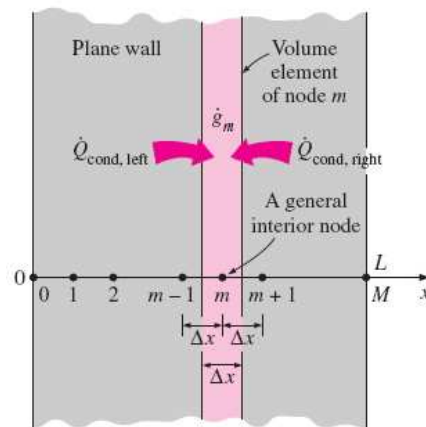
$$n = 1, 2, 3, \dots, N - 1$$

Energia-balantzearen metodoa

$\Delta x = L/M$ lodierako M eskualde

Posizioa: $x_m = m \cdot \Delta x$

Temperatura: $T(x_m) = T_m$



$$\dot{Q}_{\text{cond,left}} + \dot{Q}_{\text{cond,right}} + \dot{E}_{\text{gen,element}} = \frac{\Delta E_{\text{element}}}{\Delta t} = 0$$

$$\dot{E}_{\text{gen,element}} = \dot{e}_m \cdot V_{\text{element}} = \dot{e}_m \cdot A \cdot \Delta x$$

$$\dot{Q}_{\text{cond,left}} = k \cdot A \cdot \frac{T_{m-1} - T_m}{\Delta x}$$

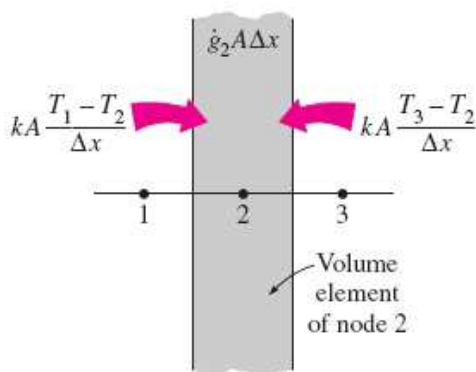
$$\dot{Q}_{\text{cond,right}} = k \cdot A \cdot \frac{T_{m+1} - T_m}{\Delta x}$$

$$k \cdot A \cdot \frac{T_{m-1} - T_m}{\Delta x} + k \cdot A \cdot \frac{T_{m+1} - T_m}{\Delta x} + \dot{e}_m \cdot A \cdot \Delta x = 0$$

$$\frac{T_{m-1} - 2 \cdot T_m + T_{m+1}}{\Delta x^2} + \frac{\dot{e}_m}{k} = 0 \quad m = 1, 2, 3, \dots, M-1$$

5.3 – DIMENTSIO BAKARREKO BERO-EROAPEN GELDIKORRA

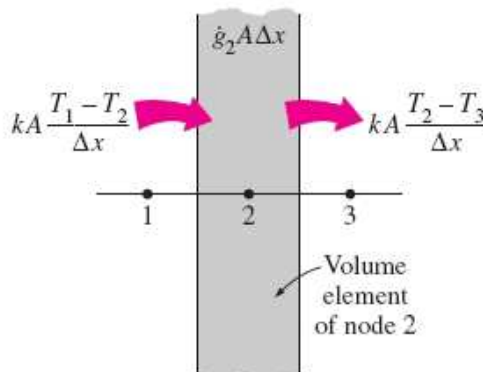
Bolumen elementuaren gainazalean eroapenerako suposatutako norantzak ez du formulazioan eraginik.



$$kA \frac{T_1 - T_2}{\Delta x} + kA \frac{T_3 - T_2}{\Delta x} + \dot{g}_2 A \Delta x = 0$$

or

$$T_1 - 2T_2 + T_3 + \dot{g}_2 \Delta x^2 / k = 0$$



$$kA \frac{T_1 - T_2}{\Delta x} - kA \frac{T_2 - T_3}{\Delta x} + \dot{g}_2 A \Delta x = 0$$

or

$$T_1 - 2T_2 + T_3 + \dot{g}_2 \Delta x^2 / k = 0$$

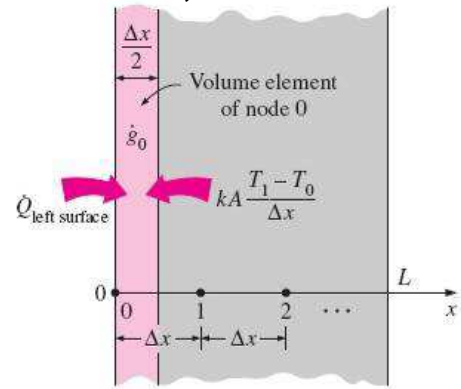
MUGALDEKO BALDINTZAK

0 nodoa $\rightarrow x = 0$ posizioa
 M nodoa $\rightarrow x = L$ posizioa } Zabalera $\Delta x/2$

Temperatura zehaztua $\left\{ \begin{array}{l} T(0) = T_0 \\ T(L) = T_M \end{array} \right.$

Konbekzio, erradiazio edo konbekzio eta erradiazio konbinatuen, bero-fluxu zehaztua

$$\dot{Q}_{\text{left surface}} + k \cdot A \cdot \frac{T_1 - T_0}{\Delta x} + \dot{e}_0 \cdot A \cdot \frac{\Delta x}{2} = 0$$

**MUGALDEKO BALDINTZAK**

Bero-fluxu zehaztua

$$\dot{q}_0 \cdot A + k \cdot A \cdot \frac{T_1 - T_0}{\Delta x} + \dot{e}_0 \cdot A \cdot \frac{\Delta x}{2} = 0$$

Kasu partikularra: Mugalde isolatua

$$k \cdot A \cdot \frac{T_1 - T_0}{\Delta x} + \dot{e}_0 \cdot A \cdot \frac{\Delta x}{2} = 0$$

Konbekzio mugaldea

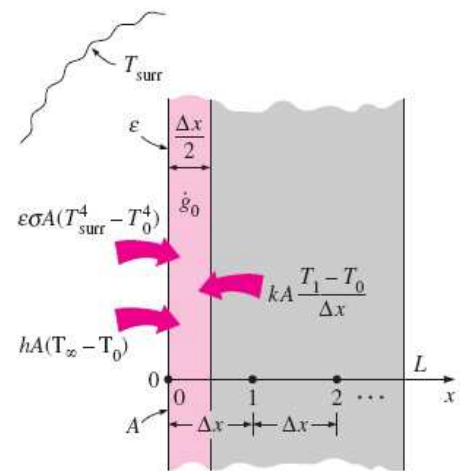
$$h \cdot A \cdot (T_\infty - T_0) + k \cdot A \cdot \frac{T_1 - T_0}{\Delta x} + \dot{e}_0 \cdot A \cdot \frac{\Delta x}{2} = 0$$

Erradiazio mugaldea

$$\varepsilon \cdot \sigma \cdot A \cdot (T_{\text{surr}}^4 - T_0^4) + k \cdot A \cdot \frac{T_1 - T_0}{\Delta x} + \dot{e}_0 \cdot A \cdot \frac{\Delta x}{2} = 0$$

MUGALDEKO BALDINTZAK

Konbektzio eta erradiazio konbinatuaren mugaldea

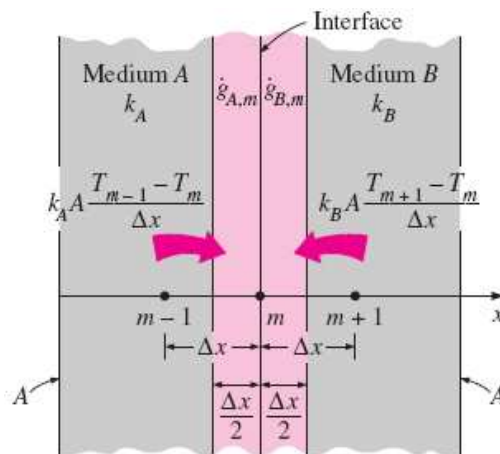


$$h \cdot A \cdot (T_{\infty} - T_0) + \varepsilon \cdot \sigma \cdot A \cdot (T_{surr}^4 - T_0^4) + k \cdot A \cdot \frac{T_1 - T_0}{\Delta x} + \dot{e}_0 \cdot A \cdot \frac{\Delta x}{2} = 0$$

$$h_{combined} \cdot A \cdot (T_{\infty} - T_0) + k \cdot A \cdot \frac{T_1 - T_0}{\Delta x} + \dot{e}_0 \cdot A \cdot \frac{\Delta x}{2} = 0$$

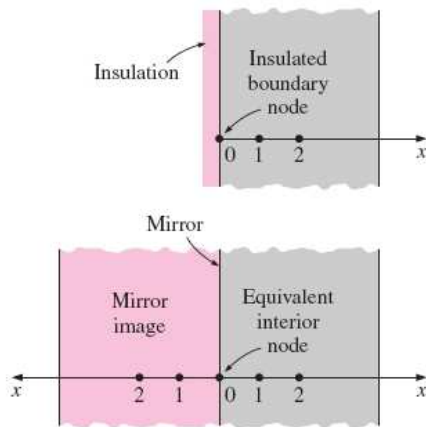
MUGALDEKO BALDINTZAK

Fasearteko mugaldea



$$k_A \cdot A \cdot \frac{T_{m-1} - T_m}{\Delta x} + k_B \cdot A \cdot \frac{T_{m+1} - T_m}{\Delta x} + \dot{e}_{A,m} \cdot A \cdot \frac{\Delta x}{2} + \dot{e}_{B,m} \cdot A \cdot \frac{\Delta x}{2} = 0$$

MUGALDE-NODO ISOLATUAK BARNE-NODOTZAT HARTZEA: ISPILU-IRUDIAREN KONTZEPTUA



$$\frac{T_{m-1} - 2 \cdot T_m + T_{m+1}}{\Delta x^2} + \frac{\dot{e}_m}{k} = 0$$

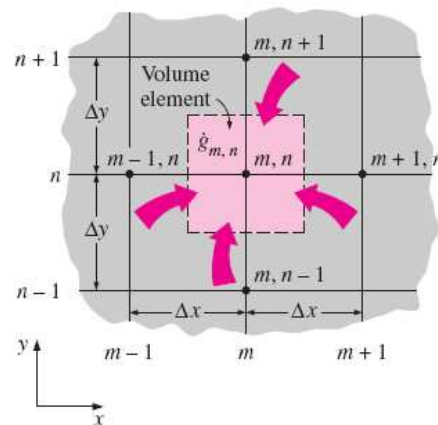
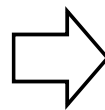
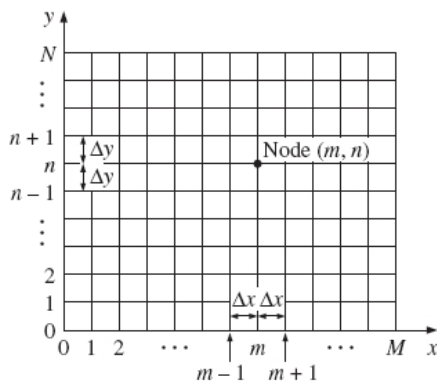


$$\frac{T_1 - 2 \cdot T_0 + T_1}{\Delta x^2} + \frac{\dot{e}_0}{k} = 0$$

Energia-balantzearen metodoaren bidez mugalde isolatuaren kasurako aztertutako adierazpen berdina.

N temperatura nodoetan N ekuazio {
 Ezabapen-metodoa
 Zuzeneko metodoa
 Iterazio-metodoa

5.4 – BI DIMENTSIOKO BERO-EROAPEN GELDIKORRA



$$\left(\text{Bero-eroapenaren abiadura ezker-, goi-, eskuin- eta behe-gainazaletan} \right) + \left(\text{Bero-sorreraren abiadura elementu barruan} \right) = \left(\text{Elementuaren energia-edukiaren aldaketaren abiadura} \right)$$

$$\dot{Q}_{\text{cond,left}} + \dot{Q}_{\text{cond,top}} + \dot{Q}_{\text{cond,right}} + \dot{Q}_{\text{cond,bottom}} + \dot{E}_{\text{gen,element}} = \frac{\Delta E_{\text{element}}}{\Delta t} = 0$$

$$k \cdot \Delta y \cdot \frac{T_{m-1,n} - T_{m,n}}{\Delta x} + k \cdot \Delta x \cdot \frac{T_{m,n+1} - T_{m,n}}{\Delta y} + k \cdot \Delta y \cdot \frac{T_{m+1,n} - T_{m,n}}{\Delta x} + k \cdot \Delta x \cdot \frac{T_{m,n-1} - T_{m,n}}{\Delta y} + \dot{e}_{m,n} \cdot \Delta x \cdot \Delta y = 0$$

$$\frac{T_{m-1,n} - 2 \cdot T_{m,n} + T_{m+1,n}}{\Delta x^2} + \frac{T_{m,n-1} - 2 \cdot T_{m,n} + T_{m,n+1}}{\Delta y^2} + \frac{\dot{e}_{m,n}}{k} = 0 \quad \begin{matrix} m = 1, 2, 3, \dots, M-1 \\ n = 1, 2, 3, \dots, N-1 \end{matrix}$$

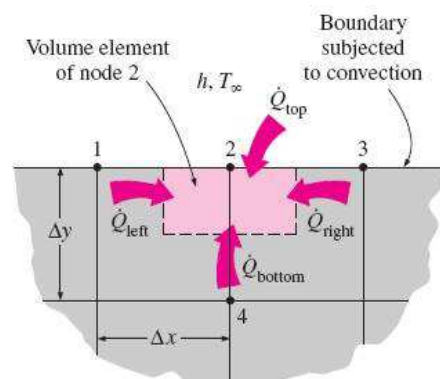
Sarea karratu itxura badauka: $\Delta x = \Delta y = l$

$$T_{m-1,n} + T_{m+1,n} + T_{m,n+1} + T_{m,n-1} - 4 \cdot T_{m,n} + \frac{\dot{e}_{m,n} \cdot l^2}{k} = 0$$

$$T_{left} + T_{top} + T_{right} + T_{bottom} - 4 \cdot T_{node} + \frac{\dot{e}_{node} \cdot l^2}{k} = 0$$

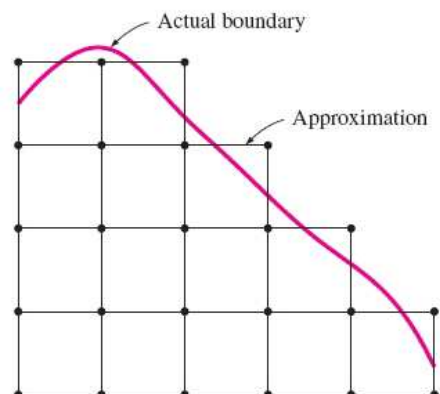
$$T_{node} = \frac{T_{left} + T_{top} + T_{right} + T_{bottom}}{4}$$

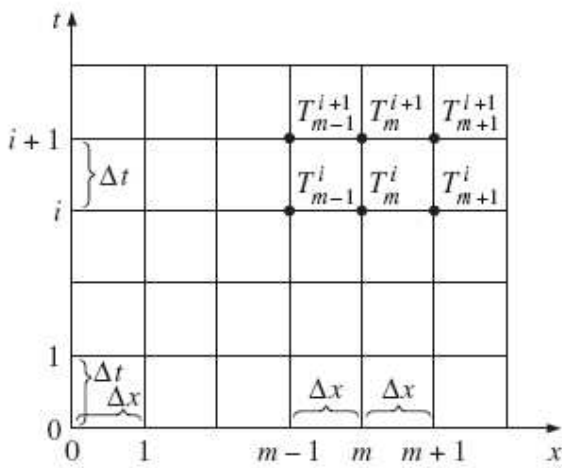
Mugalde-nodoak



5.4 TEST ARIKETA

Mugalde irregularrak





i = denbora-urratzen indizea edo zenbatzailea

$i = 0$; Hasierako baldintza

$t_i = i \cdot \Delta t$

T_m^i = m nodo eta i denbora-urratseko temperatura

$$\left(\text{Bolumen-elementuaren gainazal guztietatik barrurantz } \Delta t \text{ tartean transferitutako beroa} \right) + \left(\text{Bolumen-elementuaren barnean } \Delta t \text{ tartean sortutako beroa} \right) = \left(\text{Bolumen-elementuan } \Delta t \text{ tartean gertatutako energia-edukiaren aldaketa} \right)$$

$$\sum_{\text{All sides}} \dot{Q} + \dot{E}_{gen,element} = \frac{\Delta E_{element}}{\Delta t} = \rho \cdot V_{elem} \cdot c_p \frac{\Delta T}{\Delta t} = \rho \cdot V_{elem} \cdot c_p \frac{T_m^{i+1} - T_m^i}{\Delta t}$$

Metodo esplizitua:
$$\sum_{\text{All sides}} \dot{Q}^i + \dot{E}_{gen,element}^i = \rho \cdot V_{elem} \cdot c_p \frac{T_m^{i+1} - T_m^i}{\Delta t}$$



Ebazteko erreza.



Egonkortasuna mantentzeko Δt mugatuta.

Metodo inplizitua:
$$\sum_{\text{All sides}} \dot{Q}^{i+1} + \dot{E}_{gen,element}^{i+1} = \rho \cdot V_{elem} \cdot c_p \frac{T_m^{i+1} - T_m^i}{\Delta t}$$

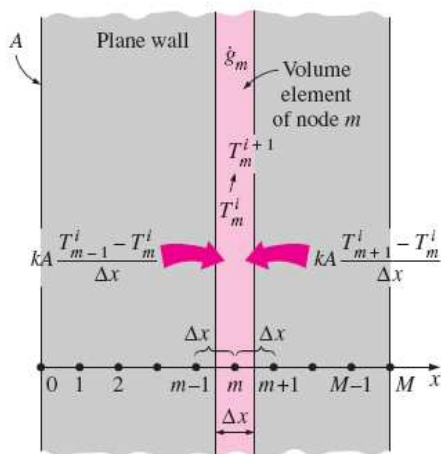


Δt ez dago mugatuta.



Nodo-temperatura guztiak aldi berean ebatzi behar dira.

Horma laua



$$k \cdot A \cdot \frac{T_{m-1}^i - T_m^i}{\Delta x} + k \cdot A \cdot \frac{T_{m+1}^i - T_m^i}{\Delta x} + \dot{e}_m \cdot A \cdot \Delta x = \rho \cdot A \cdot \Delta x \cdot c_p \frac{T_m^{i+1} - T_m^i}{\Delta t}$$

$$T_{m-1}^i - 2 \cdot T_m^i + T_{m+1}^i + \frac{\dot{e}_m \Delta x^2}{k} = \frac{\Delta x^2}{\alpha \cdot \Delta t} (T_m^{i+1} - T_m^i)$$

Egoera egonkorreko formulazioa



$$\tau = \frac{\alpha \cdot \Delta t}{\Delta x^2}$$

Horma laua

Metodo esplizitua:

$$T_{m-1}^i - 2 \cdot T_m^i + T_{m+1}^i + \frac{\dot{e}_m^i \Delta x^2}{k} = \frac{T_m^{i+1} - T_m^i}{\tau}$$

$$T_m^{i+1} = \tau \cdot (T_{m-1}^i + T_{m+1}^i) + (1 - 2\tau) \cdot T_m^i + \tau \frac{\dot{e}_m^i \Delta x^2}{k}$$

Egonkortasun-irizpidea

$$\tau = \frac{\alpha \cdot \Delta t}{\Delta x^2} \leq \frac{1}{2} \Rightarrow \Delta t \leq \frac{1}{2} \frac{\Delta x^2}{\alpha}$$

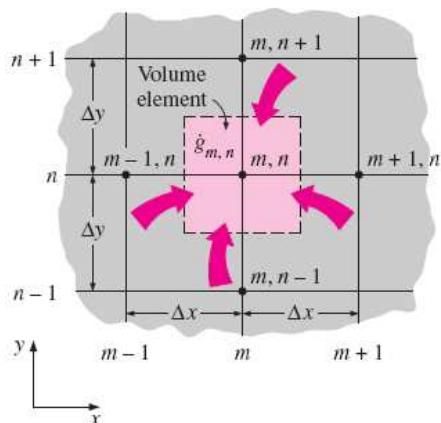
Metodo inplizitua:

$$T_{m-1}^{i+1} - 2 \cdot T_m^{i+1} + T_{m+1}^{i+1} + \frac{\dot{e}_m^{i+1} \Delta x^2}{k} = \frac{T_m^{i+1} - T_m^i}{\tau}$$

$$\tau \cdot T_{m-1}^{i+1} - (1 + 2\tau) \cdot T_m^{i+1} + \tau \cdot T_{m+1}^{i+1} + \tau \frac{\dot{e}_m^{i+1} \Delta x^2}{k} + T_m^i = 0$$

M-1 barne-nodoentzako formulazio baliogarria

Bi dimentsioko bero-eroapen iragankorra



$$k \cdot \Delta y \cdot \frac{T_{m-1,n} - T_{m,n}}{\Delta x} + k \cdot \Delta x \cdot \frac{T_{m,n+1} - T_{m,n}}{\Delta y} + k \cdot \Delta y \cdot \frac{T_{m+1,n} - T_{m,n}}{\Delta x} + k \cdot \Delta x \cdot \frac{T_{m,n-1} - T_{m,n}}{\Delta y} + \dot{e}_{m,n} \cdot \Delta x \cdot \Delta y = \rho \cdot \Delta x \cdot \Delta y \cdot c_p \cdot \frac{T_m^{i+1} - T_m^i}{\Delta t}$$

Sarea karratu itxura badu: $\Delta x = \Delta y = l$

$$T_{m-1,n} + T_{m+1,n} + T_{m,n+1} + T_{m,n-1} - 4 \cdot T_{m,n} + \frac{\dot{e}_{m,n} \cdot l^2}{k} = \frac{T_m^{i+1} - T_m^i}{\tau}$$

$$T_{left} + T_{top} + T_{right} + T_{bottom} - 4 \cdot T_{node} + \frac{\dot{e}_{node} \cdot l^2}{k} = \frac{T_{node}^{i+1} - T_{node}^i}{\tau}$$

Egoera egonkorreko formulazioa

Bi dimentsioko bero-eroapen iragankorra

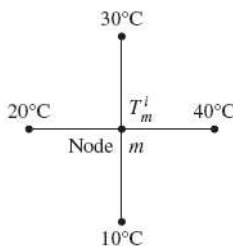
Metodo esplizitua

$$T_{left}^i + T_{top}^i + T_{right}^i + T_{bottom}^i - 4 \cdot T_{node}^i + \frac{\dot{e}_{node}^i \cdot l^2}{k} = \frac{T_{node}^{i+1} - T_{node}^i}{\tau}$$

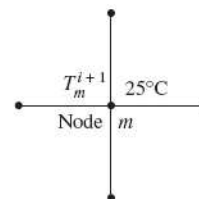
$$T_{node}^{i+1} = \tau \cdot (T_{left}^i + T_{top}^i + T_{right}^i + T_{bottom}^i) + (1 - 4 \cdot \tau) \cdot T_{node}^i + \tau \cdot \frac{\dot{e}_{node}^i \cdot l^2}{k}$$

$$T_{node}^{i+1} = \frac{T_{left}^i + T_{top}^i + T_{right}^i + T_{bottom}^i}{4}$$

Time step i:



Time step i + 1:



Egonkortasun-irizpidea

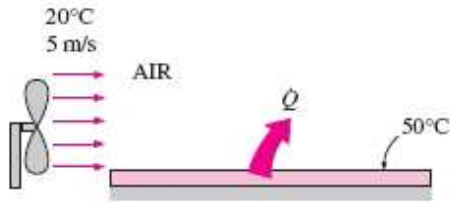
$$\tau = \frac{\alpha \cdot \Delta t}{l^2} \leq \frac{1}{4}$$

6. GAIA

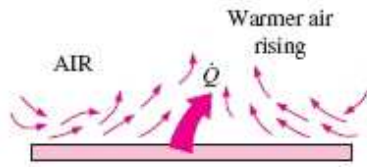
KONBEKZIOAREN OINARRIAK

6.0 - HELBURUAK

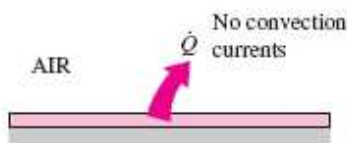
- **Konbekzioaren** mekanismo fisikoak eta sailkapena ulertu.
- Gainazalen gaineko fluxuen **abiaduraren mugalde-geruza** eta **mugalde-geruza termikoa** irudikatu.
- Reynoldsen, Prandtlen eta Nusselten **zenbaki dimentsiogabeen** ezagutza praktikoa izan.
- Fluxu **laminarrak** eta **turbulentuak** bereizi, eta fluxu turbulentuen momentu- eta bero-transferentziako mekanismoak ulertu
- Konbekzioa deskribatzen duten **ekuazio diferentzialak garatu**, masa-, momentu- eta energia-balantzeetan oinarrituta
- Konbekzio-ekuazioak **dimentsiogabetu**, eta marruskaduraren eta bero-transferentziaren koefizienteen forma **funtzionalak** lortu.
- Momentu- eta bero-transferentziaren arteko **analogiak** erabili, eta bero-transferentziaren koefizientea kalkulatu, marruskadura-koefizientetik abiatuta



(a) Forced convection



(b) Free convection



(c) Conduction

Konbekzio bero-transferentzia hurrengo parametroen araberakoa da:

Biskositate dinamikoa μ_{fluid}

Eroankortasun termikoa k_{fluid}

Dentsitatea ρ_{fluid}

Bero espezifikoa $C_{p,fluid}$

Jariakinaren abiadura V_{fluid}

Konfigurazio geometrikoa

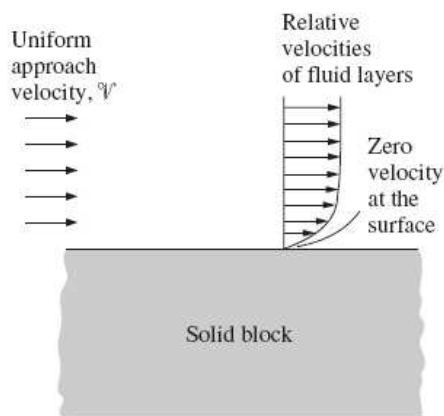
Solidoaren zimurtasuna

Jariakinaren fluxu mota (laminarra edo turbulentua)

Newtonen hozte legea

$$\dot{Q} = h \cdot A \cdot (T_s - T_\infty) \quad [W]$$

TERMOTEKNIA



Irristadurarik ezaren baldintza

Mugalde geruza

$$\dot{q}_{cond} = -k_{fluid} \cdot \left. \frac{\partial T}{\partial y} \right|_{y=0} \quad [W/m^2]$$

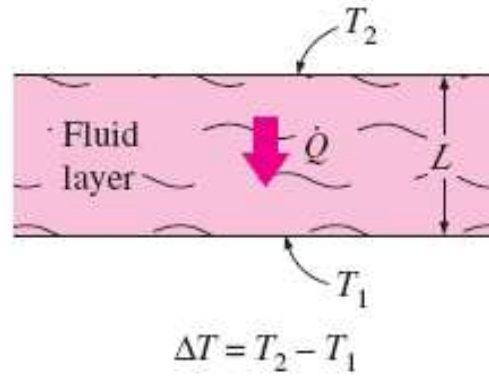
$$\dot{q}_{conv} = h \cdot (T_s - T_\infty) \quad [W/m^2]$$

$$h = \frac{-k_{fluid} \cdot (\partial T / \partial y)_{y=0}}{T_s - T_\infty} \quad [W/m^2 \cdot ^\circ C]$$

Bataz besteko konbekzio koefizientea – Konbekzio koefiziente lokala

NUSSELTEN ZENBAKIA

$$Nu = \frac{h \cdot L_c}{k}$$



$$\left. \begin{aligned} \dot{q}_{conv} &= h \cdot \Delta T \\ \dot{q}_{cond} &= k \cdot \frac{\Delta T}{L} \end{aligned} \right\} \Rightarrow \frac{\dot{q}_{conv}}{\dot{q}_{cond}} = \frac{h \cdot \Delta T}{k \cdot \Delta T / L} = \frac{h \cdot L}{k} = Nu$$

6.2 – FLUIDO-FLUXUEN SAILKAPENA

Fluxu likatsua vs ez-likatsua.

Barne-fluxua vs kanpo-fluxua.

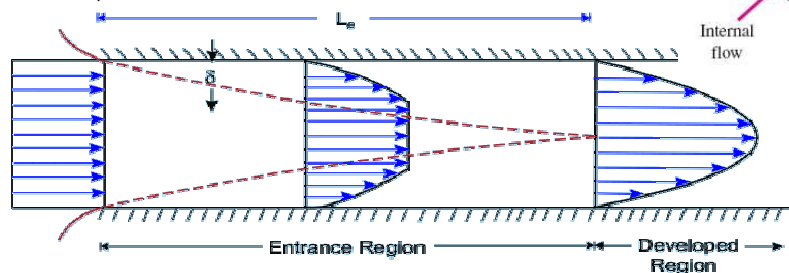
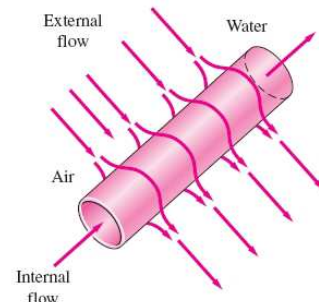
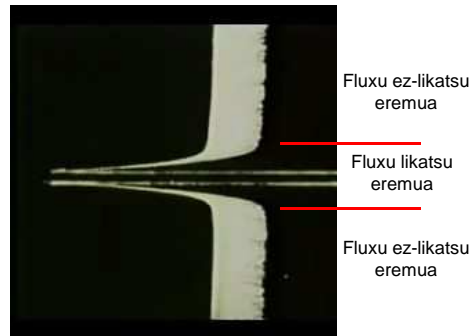
Fluxu konprimagarria vs konprimaezina.

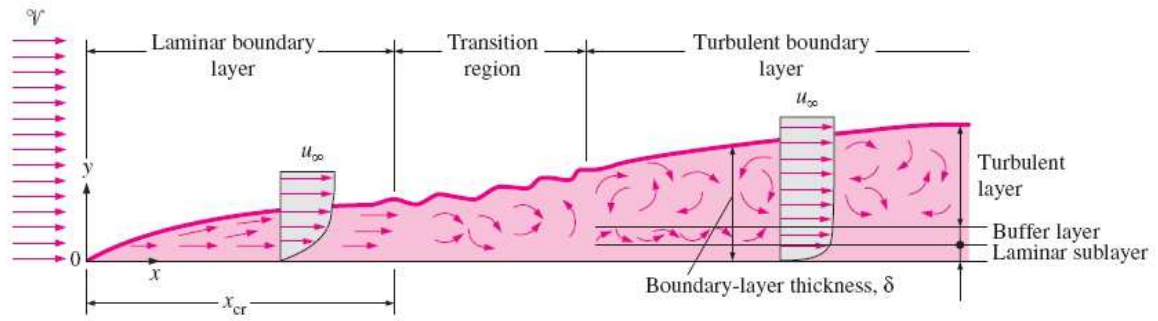
Fluxu laminarra vs turbulenta.

Fluxu naturala vs behartua.

Fluxu geldikorra vs ez-geldikorra (iragankorra).

Dimensio bakarreko, biko eta hiruko fluxuak.



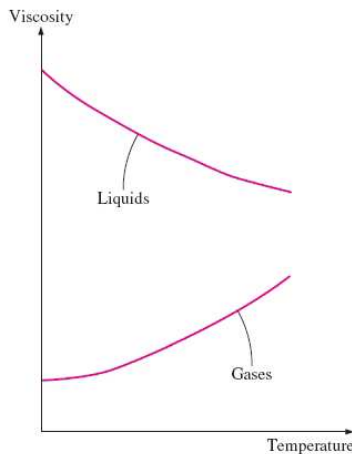


GAINAZALEKO EBAKIDURA-TENTSIOA $\tau = \mu \frac{\partial u}{\partial y} \Big|_{y=0}$ [N/m²] Jariakin
newtondarrak

Biskositate dinamikoa: μ [kg/m·s] [poise] 1 poise = 0,1 kg /m·s

Biskositate zinematikoa: $\nu = \frac{\mu}{\rho}$ [m²/s] [stoke] 1 stoke = 10⁻⁴ m²/s

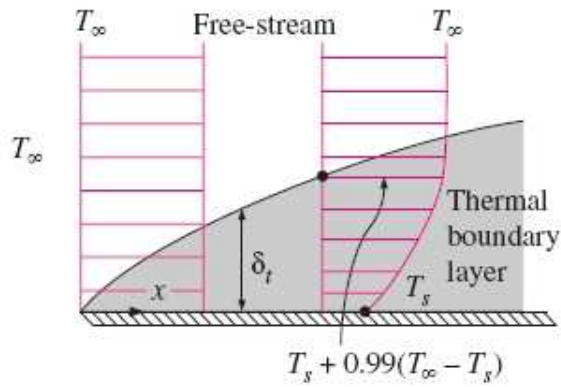
GAINAZALEKO EBAKIDURA-TENTSIOA



Fluid	Dynamic viscosity μ , kg/m · s
Glycerin:	
-20°C	134.0
0°C	12.1
20°C	1.49
40°C	0.27
Engine oil:	
SAE 10W	0.10
SAE 10W30	0.17
SAE 30	0.29
SAE 50	0.86
Mercury	0.0015
Ethyl alcohol	0.0012
Water:	
0°C	0.0018
20°C	0.0010
100°C (liquid)	0.0003
100°C (vapor)	0.000013
Blood, 37°C	0.0004
Gasoline	0.00029
Ammonia	0.00022
Air	0.000018
Hydrogen, 0°C	0.000009

$\tau_s = C_f \frac{\rho \cdot V^2}{2}$

C_f ↙ Marrusadura-koefizientea



Typical ranges of Prandtl numbers for common fluids

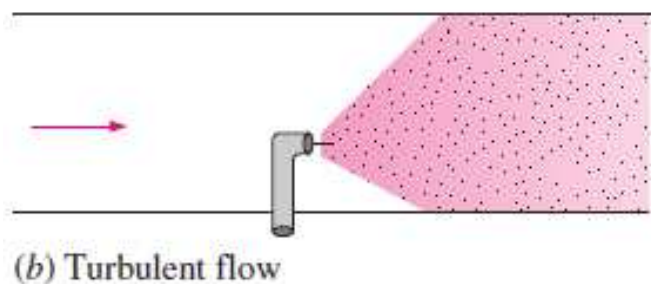
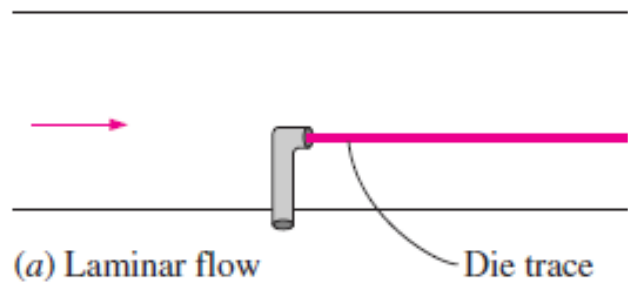
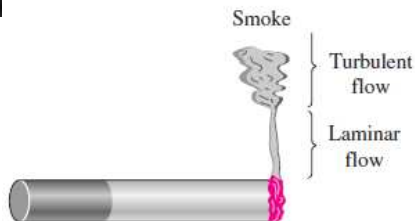
Fluid	Pr
Liquid metals	0.004–0.030
Gases	0.7–1.0
Water	1.7–13.7
Light organic fluids	5–50
Oils	50–100,000
Glycerin	2000–100,000

PRANDTLEN ZENBAKIA

$$Pr = \frac{\nu}{\alpha} = \frac{\mu \cdot c_p}{k}$$

$$Pr = \frac{\text{Momentuaren difusibitate molekularra}}{\text{Beroaren difusibitate molekularra}}$$

6.5 – FLUXU LAMINARRAK ETA TURBULENTOAK

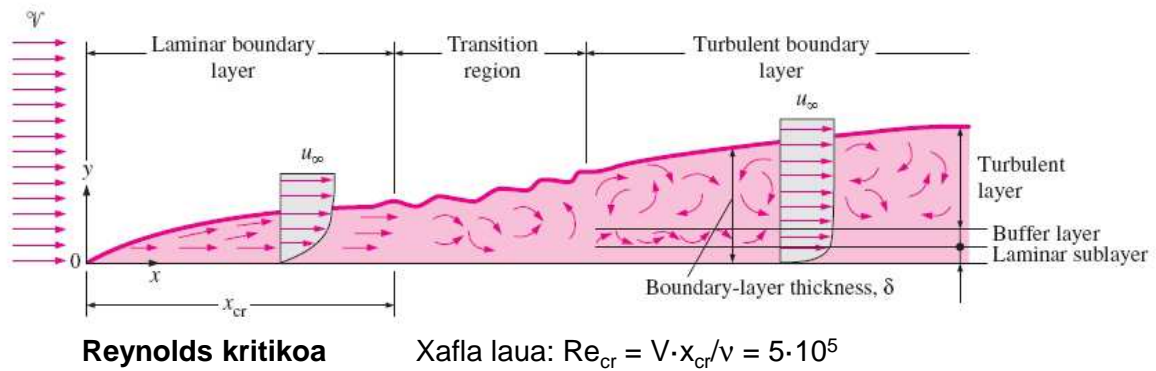


REYNOLDSSEN ZENBAKIA

$$Re = \frac{V \cdot L_c}{\nu} = \frac{\rho \cdot V \cdot L_c}{\mu}$$

$Re = \frac{\text{Inertzi indarrak}}{\text{Biskosite - indarrak}}$

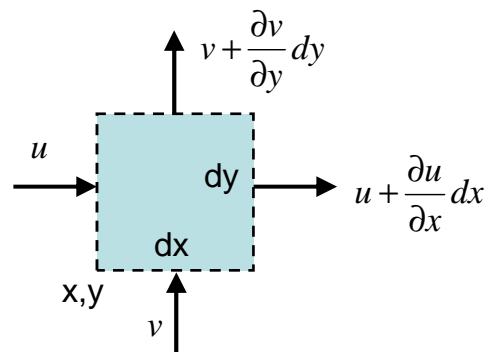
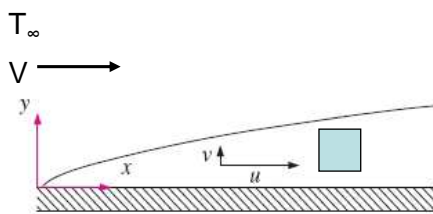
Re baxuak → Fluxu laminarra
 Re altuak → Fluxu turbulenta



6.7 – KONBEKZIO-EKUAZIO DIFERENTZIALEN GARAPENA

Hipotesiak: Fluxu geldikorra bi dimentsiotan
 Jariakin Newtondarra
 Propietateak konstante (ρ, ν, k, \dots)

JARRAITUTASUN-EKUAZIOA

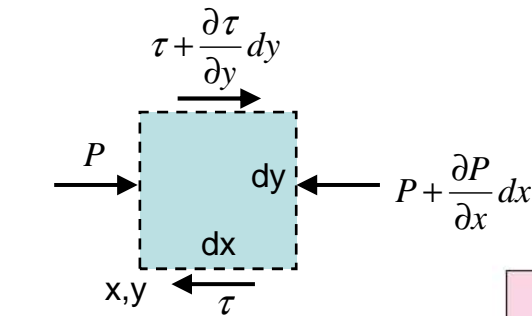


$\left[\text{Masa-emaria kontrol-bolumenaren barrurantz} \right] = \left[\text{Masa-emaria kontrol-bolumenaren kanporantz} \right]$

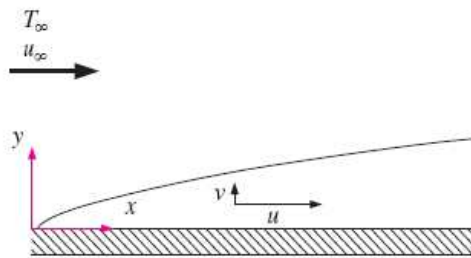
$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

MOMENTU-EKUAZIOAK

$$\left(\text{Masa} \right) \left(\begin{array}{c} \text{Azelerazioa, zehaztutako} \\ \text{noranzkoan} \end{array} \right) = \left(\begin{array}{c} \text{Noranzko horretan eragiten duen indar} \\ \text{garbia (gorputzekoa eta gainazalekoa)} \end{array} \right)$$



$$\rho \left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = \mu \frac{\partial^2 u}{\partial y^2} - \frac{\partial P}{\partial x}$$



- 1) Velocity components:
 $v \ll u$
- 2) Velocity gradients:
 $\frac{\partial v}{\partial x} \approx 0, \frac{\partial v}{\partial y} \approx 0$
 $\frac{\partial u}{\partial x} \ll \frac{\partial u}{\partial y}$
- 3) Temperature gradients:
 $\frac{\partial T}{\partial x} \ll \frac{\partial T}{\partial y}$

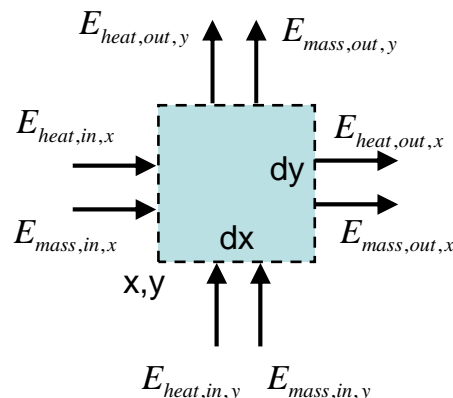
$$\frac{\partial P}{\partial y} = 0$$

ENERGIA-KONTSERBAZIOAREN EKUAZIOA

$$E_{in} - E_{out} = \Delta E_{sist} \xrightarrow{\text{Egoera egonkorrean:}} \dot{E}_{in} - \dot{E}_{out} = 0$$

$$\left(\dot{E}_{in} - \dot{E}_{out} \right)_{\text{by heat}} + \left(\dot{E}_{in} - \dot{E}_{out} \right)_{\text{by work}} + \left(\dot{E}_{in} - \dot{E}_{out} \right)_{\text{by mass}} = 0$$

$$e_{stream} = h + e_c + e_p = h + \frac{1}{2} V^2 + g \cdot z \approx h = c_p \cdot T$$



ENERGIA-KONTSERBAZIOAREN EKUAZIOA

$$\left(\dot{E}_{in} - \dot{E}_{out}\right)_{\text{by mass}} = -\rho \cdot c_p \cdot \left(u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y}\right) dx \cdot dy$$

$$\left(\dot{E}_{in} - \dot{E}_{out}\right)_{\text{by heat}} = k \cdot \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) dx \cdot dy$$

$$\left(\dot{E}_{in} - \dot{E}_{out}\right)_{\text{by work}} = \mu \cdot \Phi \cdot dx \cdot dy = \mu \cdot \left(\frac{\partial u}{\partial y}\right)^2 \cdot dx \cdot dy \quad \text{Abiadura baxuetan mesprezagarria}$$

$$\dot{E}_{in} - \dot{E}_{out} = 0 \quad \rightarrow \quad \rho \cdot c_p \cdot \left(u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y}\right) = k \cdot \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right)$$

Mugalde geruzaren barnean oso txikia

6.9 – KONBEKZIO-EKUAZIO DIMENTSIOGABEAK ETA ANTZEKOTASUNA

Aldagai guztiak dimentsiogabetuz:

$$x^* = \frac{x}{L}; \quad y^* = \frac{y}{L}; \quad u^* = \frac{u}{V}; \quad v^* = \frac{v}{V}; \quad P^* = \frac{P}{\rho \cdot V^2}; \quad T^* = \frac{T - T_s}{T_\infty - T_s};$$

Jarraitutasuna:
$$\frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*} = 0$$

Momentua:
$$u^* \frac{\partial u^*}{\partial x^*} + v^* \frac{\partial u^*}{\partial y^*} = \frac{1}{\text{Re}_L} \frac{\partial^2 u^*}{\partial y^{*2}} - \frac{\partial P^*}{\partial x^*}$$

Energia:
$$u^* \frac{\partial T^*}{\partial x^*} + v^* \frac{\partial T^*}{\partial y^*} = \frac{1}{\text{Re}_L \cdot \text{Pr}} \left(\frac{\partial^2 T^*}{\partial y^{*2}}\right)$$

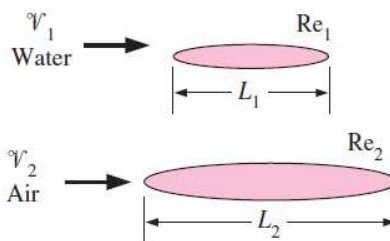
$$u^*(0, y^*) = 1; \quad u^*(x^*, 0) = 0; \quad u^*(x^*, \infty) = 1; \quad v^*(x^*, 0) = 0;$$

$$T^*(0, y^*) = 1; \quad T^*(x^*, 0) = 0; \quad T^*(x^*, \infty) = 1;$$

“Bi fenomeno fisiko *antzekoak* dira, baldin eta deskribatzen dituzten ekuazio diferentzialen eta mugalde-baldintzen forma dimentsiogabe berak badituzte.”

$$\begin{array}{l}
 L \\
 V \\
 T_{\infty} \\
 T_S \\
 \nu \\
 \alpha
 \end{array}
 \longrightarrow
 \begin{array}{l}
 Re_L \\
 Pr
 \end{array}$$

“Geometria jakin batean, antzekotasun-parametroen balio **bera** duten problemek **ebazpen berdin-berdinak** dituzte.”



