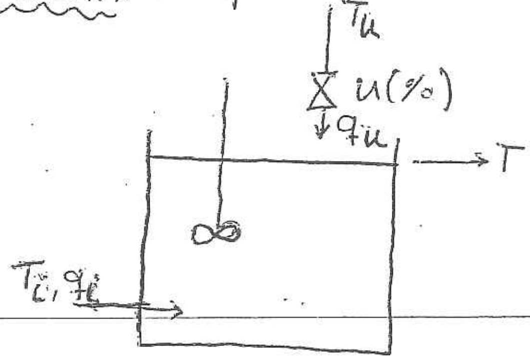
1. GAIA

KONTROL - SISTEMAK

Begizta irekiko kontrol-sistema:

1. ARIKETA : Tenperatua tankea.



Irteera : T(°C)

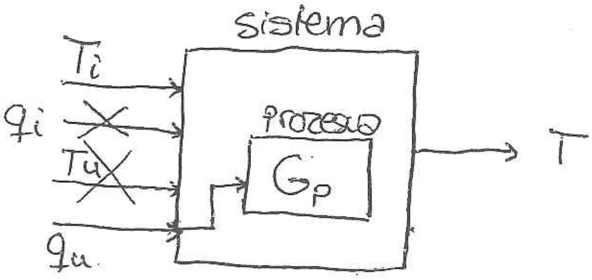
Sarrera : T_i, T_u, q_i, q_u

Aldagai kontrolatua: T(t)

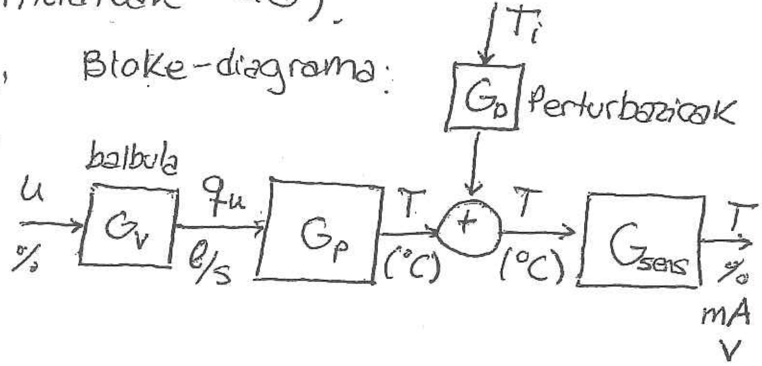
Aldagai manipulatua: u(%) balbula

Aldagai manipulatua prozesuan: q_u(t)

Tu eta qi konstante (enuntziatuak dio).



Bloke-diagrama:

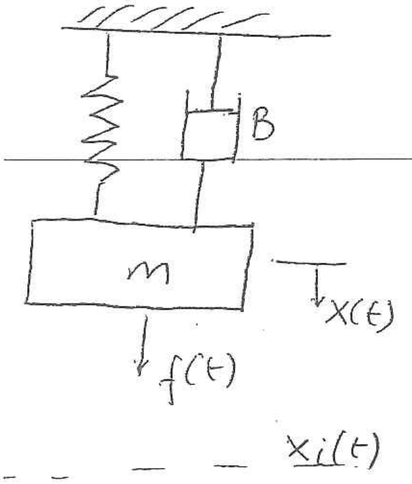


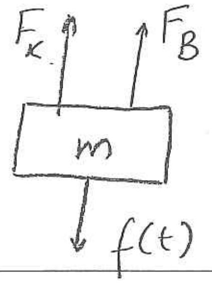
2.GAIA

EREDUA

TRANSLAZIO-SISTEMA MEKANIKOAK

1.ARIKETA (1. mintegia)





$$F_K + F_B - f(t) = m\frac{d^2x}{dt^2}$$

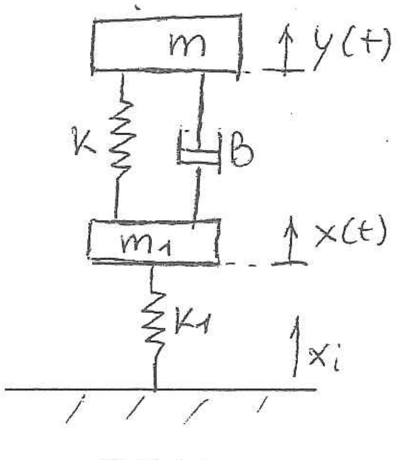
$$F_K = K(x_i(t) - x(t))$$

$$F_B = B\frac{d(x_i(t) - x(t))}{dt}$$

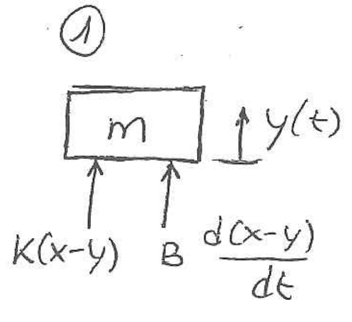
$$K\cdot x_i(t) - Kx(t) + B\frac{dx_i(t)}{dt} - B\frac{dx(t)}{dt} = m\frac{d^2x}{dt^2} + f($$

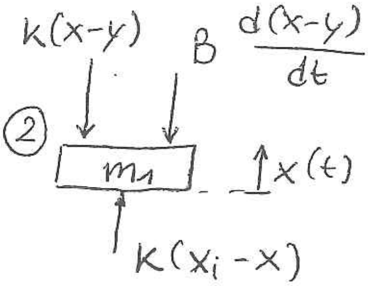
$$\left[Kx_i(t) + B\frac{dx_i(t)}{dt} = m\frac{d^2x(t)}{dt^2} + Kx(t) + B\frac{dx(t)}{dt} + f(t \right.$$

Adib:



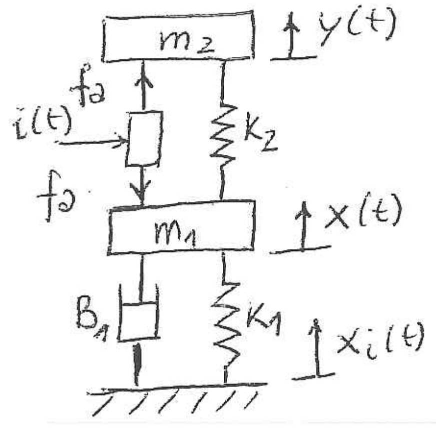
$m \gg m_1$

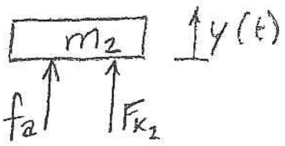


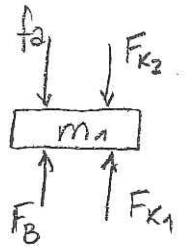


$$\begin{cases} k(x-y) + B\frac{d(x-y)}{dt} = m\cdot\frac{d^2y}{dt^2} & ① \\ k(x_i-x) - k(x-y) - B\frac{d(x-y)}{dt} = m_1\cdot\frac{d^2x}{dt} & ② \end{cases}$$

Adibidea:







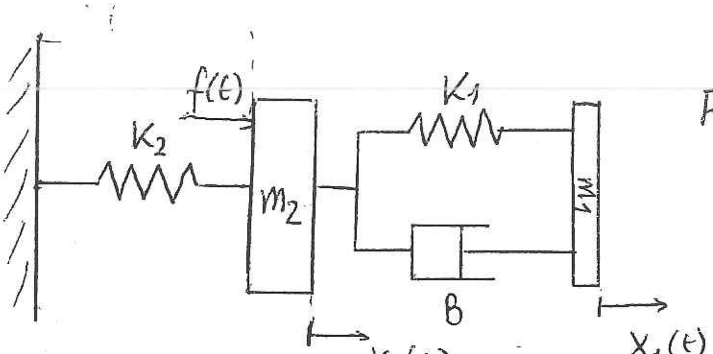
$$\begin{cases} f_a = \; ? \\ F_{K_2} = k_2(x-y) \\ f_a + F_{K_2} = m_2 \dfrac{d^2y(t)}{dt^2} \end{cases}$$

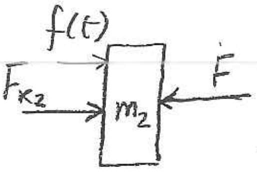
$$\begin{cases} F_B + F_{K_1} - f_a - F_{K_2} = m_1 \dfrac{d^2x(t}{dt^2} \\ F_B = B \dfrac{d(x_i - X)}{dt} \\ F_{K_1} = K_1(x_i - x) \end{cases}$$

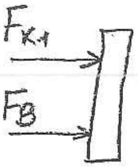
$$\begin{cases} f_a + k_2(x-y) = m_2 \dfrac{d^2y(t)}{dt^2} \\ B \dfrac{d(x_i - x)}{dt} + K_1(x_i - x) - f_a - k_2(x-y) = m_1 \dfrac{d^2x(t)}{dt^2} \end{cases} \Rightarrow$$

$$\Rightarrow \quad B \frac{d(x_i - x)}{dt} + K_1(x_i - x) = m_1 \frac{d^2x(t)}{dt^2} + m_2 \frac{d^2y(t)}{dt^2}$$

2. Ariketa: Energia - xurgatzailea (1. mintegia)







non $F = F_{K_1} + F_B$

m_2:

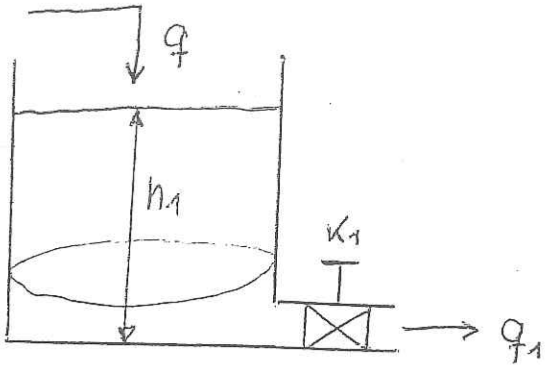
$$f(t) + F_{K_2} - F = m_2 \frac{d^2x_2(t)}{dt} \quad \text{non} \begin{cases} F_{K_2} = k_2 \cdot x_2(t) \\ f(t) \end{cases} \Rightarrow$$

m_1:

$$F_{K_1} + F_B = m_1 \frac{d^2x_1(t)}{dt} \quad \text{non} \begin{cases} F_{K_1} = K_1(x_2(t) - x_1(t)) \\ F_B = B \dfrac{d(x_2(t) - x_1(t))}{dt} \end{cases}$$

$$\Rightarrow f(t) + k_2 \cdot x_2(t) = m_2 \frac{d^2x_2(t)}{dt} + m_1 \frac{d^2x_1(t)}{dt}$$

SISTEMA HIDRAULIKOAK



Masa kontserbazioaren ekuazioa:

$$T_1 : \left.\begin{array}{l} \frac{dm}{dt} = \rho q - \rho q_1 \\ m = \rho V = \rho A \cdot h \end{array}\right\} \frac{dm}{dt} = \rho A \frac{dh}{dt} = $$

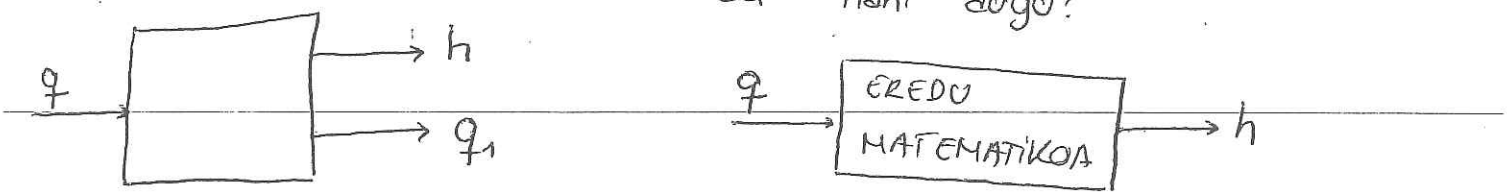
↳ $\rho = Kte$ (likidoetan)

$$= \rho q - \rho q_1 \Rightarrow$$

$$\Rightarrow A \frac{dh}{dt} = q - q_1 \quad (1)$$

Hustutze emaria : q
- → fluxu laminarra: $q_1 = K \cdot h$ (lineala)
- → fluxu zurrunbilotsua: $q_1 = K\sqrt{h}$ (ez-lineala)

Zer nahi dugu?



Laminarra:
$$\left\{\begin{array}{l} A \frac{dh}{dt} = q - q_1 \\ q_1 = K \cdot h \end{array}\right. \Rightarrow \boxed{A \frac{dh}{dt} = q - Kh}$$

Zurrunbilotsua:
$$\left\{\begin{array}{l} A \frac{dh}{dt} = q - q_1 \\ q_1 = K\sqrt{h} \end{array}\right. \Rightarrow \boxed{A \frac{dh}{dt} = q - K\sqrt{h}} \Rightarrow$$

Lineala ez denez bilatu behar dugu modu bat linealizatzeko.

⇒ Linealizazioa:

$$f(\underset{\substack{\uparrow \\ \text{deribatua}}}{h'}, \underbrace{h, q}_{\text{↳ aldagaiak}}) = 0 \quad ; \quad OP: \bar{h}', \bar{h}, \bar{q} \quad (h'_0, h_0, q_0)$$

↳ ez dago aldakuntzarik : $\frac{dh}{dt} = \bar{h}' = 0$

Ekuazio estatikoa: $\bar{q} = K\bar{h}$

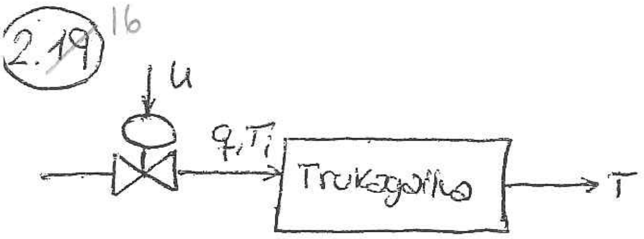
Linealizatu Taylor aplikatuz ⇒ ek. diferentziala lortu ⇒

↳ ez dugu erabiltzen, eredu sinpleago bat nahi dugu.

⇒ Laplace ⇒ $\boxed{\text{Transferentzi funtzioa.}}$

ARIKETA BILDUMA

2.~~19~~ 16



2) Aldagai esanguratsuak:

Kontrolatua : T(°C)

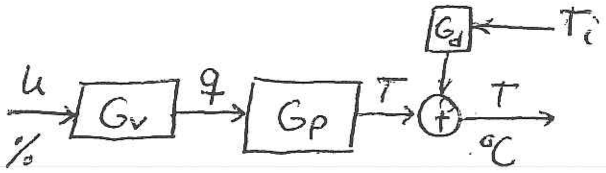
Manipulatua: u (%)

Manipulatuta prozesuan: q (l/min)

Perturbazioak : T_i (°C)

Ekuazioa: $\frac{3dT}{dt} = -6T + 8{,}8u^2 + 2T_i$

OP : $\begin{cases} T = 40°C \\ T_i = 10°C \end{cases}$



→ Ekuazio diferentziala linealizatu:

$$f(\dot{T}, T, u, T_i) = 0 \longrightarrow \frac{3dT}{dt} + 6T - 8{,}8u^2 - 2T_i = 0$$

OP : $\bar{T}_i = 10°C$, $\bar{T} = 40°C$ ⟶ (ek. diferentzialaren soluzioa)

OP-n ekuazio estatikoa: $\frac{dT}{dt} = 0 \longrightarrow 6\bar{T} - 8{,}8\bar{u}^2 - 2\bar{T}_i = 0$

$$6 \cdot 40 - 8{,}8\bar{u}^2 - 2 \cdot 10 = 0$$

$$\underline{\bar{u} = \%5}$$

→ Linealizatu (Taylor-en segidaren garapena)

$$\left.\frac{\partial f}{\partial \dot{T}}\right|_{op} \cdot \Delta\dot{T} + \left.\frac{\partial f}{\partial T}\right|_{op} \cdot \Delta T + \left.\frac{\partial f}{\partial u}\right|_{op} \cdot \Delta u + \left.\frac{\partial f}{\partial T_i}\right|_{op} \Delta T_i = 0$$

$$\left.\frac{\partial f}{\partial \dot{T}}\right|_{op} = 3 \quad , \quad \left.\frac{\partial f}{\partial T}\right|_{op} = 6 \quad , \quad \left.\frac{\partial f}{\partial u}\right|_{op} = -17{,}6\,\bar{u} \quad , \quad \left.\frac{\partial f}{\partial T_i}\right|_{op} = -2$$

$$3\Delta\dot{T} + 6\Delta T - 17{,}6\,\bar{u} \cdot \Delta u - 2\Delta T_i = 0$$

$$3\Delta\dot{T} + 6\Delta T - 88\Delta u - 2T_i = 0 \quad \text{(Lineala)}$$

$$\boxed{3\frac{dT}{dt} + 6T - 88u - 2T_i = 0} \Rightarrow \text{eredu matematikoa}$$

→ Transferentzia - funtzioa lortzeko, Laplace aplikatu (hasierako baldintzak =0)

~~$E_b(s) = K_b \cdot \Omega(s) = K_b \cdot s \cdot \Theta(s)$ (4) ≡ (9)~~

~~$T(s) = J s^2 \cdot \Theta(s) + B \cdot s \cdot \Omega(s) = J \cdot s \cdot \Theta(s) + B \cdot \Omega(s)$ (5) ≡ (10)~~

~~(9) → (8) : $E_i(s) = (R_i + s L_i) I_i(s) + K_b \cdot \Omega(s) \Rightarrow I_i(s) = \frac{E_i(s) - K_b \cdot \Omega(s}{R_i + s \cdot L_i}$~~

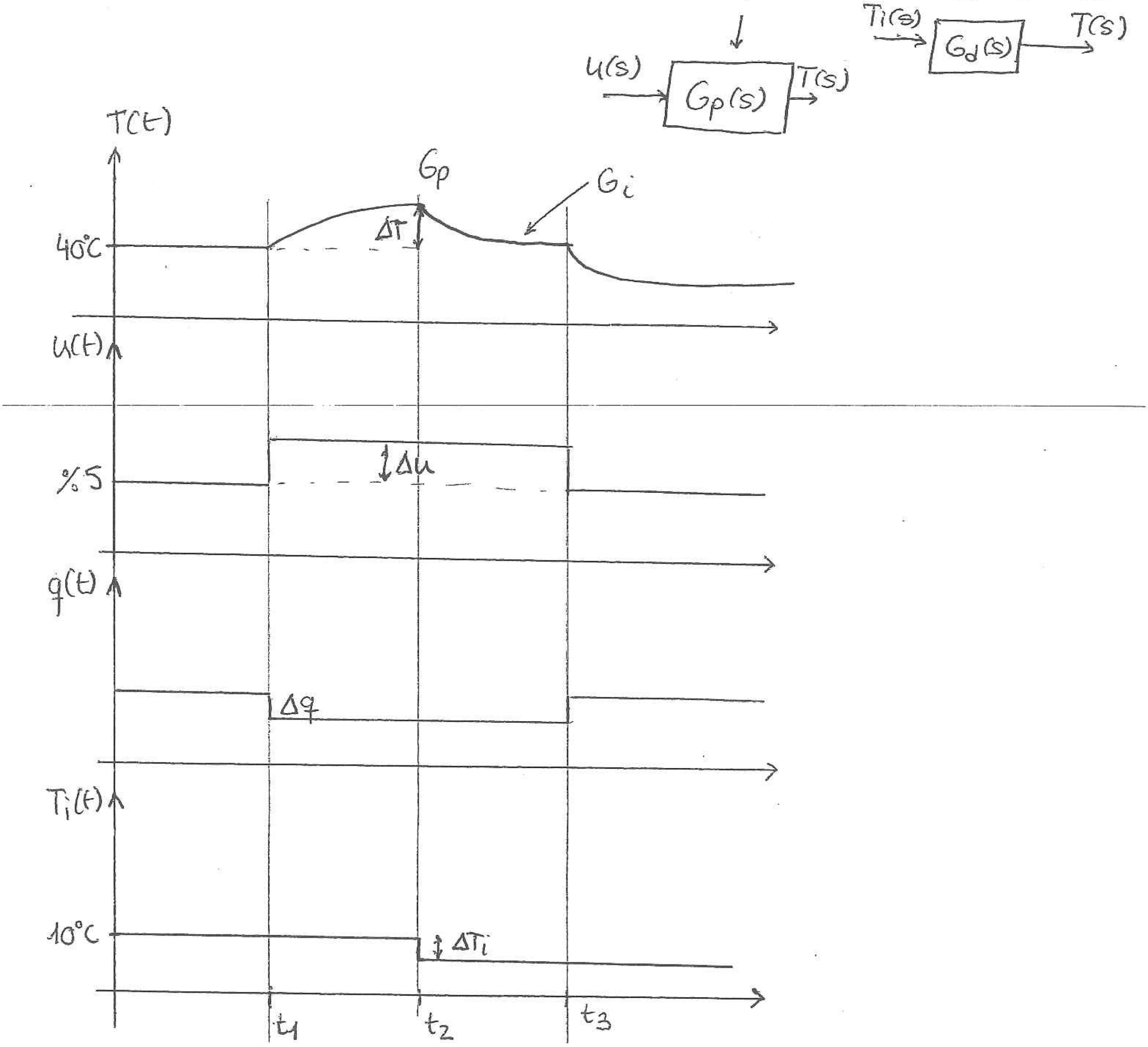
~~(11) → (7) : $T(s) = \frac{E_i(s) - K_b \cdot \Omega(s)}{R_i + s L_i} \cdot K_m$ (12)~~

$\left(T(t) \xrightarrow{\mathcal{L}} T(s)\right)$; $t \to \infty \Rightarrow s \to 0$

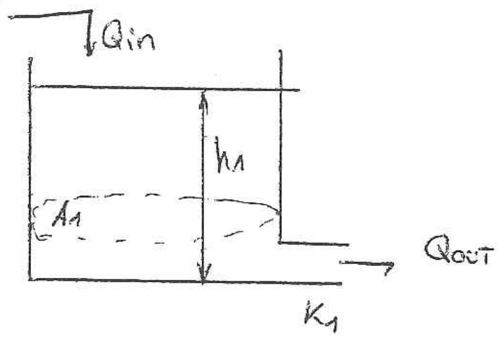
$3 \cdot s \cdot T(s) + 6T(s) - 88\,u(s) - 2T_i(S) = 0$

$(3s+6)T(s) - 88u(s) - 2T_i(s) = 0$

$$T(s) = \frac{88u(s) + 2T_i(s)}{3s+6} \Rightarrow T(s) = \underbrace{\frac{15}{0,5s+1}}\,u(s) + \underbrace{\frac{1/3}{0,5s+1}}\,T_i(s)$$



2.1



Aldagai esanguratsuak:

- Kontrolatua: $h(t)$
- Manipulazioa: $u(\%)$, (K_1)
- Manipulatua prozesuan: q_{out}
- Perturbazioa: q_{in}

Masa kontserbazioaren ekuazioa:

$$\left.\begin{array}{l}\frac{dm}{dt} = \rho q_{in} - \rho q_{out} \\ m = \rho \cdot V = \rho \cdot A_1 \cdot h_1\end{array}\right\} \frac{dm}{dt} = \cancel{\rho} A \frac{dh}{dt} = \cancel{\rho} q_{in} - \cancel{\rho} q_{out} \Rightarrow$$

$$\rightarrow A \frac{dh}{dt} = q_{in} - q_{out} \quad (1)$$

Fluxu laminarra:
$$\left\{\begin{array}{l} A \frac{dh}{dt} = q_{in} - q_{out} \\ q_{out} = K \cdot h \end{array}\right. \Rightarrow A \frac{dh}{dt} = q_{in} - Kh$$

Fluxu zurrunbilotsua:
$$\left\{\begin{array}{l} A \frac{dh}{dt} = q_{in} - q_{out} \\ q_{out} = K\sqrt{h} \end{array}\right. \Rightarrow A \frac{dh}{dt} = q_{in} - K\sqrt{h}$$

$\rightarrow$ Linealizatu: $f(\dot{h}, h, q_{in}) = 0 \rightarrow A \cdot \frac{dh}{dt} - q_{in} + K \cdot h = 0$

OP: $\bar{\dot{h}} = 0$; $\bar{h} = h$;

OP-n ekuazio estatikoa: $\frac{dh}{dt} = 0 \Rightarrow K \cdot \bar{h}_1 - \bar{q}_{in} = 0$; $\bar{q}_{in} = K\bar{h}$

Taylor: $$\left.\frac{\partial f}{\partial \dot{h}}\right|_{op} \cdot \Delta \dot{h} + \left.\frac{\partial f}{\partial h}\right|_{op} \Delta h + \left.\frac{\partial f}{\partial q_{in}}\right|_{op} \cdot \Delta q_{in} = 0$$

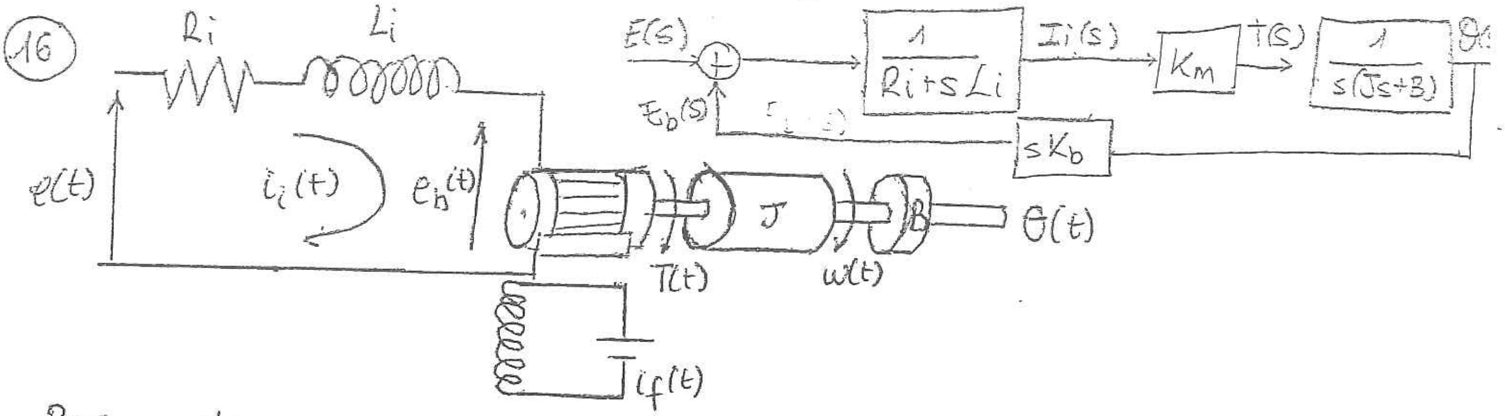
$$\left.\frac{\partial f}{\partial \dot{h}}\right|_{op} = A \; ; \; \left.\frac{\partial f}{\partial h}\right|_{op} = K \; ; \; \left.\frac{\partial f}{\partial q_{in}}\right|_{op} = -1$$

$$A \cdot \Delta \dot{h} + K \Delta h - 1 \cdot \Delta q_{in} = 0 \Rightarrow \left[A \frac{dh}{dt} + Kh - q_{in} = 0 \right] \Rightarrow \text{eredu matematikoa}$$

Laplace: $A \cdot s \cdot h(s) + K h(s) - q_{in}(s) = 0$

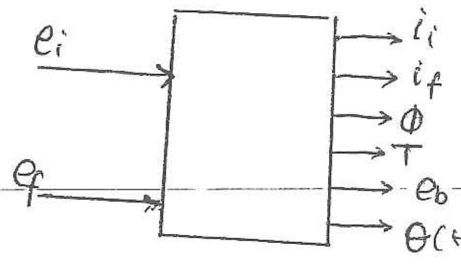
$$(As + K) h(s) = q_{in}(s) \rightarrow \frac{h(s)}{q_{in}(s)} = \frac{1}{K + As} \rightarrow \boxed{\frac{h(s)}{q_{in}(s)} = \frac{K}{1 + AKs}}$$

⑯


Bloke - diagrama;

Bere portaera adierazten duten ekuazioak ere ematen dira:

1. $\phi(t)=K_f\cdot i_f(t)$
2. $T(t)=K_i\cdot\phi(t).\,i_i(t)$ $\Big\}$ $T(t)=K_iK_f\,i_f(t)\cdot i_i(t)$
3. $e_i(t)=R_i\cdot i_i(t)+L_i\,\frac{d_i(t)}{dt}+e_b(t)$ NV=6, NE=6 $\|$ NF=0
4. $e_f(t)=R_f\cdot i_f(t)+L_f\,\frac{di(t)}{dt}$
5. $e_b(t)=K_b\cdot w(t)=K_b\cdot\frac{d\theta(t)}{dt}$
6. $J\,\frac{d^2\theta(t)}{dt^2}=T(t)-B\,\frac{d\theta}{dt}$



$\hookrightarrow w(t)=\frac{d\theta(}{dt}$

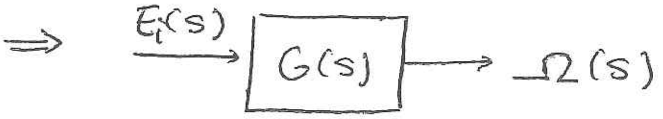
a) Indukzio bidez kontrolatzeko. ($i_f(t)=kte$) $\rightarrow$ $e_f(t)=kte$

$T(t)=K_m\cdot i_i(t)$ (2) non $K_m=K_i\cdot K_f\cdot i_f$

$e_i(t)=R_i\,i_i(t)+L_i\,\frac{d_i(t)}{dt}+e_b(t)$ (3)

$e_b(t)=K_b\cdot\frac{d\theta(t)}{dt}=K_b\cdot w(t)$ (5)

$T(t)=J\,\frac{d^2\theta(t)}{dt^2}+B\,\frac{d\theta}{dt}$ (6)



Laplace:

$T(S)=K_m\cdot I_i(S)$ (2) ≡ (7)

$E_i(S)=R_i\,I_i(s)+L_i\cdot s\cdot I_i(s)+E_b(s)$ (3) ≡ (8)

$E_b(S)=K_b\cdot S\cdot\theta(S)=K_b\cdot\Omega(S)$ (4) ≡ (9)

$T(S)=J\cdot s^2\cdot\theta(S)+B\cdot S\cdot\theta(S)=J\cdot S\cdot\Omega(S)+B\,\Omega(S)$

$(9) \rightarrow (8)$: $E_i(s) = (R_i + s \cdot L_i) I_i(s) + K_b \cdot \Omega(s)$

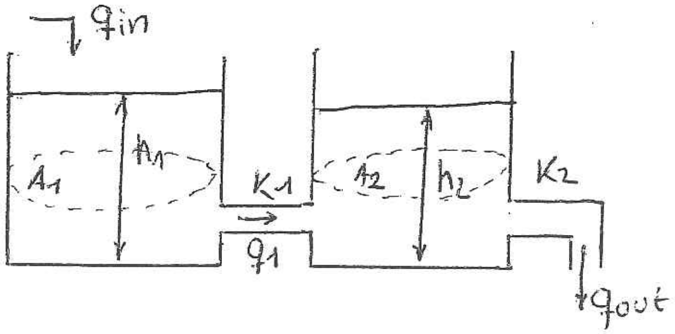
$$I_i(s) = \frac{E_i(s) - K_b \cdot \Omega(s)}{R_i + s L_i} \quad (11)$$

$(11) \rightarrow (7)$: $$T(s) = \frac{E_i(s) - K_b \cdot \Omega(s)}{R_i + s \cdot L_1} \cdot K_m \quad (12)$$

$(12) \rightarrow (10)$: $$\left[\frac{\Omega(s)}{E_i(s)} = G(s) = \frac{K_m}{(J \cdot s + B)(s L_i + R_i) + K_m \cdot K_b \cdot s}\right]$$

$\Omega(s) = G(s) \cdot E_i(s)$

2.3



$$\begin{cases} \dfrac{dV_1}{dt} = q_{in} - q_1 & (1) \\ q_1 = K_1(h_1 - h_2) & (2) \end{cases}$$

non

$$\begin{cases} \dfrac{dV_2}{dt} = q_1 - q_{out} & (3) \\ q_{out} = K_2 h_2 & (4) \end{cases}$$

$$\left.\begin{matrix} V_1 = A_1 h_1 \\ V_2 = A_2 h_2 \end{matrix}\right\} \quad \boxed{A_1, A_2} \rightarrow \text{kte}$$

(1) $A_1 \dfrac{dh_1}{dt} = q_{in} - q_1 \overset{(2)}{\Longrightarrow} A_1 \dfrac{dh_1}{dt} = q_{in} - K_1(h_1 - h_2)$

(3) $A_2 \dfrac{dh_2}{dt} = q_1 - q_{out} \overset{(4)}{\Longrightarrow} A_2 \dfrac{dh_2}{dt} = q_1 - K_2 h_2$

$\hookrightarrow q_1 = A_2 \dfrac{dh_2}{dt} + q_{out} \Rightarrow \dfrac{dq_1}{dt} = A_2 \dfrac{d^2 h_2}{dt^2} + \boxed{\dfrac{dq_{out}}{dt}} \rightarrow K_2 \dfrac{dh_2}{dt}$

(2) $h_1 = \dfrac{q_1}{K_1} + h_2 \Rightarrow \dfrac{dh_1}{dt} = \dfrac{1}{K_1} \cdot \dfrac{dq_1}{dt} + \dfrac{dh_2}{dt}$

(1) $A_1 \dfrac{dh_1}{dt} + q_1 = q_{in}$

$$A_1 \left[\frac{1}{K_1} \cdot A_2 \frac{d^2 h_2}{dt^2} + K_2 \frac{dh_2}{dt}\right] + A_2 \frac{dh_2}{dt} + K_2 h_2 = q_{in}$$

$$\left[\frac{d^2 h_2}{dt^2}\left[\frac{A_1 A_2}{K_1}\right] + \frac{dh_2}{dt}\left[\frac{A_1 K_2}{K_1} + A_1 + A_2\right] + K_2 h_2 = q_{in}\right] \Rightarrow \text{Eredu matematikoa}$$

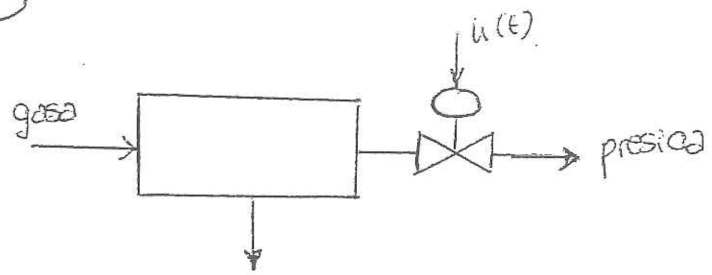
Laplace:

$$s^2 H_2(s)\left[\frac{A_1 A_2}{K_1}\right] + s H_2(s)\left[\frac{A_1 K_2}{K_1} + A_1 + A_2\right] + K_2 H_2(s) = Q_{in}(s)$$

$$H_2(s)\left[s^2\left(\frac{A_1 A_2}{K_1}\right) + s\left(\frac{A_1 K_2}{K_1} + A_1 + A_2\right) + K_2\right] = Q_{in}$$

$$\frac{H_2(s)}{Q_{in}(s)} = \frac{1}{s^2(\underline{A_1 A_2}) + s(\underline{A_1 K_2} + A_1 + A_2) + K}$$

2.~~18~~ 14



$$OP: \begin{cases} p = 3 \text{ bar} \\ u = \% 30 \end{cases}$$

$$1800 \frac{dp}{dt} = (-3p^2+30)T(t-0,4)-30$$

$$3p^2Tt - 30Tt - T \cdot 3p^2\, 0,4 + 30 \cdot 0,4T$$

⟹ Ekuazio diferentziala linealizatuz:

$$f(\dot{p}, p, T): \;=0 \longrightarrow 1800 \frac{dp}{dt} + 30 - \underbrace{(-3p^2+30)T(t-0,4)} = 0$$

$$OP: \; \bar{p} = 3 \; ; \; \bar{t} = 0,3 \qquad 1800 \frac{dp}{dt} + 30 + (3p^2-30)(Tt - T \cdot 0,4) = 0$$

OP-n ekuazio estatikoa: $\frac{dp}{dt} = 0 \longrightarrow 30 + (3\bar{p}^2 - 30)\bar{T}(\bar{t} - 0,4) = 0$

$$30 + \underbrace{(3 \cdot 9 - 30)}_{-3} \bar{T} \underbrace{(0,3 - 0,4)}_{-0,1} = 0$$

Taylor:

$$30 + 0,3\bar{T} = 0 \longrightarrow \bar{T} = -100\,°C$$

$$\left.\frac{\partial f}{\partial \dot{p}}\right|_{op} = 1800 \; ; \; \left.\frac{\partial f}{\partial p}\right|_{op} = 6pTt - 6pT \cdot 0,4 \; ; = 180 \; \left.\frac{\partial f}{\partial T}\right|_{op} = (3p^2-30)(t-0,4) = -3\,(t-0,4)$$

~~$\left.\frac{\partial f}{\partial t}\right|_{op} = 3p^2T - 30T = 300$~~

$$1800\,\Delta\dot{p} + 180\,\Delta p - 3\,(t-0,4)\,\Delta T = 0 \quad (\text{Lineala})$$

$$\Longrightarrow \boxed{1800 \frac{dp}{dt} + 180p - 3(t-0,4)T = 0}$$

$$\mathcal{L}[f(t-a)] = e^{-as} \cdot F(s)$$

Laplace: $1800s\,P(s) + 180\,P(s) - 3e^{-0,4s}\,T(s)$

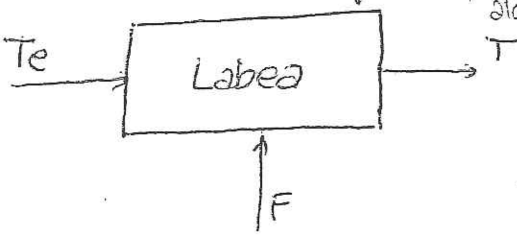
$$P(s)(1800s + 180) = 3e^{-0,4s}\,T(s)$$

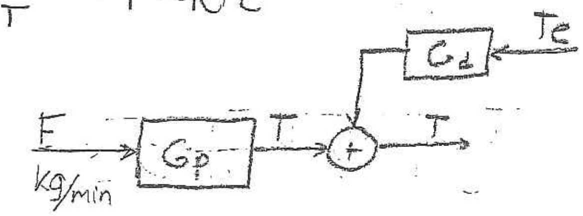
$$\boxed{\frac{P(s)}{T(s)} = \frac{3e^{-0,4s}}{1800s + 180}}$$

(30) 15 [illegible] : (Minitegan egindakoa! pixka bat aldatuta)

$T_e = 10°C$

$T = 40°C$





$$(5+3F)\frac{dT}{dt} + 2T^2 = 3FT + T_e$$

$$f(\dot T, T, F, T_e) = 0 \longrightarrow (5+3F)\frac{dT}{dt} + 2T^2 - 3FT - T_e = 0$$

OP: $\bar{T}_e = 10$; $\bar{T} = 40$ // ekuazio estatikoa OP-n: $\frac{dT}{dt} = 0$

$$\longrightarrow 2\bar{T}^2 - 3\bar{F}\bar{T} - \bar{T}_e = 0 \xrightarrow{OP} 2\cdot 40^2 - 3\cdot\bar{F}\cdot 40 - 10 = 0$$

$$3200 - 10 = 120\bar{F} \longrightarrow \bar{F} = \frac{319}{12}$$

Taylor:

$$\left.\frac{\partial f}{\partial \dot T}\right|_{op} = 5 + 3\bar{F} = 5 + \frac{319}{4} = \frac{339}{4} \quad ; \quad \left.\frac{\partial f}{\partial T}\right|_{op} = 4\bar{T} - 3\bar{F} = 160 - \frac{319}{4} = \frac{321}{4}$$

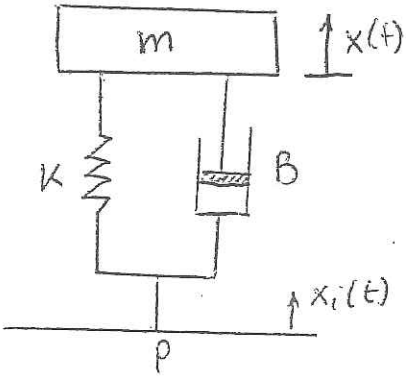
$$\left.\frac{\partial f}{\partial F}\right|_{op} = 3\underbrace{\frac{d\bar{T}}{dt}}_{\dot{\bar T} = 0} - 3T = -120 \quad ; \quad \frac{\partial f}{\partial T_e} = -1$$

$$\Rightarrow \frac{339}{4}\Delta\dot T + \frac{321}{4}\Delta T - 120\,\Delta F - \Delta T_e = 0 \quad \text{(Lineala)}$$

$$\Longrightarrow \boxed{339\frac{dT}{dt} + 321T - 480F - 4T_e = 0}$$

3 GAIA

2. Adibidea: Auto 1/4 -ren esekidura sistema.



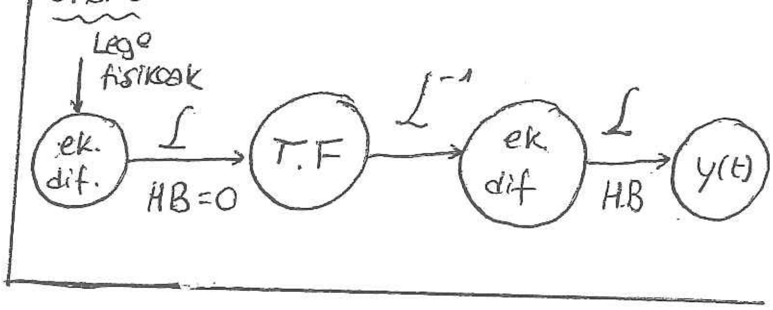
$m = 700 \text{ Kg}$

$B = 100 \ \frac{Ns}{m}$

$K = 1000 \ \frac{N}{m}$

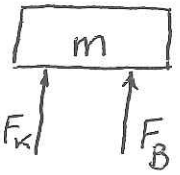
$x_i(t) = 15 \text{ cm}$

Oharra:



$$\Sigma \vec{F} = m\vec{a}$$

Lege fisikoak:



$$F_K + F_B = m \cdot a$$

$$K[x_i(t) - x(t)] + B \frac{d[x_i(t) - x(t)]}{dt} = m$$

$$Kx_i(t) - Kx(t) + B\frac{dx_i(t)}{dt} - B\frac{dx(t)}{dt} = m\frac{d^2x(t)}{dt^2}$$

k. diferentziala: $m\frac{d^2x(t)}{dt^2} + Kx(t) + B\frac{dx(t)}{dt} = Kx_i(t) + B\frac{dx_i(t)}{dt}$

$\downarrow \mathcal{L}$

$$m[s^2X(s) - sx(0) - \dot{x}(0)] + KX(s) + B[X(s)\cdot s - x(0)] = KX_i(s) +$$

$$+ B[X_i(s)\cdot s - x_i(0)] \xRightarrow{H.B=0} ms^2X(s) + sBX(s) + KX(s) = sBX_i(s) +$$

$$+ KX_i(s)$$

Transferentzia funtzioa: $X(s)[ms^2 + sB + K] = X_i(s)[sB + K]$

$$G(s) = \frac{X(s)}{X_i(s)} = \frac{sB + K}{ms^2 + sB + K}$$

8. Ariketa: sistema baten transferentzi funtzioa $G(s)$ da.

$$G(s) = \frac{Y(s)}{R(s)} = \frac{s+4}{s^2+s+1}$$

kalkulatu irteera ($y(t)$), $r(t)$ sarrera inpultsoa denean ($\underline{A=3}$) eta HB:

$y(0)=1$; $\dot{y}(0)=1$ $\hookrightarrow R(s)=3$

$$(s^2+s+1)\,Y(s) = (s+4)\,R(s) \xrightarrow{\mathcal{L}^{-1}} \frac{d^2y(t)}{dt^2} + \frac{dy(t)}{dt} + y(t) = \frac{d\,r(t)}{dt} + 4r($$

$\Longrightarrow$ Laplace:

$$s^2 Y(s) - s\,y(0) - \dot{y}(0) + s\,Y(s) - y(0) + Y(s) = sR(s) + 4R(s)$$

$$s^2 Y(s) - s - 1 + s\,Y(s) - 1 + Y(s) = (s+4)\,R(s)$$

$$(s^2+s+1)\,Y(s) - (s+2) = (s+4)\,R(s)$$

$$Y(s) = \frac{s+2}{s^2+s+1} + \frac{s+4}{s^2+s+1}R(s)$$

$$Y(s) = \frac{s+2}{s^2+s+1} + \frac{s+4}{s^2+s+1}3 = \frac{4s+14}{s^2+s+1} = 4\frac{s+\frac{1}{2}}{(s+\frac{1}{2})^2+0{,}75} + \frac{12}{\sqrt{0{,}75}}\cdot\frac{\sqrt{0{,}75}}{(s+1)^2+0{,}7}$$

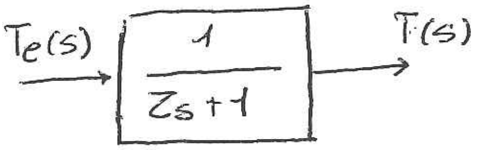
$\mathcal{L}^{-1}$

$$\left[y(t) = 4\cdot e^{-t/2}\cos(0{,}75t) + \frac{12}{\sqrt{0{,}75}}\,e^{-t/2}\sin(\sqrt{0{,}75}\,t)\right]$$

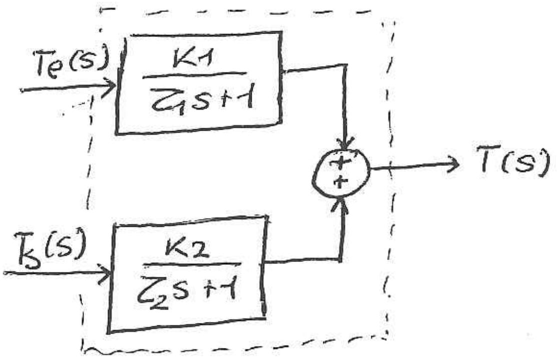
BLOKEEN ALJEBRA

Adibideak

1) $T(s) = \frac{1}{Z s+1}\cdot Te(s)$

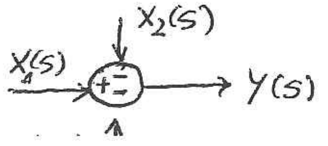


2) $T(s) = \frac{K_1}{Z_1 s+1}\,Te(S) + \frac{K_2}{Z_2 s+1}\,T_s(s)$



Propietateak

1. $Y(s) = X_1(s) - X_2(s) - X_3(s)$



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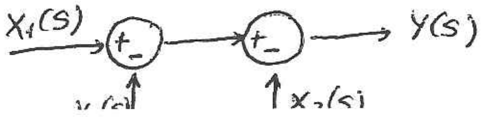
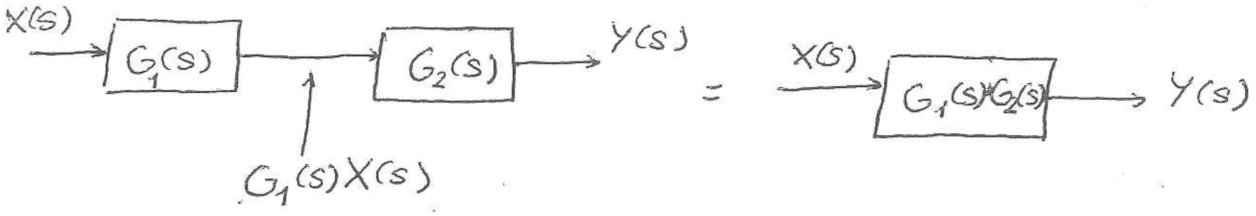
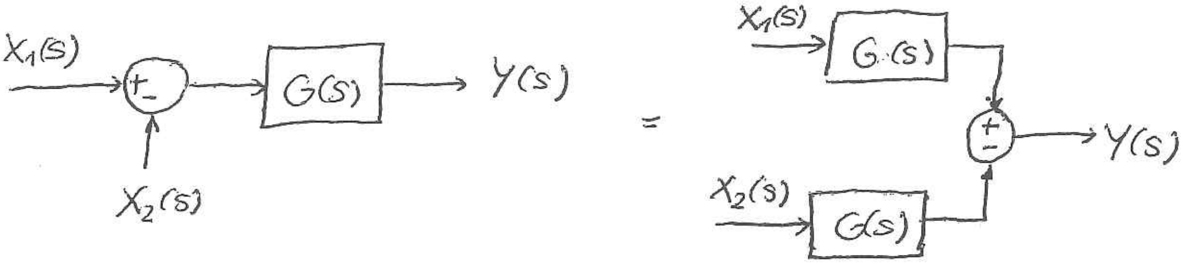


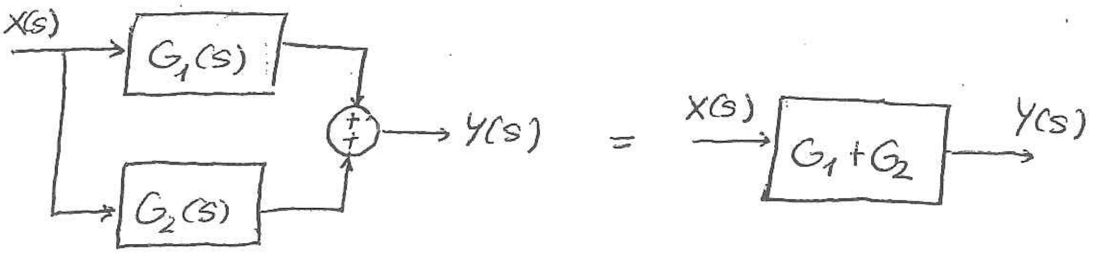
2. Asoziatibo eta konmutatiboa.