Baldin $Re_1 = Re_2$ orduan $C_{f1} = C_{f2}$

6.10 – MARRUSKADURA- ETA KONBEKZIO-KOEFIZIENTEEN FORMA FUNTZIONALAK

3 ekuazio dimentsiogabeak	}	3 funtzio ezezagun:	$u^*, v^* \text{ y } T^*$
		2 aldagai independente:	x^*, y^*
		2 parametro:	$Re_L \text{ y } Pr$

Lehenengo bi ekuazioetatik: $u^* = f_1(x^*, y^*, Re_L)$

Ebakidura-tentsioa: $\tau_s = \mu \frac{\partial u}{\partial y} \Big|_{y=0} = \frac{\mu \cdot V}{L} \frac{\partial u^*}{\partial y^*} \Big|_{y^*=0} = \frac{\mu \cdot V}{L} f_2(x^*, Re_L)$

Marruskadura-koefizientea:

$$C_{f,x} = \frac{\tau_s}{\rho \cdot V^2 / 2} = \frac{\mu \cdot V / L}{\rho \cdot V^2 / 2} f_2(x^*, Re_L) = f_3(x^*, Re_L)$$

6.10 – MARRUSKADURA- ETA KONBEKZIO-KOEFIZIENTEEN FORMA FUNTZIONALAK

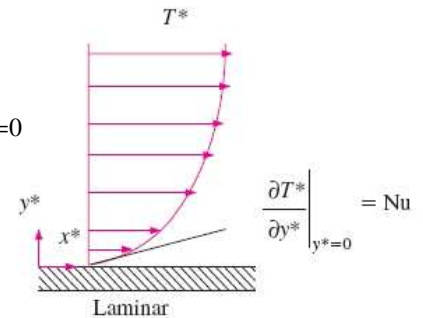
19/22

Azkenengo ekuaziotik.: $T^* = g_1(x^*, y^*, Re_L, Pr)$

Konbekzio koefizientea: $y = 0 \Rightarrow \dot{q}_{cond} = \dot{q}_{conv} \Rightarrow -k_{fluid} \cdot \frac{\partial T}{\partial y} \Big|_{y=0} = h \cdot (T_s - T_\infty)$

$$h = -\frac{k \frac{\partial T}{\partial y} \Big|_{y=0}}{T_s - T_\infty} = -\frac{k \frac{T_\infty - T_s}{L} \cdot \frac{\partial T^*}{\partial y^*} \Big|_{y^*=0}}{T_s - T_\infty} = \frac{k}{L} \frac{\partial T^*}{\partial y^*} \Big|_{y^*=0}$$

$$Nu_x = \frac{h \cdot L}{k} = \frac{\partial T^*}{\partial y^*} \Big|_{y^*=0} = g_2(x^*, Re_L, Pr)$$



0 eta 1 artean, X-ekiko integratuz

$$C_f = f_4(Re_L) \quad Nu = g_3(Re_L, Pr)$$

$$Nu = C \cdot Re_L^m \cdot Pr^n$$

6.11 – MOMENTU- ETA BERO TRANSFERENTZIAREN ARTEKO ANALOGIAK

20/22

Helburua: C_f eta Nu lortu



Reynoldsen analogia

Chilton-Colburnen analogia

Baldin $Pr = 1$ eta $\frac{\partial P^*}{\partial x^*} = 0$

Momentua:

$$u^* \frac{\partial u^*}{\partial x^*} + v^* \frac{\partial u^*}{\partial y^*} = \frac{1}{Re_L} \frac{\partial^2 u^*}{\partial y^{*2}}$$

Energia:

$$u^* \frac{\partial T^*}{\partial x^*} + v^* \frac{\partial T^*}{\partial y^*} = \frac{1}{Re_L} \frac{\partial^2 T^*}{\partial y^{*2}}$$

Mugalde baldintzak:

$$u^*(0, y^*) = 1; \quad u^*(x^*, 0) = 0; \quad u^*(x^*, \infty) = 1;$$

$$T^*(0, y^*) = 1; \quad T^*(x^*, 0) = 0; \quad T^*(x^*, \infty) = 1;$$

u^* eta T^* berdinak behar dute izan

$$\frac{\partial u^*}{\partial y^*} \Big|_{y^*=0} = \frac{\partial T^*}{\partial y^*} \Big|_{y^*=0}$$

$$C_{f,x} \frac{Re_L}{2} = Nu_x \quad (Pr = 1)$$

Reynoldsen analogia

$$\frac{C_{f,x}}{2} = St_x \quad (Pr = 1)$$

$$St = \frac{h}{\rho \cdot c_p \cdot V} = \frac{Nu}{Re_L \cdot Pr}$$

Stantonen zenbakia

Prandtl-en zenbakia zuzenduz

Reynoldsen analogia eraldatua edo
Chilton-Colburnen analogia

$$C_{f,x} \frac{Re_L}{2} = Nu_x \cdot Pr^{-1/3}$$

$$\frac{C_{f,x}}{2} = \frac{h_x}{\rho \cdot c_p \cdot V} \cdot Pr^{2/3} \equiv j_H$$

Colburnen j faktorea

Baliogarria: $0,6 < Pr < 60$ tartean

Fluxu turbulenta presio-gradienteak badaude ere

Fluxu laminarra baldin eta $\partial P^* / \partial x^* = 0$

6.12 – IRAKATSIKO EZ DIREN ATALAK

6.6 atala – Fluxu turbulentuen bero- eta momentu-transferentzia

6.8 atala – Xafla lau baten konbekzio-ekuazioen ebazpenak

7 GAIA

KANPO KONBEKZIO BEHARTUA

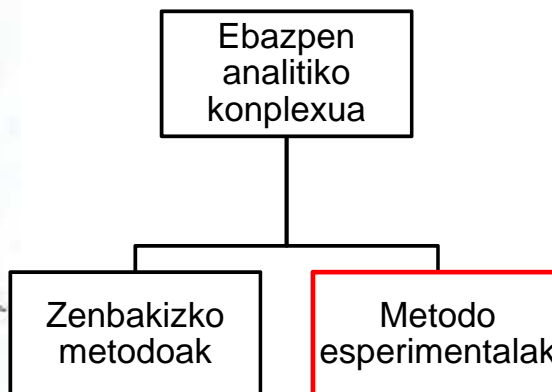
7.0 - HELBURUAK

2/23

- **Barne-** eta **kanpo-fluxuak** bereizi.
- Marruskadura- eta presio-arrastea era intuitiboan ulertzeko gaitasuna landu, eta kanpo-fluxuen **batez besteko** arraste- eta **konbekzio-koefizienteak** kalkulatu
- **Xafla lauen** gaineko fluxu laminar nahiz turbulentuen arrastea eta bero-transferentzia kalkulatu.
- Fluxu gurutzatuetan **zilindroei** eragiten zaien arraste-indarra kalkulatu, eta orobat batez besteko bero-transferentziaren koefizientea.
- **Hodi multzoetan** zeharreko fluxuen batez besteko bero-transferentziaren koefizientea kalkulatu, konfigurazio lerrokatuan nahiz mailakatuan.

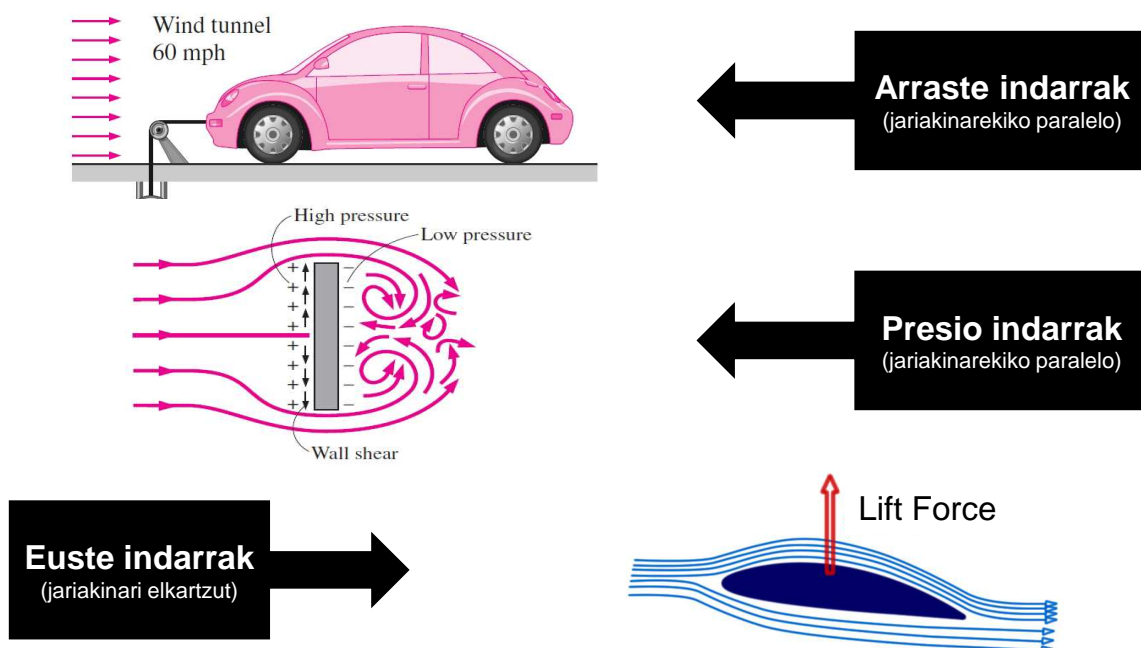
7.1 – ARRASTEA ETA BERO-TRANSFERENTZIA KANPO-FLUXUETAN

Gorputz solidoen aurkako jariakin fluxua:



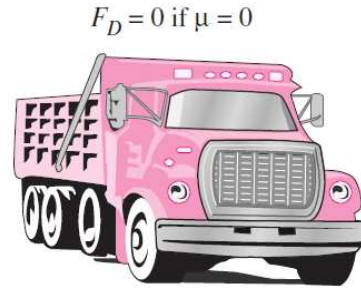
7.1 – ARRASTEA ETA BERO-TRANSFERENTZIA KANPO-FLUXUETAN

MARRUSKADURA- ETA PRESIO-ARRASTEAK



MARRUSKADURA- ETA PRESIO-ARRASTEAK

Arraste-koefizientea $\rightarrow C_D = \frac{F_D}{\frac{1}{2} \rho V^2 A}$



$$C_D = C_{D, friction} + C_{D, pressure}$$

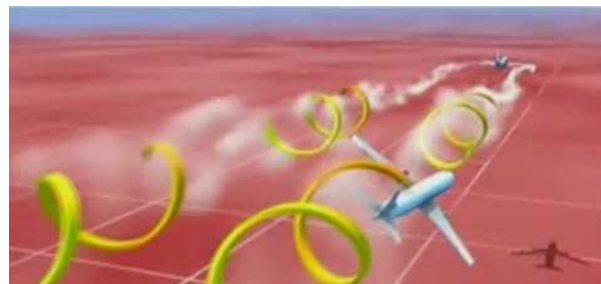
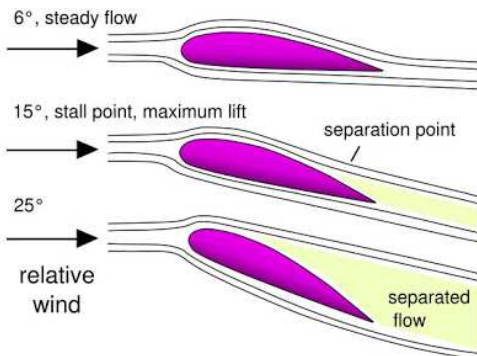
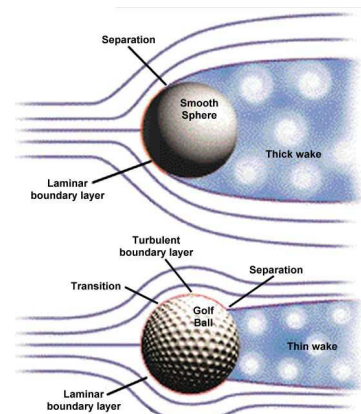
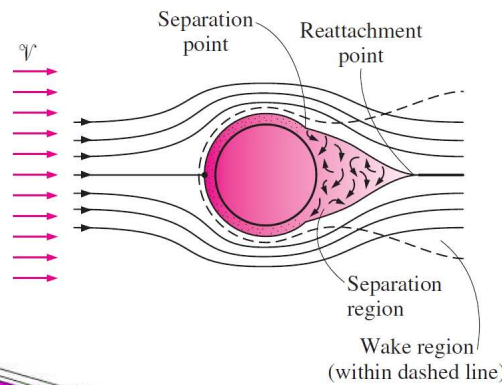
Azaleko marruskadura-arrastea (ebakidura tentsioa)

Presio-arrastea (gorputzaren formagatik)

$C_{D, pressure} = 0$
 $C_D = C_{D, friction} = C_f$
 $F_{D, pressure} = 0$
 $F_D = F_{D, friction} = F_f = C_f A \frac{\rho V^2}{2}$

MARRUSKADURA- ETA PRESIO-ARRASTEAK

Eskualde banatua (ubera)



BERO-TRANSFERENTZIA

6. GAIA $\Rightarrow Nu_x = f_1(x^*, Re_x, Pr) \Rightarrow Nu = f_2(Re_L, Pr)$

$$Nu = \frac{1}{L} \int_0^L Nu_x dx$$

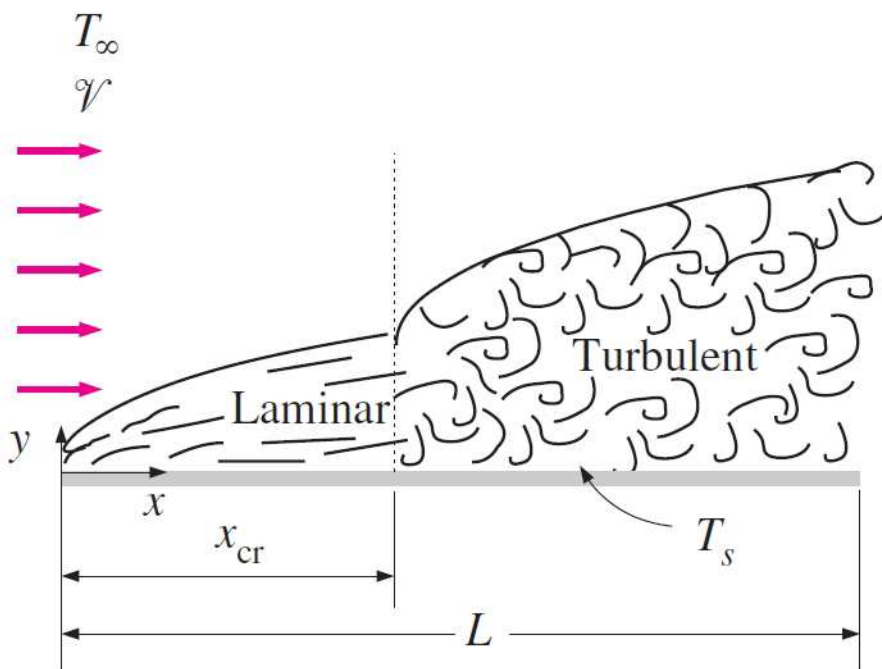
$$Nu = C Re_L^m Pr^n$$

Jariakinaren propietateak $T_f = \frac{T_s + T_\infty}{2}$ → Geruza-tenperatura

T_∞ → $\left(\frac{Pr_\infty}{Pr_s}\right)^r$ edo $\left(\frac{\mu_\infty}{\mu_s}\right)^r$

TERMOTEKNIA

7.2 – XAFLA LAUEN GAINEKO FLUXU PARALELOA



$$Re_x = \frac{Vx}{\nu}$$

$$Re_{cr} = \frac{Vx_{cr}}{\nu} = 5 \cdot 10^5$$

MARRUSKADURA-KOEFIZIENTEA

$$\delta_{v,x} = \frac{4.91x}{Re_x^{1/2}} \quad \text{eta} \quad C_{f,x} = \frac{0.664}{Re_x^{1/2}} \quad Re_x < 5 \cdot 10^5$$

$$\delta_{v,x} = \frac{0.38x}{Re_x^{1/5}} \quad \text{eta} \quad C_{f,x} = \frac{0.059}{Re_x^{1/5}} \quad 5 \cdot 10^5 \leq Re_x < 10^7$$

} Balio lokalak

$$C_f = \frac{1}{L} \int_0^L C_{f,x} dx$$

$$= \frac{1}{L} \int_0^L \frac{0.664}{Re_x^{1/2}} dx$$

$$= \frac{0.664}{L} \int_0^L \left(\frac{\gamma x}{v}\right)^{-1/2} dx$$

$$= \frac{0.664}{L} \left(\frac{\gamma}{v}\right)^{-1/2} \frac{x^{1/2}}{\frac{1}{2}} \Big|_0^L$$

$$= \frac{2 \times 0.664}{L} \left(\frac{\gamma L}{v}\right)^{-1/2}$$

$$= \frac{1.328}{Re_L^{1/2}}$$

$$C_f = \frac{1.328}{Re_L^{1/2}} \quad Re_L < 5 \cdot 10^5$$

$$C_f = \frac{0.074}{Re_L^{1/5}} \quad 5 \cdot 10^5 \leq Re_L < 10^7$$

} Bataz besteko balioak

MARRUSKADURA-KOEFIZIENTEA

$$C_f = \frac{1}{L} \left(\int_0^{x_{cr}} C_{f,x} \text{ laminar} dx + \int_{x_{cr}}^L C_{f,x} \text{ turbulent} dx \right)$$

} Fluxu mistoa

$$C_f = \frac{0.074}{Re_L^{1/5}} - \frac{1742}{Re_L} \quad 5 \cdot 10^5 \leq Re_L < 10^7$$

- Fluxu laminarra ($Re \downarrow$) $\rightarrow C_f = f(Re)$
- Fluxu guztiz turbulenta ($Re \uparrow\uparrow$) $\rightarrow C_f = f(\epsilon)$

Zimurtasuna \leftarrow

Relative roughness, ϵ/L	Friction coefficient C_f
0.0*	0.0029
1×10^{-5}	0.0032
1×10^{-4}	0.0049
1×10^{-3}	0.0084

*Smooth surface for $Re = 10^7$. Others calculated from Eq. 7-18.

$$C_f = \left(1.89 - 1.62 \log \frac{\epsilon}{L} \right)^{-2.5}$$

\Rightarrow Schlichtingen korrelazioa fluxu turbulenta dauden gainazal zimurrentzat

BERO-TRANSFERENTZIAREN KOEFIZIENTEA

$$Nu_x = \frac{h_x x}{k} = 0.332 Re_x^{0.5} Pr^{1/3}$$

$$Pr > 0.6$$

$$Re_x < 5 \cdot 10^5$$

$$Nu_x = \frac{h_x x}{k} = 0.0296 Re_x^{0.8} Pr^{1/3}$$

$$0.6 \leq Pr \leq 60$$

$$5 \cdot 10^5 \leq Re_x \leq 10^7$$

Balio lokalak
($T_s = cte$)

$$Nu = \frac{hL}{k} = 0.664 Re_L^{0.5} Pr^{1/3}$$

$$Pr > 0.6$$

$$Re_L < 5 \cdot 10^5$$

$$Nu = \frac{hL}{k} = 0.037 Re_L^{0.8} Pr^{1/3}$$

$$0.6 \leq Pr \leq 60$$

$$5 \cdot 10^5 \leq Re_L \leq 10^7$$

Bataz besteko balioak

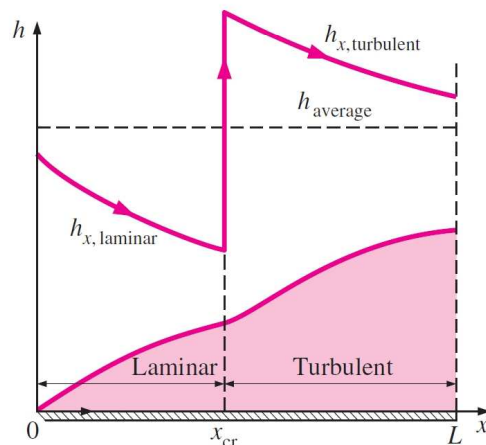
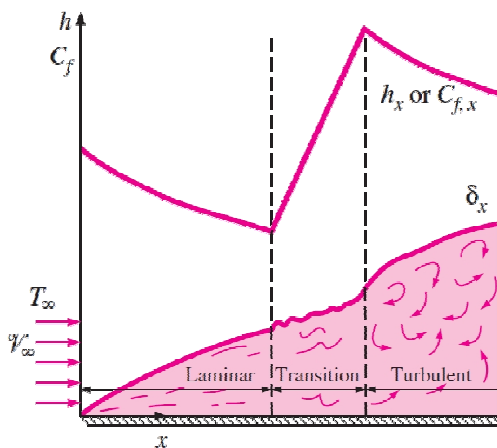
BERO-TRANSFERENTZIAREN KOEFIZIENTEA

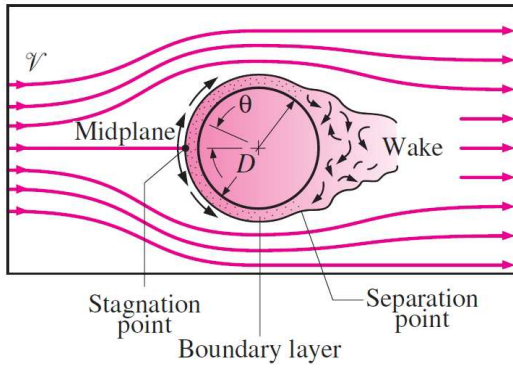
$$h = \frac{1}{L} \left(\int_0^{x_{cr}} h_{x \text{ laminar}} dx + \int_{x_{cr}}^L h_{x \text{ turbulent}} dx \right)$$

$$0.6 \leq Pr \leq 60$$

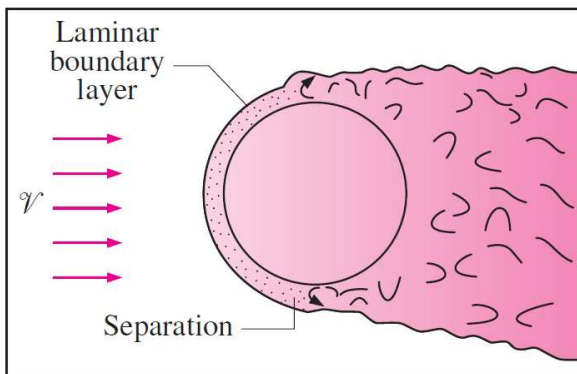
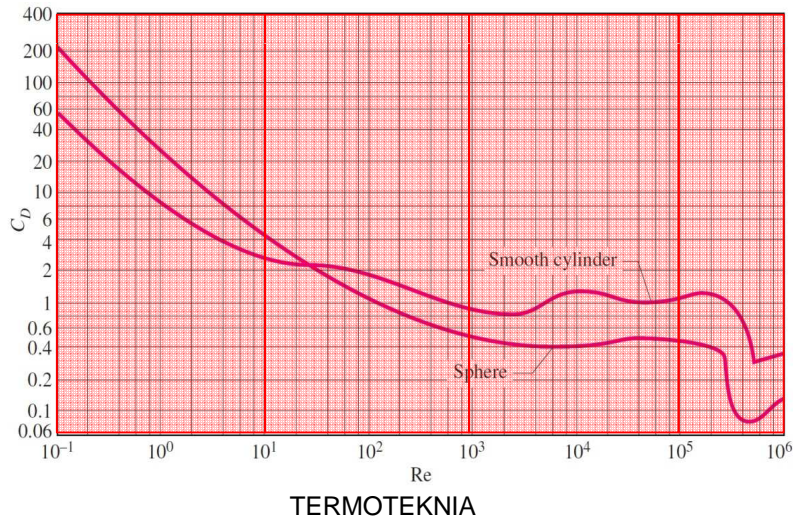
$$5 \cdot 10^5 \leq Re_L \leq 10^7$$

Fluxu mistoa

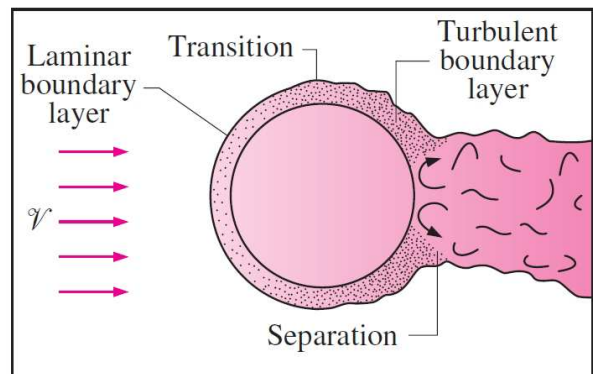




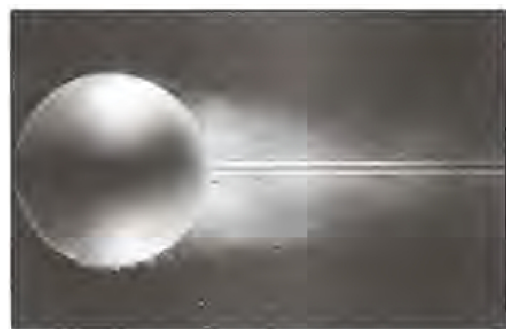
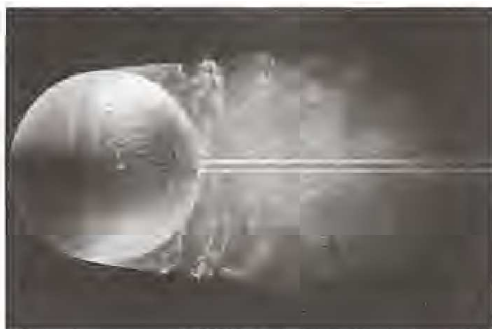
- $Re \downarrow \rightarrow$ Marruskadura indarrak
- $Re \uparrow \rightarrow$ Presio indarrak
- Tarteko $Re \rightarrow$ Biak



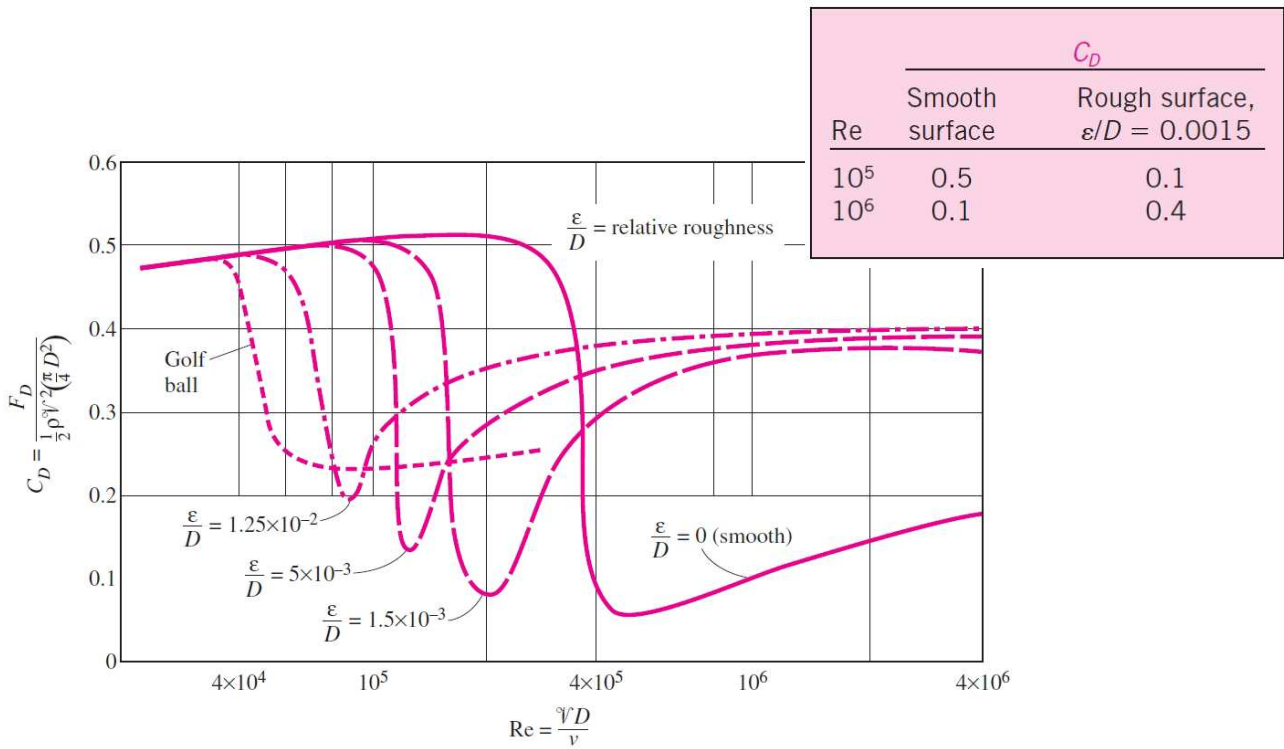
(a) Laminar flow ($Re < 2 \times 10^5$)



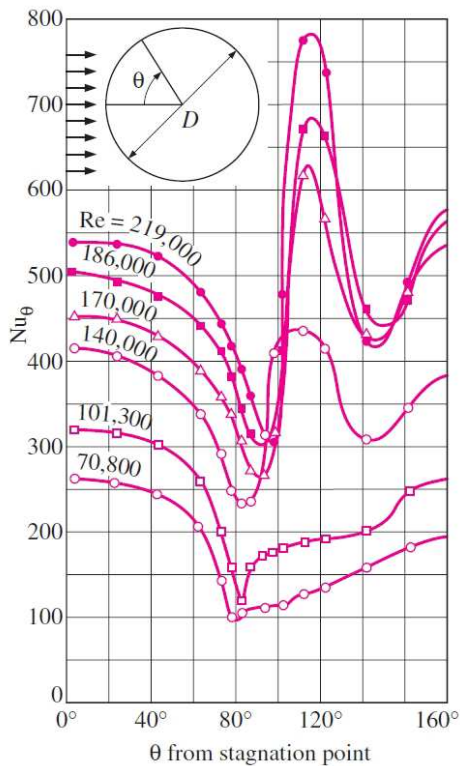
(b) Turbulence occurs ($Re > 2 \times 10^5$)



GAINAZAL-ZIMURTASUNAREN ERAGINA



BERO-TRANSFERENTZIAREN KOEFIZIENTEA



- Fluxu konplexua
- Zenbakizko ebazpena edo esperimentalak
- Re-en araberako portaera
- Batz besteko balioen erabilera

BERO-TRANSFERENTZIAREN KOEFIZIENTEA

Churchill eta Berstein
(zilindroarekiko zeharkako fluxua) →
$$Nu_{cyl} = \frac{hD}{k} = 0.3 + \frac{0.62 Re^{1/2} Pr^{1/3}}{\left[1 + (0.4/Pr)^{2/3}\right]^{1/4}} \left[1 + \left(\frac{Re}{282000}\right)^{5/8}\right]^{4/5}$$

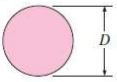
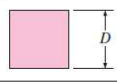
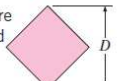
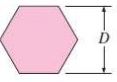
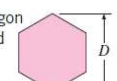

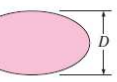
Re·Pr > 0.2

Whitaker
(esferarekiko fluxua) →
$$Nu_{sph} = \frac{hD}{k} = 2 + \left[0.4 Re^{1/2} + 0.06 Re^{2/3}\right] Pr^{0.4} \left(\frac{\mu_{\infty}}{\mu_s}\right)^{1/4}$$

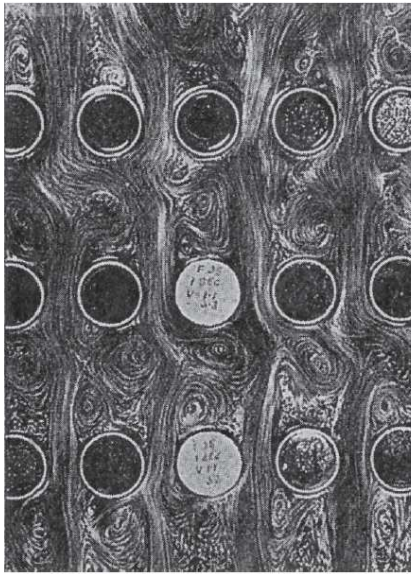
0.7 ≤ Pr ≤ 380
3.5 ≤ Re_D ≤ 80000

BERO-TRANSFERENTZIAREN KOEFIZIENTEA

$$Nu = \frac{hD}{k} = C Re^m Pr^{1/3}$$
 →

Cross-section of the cylinder	Fluid	Range of Re	Nusselt number
Circle 	Gas or liquid	0.4–4 4–40 40–4000 4000–40,000 40,000–400,000	$Nu = 0.989Re^{0.330} Pr^{1/3}$ $Nu = 0.911Re^{0.385} Pr^{1/3}$ $Nu = 0.683Re^{0.466} Pr^{1/3}$ $Nu = 0.193Re^{0.618} Pr^{1/3}$ $Nu = 0.027Re^{0.805} Pr^{1/3}$
Square 	Gas	5000–100,000	$Nu = 0.102Re^{0.675} Pr^{1/3}$
Square (tilted 45°) 	Gas	5000–100,000	$Nu = 0.246Re^{0.588} Pr^{1/3}$
Hexagon 	Gas	5000–100,000	$Nu = 0.153Re^{0.638} Pr^{1/3}$
Hexagon (tilted 45°) 	Gas	5000–19,500 19,500–100,000	$Nu = 0.160Re^{0.638} Pr^{1/3}$ $Nu = 0.0385Re^{0.782} Pr^{1/3}$
Vertical plate 	Gas	4000–15,000	$Nu = 0.228Re^{0.731} Pr^{1/3}$
Ellipse 	Gas	2500–15,000	$Nu = 0.248Re^{0.612} Pr^{1/3}$

Flow direction
↑



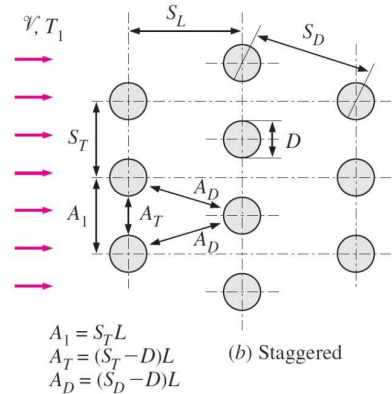
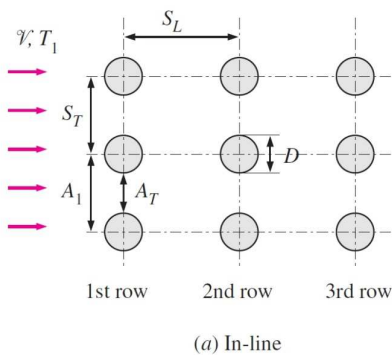
Lerrokatuta



Mailakatuta

Bero-trukagailuak:

- Barne-fluxua
- Kanpo-fluxua



$$V_{\max} = \frac{S_T}{S_T - D} V$$

$$Re_D = \frac{V_{\max} D}{\nu}$$

$$S_D = \sqrt{S_L^2 + (S_T/2)^2}$$

Baldin $2 \cdot A_D > A_T \rightarrow V_{\max} = \frac{S_T}{S_T - D} V$

Baldin $2 \cdot A_D < A_T \rightarrow V_{\max} = \frac{S_T}{2(S_D - D)} V$

$$Nu = \frac{hD}{k} = C Re_D^m Pr^n (Pr/Pr_s)^{0.25} \quad \begin{array}{l} 0.7 \leq Pr \leq 500 \\ 0 \leq Re_D \leq 2 \cdot 10^6 \end{array}$$

Arrangement	Range of Re_D	Correlation
In-line	0–100	$Nu_D = 0.9 Re_D^{0.4} Pr^{0.36} (Pr/Pr_s)^{0.25}$
	100–1000	$Nu_D = 0.52 Re_D^{0.5} Pr^{0.36} (Pr/Pr_s)^{0.25}$
	1000– 2×10^5	$Nu_D = 0.27 Re_D^{0.63} Pr^{0.36} (Pr/Pr_s)^{0.25}$
	2×10^5 – 2×10^6	$Nu_D = 0.033 Re_D^{0.8} Pr^{0.4} (Pr/Pr_s)^{0.25}$
Staggered	0–500	$Nu_D = 1.04 Re_D^{0.4} Pr^{0.36} (Pr/Pr_s)^{0.25}$
	500–1000	$Nu_D = 0.71 Re_D^{0.5} Pr^{0.36} (Pr/Pr_s)^{0.25}$
	1000– 2×10^5	$Nu_D = 0.35 (S_T/S_L)^{0.2} Re_D^{0.6} Pr^{0.36} (Pr/Pr_s)^{0.25}$
	2×10^5 – 2×10^6	$Nu_D = 0.031 (S_T/S_L)^{0.2} Re_D^{0.8} Pr^{0.36} (Pr/Pr_s)^{0.25}$

Baldin $N_L < 16 \rightarrow Nu_{D,N_L} = F Nu_D$

N_L	1	2	3	4	5	7	10	13
In-line	0.70	0.80	0.86	0.90	0.93	0.96	0.98	0.99
Staggered	0.64	0.76	0.84	0.89	0.93	0.96	0.98	0.99

Bataz besteko temperatura-diferentzia logaritmikoa:

$$\Delta T_{\ln} = \frac{\Delta T_e - \Delta T_i}{\ln[\Delta T_e / \Delta T_i]} = \frac{(T_s - T_e) - (T_s - T_i)}{\ln[(T_s - T_e) / (T_s - T_i)]}$$

Irteera-temperatura:

$$T_e = T_s - (T_s - T_i) \cdot e^{-\frac{A_s h}{\dot{m} c_p}}$$

$$\dot{Q} = h A_s \Delta T_{\ln} = \dot{m} c_p (T_e - T_i)$$

- 7.2ko azpiatala: BEROTU GABEKO HASIERA ZATIA DUEN XAFLA LAUA
- 7.2ko azpiatala: BERO-FLUXU UNIFORMEA
- 7.4ko azpiatala: PRESIO-JAITSIERA

8. GAIA

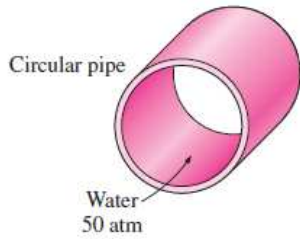
BARNEKO KONBEKZIO BEHARTUA

8.0 - HELBURUAK

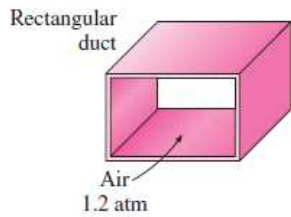
2/24

- Barne-fluxuaren **batez besteko abiadura** lortu, abiadura-profilean oinarrituta, eta **batez besteko tenperatura** lortu, tenperatura-profilean oinarrituta
- Barne-fluxuko **fluxu-eskualdeen ulermen** bisuala eduki, hala nola sarrera-eskualdearena eta erabat garatutako fluxu-eskualdearena, eta sarrera-luzera hidrodinamikoa eta termikoa kalkulatu
- Gainazal-tenperatura konstantea eta gainazaleko bero-fluxuaren baldintza konstanteak dituen hodi batean doan fluido bat **berotzeko eta hozteko prozesua aztertu**, eta **batez besteko tenperatura-diferentzia logaritmikoarekin** lan egin
- **Fluxu laminar** erabat garatuen abiadura-profilearen, presio-jaitsieraren, marruskadura-faktorearen eta Nusselten zenbakiaren **erlazio analitikoa lortu**
- **Fluxu turbulentu** erabat garatuen marruskadura-faktorea eta Nusselten zenbakia kalkulatu, erlazio enpirikoez baliatuta, eta presio-jaitsiera eta bero-transferentziaren abiadura kalkulatu

Barne-fluxuan, fluidoa hodiaren barne-gainazalek inguratua dago



← **Fluido LIKIADOA denean**

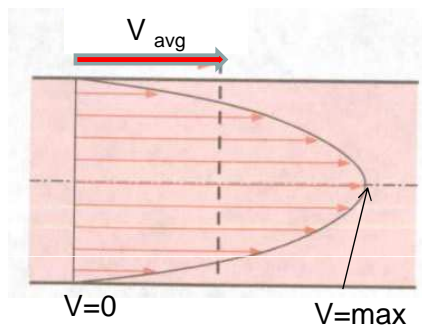


← **Fluido GASA denean**



8.2 – BATEZ BESTEKO ABIADURA ETA TEMPERATURA

BATEZ BESTEKO ABIADURA (V_{avg})



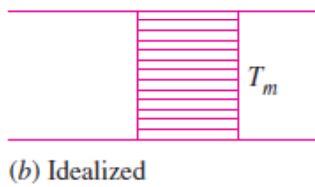
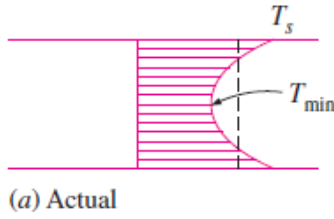
MASAREN kontserbazio-printzipioa

$$\dot{m} = \rho V_{avg} A_c = \int_{A_c} \rho u(r) dA_c$$

$$V_{avg} = \frac{2}{R^2} \int_0^R u(r) r dr$$

$u(r)$: abiadura - profila

BATEZ BESTEKO TENPERATURA (T_m)



ENERGIAREN kontserbazio-printzipioa

$$\dot{E}_{fluid} = \dot{m}c_p T_m = \int_{A_c} \rho C_p T(r)u(r)dA_c$$

$$T_m = \frac{2}{V_{avg} R^2} \int_0^R T(r)u(r)rdr$$

FLUXU LAMINARRAK ETA TURBULENTUAK HODIETAN

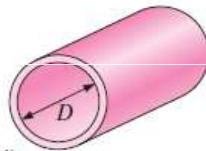
$$Re = \frac{\rho V_{avg} D}{\mu} = \frac{V_{avg} D}{\nu}$$

$$\nu = \frac{\mu}{\rho} \quad \text{Biskositate zinematikoa}$$

- $Re < 2.300$ fluxu laminarra: fluido oso biskosoak edo abiadura baxuak
- $Re > 10.000$ fluxu turbulentoa

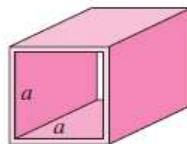


Circular tube:



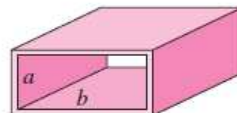
$$D_h = \frac{4(\pi D^2/4)}{\pi D} = D$$

Square duct:



$$D_h = \frac{4a^2}{4a} = a$$

Rectangular duct:

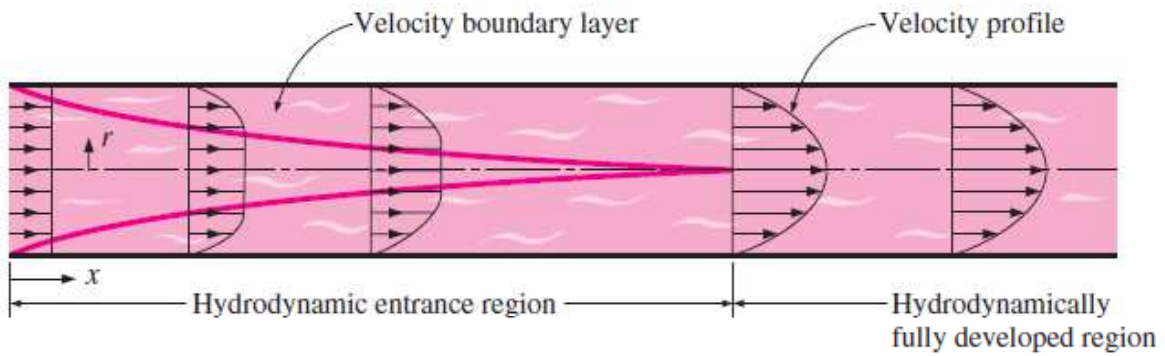


$$D_h = \frac{4ab}{2(a+b)} = \frac{2ab}{a+b}$$

Diametro Hidraulikoa

$$D_h = \frac{4A_c}{p}$$

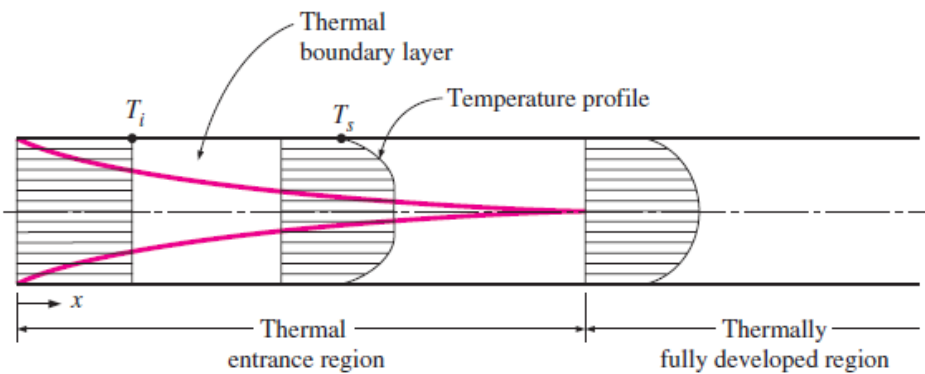
HODI BATEKO MUGALDE-GERUZA ABIADURAREN GARAPENA



Hidrodinamikoki erabat garatutako eskualdea

$$\frac{\partial u(r, x)}{\partial x} = 0 \rightarrow u = u(r)$$

HODI BATEKO MUGALDE-GERUZA TERMIKOAREN GARAPENA

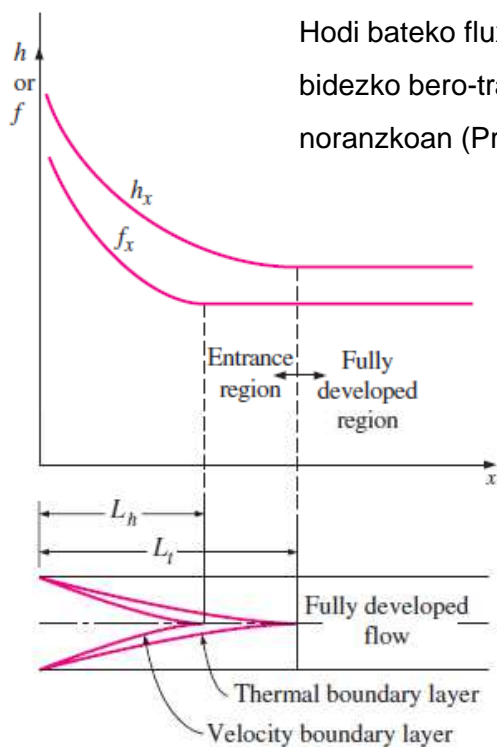


Termikoki erabat garatutako eskualdea

$$\frac{\partial}{\partial x} \left[\frac{T_s(x) - T(r, x)}{T_s(x) - T_m(x)} \right] = 0$$

Gainazaleko bero-fluxua

$$\dot{q}_s = h_x(T_s - T_m) = k \left. \frac{\partial T}{\partial r} \right|_{r=R}$$



Hodi bateko fluxuaren marruskadura-faktorearen eta konbektzio bidezko bero-transferentziaren koefizientearen aldaketa fluxuaren noranzkoan ($Pr > 1$)

L_h luzera hidrodinamikoa
 L_t luzera termikoa

SARRERA-LUZERAK EXPERIMENTALAK

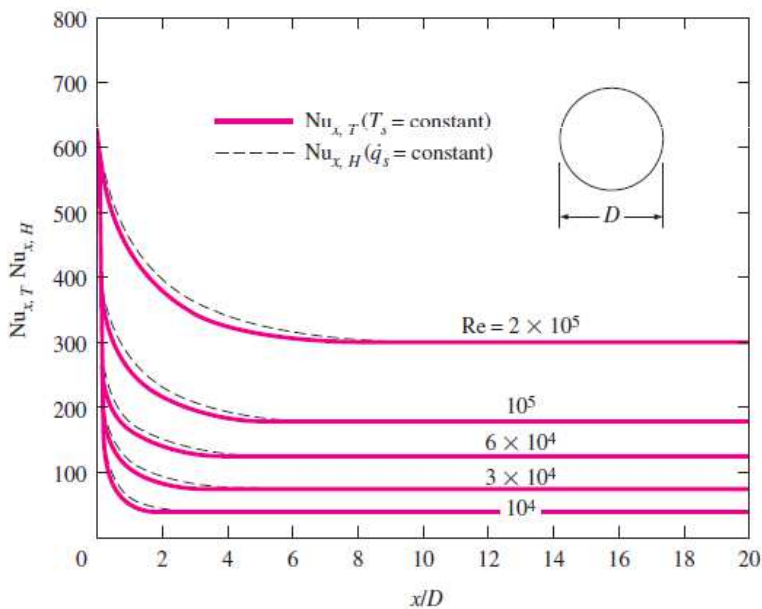
Fluxu Laminarra

$$L_{h,laminar} \approx 0,05 ReD$$

$$L_{t,laminar} \approx 0,05 Re Pr D = Pr L_{h,laminar}$$

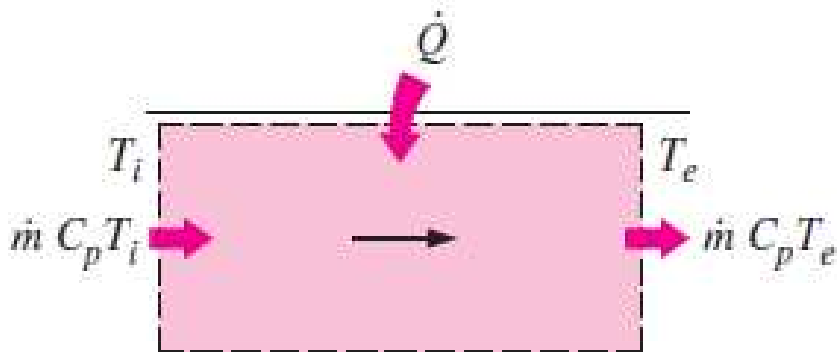
Fluxu turbulentua

$$L_{h,turbulent} \approx L_{t,turbulent} \approx 10D$$



Fluxu turbulentua duen hodi batean zeharreko Nusselten zenbaki lokalaren aldaketa