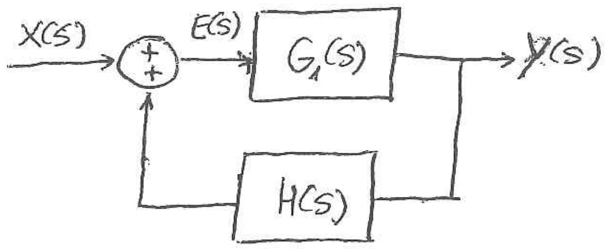
3. $Y(s) = G(s)[X_1(s) - X_2(s)] = G(s)X_1(s) - G(s)X_2(s)$



4. Blokeak paralelo

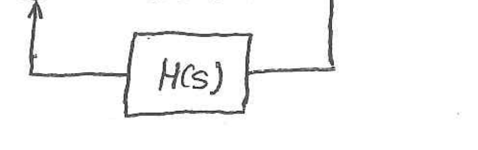


5. Berrelikadura ⊕ begizta



$$Y(s) = G(s) \cdot E(s) = G(s)[X(s) + H(s)\,Y(s)] \longrightarrow G(s) = \frac{Y(s)}{X(s)} = \frac{G_1(s)}{1 - G_1H(s)}$$

Berrelikadura ⊖ begizta:

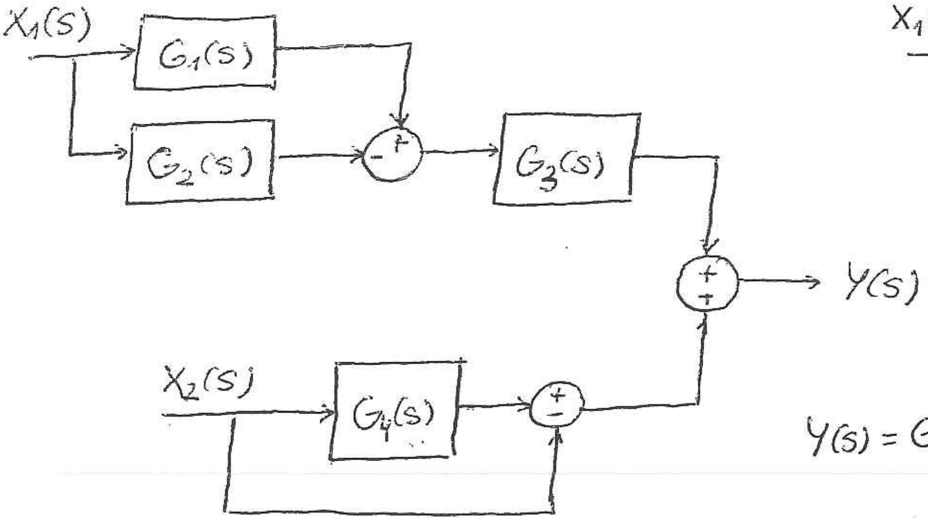


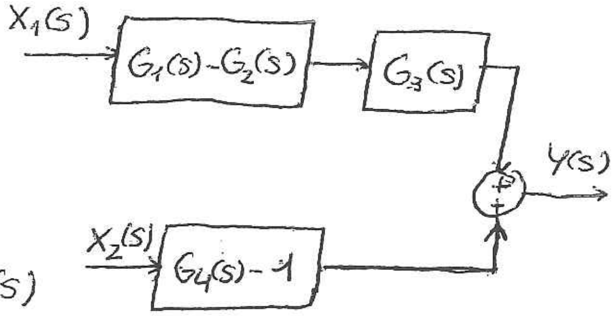
$$G(s) = \frac{Y(s)}{X(s)} = \frac{G_1(s)}{1 + G_1(s)\,H(s)}$$

Kate zuzena: $G_1(s)$

Berrelikadura - katea: $G_1(s)\,H(s)$

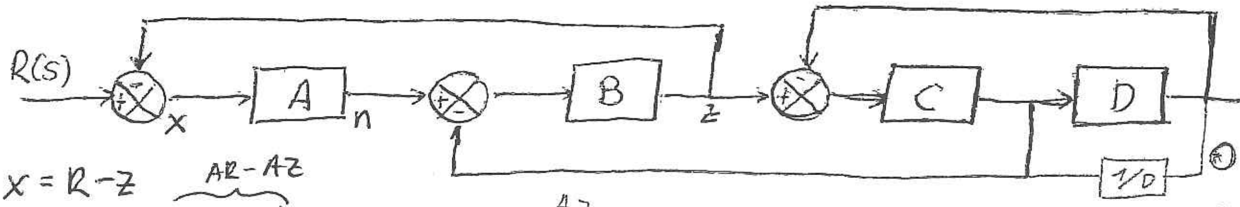
Adibidea sinplifikatuko dugu bloke diagrama:





$$Y(s) = G_3(s)\left[G_1(s) - G_2(s)\right]X_1(s) +$$
$$+ \left[G_4(s) - 1\right]X_2(s)$$

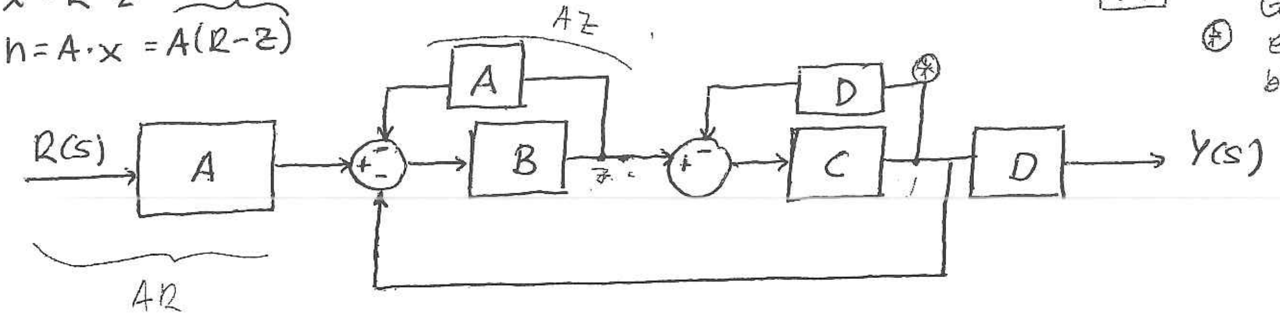
9. Ariketa (41. diapositiba)

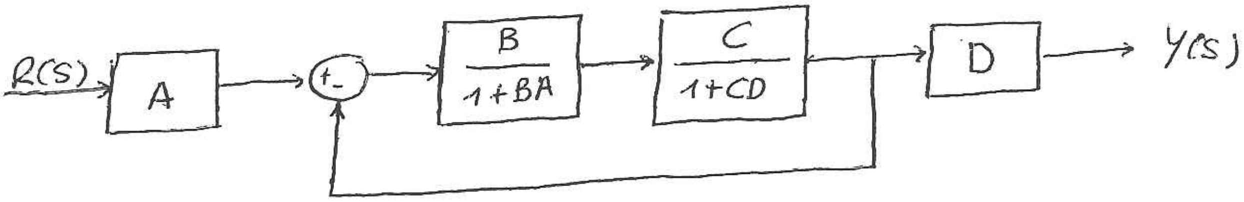


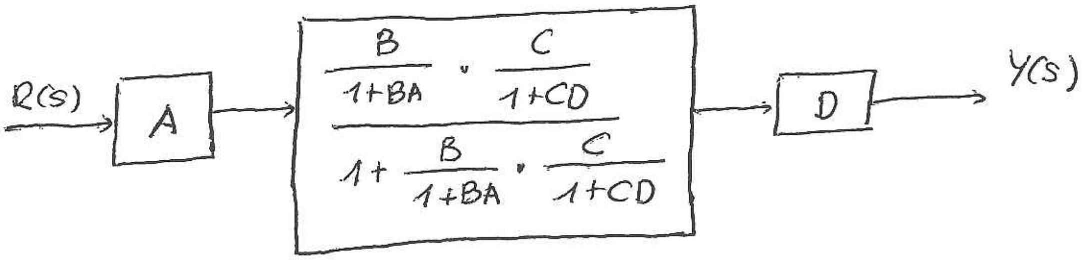
$x = R - Z$

$n = A \cdot x = \overbrace{A(R-Z)}^{AR-AZ}$

⊛ Goiko zein beheko eskemak berdinak dira.







$$G(s) = \frac{Y(s)}{R(s)} = A \cdot \frac{\dfrac{B\,C}{\cancel{(1+BA)}\,\cancel{(1+CD)}}}{\dfrac{(1+BA)(1+CD)+BC}{\cancel{(1+BA)}\,\cancel{(1+CD)}}} \cdot D = \frac{A\,B\,C\,D}{(1+BA)(1+CD)+BC}$$

3. Ariketa (13. diapositiba) (2a))

$$\frac{d^2x(t)}{dt^2} + 3\frac{dx(t)}{dt} + 6x(t) = f(t)$$

$f(t) = 2N$; $x(0)=0$; $\dot{x}(0)=3$; $x(t)?$

↓ Laplace

$$s^2X(s) - \cancelto{0}{sX(0)} - \dot{x}(0) + 3(sX(s) - \cancelto{0}{X(0)}) + 6X(s) = \overbrace{F(s)}^{\frac{2}{s}}$$

$$s^2X(s) - 3 + 3sX(s) + 6X(s) = \frac{}{s}$$

$$X(s)\cdot(s^2+3s+6) = \frac{2}{s} + 3 \longrightarrow X(s) = \frac{2+3s}{s(s^2+3s+6)} = \ldots$$

$$= \frac{A}{s} + \frac{Bs+C}{s^2+3s+6} \Rightarrow 2+3s = A(s^2+3s+6) + Bs^2 + Cs$$

• $2 = 6A \rightarrow A = 1/3$ • $3 = \overbrace{\frac{1}{3}\cdot 3}^{1} + C \rightarrow C=2$ • $0 = \frac{1}{3}+B \rightarrow B = -1/3$

$$X(s) = \frac{1/3}{s} + \frac{-1/3\,s + 2}{s^2+3s+6} = \frac{1/3}{s} - \frac{1}{3}\cdot\frac{s-6}{s^2+3s+6} = \frac{1/3}{s} - \frac{1}{3}\cdot\frac{s-6}{(s+\frac{3}{2})^2+\frac{15}{4}}$$

— — —

ARIKETA BILDUMA

① $\ddot{x}(t) + 3\dot{x}(t) + 6x = 6$; $x(0)=0$; $\dot{x}(0)=3$

↓ $\mathcal{L}$

$$s^2X(s) - \cancelto{0}{sX(0)} - \dot{x}(0) + 3(sX(s) - \cancelto{0}{X(0)}) + 6X(s) = \frac{6}{s}$$

$$X(s)[s^2+3s+6] = \frac{6}{s}+3$$

$$X(s)[s^2+3s+6] = \frac{6+3s}{s} \Rightarrow X(s) = \frac{6+3s}{s(s^2+3s+6)} =$$

$$= \frac{A}{s} + \frac{Bs+C}{s^2+3s+6} \Rightarrow 6+3s = A(s^2+3s+6) + Bs^2 + Cs$$

• $6=6A \rightarrow A=1$

• $3=3A+C \rightarrow C=0$ $\quad X(s) = \frac{1}{s} + \frac{-s}{s^2+3s+6} = \frac{1}{s} - \left[\frac{s}{(s+\frac{3}{2})^2+\frac{15}{4}}\right] =$

• $0=A+B \rightarrow B=-1$

$$= \frac{1}{s} - \left[\frac{s+3/2}{(s+\frac{3}{2})^2+\frac{15}{4}} - \frac{3/\cancel{2}\cdot\frac{\sqrt{15}}{\;}}{\sqrt{15}(s+\frac{3}{2})^2+\frac{15}{4}}\right]; \left[x(t) = 1 - e^{-\frac{3}{2}t}\cos\left(\sqrt{\frac{15}{4}}t\right) + \frac{3}{\sqrt{15}}e^{-\frac{3}{2}t}\sin\left(\sqrt{\frac{15}{4}}t\right.\right.$$

③

$$X(s) = \frac{1}{s(s^2+2s+2)} = \frac{A}{s} + \frac{Bs+C}{s^2+2s+2} \Rightarrow 1 = A(s^2+2s+2) + Bs^2 + Cs$$

- $1 = 2A \longrightarrow A = 1/2$
- $0 = \frac{1}{\cancel{2}}\cdot\cancel{2} + C \longrightarrow C = -1$
- $0 = \frac{1}{2} + B \longrightarrow B = -1/2$

$$X(s) = \frac{1/2}{s} - \frac{1/2\, s + \ }{s^2+2s+2} =$$

$$= \frac{1/2}{s} - \frac{\frac{1}{2}s + 1}{(s+1)^2+1} =$$

$$= \frac{1/2}{s} - \left[\frac{1}{2}\cdot\frac{s+2}{(s+1)^2+1}\right] = \frac{1/2}{s} - \frac{1}{2}\left[\frac{s+1}{(s+1)^2+1} + \frac{1}{(s+1)^2+1}\right] \Rightarrow$$

$$\boxed{(t) = \frac{1}{2} - \frac{1}{2}e^{-t}\cos t - \frac{1}{2}e^{-t}\sin t}$$

$s^2 + 4s + 3$

$$X(s) = \frac{5(s+2)}{s^2(s+1)(s+3)} = \frac{A}{s+1} + \frac{B}{s+3} + \frac{Cs+D}{s^2} \Rightarrow$$

$$\Rightarrow 5(s+2) = As^3 + A3s^2 + Bs^3 + Bs^2 + Cs^3 + 4Cs^2 + 3Cs + Ds^2 + 4Ds + 3D$$

- $10 = 3D \longrightarrow D = 10/3$
- $5 = 3\cdot\ + 4\cdot\frac{10}{3} \longrightarrow C = -\frac{25}{9}$
- $0 = 3A + B - 4\cdot\frac{25}{9} + \frac{10}{3}$
- $0 = A + B + \left(-\frac{25}{9}\right)$

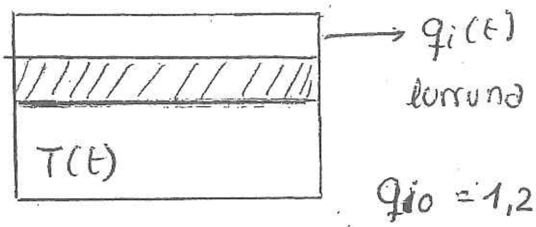
$\Bigg\|$ $A = \frac{5}{2}$

$B = \frac{5}{18}$

$$X(s) = \frac{5}{2}\cdot\frac{1}{s+1} + \frac{5}{18}\,\frac{1}{s+3} + \underbrace{\frac{-25/9\, s + \frac{10}{3}}{s^2}}_{\hookrightarrow\ -\frac{25/9}{s} + \frac{10/3}{s^2}}$$

$$\boxed{X(t) = \frac{5}{2}e^{-t} + \frac{5}{18}e^{-3t} - \frac{25}{9} + \frac{10}{3}t}$$

⑤



$$-45 q_i(t)\frac{dT(t)}{dt} + 400 q_i(t) =$$
$$= -\frac{d^2q_i(t)}{dt^2} + \frac{T^2(t)}{20} - 2q_i(t)\frac{dq_i(t)}{dt^2}$$

$q_{io} = 1{,}25 \frac{l}{min}$

$(\ddot{q}_i, \dot{q}_i, q_i, \dot{T}, T) = 0$; OP: $\bar{q}_i = q_{io} = 1{,}25$; $\bar{T} = T_0$

$\hookrightarrow -45 q_i \frac{dT}{dt} + 400 q_i + \frac{d^2q_i}{dt^2} - \frac{T^2}{20} + 2q_i\frac{dq_i}{dt} = 0$

$$\left.\begin{matrix}\frac{dT}{dt} = 0 \\ \frac{dq_i}{dt} = 0\end{matrix}\right\} \Rightarrow 400 q_{io} - \frac{T_0^2}{20} = 0 \rightarrow 400 \cdot 20 \overset{1,25}{q_{io}} = T_0^2$$

$T_0 = 100$

Taylor: $\left.\frac{\partial f}{\partial \dot{q}_i}\right|_{op} \Delta\dot{q}_i + \left.\frac{\partial f}{\partial q_i}\right|_{op} \Delta q_i + \left.\frac{\partial f}{\partial \dot{T}}\right|_{op} \Delta\dot{T} + \left.\frac{\partial f}{\partial T}\right|_{op} \Delta T + \left.\frac{\partial f}{\partial \ddot{q}_i}\right|_{op} \Delta\ddot{q}_i =$

$\Delta\ddot{q}_i + 2q_{io}\Delta\dot{q}_i + (-45 \overset{0}{\cancel{\dot{T}_0}} + 400 + 2\overset{0}{\cancel{\dot{q}_{io}}})\Delta q_i + (-45 q_{io})\Delta\dot{T} - \frac{T_0}{10}\Delta T = 0$

$\frac{d^2q_i}{dt} + 2{,}5\frac{dq_i}{dt} + 500 q_i - 56{,}25\frac{dT}{dt} - 10T = 0$

$(\mathcal{L}$

$s^2 Q_i(s) + 2{,}5 s Q_i(s) + 400 Q_i(s) - 56{,}25 s T(s) - 10T(s) = 0$

$Q_i(s)[s^2 + 2{,}5s + 400] = T(s)[56{,}25 s + 10]$

$$\left[G(s) = \frac{Q_i(s)}{T(s)} = \frac{56{,}25 s + 10}{s^2 + 2{,}5s + 400}\right] \quad \boxed{C)}$$

6) $20\frac{d^2f(t)}{dt^2} + 80\frac{df(t)}{dt} - 6\frac{dx(t)}{dt} + 20 f(t) = 2\frac{d^3x(t)}{dt^3} + 10\frac{d^2x(t)}{dt^2} + 6\frac{dx(t)}{dt} - 40f$

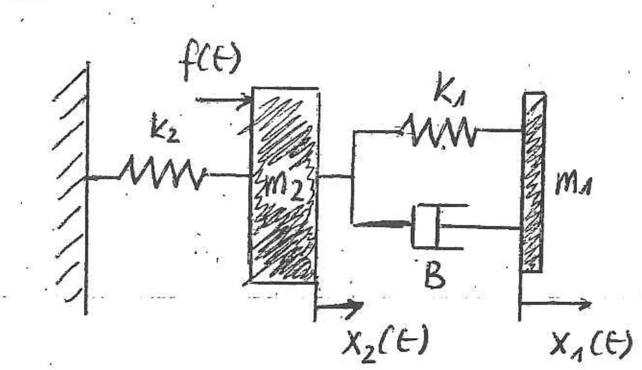
Laplace: $20 s^2 F(s) + 80 \cdot s F(s) - 6 s X(s) + 20F(s) = 2s^3X(s) + 10s^2X(s) + 6sX(s)$
$- 40F(s)$

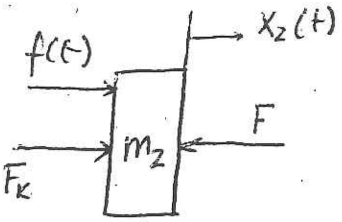
$F(s)[20s^2 + 80s + 40 + 20] = X(s)[2s^3 + 10s^2 + 12s]$

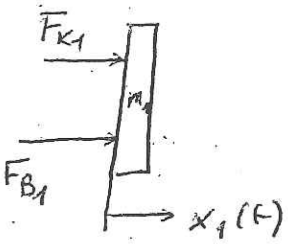
$\underset{10}{\cancel{20}} F(s)[s^2 + 4s + 3] = \cancel{2} X(s)[s^3 + 5s^2 + 6s]$

$$\left[G(s) = \frac{X(s)}{F(s)} = \frac{10(s^2+4s+3)}{s^3+5s^2+6s}\right] \quad \boxed{a)}$$

⑨







non $F = F_{K_1} + F_{B_1}$

(m_1) : $\Sigma F = m_1 \cdot \vec{a}$

$$F_{K_1} + F_{B_1} = m_1 \frac{d^2 x_1(t)}{dt^2}$$

$$K_1[x_2(t)-x_1(t)] + B\frac{d[x_2(t)-x_1(t)]}{dt} = m_1 \frac{d^2 x_1(t)}{dt}$$

$$K_1 x_2(t) + B\frac{dx_2(t)}{dt} = m_1\frac{d^2 x_1(t)}{dt} + K_1 x_1(t) + B\frac{dx_1(t)}{dt} \quad (1)$$

(m_2) :

$$f(t) + F_{K_2} - F_{K_1} - F_{B_1} = m_2 \frac{d^2 x_2(t)}{dt^2},$$

$$f(t) + K_2[-x_2(t)] - K_1[x_2(t)-x_1(t)] - B\frac{d[x_2(t)-x_1(t)]}{dt} = m_2\frac{d^2 x_2(t)}{dt},$$

$\downarrow \mathcal{L}$

$$(1):\ kX_2(s) + sBX_2(s) = m_1 s^2 X_1(s) + kX_1(s) + sBX_1(s)$$

$$X_2(s)[K_1+sB] = X_1(s)[m_1 s^2 + k_1 + sB] \Rightarrow X_1(s) = \frac{K_1+sB}{m_1 s^2 + k_1 + sB} X_2(s$$

$\downarrow \mathcal{L}$

$$(2):\ F(s) + K_1 X_1(s) + sBX_1(s) = m_2 s^2 X_2(s) + K_2 X_2(s) + k_1 X_2(s) + sBX_2($$

$$F(s) + X_1(s)[K_1 + sB] = X_2(s)[m_2 s^2 + k_2 + K_1 + sB]$$

$\hookrightarrow X_1(s)$ Ordezkatuz

$$F(s) + X_2(s)\frac{[k_1+sB]}{m_1 s^2 + K_1 + sB}\cdot[K_1+sB] = X_2(s)[m_2 s^2 + K_2 + K_1 + sB]$$

$$F(s) = X_2(s)\left[m_2 s^2 + k_2 + K_1 + sB - \frac{(K_1+sB)^2}{m_1 s^2 + K_1 + sB}\right]$$

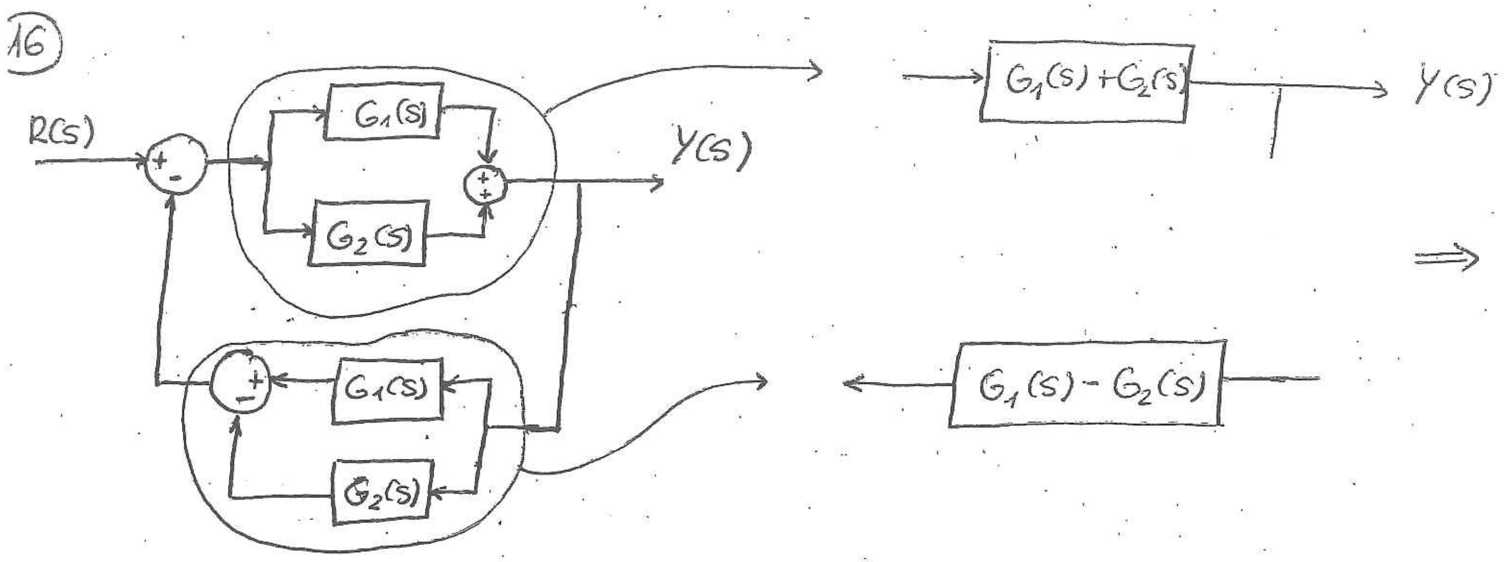
$$\left[G_2(s) = \frac{X_2(s)}{F(s)} = \frac{1}{m_2 s^2 + K_2 + K_1 + sB - \frac{(K_1+sB)^2}{m_1 s^2 + K_1 + sB}}\right]$$

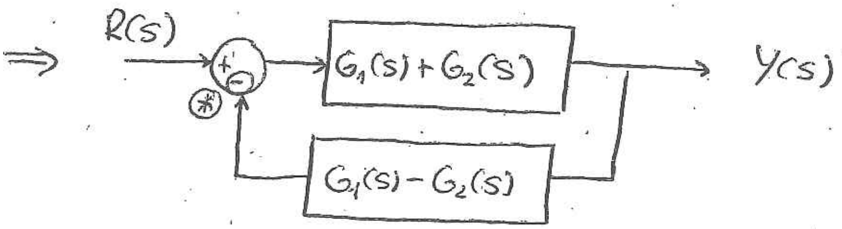
$$X_1(s) = \frac{(K_1+sB)}{(m_1 s^2 + K_1 + sB)} \cdot F(s) \cdot \frac{1}{m_1 s^2 + K_1 + sB + K_2 - \frac{(K_1+sB)^2}{m_1 s^2 + K_1 + sB}}$$

$$\left[G_1(s) = \frac{X_1(s)}{F(s)} = \frac{(K_1+sB)}{(m_1 s^2 + K_1 + sB)\left(m_1 s^2 + K_2 + K_1 + sB - \frac{(K_1+sB)^2}{m_1 s^2 + K_1 + sB}\right)}\right]$$

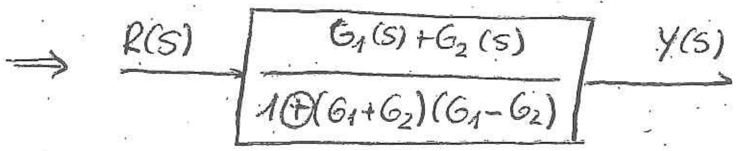
BLOKEEN DIAGRAMEN ALJEBRAKO ARIKETAK

⑯





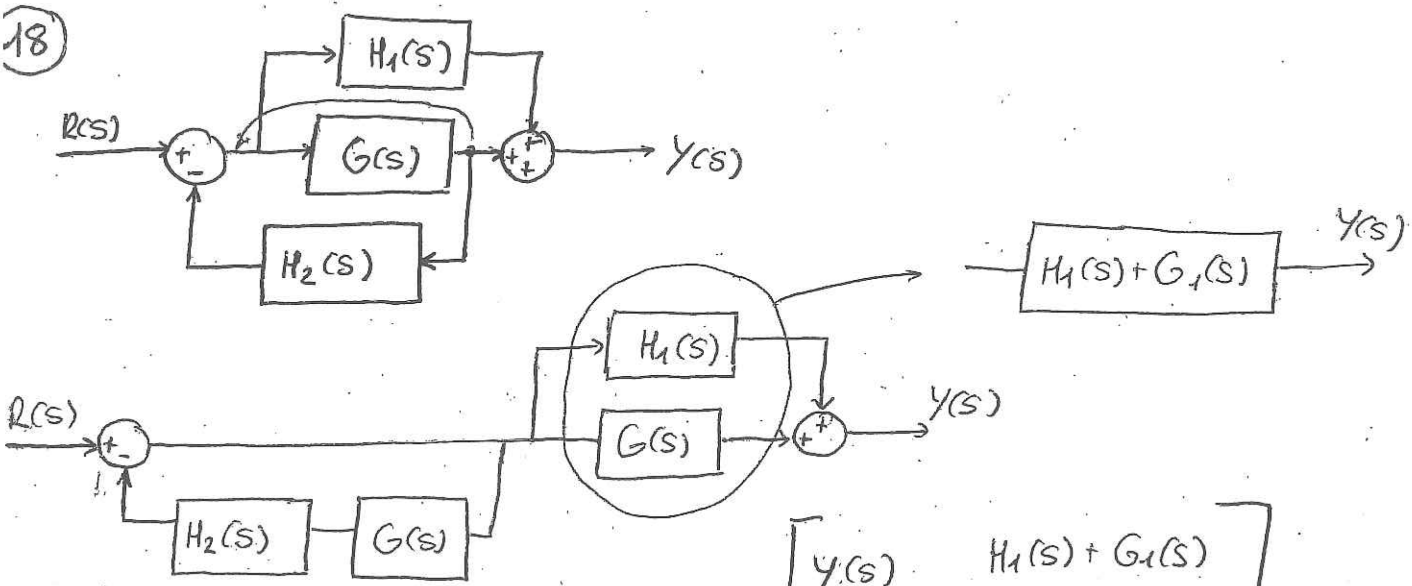
Berrelikadura - begizta baten murrizketa balean sinplifikatu dugu.



$$\left[\frac{Y(s)}{R(s)} = \frac{G_1(s) + G_2(s)}{1 + G_1^2(s) - G_2^2(s)} \right]$$

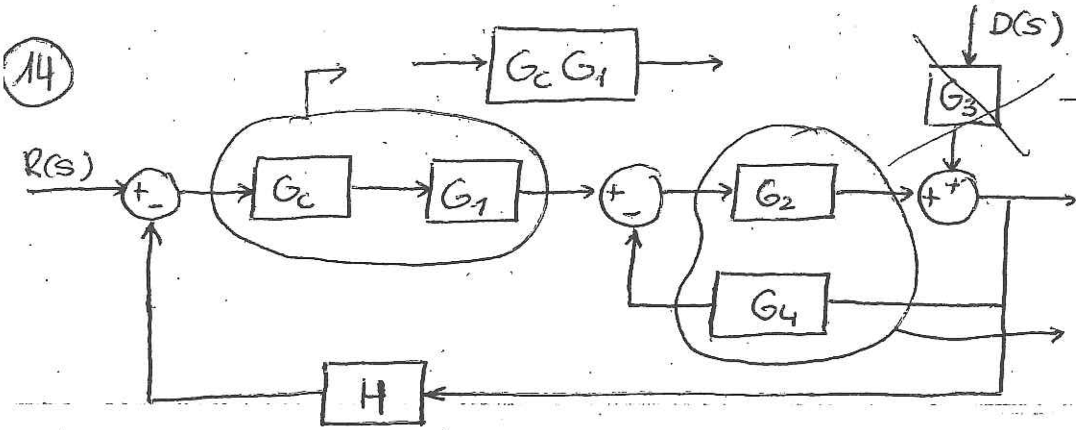
⊛ seinaleen batukarian ⊖ bat badago berrelikaduran, gero ⊕ jarri behar dugu.

⑱



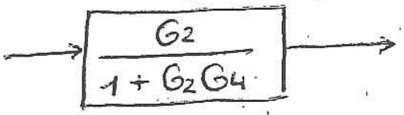
$$\left[\frac{Y(s)}{R(s)} = \frac{H_1(s) + G_1(s)}{1 + H_2(s) \cdot G(s)} \right]$$

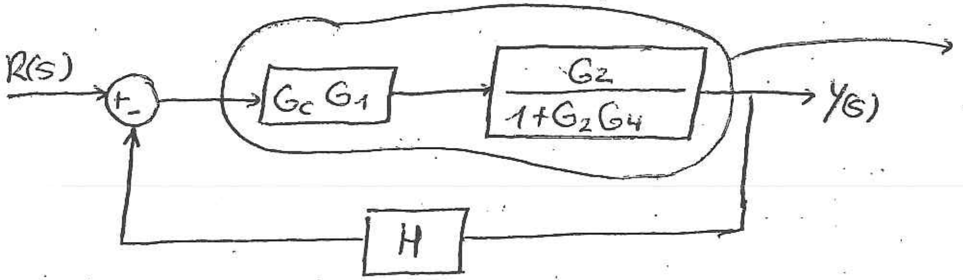
⑭

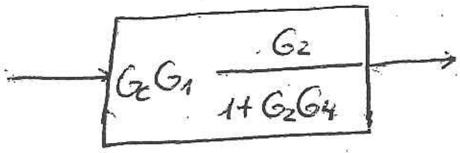


$\Rightarrow \left\{ \frac{Y(s)}{R(s)} \right.$ lortzeko: $D(s)=0$

Berrelikadura



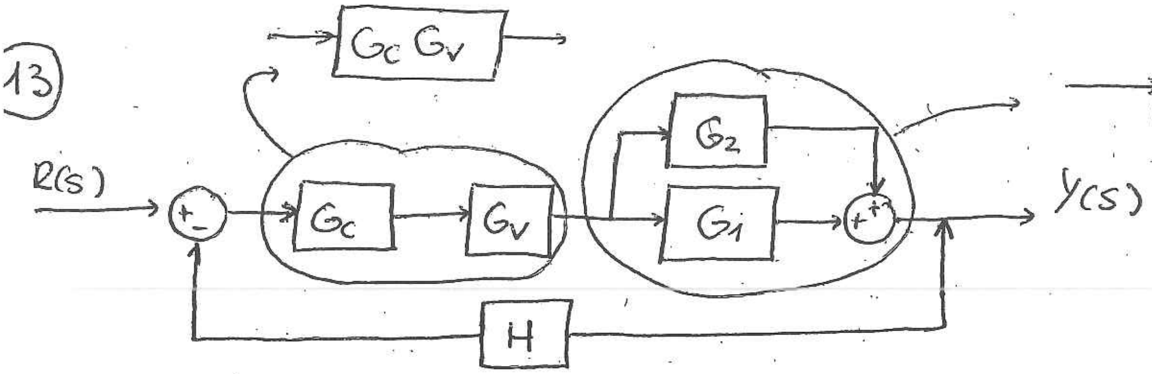


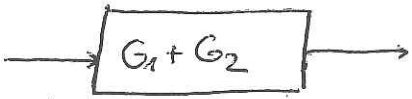


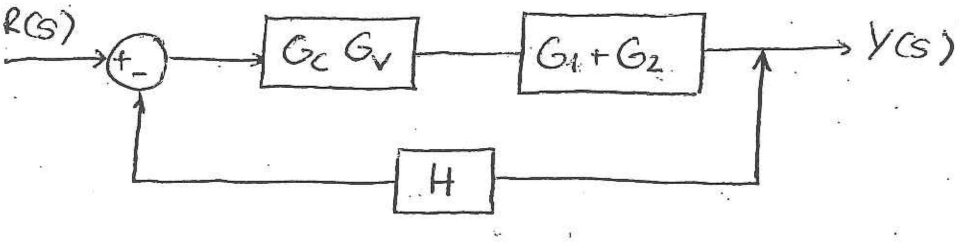
Berrelikadura:

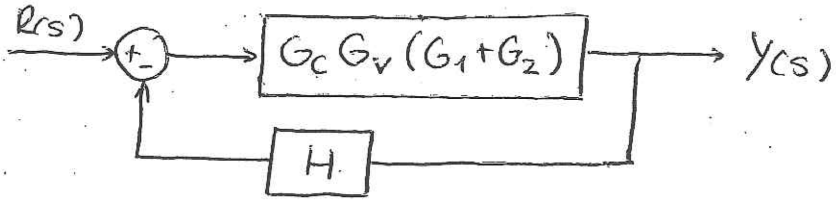
$$\frac{Y(s)}{R(s)} = \frac{G_cG_1 \frac{G_2}{\cancel{1+G_2G_4}}}{1+H\cdot G_cG_1 \frac{G_2}{\cancel{1+G_2G_4}}} = \frac{G_cG_1G_2}{1+G_2G_4+HG_cG_1G_2}$$

⑬



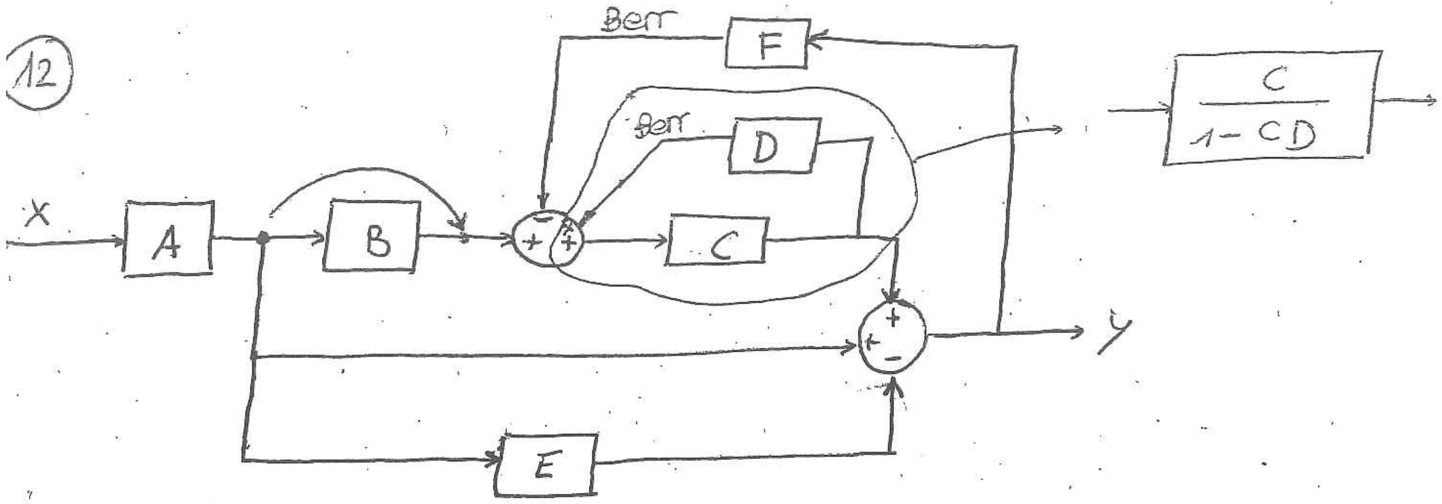


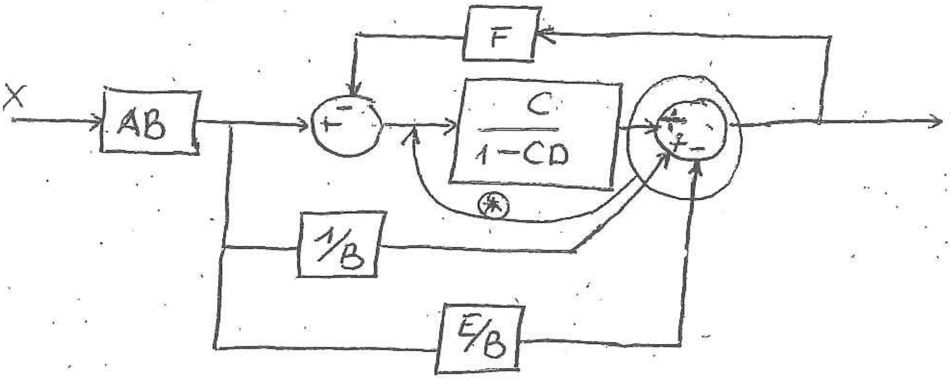




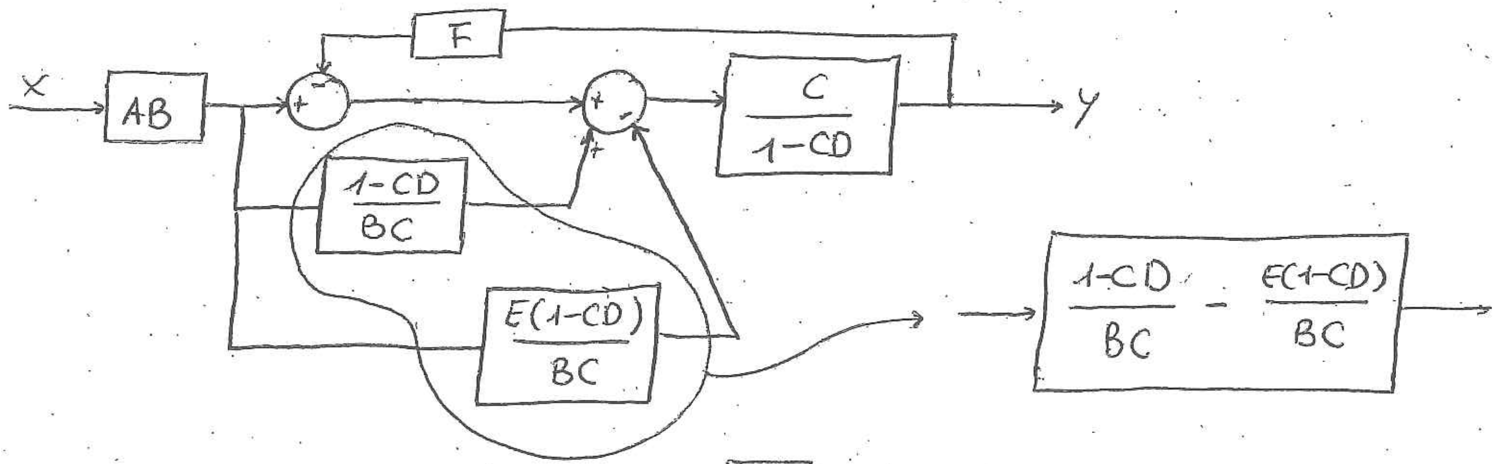
$$\left[\frac{Y(s)}{R(s)} = \frac{G_cG_v(G_1+G_v)}{1+HG_cG_v(G_1+G_v)} \right]$$

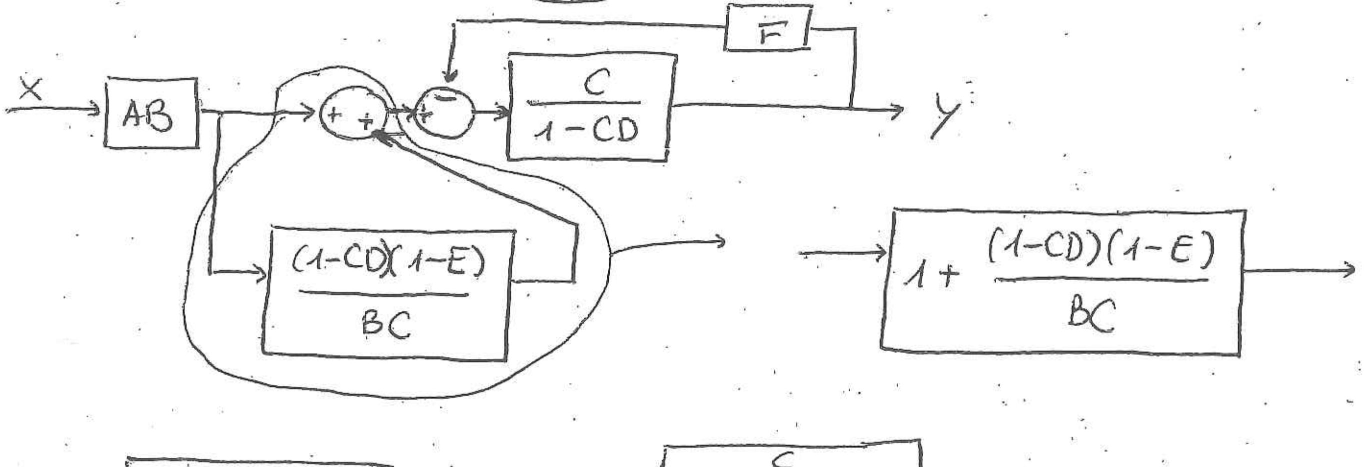
⑫

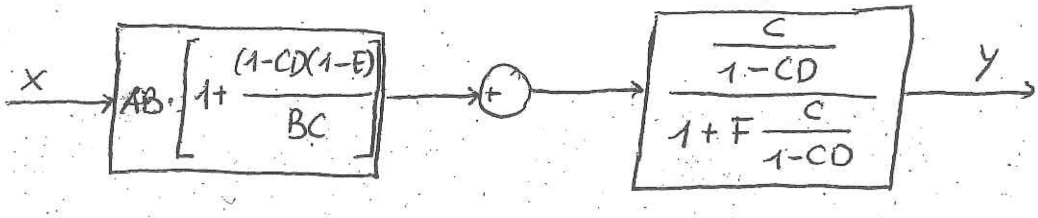




⊛ Hau egitean beheko adarrak $\left(\frac{C}{1-CD}\right)$ -gatik zatitu behar dugu.

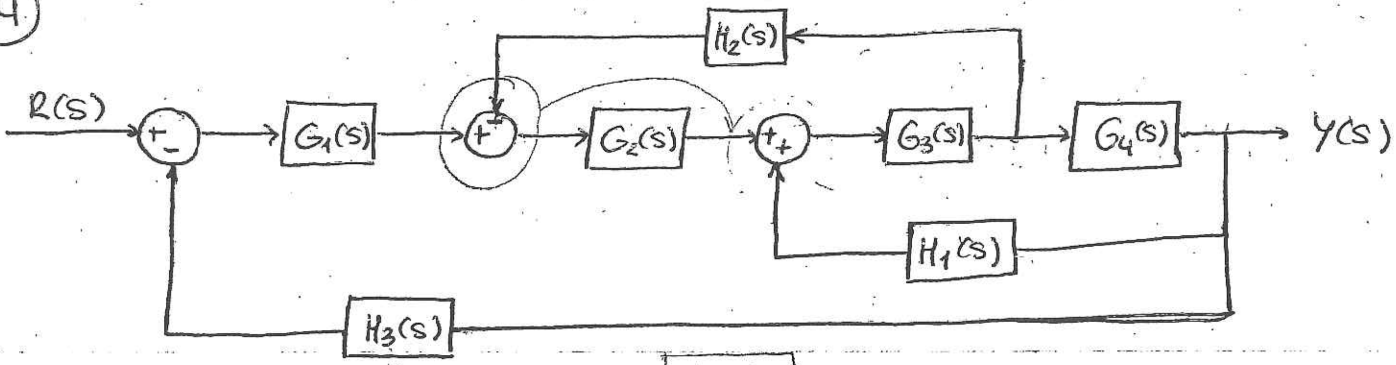


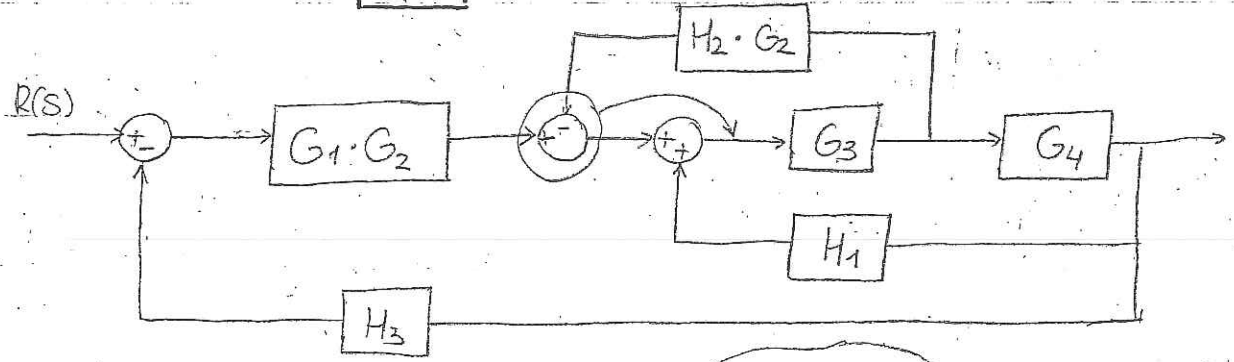


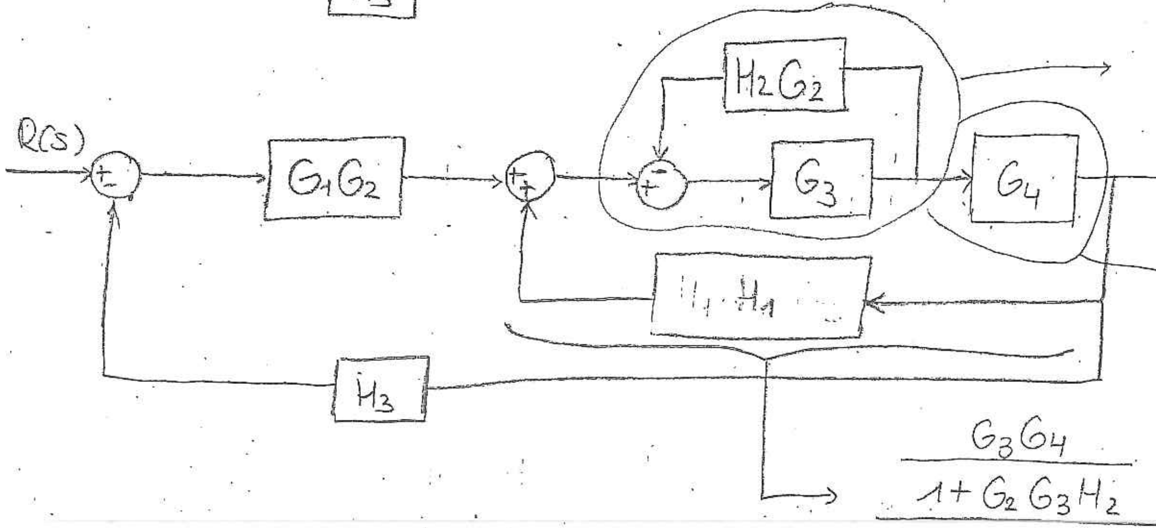


$$\boxed{\frac{Y}{X} = AB\left[1 + \frac{(1-CD)(1-E)}{BC}\right] \cdot \frac{C}{1-CD+CF} = \frac{ABC + (1-CD)(1-E)}{1-CD+CF}}$$

⑲



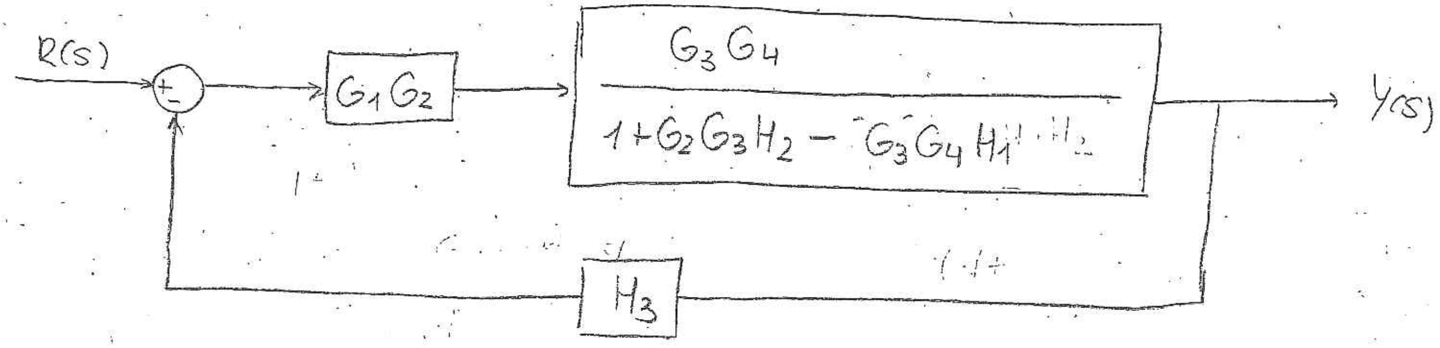




$$\frac{G_3}{1+G_2G_3H_2}$$

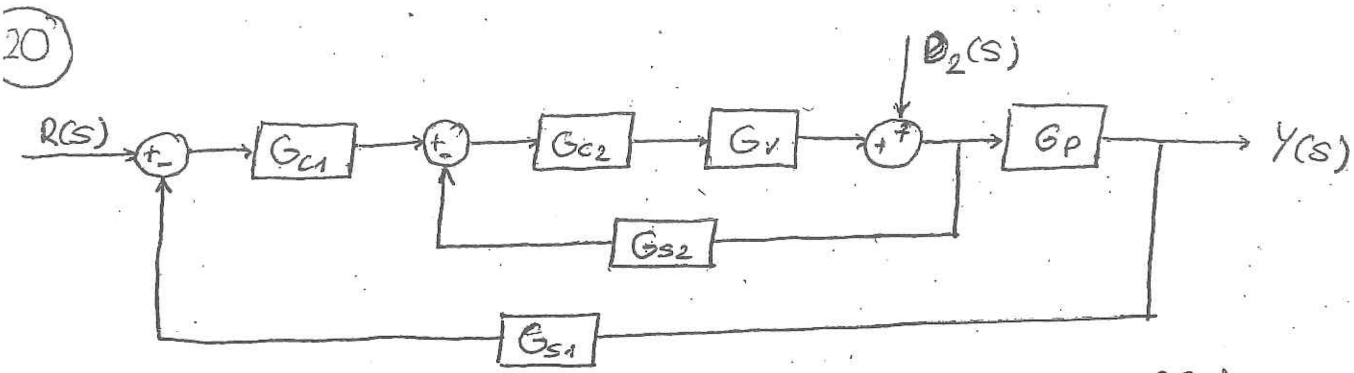
$$\frac{G_3G_4}{1+G_2G_3H_2}$$

$$\frac{\frac{G_3G_4}{1+G_2G_3H_2}}{1-\frac{G_3G_4H_1}{1+G_2G_3H_2}}$$



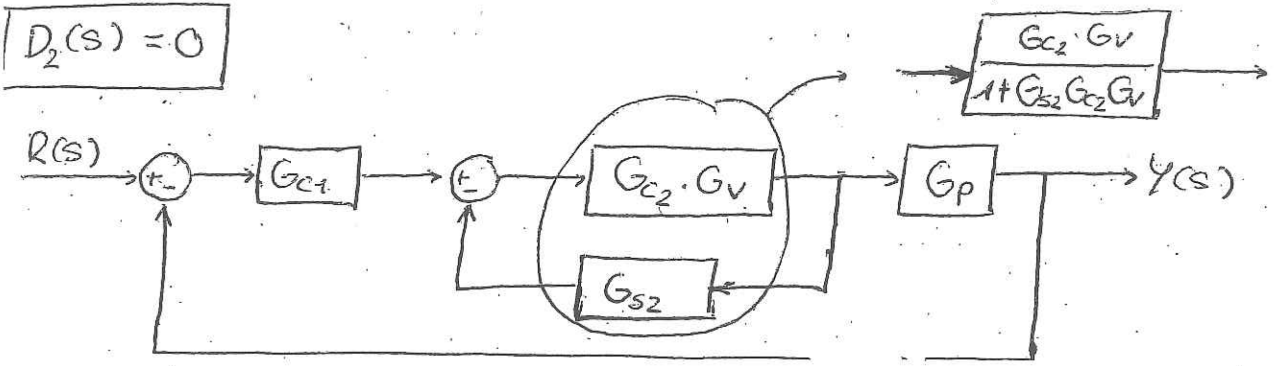
$$\frac{Y(s)}{R(s)} = \frac{G_1G_2G_3G_4}{1+G_2G_3H_2-G_3G_4H_1+G_3G_4H_3G_1G_2}$$