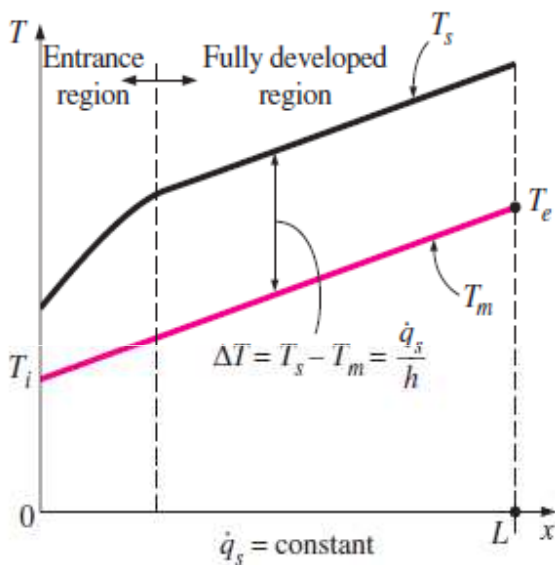
Hodi bateko fluido baten fluxu geldikorren energia-kontserbazioaren ekuazioa



$$\dot{Q} = \dot{m}c_p(T_e - T_i) \quad (\text{W})$$

$$\dot{q}_s = h_x(T_s - T_m) \quad (\text{W/m}^2)$$

GAINAZALEKO BERO-FLUXU KONSTANTEA ($q_s = \text{konstantea}$)



$$\dot{Q} = \dot{q}_s A_s = \dot{m}c_p(T_e - T_i) \quad (\text{W}) \quad T_e = T_i + \frac{\dot{q}_s A_s}{\dot{m}c_p}$$

$$\dot{q}_s = h \cdot (T_s - T_m) \rightarrow T_s = T_m + \frac{\dot{q}_s}{h}$$

$$h = kte \rightarrow T_s - T_m = kte$$

$$\dot{m}c_p dT_m = \dot{q}_s (pdx) \rightarrow \frac{dT_m}{dx} = \frac{\dot{q}_s p}{\dot{m}c_p} = kte$$

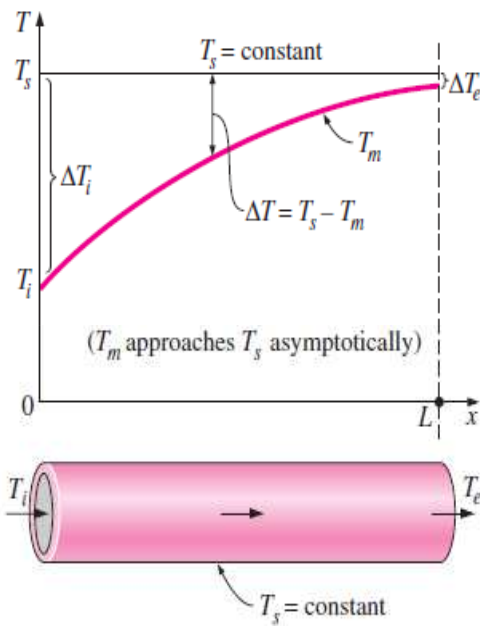
$$\frac{\partial T}{\partial x} = \frac{dT_s}{dx} = \frac{dT_m}{dx} = \frac{\dot{q}_s p}{\dot{m}c_p} = kte$$

Tutu zirkularra:

$$\frac{\partial T}{\partial x} = \frac{dT_s}{dx} = \frac{dT_m}{dx} = \frac{2\dot{q}_s}{\rho V_{\text{avg}} c_p R} = kte$$

$$p = 2\pi R \text{ eta } \dot{m} = \rho V_{\text{avg}} A_c$$

GAINAZAL-TEMPERATURA KONSTANTEA ($T_s = \text{konstantea}$)



$$\dot{Q} = hA_s \Delta T_{\text{avg}} = hA_s (T_s - T_m)_{\text{avg}} \quad (\text{W})$$

$$\Delta T_{\text{avg}} \approx \Delta T_{\text{am}} = \frac{\Delta T_i + \Delta T_e}{2} = \frac{(T_s - T_i) + (T_s - T_e)}{2}$$

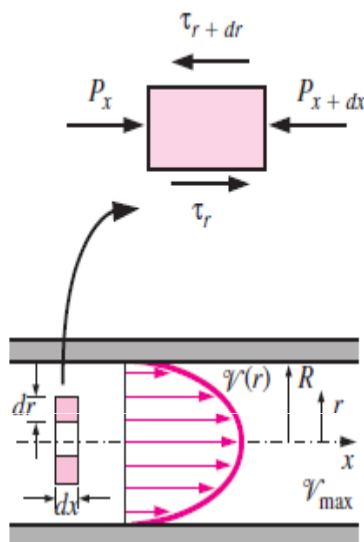
Energia Balantzea:

$$\dot{m}c_p dT_m = h(T_s - T_m)dA_s$$

$$T_e = T_s - (T_s - T_i) \exp(-hA_s / \dot{m}c_p)$$

$$\Delta T_{\text{ln}} = \frac{(T_s - T_e) - (T_s - T_i)}{\ln[(T_s - T_e)/(T_s - T_i)]} = \frac{\Delta T_e - \Delta T_i}{\ln(\Delta T_e / \Delta T_i)}$$

8.5 – FLUXU LAMINARRA HODIETAN



Indar-balantzeak:

$$(2\pi r dr P)_x - (2\pi r dr P)_{x+dx} + (2\pi r dx \tau)_r - (2\pi r dx \tau)_{r+dr} = 0$$

Lim dr, dx → 0 , ordezkapena $\tau = -\mu(du / dr)$

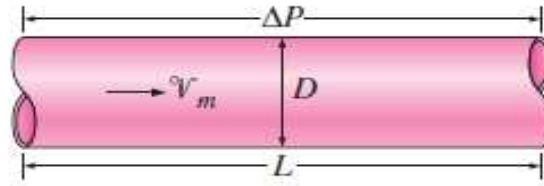
$$\frac{dP}{dx} = -\frac{2\tau_w}{R} = kte \quad \text{berrantolatuz eta bi aldiz integratuz}$$

$$u(r) = \frac{1}{4\mu} \left(\frac{dP}{dx} \right) + C_1 \ln r + C_2$$

$$\left. \begin{aligned} u(r) &= -\frac{R^2}{4\mu} \left(\frac{dP}{dx} \right) \left(1 - \frac{r^2}{R^2} \right) \\ V_{\text{avg}} &= \frac{2}{R^2} \int_0^R u(r)rdr = -\frac{R^2}{8\mu} \left(\frac{dP}{dx} \right) \end{aligned} \right\} \begin{aligned} u(r) &= 2V_{\text{avg}} \left(1 - \frac{r^2}{R^2} \right) \\ \text{non } r=0 &\Rightarrow u_{\text{max}} = 2V_{\text{avg}} \end{aligned}$$

PRESIO-GALERA (ΔP)

$$\frac{dP}{dx} = \frac{P_2 - P_1}{L}$$



Fluxu Laminarra →

$$\Delta P = P_1 - P_2 = \frac{8\mu L V_{avg}}{R^2} = \frac{32\mu L V_{avg}}{D^2}$$

Presio Galera →

$$\Delta P_L = f \frac{L}{D} \frac{\rho V_{avg}^2}{2}$$

f = Darcy-ren marruskadure faktorea →

$$f = \frac{8\tau_w}{\rho V_{avg}^2}$$

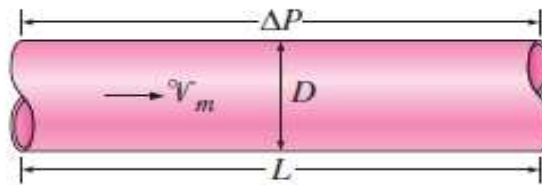
Hodi zirkularra, Laminarra →

$$f = \frac{64\mu}{\rho D V_{avg}} = \frac{64}{Re}$$

KARGA-GALERA (h_L)

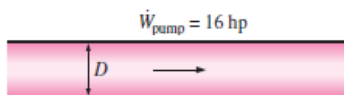
$$\Delta P = \rho g h$$

Karga Galera →

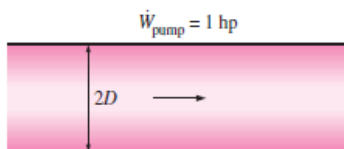


$$h_L = \frac{\Delta P_L}{\rho g} = f \frac{L}{D} \frac{V_{avg}^2}{2g}$$

PONPATZE-POTENTZIA



$$W_{pump,L} = \dot{V} \Delta P_L = \dot{V} \rho g h_L = \dot{m} g h_L$$

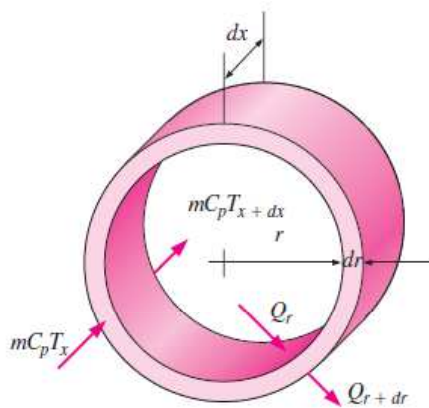


$$V_{avg} = \frac{(P_1 - P_2) R^2}{8\mu L} = \frac{\Delta P D^2}{32\mu L}$$

Poiseuille-ren Legea →

$$\dot{V} = V_{avg} A_c = \frac{(P_1 - P_2) R^2}{8\mu L} \pi R^2 = \frac{\Delta P \pi D^4}{128\mu L}$$

TENPERATURA-PROFILA ETA NUSSELTEN ZENBAKIA



Fluxu laminarra geldikorra, $u=u(r)$

Energia balantzea

$$\dot{m}c_p T_x - \dot{m}c_p T_{x+dx} + \dot{Q}_r - \dot{Q}_{r+dr} = 0$$

$$u \frac{\partial T}{\partial x} = - \frac{1}{2\rho c_p \pi dx} \frac{\partial \dot{Q}}{\partial r}$$

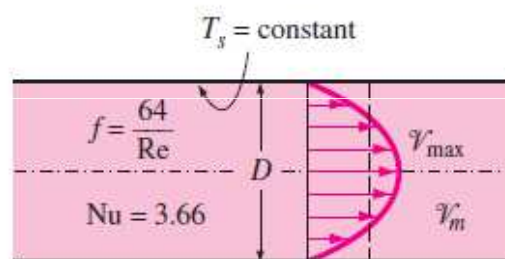
$$u \frac{\partial T}{\partial x} = \frac{\alpha}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right)$$

GAINAZALEKO BERO-FLUXU KONSTANTEA ($q_s = kte$)

$$Nu = \frac{hD}{k} = 4,36$$

GAINAZAL-TENPERATURA KONSTANTEA ($T_s = kte$)


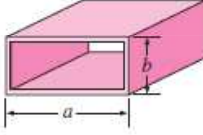
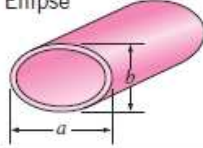
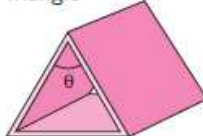
$$Nu = \frac{hD}{k} = 3,66$$



Fully developed laminar flow

FLUXU LAMINARRA HODI EZ-ZIRKULARRETAN

$$Nu = \frac{hD}{k} \Rightarrow$$

Tube Geometry	a/b or θ°	Nusselt Number		Friction Factor f
		T _s = Const.	q̇ _s = Const.	
Circle 	—	3.66	4.36	64.00/Re
Rectangle 	a/b			
	1	2.98	3.61	56.92/Re
	2	3.39	4.12	62.20/Re
	3	3.96	4.79	68.36/Re
	4	4.44	5.33	72.92/Re
	6	5.14	6.05	78.80/Re
	8	5.60	6.49	82.32/Re
	∞	7.54	8.24	96.00/Re
Ellipse 	a/b			
	1	3.66	4.36	64.00/Re
	2	3.74	4.56	67.28/Re
	4	3.79	4.88	72.96/Re
	8	3.72	5.09	76.60/Re
	16	3.65	5.18	78.16/Re
Triangle 	θ			
	10°	1.61	2.45	50.80/Re
	30°	2.26	2.91	52.28/Re
	60°	2.47	3.11	53.32/Re
	90°	2.34	2.98	52.60/Re
	120°	2.00	2.68	50.96/Re

GARAPEN-FASEKO FLUXU LAMINARRA SARRERA-ESKUALDEAN

Hodi zirkularra, L, T_s=kte

$$Nu = 3,66 + \frac{0,0065(D/L) Re Pr}{1 + 0,04[(D/L) Re Pr]^{2/3}}$$

Hodi zirkularra, L, T_s=kte, T_s>>T_{jariakin} denean

$$Nu = 1,86 + \left(\frac{Re Pr D}{L}\right)^{1/3} \left(\frac{\mu_b}{\mu_s}\right)^{0,14}$$

Plaka paralelo isotermoen arteko fluxua, L, Re ≤ 2800

$$Nu = 7,54 + \frac{0,03(D_h/L) Re Pr}{1 + 0,016[(D_h/L) Re Pr]^{2/3}}$$

HODI LEUNA

Petukhov → $f = (0,790 \ln Re - 1,64)^{-2}$ $3000 < Re < 5 \times 10^6$

Chilton-Colburn → $Nu = 0,125 \cdot f \cdot Re Pr^{1/3}$

Colburn → $Nu = 0,023 Re^{0,8} Pr^{1/3}$ $\left(\begin{array}{l} 0,7 \leq Pr \leq 160 \\ Re > 10\,000 \end{array} \right)$

Dittus-Boelter → $Nu = 0,023 Re^{0,8} Pr^n$ $\left(\begin{array}{l} n = 0,4 \text{ heating} \\ n = 0,3 \text{ cooling} \end{array} \right)$

Sieder-Tate → $Nu = 0,027 Re^{0,8} Pr^{1/3} \left(\frac{\mu}{\mu_s} \right)^{0,14}$ $\left(\begin{array}{l} 0,7 \leq Pr \leq 17\,600 \\ Re \geq 10\,000 \end{array} \right)$

Re > 10 000 Y HODI LEUNA ETA ZIMURRAK

Gnielinski → $Nu = \frac{(f/8)(Re-1000)Pr}{1+12,7(f/8)^{0,5}(Pr^{2/3}-1)}$ $\left(\begin{array}{l} 0,5 \leq Pr \leq 2\,000 \\ 3 \times 10^3 < Re < 5 \times 10^6 \end{array} \right)$

GAINAZAL ZIMURRAK

Colebrook → $\frac{1}{\sqrt{f}} = -2,0 \log \left(\frac{\epsilon/D}{3,7} + \frac{2,51}{Re \sqrt{f}} \right)$

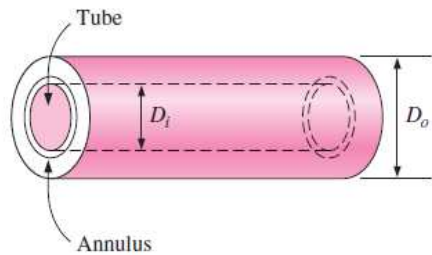
Moody → $\frac{1}{\sqrt{f}} = -1,8 \log \left(\frac{6,9}{Re} + \left(\frac{\epsilon/D}{3,7} \right)^{1,11} \right)$

Equivalent roughness values for new commercial pipes*

Material	Roughness, ϵ	
	ft	mm
Glass, plastic	0 (smooth)	
Concrete	0.003–0.03	0.9–9
Wood stave	0.0016	0.5
Rubber, smoothed	0.000033	0.01
Copper or brass tubing	0.000005	0.0015
Cast iron	0.00085	0.26
Galvanized iron	0.0005	0.15
Wrought iron	0.00015	0.046
Stainless steel	0.000007	0.002
Commercial steel	0.00015	0.045

*The uncertainty in these values can be as much as ±60 percent.

FLUXUA HODI ZENTROKIDEEN ARTEKO ERAZTUN-FORMAKO ESPAZIOAN



$$D_h = \frac{4A_c}{p} = D_o - D_i$$

$$Nu_i = \frac{h_i D_h}{k} \quad Nu_o = \frac{h_o D_h}{k}$$

Erabat garatutako fluxu turbulentueta \rightarrow

Gnielinski

Gnielinski ekuazioaren opezpena:

$$Nu_i = F_i Nu_{\text{Gnielinski}} \quad F_i = 0,86 \left(\frac{D_i}{D_o} \right)^{-0,16} \quad \text{Kanpoko horma adiabatikoa}$$

$$Nu_o = F_o Nu_{\text{Gnielinski}} \quad F_o = 0,86 \left(\frac{D_i}{D_o} \right)^{-0,16} \quad \text{Barneko horma adiabatikoa}$$

8.7 – IRAKATSIKO EZ DIREN ATALAK

9. GAIA

KONBEKZIO NATURALA

9.0 - HELBURUAK

2/24

- Konbekzio naturalaren **mekanismo fisikoa** ulertu
- Konbekzio naturala deskribatzeko **ekuazioak** garatu, eta horiek **dimentsiogabetuz**, **Grashofen** zenbaki dimentsiogabea lortu.
- Plano bertikal, horizontal eta inklinatuetako, nahiz zilindro eta esferetako konbekzio naturalaren **Nusselten zenbakia** ebaluatu.
- **Gainazal hegaldunen** konbekzio naturala aztertu, eta **hegal-tarte optimoa** kalkulatu.
- **Itxituren barneko** konbekzio naturala aztertu, hala nola beira bikoitzeko leihoetakoa.
- **Konbekzio natural eta behartu konbinatua** kontuan izan, eta mota bakoitzaren garrantzi erlatiboa neurtu.

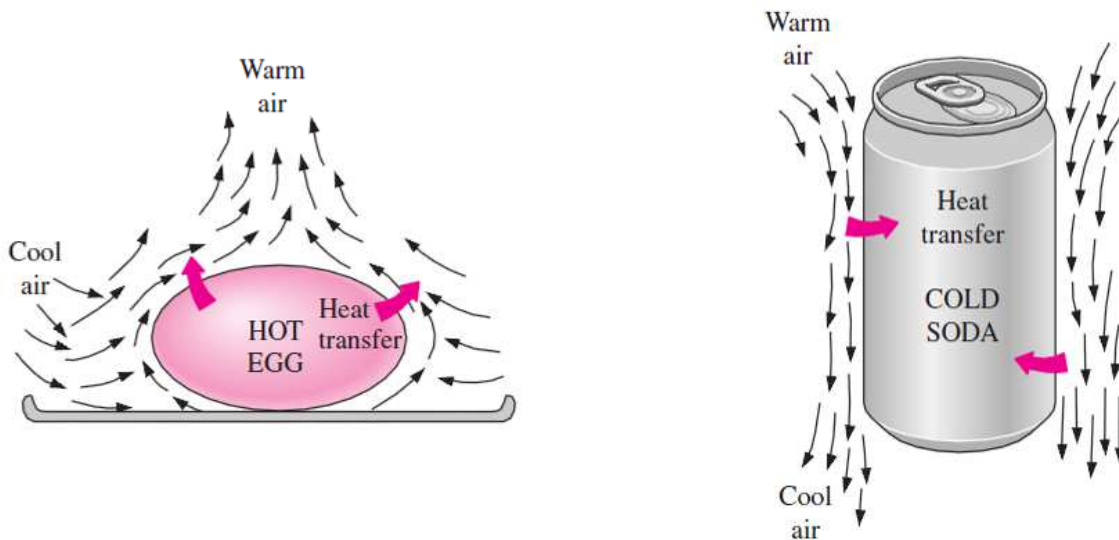
JARIAKINAREN TENPERATURA ALDAKETAREN ONDORIOZ, DENTSITATE ALDAKETA



KONBEKZIO NATURALAREN KORRONTEA



KONBEKZIO NATURALEKO BERO-TRANSFERENTZIA

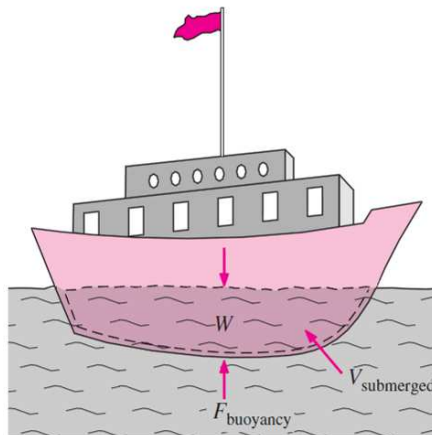


ARKIMEDESEN PRINTZIOA (FLOTAZIO EFEKTUA)

$$F_{buoyancy} = \rho_{fluid} g V_{submerged}$$

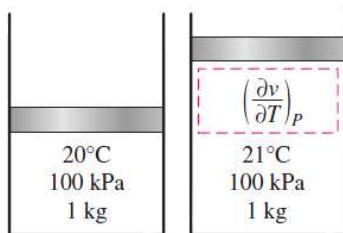
$$F_{net} = W - F_{buoyancy}$$

$$= \rho_{body} g V_{body} - \rho_{fluid} g V_{submerged}$$

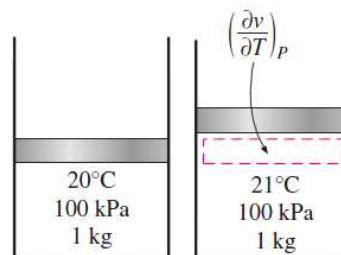


INTERESA → Jariakin baten dentsitate aldakuntza temperaturarekiko presio konstantepean

PROPIETATEA → **DILATAZIO KOEFIZIENTE BOLUMETRIKOA: β**



(a) A substance with a large β



(b) A substance with a small β

DILATAZIO KOEFIZIENTE BOLUMETRIKOA: β

$$\beta = \frac{1}{v} \left(\frac{\partial v}{\partial T} \right)_p = - \frac{1}{\rho} \left(\frac{\partial \rho}{\partial T} \right)_p \quad (1/K)$$

$$\beta \approx - \frac{1}{\rho} \frac{\Delta \rho}{\Delta T} = - \frac{1}{\rho} \frac{\rho_\infty - \rho}{T_\infty - T} \quad \Rightarrow \quad \rho_\infty - \rho = \rho \beta (T - T_\infty) \quad (P \text{ konstantepean})$$

GAS IDEALA
($P = \rho RT$)

$$\beta_{ideal\ gas} = \frac{1}{T} \quad (1/K)$$

Non T temperatura termodinamikoa den

¿ESPAZIOAN KONBEKZIO NATURALA DAGO?



Ez dago grabitate esanguratsurik, eta, beraz, espazio-ontzietan ezin da egon konbekzio natural bidezko bero-transferentziarik, ezta espazio-ontzia airez beteta badago ere.

¿NOLA EZARTZEN DA FLUXU MASIKOA KONBEKZIO NATURALEAN?

Emaria, goranzko bultzadaren eta marruskaduraren balantze dinamikoak finkatzen du.

9.2 – HIGIDURAREN EKUAZIOA ETA GRASHOFEN ZENBAKIA

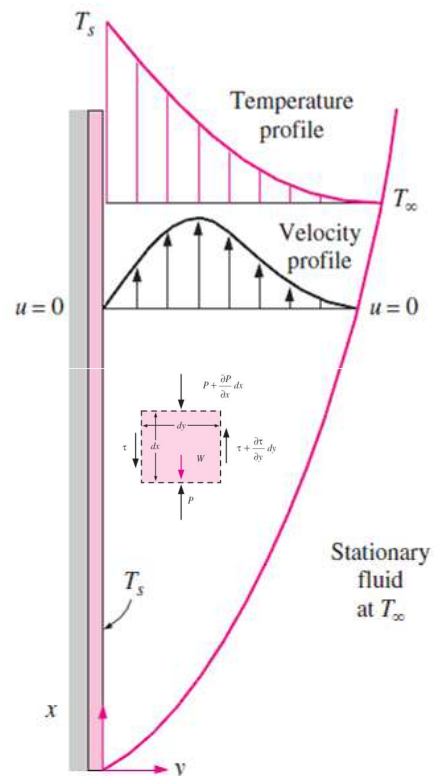
AIRETAN DAGOEN XAFLA BERO BATEN GAINEKO KONBEKZIO NATURALAREN ISOTERMAK ETA ABIADURA PROFILAK



(a) Laminar flow



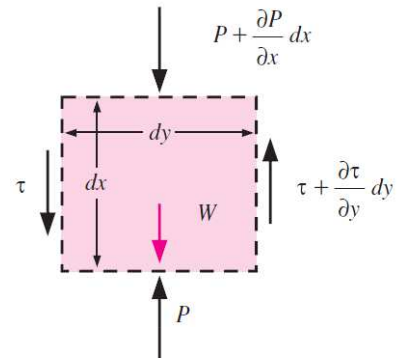
(b) Turbulent flow



6 GAIA (13/22 diapositiba), **KONBEKZIO BEHARTUARENTZAKO MOMENTUAREN KONTSERBAZIOA** (indarren balantzea)

Jariakinaren masaren azalera zioa Marruskadura indarrak Presio indarrak (haizegailua edo ponpa)

$$\rho \left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = \mu \frac{\partial^2 u}{\partial y^2} - \frac{\partial P}{\partial x}$$



KONBEKZIO NATURALARENTZAKO MOMENTUAREN KONTSERBAZIOA (indarren balantzea)

$$\rho \left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = \mu \frac{\partial^2 u}{\partial y^2} - \frac{\partial P}{\partial x} + \rho g$$

Goranzko bultzadaren indarrak

PRESIOARI DAGOZKION INDARRAK BAKARRIK HIDROSTATIKOAK DIRA $\Rightarrow \frac{\partial P_\infty}{\partial x} = -\rho_\infty g$

TERMOTEKNIA

KONBEKZIO NATURALA GOBERNATZEN DUEN EKUAZIO DIFERENTZIALEN LABURPENA

MASAREN KONTSERBAZIOA (6 Gaia - 12/22 diapositiba)

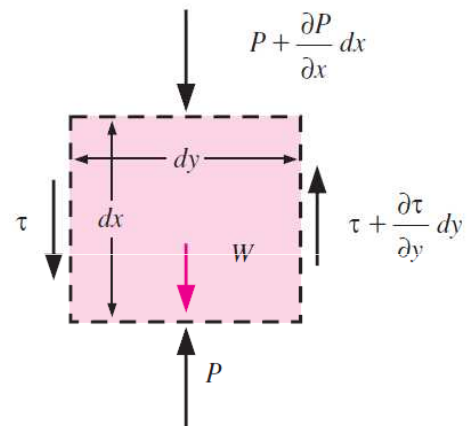
$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

MOMENTUAREN KONTSERBAZIOA

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{\mu}{\rho} \frac{\partial^2 u}{\partial y^2} + g\beta(T - T_\infty)$$

ENERGIAREN KOPNTSERBAZIOA (6 Gaia - 15/22 diapositiba)

$$\rho \cdot c_p \cdot \left(u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = k \cdot \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right)$$



TERMOTEKNIA

GRASHOFEN ZENBAKIA

HIRU EKUAZIO DIFERENTZIALAK ETA MUGALDEKO BALDINTZAK DIMENSTIOGABETUZ

$$x^* = \frac{x}{L_c} \quad y^* = \frac{y}{L_c} \quad u^* = \frac{u}{V_\infty} \quad v^* = \frac{v}{V_\infty} \quad \text{eta} \quad T^* = \frac{T - T_\infty}{T_s - T_\infty}$$

MOMENTUAREN EKUAZIO DIFERENTZIAL DIMENSTIOGABETUA

$$u^* \frac{\partial u^*}{\partial x^*} + v^* \frac{\partial u^*}{\partial y^*} = \left[\frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2} \right] \frac{T^*}{Re_L^2} + \frac{1}{Re_L} \frac{\partial^2 u^*}{\partial y^{*2}}$$

GRASHOFEN ZENBAKIA

$$Gr_L = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2}$$

- Konbekzio naturalaren efektuak adierazten du.
- Goranzko bultzaden eta biskositate-indarren arteko erlazioa adierazten du.

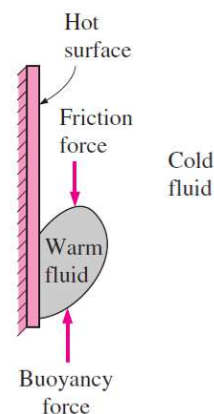
OHARRA: Reynolds (Re) inertzia-indar eta biskositate-indarren arteko erlazioa adierazten du

TERMOTEKNIKIA

9.2 - HIGIDURAREN EKUAZIOA ETA GRASHOFEN ZENBAKIA

GRASHOFEN ZENBAKIA

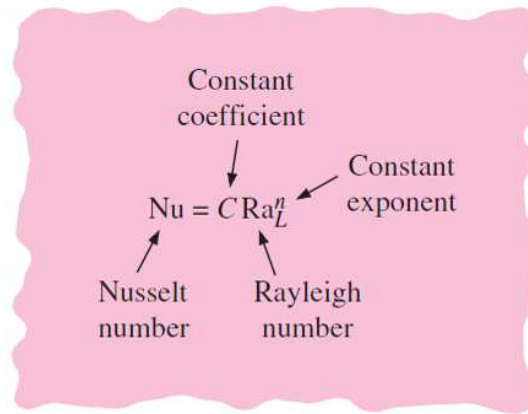
- Grashofen zenbakia da fluidoan eragiten duten goranzko bultzadaren eta kontrako biskositate-indarraren magnitude erlatiboan neurria.
- Konbekzio naturalean fluxua laminarra edo turbulenta den adierazteko balio du.



KONBEKZIO NATURALA vs. KONBEKZIO BEHARTUA

- Konbekzio naturalaren eragina baztergarria da $Gr_L/Re_L^2 < 0,1$ baldin bada.
- Konbekzio behartuaren eragina baztergarria da $Gr_L/Re_L^2 > 10$ baldin bada.
- Bien eragina esanguratsua da eta kontuan hartu behar dira $0,1 > Gr_L/Re_L^2 > 10$ baldin bada

TERMOTEKNIKIA



$$Nu = \frac{hL_c}{k} = C(Gr_L Pr)^n = C Ra_L^n$$

FLUIDOAREN PROPIETATEAK
GERUZA-TEMPERATURAN

$$Ra_L = Gr_L Pr = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2} Pr$$

$$T_f = \frac{T_s + T_\infty}{2}$$

$$\dot{Q}_{conv} = hA_s(T_s - T_\infty) \quad (W)$$

9.3 - GAINAZALEN GAINEKO KONBEKZIO NATURALA

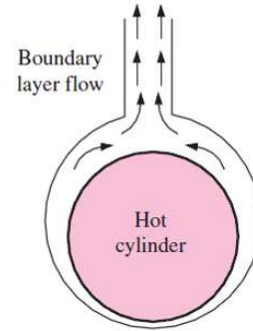
Empirical correlations for the average Nusselt number for natural convection over surfaces

Geometry	Characteristic length L_c	Range of Ra	Nu
Vertical plate 	L	10^4-10^9 10^9-10^{13} Entire range	$Nu = 0.59Ra_L^{1/4}$ (9-19) $Nu = 0.1Ra_L^{1/3}$ (9-20) $Nu = \left\{ 0.825 + \frac{0.387Ra_L^{1/6}}{[1 + (0.492/Pr)^{9/16}]^{1/4}} \right\}^2$ (9-21) (complex but more accurate)
Inclined plate 	L		Use vertical plate equations for the upper surface of a cold plate and the lower surface of a hot plate Replace g by $g \cos\theta$ for $Ra < 10^9$
Horizontal plate (Surface area A and perimeter p) (a) Upper surface of a hot plate (or lower surface of a cold plate) 	A_s/p	10^4-10^7 10^7-10^{11}	$Nu = 0.54Ra_L^{1/4}$ (9-22) $Nu = 0.15Ra_L^{1/3}$ (9-23)
(b) Lower surface of a hot plate (or upper surface of a cold plate) 		10^5-10^{11}	$Nu = 0.27Ra_L^{1/4}$ (9-24)
Vertical cylinder 	L		A vertical cylinder can be treated as a vertical plate when $D \geq \frac{35L}{Gr_L^{1/4}}$
Horizontal cylinder 	D	$Ra_D \approx 10^{12}$	$Nu = \left\{ 0.6 + \frac{0.387Ra_D^{1/6}}{[1 + (0.559/Pr)^{9/16}]^{1/4}} \right\}^2$ (9-25)
Sphere 	D	$Ra_D \approx 10^{11}$ $(Pr \approx 0.7)$	$Nu = 2 + \frac{0.589Ra_D^{1/4}}{[1 + (0.469/Pr)^{9/16}]^{1/4}}$ (9-26)

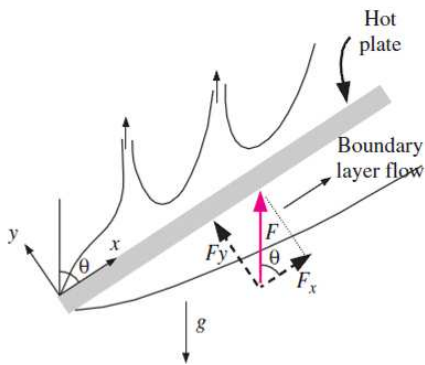
NOLA MUGITZEN DA JARIAKINA?



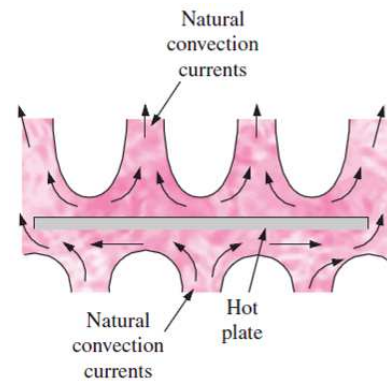
ZILINDRO HORIZONTALA



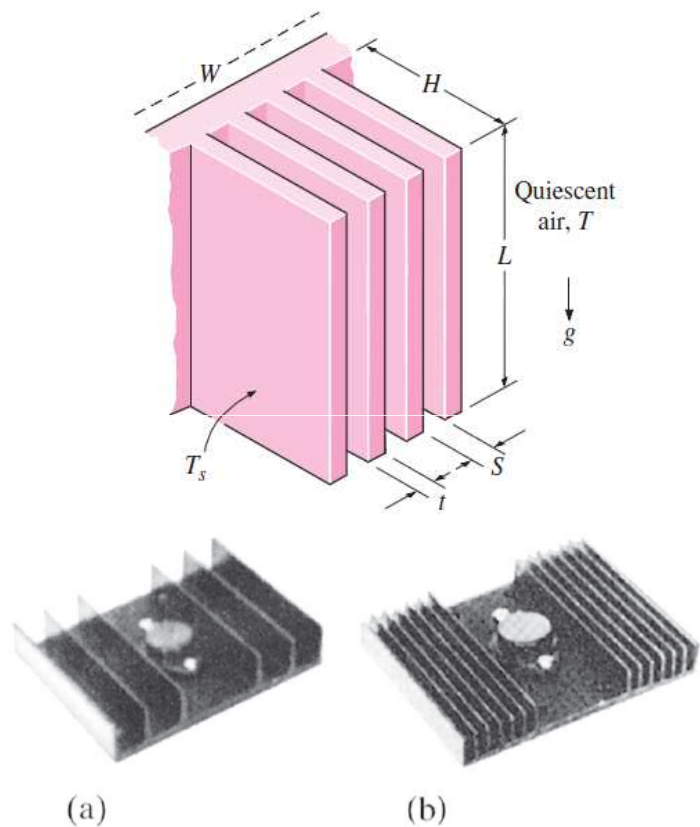
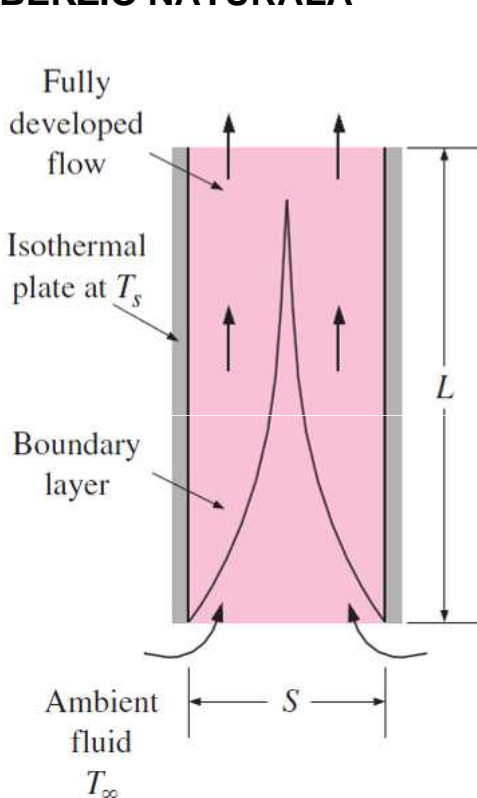
XAFLA INKLINATUA



XAFLA HORIZONTALA



9.4 – GAINAZAL HEGALDUNEN ETA ZIRKUITU INPRIMATUEN KONBEKZIO NATURALA



9.4 - GAINAZAL HEGALDUNEN ETA ZIRKUITU INPRIMATUEN KONBEKZIO NATURALA

15/24

KONBEKZIO NATURAL BIDEZKO HOZTEA GAINAZAL HEGALDUNETAN ($T_s = kte$)

$$Ra_s = \frac{g\beta(T_s - T_\infty)S^3}{\nu^2} Pr \quad \text{eta} \quad Ra_L = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2} Pr = Ra_s \frac{L^3}{S^3}$$

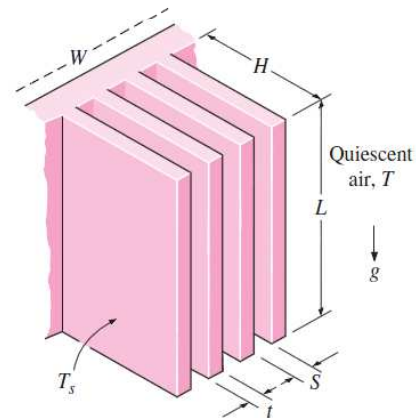
$$T_s = kte: \quad Nu = \frac{hS}{k} = \left[\frac{576}{(Ra_s S/L)^2} + \frac{2,873}{(Ra_s S/L)^{0,5}} \right]^{-0,5}$$

$$S_{opt} = 2,714 \left(\frac{S^3 L}{Ra_s} \right)^{0,25} = 2,714 \frac{L}{Ra_L^{0,25}}$$

$$Nu = \frac{hS_{opt}}{k} = 1,307$$

$$\dot{Q} = h(2nLH)(T_s - T_\infty)$$

$$non \begin{cases} n = W / (S + t) \approx W / S \\ T_f = \frac{T_s + T_\infty}{2} \end{cases}$$



9.4 - GAINAZAL HEGALDUNEN ETA ZIRKUITU INPRIMATUEN KONBEKZIO NATURALA

16/24

KONBEKZIO NATURAL BIDEZKO HOZTEA ZIRKUITU INPRIMATU BERTIKALETAN ($q_s = kte$)

$$Ra_s^* = \frac{g\beta \dot{q}_s S^4}{k\nu^2} Pr$$

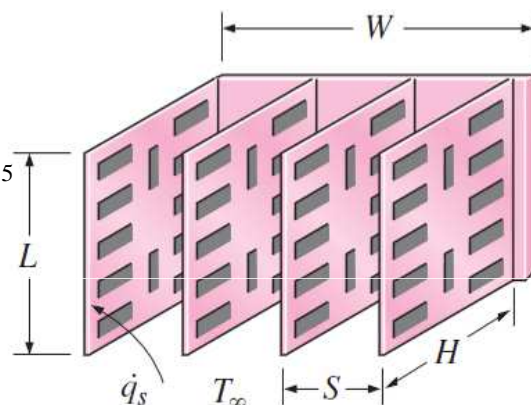
$$Nu_L = \frac{h_L S}{k} = \left[\frac{48}{Ra_s^* S/L} + \frac{2,51}{(Ra_s^* S/L)^{0,4}} \right]^{-0,5}$$

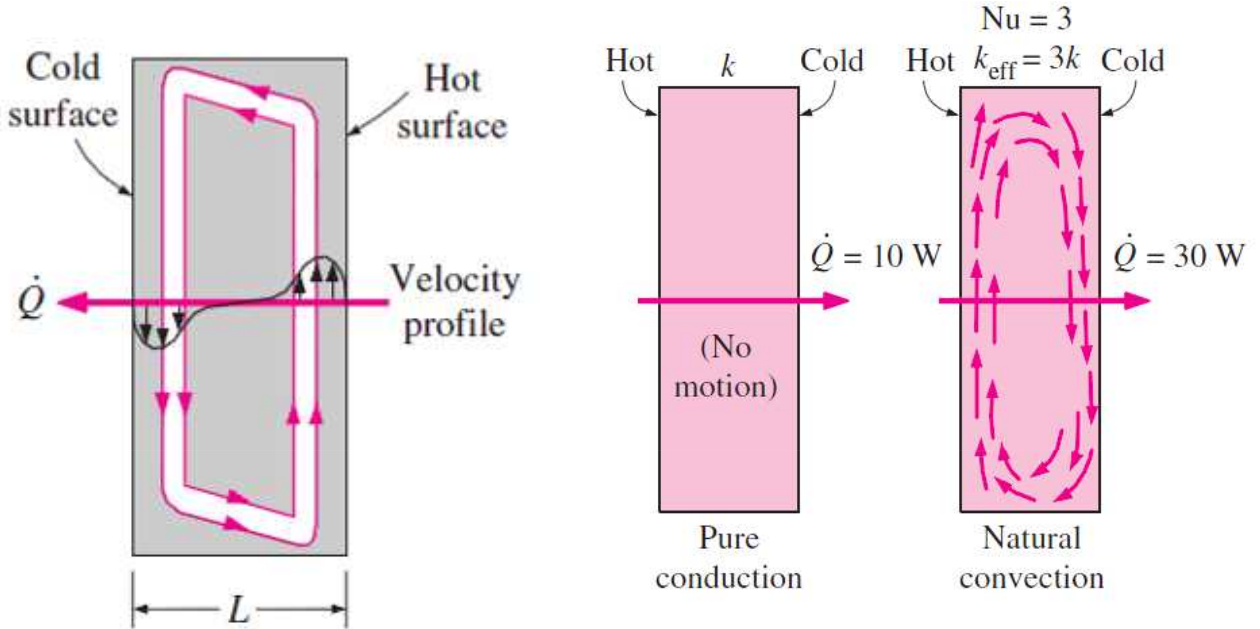
$$q_s = kte: \quad S_{opt} = 2,12 \left(\frac{S^4 L}{Ra_s^*} \right)^{0,2}$$

$$\dot{Q} = \dot{q}_s \dot{A}_s = \dot{q}_s (2nLH)$$

$$\dot{q}_s = h_L (T_L - T_\infty)$$

$$non \begin{cases} n = W / (S + t) \approx W / S \\ T_f = \frac{T_L + T_\infty}{2} \end{cases}$$



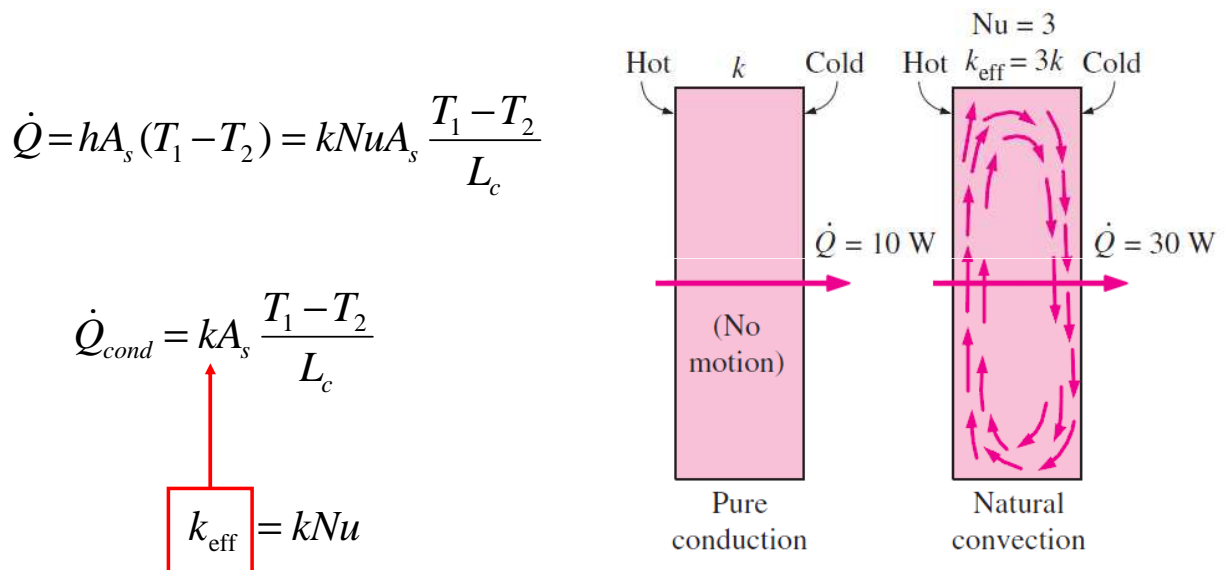


- L_c gainazal bero eta hotzaren arteko distantzia da.
- T_1 eta T_2 gainazal beroaren eta gainazal hotzaren temperaturak dira hurrenez-hurren.
- 9.5 PUNTUKO KASU GUZTIETAN: Jariakinaren propietateak batz besteko tenperaturan kalkulatu behar dira.

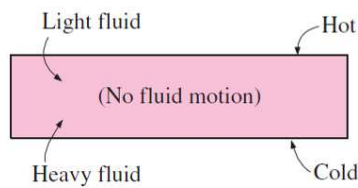
$$Ra_L = \frac{g\beta(T_1 - T_2)L_c^3}{\nu^2} Pr$$

$$T_{avg} = \frac{(T_1 + T_2)}{2}$$

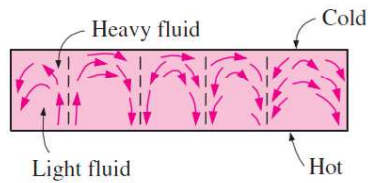
EROANKORTASUN TERMIKO ERAGINKORRA



ITXITURA ANGELUZUZEN HORIZONTALAK



(a) Hot plate at the top

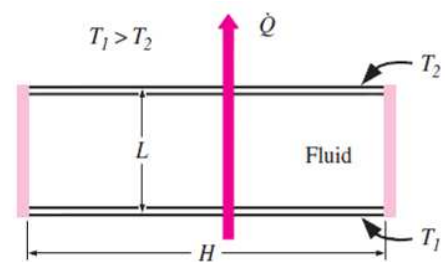


(b) Hot plate at the bottom

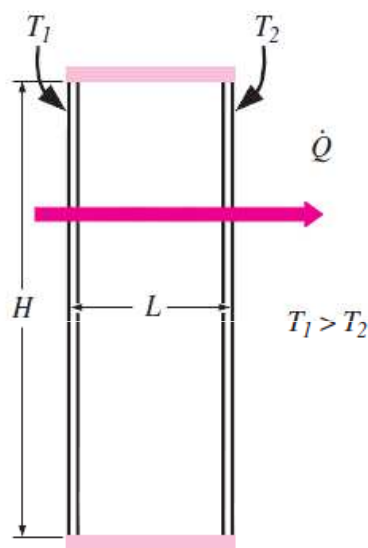
- Xafla beroena goian dagoen kasuetarako, bero transferentzia beherantz gertatzen da eroapenez ez baita konbekzio-korronterik izaten ($Nu=1$).

$$Nu = 1 + 1,44 \left[1 - \frac{1708}{Ra_L} \right]^+ + \left[\frac{Ra_L^{1/3}}{18} - 1 \right]^+ \quad Ra_L < 10^8$$

[]⁺ notazioak adierazten du kortxete arteko kantitatea negatiboa bada, zero jarri beharko litzatekeela.



ITXITURA ANGELUZUZEN BERTIKALAK



$$Nu = 0,18 \left(\frac{Pr}{0,2 + Pr} Ra_L \right)^{0,29}$$

$1 < H / L < 2$
any Prandtl number
 $Ra_L Pr / (0,2 + Pr) > 10^3$

$$Nu = 0,22 \left(\frac{Pr}{0,2 + Pr} Ra_L \right)^{0,28} \left(\frac{H}{L} \right)^{-1/4}$$

$2 < H / L < 10$
any Prandtl number
 $Ra_L < 10^{10}$

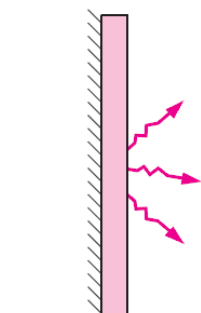
$$Nu = 0,42 Ra_L^{1/4} Pr^{0,012} \left(\frac{H}{L} \right)^{-0,3}$$

$10 < H / L < 40$
 $1 < Pr < 2 \times 10^4$
 $10^4 < Ra_L < 10^7$

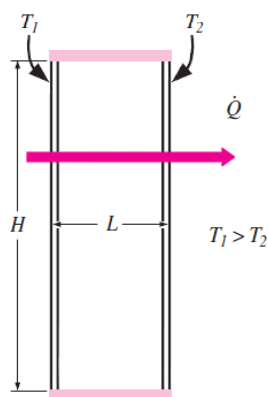
$$Nu = 0,46 Ra_L^{1/3}$$

$1 < H / L < 40$
 $1 < Pr < 20$
 $10^6 < Ra_L < 10^9$

KONBEKZIO NATURALAREN ETA ERRADIAZIOAREN KONBINAZIOA



$$\dot{Q}_{total} = \dot{Q}_{conv} + \dot{Q}_{rad} \begin{cases} \dot{Q}_{rad} = \epsilon \sigma A_s (T_s^4 - T_{surr}^4) \\ \dot{Q}_{conv} = h A_s (T_s - T_\infty) \end{cases}$$



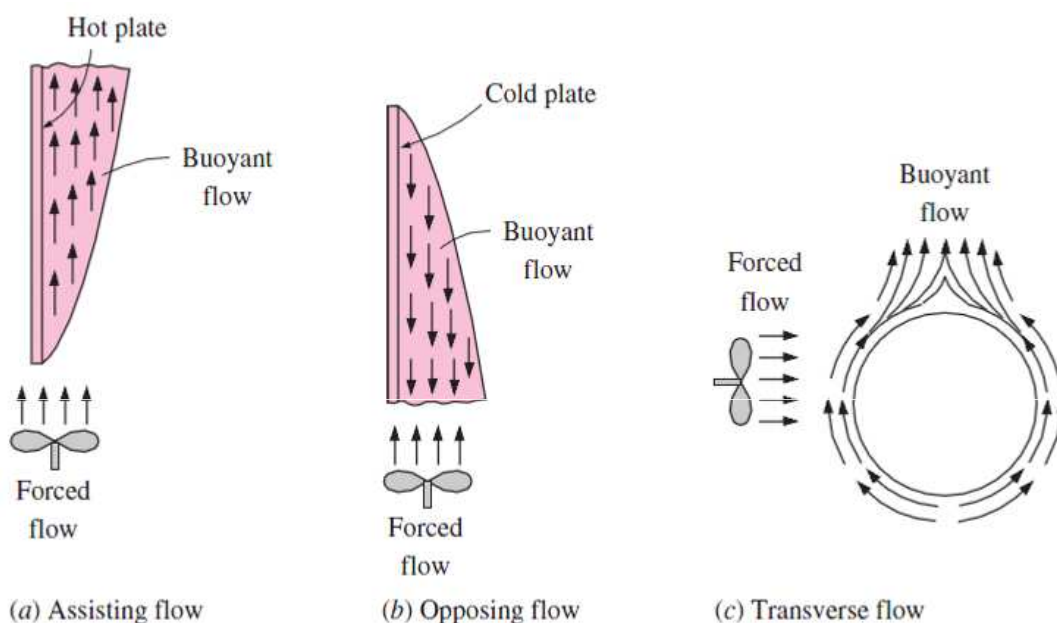
$$\dot{Q}_{total} = \dot{Q}_{conv} + \dot{Q}_{rad}$$

$$\dot{Q}_{rad} = \frac{\sigma A_s (T_1^4 - T_2^4)}{1/\epsilon_1 + 1/\epsilon_2 - 1} = \epsilon_{effective} \sigma A_s (T_1^4 - T_2^4)$$

$$\epsilon_{effective} = \frac{1}{1/\epsilon_1 + 1/\epsilon_2 - 1}$$

$$\dot{Q}_{conv} = h A_s (T_1 - T_2)$$

9.6 – KONBEKZIO NATURAL ETA BEHARTUAREN KONBINAZIOA



Konbekzio naturalak areagotu edo inhibitu egin dezake konbekzio bidezko bero-transferentzia, goranzko bultzadak eragindako mugimenduaren eta konbekzio behartuak eragindako mugimenduaren noranzko erlatiboaren arabera

- Konbekzio naturalaren eragina baztergarria da $Gr_L/Re_L^2 < 0,1$ baldin bada.
- Konbekzio behartuaren eragina baztergarria da $Gr_L/Re_L^2 > 10$ baldin bada.
- Bien eragina esanguratsua da eta kontuan hartu behar dira $0,1 > Gr_L/Re_L^2 > 10$ baldin bada.

$$Nu_{combined} = (Nu_{forced}^n \pm Nu_{natural}^n)^{1/n} \quad \text{where } 3 < n < 4$$

- Aldeko fluxuan, bultzadak eragindako mugimendua mugimendu behartuaren **noranzko berekoa** da. Horrenbestez, konbekzio naturalak konbekzio behartuaren **alde** egiten du, eta bero-transferentzia areagotzen du.
- Kontrako fluxuan, bultzadak eragindako mugimenduak mugimendu behartuaren **kontrako noranzkoa** du. Horrenbestez, konbekzio naturalak konbekzio behartua **oztopatzen** du, eta bero-transferentzia txikitzen du.
- Zeharkako fluxuan, bultzadaren mugimendua mugimendu behartuarekiko perpendikularra da. Zeharkako fluxuak gehiago nahasten du fluidoa, eta, beraz, areagotu egiten du bero-transferentzia.

9.7 – IRAKATSIKO EZ DIREN ATALAK

- 9.4eko azpiatala: XAFLEN ARTEKO ESPAZIOKO MASA-EMARIA
- 9.5eko azpiatala: ITXITURA ANGELUZUZEN INKLINATUAK
- 9.5eko azpiatala: ZILINDRO ZENTROKIDEAK
- 9.5eko azpiatala: ESFERA ZENTROKIDEAK

10. GAIA

IRAKITEA ETA KONDENTSAZIOA

10.0 - HELBURUAK

2/28

- **Lurrunketa** eta **irakitea** desberdindu, eta zenbait irakite modu bereizi,
- **Irakite-kurba** eta irakite-kurbaren eskualdeetako irakite-erregimenak ondo ulertu
- **Bero-fluxua** kalkulatu eta, orobat, **irakite nukleatuari** lotutako balio kritikoa, eta irakiteprozesuko bero-transferentzia indartzeko metodoak aztertu
- Xafla bertikalen gaineko **geruza-erako kondentsazio laminarreko** bero-transferentziaren koefizientearen erlazioa garatu
- Xafla inklinatuetan eta horizontaletan, zilindro bertikaletan eta horizontaletan, esferetan eta hodi multzoetan gertatzen den **kondentsazioko bero-fluxua** kalkulatu
- **Tanta-erako kondentsazioa** aztertu eta haren ziurgabetasun batzuk ulertu.



Lurrunketa

Likido-lurrun faseartea

Lurrun-presioa <math>P < P_{sat}</math> ($T_{jariakin}$)

Irakitea

Solido-likido faseartea

Gainazalarekin kontaktuan dagoen likidoa $T_s > T_{sat}$

Lurrun-burbuilak azkar sortzen dira, zeinak gainazaletik askatzen baitira neurri jakin bat hartzen dutenean eta likidoaren gainazal askera igotzen saiatzen baitira

$$\dot{q}_{boiling} = h \cdot (T_s - T_{sat}) = h \cdot \Delta T_{excess} \quad [W/m^2]$$

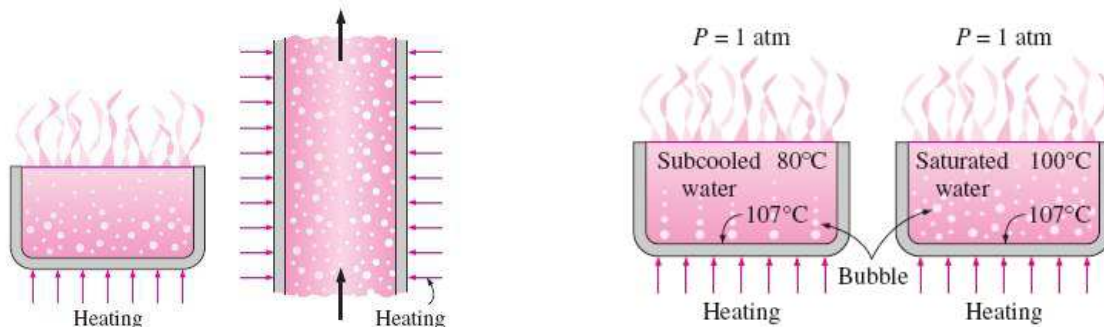
Konbekzio behartu eta naturala (fase bakarra) Propietateak: μ, ρ, k eta c_p

Irakitea Propietateak: μ, ρ, k eta c_p (likido eta lurrunarena).

h_{fg} = Lurruntze-bero sorra.

σ = Gainazal-tentsioa.

Oreka-termodinamiko eza: $P_v \neq P_l$ $T_v \neq T_l$



Tanga-erako irakitea

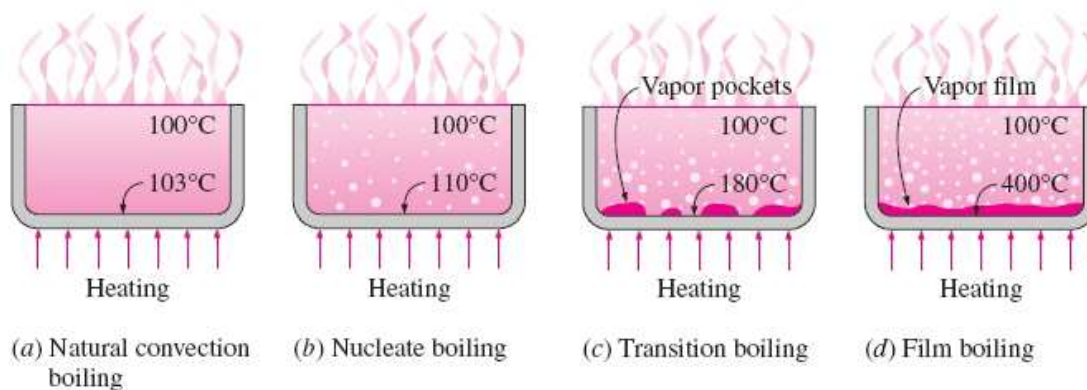
Fluxu-erako irakitea (irakitea konbekzio behartuan)

Irakite azpihoztua (lokala)

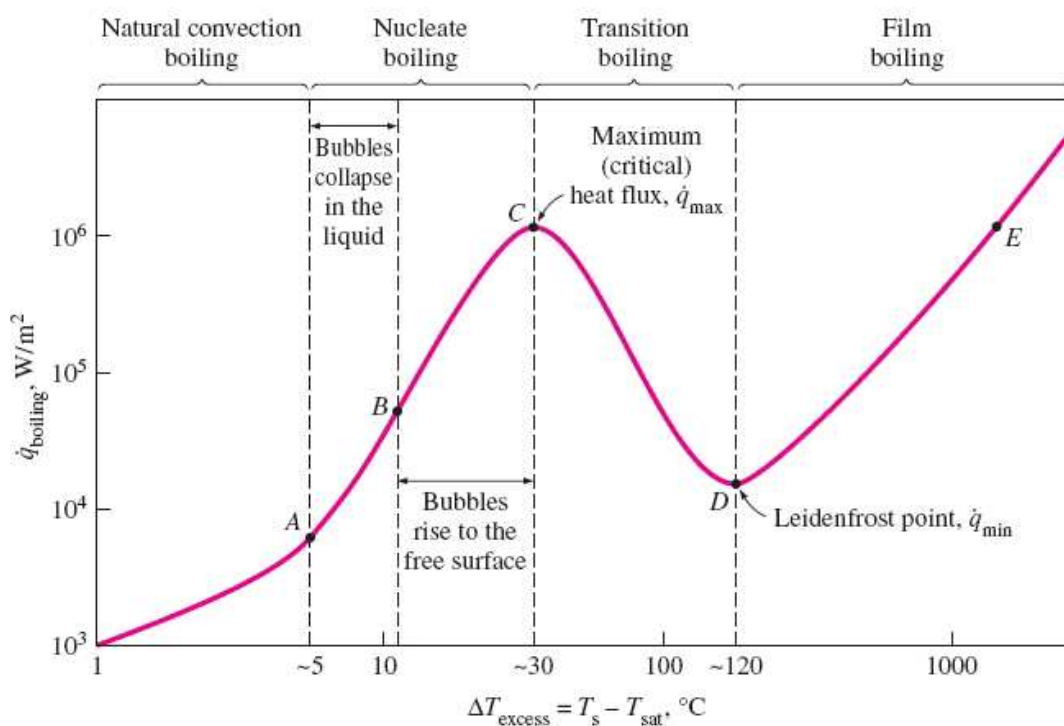
Irakite ase (globala)

IRAKITE-ERREGIMENAK ETA IRAKITE-KURBA

Irakite prozesuak ΔT_{excess} -ren arabera forma desberdinak har ditzake



IRAKITE-ERREGIMENAK ETA IRAKITE-KURBA



IRAKITE-ERREGIMENAK ETA IRAKITE-KURBA

Konbekzio naturaleko irakitea (A puntura arte)

Likidoaren gainberoketa txikia (2-6 °C).

Konbekzio naturala

Irakite nukleatua (A eta C puntuen artean)

Lehen burbuilak sortzen dira.

2 eskualde desberdin:

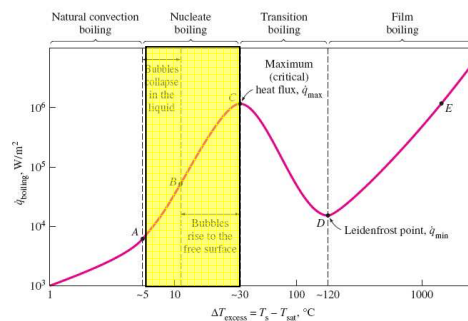
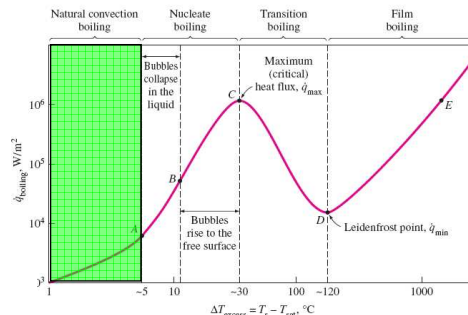
A-B: Burbuila isolatuak.

Likidoaren mugimendua

B-C: Lurrun-zutabe jarraituak sortzen dira.

Likidoaren mugitzea + lurrunketa efektuak

C: Bero-fluxu kritikoa \dot{q}_{max}



IRAKITE-ERREGIMENAK ETA IRAKITE-KURBA

Trantsizio-irakitea (C eta D puntuen artean)

Lurrun-geruza batek gainazalaren parte bat estaltzen du.

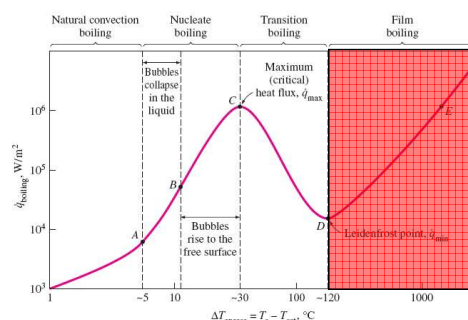
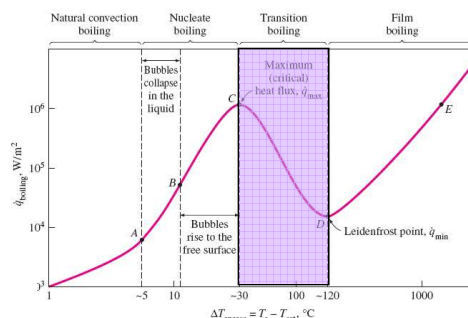
Irakite nukleatutik → Geruza-erako irakitera.

Geruza-erako irakitea (D puntutik aurrera)

Lurrun geruza jarraitu eta egonkorra.

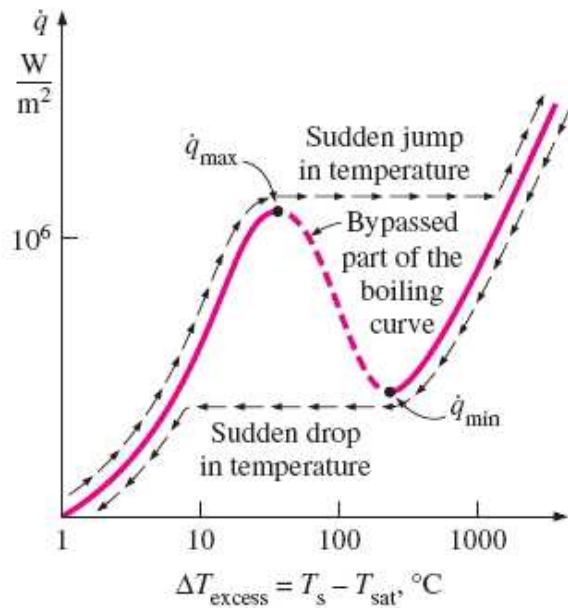
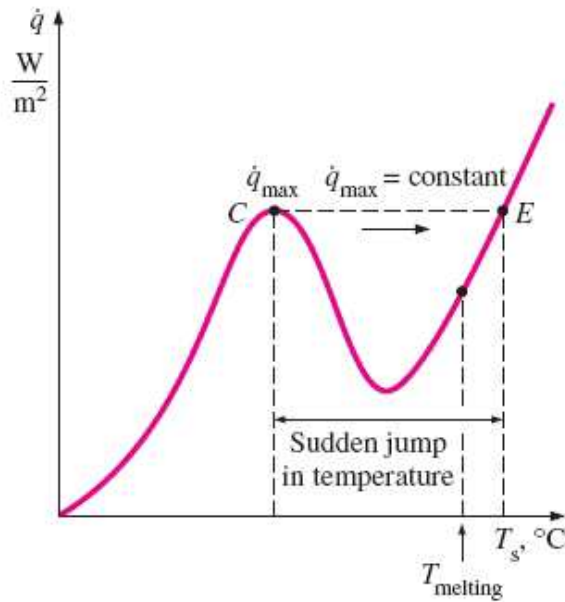
D: Leidenfrost puntua \dot{q}_{min}

Lurrun-geruzan erradiazio bero-transferentzia.



IRAKITE-ERREGIMENAK ETA IRAKITE-KURBA

Irakite-kurba errealak



TANGA-ERAKO KORRELAZIOAK

IRAKITE-PROZESUKO

BERO-TRANSFERENTZIAKO

Konbekzio naturaleko irakitea → 9. GAIA

Irakite nukelatua

Rohsenow (propietateak T_{sat}) →
$$\dot{q}_{nucleate} = \mu_l \cdot h_{fg} \cdot \left[\frac{g \cdot (\rho_l - \rho_v)}{\sigma} \right]^{1/2} \left[\frac{C_{pl} \cdot (T_s - T_{sat})}{C_{sf} \cdot h_{fg} \cdot Pr_l^n} \right]^3$$

Errorea:
 ± 100 % q-n
 ± 30 % ΔT_{excess} -n

- $\dot{q}_{nucleate}$ = nucleate boiling heat flux, W/m²
- μ_l = viscosity of the liquid, kg/m · s
- h_{fg} = enthalpy of vaporization, J/kg
- g = gravitational acceleration, m/s²
- ρ_l = density of the liquid, kg/m³
- ρ_v = density of the vapor, kg/m³
- σ = surface tension of liquid–vapor interface, N/m
- C_{pl} = specific heat of the liquid, J/kg · °C
- T_s = surface temperature of the heater, °C
- T_{sat} = saturation temperature of the fluid, °C
- C_{sf} = experimental constant that depends on surface–fluid combination
- Pr_l = Prandtl number of the liquid
- n = experimental constant that depends on the fluid

**TANGA-ERA KO
KORRELAZIOAK**

Surface tension of liquid–vapor interface for water

T, °C	σ, N/m*
0	0.0757
20	0.0727
40	0.0696
60	0.0662
80	0.0627
100	0.0589
120	0.0550
140	0.0509
160	0.0466
180	0.0422
200	0.0377
220	0.0331
240	0.0284
260	0.0237
280	0.0190
300	0.0144
320	0.0099
340	0.0056
360	0.0019
374	0.0

IRAKITE-PROZESUKO

BERO-TRANSFERENTZIAKO

Substance and Temp. Range	Surface Tension, σ, N/m* (T in °C)
Ammonia, –75 to –40°C:	0.0264 + 0.000223T
Benzene, 10 to 80°C:	0.0315 – 0.000129T
Butane, –70 to –20°C:	0.0149 – 0.000121T
Carbon dioxide, –30 to –20°C:	0.0043 – 0.000160T

Values of the coefficient C_{sf} and n for various fluid–surface combinations

Fluid-Heating Surface Combination	C_{sf}	n
Water–copper (polished)	0.0130	1.0
Water–copper (scored)	0.0068	1.0
Water–stainless steel (mechanically polished)	0.0130	1.0
Water–stainless steel (ground and polished)	0.0060	1.0
Water–stainless steel (teflon pitted)	0.0058	1.0
Water–stainless steel (chemically etched)	0.0130	1.0
Water–brass	0.0060	1.0
Water–nickel	0.0060	1.0
Water–platinum	0.0130	1.0
n-Pentane–copper (polished)	0.0154	1.7

**TANGA-ERA KO
KORRELAZIOAK**

IRAKITE-PROZESUKO

BERO-TRANSFERENTZIAKO

Puntako bero-fluxua (\dot{q}_{max})

Kutateladze y Zuber

$$\dot{q}_{max} = C_{cr} \cdot h_{fg} \cdot [\sigma \cdot g \cdot \rho_v^2 (\rho_l - \rho_v)]^{1/4}$$

Values of the coefficient C_{cr} for use in Eq. 10–3 for maximum heat flux (dimensionless parameter $L^* = L[g(\rho_l - \rho_v)/\sigma]^{1/2}$)

Heater Geometry	C_{cr}	Charac. Dimension of Heater, L	Range of L^*
Large horizontal flat heater	0.149	Width or diameter	$L^* > 27$
Small horizontal flat heater ¹	$18.9K_1$	Width or diameter	$9 < L^* < 20$
Large horizontal cylinder	0.12	Radius	$L^* > 1.2$
Small horizontal cylinder	$0.12L^{*-0.25}$	Radius	$0.15 < L^* < 1.2$
Large sphere	0.11	Radius	$L^* > 4.26$
Small sphere	$0.227L^{*-0.5}$	Radius	$0.15 < L^* < 4.26$

¹ $K_1 = \sigma/[g(\rho_l - \rho_v)A_{heater}]$

TANGA-ERAKO
KORRELAZIOAK

IRAKITE-PROZESUKO

BERO-TRANSFERENTZIAKO

Bero-fluxu minimoa (\dot{q}_{min})

Zuber
(xafla handi
horizontala)

$$\dot{q}_{min} = 0,09 \cdot \rho_v \cdot h_{fg} \cdot \left[\frac{\sigma \cdot g \cdot (\rho_l - \rho_v)}{(\rho_l + \rho_v)^2} \right]^{\frac{1}{4}}$$

Errorea ~ 50 % edo handiagoa

Geruza-erako irakitea

Bromley
(zilindro horizontala
edo esfera)

$$\dot{q}_{film} = C_{film} \left[\frac{g k_v^3 \rho_v (\rho_l - \rho_v) [h_{fg} + 0,4 \cdot C_{pv} (T_s - T_{sat})]}{\mu_v D (T_s - T_{sat})} \right]^{\frac{1}{4}}$$

$$C_{film} = \begin{cases} 0,62 \text{ zilindro horizontaletan} \\ 0,67 \text{ esferetan} \end{cases}$$

Propietateak $\begin{cases} \text{Lurrun-geruzaren temperatura} & T_f = \frac{T_s + T_{sat}}{2} \\ \text{Likidoa eta } h_{fg} \text{ asetasun temperatura.} \end{cases}$

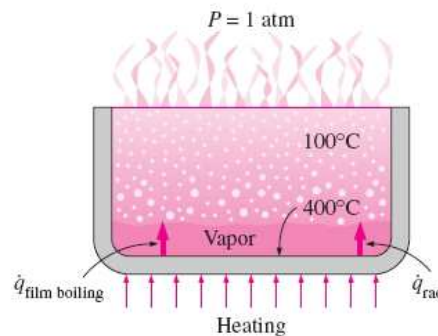
TANGA-ERAKO
KORRELAZIOAK

IRAKITE-PROZESUKO

BERO-TRANSFERENTZIAKO

Geruza-erako irakitea

• $T_s \uparrow \rightarrow$ Erradiazioa



$$\dot{q}_{rad} = \epsilon \cdot \sigma \cdot (T_s^4 - T_{sat}^4)$$

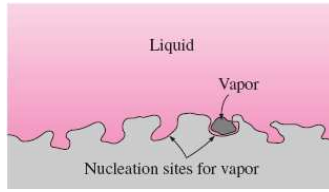
Baldin $\dot{q}_{rad} < \dot{q}_{film}$

Bromley

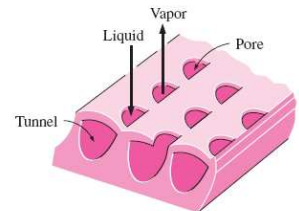
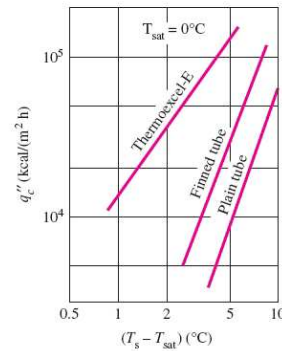
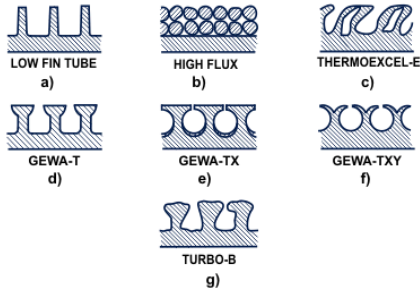
$$\dot{q}_{total} = \dot{q}_{film} + \frac{3}{4} \dot{q}_{rad}$$

TANGA-ERAKO IRAKITE-PROZESUKO BERO-TRANSFERENTZIA INDARTZEZA

Zimurtasuna eta hautsa
(gainazal irregulartasunak)



Bero-transferentzia hobetzen duten gainazalak



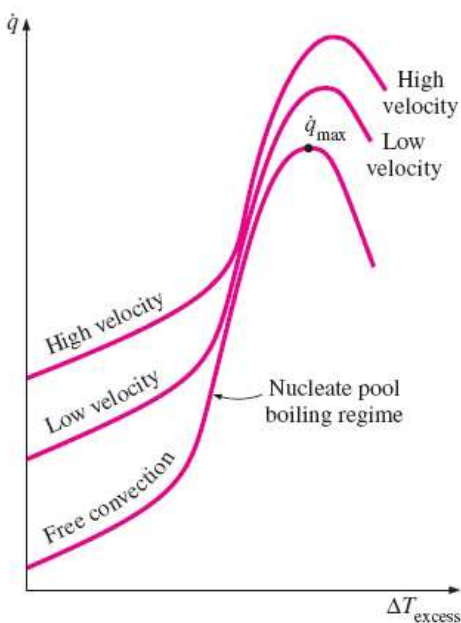
Mekanikoki eraginda

Gainazal-bibrazioa

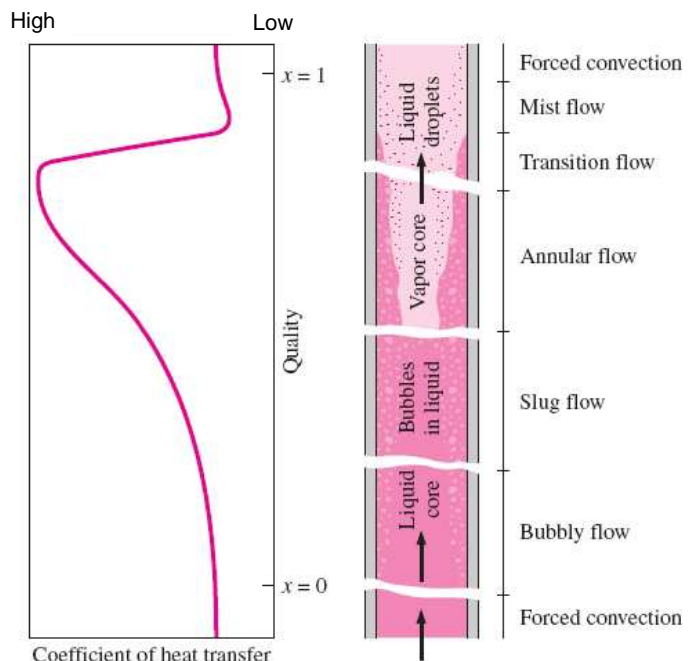
10.3 – FLUXU-ERAKO IRAKITEA

Konbekzio behartua eta tanga-erako irakitearen efektu bateratuak

Kanpo fluxu-erako irakitea



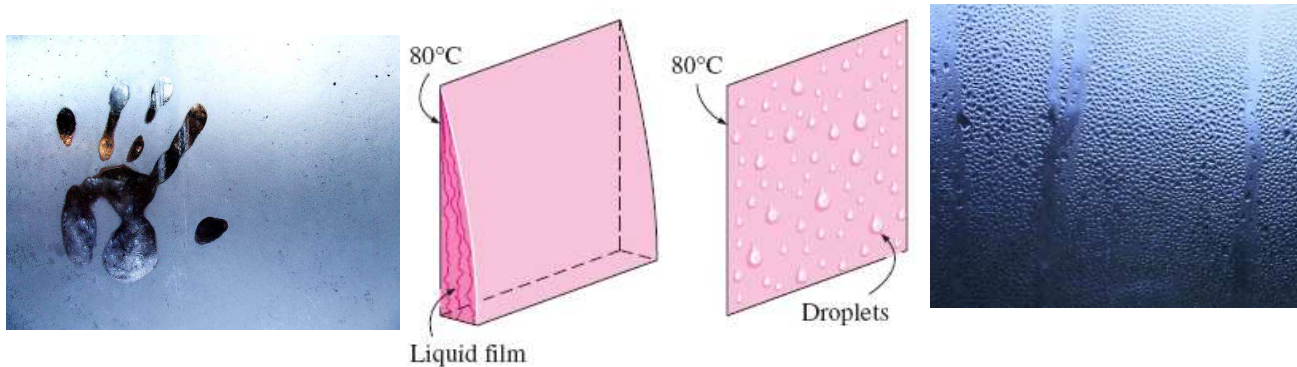
Barne fluxu-erako irakitea



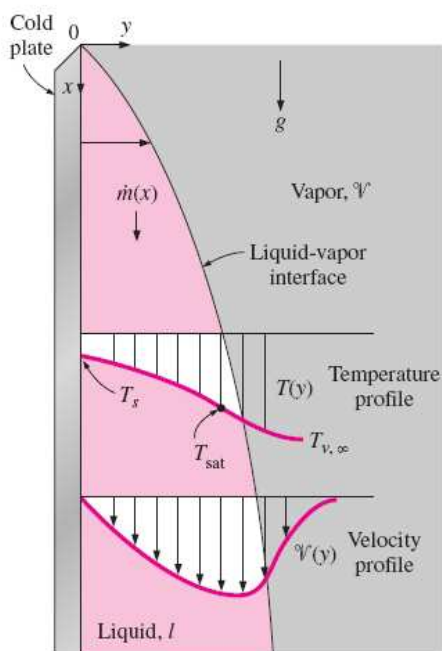
Estimazio gordina:

$$\dot{q}_{flow\ boiling} = \dot{q}_{forced\ convection} + \dot{q}_{pool\ boiling}$$

- Lurrin baten temperatura haren T_{sat} -tik behera jaistean.
- Gainazal solidoen gaineko kondentsazioa.
- Bi motakoak: **Geruza-erako kondentsazioa** **Tanta-erako kondentsazioa**



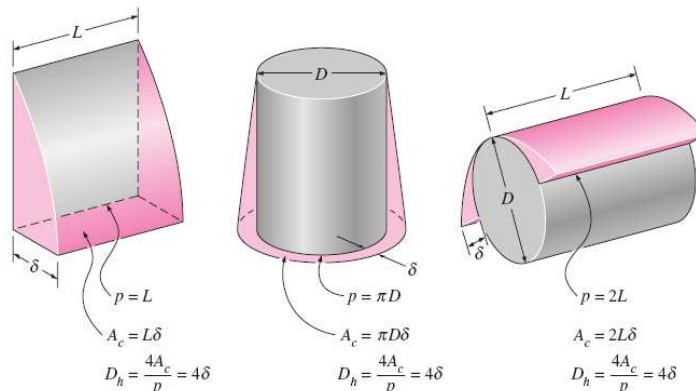
10.5 – GERUZA-ERAKO KONDENTSAZIOA



	T	V
Solido – likido faseartea	T_s	0
Likido – lurrin faseartea	T_{sat}	V_{max}

$$Re = \frac{D_h \rho_l V_l}{\mu_l} = \frac{4 \cdot \dot{m}}{p \mu_l}$$

\dot{m} = Beheko aldeko kondentsatuaren masa emaria [kg/s]



Kondentsatuak T_s eta T_{sat} tarteko tenperatura batera azpizozten da.

Rohsenow
(Lurrunketa bero sorra eraldatua)

$$h_{fg}^* = h_{fg} + 0,68c_{pl}(T_{sat} - T_s)$$

T_v tenperatura bateko lurrun gainberotuan:

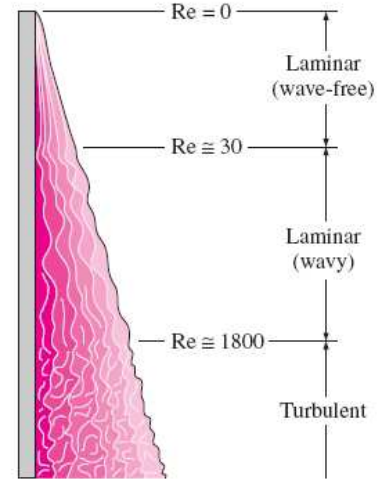
$$h_{fg}^* = h_{fg} + 0,68c_{pl}(T_{sat} - T_s) + c_{pv}(T_v - T_{sat})$$

$$\dot{Q}_{condens} = h \cdot A_s \cdot (T_{sat} - T_s) = \dot{m} \cdot h_{fg}^*$$

$$Re = \frac{4 \cdot \dot{Q}_{condens}}{p \cdot \mu_l \cdot h_{fg}^*} = \frac{4 \cdot h \cdot A_s \cdot (T_{sat} - T_s)}{p \cdot \mu_l \cdot h_{fg}^*}$$

Likidoaren propietateak: $T_f = \frac{T_{sat} + T_s}{2}$

$$h_{fg} = h_{fg}(T_{sat})$$



GERUZA-ERA KO KONDENTSATZIOKO BERO-TRANSFERENTZIAREN KORRELAZIOAK

1 – Xafila bertikalak – Fluxu laminarra

T_s eta T_{sat} konstanteak. Kondentsatuan aldaketa lineala.

Bero-transferentzia eroapen bidezkoa.

Lurrunaren abiadura txikia (arrasterik gabe)

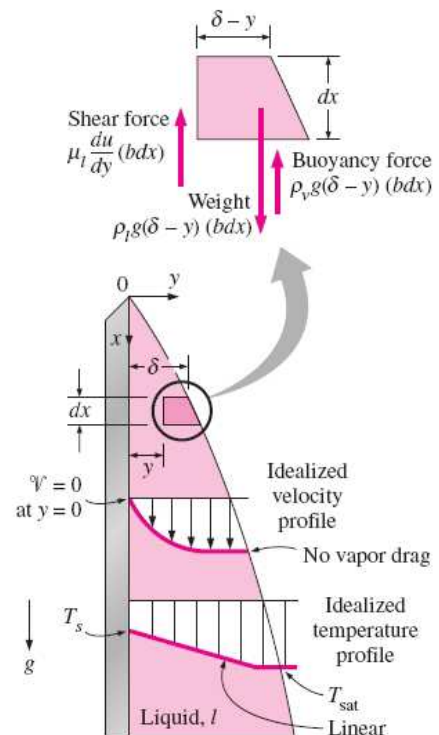
Kondentsatu-fluxu laminarra eta propietateak konstanteak.

Kondentsatu-geruzaren azelerazio baztergarria.

$$\sum F_x = m \cdot a_x = 0$$

$$\frac{du}{dy} = \frac{g(\rho_l - \rho_v)(\delta - y)}{\mu_l} \Rightarrow$$

$$u(y) = \frac{g(\rho_l - \rho_v)}{\mu_l} \left(y\delta - \frac{y^2}{2} \right)$$



GERUZA-ERA KO KONDENTSATZIOKO BERO-TRANSFERENTZIAREN KORRELAZIOAK

1 – Xafla bertikalak – Fluxu laminarra

$$\dot{m}(x) = \int_A \rho_l u(y) dA = \int_0^\delta \rho_l u(y) b dy \Rightarrow \dot{m}(x) = \frac{gb\rho_l(\rho_l - \rho_v)}{3\mu_l} \delta^3$$

$$d\dot{Q} = h_{fg} \cdot d\dot{m} = k_l \cdot (bdx) \frac{T_{sat} - T_s}{\delta} \Rightarrow \delta(x) = \left[\frac{4\mu_l k_l (T_{sat} - T_s)}{g\rho_l(\rho_l - \rho_v) h_{fg}} x \right]^{1/4}$$

$$\dot{q}_x = h_x \cdot (T_{sat} - T_s) = k_l \frac{T_{sat} - T_s}{\delta} \Rightarrow h_x = \frac{k_l}{\delta(x)}$$

$$h_x = \left[\frac{g\rho_l(\rho_l - \rho_v) h_{fg} k_l^3}{4\mu_l (T_{sat} - T_s) x} \right]^{1/4}$$

$$h_{vert} = \frac{1}{L} \int_0^L h_x dx = \frac{4}{3} h_{x=L} \Rightarrow h = 0,943 \left[\frac{g\rho_l(\rho_l - \rho_v) h_{fg} k_l^3}{\mu_l (T_{sat} - T_s) L} \right]^{1/4}$$

GERUZA-ERA KO KONDENTSATZIOKO BERO-TRANSFERENTZIAREN KORRELAZIOAK

1 – Xafla bertikalak – Fluxu laminarra

$$h_{vert} = 0,943 \left[\frac{g\rho_l(\rho_l - \rho_v) h_{fg}^* k_l^3}{\mu_l (T_{sat} - T_s) L} \right]^{1/4} \quad [\text{W/m}^2 \cdot ^\circ\text{C}] \quad 0 < \text{Re} < 30$$

Hurrengo ekuazioak kontutan hartuz eta $\rho_v \ll \rho_l$ suposatuz

$$\left\{ \begin{array}{l} \text{Re} = \frac{4 \cdot \dot{m}}{p\mu_l} \quad \dot{m}(x) = \frac{gb\rho_l(\rho_l - \rho_v)}{3\mu_l} \delta^3 \\ h_x = \frac{k_l}{\delta(x)} \quad h_{vert} = \frac{4}{3} h_{x=L} \end{array} \right.$$

$$\text{Re} \cong \frac{4g\rho_l(\rho_l - \rho_v)\delta^3}{3\mu_l^2} = \frac{4g\rho_l^2}{3\mu_l^2} \left(\frac{k_l}{h_{x=L}} \right)^3 = \frac{4g}{3v_l^2} \left(\frac{k_l}{3h_{vert}/4} \right)^3$$

$$h_{vert} \cong 1,47 k_l \text{Re}^{-1/3} \left(\frac{g}{v_l^2} \right)^{1/3} \quad \begin{array}{l} 0 < \text{Re} < 30 \\ \rho_v \ll \rho_l \end{array}$$

GERUZA-ERA KO KONDENTSATZIOKO BERO-TRANSFERENTZIAREN KORRELAZIOAK

1 – Xaf la bertikalak – Fluxu laminar izurtua

Kutateladze

$$h_{vert,wavy} = \frac{k_l Re}{1,08 Re^{1,22} - 5,2} \left(\frac{g}{v_l^2} \right)^{1/3} \quad 30 < Re < 1800 \quad \rho_v \ll \rho_l$$

$$h_{vert,wavy} = 0,8 Re^{0,11} h_{vert}$$

$$Re_{vert,wavy} = \left[4,81 + \frac{3,70 L k_l (T_{sat} - T_s)}{\mu_l h_{fg}^*} \left(\frac{g}{v_l^2} \right)^{1/3} \right]^{0,820} \quad \rho_v \ll \rho_l$$

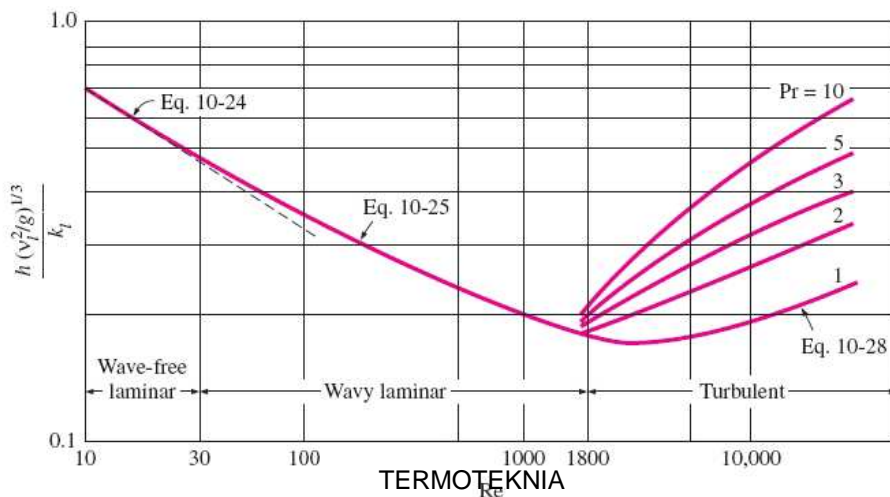
GERUZA-ERA KO KONDENTSATZIOKO BERO-TRANSFERENTZIAREN KORRELAZIOAK

1 – Xaf la bertikalak – Fluxu turbulentu a

Labuntsov

$$h_{vert,turb} = \frac{k_l Re}{8750 + 58 Pr^{-0,5} (Re^{0,75} - 253)} \left(\frac{g}{v_l^2} \right)^{1/3} \quad Re > 1800 \quad \rho_v \ll \rho_l$$

$$Re_{vert,turb} = \left[\frac{0,0690 L k_l Pr^{0,5} (T_{sat} - T_s)}{\mu_l h_{fg}^*} \left(\frac{g}{v_l^2} \right)^{1/3} - 151 Pr^{0,5} + 253 \right]^{4/3}$$

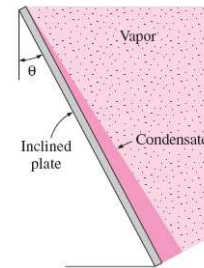


GERUZA-ERA KO KONDENTSAZIOKO BERO-TRANSFERENTZIAREN KORRELAZIOAK

2 – Xafla inklinatuak

$$g = g \cdot \cos \theta \Rightarrow h_{inclined} = h_{vert} (\cos \theta)^{1/4}$$

(Laminarra)



3 – Hodi bertikalak Baldin $D \gg \delta$ xafla laueko korrelazio bera erabili

4 – Hodi horizontalak eta esferak

$$h_{horiz} = 0,729 \left[\frac{g \rho_l (\rho_l - \rho_v) h_{fg}^* k_l^3}{\mu_l (T_{sat} - T_s) D} \right]^{1/4} \Rightarrow \frac{h_{vert}}{h_{horiz}} = 1,29 \left(\frac{D}{L} \right)^{1/4}$$

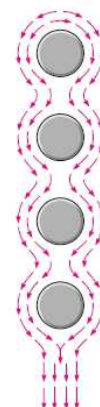
$$L = 2,77 D$$

$$h_{sph} = 0,815 \left[\frac{g \rho_l (\rho_l - \rho_v) h_{fg}^* k_l^3}{\mu_l (T_{sat} - T_s) D} \right]^{1/4}$$

GERUZA-ERA KO KONDENTSAZIOKO BERO-TRANSFERENTZIAREN KORRELAZIOAK

4 – Hodi multzo horizontalak

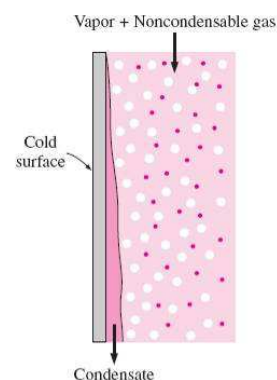
$$h_{horiz, N} = 0,729 \left[\frac{g \rho_l (\rho_l - \rho_v) h_{fg}^* k_l^3}{\mu_l (T_{sat} - T_s) ND} \right]^{1/4} = \frac{1}{N^{1/4}} h_{horiz, 1tubo}$$

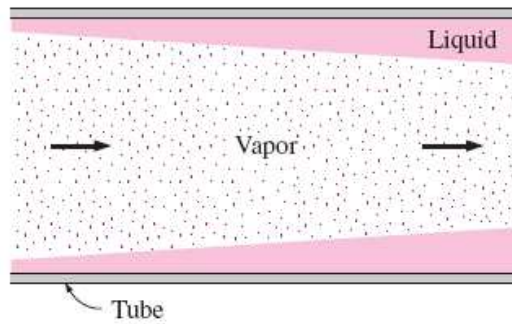


LURRUN ABIADURA EFEKTUA

Baldin V_{lurrun} beherantz $\rightarrow V_{likido} \uparrow \rightarrow \delta \downarrow \rightarrow Q \uparrow$
 Baldin V_{lurrun} gorantz $\rightarrow V_{likido} \downarrow \rightarrow \delta \uparrow \rightarrow Q \downarrow$

GAS KONDENTSAEZINAK KONDENTSA DOREETAN





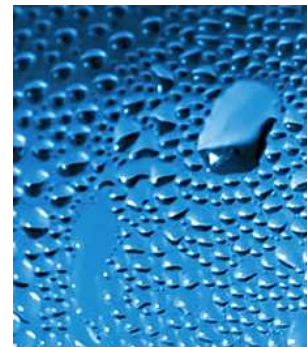
Baldin $Re_{vapor} = \left(\frac{\rho_v V_v D}{\mu_v} \right)_{inlet} < 35000$

Chato $h_{internal} = 0,555 \left[\frac{g \rho_l (\rho_l - \rho_v) k_l^3}{\mu_l (T_{sat} - T_s)} \left(h_{fg} + \frac{3}{8} c_{pl} (T_{sat} - T_s) \right) \right]^{1/4}$

10.7 – TANTA-ERAKO KONDENTSATZIOA

Gehigarri kimikoak

Metal nobleak



Griffith (Lurruna + Kupre gainazala) $h_{dropwise} = \begin{cases} 51104 + 2044 \cdot T_{sat} & 22^\circ \text{ C} < T_{sat} < 100^\circ \text{ C} \\ 255310 & T_{sat} > 100^\circ \text{ C} \end{cases}$

10.8 – IRAKATSIKO EZ DIREN ATALAK

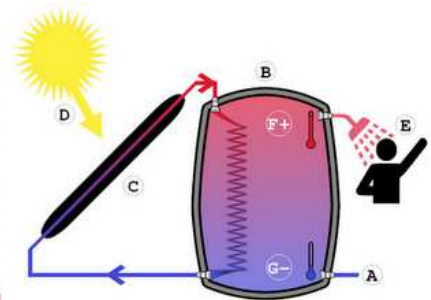
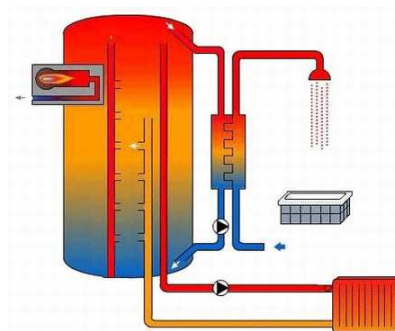
11. GAIA

BERO-TRUKAGAILUAK

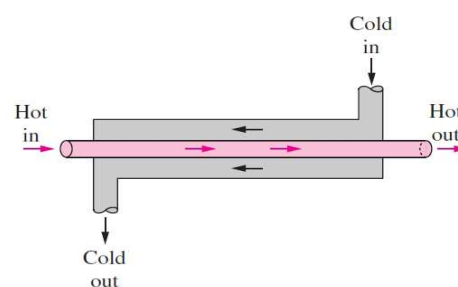
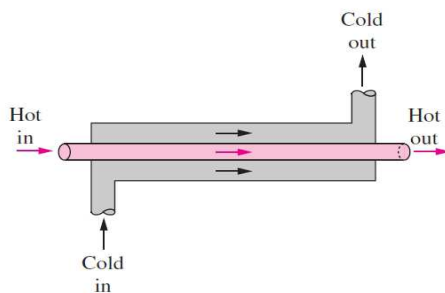
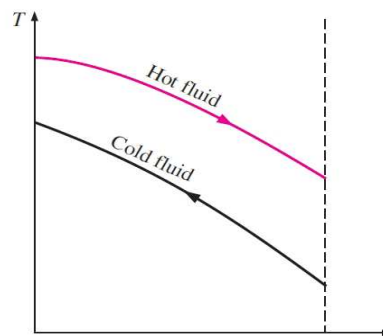
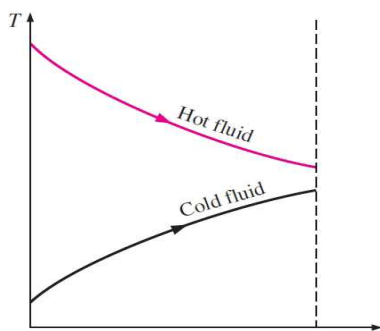
11.0 - HELBURUAK

2/29

- **Bero-trukagailu motak** bereizi eta sailkatu
- Gainazaletan metakinak pilatzeak duen eraginaz jabetu eta bero-trukagailu baten **bero-transferentziaren koefiziente orokorra** kalkulatu
- Bero-trukagailuen **energia-analisi orokorrak** egin
- **LMTD metodoan** erabiltzeko **batez besteko temperatura-diferentzia logaritmikoaren** erlazioa lortu, eta bero-trukagailu desberdinetarako moldatu, **zuzenketa-faktoreak** erabiliz
- Eraginkortasun-erlazioak garatu, eta bero-trukagailuak **eraginkortasun-NTU metodoarekin** analizatu, irteera-tenperatura ezezaguna denean
- Bero-trukagailuak aukeratzeko **oinarrizko irizpideak** zein diren jakin.