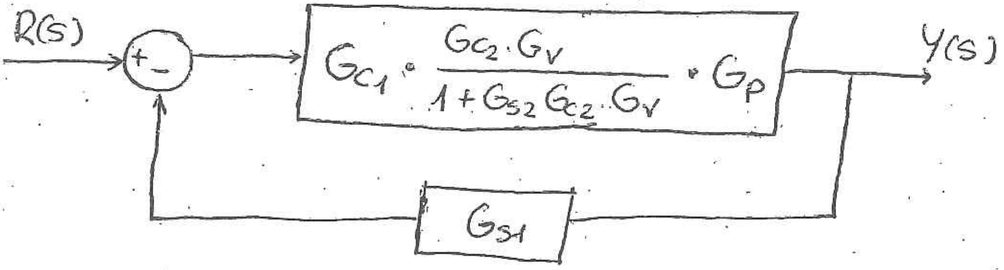
20



Y(s)-k bi adierazpide ezberdin izango ditu: $\left\{ \begin{array}{l} R(s)=0 \\ D_2(s)=0 \end{array} \right.$

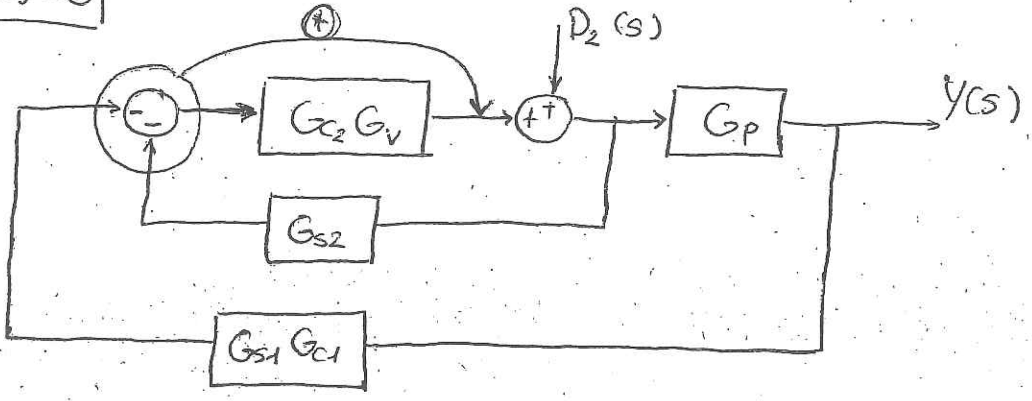
$\boxed{D_2(s)=0}$



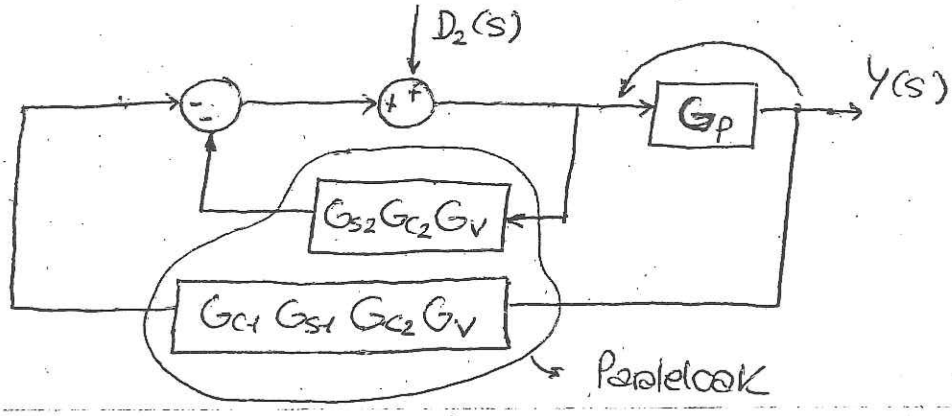


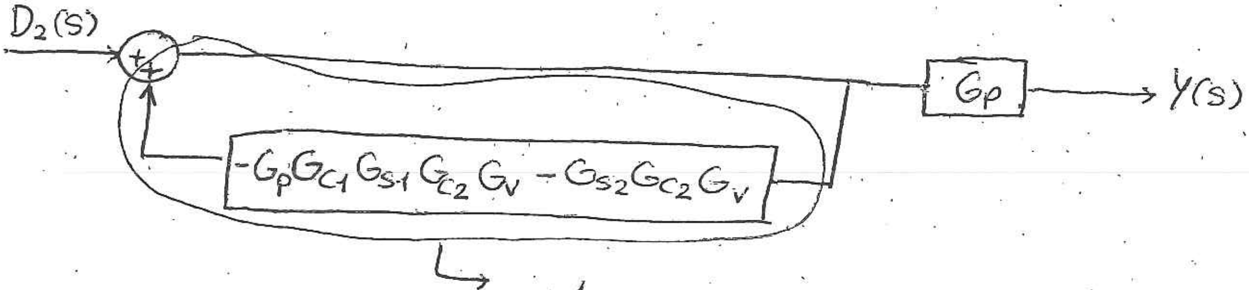
$$Y(s)_1 = R(s)\cdot\frac{\dfrac{G_{c1}G_{c2}G_vG_p}{1+G_{s2}G_{c2}G_v}}{1+G_{s1}\dfrac{G_{c1}G_{c2}G_vG_p}{1+G_{s2}G_{c2}G_v}} = \frac{G_{c1}\,G_{c2}\,G_v\,G_p\cdot R(s)}{1+G_{c2}G_vG_{s2}+G_{s1}G_{c1}G_{c2}G_vG_p}$$

$\boxed{R(s)=0}$



(*) Hau egiteko $\left.\begin{array}{l}\boxed{G_{s2}}\\ \boxed{G_{s1}G_{c1}}\end{array}\right\}$ blokeak $G_{c2}G_v$ -gatik biderkatu behar d





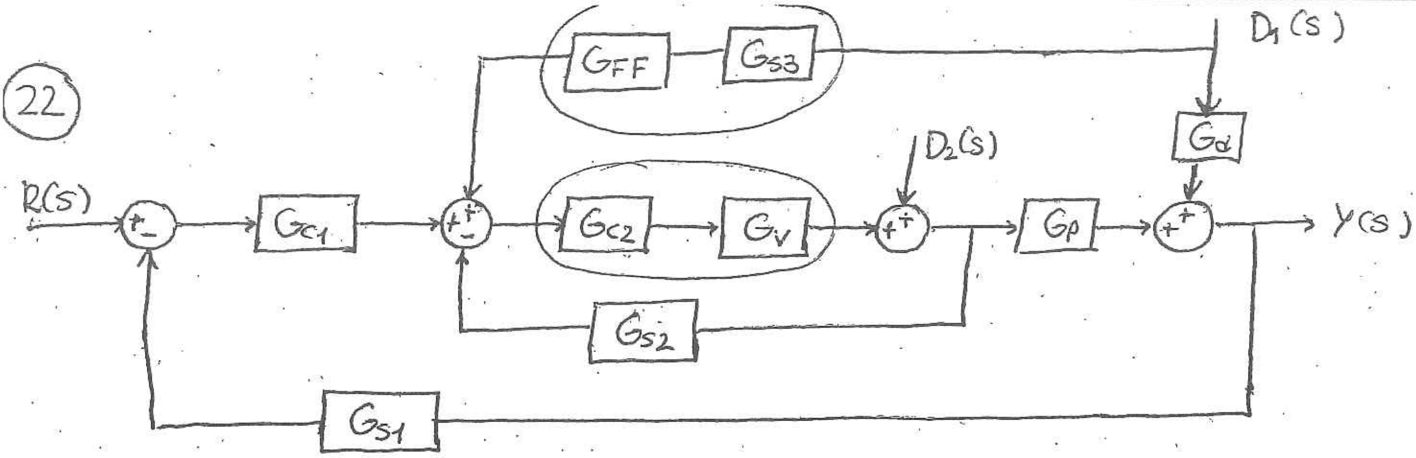
$$\frac{1}{1 - 1\cdot(-G_{c1}G_{s1}G_{c2}G_vG_p - G_{s2}G_{c2}G_v)}$$

$$Y(s)_2 = \underbrace{D(s)\cdot G_p \cdot \frac{1}{1+G_{c1}G_{s1}G_{c2}G_vG_p + G_{s2}G_{c2}G_v}}$$

$$Y(s) = Y(s)_1 + Y(s)_2 = \frac{G_{c1}G_{c2}G_vG_p}{1+G_{c2}G_vG_{s2}+G_{s1}G_{c2}G_{c1}G_vG_p}R(s) +$$

$$\left[Y(s) = \frac{R(s)\cdot G_{c1}G_{c2}G_vG_p + D(s)G_p}{1+G_{c2}G_vG_{s2}+G_{s1}G_{c2}G_{c1}G_vG_p} \right]$$

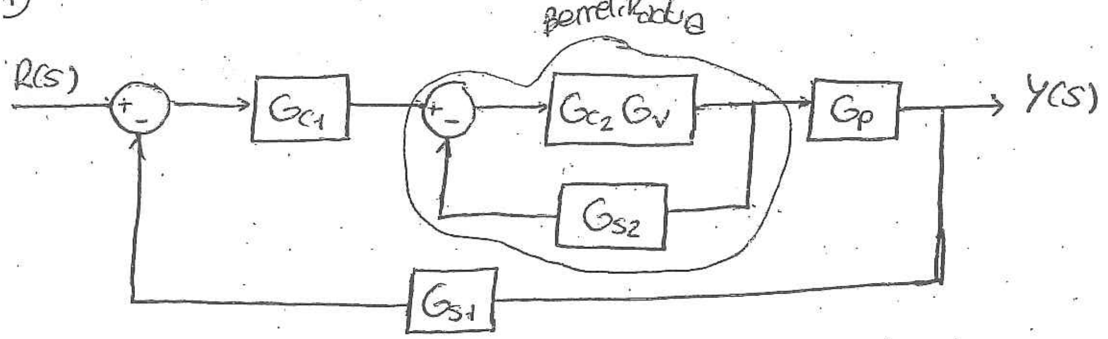
㉒

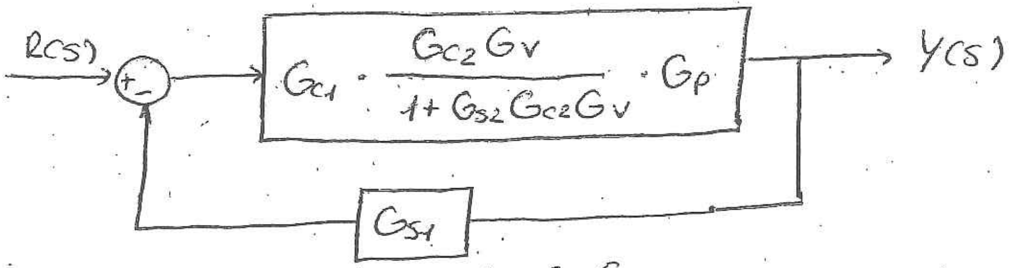


Hiru sarreretatik bakarrik batekin egin behar dugu lan, beraz, hiru Kasu ezberdin landuko ditugu:

$D_1(s)=0 \wedge D_2(s)=0$ ①
$D_1(s)=0 \wedge R(s)=0$ ②
$D_2(s)=0 \wedge R(s)=0$ ③

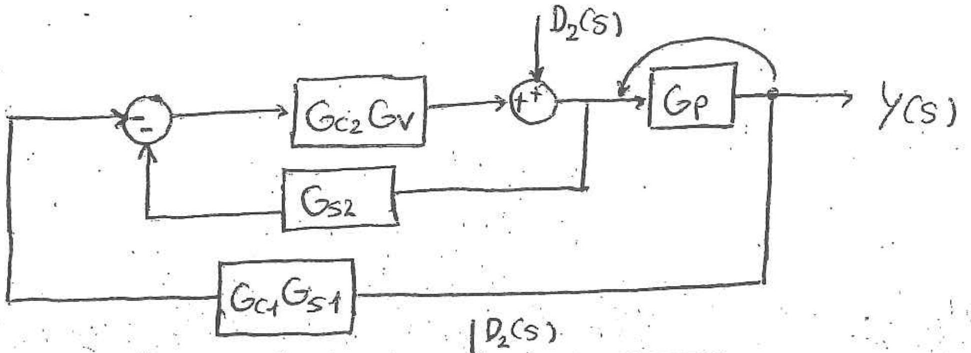
1)

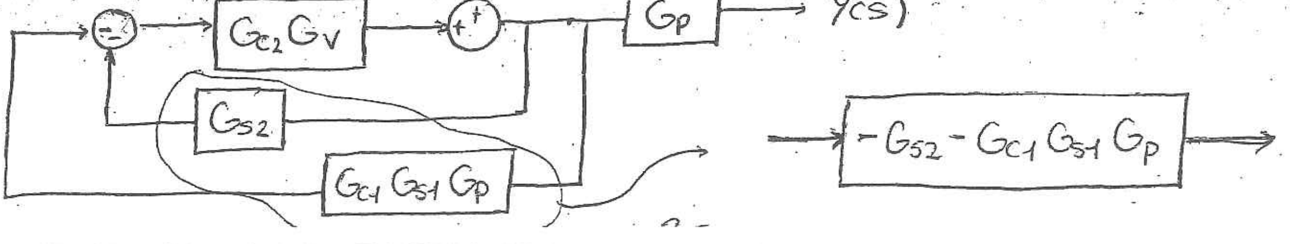


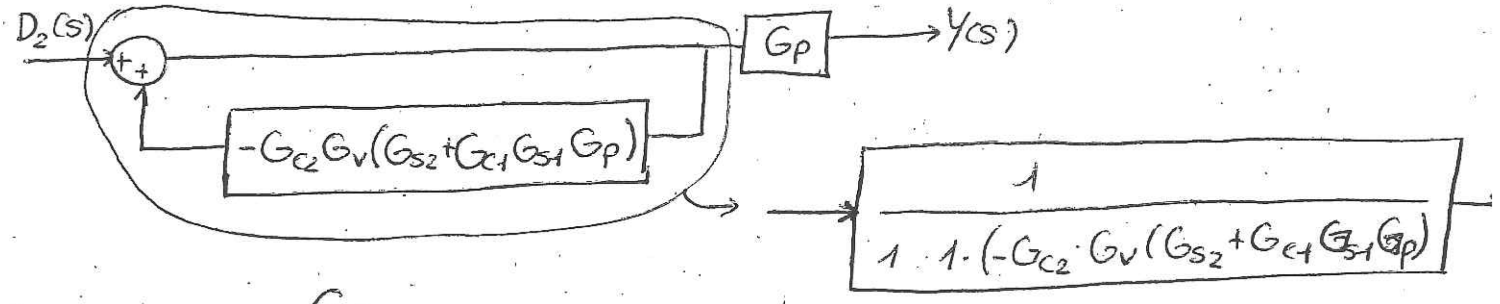


$$Y_1(s) = R(s) \cdot \frac{\dfrac{G_{c1} G_{c2} G_v G_p}{1+G_{s2} G_{c2} G_v}}{1+\dfrac{G_{c1} G_{c2} G_{s1} G_v G_p}{1+G_{s2} G_{c2} G_v}} = \frac{G_{c1} G_{c2} G_v G_p}{1+G_{s2} G_{c2} G_v + G_{c1} G_{c2} G_{s1} G_v G_p} R(s)$$

)

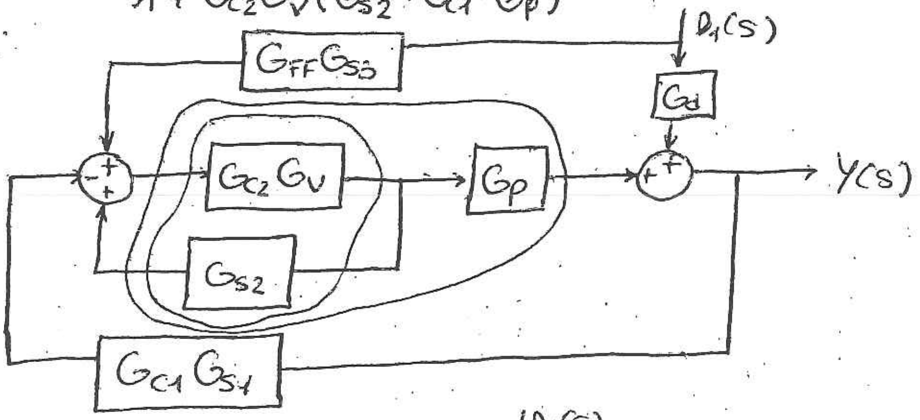


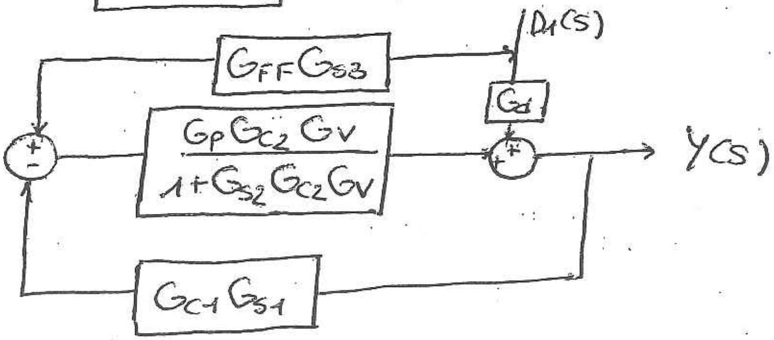


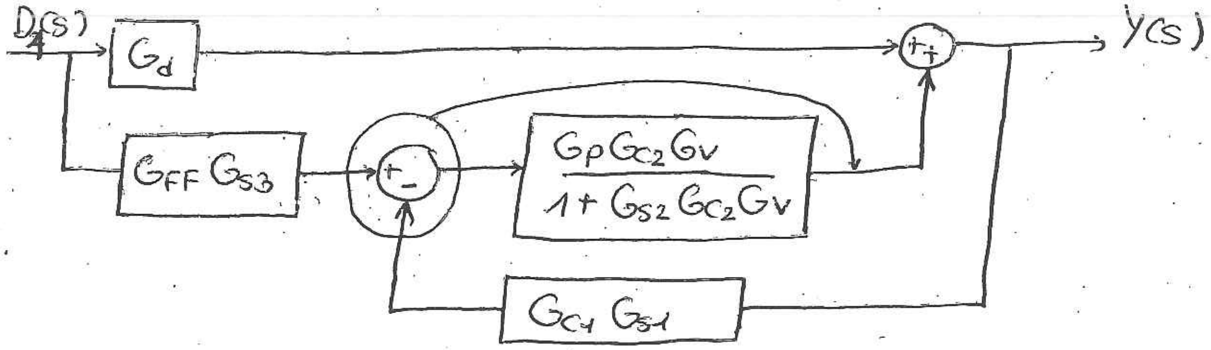


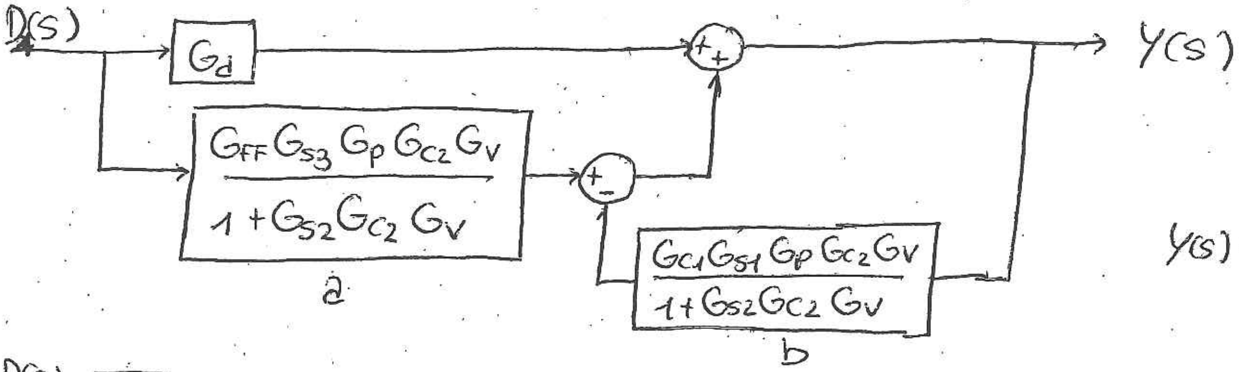
$$Y_2(s) = \frac{G_p}{1 + G_{c2}G_v(G_{s2} + G_{c1}\cdot G_p)}\, D_2(s)$$

③

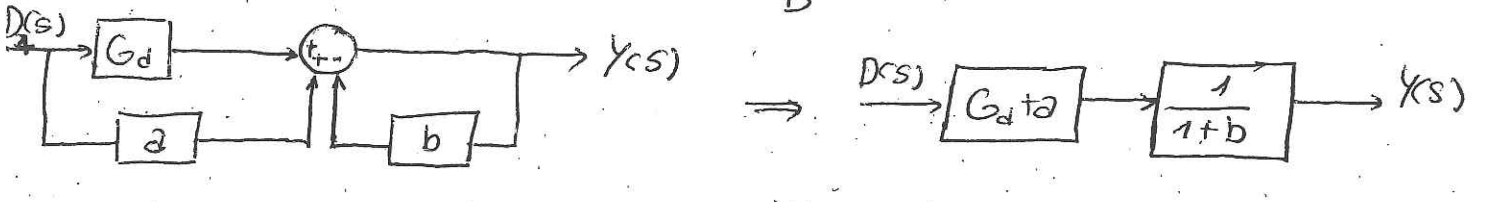




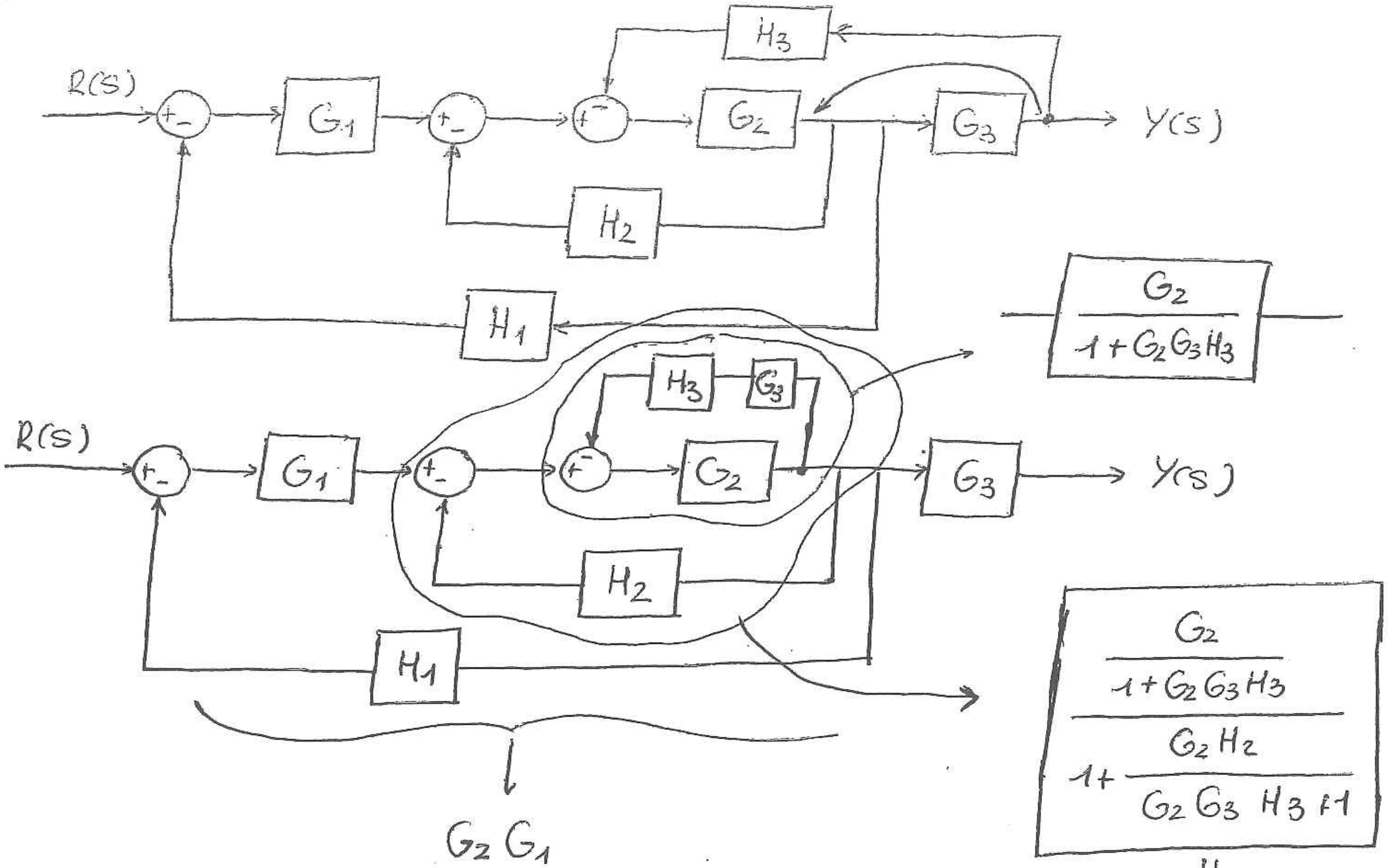




$$Y(s) = \frac{G_d + a}{1 + b} \cdot D_1(s$$



10 Ariketa (42. diapositiba)



$$\boxed{\frac{G_2}{1+G_2G_3H_3}}$$

$$\boxed{\frac{\frac{G_2}{1+G_2G_3H_3}}{1+\frac{G_2H_2}{G_2G_3H_3+1}}} = \frac{G_2}{G_2G_3H_3+G_2H_2+1}$$

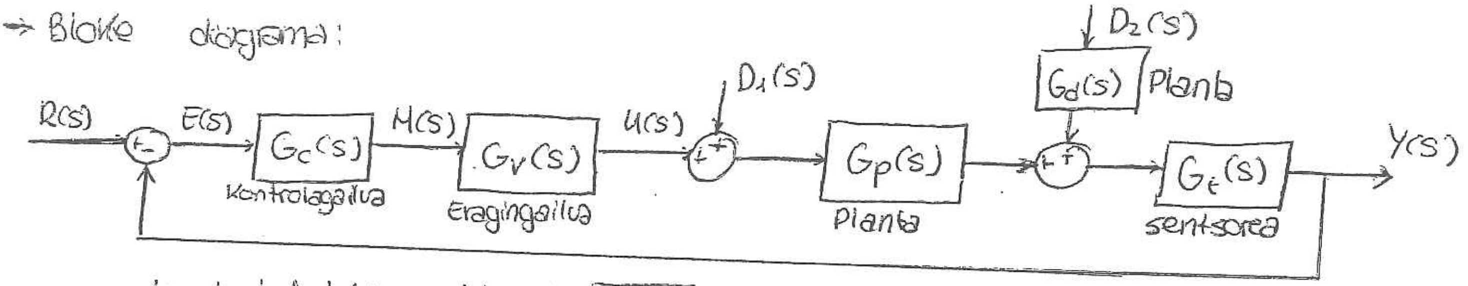
$$\frac{\frac{G_2G_1}{G_2G_3H_3+G_2H_2+1}}{1+\frac{G_2G_1H_1}{G_2G_3H_3+G_2H_2+1}}$$

$$\boxed{G(s)=\frac{Y(s)}{R(s)}=\frac{G_1G_2G_3}{G_2G_3H_3+G_2H_2+G_1G_2H_1+1}}$$

6. GAIA

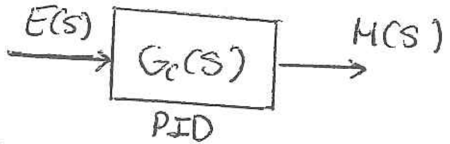
I) - KONTROLAGAILUEN DISEINUA

→ Bloke diagrama:



- kontrolatutako aldagai: $\boxed{y(t)}$
- Manipulatutako aldagaia: $\boxed{m(t)}$
- Manipulatutako prozesuko aldagaia: $\boxed{u(t)}$
- Perturbazioak: $\boxed{D_1(t) \text{ eta } D_2(t)}$

• PID: AKZIOAK ETA PARAMETROAK



PID kontrolagailu ideala:

$$m(t) = K_c\left[e(t) + \frac{1}{T_i}\int e(t)\,dt + T_d\,\frac{de(t)}{dt}\right]$$

⊛ Akzioak:

- Errorearekiko proportzionala.
- Errorearen deribazioarekiko proportzionala.
- Errorearen integrazioarekiko proportzionala.

⊛ Parametroak

- K_c: Irabazpen proportzionala
- T_d: Deribatze - denbora
- T_i: Integratze - denboa.

↳ P AKZIOA: P KONTROLA $m(t) = K_c\, e(t)$

- %X -ko erroreak, $m = K_c \times (\%)$ kontrol ekintza sortarazten du eragingailuaren seinalean.
- sistema baten erantzuna azkartzeko erabiltzen da.
- Limiteak:
  - ↳ Egonkortasuna (K_{cr})
  - ↳ Asetasuna.

↳ I: INTEGRAZIO AKZIOA $m(t) = \frac{K_c}{T_i} \int e(t)\, dt$

- Errorea zero ez den bitartean, ekintza integralean eboluzionatzen du.
- Sistema berrelikatzaren mota handitzeko erabilia.
- Ekintza integrala ↑ ⇒ T_i ↓ ⇒ abiadura handiagoz kenduko da errorea.
- Ez da inoiz bakarrik erabiltzen, akzio proportzionalarekin erabiltzen da beti. (PI)

↳ DERIBAZIO AKZIOA: D $m(t) = K_c T_d \frac{de(t)}{dt}$

- Aurreratze akzioa da
- Egoera egonkorrean ez du eraginik (errorea ⇒ ,)

- ALGORITMO ERABILENAK

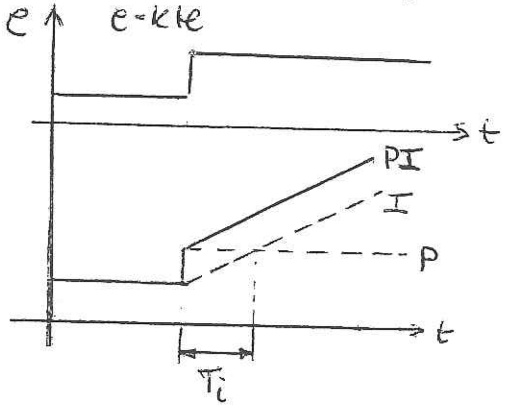
⊛ P : $\boxed{m(t) = K_c [e(t)]}$

Bere eragina ET-n ikus daiteke:

- 2. ordeneko sistemak ez dira desegonkortzen P batekin.
- Goi-ordeneko sistemetan, K-ren balioak muga bat du eta hortik aurrera sistema desegonkortzen da.

⊛ PI : $\boxed{m(t) = \bar{m} + K_c\left(e(t) + \frac{1}{T_i}\int e(t)\, dt\right)}$

$$TF: \; G_c(s) = \frac{K_c}{T_i} \cdot \frac{1 + T_i\, s}{s} = \frac{K_c\,(s + 1/T_i)}{s} = G_{PI}(s)$$



T_i: errore kte baten aurrean, ekintza integralak, ekintza proportzionalak hartzen duen balioa berdintzeko behar duen denbora da.

T_i ↓ ⇒ I ↑ (ekintza integrala)

↱ ET-ren forman eragina hango du.

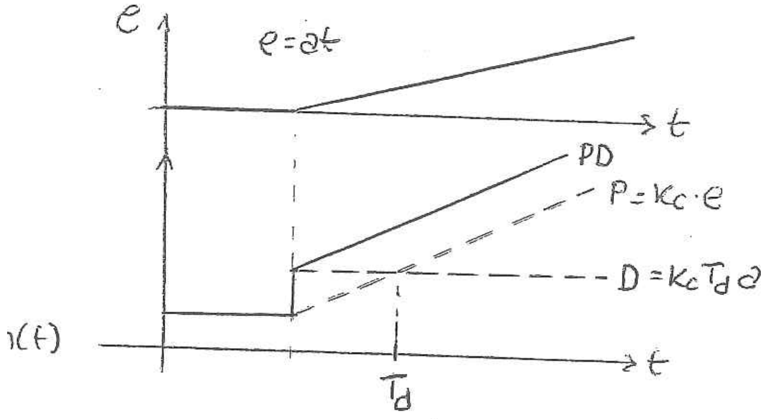
- Polo bat jatorrian eta zero bat $(s = -\frac{1}{T_i})$ sartzen ditu.
  ↳ ($s=0$, integratzailea) sistema mota handitu baina egonkortasuna kaltetzen d
- Orokorrean, zeroaren eragina ET ezkerretara mugarazko du, sistema egonkortuz.
- T_i-ren balioa iragankorreko eskakizunak betetzeko helburuarekin finka daitek

- Ikus dezagun zelan aldatzen den ET T_i-ren balioarekin:
  - Lehen ordeneko sistema: $G_{BA}(s) = \frac{k_c(s+z_i)}{s} \cdot \frac{k}{(s+p)}$; $z_i = \frac{1}{T_i}$
    - ↳ PI-aren zeroa dominatzailea ⇒ $z_i < p$ (20. diapositiba)
    - ↳ PI-aren zeroa dominatzailea ez denean → $z_i > p$ (19. diap.)
  - Bigarren ordeneko sistema: → $G_{BA}(s) = \frac{k_c(s+z_i)}{s} \cdot \frac{k}{(s+p_1)(s+p_2)}$; $z_i = 1/T_i$
    - ↳ PI-aren zeroa EZ dominatzailea: $z_i < p_j$ (21. diap.)
    - ↳ PI-aren zeroa EZ dominatzailea $z_i < Er(p_{1/2})$ (22. diap)

      → $G_{BA}(s) = \frac{k_c(s+z_i)}{s} \cdot \frac{k\omega_n^2}{s^2+2\delta\omega_n+\omega_n^2}$; $z_i = \frac{1}{T_i}$

* PD: $\boxed{m(t) = \bar{m} + k_c\left(e(t) + T_d \frac{de(t)}{dt}\right)}$

TF: $G_c(s) = k_c(1+T_d s)\,E(s) = G_{PD}(s)$



T_d: ekintza proportzionalak ekintza deribatiboaren balio bera lortzeko behar duen denbora (arrapala sarreraren aurrean).

$e \uparrow$ ⇒ kontrol-seinalea azkartu
$e \downarrow$ ⇒ kontrol-seinalea moteldu.

- Deribazio akzioa erabiltzen da bere eragin egonkortzaileak k_c handitzeko aukera ematen duelako.
- zeroa sartzen du, $s = -\frac{1}{T_d}$, eta bere kokapena T_d-ren araberakoa da. ET-ren forman eragina izango du, orokorrean ET ezkerrerantz mugiarazi eta sistema egonkortuz.
- T_d-ren balioa iragankorreko eskakizunak betetzeko helburuarekin finka daiteke.
- Ez du eraginik iraunkorrean.
- Ikus dezagun zelan aldatzen den ET T_d-ren balioarekin:
  - Lehen ordeneko sistema: $G_{BA}(s) = k_c(s+z_d)\frac{k}{(s+p)}$; $z_d = \frac{1}{T_d}$

↳ PD-ren zeroa EZ dominatzailea → $z_d > p$ (28. diap)

↳ PD-ren zeroa dominatzailea → $z_d < p$ (29. diap)

- Bigarren ordeneko sistema: (gainmoteldua) $G_{BA}(s) = K_c(s+z_d)\frac{K}{(s+p_1)(s+p_2)}$; $z_d = \frac{1}{T_d}$

↳ PD-ren zeroa EZ dominatzailea ⇒ $z_d > p_1, p_2$ (30. diap)

↳ PD-ren zeroa dominatzailea ⇒ $z_d < p_1, p_2$ (31. diap)

⊛ PID:

$$\boxed{m(t) = \bar{m} + K_c\left[e(t) + \frac{1}{T_i}\int_0^{\tau} e(\tau)\,d\tau + T_d\frac{de(t)}{dt}\right]}$$

TF: $G_c(s) = K_c\left[1 + \frac{1}{T_i s} + T_d s\right] = K_c\frac{T_i s + 1 + T_i T_d s^2}{T_i s} = G_{PID}(s)$

- Ezin da fisikoki inplementatu (poloak baino zero gehiago dago
- Maiztasun handiko zaratarekiko sentikortasun handia.
- $T_i > 4T_d$ denean ⇒ zero errealak.
- PID-aren eragina ET-n:
  - Integratzaileak egonkortasuna kaltetzen du. (Ku txikiagoa P-rena bain
  - Zeroen eragina: ET ezkerrerata mugaraztea ⇒ egonkortu.
  - Zeroak dominatzaileagoak ⇒ eragin egonkortzaile handiagoa.

II) -KONTROLAGAILUEN DISEINUA

- SARRERA

→ 1.URRATSA: Algoritmoaren aukeraketa.

⊛ P erabili: - Egoera iraunkorreko errorea onartzen denean.
- Sistema berrelikatua 1. motakoa bada.

⊛ PD erabili: - Iragankorreko eskakizunak betetzeko beharra dagoenean. Hau da, ET eta erantzun-eskakizunen artean intersekziorik ez dagoenean.

⊛ PI erabili: - Egoera iraunkorreko errorerik onartzen ez denean (derrigorrezkoa denean sistema mota handitzea)

⊛ PID erabili: - Zaratarik ez badago eta abiadura handitu nahi bada.
- Dinamika motela izatekotan, D ekintzak erantzuna egonkortuko du, K_c igotzea baimenduz.
- ET-n PI batekin iragankorreko eskakizunak beteko ez direla ikusten bada (intersekziorik ez badago)

→ Sintonizazio metodoak:

1) Froga eta errorea
2) Diseinu analitikoa ereduetan oinarrituta.
3) Esperimentuetan oinarrituak: Ziegler Nichols < BIreka / BItxia

- 1-FROGA ETA ERROREA

K_c-ren balio txikietatik abiatuta, eta akzio integralik gabe (T_i handia)

① K_c handitu kurbak nahi dugun forma lortu arte, kontrol-seinalearen balioak oso handiak ez direnik.

② T_d handitu erantzuna hobetzeko (K_c handitzea baimentzen du)

1 eta 2 errepikatu erantzunaren kurbak nahi dugun forma izan arte.

③ T_i txikitu egoera egonkorreko errorea kendu arte. (oszilazioak ekiditu)

2- DISEINU ANALITIKOA EREDUETAN OINARRITUTA

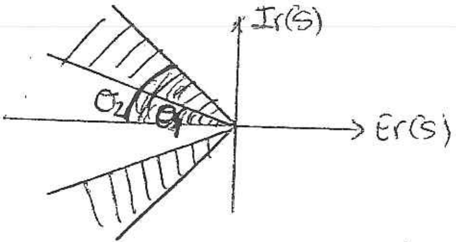
Erantzun eskakizunak: begizta itxiko sistemaren polo nagusiak s planoan non kokatu behar diren (eremua) definitzen dute.

→ EGOERA IRAGANKORREKO ESKAKIZUNAK

- M_p balio tartea

  Polo dominatzaileek ardatz errealarekiko izan behar duten angelu maximoa eta minimoa definitzen dute. Horrela, polo hauek izan behar duten δ mugatzen dute.

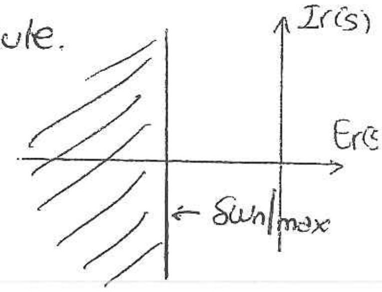
  $$\cos\Theta_1 = \delta_1 \qquad \cos\Theta_2 = \delta \implies \delta_2 < \delta < \delta_1$$



- t_s

  Polo dominatzaileen (konplexu konjokatuak badira) edo polo erreal dominatzailearen zati errealaren balio maximoa definitzen dute. Polo hauen δ mugatzen dute.

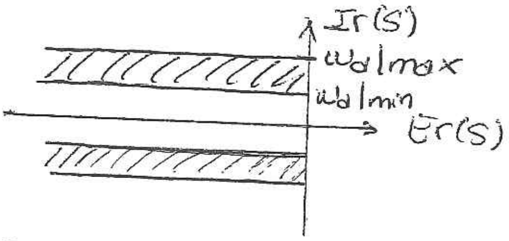
  $$t_s \leq \text{balioa} \implies \delta\omega_n \geq \text{balioa}$$
  $$\tau \leq \text{balioa}$$



- t_p

  Polo dominatzaileen zati irudikariak har dezakeen balio maximoa mugatzen du.

  $$\omega_d|_{min} \leq \omega_d \leq \omega_d|_{max}$$



- Erreferentzia seinalearen jarraipena → sistema berelikatu mota definitzen du.

- Sarrera mota bati jarraitzean eman dezakeen errore maximoa → → balio minimoa finkatzen du ω_n-rentzako.

→ DISEINURAKO JARRAIBIDEAK

- Sistema-mota handitu behar bada iraunkorreko eskakizunak betetzeko, I akzioa beharrezkoa da.

↳ Zeroen hautaketa:

*Sistemaren polo nagusiak baliogabetzeko → PI kontrolagailuak gehitzen duen zeroa (PIDak bi zero) polo nagusia baliogabetzeko erabiltzen da → Geldiena baliogabetu → Ti finkatu (edo Ti eta T_d PIDarekin) eta K_c parametroa iragankorreko eskakizun bat betetzeko erabiliko da.

* Polo nagusiak zeroekin baliogabetuta, ez dago intersekziorik ET-aren eta eskakizunen eremuaren artean → It etu: zeroak beste nolabait kokatu intersekzioa lortu arte.

- Kc hautatzeko:

BItxiko ek. karakteristikoa hautatutako kontrolagailuarekin = Desio den ek. karakteristikoa (poloak eskakizunen eremuan)

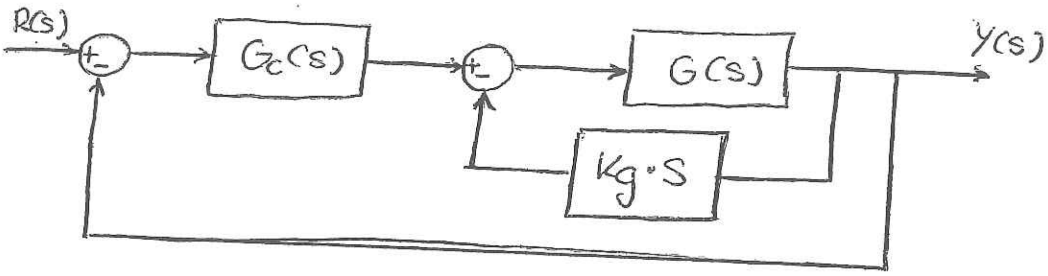
- Diseinuaren emaitzak frogatu.

Zeroek eragina dute portaeran:

  - Zeroa dominatzailea → Mp espero dena baino handiagoa
  - Zeroa desegonkorra → t_s ↑ (sistemak hasieran kontrako zentzuan erantzungo duelako)

→ ABIADURAREN BERRELIKADURA

- P kontrola + Abiaduraren berrelikadura



- Barne begizta bat txertatzen da irteeraren deribatua berrelikatu
- Egonkortasuna hobetuko dugu Kg terminoarekin, beraz, egoera iragankorreko eskakizun bat betetzeko erabil daiteke.
- K_c eta Kg, egoera egonkorreko eta iraunkorreko eskakizun bana betetzeko erabil daitezke.

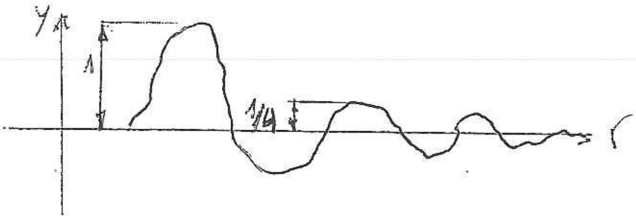
* ESPERIMENTUETAN OINARRITUTAKO METODOAK

Kontrolagailuen parametroak kalkulatzeko erabiltzen dira taulak edo esperimentu baten erantzunetik estimatutako ezaugarri dinamikoetatik dedukitutako formulak.

→ ERANTZUN OSOAN OINARRITUTA ($t=0$ eta $t \to \infty$ tartean)

Errorearen integralean oinarritua. Aukeratutako sintonizazioak, adierazle bat minimizatzen du.

→ ERANTZUNAREN EZAUGARRI PUNTUALETAN OINARRITUTA



Moteldura erlazioa $\frac{1}{4}$ bada:

$$\boxed{\text{ZIEGLER-NICHOLS-en METODOA}}$$

* BEGIZTA IREKIAN

$$G(s) = \frac{ke^{-t_m s}}{1+\tau s} \Rightarrow \text{TAULAK}$$

- PID ez interaktiboa.
- Erabilera tartea: $0{,}1 < y = \frac{t_m}{\tau} < 1$

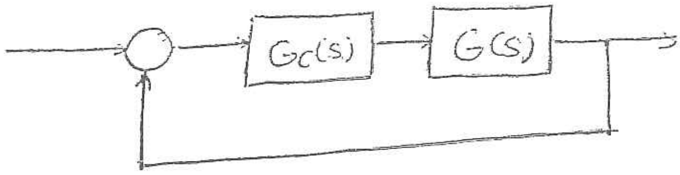
* BEGIZTA ITXIAN

Begizta itxiko sistema kritikoki egonkor egiten duen Kc-ren balioa aurkitzean datza, Ku. Horretarako R-H-en irizpidea erabiltzen da.

Ku: azken irabazpena
Tu: oszilazioaren periodoa ⇒ TAULAK

5. eta 6. GAIKO ARIKETAK

⑲ $G(s) = \dfrac{0{,}5}{(s+1)(s+5)}$



Baldintzak:

$t_s(\%2) \leq 2s$

$M_p \leq \%4{,}3$

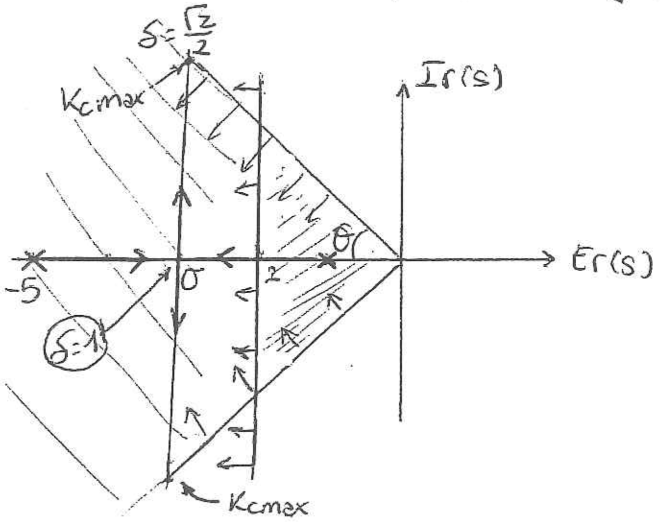
$e_{ss} \leq \%35$

1) Eskakizunaren eremua

$$t_s = \frac{4}{\delta\omega_n} \leq 2 \Rightarrow \delta\omega_n \geq 2$$

$$M_p = e^{\frac{-\pi\delta}{\sqrt{1-\delta^2}}} \leq 0{,}043 \Rightarrow \delta \geq \sqrt{\frac{(\ln 0{,}043)^2}{\pi^2+(\ln 0{,}043)^2}} = 0{,}707 = \frac{\sqrt{2}}{2}$$

$\hookrightarrow \theta = 45°$



2) I akzioa behar da?

Sistema berrelikatu mota: 0 (GH-ren integradore kopurua)

$\hookrightarrow$ Ez da behar errorea onartzen delako.

P kontrola probatu. Parametro bakarra dago (K_c)

2 eskakizun.

3) ET marraztu eta intersekzioa.

$n=2$ (2 polo) poloak: $s=-1$; $s=-5$

$\hookrightarrow$ adar kopurua

$\left.\begin{array}{l} \sigma = -3 \\ \theta = \pm -\frac{\pi}{2} \end{array}\right\}$ Intersekzioa dago (σ eskakizunaren eremuaren barruan dago)

4) K_c?