• Hodi bikoitzeko bero-trukagailuak → Konfiguraziorik sinpleena

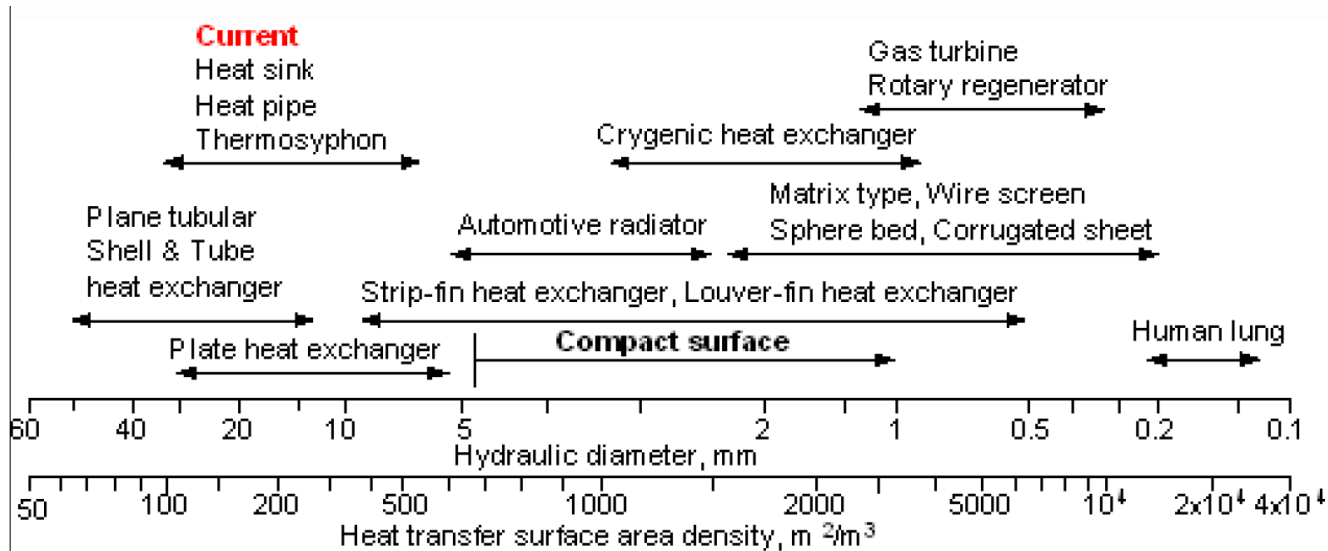


Fluxu paraleloa

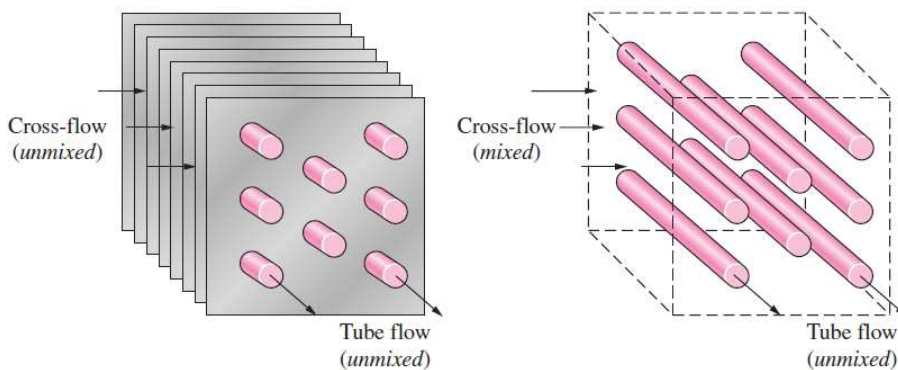
Kontrako fluxua

- Bero trukagailu trinkoa → $\beta = \frac{A_s}{Vol} > 700 \text{ m}^2/\text{m}^3$

Azalera-dentsitatea

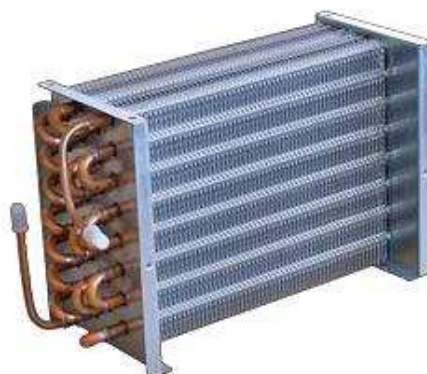


- Fluxu-gurutzatuko bero trukagailua

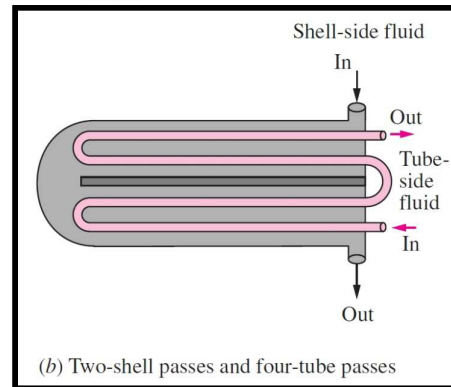
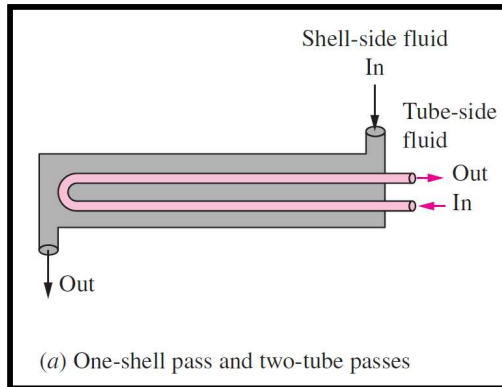
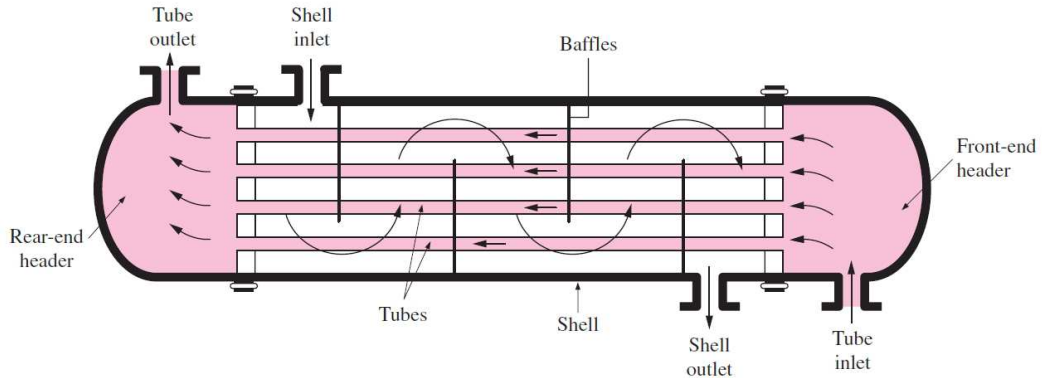


(a) Both fluids unmixed

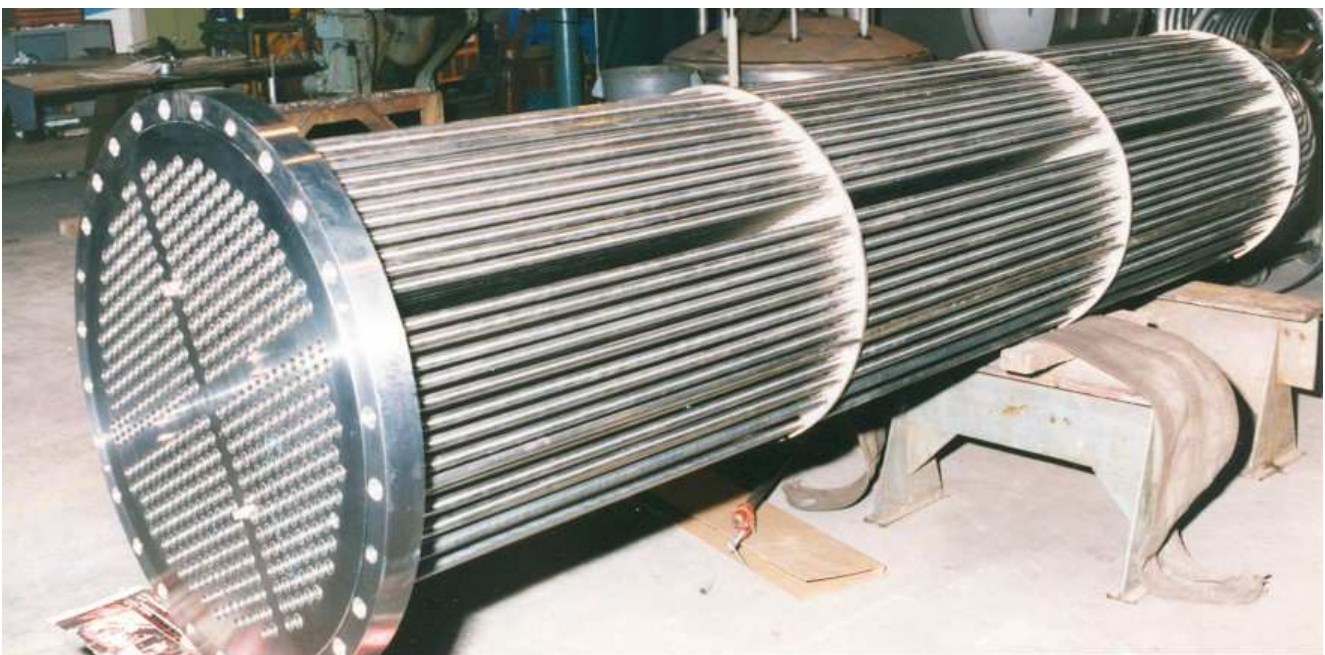
(b) One fluid mixed, one fluid unmixed



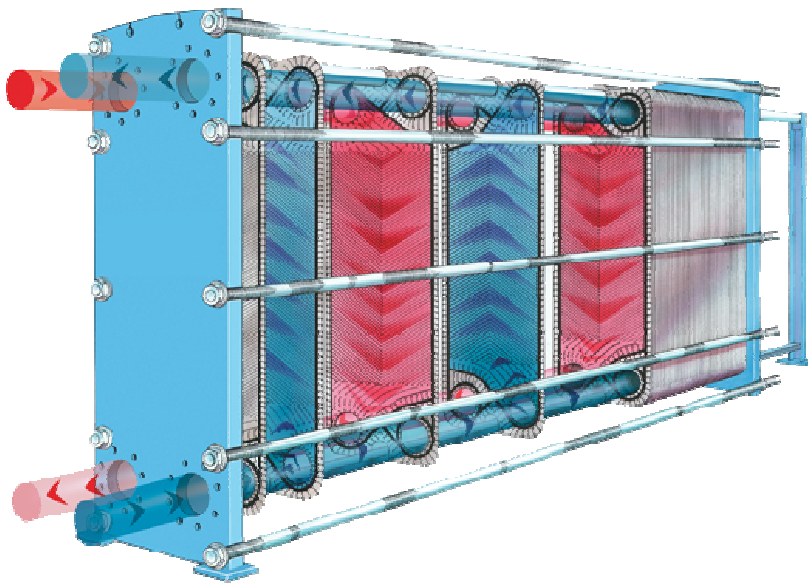
- Karkasa eta hodi erako bero-trukagailua



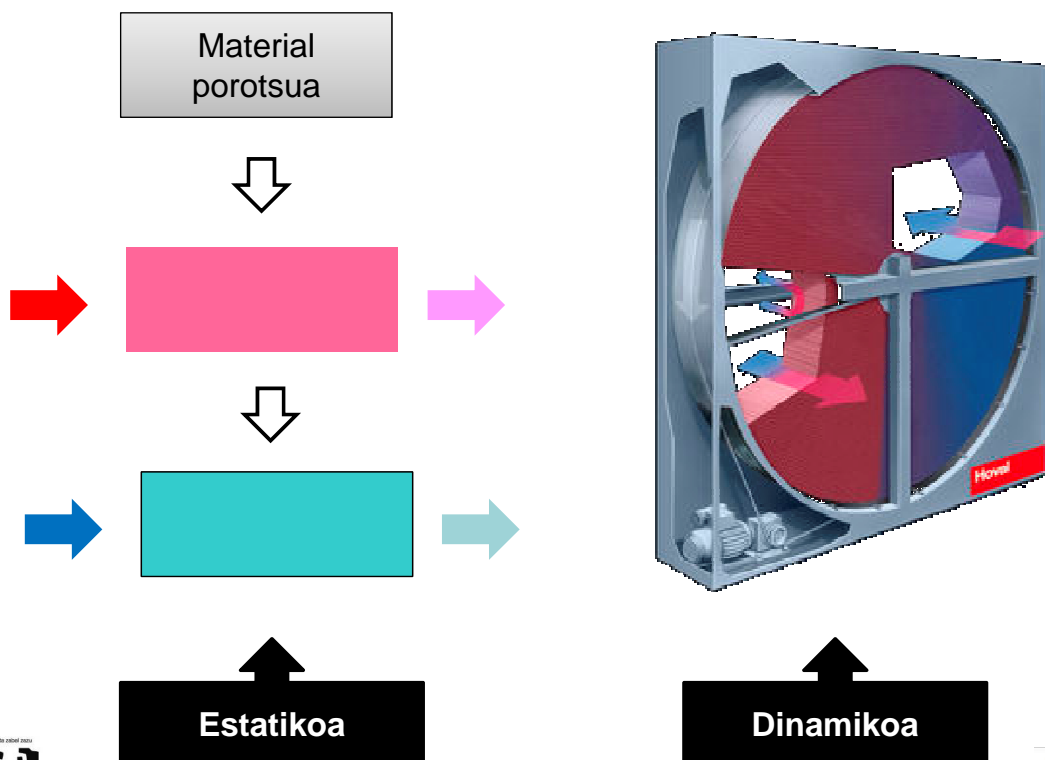
- Karkasa eta hodi erako bero-trukagailua



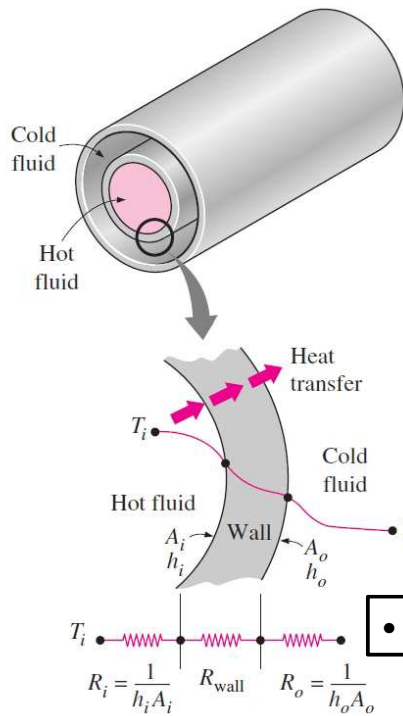
- Xafla eta bastidore bero-trukagailuak



- Birsorgailu bero-trukagailuak



Bero-trukagailuetan → Kondukzio, konbekzio eta erradiazio motako bero-transferentziak (azken biak orokorrean konbinatzen dira).



$$R_{total} = R_i + R_{wall} + R_o \Rightarrow$$

$$\Rightarrow R_{total} = \frac{1}{h_i A_i} + \frac{\ln(D_o/D_i)}{2\pi k L} + \frac{1}{h_o A_o}$$

$$\dot{Q} = \frac{\Delta T}{R_{total}} = UA\Delta T = U_i A_i \Delta T = U_o A_o \Delta T$$

• Baldin $R_{wall} \cong 0$: $\rightarrow A \cong A_i \cong A_o \rightarrow \frac{1}{U} = \frac{1}{h_i} + \frac{1}{h_o}$

Representative values of the overall heat transfer coefficients in heat exchangers

Type of heat exchanger	$U, \text{W/m}^2 \cdot \text{°C}^*$
Water-to-water	850-1700
Water-to-oil	100-350
Water-to-gasoline or kerosene	300-1000
Feedwater heaters	1000-8500
Steam-to-light fuel oil	200-400
Steam-to-heavy fuel oil	50-200
Steam condenser	1000-6000
Freon condenser (water cooled)	300-1000
Ammonia condenser (water cooled)	800-1400
Alcohol condensers (water cooled)	250-700
Gas-to-gas	10-40
Water-to-air in finned tubes (water in tubes)	30-60 [†]
	400-850 [†]
Steam-to-air in finned tubes (steam in tubes)	30-300 [†]
	400-4000 [‡]

*Multiply the listed values by 0.176 to convert them to Btu/h · ft² · °F.

[†]Based on air-side surface area.

[‡]Based on water- or steam-side surface area.

Hegal laburra ($T_s = \text{cte}$) → $A_s = A_{unfinned} + A_{fin}$

Hegal luzea ($T_s \neq \text{cte}$) → $A_s = A_{unfinned} + \eta_{fin} A_{fin}$

METAKETA-FAKTOREA (zikintze faktorea)

Metaketa motak:

- ✓ Metakin solidoen prezipitatzea
- ✓ Korrosioa eta kimikoa
- ✓ Biologikoa



Metaketa faktorea R_f
(erresistentzia gehigarria)

(Source: Tubular Exchange Manufacturers Association.)

Fluid	$R_f, m^2 \cdot ^\circ C/W$
Distilled water, sea water, river water, boiler feedwater:	
Below 50°C	0.0001
Above 50°C	0.0002
Fuel oil	0.0009
Steam (oil-free)	0.0001
Refrigerants (liquid)	0.0002
Refrigerants (vapor)	0.0004
Alcohol vapors	0.0001
Air	0.0004

$$R_{total} = \frac{1}{UA_s} = \frac{1}{h_i A_i} + \frac{R_{f,i}}{A_i} + \frac{\ln(D_o/D_i)}{2\pi k L} + \frac{R_{f,o}}{A_o} + \frac{1}{h_o A_o}$$

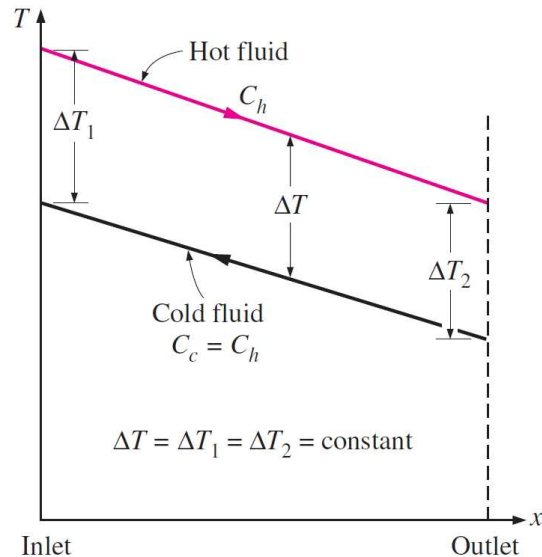
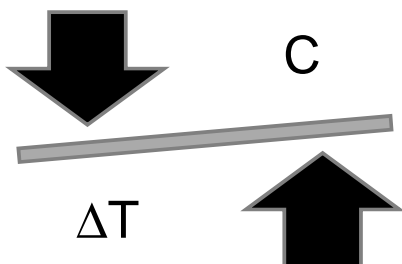
11.3 – BERO-TRUKAGAILUEN ANALISIA

$\dot{Q} = \dot{m}_c c_{pc} (T_{c,out} - T_{c,in})$ ➡ **Jariakin hotza** (berotzen dena)

$\dot{Q} = \dot{m}_h c_{ph} (T_{h,in} - T_{h,out})$ ➡ **Jariakin beroa** (hozten dena)

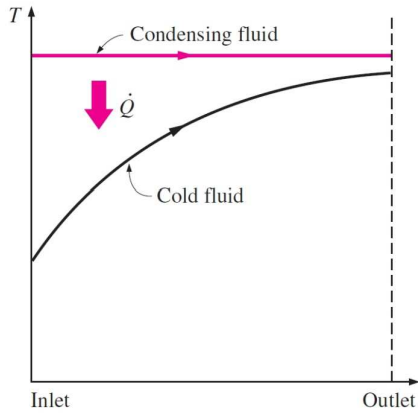
Bero-ahalmena

$$C_i = \dot{m}_i c_{pi} \begin{cases} C_h = \dot{m}_h c_{ph} \\ C_c = \dot{m}_c c_{pc} \end{cases}$$

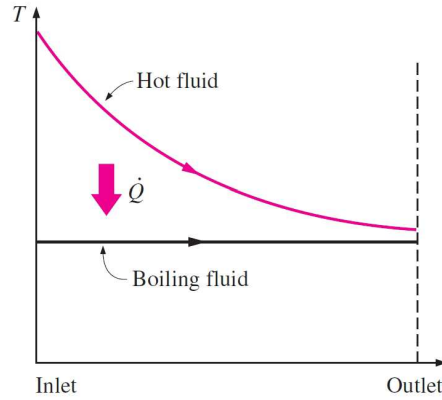


• Bero-transferentzia sentikorra (ΔT) $\implies \dot{Q} = \dot{m}c_p \Delta T$

• Bero-transferentzia sorra ($T = cte$) $\implies \dot{Q} = \dot{m}h_{fg}$



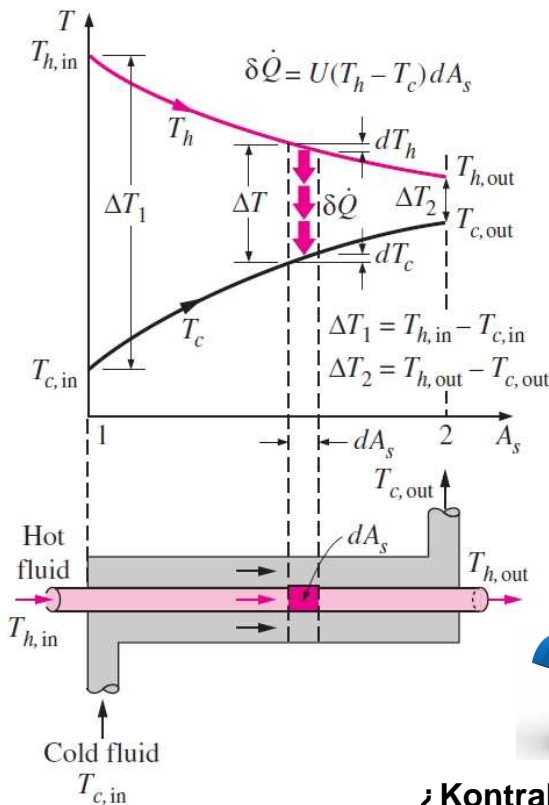
(a) Condenser ($C_h \rightarrow \infty$)



(b) Boiler ($C_c \rightarrow \infty$)

$$\dot{Q} = UA_s \Delta T_m$$

11.4 – BATEZ BESTEKO TEMPERATURA-DIFERENTZIA LOGARITMIKOAREN METODOA



$$\begin{cases} \delta \dot{Q} = -\dot{m}_h c_{ph} dT_h \\ \delta \dot{Q} = \dot{m}_c c_{pc} dT_c \\ \delta \dot{Q} = U(T_h - T_c) dA_s \end{cases}$$



$$\dot{Q} = UA_s \Delta T_{lm}$$

$$\Delta T_{lm} = \frac{\Delta T_{in} - \Delta T_{out}}{\ln\left(\frac{\Delta T_{in}}{\Delta T_{out}}\right)}$$

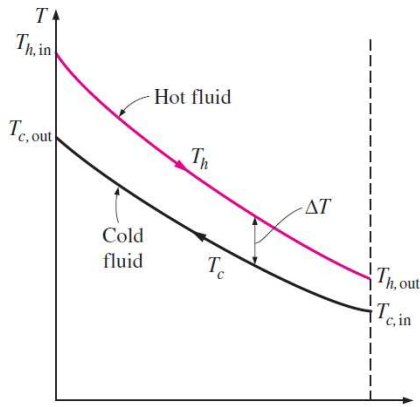


¿Kontrako fluxuan?

OHARRA: $\Delta T_{lm} < \Delta T_{am}$

11.4 – BATEZ BESTEKO TEMPERATURA-DIFERENTZIA LOGARITMIKOAREN METODOA

KONTRAKO FLUXUKO BERO-TRUKAGAILUAK

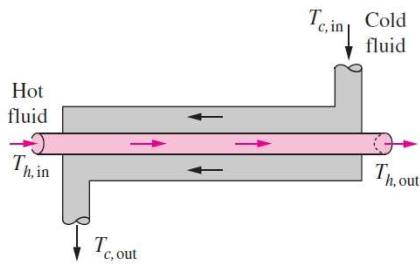


Sarrerako eta irteerako tenperaturak berdinak izanik:

$$\Delta T_{lm,CF} > \Delta T_{lm,PF}$$

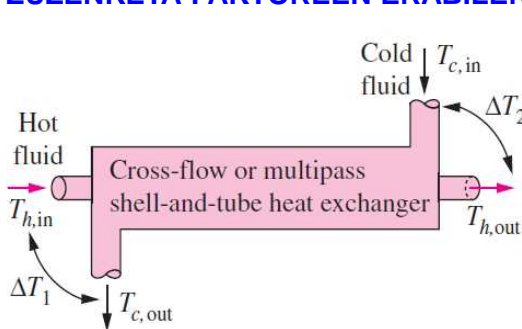


$$C_h = C_c \rightarrow \Delta T_{lm} ?$$



11.4 – BATEZ BESTEKO TEMPERATURA-DIFERENTZIA LOGARITMIKOAREN METODOA

IRAGANALDI ANITZEKO ETA FLUXU GURUTZATUKO BERO TRUKAGAILUAK: ZUZENKETA-FAKTOREEN ERABILERA



Heat transfer rate:

$$\dot{Q} = UA_s F \Delta T_{lm,CF}$$

where

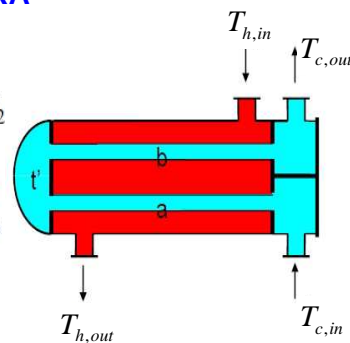
$$\Delta T_{lm,CF} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1 / \Delta T_2)}$$

$$\Delta T_1 = T_{h,in} - T_{c,out}$$

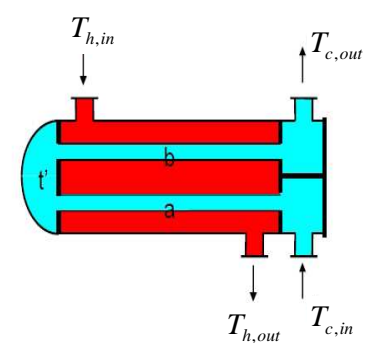
$$\Delta T_2 = T_{h,out} - T_{c,in}$$

and

$$F = \dots \text{ (Fig. 13-18)}$$



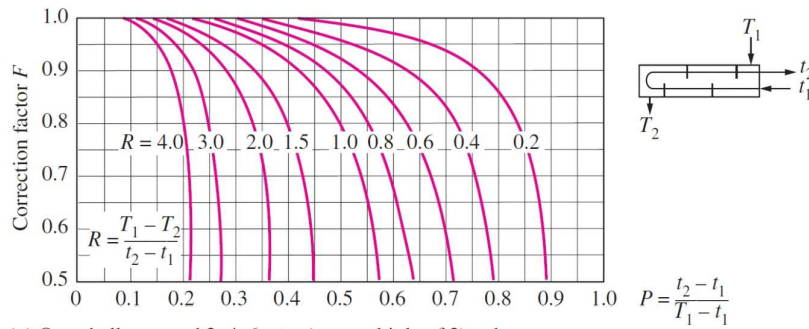
- a: Parallel flow
- b: Counter flow



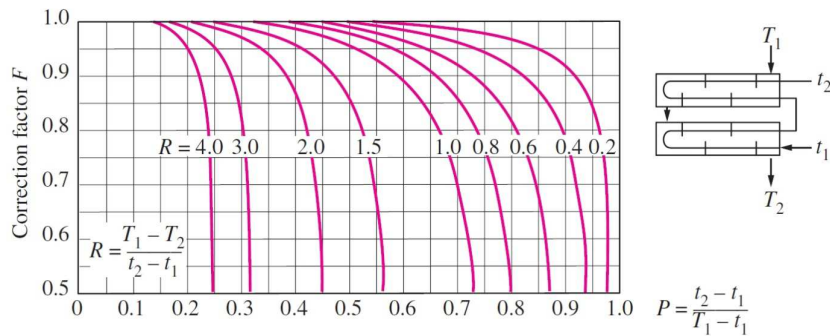
- a: Parallel flow
- b: Counter flow

11.4 – BATEZ BESTEKO TENPERATURA-DIFERENTZIA LOGARITMIKOAREN METODOA

IRAGANALDI ANITZEKO ETA FLUXU GURUTZATUKO BERO TRUKAGAILUAK: ZUZENKETA-FAKTOREEN ERABILERA



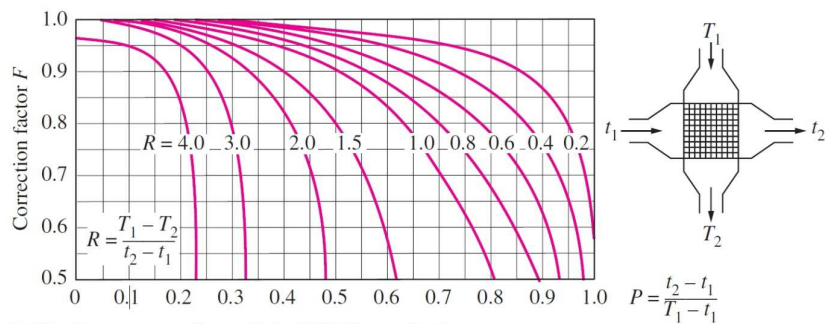
(a) One-shell pass and 2, 4, 6, etc. (any multiple of 2), tube passes



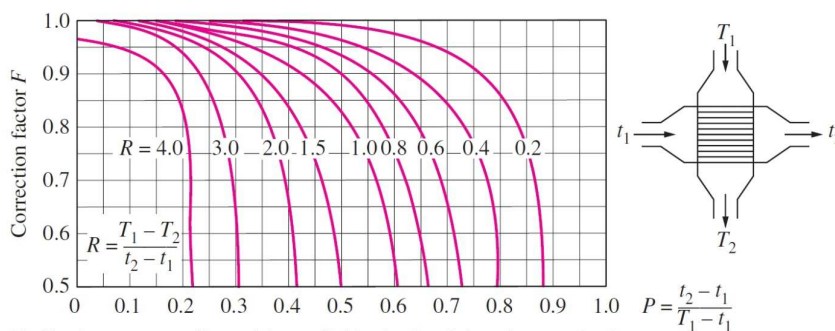
(b) Two-shell passes and 4, 8, 12, etc. (any multiple of 4), tube passes

11.4 – BATEZ BESTEKO TENPERATURA-DIFERENTZIA LOGARITMIKOAREN METODOA

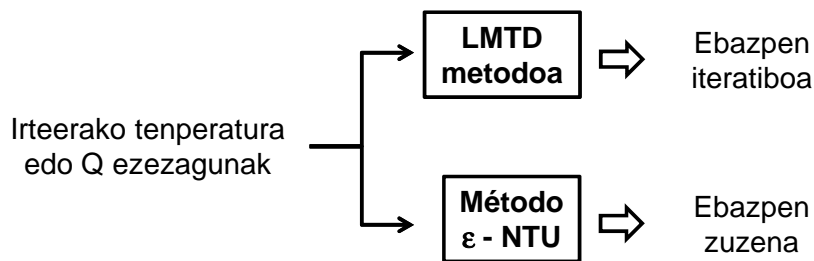
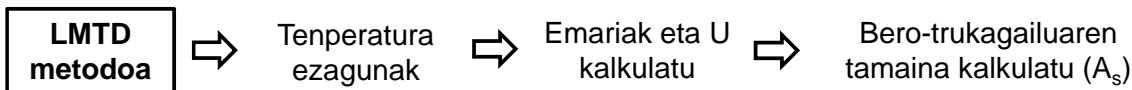
IRAGANALDI ANITZEKO ETA FLUXU GURUTZATUKO BERO TRUKAGAILUAK: ZUZENKETA-FAKTOREEN ERABILERA



(c) Single-pass cross-flow with both fluids *unmixed*

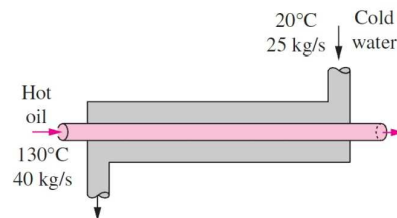


(d) Single-pass cross-flow with one fluid *mixed* and the other *unmixed*



$$\epsilon = \frac{\text{Bero - transferentzia abiadura erreala}}{\text{Bero - transferentzia abiadura maximoa}} = \frac{\dot{Q}}{\dot{Q}_{\max}}$$

$$\dot{Q}_{\max} = C_{\min} (T_{h,in} - T_{c,in})$$



$$C_c = \dot{m}_c C_{pc} = 104.5 \text{ kW/}^\circ\text{C}$$

$$C_h = \dot{m}_h C_{ph} = 92 \text{ kW/}^\circ\text{C}$$

$$C_{\min} = 92 \text{ kW/}^\circ\text{C}$$

$$\Delta T_{\max} = T_{h,in} - T_{c,in} = 110^\circ\text{C}$$

$$\dot{Q}_{\max} = C_{\min} \Delta T_{\max} = 10,120 \text{ kW}$$

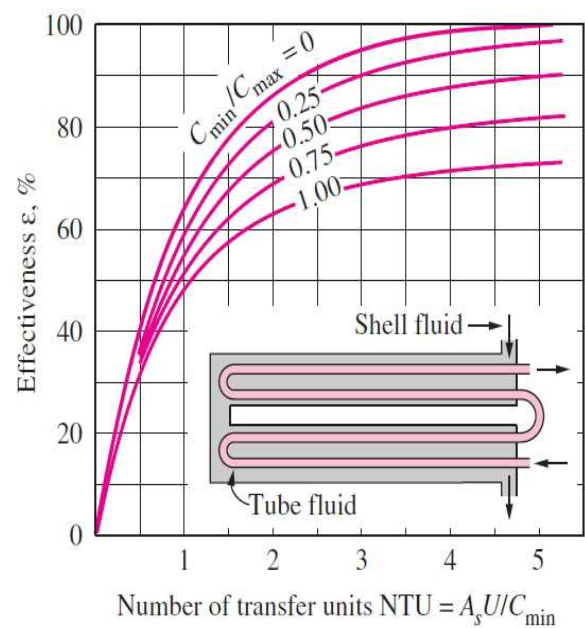
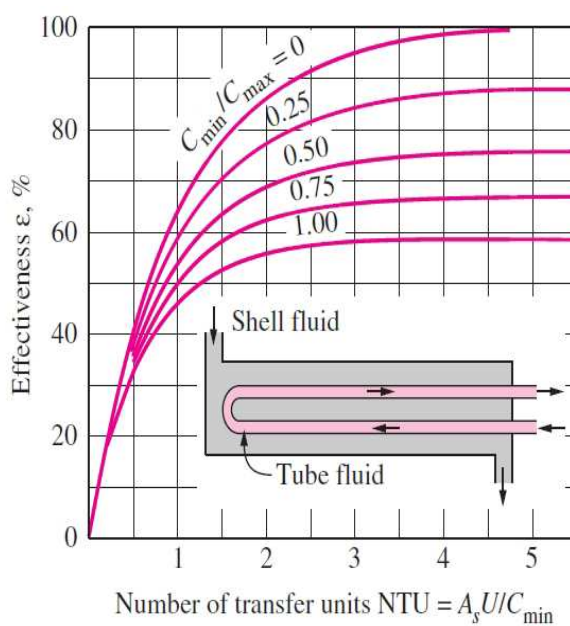
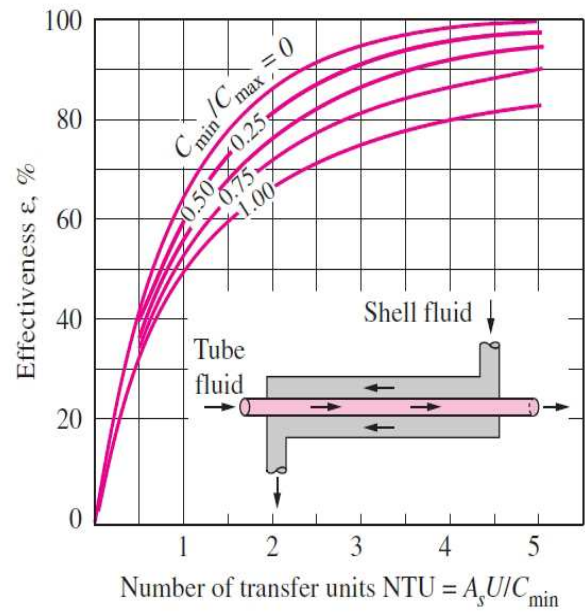
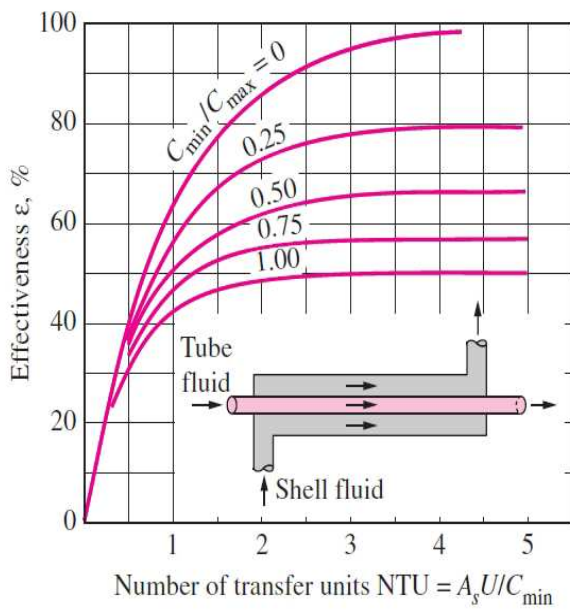
$$NTU = \frac{UA_s}{C_{\min}}$$

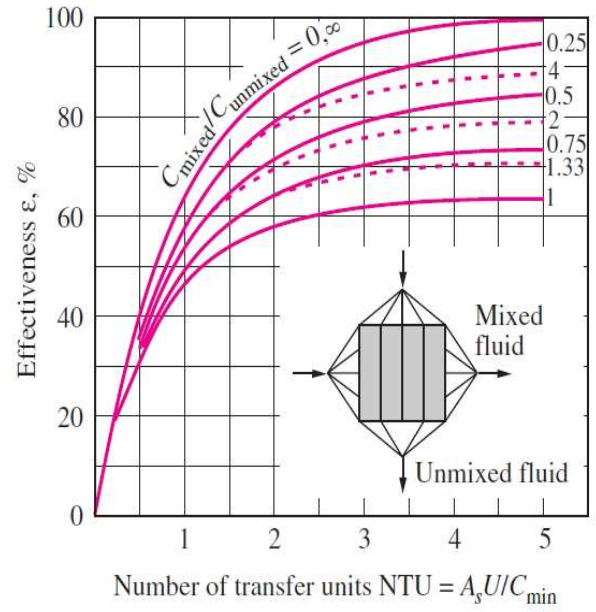
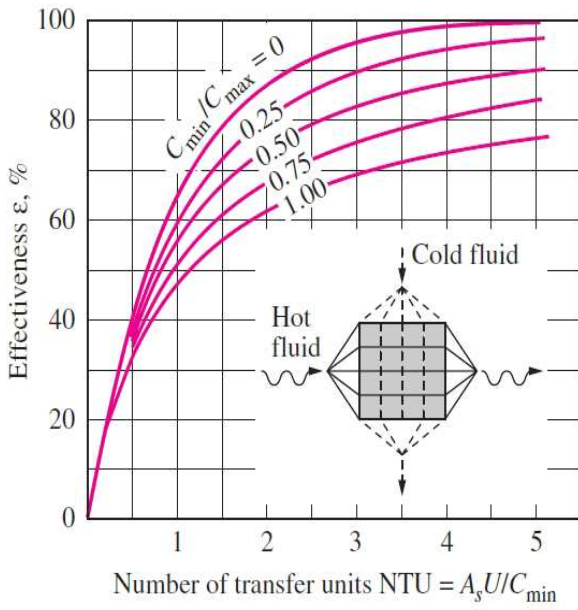
$$c = \frac{C_{\min}}{C_{\max}}$$

$$\epsilon = \epsilon(NTU, c)$$

Effectiveness relations for heat exchangers: $NTU = UA_s/C_{\min}$ and $c = C_{\min}/C_{\max} = (\dot{m}C_p)_{\min}/(\dot{m}C_p)_{\max}$ (Kays and London, Ref. 5.)

Heat exchanger type	Effectiveness relation
1 <i>Double pipe:</i> Parallel-flow	$\epsilon = \frac{1 - \exp[-NTU(1 + c)]}{1 + c}$
Counter-flow	$\epsilon = \frac{1 - \exp[-NTU(1 - c)]}{1 - c \exp[-NTU(1 - c)]}$
2 <i>Shell and tube:</i> One-shell pass 2, 4, . . . tube passes	$\epsilon = 2 \left\{ \frac{1 + c + \sqrt{1 + c^2}}{1 - \exp[-NTU\sqrt{1 + c^2}]} \right\}^{-1}$
3 <i>Cross-flow (single-pass)</i> Both fluids unmixed	$\epsilon = 1 - \exp \left\{ \frac{NTU^{0.22}}{c} [\exp(-c NTU^{0.78}) - 1] \right\}$
C_{\max} mixed, C_{\min} unmixed	$\epsilon = \frac{1}{c} (1 - \exp \{1 - c[1 - \exp(-NTU)]\})$
C_{\min} mixed, C_{\max} unmixed	$\epsilon = 1 - \exp \left\{ -\frac{1}{c} [1 - \exp(-c NTU)] \right\}$
4 <i>All heat exchangers with $c = 0$</i>	$\epsilon = 1 - \exp(-NTU)$

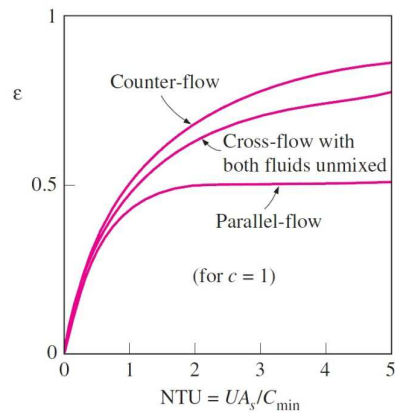




Iruzkina:

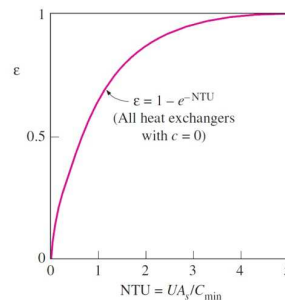
1. $NTU \downarrow \rightarrow \Delta \epsilon \uparrow$, baina $NTU \uparrow \rightarrow \Delta \epsilon \downarrow \rightarrow$ (justifikazio ekonomikoa $NTU < 3$)

2. NTU eta c zehatz batzuetarako \rightarrow



3. $NTU < 0.3$ denean $\rightarrow \epsilon = \epsilon(NTU)$

4. $c = 0$ denean $\rightarrow \epsilon_{max} = 1 - \exp(-NTU)$



- U-ren kalkulua %30-eko ziurgabetasuna → Bero-trukagailuen gairak gaitzesionaketa

- $\varepsilon \uparrow \rightarrow \Delta P \uparrow$



Jariakin biskosoena:

¿Hodien barnealdetik edo karkasatik?

BERO-TRANSFERENTZIAREN ABIADURA

$$\dot{Q}_{\max} = \dot{m}c_p (T_{in} - T_{out})$$

KOSTUA

- Eskarapeko bero-trukagailuaren kostua >> Seriean egindako bero-trukagailuaren kostua
- Operazio + Mantentze lanak

PONPATZE-POTENTZIA

- Operazio lanak = Ponpatze potentzia [kW] x Operazio-orduak [h] x Elektrizitatearen kostua energia unitateko [€/kWh]
- Operazio lanak vs Hasierako kostua
- Abiadura muga → Higadura, zarata, bibrazioak eta presio galerak txikitzeko.

TAMAINA ETA PISUA

- Tamaina \uparrow → Prezioa \uparrow

MOTA

- Erabiliko ditugun jariakin moten eta daukagun espazioaren araberakoa.

MATERIALAK

- Tentsio-efektu termikoak (dilatazioak)
- Korrosioarekiko erresistentzia

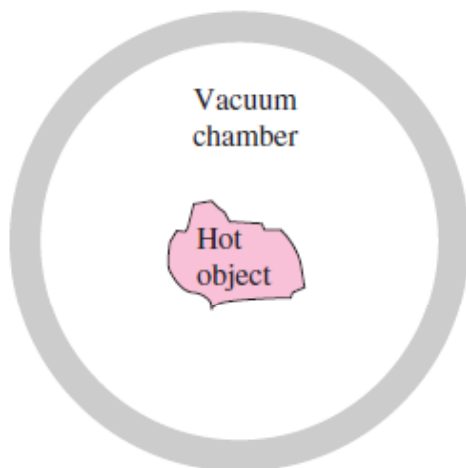
12. GAIA

ERRADIAZIO TERMIKOAREN OINARRIAK

12.0 - HELBURUAK

2/34

- **Erradiazio elektromagnetikoa** sailkatu, eta **erradiazio termikoa** identifikatu
- **Gorputz beltz** idealizatua ulertu, eta gorputz beltzaren emisio-ahalmen totala eta espektrala kalkulatu
- Uhin-luzeraren tarte jakin batean **igorritako erradiazio-frakzioa** kalkulatu, gorputz beltzaren erradiazio-funtzioak erabiliz
- **Erradiazio-intentsitatearen** kontzeptua ulertu, eta magnitude direkzional espektralak definitu, intentsitatea erabiliz
- **Emisibitate, absortibitate, erreflektibitate eta transmisibitatearen** propietateak ondo ulertu, oinarri espektral, direkzional eta totalekin
- **Kirchhoffen legea** aplikatu emisibitate ezaguneko gainazal baten absortibitatea kalkulatzeko
- **Erradiazio atmosferikoaren** eredia egin zero-tenperatura eraginkor bat erabilita, eta **berotegi-efektuaren** garrantziaz jabetu.



Erradiazioa

Ez du ingurune material baten beharra.

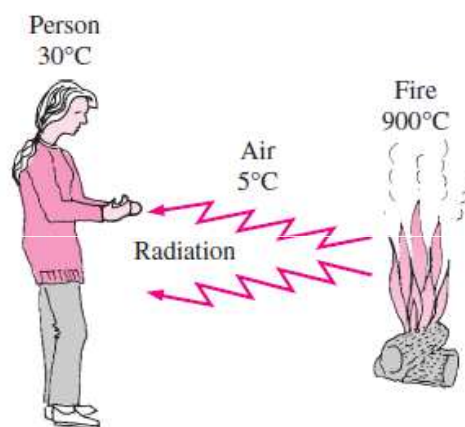
Ez da moteltzen hutsean.

Energia-transferentzia lasterrena da (argiaren abiadura du).

Solidoetan, likidoetan eta gasetan gertatzen da.



Nola ailegutzen da Eguzkiaren energia Lurrera?



Erradiazio bidezko bero-transferentzia ingurune hotzago batek banatutako bi gorputzen artean gerta daiteke

FUNDAMENTU TEORIKOA

Uhin elektromagnetiko edo erradiazio elektromagnetiko materiak atomoen edo molekulen konfigurazio elektronikoaren aldaketen ondorioz igortzen duen energia adierazten dute

Fotoi izeneko energia-pakete multzoen hedatzen dituzte

$$\lambda = \frac{c}{\nu}$$

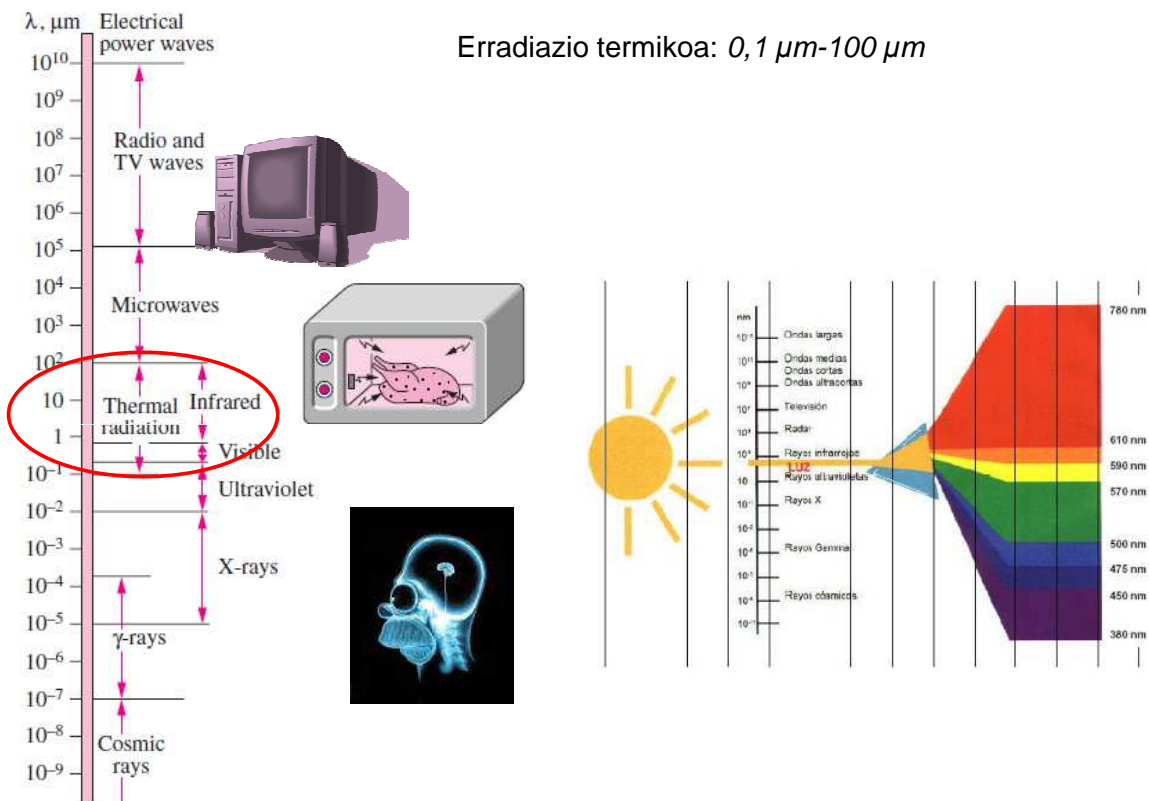
$$c = \frac{c_0}{n}$$

$$e = h\nu = \frac{hc}{\lambda}$$

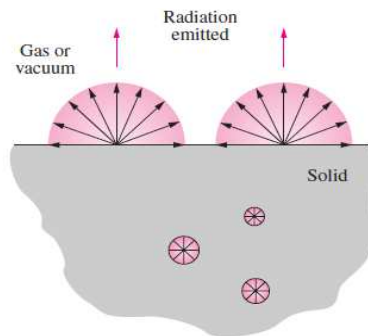
- λ**
 - Uhin luzera
 - 1 μm = 10⁻⁶ m
- C**
 - C: uhin batek ingurune horretan duen hedatze-abiadura [m/s]
 - C₀: 2,9979x10⁸ m/s (argiaren abiadura hutsean)
- n**
 - Maiztasuna
 - Iturriaren menpekkoa da, eta hedatzen den ingurunearekiko independentea da
- h**
 - Hz
 - Errefrakzio indizea
 - n=1 airea; n=1,5 beira; n=1,33 ura
- e**
 - Fotoiaren energia
- h**
 - Planck-en konstantea
 - 6,6256x10⁻³⁴ J·s

12.2 – ERRADIAZIO TERMIKOA

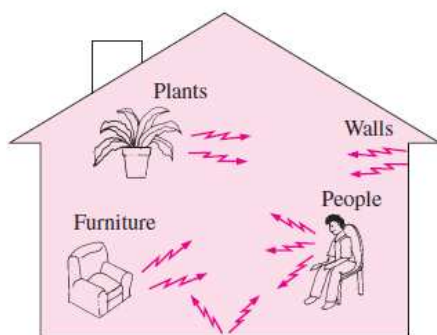
Erradiazio termikoa: 0,1 μm-100 μm



- Fenomeno bolometrikoa da.
- Solido opakoetan (gardenak ez direnak): metalak, zura eta harria: gainazal fenomenoa da.



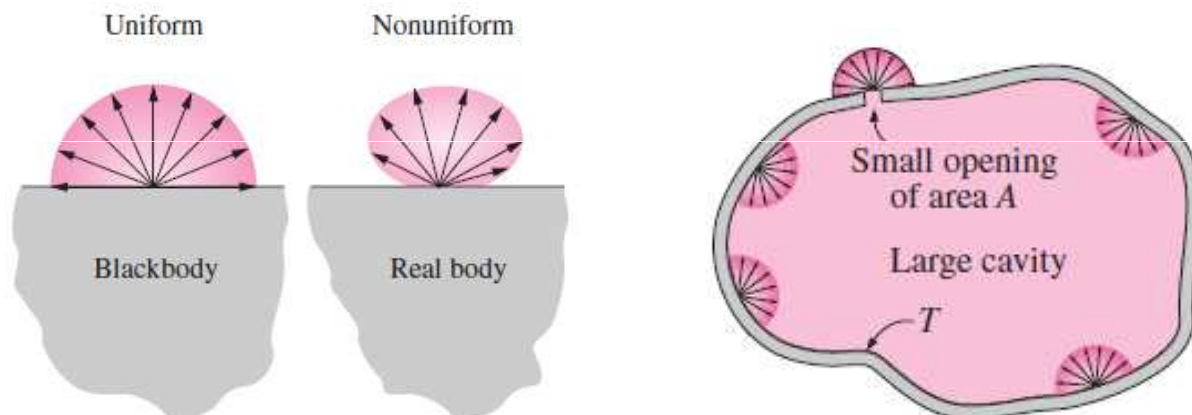
- $T \uparrow \rightarrow Q_{\text{rad}} \uparrow$
- Zerotik gorako tenperatura termodinamikoa (edo absolutua) duen gorputz batek norabide guztietan igortzen du uhin-luzera tarte zabal bateko erradiazioa. ($T > 0 \text{ K}$).



Zein da tenperatura batean gorputz batek igor dezakeen erradiazio maximoa?

12.3 – GORPUTZ BELTZAREN ERRADIAZIOA

- Erradiazio-igorle eta -xurgatzaile perfektua da
- Jasotzen duen erradiazio guztia xurgatzen du, uhin-luzera eta norabidea edozein direla ere.
- Igorle difusoa da: modu uniformearen igortzen du erradiazio-energia norabide guztietan,



PLANCK-EN LEGEA (HUTSA ETA GAS-ENTZAT)

Gorputz beltzaren emisio-ahalmen espektrala.

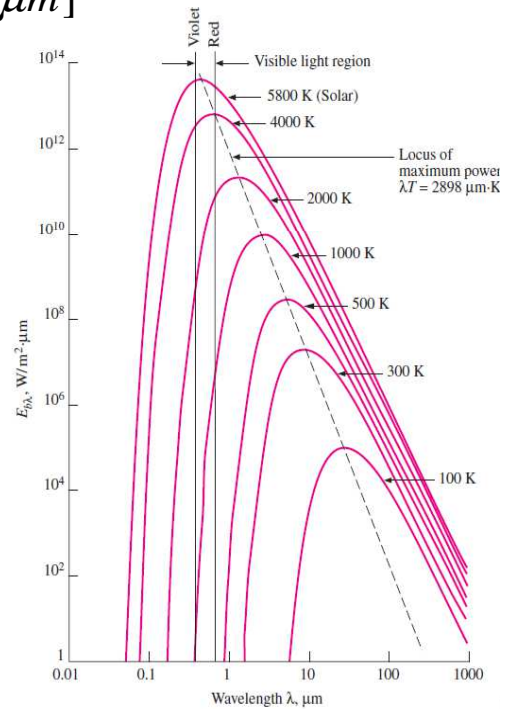
$$E_{b\lambda}(\lambda, T) = \frac{C_1}{\lambda^5 [\exp(C_2/\lambda T) - 1]} \quad [W / m^2 \mu m]$$

$$C_1 = 2 \pi h c_0^2 = 3,74177 \times 10^8 [W \mu m^4 / m^2]$$

$$C_2 = h c_0 / k = 1,43878 \times 10^4 [\mu m K]$$

$$k = 1,38065 \times 10^{-23} [J / K]$$

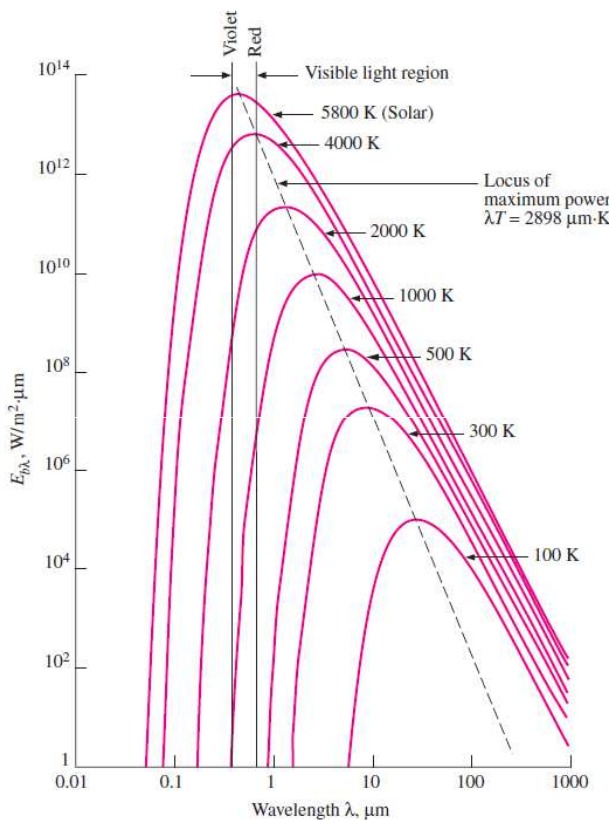
Beste ingurunetarako: $C_1 = \frac{C_1}{n^2}$
 n: errefrakzio indizea



WIEN-EN DESPLAZAMENDU LEGEA

Erradiazioaren emisio-kurben gailurraren kokapena.

$$(\lambda \cdot T)_{\max \text{ power}} = 2897,8 [\mu m K]$$



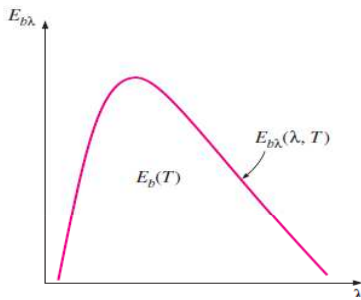
Eguzkiaren gainazala 5800K

$$\lambda_{\max} = 2897,8 / 5800 = 0,50 \mu m$$

tarte ikusgaiaren erdian

STEFAN-BOLTZMANN-EN LEGEA

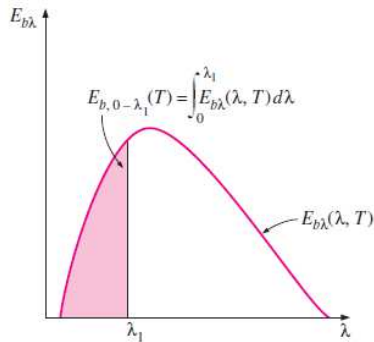
Gorputz beltzaren emisio-ahalmen totala ematen da, uhin-luzera guztietan igorritako erradiazioaren batura totala delarik.



$$E_b(T) = \int_0^{\infty} E_{b\lambda}(\lambda, T) d\lambda = \sigma T^4 \quad (W/m^2)$$

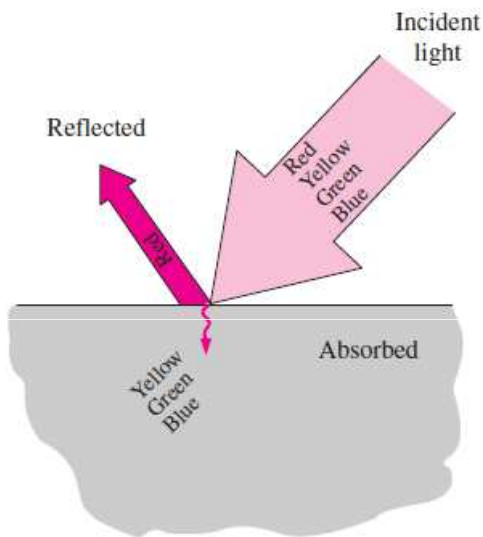
$$E_b(T) = \sigma T^4 \quad (W/m^2)$$

σ : Stefan-Boltzmann-en kte
 $5,67 \times 10^{-8} W/m^2 K^4$



$$E_{b,0-\lambda}(T) = \int_0^{\lambda} E_{b\lambda}(\lambda, T) d\lambda \quad (W/m^2)$$

$$f_{\lambda}(T) = \frac{\int_0^{\lambda} E_{b\lambda}(\lambda, T) d\lambda}{\sigma T^4} \rightarrow \text{Gorputz-beltz erradiazio funtzioa}$$

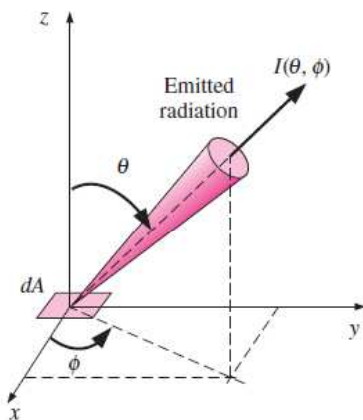


Zergatik elurra gainazal zuriak zuri ikusten dira?

Eta landareen hostoak berde?

ERRADIAZIO INTENTSITATEA (I)

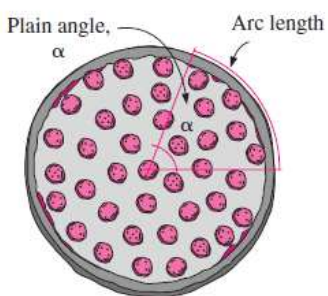
Espazioko norabide jakin batean igorritako (edo jasotako) erradiazioa norabide zenitalean θ eta azimutalean ϕ



KASUA

Gorputz beltza: Igorle difusoa: igorritako erradiazioa berdina da norabide guztietan, beraz, ez da norabidearen araberakoa

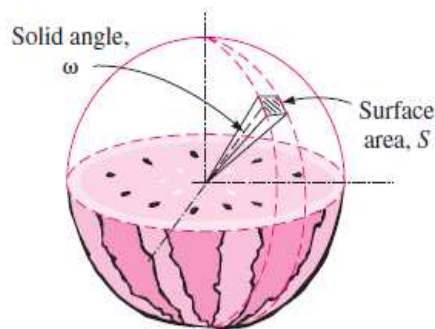
ANGELU SOLIDOA



A slice of pizza of plain angle α

Angelu laua:

$$\alpha = \frac{\text{arc length}}{\text{radius}}$$

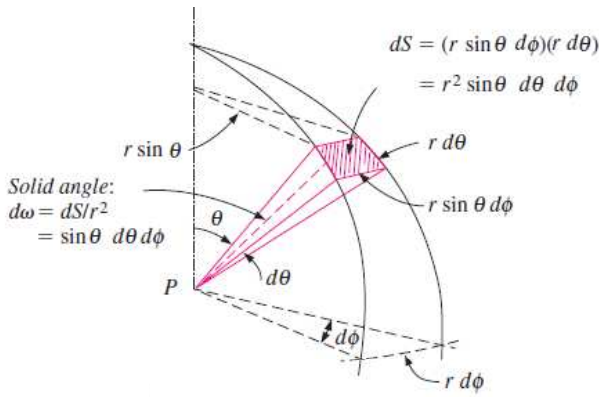


A slice of watermelon of solid angle ω

Angelu solidoa:

$$\omega = \frac{\text{Surface area}}{\text{radius}^2}$$

ANGELU SOLIDOA



$$d\omega = \frac{dS}{r^2} = \sin \theta d\theta d\phi$$

$$d\omega = \frac{dA_n}{r^2} = \frac{dA \cos \alpha}{r^2}$$

Hemisferioaren angelu solidoa

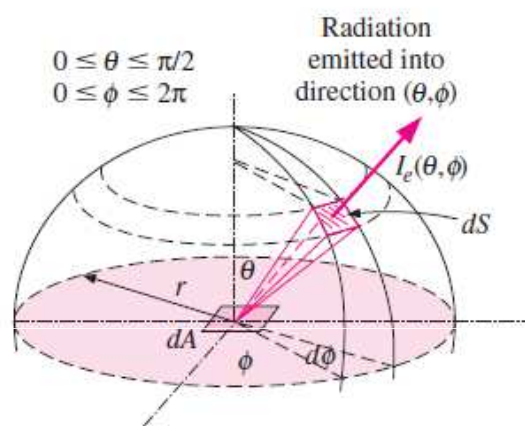
$$\omega = \int_{\text{hemisphere}} d\omega = \int_{\theta=0}^{\pi/2} \int_{\phi=0}^{2\pi} \sin \theta d\theta d\phi = 2\pi$$

$$S = \int_{\text{sphere}} dS = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} r^2 \sin \theta d\theta d\phi = 2\pi r^2 \int_{\theta=0}^{\pi} \sin \theta d\theta = 4\pi r^2$$

Baldin $r=1 \rightarrow S=4\pi \rightarrow \omega=4\pi$ [sr]

Hemisferioa: $\omega=2\pi$ [sr]

IGORRITAKO ERRADIAZIOAREN INTENTSITATEA [$I_e(\theta, \phi)$]



$$I_e(\theta, \phi) = \frac{d\dot{Q}_e}{dA \cos \theta d\omega} = \frac{d\dot{Q}_e}{dA \cos \theta \sin \theta d\theta d\phi} \quad [W / m^2 sr]$$

ERRADIAZIO FLUXUAK: EMISIO-AHALMENA; IRRADIAZIOA; ERRADIOSITATEA

1- EMISIO-AHALMENA (E)

Igorritako erradiazio fluxua da energiaren igortze-abiadura, gainazal igorlearen azalera unitateko

$$E = \int_{\text{hemisphere}} dE = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi/2} I_e(\theta, \phi) \cos \theta \sin \theta d\theta d\phi \quad [W / m^2]$$

KASUAK:

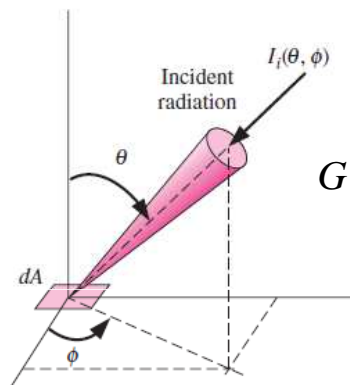
Modu difusoan igortzen duen gainazala: $I_e = \text{cte}$ $E = \pi I_e \quad [W / m^2]$

Gorputz beltza: Igorle difusoa $E_b = \pi I_b \quad [W / m^2]$

$$I_b(T) = \frac{E_b(T)}{\pi} = \frac{\sigma T^4}{\pi} \quad [W / m^2 \text{ sr}]$$

2-IRRADIAZIOA (G)

Gainazal batera norabide guztietatik **iristen** den erradiazio-fluxua da irradiazio,



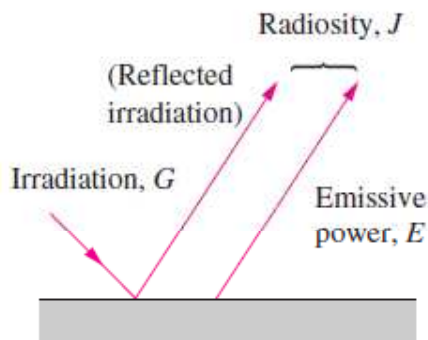
$$G = \int_{\text{hemisphere}} dG = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi/2} I_i(\theta, \phi) \cos \theta \sin \theta d\theta d\phi \quad [W / m^2]$$

KASUA:

Erradiazio intzidentea difusoa: $I_i = \text{kte}$ $G = \pi I_i \quad [W / m^2]$

3- ERRADIOSITATEA (J)

Gainazal baten azalera unitate batetik norabide guztietan irteten den erradiazio energiaren abiadura da



$$J = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi/2} I_{e+r}(\theta, \phi) \cos \theta \sin \theta d\theta d\phi \quad [W / m^2]$$

KASUA

Igorle eta isolatzaile difusoa: $J = \pi I_{e+r} \quad [W / m^2]$

Gainazal beltza: $J_b = \pi I_e = \sigma T^4 \quad [W / m^2]$

MAGNITUDE ESPEKTRALAK

Erradiazioaren-aldaketak uhin-luzeraren arabera

$$I_{\lambda,e}(\lambda, \theta, \phi) = \frac{d\dot{Q}_e}{dA \cos \theta d\omega d\lambda} \quad [W / m^2 sr \mu m]$$

$$E_\lambda = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi/2} I_{\lambda,e}(\lambda, \theta, \phi) \cos \theta \sin \theta d\theta d\phi \quad [W/m^2 \mu m]$$

KASUA

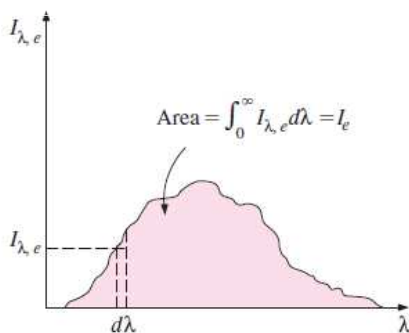
Gainazal eta erradiazio intzidente difusoa bada:

$$E_\lambda = \pi I_{\lambda,e} \quad G_\lambda = \pi I_{\lambda,i} \quad J_\lambda = \pi I_{\lambda,e+r}$$

Gorputz beltza :

$$I_{\lambda,b}(\lambda, T) = \frac{2hc_0^2}{\lambda^5 [\exp(hc_0 / \lambda KT) - 1]} \quad [W / m^2 sr \mu m]$$

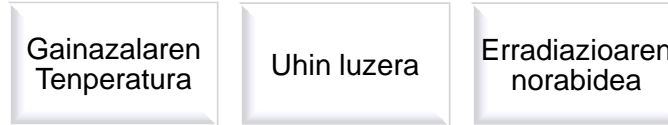
$$E_{b,\lambda}(\lambda, T) = \pi I_{b,\lambda}(\lambda, T)$$



EMISIBITATEA (ϵ)

Gainazalak tenperatura jakin batean igorritako erradiazioaren eta gorputz beltz batek tenperatura berean igorritako erradiazioaren arteko arrazoa.

Gorputz beltza $\epsilon = 1$ Gainazal erreala $0 < \epsilon < 1$



EMISIBITATE DIREKZIONAL ESPEKTRALA EMISIBITATE DIREKZIONAL TOTALA

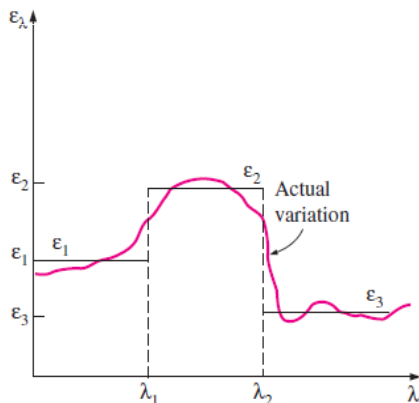
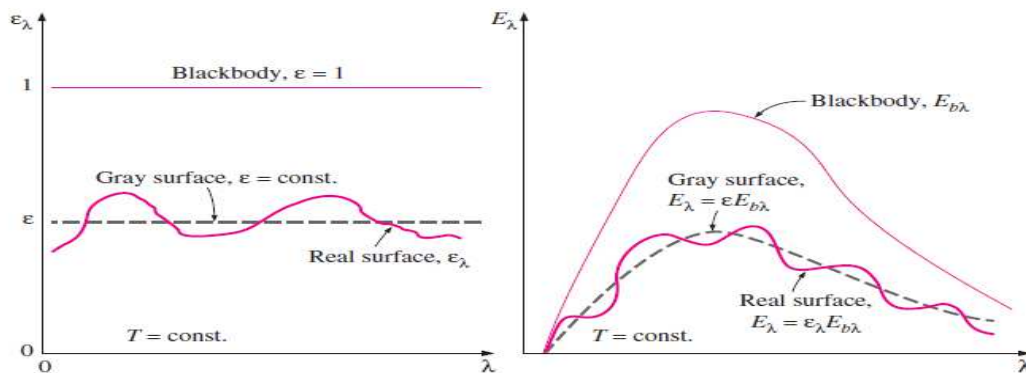
$$\epsilon_{\lambda\theta}(\lambda, \theta, \phi, T) = \frac{I_{\lambda,e}(\lambda, \theta, \phi, T)}{I_{b\lambda}(\lambda, T)}$$

$$\epsilon_{\theta}(\theta, \phi, T) = \frac{I_e(\theta, \phi, T)}{I_b(T)}$$

EMISIBITATE HEMISFERIKO ESPEKTRALA EMISIBITATE HEMISFERIKO TOTALA

$$\epsilon_{\lambda}(\lambda, T) = \frac{E_{\lambda}(\lambda, T)}{E_{b\lambda}(\lambda, T)}$$

$$\epsilon(T) = \frac{E(T)}{E_b(T)}$$



$$\epsilon_{\lambda} = \begin{cases} \epsilon_1 = \text{constant}, & 0 \leq \lambda < \lambda_1 \\ \epsilon_2 = \text{constant}, & \lambda_1 \leq \lambda < \lambda_2 \\ \epsilon_3 = \text{constant}, & \lambda_2 \leq \lambda < \infty \end{cases}$$

$$\epsilon(T) = \frac{\epsilon_1 \int_0^{\lambda_1} E_{b\lambda} d\lambda}{E_b} + \frac{\epsilon_2 \int_{\lambda_1}^{\lambda_2} E_{b\lambda} d\lambda}{E_b} + \frac{\epsilon_3 \int_{\lambda_2}^{\infty} E_{b\lambda} d\lambda}{E_b} \Rightarrow$$

$$\Rightarrow \epsilon(T) = \epsilon_1 \cdot f_{0-\lambda_1} + \epsilon_2 \cdot (f_{0-\lambda_2} - f_{0-\lambda_1}) + \epsilon_3 \cdot (1 - f_{0-\lambda_2})$$

EMISIBITATEA (ϵ)

Real surface:
 $\epsilon_{\theta} \neq \text{constant}$
 $\epsilon_{\lambda} \neq \text{constant}$

Diffuse surface:
 $\epsilon_{\theta} = \text{constant}$

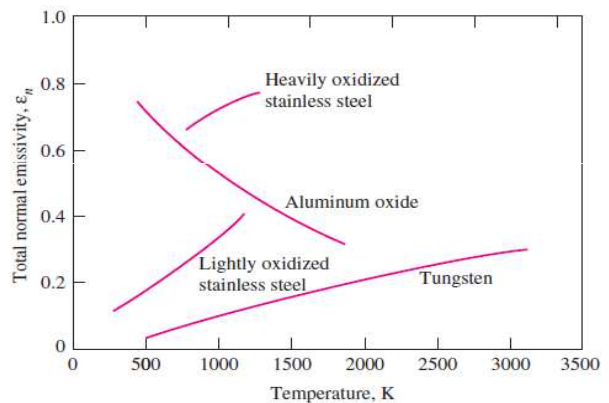
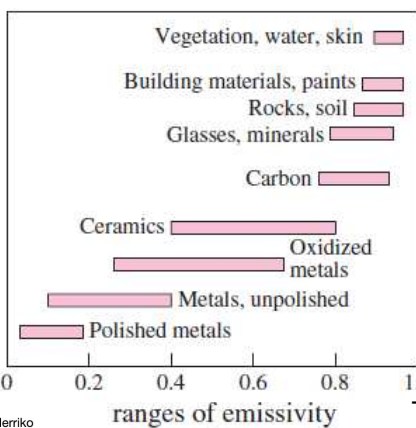
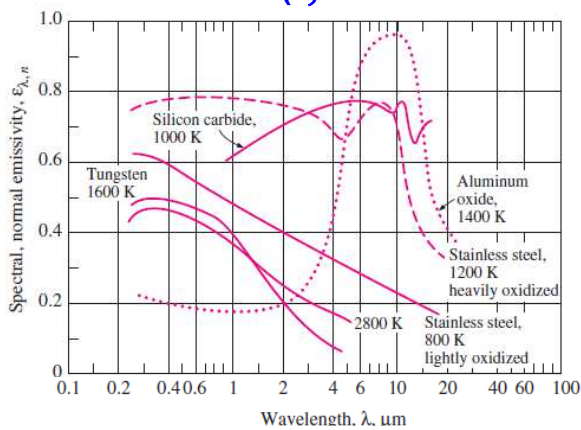
Gray surface:
 $\epsilon_{\lambda} = \text{constant}$

Diffuse, gray surface:
 $\epsilon = \epsilon_{\lambda} = \epsilon_{\theta} = \text{constant}$

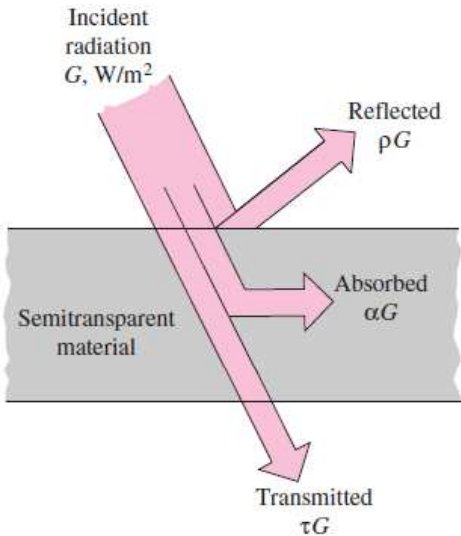
Gainazal Difusoa
 Bere propietateak norabidearekiko independenteak baldin badira

Gainazal Grisa
 Bere propietateak uhin luzerarekiko independenteak baldin badira

EMISIBITATEA (ϵ)



ABSORTIBITATEA (α), ERREFLEKTIBITATEA (ρ) ETA TRASMISIBITATEA (τ)



Absorptivity: $\alpha = \frac{\text{Absorbed radiation}}{\text{Incident radiation}} = \frac{G_{abs}}{G}, \quad 0 \leq \alpha \leq 1$

Reflectivity: $\rho = \frac{\text{Reflected radiation}}{\text{Incident radiation}} = \frac{G_{ref}}{G}, \quad 0 \leq \rho \leq 1$

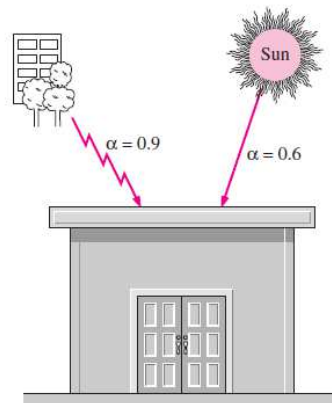
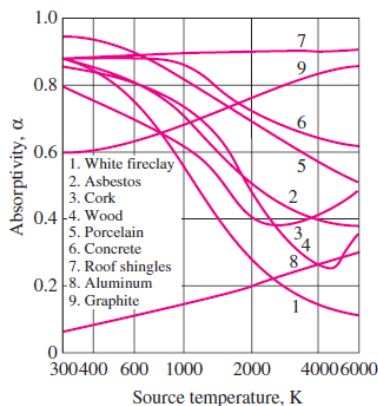
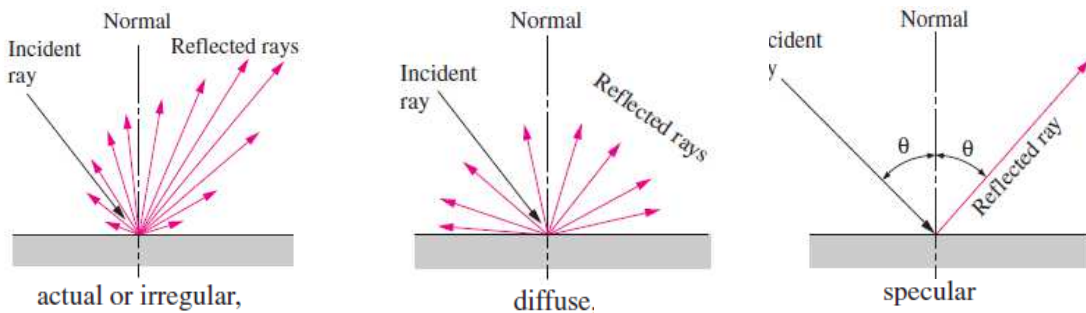
Transmissivity: $\tau = \frac{\text{Transmitted radiation}}{\text{Incident radiation}} = \frac{G_{tr}}{G}, \quad 0 \leq \tau \leq 1$

$$G_{abs} + G_{ref} + G_{tr} = G$$

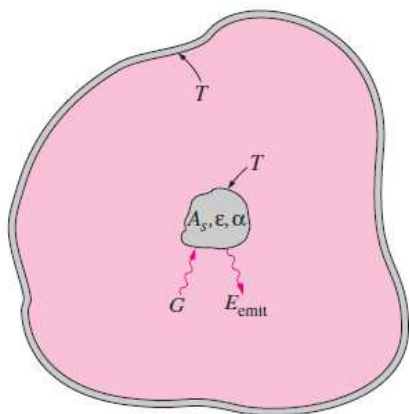
$$\alpha + \rho + \tau = 1$$

Gainazal beltza	Gainazal ispilua	Gainazal gardena	Gainazal opaka	Gainazal matea
<ul style="list-style-type: none"> $\alpha = 1$ $\rho = \tau = 0$ 	<ul style="list-style-type: none"> $\rho = 1$ $\alpha = \tau = 0$ 	<ul style="list-style-type: none"> $\tau = 1$ $\alpha = \rho = 0$ 	<ul style="list-style-type: none"> $\tau = 0$ $\alpha + \rho = 1$ 	<ul style="list-style-type: none"> $\rho = 0$ $\alpha + \tau = 1$

ABSORTIBITATEA (α), ERREFLEKTIBITATEA (ρ) ETA TRASMISIBITATEA (τ)



KIRCHHOFF-EN LEGEA



$$\left. \begin{aligned} G_{abs} &= \alpha G = \alpha \sigma T^4 \\ E_{emit} &= \varepsilon \sigma T^4 \end{aligned} \right\} \begin{array}{l} \text{Oreka termikoa:} \\ \varepsilon(T) = \alpha(T) \end{array}$$

Baldintza:

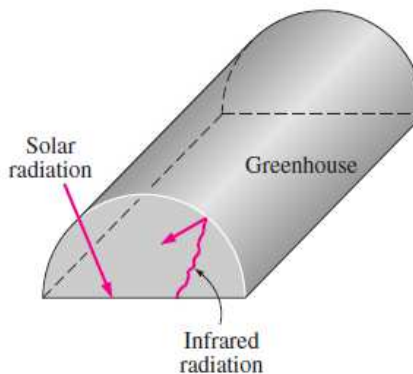
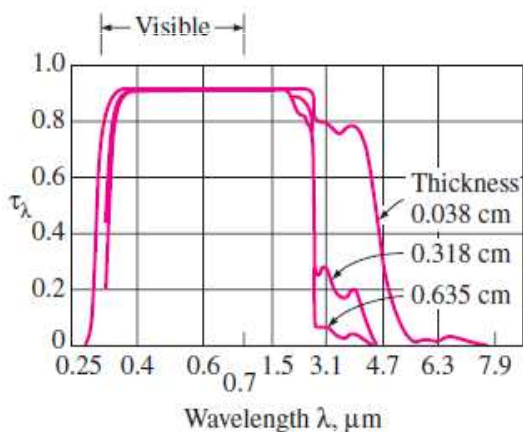
T gainazala = T irradiazio-iturriaren temperatura

BEROTEGI EFEKTUA



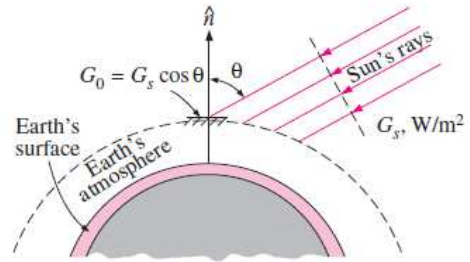
Zergatik autoa egun eguzkitsu batean eguzkitan uztean autoaren barrua kanpoko airea baino askoz gehiago berotzen da?

Zer da Lurraren berotegi efektua?

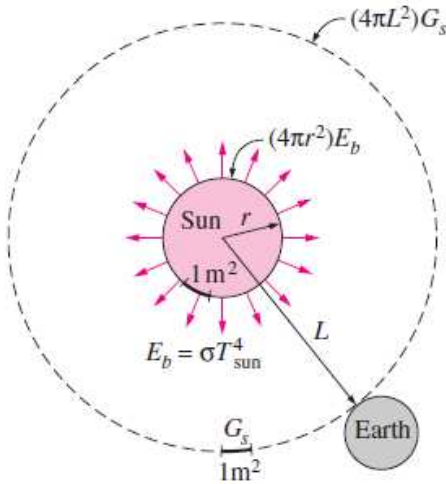


EGUZKI EZAUGARRIAK

- D = $1,39 \times 10^9$ m
- L = $1,50 \times 10^{11}$ m lurretik eguzkira
- $E_{\text{eguzkia}} = 3,8 \times 10^{26}$ W
- Lurrera ailegatzen dena $E = 1,7 \times 10^{17}$ W
- Nukleoaren T = 40 000 000 K
- Kanpoaldeko T = 5 800 K



Eguzki irradiazio totala: Lurraren atmosferara iristen den eguzki energia $G_s = 1373 \text{ W/m}^2$

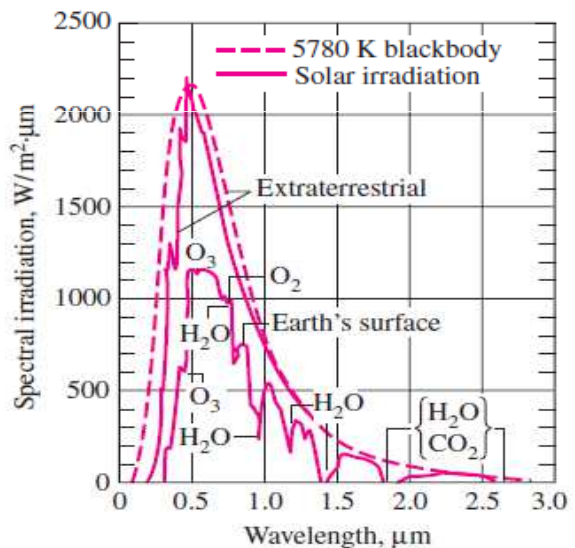


ATMOSFERAREN ABSORTZIOA

- Eguzki-erradiazioak atmosfera zeharkatzean nabarmen *moteltzen da*
- Atmosferaren %99 lurrazaletik 30 km-ko distantziaren barruan dago
- O₂: absortzioa $\lambda = 0,76 \mu\text{m}$
- O₃ (ozonoa): absortzioa $\lambda = 0,30 \mu\text{m}$ (ultramorea)
- H₂O y CO₂: absortzioa $\lambda = 1,5 \mu\text{m}$ (infragorri)

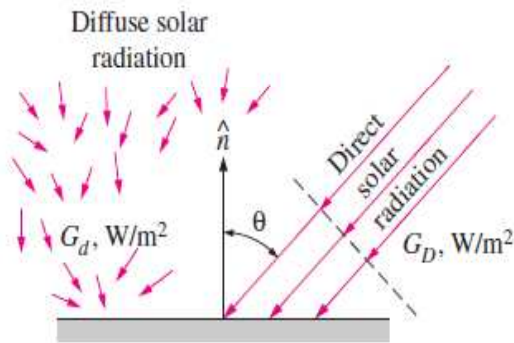
Lurraren gainazalean

- Eguzki-energia : 950 W/m^2
- Uhin luzeera: $0,3\text{-}2,5 \mu\text{m}$



Eguzki Erradiazio Zuzena G_D : Lurrazalera atmosferak dispertsatu edo xurgatu gabe iristen den eguzki-erradiazioaren zatia.