K_c minimoa σ puntuan.

t_s betetzeko $\longrightarrow$ Polo konplexuak $0<\delta<1$

Ek. karakteristikoa: $1+G(s)\cdot H(s)=0$; $1+K_c\dfrac{0{,}5}{(s+1)(s+5)}=0 \Rightarrow$

$\Rightarrow s^2+6s+5+0{,}5K_c=0$ $\qquad 6=2\delta\omega_n \Rightarrow \omega_n=3$ rad/s

$\delta=1 \longrightarrow s^2+6s+5+0{,}5K_c=0$ $\qquad K_c\cdot 0{,}5=\omega_n^2 \Rightarrow K_c=8$

$\delta=\frac{\sqrt{2}}{2} \rightarrow s^2+6s+(5+0{,}5K_c)=0 \left\{\begin{array}{l} 6=2\cdot\frac{\sqrt{2}}{2}\cdot\omega_n \rightarrow \omega_n=6/\sqrt{2} \text{ rad/s} \\ K_c\cdot 0{,}5+5=\frac{36}{2} \Rightarrow K_c=26 \end{array}\right.$ $\qquad 8<K_c<26$

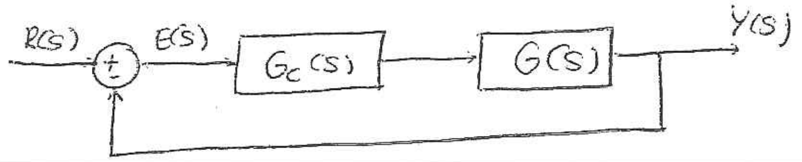
Lortutako K_c-rekin errorearen eskakizuna betetzen da?

$$k_p = \lim_{s\to 0} G(s)\cdot H(s) = \lim_{s\to 0} \frac{K_c \cdot 0{,}5}{(s+1)(s+5)} = \frac{K_c \cdot 0{,}5}{5} = 0{,}1K_c$$

$$e_{ss} = \frac{1}{1+0{,}1K_c} \leq 0{,}35 \Rightarrow K_c \geq 18{,}6 \Rightarrow \boxed{18{,}6 \leq K_c \leq 26}$$

4. Adibidea (30. diapositiba, 6 II)

$$G(s) = \frac{0{,}5}{(s+1)(s+2)}$$



$t(\%) \leq 2s$

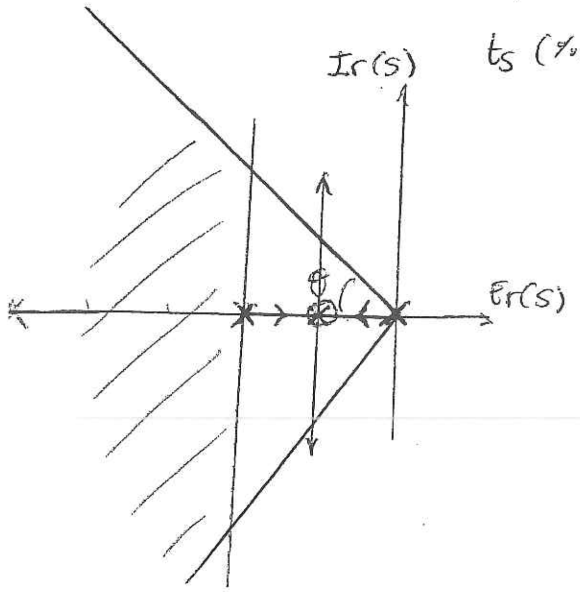
$M_p \leq \% 4{,}3$

$e_{ss} = 0$

1) Eskakizunaren eremua

$M_p \leq \% 4{,}3 \longrightarrow \delta \geq 0{,}707 = \frac{\sqrt{2}}{2} \longrightarrow \theta = 45°$

$t_s\ (\%2) \leq 2s \longrightarrow \delta\omega_n \geq 2$



2) I?

Sistema bereizikatu mota: 0

1 motako sistema bereizikatua izateko,

$e_{ss}=0$ izateko, integradore bat sartu

behar dugu ⟶ PI

$$u(t) = K_c\left(e(t) + \frac{1}{T_i}\int e(t)dt\right)$$

$$\downarrow \mathcal{L}$$

$$U(s) = K_c\left(1 + \frac{1}{T_i s}\right)E(s) \rightarrow G_c(s) = \frac{K_c(1+T_i s)}{T_i s} = \frac{K_c(s+1/T_i)}{s}$$

3) ET egin PI kontrolagailuarekin

$$G(s)\cdot H(s) = \frac{0{,}5K_c\left(s+\frac{1}{T_i}\right)}{s(s+1)(s+2)}$$

poloak: $s=0$; $s=-1$; $s=-2$

Adar-kopurua = GH-ren polo-kopurua = 3 ⟶ $\begin{cases} n=3 \\ m=1 \end{cases}$ (zero-kopurua)

~~$T_i<1$~~ kontrolagailuaren zeroa kokatzeko irizpidea: Polo dominatzailearen gainean, $T_i=1$

ET; Er(s)-n: (-2,0)

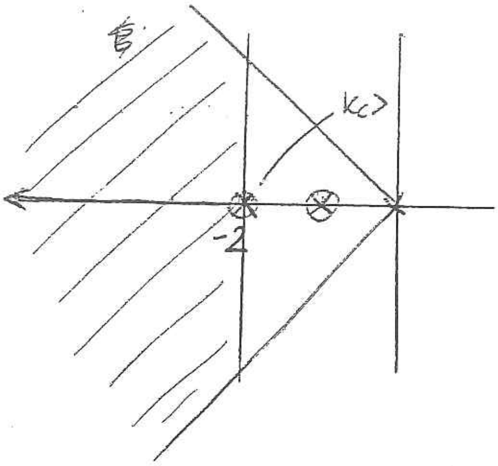
Asintotak: $\sigma = \pm \frac{(2K+1)\pi}{n-m} = \frac{\pi}{2}$ zentroidea: $\sigma = -1$

Intersekzioa dago ET eta eskakizunetan?? <u>EZ</u> ⟶ D behar dugu (zero bat gehiago)

4) PID-ren Erroen Tokia

$$G_{PID}(s) = \frac{K\cdot(s+z_1)(s+z_2)}{s}$$

ET: $\begin{cases} n=3 \\ m=2 \end{cases}$ Er(s)-n : $(-\infty,0)$



5) K_c-ren balioa:

$$G(s)\cdot H(s) = \frac{K\cancel{(s+1)}\cancel{(s+2)}}{s\cancel{(s+1)}\cancel{(s+2)}} = \frac{K}{s} \quad \text{non } K=0{,}5K_c$$

Ek. Karak:

$$1+G(s)H(s)=0 \rightarrow 1+\frac{K}{s}=0$$

$$s+K=0 \rightarrow s=-K \Rightarrow K=2 \Rightarrow \boxed{K_c \geq 4}$$

$$G(s)H(s) = \frac{2(s+1)(s+2)}{s(s+1)(s+2)} = \frac{4(s+1)(s+2)}{s}$$

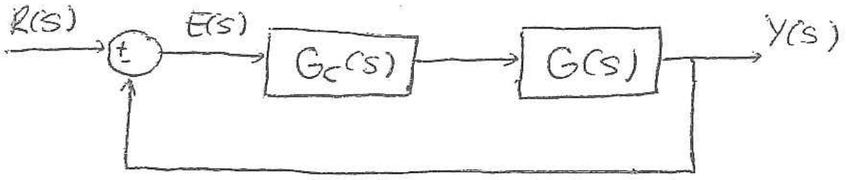
→PID

$$u(t) = K_c\left(1+\frac{1}{T_i s}+T_d s\right) = \frac{K_c(T_i s+1+T_dT_is^2)}{T_i s} = \frac{K_c\left(T_d\left(s^2+\frac{1}{T_d}s+\frac{1}{T_dT_i}\right)\right)}{s}$$

$$\frac{4(s^2+3s+2)}{s} = \frac{K_c\left(T_d s^2+\frac{1}{T_d}s+\frac{1}{T_dT_i}\right)}{s}$$

$$\begin{cases} 1/T_d = 3 \Rightarrow \boxed{T_d = 1/3} \\ \frac{1}{T_dT_i} = 2 \Rightarrow \frac{3}{T_i} = 2 \rightarrow \boxed{T_i = \frac{3}{2}} \end{cases}$$

⑳ (3. Adibidea - 26. orap - 6 II) $G(s) = \frac{0{,}5}{(s+1)(s+5)}$

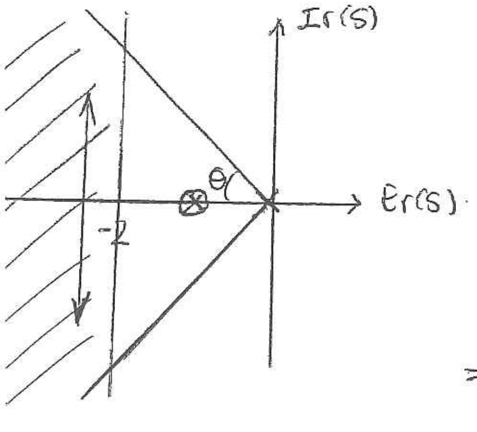


1) Eskakizunaren eremua

$M_p \leq 0{,}43 \rightarrow \delta \geq 0{,}707$

$t_s \leq 2s \rightarrow \delta\omega_n \geq 2$

$M_p \leq 4{,}3$ (%) ; $e_{ss}=0$; $t_s(\%2) \leq 2s$



2) I?

$e_{ss}=0$ denez → 1 motako sistema lortu — I beteheko ⇒ PI

3) ET egin PI kontrolagailuan:

$$G_{PI}(s) = \frac{K_c(s+z)}{s} \quad \text{non } z=\frac{1}{T_i}$$

$$\Rightarrow G_{BA}(s) = \frac{0{,}5K_c(s+1/T_i)}{s(s+1)(s+5)}$$ poloak: $s=0$, $s=-1$, $s=-5$

kontrolagailuaren zeroarekin polo dominatzailea baliogabetuko dugu:

$$z=\frac{1}{T_i}=1 \rightarrow T_i=1$$

Intersekzioa dagoela ikus daiteke ⇒ PI kontrolagailua erabil daiteke

4) K_c-ren kalkula.

$$G_{BA}(s) = \frac{0{,}5\,K_c\,\cancel{(s+1)}}{s\cancel{(s+1)}(s+5)} = \frac{0{,}5\,K_c}{s^2+5s}$$

$$G_{BC}(s) = \frac{K_c}{s^2+5s+0{,}5K_c}$$ → Ek. karakteristikoa: $s^2+0{,}5K_c+5s=0$

$$s^2+5s+0{,}5K_c = s^2+2\delta\omega_n s+\cancel{0{,}5}\,\omega_n^2 \left\{ \begin{array}{l} 5=2\delta\omega_n \\ 0{,}5K_c=\omega_n^2 \\ \delta<1 \Rightarrow \text{kritikoki egonkorra.} \end{array} \right.$$

Polo konplexu konjokatuak suposatu ditugunez $0<\delta<1$, K_c-ren balio kritikoak lortzeko: $0{,}707<\delta<1$

→ K_c minimoa lortzeko

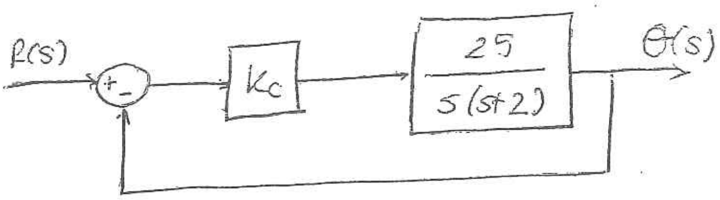
↳ K_c maximoa lortzeko

$$\left\{ \begin{array}{l} 5=2\omega_n \rightarrow \omega_n \geqslant 5/2 \text{ rad/s} \\ 0{,}5K_c=\dfrac{25}{4} \rightarrow K_c \geqslant 12{,}5 \end{array} \right.$$

$$\delta>0{,}707 \rightarrow \left\{ \begin{array}{l} 5=\dfrac{\sqrt{2}}{\cancel{2}}\cdot\cancel{2}\,\omega_n \rightarrow \omega_n \leqslant 2{,}82 \text{ rad/s} \\ 0{,}5K_c=\dfrac{25}{2} \rightarrow K_c \leqslant 16 \end{array} \right.$$

$$G_{PI}(s) = \frac{K_c(s+1/T_i)}{s} \Rightarrow \left\{ \begin{array}{l} T_i=1 \\ 12{,}5 \leqslant K_c \leqslant 16 \end{array} \right.$$

Ariketa (39. diapositiba)



$\delta = 0,7$

$e_{ss} = 0,1$ (arrapala unitarioa)

Motorraren TF: $G(s) = \frac{25}{s(s+2)}$

a) Posible al da eskakizunak betetzea kontrol proportzionalarekin?

$e_{ss} = \frac{1}{K_v}$ non $K_v = \lim_{s\to 0} s\, H(s)\cdot G(s) = \lim_{s\to 0} \cancel{s} \frac{25K_c}{\cancel{s}(s+2)} = 12,.\ K_c$

$0,1 = \frac{1}{12,5 K_c} \Rightarrow \underline{K_c = 0,8}$

Begizta itxiko TF: $\frac{\theta(s)}{R(s)} = \frac{\frac{25K_c}{s(s+2)}}{1+\frac{25K_c}{s(s+2)}} = \frac{25K_c}{s^2+2s+25K_c}$

Ek. karakteristikoa: $s^2+2s+25K_c = 0$

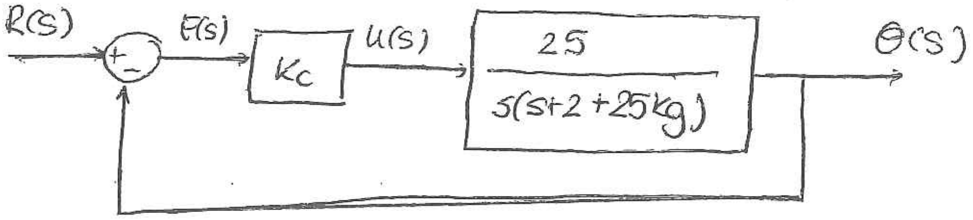
$s^2+2\delta\omega_n s+\omega_n^2 = 0 \quad (0<\delta<1)$

$$\begin{cases} 2 = 2\delta\omega_n \rightarrow 1 = \delta\omega_n \rightarrow \frac{1}{0,7} = \omega_n \longrightarrow \omega_n = 1,414 \text{ rad/s} \\ 25K_c = \omega_n^2 \longrightarrow K_c = \frac{1,414^2}{25} = 0,08 \longrightarrow e_{ss} > 0,1 \Rightarrow \text{Ez dira eskakizunak betetzen.} \end{cases}$$

b) Eta, abiaduraren berrelikadura gehitzen bada?

Abiaduraren berrelikadura:

Barruko begizta $\Rightarrow \frac{\theta(s)}{U(s)} = \frac{\frac{25}{s(s+2)}}{1+\frac{25K_g s}{s(s+2)}} = \frac{25}{s(s+2+25K_g)}$



Begizta itxiko TF: $\frac{\theta(s)}{R(s)} = \frac{25K_c}{s^2+(2+25K_g)s+25K_c} \Rightarrow \begin{cases} 25K_c = \omega_n^2 \ (1) \\ 2+25K_g = 2\delta\omega_n \ (2) \\ \delta = 0,7 \ (3) \end{cases}$

$K_v = \lim_{s\to 0} \cancel{s}\cdot \frac{25K_c}{\cancel{s}(s+2+25K_g)} = \frac{25K_c}{2+25K_g} \Rightarrow e_{ss} = \frac{1}{K_v} = \frac{2+25K_g}{25K_c} = 0,1 \quad (4)$

$2+25k_g = 2 \cdot 0{,}7\, \omega_n$

$\omega_n = \sqrt{25K_c} = 5\sqrt{K_c}$

$2+25k_g = 0{,}1 \cdot 25K_c$

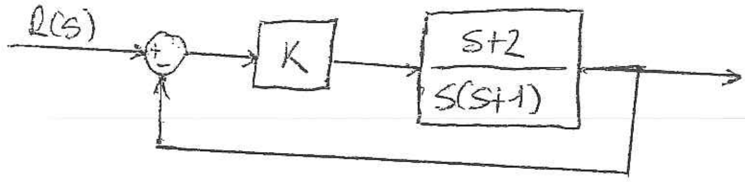
$$\Rightarrow \left.\begin{aligned} 2+25k_g &= 1{,}4 \cdot 5\sqrt{K_c} \\ 2+25k_g &= 0{,}1 \cdot 25k_c \end{aligned}\right\} \boxed{\begin{aligned} K_c &= 7{,}8 \\ k_g &= 0{,}7 \end{aligned}}$$

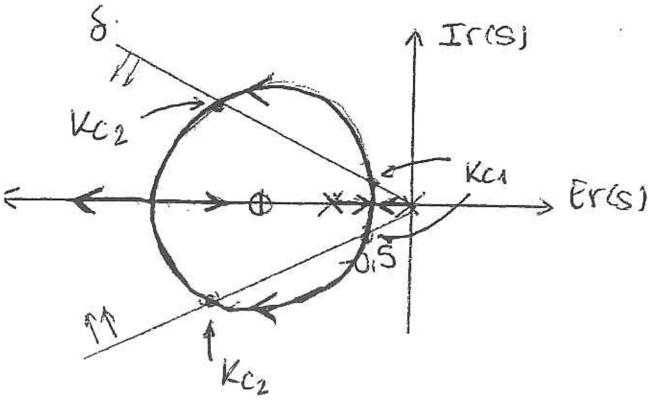
Ariketa (44. diapositiboa)

M_p ahal den txikiena: %1

$M_p \leq \%1 \longrightarrow M_p \leq 0{,}01 = e^{\frac{\pi\delta}{\sqrt{1-\delta^2}}}$

$\delta \geq 0{,}826$





$G(s)H(s) = \frac{K_c(s+2)}{s(s+1)}$

$n=2 \longrightarrow$ Adar bi

$m=1 \longrightarrow$ Adar bat zerora beste infinitura

ET ardatz Er-n:

$\theta = \frac{(2k+1)\pi}{n-m} = \pm\pi$

$\boxed{0 < K < K_{c_1}}$; $\boxed{K_c \geq K_{c_2}}$

1) EK. karakteristikoa: $1 + G_c(s)G_p(s) = s^3 + 3s^2 + 3s + 1 + \frac{K_c}{8}$

R-H taula:

s^3	1	3
s^2	3	$1+\frac{K_c}{8}$
s	b_1	0
s^0	c_1	

$b_1 = \frac{9 - 1 - \frac{K_c}{8}}{3} = \frac{8}{3} - \frac{K_c}{24}$

$c_1 = 1 + \frac{K_c}{8}$

BB: $1 + \frac{K_c}{8} > 0 \rightarrow K_c > -8 \rightarrow K_c > 0$

BN: $\frac{8}{3} - \frac{K_c}{24} > 0 \rightarrow K_c < 64$

$\boxed{0 < K_c < 64}$

Kritikoki egonkorra: $\boxed{K_c = 64 = K_u}$

s^3	1	3
s^2	3	9
s	0	0
s^0	9	

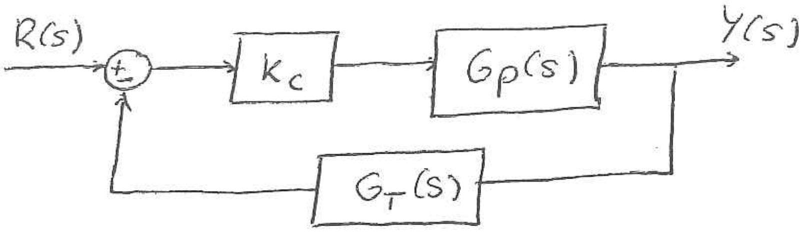
$\longrightarrow P(s) = 3s^2 + 9 \longrightarrow s = \pm\sqrt{3}j = \pm 1{,}73j$

$\omega_u = 1{,}73$ rad/s

$\boxed{T_u = \frac{2\pi}{\omega_u} = \frac{2\pi}{\sqrt{3}} = 3{,}6}$

② $G_p(s) = \frac{2,5}{1+2s}$; $G_T(s) = \frac{k_T}{1+Z_T s}$; $k_c = 2$; $k_v = 1$

a) Bloke diagrama.



b)

$$G_{BC}(s) = \frac{2\cdot\frac{2,5}{1+2s}}{1+\frac{5}{1+2s}\cdot\frac{k_T}{1+Z_T s}} = \frac{5(1+Z_T s)}{\underbrace{(1+2s)(1+Z\cdot s)}_{1+2s+Z_T s+2Z_T s^2}+5k_T} \overset{Z_T=1}{=} \frac{5(1+s)}{2s^2+3s+(1+5k_T}$$

Ek. Karakteristikoa: $2s^2+3s+(1+5k_T)=0 \equiv s^2+2\delta\omega_n s+\omega_n^2$

$$\begin{cases} 3 = 2\delta\omega_n \\ 1+k_T = \omega_n^2 \end{cases}$$

$$s_{1,2} = \frac{-3\pm\sqrt{9-8(1+5k_T)}}{4}$$

↳ $\sqrt{9-8(1+5k_T)} \geq 0$ (polo errealak)

$9 \geq 8+40k_T$

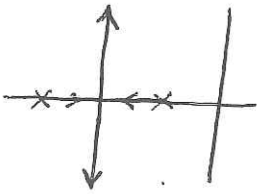
$1 \geq 40k_T \Longrightarrow \boxed{0,025 \geq k_T}$

↳ $\sqrt{9-8(1+5k_T)} \rightsquigarrow 9-8(1+5k_T) < 0$ (polo konplexu konjokatuak)

$\boxed{k_T > 0,025}$

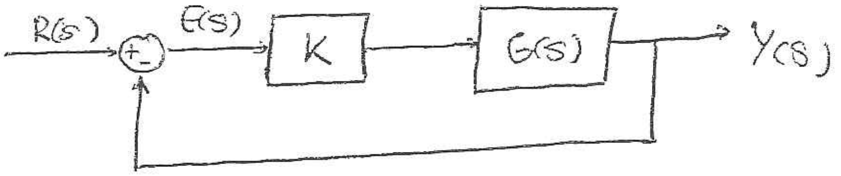
k_T-ren balio kritikoa : $k_T = 0,025$

- $k_T > 0,025 \rightarrow k_T\uparrow \rightarrow \omega_n\uparrow \rightarrow \delta\downarrow \rightarrow \delta\omega_n = kte$
(sistema kritikoki egonkorra)



- $k_T < 0,025 \rightarrow k_T\downarrow \rightarrow \omega_n\downarrow \rightarrow \delta\uparrow$

③



$$G(s)=\frac{(s+1)(s+3)}{s(s+2)(s+4)}$$

↳ integradorea

a)

$$G_{BC}(s)=\frac{K(s+1)(s+3)}{\underbrace{s(s+2)(s+4)}_{s^2+6s+8}+K\underbrace{(s+1)(s+3)}_{s^2+4s+3}}=\frac{k(s+1)(s+3)}{s^3+6s^2+8s+ks^2+K4s+3K}=$$

$$=\frac{k(s+1)(s+3)}{s^3+(6+k)s^2+(8+4k)s+3K}$$ → sistema egonkorra

Sistema mota ikusteko sistema irekian egingo dugu. Kasu honetan integradore bat daukagu: 1. MOTA

b) Ebazpena: $G_{BC}(0)=\frac{3K}{3K}=1$

c)

maila + 1. MOTA : $e_{ss}=0$ → $\boxed{e_{ss_{maila}}=0}$

arapala + 1. MOTA: $e_{ss}=\frac{1}{K_v}$

$$K_v=\lim_{s\to 0} s\cdot\underbrace{G(s)H(s)}_{G_{BA}(s)}=\frac{3K}{8}\rightarrow\boxed{e_{ss_{arapala}}=\frac{8}{3K}}$$

④ $G(s)=\frac{1}{(s+2)(s+10)}$

a) • $k=7$

$$G_{BC}(s)=\frac{7}{(s+2)(s+10)+7}=\frac{7}{s^2+12s+27}=\frac{Y(s)}{R(s)}$$ non $R(s)=\frac{1}{s}$

$$Y(s)=\frac{1}{s}\cdot\frac{7}{\underbrace{s^2+12s+27}_{(s+3)(s+9)}}=\frac{A}{s}+\frac{B}{s+3}+\frac{C}{s+9}=\frac{A(s+3)(s+9)+Bs(s+9)+Cs(}{s(s+3)(s+9)}$$

•) $7=A\cdot 27\rightarrow A=0{,}26$

s) $0=12A+9B+3C$ } $B=-0{,}39$

s^2) $0=A+B+C$ } $C=0{,}13$

$$Y(s)=\frac{0{,}26}{s}\cdot\frac{0{,}39}{s+3}+\frac{0{,}13}{s+9}$$

$\mathcal{L}^{-1}$

$$\boxed{y(t)=0{,}26-0{,}39e^{-3t}+0{,}13e^{-9t}}$$

• $K=20$

$$G_{BC}(s) = \frac{20}{s^2+12s+40} = \frac{Y(s)}{R(s)} \rightarrow Y(s) = \frac{1}{s}\cdot\frac{20}{s^2+12s+40} = \frac{A}{s} + \frac{Bs+C}{s^2+12s+40}$$

$$= \frac{A(s^2+12s+40)+(Bs+C)s}{s(s^2+12s+40)}$$

$\bullet)\ 20 = A\cdot 40 \rightarrow A = 0{,}5$

$s)\ 0 = 6 + C \rightarrow C = -6 \Rightarrow Y(s) = \frac{0{,}5}{s} \cdot \frac{0{,}5s+6}{s^2+12s+40} =$

$s^2)\ 0 = 0{,}5 + B \rightarrow B = -0{,}5$

$$= \frac{0{,}5}{s} - \frac{0{,}5s+6}{(s+6)^2+4}$$

$$\boxed{y(t) = 0{,}5 - 0{,}5\cos(2t)e^{-6t} - 3\,\text{sen}(2t)\,e^{-6t}}$$

b)

• $k=7$

$$e_{ss} = \frac{1}{1+k_p} \quad \text{non} \quad k_p = \lim_{s\to 0} s\cdot\overbrace{G(s)H(s)}^{G_{BA}(s)} = \lim_{s\to 0}\frac{7}{(s+2)(s+10)} = \frac{7}{20} = 0{,}35$$

$$\boxed{e_{ss} = \frac{1}{1+0{,}35} = 0{,}74}$$

• $k=20$

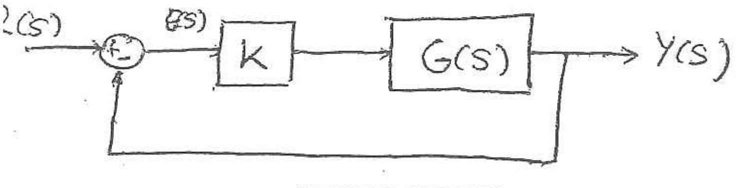
$$k_p = \lim_{s\to 0}\frac{20}{(s+2)(s+10)} = 1 \Rightarrow \boxed{e_{ss} = 0{,}5}$$

c)