Eguzki Erradiazio Difusoa G_d : erradiazio barreiatua lurrazalera modu uniformean iristen da norabide guztietatik



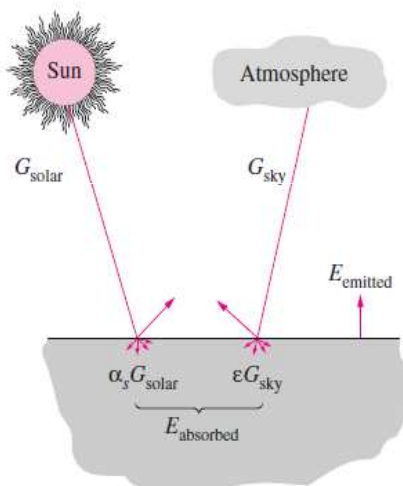
$$G_{solar} = G_D \cos \theta + G_d \quad [W / m^2]$$

θ : intzidentzia angelua

ATMOSFERAKO IGORPENAK

H₂O eta CO₂: igorri $\lambda = 5-8 \mu m$

Zeru tenperatura eraginkorra (T_{sky}) : baldintza atmosferikoen araberakoa da 230-285 K



$$G_{sky} = \sigma T_{sky}^4 \quad [W / m^2]$$

Kirchhoffen legea: $\epsilon = \alpha$

$$E_{sky,abs} = \alpha G_{sky} = \alpha \sigma T_{sky}^4 = \epsilon \sigma T_{sky}^4 \quad [W / m^2]$$

$$q_{net,rad} = \sum E_{abs} - \sum E_{emitted}$$

$$q_{net,rad} = E_{solar,abs} + E_{sky,abs} - E_{emit}$$

$$q_{net,rad} = \alpha_s G_{solar} + \epsilon \sigma T_{sky}^4 - \epsilon \sigma T_s^4$$

$$q_{net,rad} = \alpha_s G_{solar} + \epsilon \sigma (T_{sky}^4 - T_s^4) \quad [W / m^2]$$

Comparison of the solar absorptivity α_s of some surfaces with their emissivity ϵ at room temperature

Surface	α_s	ϵ
Aluminum		
Polished	0.09	0.03
Anodized	0.14	0.84
Foil	0.15	0.05
Copper		
Polished	0.18	0.03
Tarnished	0.65	0.75
Stainless steel		
Polished	0.37	0.60
Dull	0.50	0.21
Plated metals		
Black nickel oxide	0.92	0.08
Black chrome	0.87	0.09
Concrete	0.60	0.88
White marble	0.46	0.95
Red brick	0.63	0.93
Asphalt	0.90	0.90
Black paint	0.97	0.97
White paint	0.14	0.93
Snow	0.28	0.97
Human skin (caucasian)	0.62	0.97



Eguzki kolektoreak ze materialarekin egiten dira?

Eta kamioi hoztaileen kanpo-gainazalak?

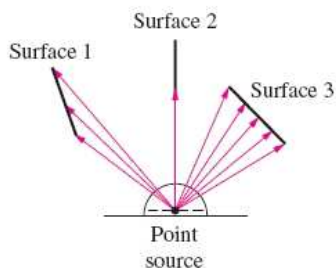
13. GAIA

ERRADIAZIO BIDEZKO BERO- TRANSFERENTZIA

13.0 - HELBURUAK

2/30

- **Ikuspen-faktorea** definitu, eta erradiazio bidezko bero-transferentziaren kalkuluetan duen garrantzia ulertu
- Ikuspen-faktorearen erlazioak garatu, eta itxitura bateko **ikuspen-faktore** ezezagunak **kalkulatu**, erlazio horiek erabiliz
- **Gainazal beltzen** arteko erradiazio bidezko **bero-transferentzia** kalkulatu
- Itxitura bateko gainazal difuso eta grisen arteko erradiazio bidezko bero-transferentzia kalkulatu, **erradiositate** kontzeptuaz baliatuz
- Bi zonako **itxiturako gainazalen** arteko erradiazio bidezko **bero-transferentziaren abiadura** garbiaren erlazioak lortu, bi xafla paralelo handi, bi zilindro zentrokide luze eta bi esfera zentrokide dituztenak barne
- **Erradiazio-babesgarriek** bi gainazalen arteko erradiazio bidezko bero-transferentzian duten eragina kuantifikatu, eta erradiazio-efektuak temperatura-neurketetan duen garrantziaz jabetu
- **Gainazal gris eta gas igoerle eta xurgatzaileen** arteko erradiazio bidezko **bero-transferentzia** kalkulatu.



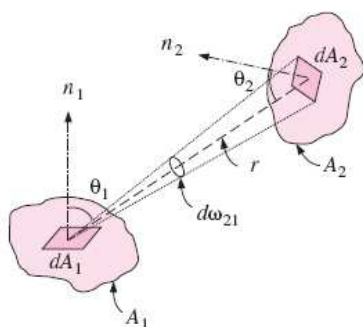
$$Q_{\text{rad}} = f \text{ (orientazioa, propietateak, tenperaturak)}$$



Ikuspen-faktorea (F_{ij} ; $F_{i \rightarrow j}$)

Forma-faktore, konfigurazio-faktore eta angelu-faktore ere baderitzo.

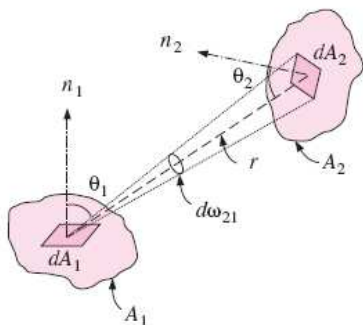
F_{ij} = *i* gainazaletik abiatzen den erradiazioetik *j* gainazalera zuzenean iristen den frakzioa.



$$\dot{Q}_{dA_1 \rightarrow dA_2} = I_1 \cos \theta_1 dA_1 d\omega_{21}$$

$$\dot{Q}_{dA_1 \rightarrow dA_2} = I_1 \cos \theta_1 dA_1 \frac{dA_2 \cos \theta_2}{r^2}$$

$$\dot{Q}_{A_1 \rightarrow A_2} = \int_{A_2} \int_{A_1} \frac{I_1 \cos \theta_1 \cos \theta_2}{r^2} dA_1 dA_2$$



$$\dot{Q}_{dA_1} = J_1 dA_1 = \pi I_1 dA_1$$

Erradiositatea

$$\dot{Q}_{A_1} = J_1 A_1 = \pi I_1 A_1$$

$$F_{12} = F_{A_1 \rightarrow A_2} = \frac{\dot{Q}_{A_1 \rightarrow A_2}}{\dot{Q}_{A_1}} = \frac{1}{A_1} \int_{A_2} \int_{A_1} \frac{\cos \theta_1 \cos \theta_2}{\pi r^2} dA_1 dA_2$$

$$F_{21} = F_{A_2 \rightarrow A_1} = \frac{\dot{Q}_{A_2 \rightarrow A_1}}{\dot{Q}_{A_2}} = \frac{1}{A_2} \int_{A_2} \int_{A_1} \frac{\cos \theta_1 \cos \theta_2}{\pi r^2} dA_1 dA_2$$

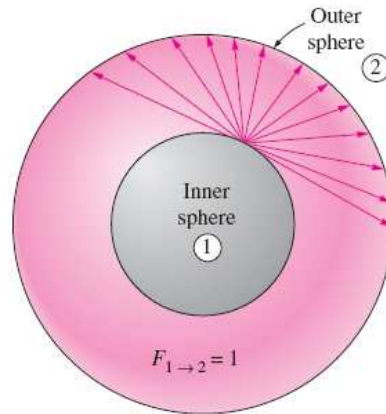
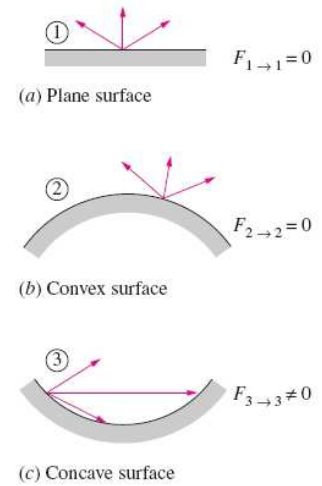
$$A_1 F_{12} = A_2 F_{21}$$

Elkarrekotasun-erlazioa

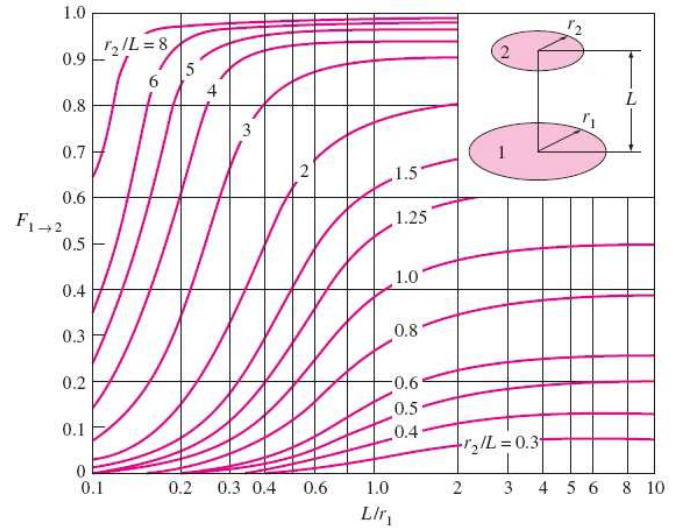
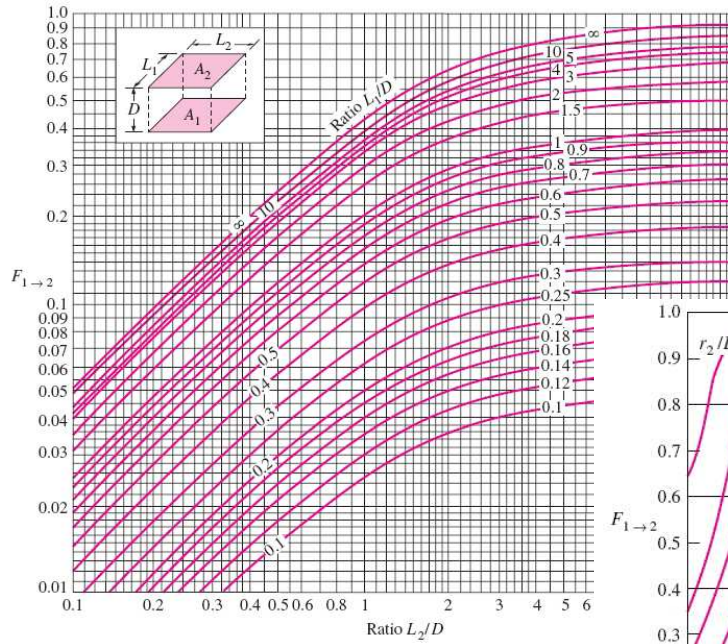
Baldin $j = i \rightarrow F_{ii} = i$ gainazaletik abiatu eta gainazal berera zuzenean iristen den erradiazio-fraczioa

Gainazal lau edo ganbilak $\rightarrow F_{ii} = 0$

Gainazal ahurak $\rightarrow F_{ii} \neq 0$



Geometry	Relation	Geometry	Relation
Aligned parallel rectangles 	$\bar{X} = X/L$ and $\bar{Y} = Y/L$ $F_{i \rightarrow j} = \frac{2}{\pi \bar{X} \bar{Y}} \left\{ \ln \left[\frac{(1 + \bar{X}^2)(1 + \bar{Y}^2)}{1 + \bar{X}^2 + \bar{Y}^2} \right]^{1/2} + \bar{X}(1 + \bar{Y}^2)^{1/2} \tan^{-1} \frac{\bar{X}}{(1 + \bar{Y}^2)^{1/2}} + \bar{Y}(1 + \bar{X}^2)^{1/2} \tan^{-1} \frac{\bar{Y}}{(1 + \bar{X}^2)^{1/2}} - \bar{X} \tan^{-1} \bar{X} - \bar{Y} \tan^{-1} \bar{Y} \right\}$	Parallel plates with midlines connected by perpendicular line 	$W_i = w_i/L$ and $W_j = w_j/L$ $F_{i \rightarrow j} = \frac{[(W_i + W_j)^2 + 4]^{1/2} - (W_j - W_i)^2 + 4]^{1/2}}{2W_j}$
Coaxial parallel disks 	$R_i = r_i/L$ and $R_j = r_j/L$ $S = 1 + \frac{R_j^2}{R_i^2}$ $F_{i \rightarrow j} = \frac{1}{2} \left\{ S - \left[S^2 - 4 \left(\frac{r_j}{r_i} \right)^2 \right]^{1/2} \right\}$	Inclined plates of equal width and with a common edge 	$F_{i \rightarrow j} = 1 - \sin \frac{1}{2} \alpha$
Perpendicular rectangles with a common edge 	$H = Z/X$ and $W = Y/X$ $F_{i \rightarrow j} = \frac{1}{\pi W} \left\{ W \tan^{-1} \frac{1}{W} + H \tan^{-1} \frac{1}{H} - (H^2 + W^2)^{1/2} \tan^{-1} \frac{1}{(H^2 + W^2)^{1/2}} + \frac{1}{4} \ln \left\{ \frac{(1 + W^2)(1 + H^2)}{1 + W^2 + H^2} \right\} \times \left[\frac{W^2(1 + W^2 + H^2)}{(1 + W^2)(W^2 + H^2)} \right]^{1/2} \times \left[\frac{H^2(1 + H^2 + W^2)}{(1 + H^2)(H^2 + W^2)} \right]^{1/2} \right\}$	Perpendicular plates with a common edge 	$F_{i \rightarrow j} = \frac{1}{2} \left\{ 1 + \frac{w_j}{w_i} - \left[1 + \left(\frac{w_j}{w_i} \right)^2 \right]^{1/2} \right\}$
		Three-sided enclosure 	$F_{i \rightarrow j} = \frac{w_i + w_j - w_k}{2w_i}$



13.2 – IKUSPEN-FAKTOREAREN ERLAZIOAK

N gainazalez osatutako itxitura → N² ikuspen-faktore

1 – Elkarrekotasun-erlazioa

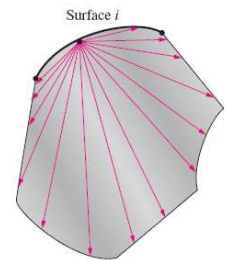
$$A_i F_{ij} = A_j F_{ji}$$

2 – Batuketa-araua

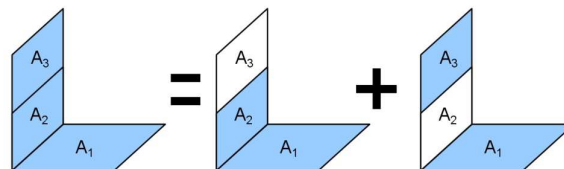
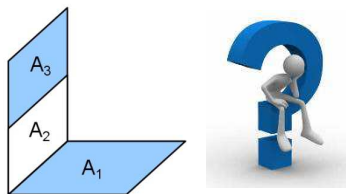
$$\sum_{j=1}^N F_{i \rightarrow j} = 1$$

$$F_{11} + F_{12} + \dots + F_{1N} = 1$$

$$F_{11} + F_{21} + \dots + F_{N1} \neq 1$$



3 – Gainjartze-araua



$$F_{1 \rightarrow (2,3)} = F_{1 \rightarrow 2} + F_{1 \rightarrow 3}$$

$$F_{1 \rightarrow 3} = F_{1 \rightarrow (2,3)} - F_{1 \rightarrow 2}$$

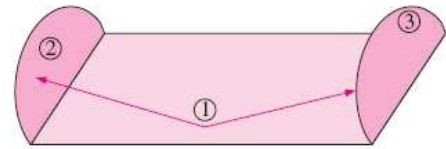
$$F_{(2,3) \rightarrow 1} = \frac{A_2 F_{2 \rightarrow 1} + A_3 F_{3 \rightarrow 1}}{A_2 + A_3}$$

4 – Simetria-araua

j eta k gainazalak i-rekiko simetrikoak badira

$$F_{i \rightarrow j} = F_{i \rightarrow k}$$

$$F_{j \rightarrow i} = F_{k \rightarrow i}$$

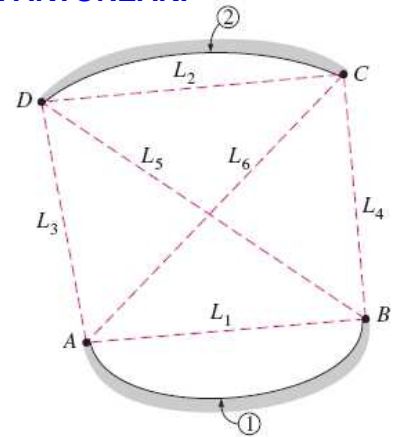


$F_{1 \rightarrow 2} = F_{1 \rightarrow 3}$
(Also, $F_{2 \rightarrow 1} = F_{3 \rightarrow 1}$)

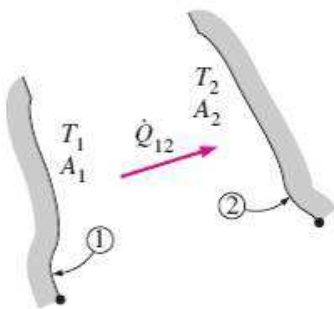
LUZERA INFINITUKO GAINAZALEN ARTEKO IKUSPEN-FAKTOREAK: HARI GURUTZATUEN METODOA

$$F_{1 \rightarrow 2} = \frac{(L_5 + L_6) - (L_3 + L_4)}{2 \cdot L_1}$$

$$F_{i \rightarrow j} = \frac{\sum (\text{Hari gurutzatuak}) - \sum (\text{Hari gurutzatu gabeak})}{2x(i \text{ gainazaleko haria})}$$



13.3 – ERRADIAZIO BIDEZKO BERO-TRANSFERENTZIA: GAINAZAL BELTZAK



$$\dot{Q}_{1 \rightarrow 2} = \left(\begin{array}{l} 1 \text{ gainazal osotik} \\ \text{irteten den eta 2} \\ \text{gainazalera iristen} \\ \text{den erradiazioa} \end{array} \right) - \left(\begin{array}{l} 2 \text{ gainazal osotik} \\ \text{irteten den eta 1} \\ \text{gainazalera iristen} \\ \text{den erradiazioa} \end{array} \right)$$

$$\dot{Q}_{1 \rightarrow 2} = A_1 \sigma T_1^4 F_{1 \rightarrow 2} - A_2 \sigma T_2^4 F_{2 \rightarrow 1}$$

Elkarrekinkotasun-erlazioa erabilita: $\dot{Q}_{1 \rightarrow 2} = A_1 F_{1 \rightarrow 2} \sigma (T_1^4 - T_2^4)$

N gainazal beltzek osatutako itxura batean:

$$\dot{Q}_i = \sum_{j=1}^N \dot{Q}_{i \rightarrow j} = \sum_{j=1}^N A_i F_{i \rightarrow j} \sigma (T_i^4 - T_j^4) \quad [\text{W}]$$

$$\dot{Q}_{i \rightarrow i} =$$

13.4 – ERRADIAZIO BIDEZKO BERO-TRANSFERENTZIA: GAINAZAL DIFUSO GRISAK

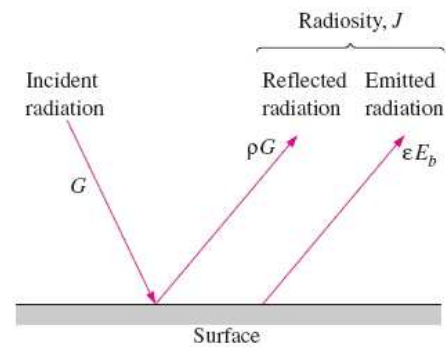
Hipotesiak: Gainazal opakuak, difusoak, grisak eta isotermikoak.

Erradiazio uniformea.

ERRADIOSITATEA

“Gainazal batetik denbora eta azalera unitateko irteten den erradiazio-energia”

$$J_i = \left(\begin{array}{l} i \text{ gainazal} \\ \text{igorritako} \\ \text{erradiazioa} \end{array} \right) + \left(\begin{array}{l} i \text{ gainazal} \\ \text{islatutako} \\ \text{erradiazioa} \end{array} \right)$$



$$J_i = \epsilon_i \cdot E_{bi} + \rho_i \cdot G_i$$

$$\left. \begin{array}{l} \text{Gainazal grisa} \rightarrow \epsilon_i = \alpha_i \\ \text{Gainazal opakua} \rightarrow \alpha_i + \rho_i = 1 \end{array} \right\} \Rightarrow J_i = \epsilon_i \cdot E_{bi} + (1 - \epsilon_i) \cdot G_i \text{ [W/m}^2\text{]}$$

Gorputz beltz batean: $J_{bi} = E_{bi} = \sigma \cdot T_i^4$

13.4 – ERRADIAZIO BIDEZKO BERO-TRANSFERENTZIA: GAINAZAL DIFUSO GRISAK

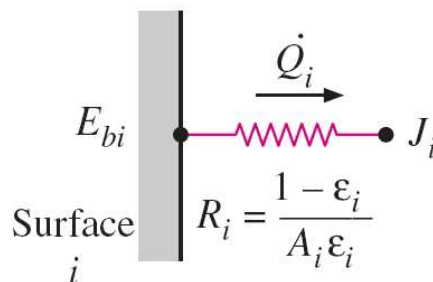
ERRADIAZIO BIDEZKO BERO-TRANSFERENTZIA NETOA:

$$\dot{Q}_i = \left(\begin{array}{l} i \text{ gainazal} \\ \text{osotik irteten} \\ \text{den erradiazioa} \end{array} \right) - \left(\begin{array}{l} i \text{ gainazal} \\ \text{osora iristen} \\ \text{den erradiazioa} \end{array} \right)$$

$$\dot{Q}_i = A_i (J_i - G_i) = A_i \left(J_i - \frac{J_i - \epsilon_i E_{bi}}{1 - \epsilon_i} \right) = \frac{A_i \epsilon_i}{1 - \epsilon_i} (E_{bi} - J_i)$$

Ohm-en legearen araberako analogia eginez:

$$\dot{Q}_i = \frac{E_{bi} - J_i}{R_i}$$



13.4 – ERRADIAZIO BIDEZKO BERO-TRANSFERENTZIA: GAINAZAL DIFUSO GRISAK

ERRADIAZIO BIDEZKO BERO-TRANSFERENTZIA NETOA: $\dot{Q}_i = \frac{E_{bi} - J_i}{R_i}$

$$R_i = \frac{1 - \epsilon_i}{A_i \epsilon_i}$$

Erradiazio gainazal erresistentzia

Gorputz beltz batean?



Gainazal berrerradiatzailea → Gainazalak irabazten duen adina erradiazio-energia galtzen du → $\dot{Q}_i = 0$

Egoera egonkorrean eta konbektzio mesprezagarriarekin

↓
Gainazal adiabatikoa

$$J_i = E_{bi} = \sigma T_i^4$$

TERMOTEKANIA

13.4 – ERRADIAZIO BIDEZKO BERO-TRANSFERENTZIA: GAINAZAL DIFUSO GRISAK

BI GAINAZALEN ARTEKO ERRADIAZIO BIDEZKO BERO-TRANSFERENTZIA

$$\dot{Q}_{i \rightarrow j} = \left(\begin{array}{l} i \text{ gainazal osotik irteten} \\ \text{den eta } j \text{ gainazalera} \\ \text{iristen den erradiazioa} \end{array} \right) - \left(\begin{array}{l} j \text{ gainazal osotik irteten} \\ \text{den eta } i \text{ gainazalera} \\ \text{iristen den erradiazioa} \end{array} \right)$$

$$\dot{Q}_{i \rightarrow j} = A_i J_i F_{i \rightarrow j} - A_j J_j F_{j \rightarrow i} \quad [\text{W}]$$

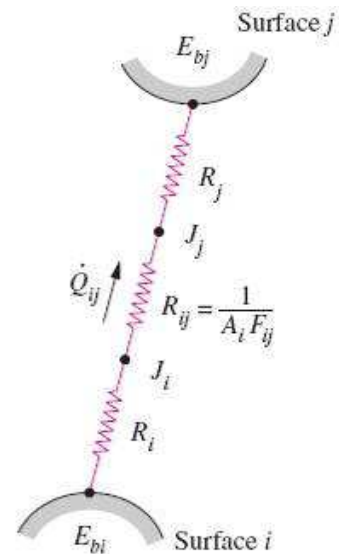
Elkarrekotasun-erlazioa erabilita: $\dot{Q}_{i \rightarrow j} = A_i F_{i \rightarrow j} (J_i - J_j)$

Ohmen legearen analogia eginda:

$$\dot{Q}_{i \rightarrow j} = \frac{J_i - J_j}{R_{i \rightarrow j}} \quad [\text{W}]$$

$$R_{i \rightarrow j} = \frac{1}{A_i F_{i \rightarrow j}}$$

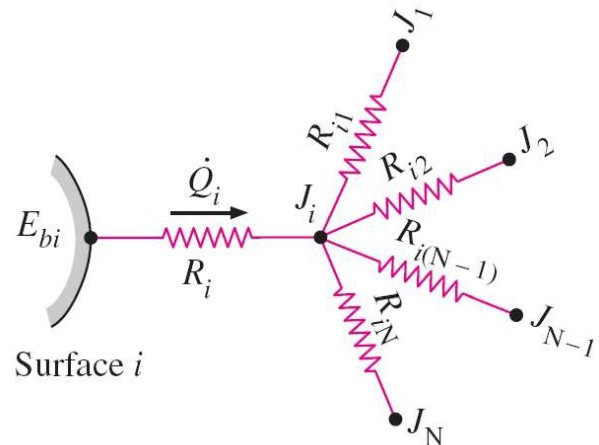
Espazio-erresistentzia



BI GAINAZALEN ARTEKO ERRADIAZIO BIDEZKO BERO-TRANSFERENTZIA

$$\dot{Q}_i = \sum_{j=1}^N \dot{Q}_{i \rightarrow j} = \sum_{j=1}^N A_i F_{i \rightarrow j} (J_i - J_j)$$

$$\dot{Q}_i = \sum_{j=1}^N \frac{J_i - J_j}{R_{i \rightarrow j}} \quad [\text{W}]$$



$$\frac{E_{bi} - J_i}{R_i} = \sum_{j=1}^N \frac{J_i - J_j}{R_{i \rightarrow j}} \quad [\text{W}]$$

ERRADIAZIO PROBLEMAK EBAZTEKO METODOAK

Gainazal bakoitzaren tenperatura T_i edo bero-transferentziaren abiadura netoa \dot{Q}_i eman behar dira.

Metodo zuzena (*matriziala*)

\dot{Q}_i bero-transferentziaren abiadura netoa zehaztua duten gainazaletan:

$$\dot{Q}_i = A_i \sum_{j=1}^N F_{i \rightarrow j} (J_i - J_j)$$

T_i tenperatura zehaztua duten gainazaletan:

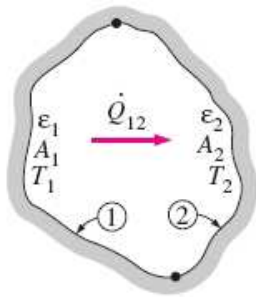
$$\sigma \cdot T_i^4 = J_i + \frac{1 - \epsilon_i}{\epsilon_i} \sum_{j=1}^N F_{i \rightarrow j} (J_i - J_j)$$

N ekuazio eta N ezezagenez osatutako sistema (J_1, J_2, \dots, J_N)

Sare-metodoa

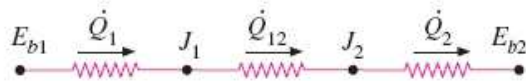
Sare elektrikoan analogian oinarrituta

ERRADIAZIO BIDEZKO BERO-TRANSFERENTZIA BI GAINAZALEKO ITXITURETAN



$$\dot{Q}_{12} = \dot{Q}_1 = -\dot{Q}_2$$

$$\dot{Q}_{12} = \frac{E_{b1} - E_{b2}}{R_1 + R_{12} + R_2}$$

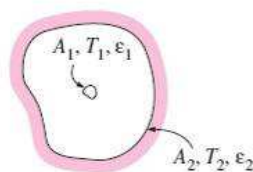


$$R_1 = \frac{1 - \epsilon_1}{A_1 \epsilon_1} \quad R_{12} = \frac{1}{A_1 F_{12}} \quad R_2 = \frac{1 - \epsilon_2}{A_2 \epsilon_2}$$

$$\dot{Q}_{12} = \frac{\sigma \cdot (T_1^4 - T_2^4)}{\frac{1 - \epsilon_1}{A_1 \epsilon_1} + \frac{1}{A_1 F_{12}} + \frac{1 - \epsilon_2}{A_2 \epsilon_2}}$$

ERRADIAZIO BIDEZKO BERO-TRANSFERENTZIA BI GAINAZALEKO ITXITURETAN

Small object in a large cavity

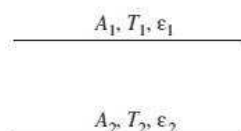


$$\frac{A_1}{A_2} \approx 0$$

$$F_{12} = 1$$

$$\dot{Q}_{12} = A_1 \sigma \epsilon_1 (T_1^4 - T_2^4)$$

Infinitely large parallel plates

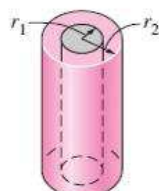


$$A_1 = A_2 = A$$

$$F_{12} = 1$$

$$\dot{Q}_{12} = \frac{A \sigma (T_1^4 - T_2^4)}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1}$$

Infinitely long concentric cylinders

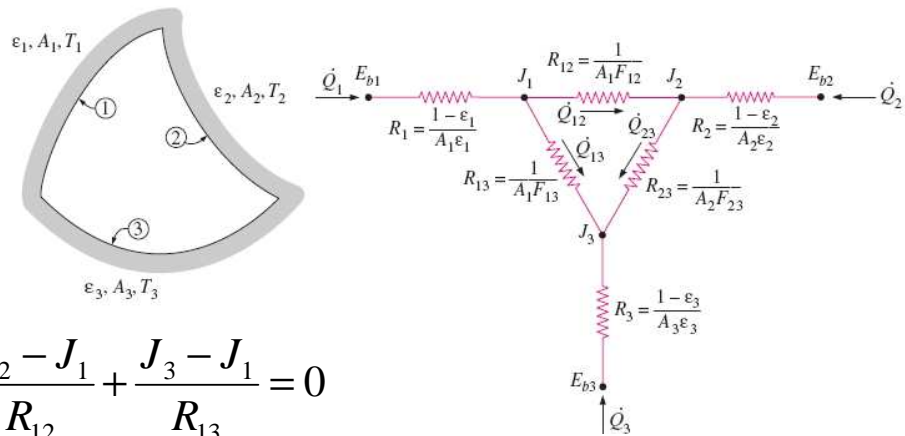


$$\frac{A_1}{A_2} = \frac{r_1}{r_2}$$

$$F_{12} = 1$$

$$\dot{Q}_{12} = \frac{A_1 \sigma (T_1^4 - T_2^4)}{\frac{1}{\epsilon_1} + \frac{1 - \epsilon_2}{\epsilon_2} \left(\frac{r_1}{r_2}\right)}$$

ERRADIAZIO BIDEZKO BERO-TRANSFERENTZIA HIRU GAINAZALEKO ITXITURETAN

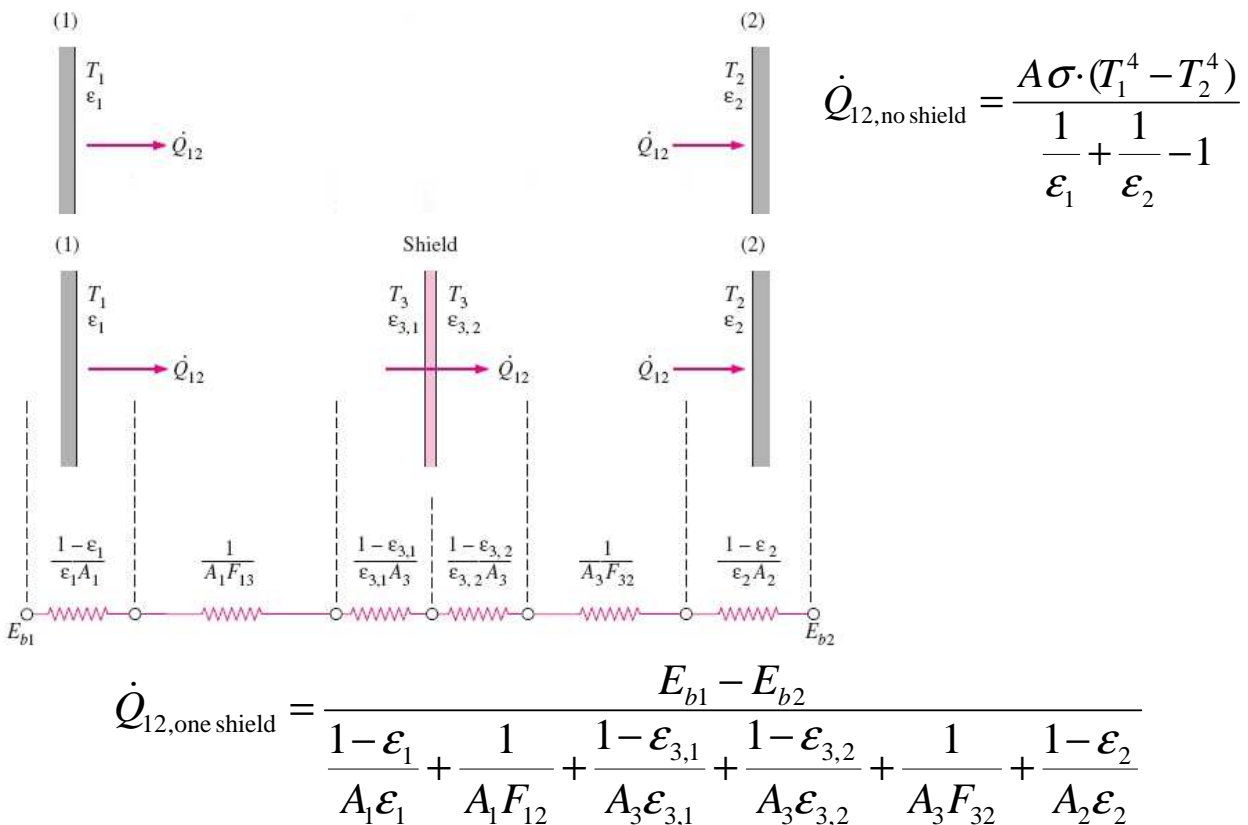


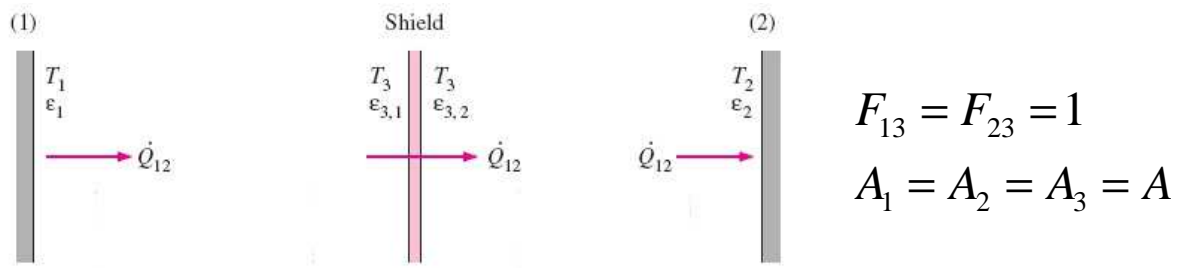
$$\begin{cases} \frac{E_{b1} - J_1}{R_1} + \frac{J_2 - J_1}{R_{12}} + \frac{J_3 - J_1}{R_{13}} = 0 \\ \frac{J_1 - J_2}{R_{12}} + \frac{E_{b2} - J_2}{R_2} + \frac{J_3 - J_2}{R_{23}} = 0 \\ \frac{J_1 - J_3}{R_{13}} + \frac{J_2 - J_3}{R_{23}} + \frac{E_{b3} - J_3}{R_3} = 0 \end{cases}$$

Temperaturaren ordeaz Q_i erradiazio bidezko bero-transferentziaren abiadura garbia zehazten denean

$$\frac{E_{bi} - J_i}{R_i} \Rightarrow \dot{Q}_i$$

13.5 – ERRADIAZIO-BABESGARRIAK ETA ERRADIAZIO-EFEKTUAK 20/30



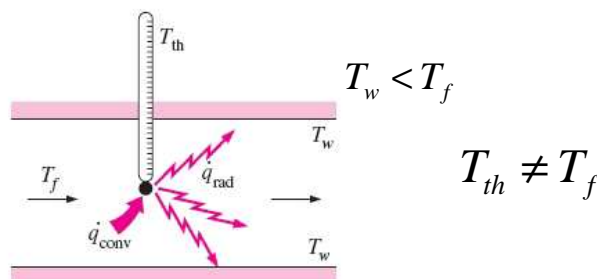


$$\dot{Q}_{12, \text{one shield}} = \frac{A\sigma \cdot (T_1^4 - T_2^4)}{\left(\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1\right) + \left(\frac{1}{\epsilon_{3,1}} + \frac{1}{\epsilon_{3,2}} - 1\right)}$$

$$\dot{Q}_{12, N \text{ shields}} = \frac{A\sigma \cdot (T_1^4 - T_2^4)}{\left(\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1\right) + \left(\frac{1}{\epsilon_{3,1}} + \frac{1}{\epsilon_{3,2}} - 1\right) + \dots + \left(\frac{1}{\epsilon_{N,1}} + \frac{1}{\epsilon_{N,2}} - 1\right)}$$

$$\epsilon_1 = \epsilon_2 = \dots = \epsilon_{N,1} = \epsilon_{N,2} = \epsilon \quad \Rightarrow \quad \dot{Q}_{12, N \text{ shields}} = \frac{A\sigma \cdot (T_1^4 - T_2^4)}{(N+1)\left(\frac{1}{\epsilon} + \frac{1}{\epsilon} - 1\right)} = \frac{1}{N+1} \dot{Q}_{12, \text{no shield}}$$

ERRADIAZIOAREN ERAGINA TEMPERATURA-NEURKETETAN



Orekan:

$$h \cdot (T_f - T_{th}) = \epsilon_{th} \sigma (T_{th}^4 - T_w^4)$$

$$T_f = T_{th} + \frac{\epsilon_{th} \sigma (T_{th}^4 - T_w^4)}{h} \quad [\text{K}]$$

Hutsean, Ar, He, O₂ eta N₂ → Erradiazio termikoarekiko gardenak.

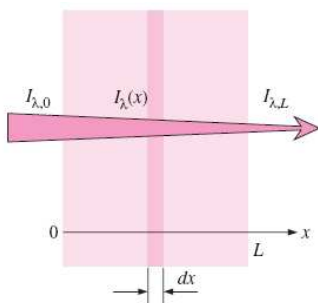
H₂O, CO₂, CO, SO₂ y H_nC_m → Erradiazio prozesuetan parte hartu. Absortzioa tenperatura ertainetan eta absortzioa eta igorpena tenperatura altuetan.

↓
Errekuntza-gasak

Zailtasunak

- Gas-erradiazioa fenomeno bolumentrikoa da.
- Gasek zenbait uhin-luzera tarte estuetan igortzen eta xurgatzen dute erradiazioa.
- Gas-nahaste bateko osagaien igortze- eta xurgatze-ezaugarriak ere tenperaturaren, presioaren eta gas-nahasturaren konposizioaren araberrakoak dira.

INGURUNE PARTE-HARTZAILE BATEN ERRADIAZIO-PROPIETATEAK



$$dI_{\lambda}(x) = -k_{\lambda} I_{\lambda}(x) dx \quad \text{Beer-en legea}$$

Absortzio espektralaren koefizienteak [m⁻¹]

$$\frac{I_{\lambda,L}}{I_{\lambda,0}} = e^{-k_{\lambda}L} = \tau_{\lambda} \quad \text{Transmisibitate espektrala}$$

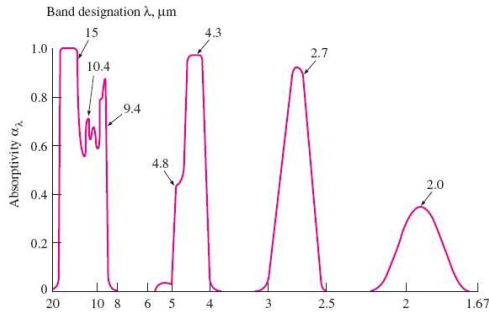
Dispertzatzen ez duten inguruak (ρ = 0) $\alpha_{\lambda} + \tau_{\lambda} = 1$

Absortzio espektrala $\alpha_{\lambda} = 1 - e^{-k_{\lambda}L}$

Emisibitate espektrala $\epsilon_{\lambda} = \alpha_{\lambda} = 1 - e^{-k_{\lambda}L}$

Baldin $k_{\lambda}L \uparrow \rightarrow \epsilon_{\lambda} \approx \alpha_{\lambda} \approx 1$ Ingurune optikoki lodia ≡ Gainazal beltza

GASEN ETA GAS-NAHASTEEN EMISIBITATEA ETA ABSORTIBITATEA

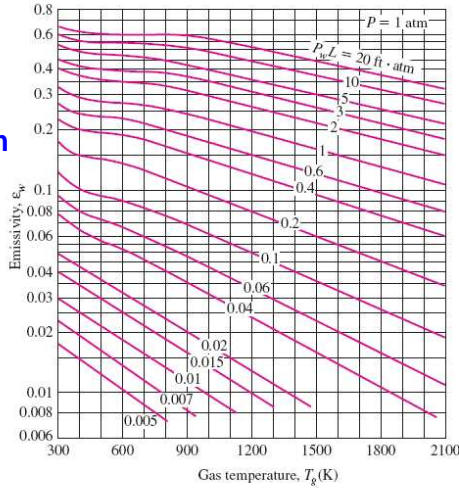


Absortzioaren banda-izaera → Ez-grisa

Bi metodo:

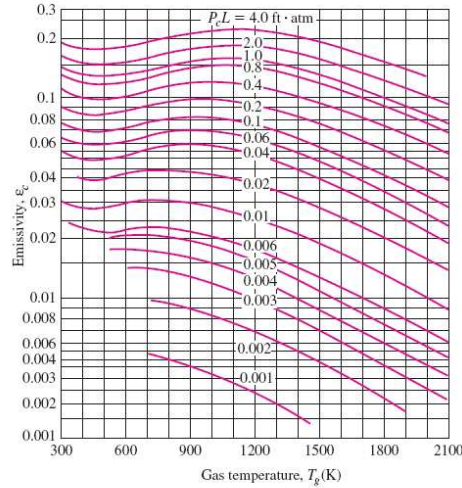
- 1) Banda-eredua
- 2) Gas gris baten absortibitate eta emisibitate erangikorak kalkulatu.

H₂O
P_T = 1 atm



(a) H₂O

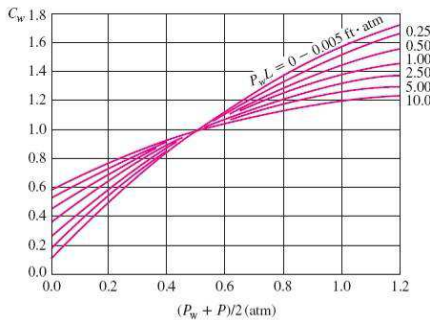
CO₂
P_T = 1 atm



(b) CO₂

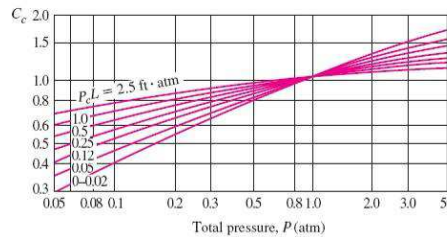
GASEN ETA GAS-NAHASTEEN EMISIBITATEA ETA ABSORTIBITATEA

H₂O
P_T ≠ 1 atm

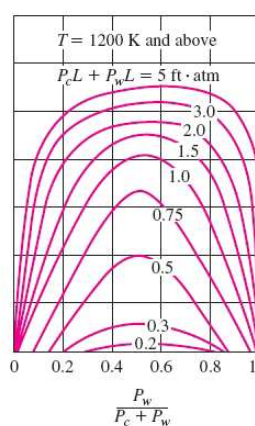
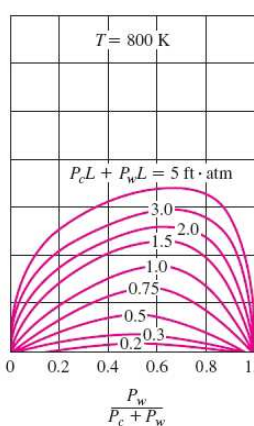
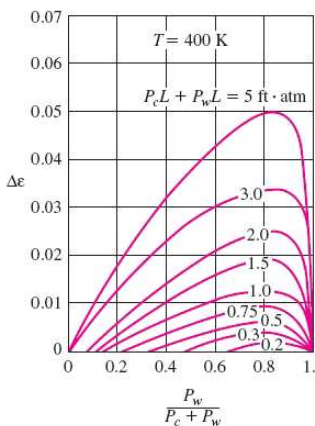


$$\epsilon_w = C_w \cdot \epsilon_{w,1atm}$$

CO₂
P_T ≠ 1 atm



$$\epsilon_c = C_c \cdot \epsilon_{c,1atm}$$



$$\epsilon_g = \epsilon_w + \epsilon_c - \Delta\epsilon$$

Emisibitatearen zuzenketa faktorea

13.6 – ERRADIAZIO-TRUKEA GAS IGORLEEKIN ETA XURGATZAILEEKIN

GASEN ETA GAS-NAHASTEEN EMISIBITATEA ETA ABSORTIBITATEA

Geometria ez hemisferikoetan → Batez besteko izpi luzera L

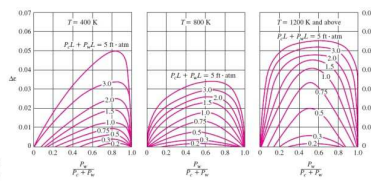
Mean beam length L for various gas volume shapes

Gas Volume Geometry	L
Hemisphere of radius R radiating to the center of its base	R
Sphere of diameter D radiating to its surface	$0.65D$
Infinite circular cylinder of diameter D radiating to curved surface	$0.95D$
Semi-infinite circular cylinder of diameter D radiating to its base	$0.65D$
Semi-infinite circular cylinder of diameter D radiating to center of its base	$0.90D$
Infinite semicircular cylinder of radius R radiating to center of its base	$1.26R$
Circular cylinder of height equal to diameter D radiating to entire surface	$0.60D$
Circular cylinder of height equal to diameter D radiating to center of its base	$0.71D$
Infinite slab of thickness D radiating to either bounding plane	$1.80D$
Cube of side length L radiating to any face	$0.66L$
Arbitrary shape of volume V and surface area A_s radiating to surface	$3.6V/A_s$

13.6 – ERRADIAZIO-TRUKEA GAS IGORLEEKIN ETA XURGATZAILEEKIN

GASEN ETA GAS-NAHASTEEN EMISIBITATEA ETA ABSORTIBITATEA

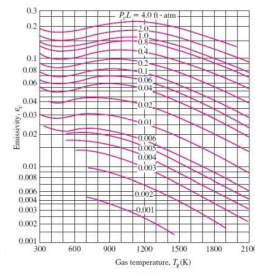
$$\alpha_g = \alpha_w + \alpha_c - \Delta\alpha$$



$$\Delta\alpha = \Delta\epsilon$$

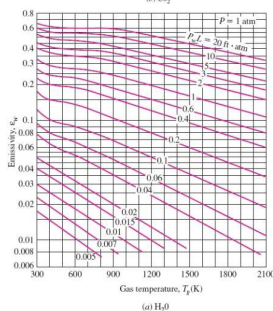
$$T_s$$

CO₂



$$\alpha_c = C_c \cdot \left(\frac{T_g}{T_s}\right)^{0.65} \cdot \epsilon_c(T_s, P_c L \frac{T_s}{T_g})$$

H₂O




$$\alpha_w = C_w \cdot \left(\frac{T_g}{T_s}\right)^{0.45} \cdot \epsilon_w(T_s, P_w L \frac{T_s}{T_g})$$

GASEN ETA GAS-NAHASTEEN EMISIBITATEA ETA ABSORTIBITATEA

Gas batek gainazalera igorritakoa

Gas batek xurgatutakoa gainazala beltza izanik

$$\dot{Q}_{g,e} = \varepsilon_g A_S \sigma T_g^4 \qquad \dot{Q}_{g,a} = \alpha_g A_S \sigma T_S^4$$


$$\dot{Q}_{net,black} = A_S \sigma (\varepsilon_g T_g^4 - \alpha_g T_S^4)$$

Gainazala beltza ez balitz ($\varepsilon > 0,7$)

$$\dot{Q}_{net,gray} = \frac{1 + \varepsilon_s}{2} \dot{Q}_{net,black} = \frac{1 + \varepsilon_s}{2} A_S \sigma (\varepsilon_g T_g^4 - \alpha_g T_S^4)$$

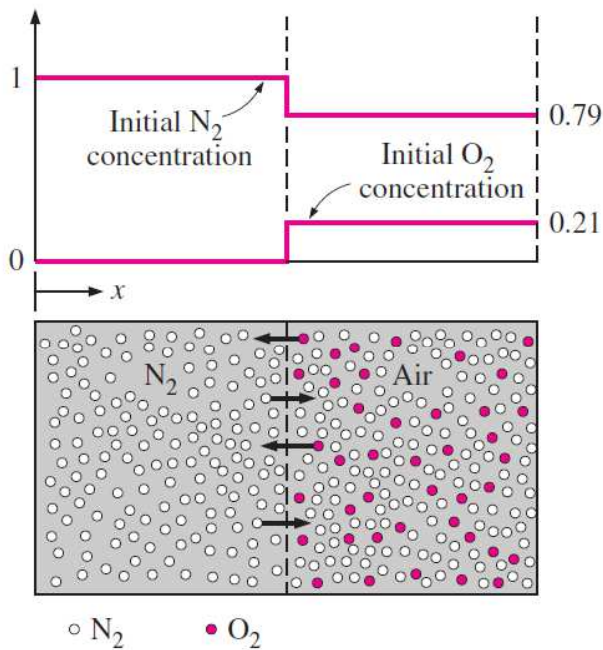
14. GAIA

MASA TRANSFERENTZIA

14.0 - HELBURUAK

2/31

- **Kontzentrazio-gradiente**a eta **masa-transferentziako** mekanismo fisikoak ulertu
- **Bero-** eta **masa-transferentziaren** arteko **analogiak** ezagutu
- Toki bateko kontzentrazioa masan edo moletan oinarrituta deskribatu, eta difusio-abiadura eta kontzentrazio-gradiente lotu, **Fick-en legearen** bidez
- Baldintza **geldikorretan**, geruza lau batean zeharreko **masa-difusioaren** abiadura kalkulatu
- **Ur-lurrunaren migrazioa** eraikinetan
- **Konbekzio** bidezko masa-transferentzia kalkulatu
- Aldibereko bero- eta masa-transferentzia aztertu.

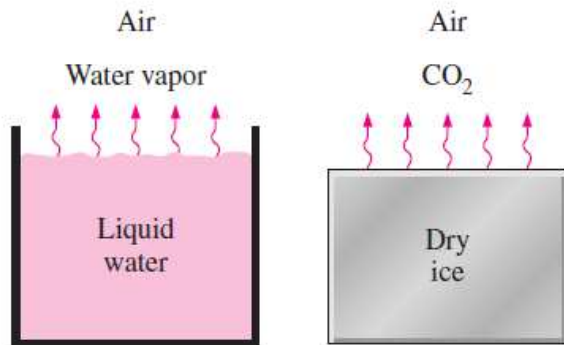


$$\dot{m} = -D_{AB} \cdot A \frac{dC}{dx}$$

D_{AB} : Espezien difusio-koefizientea
 dC/dx : kontzentrazio gradientea

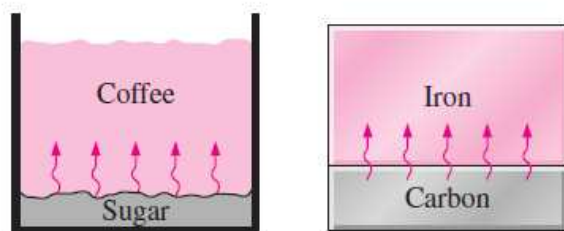
Fenomenoa	Ezaugarri garraiatzailea	Indar bultzatzailea	Legea	Ekuazioa	Garraio koefizientea
Difusioa	Materia	Kontzentrazio diferentzia	Fick	$m = -D_{AB} A dC/dx$	Difusio koefizientea D_{AB}
Kondukzio termikoa	Energia	Temperatura diferentzia	Fourier	$Q = -k A dT/dx$	Konduktibitate termikoa k
Kondukzio elektrikoa	Elektroiak	Potentzial diferentzia	Ohm	$I = -\sigma A dV/dx$	Konduktibitate elektrikoa σ
Biskositatea	Momentu kantitatea	Abiadura diferentzia	Newton	$F = -\mu A dv/dx$	Biskositatea μ

MASA TRANSFERENTZIA: LIKIDOAK, SOLIDOAK ETA GASAK



(a) Liquid to gas

(b) Solid to gas



(c) Solid to liquid

(d) Solid to solid

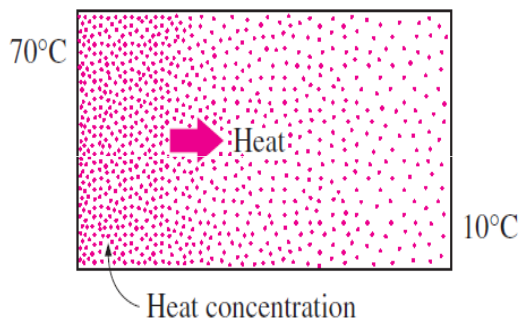
FAKTOREAK:

- Kontzentrazioa
- Temperatura
- Tarte molekularra

14.2 – BERO- ETA MASA-TRANSFERENTZIEN ARTEKO ANALOGIA

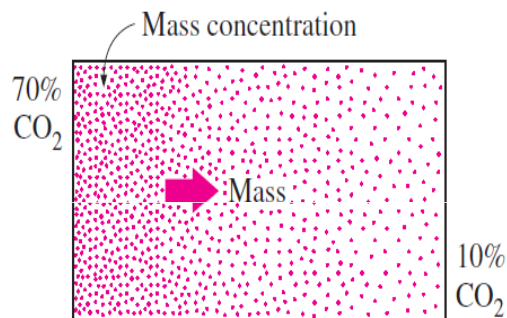
TENPERATURA

Bero Transferentzia



Indar bultzatzailea:
Temperatura gradientea

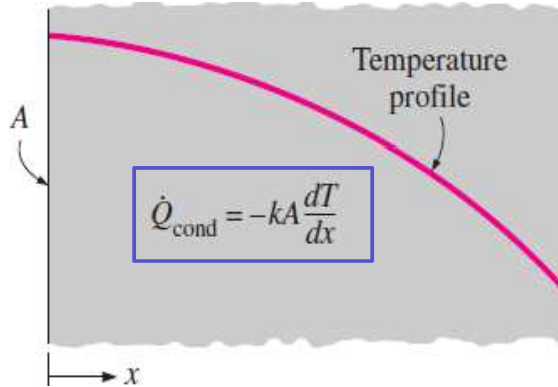
Masa Transferentzia



Indar bultzatzailea:
Kontzentrazio diferentzia

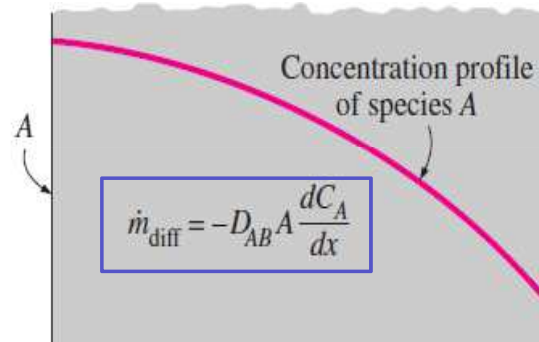
EROAPENA

Bero Transferentzia



Fourier-en Legea

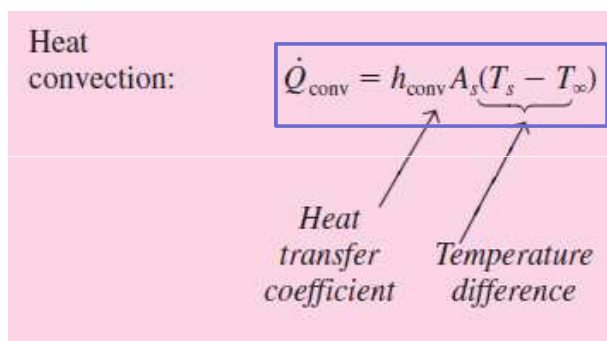
Masa Transferentzia



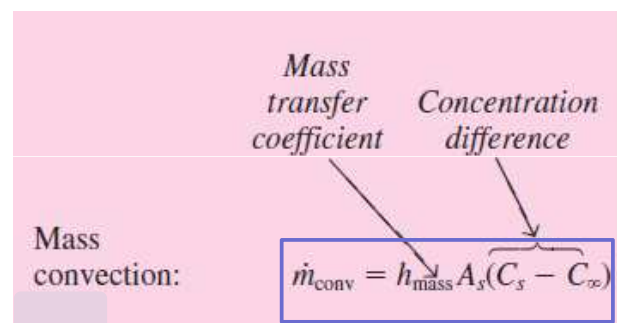
Fick-en Legea **Difusibitatea**

KONBEKZIOA

Bero Transferentzia

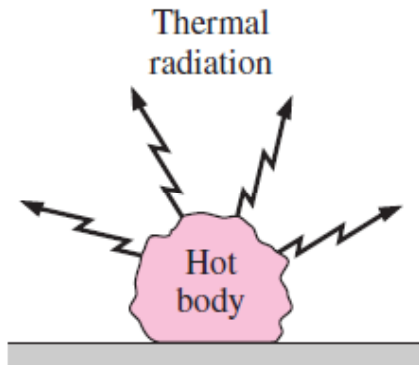


Masa Transferentzia

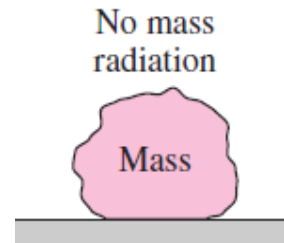


ERRADIAZIOA

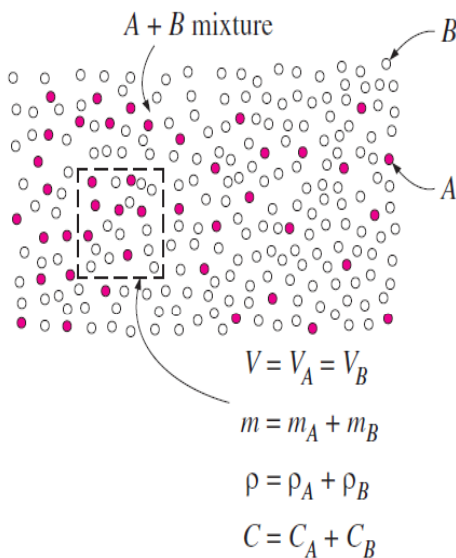
Bero Transferentzia



Masa Transferentzia



14.3 – MASA DIFUSIOA



MASAN OINARRITUA:

Dentsitatea

$$\rho_i = m_i / V \text{ [kg / m}^3\text{]}$$

$$\rho = m / V$$

Masa Frakzioa

$$w_i = \frac{m_i}{m} = \frac{\rho_i}{\rho}$$

$$0 < w < 1 \quad \sum w_i = 1$$

MOLETAN OINARRITUA:

Dentsitate Molarra

$$C_i = N_i / V \text{ [kmol / m}^3\text{]}$$

$$C = N / V$$

Frakzio Molarra

$$y_i = \frac{N_i}{N} = \frac{C_i}{C}$$

$$0 < y < 1 \quad \sum y_i = 1$$

BIEN ARTEKO ERLAZIOA:

$$C_i = \rho_i / M_i$$

$$C = \rho / M$$

M : masa molarra $M = m / N$

$$w_i = \frac{\rho_i}{\rho} = \frac{C_i \cdot M_i}{C \cdot M} = y_i \frac{M_i}{M}$$

GAS NAHASTURA IDEALAK

Presio Partziala: P_i

Gas presio partzialen batura : $P = \sum P_i \implies$ **Dalton-en Legea Presio gehigarriak**

$$\frac{P_i}{P} = \frac{N_i \cdot R_u \cdot T / V}{N \cdot R_u \cdot T / V} = \frac{N_i}{N} = y_i$$

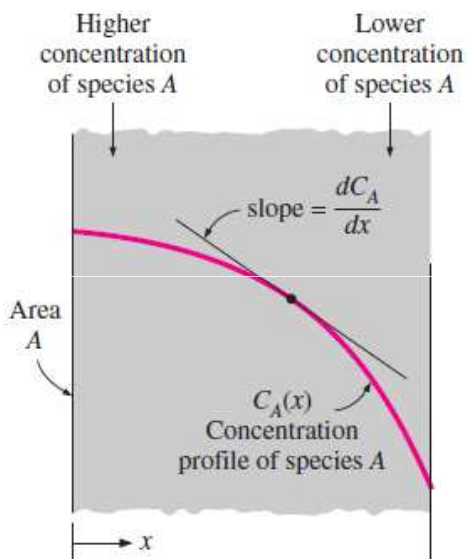
2 mol A
6 mol B
 $P = 120 \text{ kPa}$

A mixture of two ideal gases A and B

$$y_A = \frac{N_A}{N} = \frac{2}{2 + 6} = 0.25$$

$$P_A = y_A P = 0.25 \times 120 = 30 \text{ kPa}$$

FICK-EN DIFUSIO LEGEA: BI ESPEZIEZ OSATUTAKO INGURUNE GELDIA



Masan Oinarritua

$$\begin{aligned} \dot{m}_{diff} &= -\rho A D_{AB} \frac{dw_A}{dx} \\ &= -\rho A D_{AB} \frac{d(\rho_A/\rho)}{dx} \\ &= -A D_{AB} \frac{d\rho_A}{dx} \quad (\text{if } \rho = \text{constant}) \end{aligned}$$

Moletan Oinarritua

$$\begin{aligned} \dot{N}_{diff, A} &= -C A D_{AB} \frac{dy_A}{dx} \\ &= -C A D_{AB} \frac{d(C_A/C)}{dx} \\ &= -A D_{AB} \frac{dC_A}{dx} \quad (\text{if } C = \text{constant}) \end{aligned}$$

DIFUSIO KOEFIZIENTEA (D_{AB})

Modu Esperimentalean kalkulatu $D_{H_2O-Air} = 1,87 \cdot 10^{-10} \frac{T^{2,072}}{P}$ [m²/s], 280K < T < 450K
T[K] eta P[atm]

- Gas-etan: nahasturaren konposizioarekiko independentea: $T \uparrow \rightarrow D_{AB} \uparrow$
 $P \uparrow \rightarrow D_{AB} \downarrow$

$$D_{AB} \cdot \alpha \frac{T^{3/2}}{P} \text{ edo } \frac{D_{AB,1}}{D_{AB,2}} = \frac{P_2}{P_1} \left(\frac{T_1}{T_2} \right)^{3/2}$$

- Solidoak eta likidoak: $T \uparrow \rightarrow D_{AB} \uparrow$

Binary diffusion coefficients of some gases in air at 1 atm pressure (from Mills, Ref. 13, Table A.17a, p. 869)

Binary Diffusion Coefficient, *
m²/s × 10⁵

(a) Diffusion through Liquids

Substance A (Solute)	Substance B (Solvent)	T, K	D_{AB} , m ² /s
Ammonia	Water	285	1.6×10^{-9}
Benzene	Water	293	1.0×10^{-9}
Carbon dioxide	Water	298	2.0×10^{-9}
Chlorine	Water	285	1.4×10^{-9}

(b) Diffusion through Solids

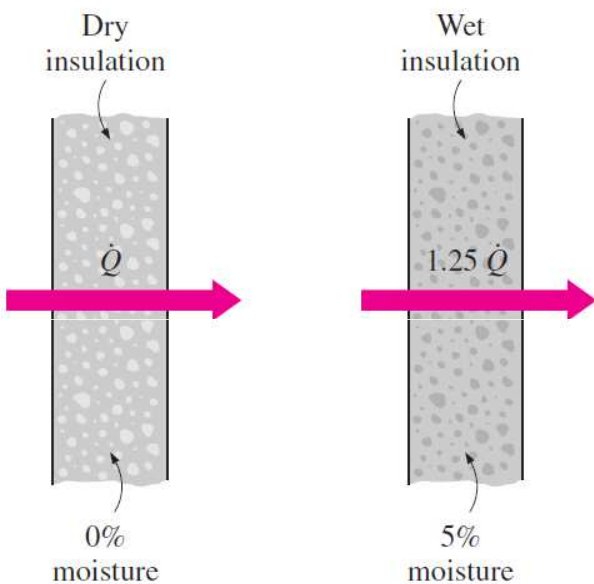
Substance A (Solute)	Substance B (Solvent)	T, K	D_{AB} , m ² /s
Carbon dioxide	Natural rubber	298	1.1×10^{-10}
Nitrogen	Natural rubber	298	1.5×10^{-10}
Oxygen	Natural rubber	298	2.1×10^{-10}
Helium	Pyrex	773	2.0×10^{-12}

T, K	O ₂	CO ₂	H ₂	NO
200	0.95	0.74	3.75	0.88
300	1.88	1.57	7.77	1.80
400	5.25	2.63	12.5	3.03
500	4.75	3.85	17.1	4.43
600	6.46	5.37	24.4	6.03
700	8.38	6.84	31.7	7.82
800	10.5	8.57	39.3	9.78
900	12.6	10.5	47.7	11.8
1000	15.2	12.4	56.9	14.1
1200	20.6	16.9	77.7	19.2
1400	26.6	21.7	99.0	24.5
1600	33.2	27.5	125	30.4
1800	40.3	32.8	152	37.0
2000	48.0	39.4	180	44.8

del F

35

14.6 – UR-LURRUNAREN MIGRAZIOA ERAIKINETAN

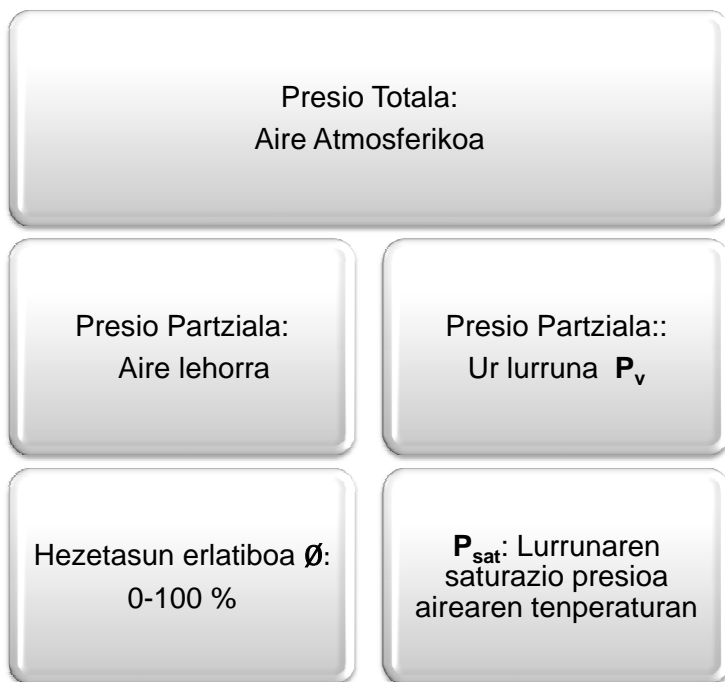


Eraikuntzako joskerako hezetasunaren eragina:

- Zuran: Dimentsioak, ltxura, ustelketak
- Metalak: korrosioa eta herdoiltzea
- Pintura: askatzea
- Lizuna: agertu

Lurrun heziak:

Material iragazgaitzak
Lurrun moteltzaileak



Saturation pressure of water at various temperatures

Temperature, °C	Saturation Pressure, Pa
-40	13
-36	20
-32	31
-28	47
-24	70
-20	104
-16	151
-12	218
-8	310
-4	438
0	611
5	872
10	1,228
15	1,705
20	2,339
25	3,169
30	4,246
35	5,628
40	7,384
50	12,349
100	101,330
200	1.55×10^6
300	8.58×10^6

$$P_v = \phi \cdot P_{sat}$$

GERUZA LAU BATEAN ZEHARREKO HEZETASAUNAREN MASA EMARIA:

$$\dot{m}_v = P \cdot A \frac{P_{v,1} - P_{v,2}}{L} = P \cdot A \frac{\phi_1 \cdot P_{sat,1} - \phi_2 \cdot P_{sat,2}}{L} \quad [\text{kg} / \text{s}]$$

Non: P lurrun-iragazkortasuna (ng/s m Pa) ($1\text{ng}=10^{-12}\text{kg}$)

Permeantzia (M) $M = \frac{P}{L} \quad [\text{kg} / \text{s m}^2 \text{ Pa}]$

Lurrun erresistentzia (R_v) $R_v = \frac{1}{M} = \frac{L}{P}$

Egitura konposatua:

$$R_{v,total} = R_{v,1} + R_{v,2} + \dots + R_{v,n} = \sum R_{v,i}$$

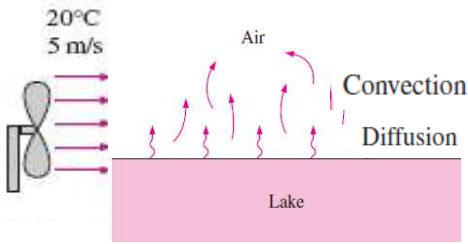
Lurrun trasmisioa egitura konposatuan:

$$\dot{m}_v = A \frac{\Delta P_v}{R_{v,total}} \quad [\text{kg} / \text{s}]$$

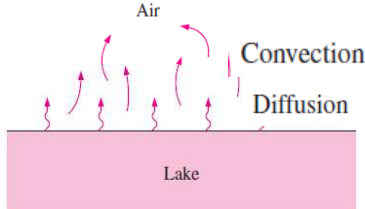
Sartzen / irteten den hezetasuna:
Difusioz <<< aire infiltrazioz

Materials and Its Thickness	Permeance ng/s · m ² · Pa
Concrete (1:2:4 mix, 1 m)	4.7
Brick, masonry, 100 mm	46
Plaster on metal lath, 19 mm	860
Plaster on wood lath, 19mm	630
Gypsum wall board, 9.5 mm	2860
Plywood, 6.4 mm	40-109
Still air, 1 m	174
Mineral wool insulation (unprotected), 1 m	245
Expanded polyurethane insulation board, 1 m	0.58-2.3
Aluminum foil, 0.025 mm	0.0
Aluminum foil, 0.009 mm	2.9
Polyethylene, 0.051 mm	9.1
Polyethylene, 0.2 mm	2.3
Polyester, 0.19 mm	4.6
Vapor retarder latex paint, 0.070 mm	26
Exterior acrylic house and trim paint, 0.040 mm	313
Building paper, unit mass of 0.16-0.68 kg/m ²	0.1-2400

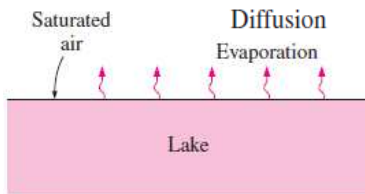
Gainazal baten eta mugitzen ari den fluido baten arteko masa transferentzia



a) Forced convection



b) Free convection



c) Diffusion

Konbektzio masa-transferentzia hurrengo parametroen arabaerakoa da:

- Gainazalaren forma
- Fluxu erregimena
- Fluidoaren abiadura
- Fluidoaren propietateen aldaketak
- Konposizio aldaketak

Erlazio Esperimentalak

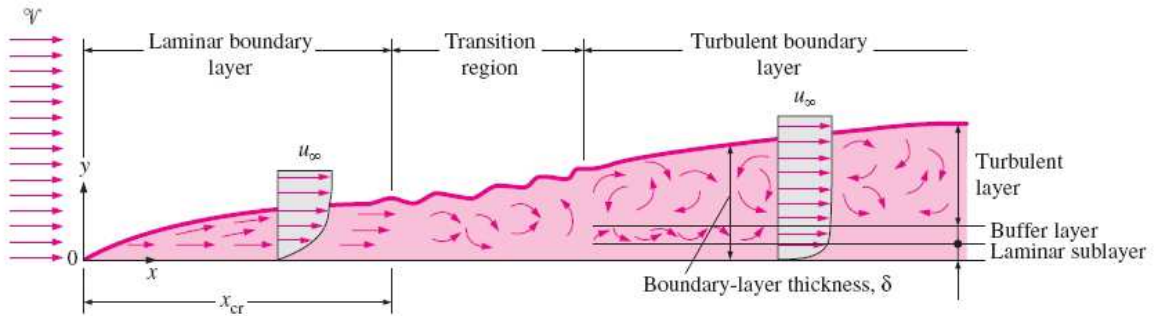
Bero Transferentzia

Masa Transferentzia

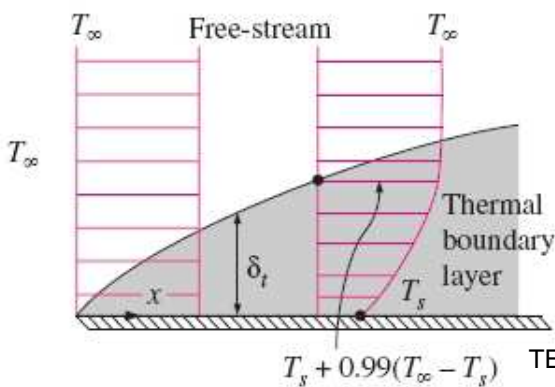
$$\dot{Q}_{conv} = h_{conv} \cdot A_s (T_s - T_\infty) \quad \dot{m}_{conv} = h_{mass} \cdot A_s (\rho_{A,s} - \rho_{A,\infty})$$

$$\dot{m}_{conv} = h_{mass} \cdot \rho A_s (w_{A,s} - w_{A,\infty})$$

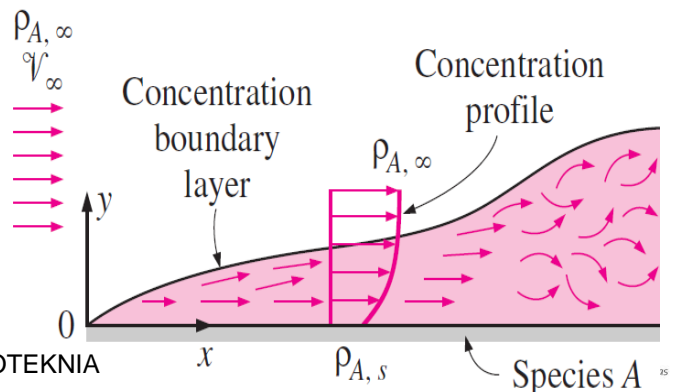
ABIADURA MUGALDE GERUZA



BERO MUGALDE GERUZA



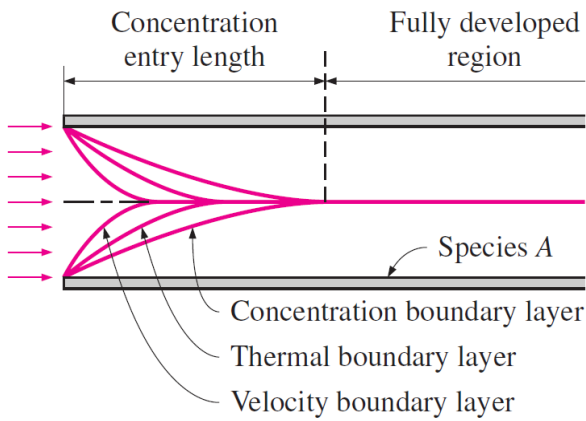
KONTZENTRAZIO MUGALDE GERUZA



KONTZENTRAZIO MUGALDE GERUZA

Kanpo Fluxua $\frac{\rho_{A,S} - \rho_A}{\rho_{A,S} - \rho_{A,\infty}} = 0,99$

Barne Fluxua



Erabat garatutako eskualdea

$$\frac{\partial}{\partial x} \left(\frac{\rho_{A,S} - \rho_A}{\rho_{A,S} - \rho_{A,b}} \right) = 0$$

Batez besteko dentsitate globala

$$\rho_{A,b} = \frac{1}{A_c \cdot V_{ave}} \int_{A_c} \rho_A \cdot V \cdot dA_c$$

Bero Transferentzia

$$Pr = \frac{v}{\alpha} = \frac{\mu \cdot c_p}{k}$$

Prandtl Zenbakia = $\frac{\text{Momentu difusibitatea}}{\text{Bero difusibitatea}}$

Masa Transferentzia

$$Sc = \frac{v}{D_{AB}}$$

Schmidt Zenbakia = $\frac{\text{Momentu difusibitatea}}{\text{Masa difusibitatea}}$

FLUXU LAMINARRA



$Pr \approx 1$

$Sc \approx 1$

$$Le = \frac{Sc}{Pr} = \frac{\alpha}{D_{AB}}$$

Lewis Zenbakia = $\frac{\text{Bero difusibitatea}}{\text{Masa difusibitatea}}$

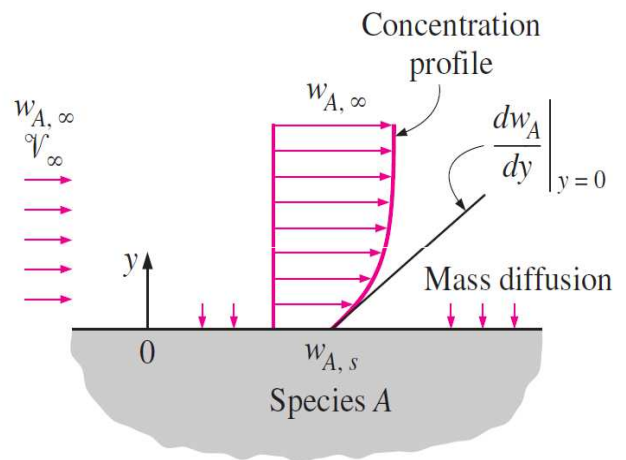
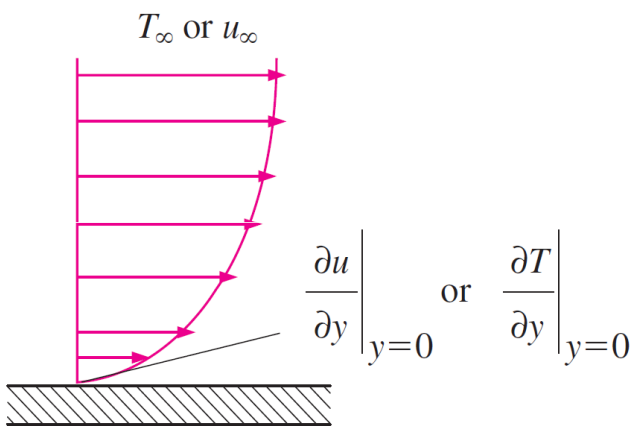
FLUXU LAMINARREAN, ABIADURAREN ETA KONTZENTRAZIOAREN MUGALDE-GERUZEN NAHIZ MUGALDE-GERUZA TERMIKOAREN LODIERA ERLATIBOAK

$$\frac{\delta_{velocity}}{\delta_{thermal}} = Pr^n \quad \frac{\delta_{velocity}}{\delta_{concentration}} = Sc^n \quad \frac{\delta_{thermal}}{\delta_{concentration}} = Le^n$$

Kasu gehienetan → n=1/3

Bero Transferentzia

Masa Transferentzia



$$h_{conv} = \frac{-k_{fluid} \cdot (\partial T / \partial y)_{y=0}}{T_s - T_\infty} \quad [W/m^2 \cdot ^\circ C]$$

$$h_{mass} = \frac{-D_{AB} \cdot (\partial w_A / \partial y)_{y=0}}{w_{A,s} - w_{A,\infty}} \quad [m / s]$$

Bero Transferentzia

NUSSELT-EN ZENBAKIA

$$Nu = \frac{h_{conv} \cdot L_c}{k}$$

STANTON-EN BERO
TRANSF. ZENBAKIA

$$St = \frac{h_{conv}}{\rho \cdot V \cdot C_p} = \frac{Nu}{Re \cdot Pr}$$

KONBEKZIO BEHARTUA

$$Nu = f(Re, Pr)$$

KONBEKZIO NATURALA

$$Nu = f(Gr, Pr)$$



Masa Transferentzia

SHERWOOD-EN ZENBAKIA

$$Sh = \frac{h_{mass} \cdot L_c}{D_{AB}}$$

STANTON-EN MASA
TRANSF. ZENBAKIA

$$St_{mass} = \frac{h_{mass}}{V} = \frac{Sh}{Re \cdot Sc}$$

$$Sh = f(Re, Sc)$$

$$Sh = f(Gr, Sc)$$



Bero Transferentzia

 T
 h_{conv}
 $\delta_{thermal}$

$$Re = \frac{V \cdot L_c}{\nu}$$

$$Gr = \frac{g \cdot \beta \cdot (T_s - T_\infty) L_c^3}{\nu^2}$$

$$Pr = \frac{\nu}{\alpha}$$

$$St = \frac{h_{conv}}{\rho \cdot V \cdot c_p}$$

$$Nu = \frac{h_{conv} \cdot L_c}{k}$$

$$Nu = f(Re, Pr)$$

$$Nu = f(Gr, Pr)$$



Masa Transferentzia

 C, y, ρ, w
 h_{mass}
 $\delta_{concentration}$

$$Re = \frac{V \cdot L_c}{\nu}$$

$$Gr = \frac{g (\rho_s - \rho_\infty) L_c^3}{\rho \nu^2}$$

$$Sc = \frac{\nu}{D_{AB}}$$

$$St_{mass} = \frac{h_{mass}}{V}$$

$$Sh = \frac{h_{mass} \cdot L_c}{D_{AB}}$$

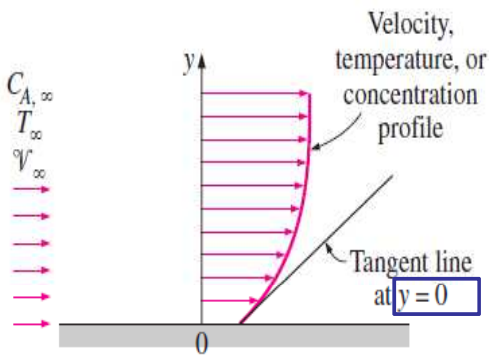
$$Sh = f(Re, Sc)$$

$$Sh = f(Gr, Sc)$$



MARRUSKADURAREN, BERO-TRANSFERENTZIAREN ETA MASA-TRANSFERENTZIAREN KOEFIZIENTEEN ARTEKO ANALOGIA

KANPO KLUXUA



Horma marruskadura $\tau_s = \mu \frac{\partial u}{\partial y} \Big|_{y=0} = \frac{C_f}{2} \rho \cdot V_\infty^2$

Bero Transf $\dot{q}_s = -k \frac{\partial T}{\partial y} \Big|_{y=0} = h_{heat} (T_s - T_\infty)$

Masa Transf $j_{A,s} = -D_{AB} \frac{\partial w_A}{\partial y} \Big|_{y=0} = h_{mass} (w_{A,s} - w_{A,\infty})$

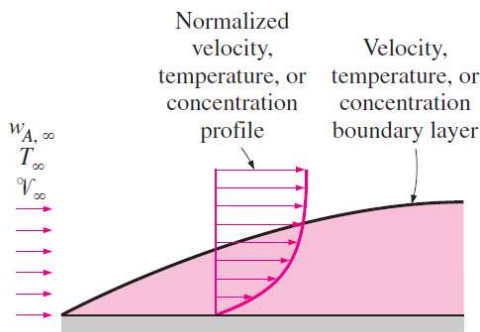
BARNE FLUXUA

Horma marruskadura $\frac{d(u/V_\infty) \partial u}{d(y/L)} \Big|_{y=0} = \frac{C_f}{2} \frac{\rho \cdot V_\infty \cdot L}{\mu} = \frac{f}{2} Re$

Bero Transf $\frac{d[(T - T_s)/(T_\infty - T_s)]}{d(y/L)} \Big|_{y=0} = \frac{h_{heat} \cdot L}{k} = Nu$

Masa Transf $\frac{d[(w_A - w_{A,s})/(w_{A,\infty} - w_{A,s})]}{d(y/L)} \Big|_{y=0} = \frac{h_{mass} \cdot L}{D_{AB}} = Sh$

REYNOLDS-EN ANALOGIA $Pr \sim Sc \sim 1$ KASURAKO



$$C_f \frac{Re_L}{2} = Nu = Sh$$

$$\frac{C_f}{2} = St = St_{mass} \quad (Pr = Sc = 1)$$

$$St = \frac{h_{conv}}{\rho \cdot V \cdot C_p} \quad St_{mass} = \frac{h_{mass}}{V}$$

Reynolds analogy
 $v = \alpha = D_{AB}$
 (or $Pr = Sc = Le$)

CHILTON-COLBURN-EN ANALOGIA

Non $0,6 < Pr < 60$ eta $0,6 < Sc < 3\,000$

$$\frac{C_f}{2} = \frac{h_{conv}}{\rho \cdot V \cdot c_p} \cdot Pr^{2/3} = \frac{h_{mass}}{V} \cdot Sc^{2/3}$$

Chilton-Colburn Analogy
 General:

$$h_{mass} = \frac{h_{heat}}{\rho C_p} \left(\frac{D_{AB}}{\alpha} \right)^{2/3} = \frac{1}{2} f^{\varphi} V \left(\frac{D_{AB}}{\nu} \right)^{2/3}$$

Special case: $v = \alpha = D_{AB}$

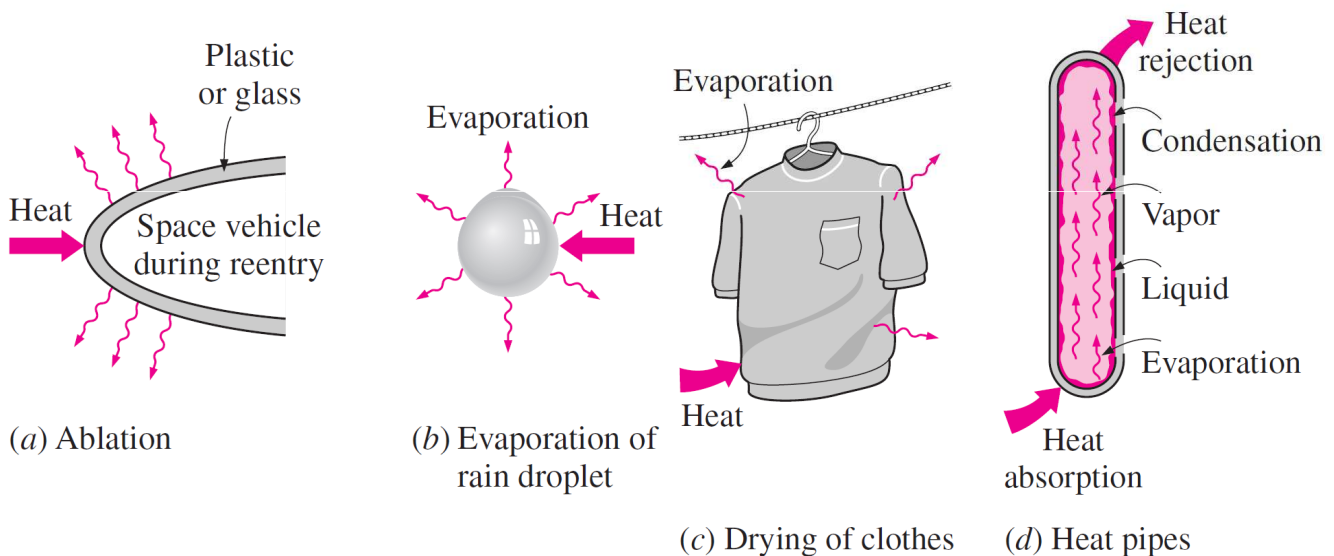
$$h_{mass} = \frac{h_{heat}}{\rho C_p} = \frac{1}{2} f^{\varphi} V$$

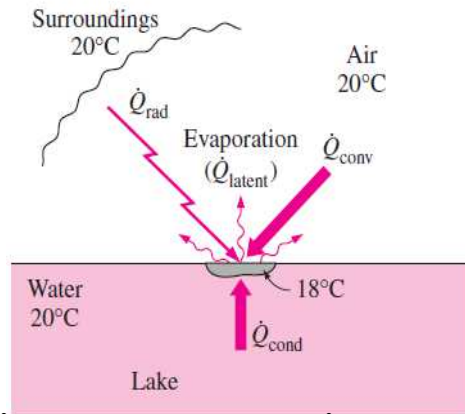
Sherwood number relations in mass convection for specified concentration at the surface corresponding to the Nusselt number relations in heat convection for specified surface temperature

Convective Heat Transfer	Convective Mass Transfer
1. Forced Convection over a Flat Plate	
(a) Laminar flow ($Re < 5 \times 10^5$) $Nu = 0.664 Re_L^{0.5} Pr^{1/3}, \quad Pr > 0.6$	$Sh = 0.664 Re_L^{0.5} Sc^{1/3}, \quad Sc > 0.5$
(b) Turbulent flow ($5 \times 10^5 < Re < 10^7$) $Nu = 0.037 Re_L^{0.8} Pr^{1/3}, \quad Pr > 0.6$	$Sh = 0.037 Re_L^{0.8} Sc^{1/3}, \quad Sc > 0.5$
2. Fully Developed Flow in Smooth Circular Pipes	
(a) Laminar flow ($Re < 2300$) $Nu = 3.66$	$Sh = 3.66$
(b) Turbulent flow ($Re > 10,000$) $Nu = 0.023 Re^{0.8} Pr^{0.4}, \quad 0.7 < Pr < 160$	$Sh = 0.023 Re^{0.8} Sc^{0.4}, \quad 0.7 < Sc < 160$
3. Natural Convection over Surfaces	
(a) Vertical plate	
$Nu = 0.59(Gr Pr)^{1/4}, \quad 10^5 < Gr Pr < 10^9$	$Sh = 0.59(Gr Sc)^{1/4}, \quad 10^5 < Gr Sc < 10^9$
$Nu = 0.1(Gr Pr)^{1/3}, \quad 10^9 < Gr Pr < 10^{13}$	$Sh = 0.1(Gr Sc)^{1/3}, \quad 10^9 < Gr Sc < 10^{13}$
(b) Upper surface of a horizontal plate	
Surface is hot ($T_s > T_\infty$)	
$Nu = 0.54(Gr Pr)^{1/4}, \quad 10^4 < Gr Pr < 10^7$	Fluid near the surface is light ($\rho_s < \rho_\infty$) $Sh = 0.54(Gr Sc)^{1/4}, \quad 10^4 < Gr Sc < 10^7$
$Nu = 0.15(Gr Pr)^{1/3}, \quad 10^7 < Gr Pr < 10^{11}$	$Sh = 0.15(Gr Sc)^{1/3}, \quad 10^7 < Gr Sc < 10^{11}$
(c) Lower surface of a horizontal plate	
Surface is hot ($T_s > T_\infty$)	
$Nu = 0.27(Gr Pr)^{1/4}, \quad 10^5 < Gr Pr < 10^{11}$	Fluid near the surface is light ($\rho_s < \rho_\infty$) $Sh = 0.27(Gr Sc)^{1/4}, \quad 10^5 < Gr Sc < 10^{11}$

14.10 – ALDIBEREKO BERO- ETA MASA-TRANSFERENTZIA

- MASA-TRANSFERENTZIAKO prozesu asko MODU ISOTERMIKOAN gertatzen dira: EZ da bero-transferentziarik izaten
- INGENIARITZA-APLIKAZIO-ETAN: Likido bat lurrundu egiten da eta lurruna inguruko gasean barreiatzen da h_{fg} -ren bitartez





a) $T_{air} = T_{water} = 20^{\circ}C$

Aire asea $\rightarrow \phi = 100\%$

Ez bero ez masa transferentziarik

b) $T_{air} = T_{water} = 20^{\circ}C$

Aire EZ asea $\rightarrow \phi < 100\%$

Masa transferentzia

$$\dot{Q}_{sensible,transferred} = \dot{Q}_{latent,absorbed}$$

$$\dot{Q} = \dot{m}_v \cdot h_{fg} \quad \dot{Q} = \sum \dot{Q}_{cond} + \dot{Q}_{conv} + \dot{Q}_{rad} \quad \underline{Q_{cond} \approx 0 \text{ eta } Q_{rad} \approx 0 \text{ bada}}$$

$$\dot{Q}_{conv} = \dot{m}_v \cdot h_{fg} \rightarrow h_{conv} \cdot A_s (T_{\infty} - T_s) = \frac{h_{conv} \cdot A_s \cdot h_{fg}}{C_p \cdot Le^{2/3}} \frac{M_v}{M} \frac{P_{v,s} - P_{v,\infty}}{P}$$

$$T_s = T_{\infty} - \frac{h_{fg}}{C_p \cdot Le^{2/3}} \frac{M_v}{M} \frac{P_{v,s} - P_{v,\infty}}{P}$$

Various expressions for evaporation rate of a liquid into a gas through an interface area A_s under various approximations (subscript v stands for vapor, s for liquid–gas interface, and ∞ away from surface)

Assumption	Evaporation Rate
General	$\dot{m}_v = h_{mass} A_s (\rho_{v,s} - \rho_{v,\infty})$
Assuming vapor to be an ideal gas, $P_v = \rho_v R_v T$	$\dot{m}_v = \frac{h_{mass} A_s}{R_v} \left(\frac{P_{v,s}}{T_s} - \frac{P_{v,\infty}}{T_{\infty}} \right)$
Using Chilton–Colburn analogy, $h_{heat} = \rho C_p h_{mass} Le^{2/3}$	$\dot{m}_v = \frac{h_{heat} A_s}{\rho C_p Le^{2/3} R_v} \left(\frac{P_{v,s}}{T_s} - \frac{P_{v,\infty}}{T_{\infty}} \right)$
Using $\frac{1}{T_s} - \frac{1}{T_{\infty}} \approx \frac{1}{T}$, where $T = \frac{T_s + T_{\infty}}{2}$ and $P = \rho RT = \rho (R_u/M) T$	$\dot{m}_v = \frac{h_{heat} A_s}{C_p Le^{2/3}} \frac{M_v}{M} \frac{P_{v,s} - P_{v,\infty}}{P}$

- 14.4 atala: MUGALDE-BALDINTZAK
- 14.5 atala : HORMA BATEAN ZEHARREKO MASA-DIFUSIO GELDIKORRA
- 14.7 atala : MASA-DIFUSIO IRAGANKORRA
- 14.8 atala : DIFUSIOA MUGIMENDUAN DAGOEN INGURUNE BATEAN