$K\uparrow \Rightarrow \omega_n\uparrow \rightarrow t_s\downarrow$

$\Rightarrow \begin{cases} t_{s_7} > t_{s_{20}} \\ k=7 \text{ bada sistema motelagoa da} \end{cases}$

⑥ $G(s) = \frac{s+1}{s(s+3)}$



a) $G_{BC}(s) = \frac{(s+1)K}{s(s+3)+k(s+1)} =$

$$= \frac{k(s+1)}{s^2+(3+k)s+K}$$

$$s_{1,2} = \frac{-(3+k) \pm \sqrt{(3+k)^2-4k}}{2} = \frac{-3-k\pm\sqrt{9+k^2+2k}}{2}$$

Polo dominatzailearen $Z = 2s$ $(p_1) \Rightarrow p_1 = -\frac{1}{2} = -0{,}5$

$$-0{,}5 = \frac{-3-k \quad \sqrt{9+k^2+2k}}{2} \longrightarrow \underbrace{-1+3+k}_{2+k} = \sqrt{9+k^2+2k}$$

$$4+\cancel{k^2}+4k = 9+\cancel{k^2}+2k$$

$$2k = 5 \rightarrow \boxed{k = 2{,}5}$$

$$\boxed{P_2 = \frac{-3-2{,}5 \pm \sqrt{9+6{,}25+5}}{2} = -5}$$

b)

Maila unitario + 1. MOTA: $\boxed{e_{ss} = 0}$

Arrapala unitario + 1 MOTA: $e_{ss} = \frac{1}{K_v} = \frac{1}{\lim \cancel{s} \cdot \frac{K(s+1)}{\cancel{s}(s+3)}} = \frac{1}{\frac{K}{3}} = \frac{6}{5} = 1{,}2$

c)

$$Y(s) = \frac{2{,}5(s+1)}{\underbrace{s^2+5{,}5s+2{,}5}_{(s+5)(s+0{,}5)}} \cdot \frac{1}{s} = \frac{A}{s} + \frac{B}{s+5} + \frac{C}{s+0{,}5}$$

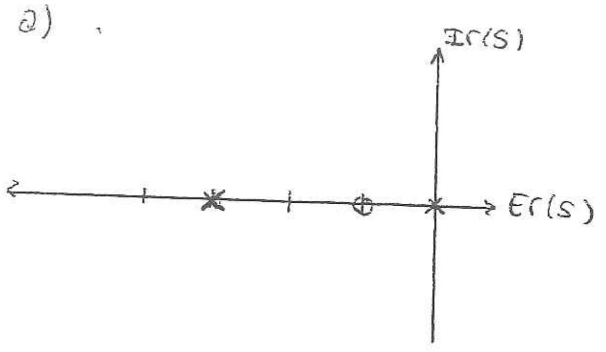
$$\Rightarrow \begin{cases} A = 1 \\ B = -0{,}44 \\ C = -0{,}55 \end{cases} \qquad Y(s) = \frac{-0{,}44}{(s+5)} - \frac{0{,}55}{(s+0{,}5)} + \frac{1}{s}$$

$$\boxed{y(t) = 1 - 0{,}44e^{-5t} - 0{,}55e^{-0{,}5t}}$$

⑥ $G(s)=\dfrac{s+1}{s(s+3)} \longrightarrow$ 1. MOTA

$\llcorner$ integradores

a)



$$G_{BC}(s)=\frac{K(s+1)}{s^2+3s+Ks+K}=\frac{K(s+1)}{s^2+(3+K)s+K}$$

Ek. karakteristikoa:

$$s^2+(3+K)s+K=0$$

$$s_{1,2}=\frac{-3-K\pm\sqrt{9+K^2+6K-4K}}{2}=\frac{-3-K\pm\sqrt{9+K^2+2K}}{2}$$

τ ; polo dominahaikearena : 2

$$G(s)=\frac{K}{\tau s+1}\Rightarrow s_1=-\frac{1}{\tau}=-0{,}5\Rightarrow -0{,}5=\frac{-3-K\pm\sqrt{9+K^2+2K}}{2}$$

$$\boxed{s_2=\frac{-3-2{,}5-\sqrt{9+(2{,}5)^2+2\cdot2{,}5}}{2}=-5}$$

$$-1=-3-K\pm\sqrt{9+K^2+2K}$$

$$4+\cancel{K^2}+4K=+(9+\cancel{K^2}+2K)$$

$$2K=5\Rightarrow\boxed{K=2{,}5}$$

b)

$$G_{BC}(s)=\frac{2{,}5(s+1)}{s^2+5{,}5s+2{,}5}\quad;\quad G_{BA}(s)=\frac{2{,}5(s+1)}{s(s+3)}$$

1. MOTA < (espaloi) $\boxed{e_{ss}=0}$; $e_{ss}=\frac{1}{K_v}$ (arapala)

$$\boxed{e_{ss}=\frac{1}{\lim\limits_{s\to0}\cancel{s}\cdot\frac{2{,}5(s+1)}{\cancel{s}(s+3)}}=\frac{1}{\frac{2{,}5}{3}}=\frac{6}{5}=1{,}2}$$

c)

$$Y(s)=R(s)\cdot G_{BC}(s)\Longrightarrow Y(s)=\frac{1}{s}\cdot\frac{2{,}5(s+1)}{s^2+5{,}5s+2{,}5}=\frac{1}{s}\frac{2{,}5(s+1)}{(s+0{,}5)(s+5}$$

$\llcorner$ maila unitarioa: $\frac{1}{s}$

$$\frac{1}{s}\cdot\frac{2{,}5(s+1)}{(s+0{,}5)(s+5)}=\frac{A}{s}+\frac{B}{s+0{,}5}+\frac{C}{s+5}$$

$$\begin{cases}A=1\\2{,}5=5{,}5+5B+0{,}5C\\0=1+B+C\end{cases}\Rightarrow\begin{matrix}B=-0{,}55\\C=-0{,}44\end{matrix}$$

$$Y(s)=\frac{1}{s}+\frac{-0{,}55}{s+0{,}5}-\frac{0{,}44}{s+5}$$

$\downarrow \mathcal{L}^{-1}$

$$\boxed{y(t)=1-0{,}55\,e^{-0{,}5t}-0{,}44\,e^{-5t}}$$

⑦ $G(s) = \frac{1}{s-2}$

a) egonkora da?

Ez da sistema egonkora $s=2$ poloa zati erreälean dagoelako.

b)

$$G_{BC}(s) = \frac{K}{s-2+K} \Rightarrow K-2>0 \text{ bada egonkora da } \boxed{K>2}$$

c) $\tau = 0{,}1s$

$$G(s) = \frac{K}{\tau s+1} \Rightarrow s = -\frac{1}{0{,}1} = -10 \Rightarrow -10-2+K=0 \quad \boxed{K=12}$$

edo

$$G_{BC}(s) = \frac{\frac{K}{K-2}}{\underbrace{\frac{s}{K-2}}_{\tau s}+1} \Rightarrow 0{,}1 = \frac{1}{K-2} \rightarrow \boxed{K=12}$$

d)

$$e_{ss} = \frac{1}{1+K_p} \quad \text{non} \quad \left[K_p = \lim_{s\to 0} \underbrace{G(s)\cdot \cancel{H(s)}^{1}}_{G_{BA}(s)} = \lim_{s\to 0} \frac{12}{s\cdot} = \frac{12}{-2} = -6\right]$$

⑧ $G(s) = \frac{s+2}{(s-2)(s+4)}$

a) sistema hau ez da egonkora $s=2$ poloa zati erreälean dagoelako → Berrelikadura behar du.

b)

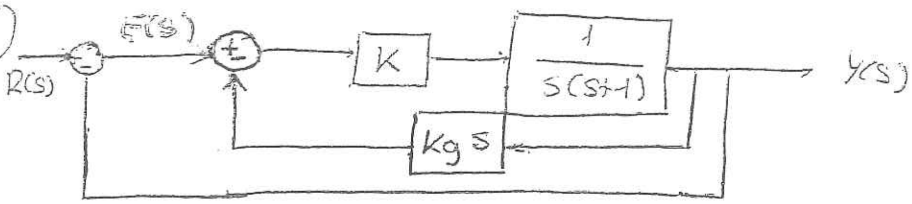
$$G_{BC}(s) = \frac{K(s+2)}{s^2+2s-8+Ks+2K} = \frac{K(s+2)}{s^2+(2+K)s+(2K-8)}$$

Ek. karakteristikoa: $s^2+(2+K)s+(2K-8)=0$

$$s_{1,2} = \frac{-(2+K)\pm\sqrt{4+K^2+4K-8K+32}}{2} = \frac{-(2+K)\pm\sqrt{K^2+\ldots}}{2}$$

$$s_1 = -\frac{1}{\tau} = -1 \Rightarrow -1 = \frac{-2-K\pm\sqrt{K^2+36+(-4K)}}{2} \rightarrow \cancel{K^2} = \cancel{K^2}+36-4K \quad \boxed{K=9}$$

⑨



a) $\delta=0{,}5 \rightarrow k?$

$$G_{BC}(s) = \frac{K}{s^2+s+K} = \frac{k\omega_n}{s^2+2\delta\omega_n s+\omega_n^2} \quad \left\{ \begin{array}{l} 2\delta\omega_n = 1 \Rightarrow \omega_n = 1 \\ \omega_n^2 = K \rightarrow \boxed{K=1} \\ \delta = 0{,}5 \end{array} \right.$$

b) ess (arapala) (sistema 1. MOTAKOA)

$$\left[e_{ss} = \frac{1}{K_v} = \frac{1}{\lim\limits_{s\to 0} \cancel{s} \cdot \frac{K}{\cancel{s}(s+1)}} = \frac{1}{1} = 1 \right]$$

c) $e_{ss}=0{,}1$

$$e_{ss} = 0{,}1 = \frac{1}{K_v} = \frac{1}{K} \Rightarrow \boxed{K=10}$$

$$K = \omega_n^2 \Rightarrow \quad 10 = \omega_n^2 \rightarrow \omega_n = \sqrt{10} \rightarrow \quad 2\delta\omega_n = 1$$

$$2\delta\sqrt{10} = 1$$

$$\boxed{\delta = \frac{1}{2\sqrt{10}} = 0{,}158}$$

d) $K_g?$ $\delta=0{,}5$ $(K=10)$

$$G_{BA}(s) = \frac{\frac{K}{s(s+1)}}{1+K_g \cdot s \cdot \frac{K}{s(s+1)}} = \frac{K}{s^2+s+K_g \cdot K s} = \frac{K}{s^2+(K_g \cdot K+1)s}$$

$$G_{BC}(s) = \frac{K^{=10}}{s^2+(10K_g+1)s+10} \quad \left\{ \begin{array}{l} 10K_g+1 = 2\delta\omega_n \\ \omega_n^2 = 10 \\ \quad \hookrightarrow \omega_n = \sqrt{10} \end{array} \right. \quad \Rightarrow \quad \begin{array}{l} 10K_g = 2\frac{1}{2}\cdot\sqrt{10}-1 \\ K_g = \frac{\sqrt{10}-1}{10} \end{array}$$

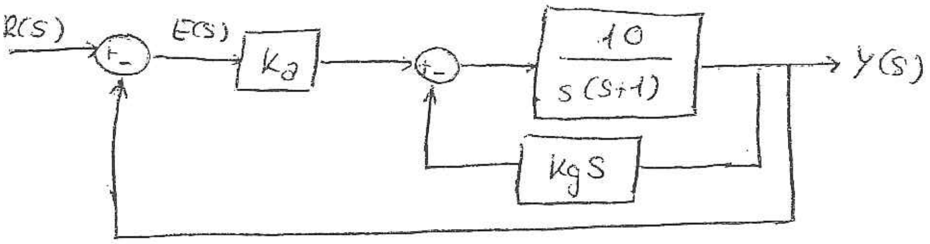
$$\boxed{K_g = 0{,}216}$$

e)

$$e_{ss} = \frac{1}{k_v} \Rightarrow K_v = \lim_{s\to 0} \cancel{s} \cdot \frac{K}{\cancel{s}(s+10 \cdot 0{,}216+1)} = \frac{10}{10\cdot 0{,}216+1} = 3{,}16$$

$$e_{ss} = \frac{1}{3{,}16} = 0{,}316$$

⑩



$K=10$

a) $\delta=0{,}5$; $e_{ss}=0{,}1$ (arrapala) Posiblea da sistema 1.MOTA-koa delako

b) K_a?

$$G_{BC}(s)=\frac{10K_a}{s^2+s(1+10K_g)+10K_a} \quad ; \quad G_{BA}(s)=\frac{10K_a}{s^2+(10K_g+1)s}$$

- $\delta=0{,}5$

$$\left.\begin{aligned}(1+10K_g)&=2\cdot0{,}5\,\omega_n\\ \omega_n&=\sqrt{10K_a}\end{aligned}\right\}\rightarrow 1+10K_g=\sqrt{10K_a}$$

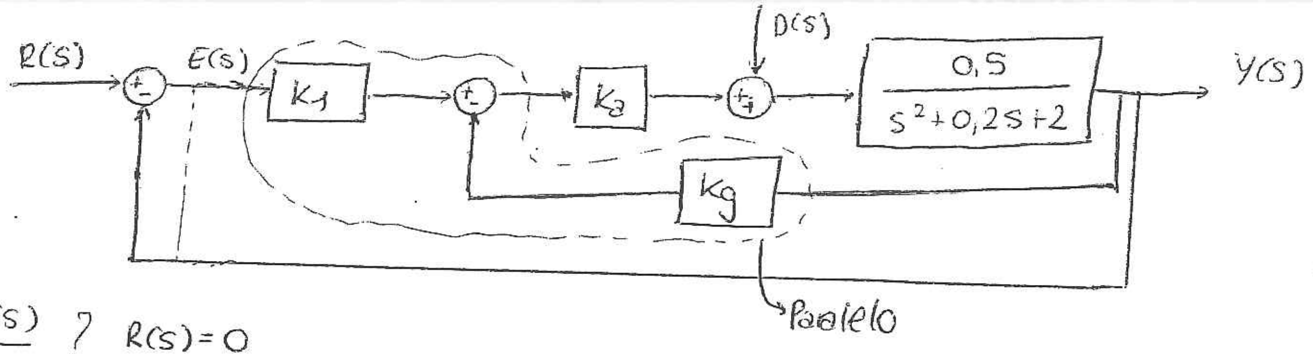
- $e_{ss}=0{,}1$

$$0{,}1=\frac{1}{K_v}\Rightarrow K_v=10=\lim_{s\to0}\cancel{s}\,\frac{10K_a}{\cancel{s}(s+10K_g+1)}=\frac{10K_a}{10K_g+1}\rightarrow 10K_g+1=K_a$$

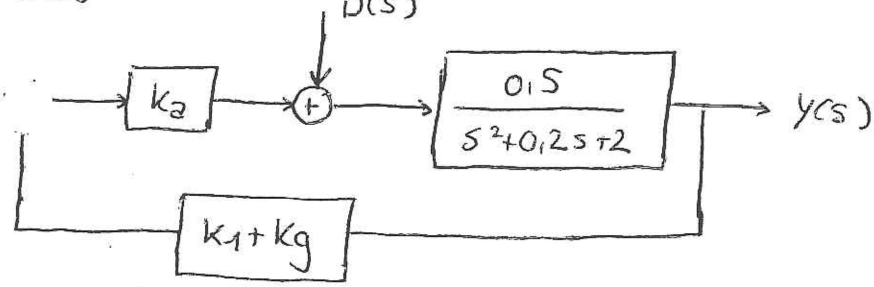
$$\rightarrow K_a=\sqrt{10K_a}$$

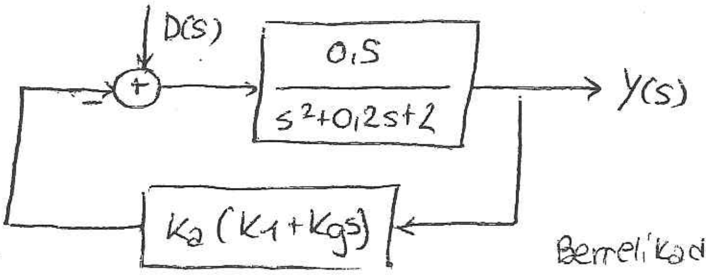
$$K_a^2=10\cancel{K_a}\rightarrow\boxed{K_a=10}$$

⑪



a) $\frac{Y(s)}{D(s)}$? $R(s)=0$





Berrelikadura

$$\Rightarrow G_{BC}=\frac{Y(s)}{D(s)}=\frac{0{,}5}{s^2+0{,}2s+2+0{,}5K_a(K_1}$$

b) $D(s) = \frac{1}{s}$; $\delta = 0{,}5$; $ss : 0{,}1$; K_a ? K_1 ? K_g ?

$$Y(s) = \frac{1}{s} \cdot \frac{0{,}5}{s^2 + 0{,}2s + 0{,}5K_a K_g s + (2 + 0{,}5K_1 K_a)}$$

$$\left.\begin{array}{l} 0{,}2 + 0{,}5K_a K_g = 2 \cdot \frac{1}{2} \cdot \omega_n \\ \omega_n = \sqrt{2 + 0{,}5K_1 K_a} \end{array}\right\} \quad 0{,}25K_a^2 K_g^2 + 0{,}2K_a K_g - 0{,}5K_a K_1 = 1{,}96$$

$$Y_{ss} < 0{,}1 \Rightarrow K \cdot u < 0{,}1 \rightarrow K < 0{,}1$$

$$K = \frac{0{,}5}{2 + 0{,}5K_1 K_a} < 0{,}1 \Rightarrow \boxed{K_1 K_a > 6}$$

$$\hookrightarrow 0{,}25K_a^2 K_g^2 + 0{,}2K_a K_g - 0{,}5 \cdot 6 = 1{,}96$$

$$K_a K_g (0{,}25K_a K_g + 0{,}2) = 4{,}96$$

$$K_a K_g = \frac{-0{,}2 \pm \sqrt{0{,}2^2 + 4 \cdot 0{,}25 \cdot 4{,}96}}{0{,}5} \left\langle \begin{array}{l} 4{,}07 \\ -4{,}87 \end{array} \right. \Rightarrow \boxed{K_a K_g > 4{,}07}$$

⑫ $G(s) = \frac{100}{s^2 + 8s + 100}$; $D = 20$ m $\Rightarrow 56 \frac{km}{h} \Rightarrow T_d$ 2 volto gehienez

$$G_c(s) = \frac{K_a}{0{,}2s + 1}$$

b) K_a ?

$$\theta(s) = R(s) \cdot \frac{G_c \cdot G}{1 + G_c \cdot G} + T_d(s) \frac{G}{1 + G_c \cdot G} = \frac{100K_a R(s) + 100(0{,}2s+1)T_d(s)}{(0{,}2s+1)(s^2+8s+100) + 100K_a}$$

Ek. karakteristikoa: $(0{,}2s+1)(s^2+8s+100) + 100K_a = 0$

$0{,}2s^3 + 2{,}6s^2 + 28s + 100 + 100K_a = 0$

B.B. $100 + 100K_a > 0 \rightarrow K_a > -1$

$$\begin{array}{c|cc} s^3 & 0{,}2 & 28 \\ s^2 & 2{,}6 & 100+100K_a \\ \hline s & b_1 & 0 \\ s^0 & c_1 & \end{array}$$

$$b_1 = \frac{28 \cdot 2{,}6 - 0{,}2 \cdot 100(1+K_a)}{2{,}6} = 28 - 7{,}692(1+K_a)$$

BN: $28 - 7{,}692(1+K_a) > 0$

$3{,}64 > 1 + K_a \rightarrow K_a < 2{,}64$

$$\boxed{-1 < K_a < 2{,}64}$$

a) $K_s = 0$

$T_d = 56 \frac{km}{h} \rightarrow u = 2$

$$\frac{\theta(s)}{T_d(s)} = G(s) = \frac{100}{s^2+8s+100} \quad \left\} \begin{array}{l} K\omega_n^2 = 100 \\ \omega_n^2 = 100 \end{array} \right. \Rightarrow K = 1$$

$$\left[\theta_{ss} = K \cdot u = 1 \cdot 2 = 2\right]$$

c) $e_{ss} < 2° \rightarrow K_a?$

$2° \text{ ——— } x$

$180° \text{ ——— } \pi \text{ rad}$

$x = 0{,}035 \text{ rad} > e_{ss}$

$$e_{ss} = \frac{1}{1+K_p} \quad \text{non} \quad K_p = \lim_{s\to 0} \frac{100(0{,}2s+1)}{(0{,}2s+1)(s^2+8s+100)+100K_a} = \frac{100}{100+100K_a}$$

⑬

maila sarrera , 2 anplitudekoa ⇒ $\Delta u = 2$

H(s) kaptadorea ⇒ maila unitario sarrera eman → $z(t) = (3-3e^{-t})$ (erantzuna)

errorea = $r(t) - z(t)$

$y_{ss} = 2{,}3 - 0{,}3 = 2 \rightarrow K = \frac{y_{ss}}{\Delta u} = 1$

$t_p = 3{,}6 \quad ; \quad y(t_p) = 2{,}3$

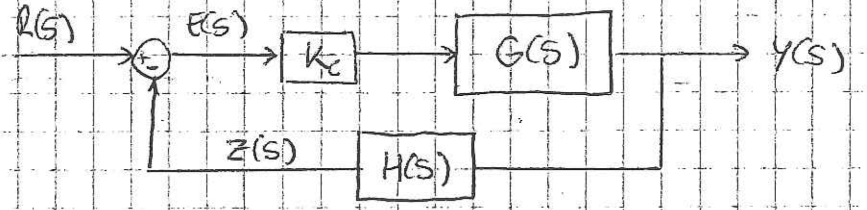
$$M_p = \frac{y(t_p) - y_{ss}}{y_{ss}} = \frac{2{,}3-2}{2{,}3} = 0{,}13$$

$$M_p = e^{-\pi\delta/\sqrt{1-\delta^2}} \Rightarrow \delta = 0{,}5446$$

$$t_p = \frac{\pi}{\omega_n\sqrt{1-\delta^2}} \Rightarrow \omega_n = \frac{\pi}{t_p\sqrt{1-\delta^2}} = 1{,}04 \text{ rad/s}$$

$$\left[G(s) = \frac{1{,}0816}{s^2+1{,}13s+1{,}0816}\right]$$

$$\left[R(s) = 1/s\right]$$



$$z(t) = 3 - 3e^{-t} \xrightarrow{\mathcal{L}} Z(s) = \frac{3}{s(s+1)} \Rightarrow \left[H(s) = \frac{Z(s)}{1/s} = \frac{3}{s+1}\right]$$

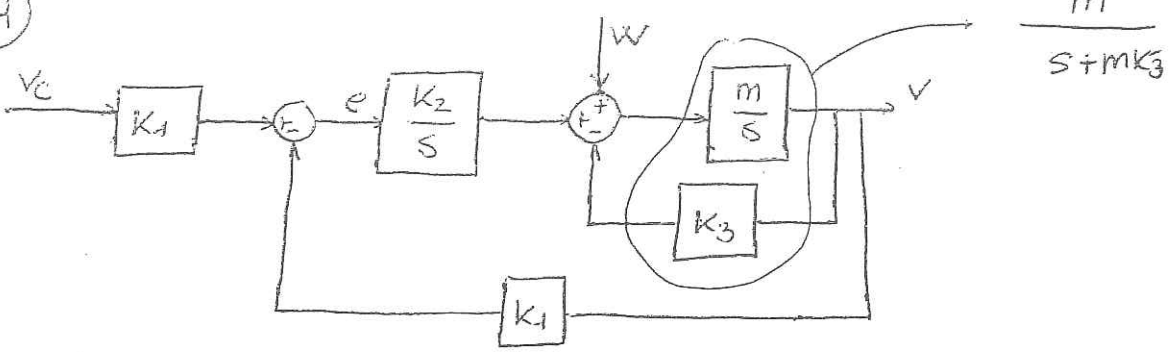
c) r(t)-n maila unitarioa eman

$$t_s(\%2) = \frac{\pi}{0{,}155} \Rightarrow \frac{4}{\delta\omega_n} = \frac{\pi}{0{,}155} \Rightarrow \delta\omega_n = 0{,}197$$

$$E(s) = \frac{1}{s} - \frac{3}{s(s+1)} = \frac{s-2}{s(s+1)}$$

Ek. karakteristikoa: $1+$

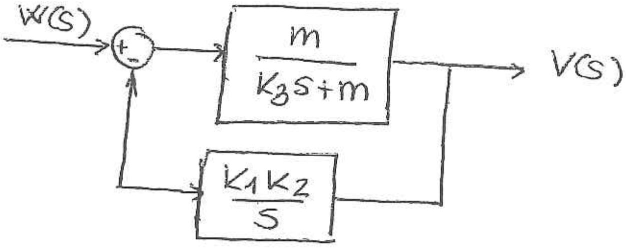
⑭



a) $\frac{V(s)}{V_c(s)}$? $(W(s)=0)$

$$\frac{V(s)}{V_c(s)} = \frac{\frac{K_2}{s}\cdot\frac{m}{s+K_3m}}{1+K_1\cdot\frac{K_2}{s}\cdot\frac{m}{s+K_3m}}\cdot K_1 = \frac{mK_1K_2}{s^2+mK_3s+mK_1K_2}$$

b) $\frac{V(s)}{W(s)}$? $(V_c(s)=0)$



$$\frac{V(s)}{W(s)} = \frac{\frac{m}{s+K_3m}}{1+\frac{K_1K_2}{s}\cdot\frac{m}{s+K_3m}} =$$

$$= \frac{ms}{s^2+K_3ms+mK_1K_2}$$

c) $V_c(s)=0$ $\rightarrow$ $W(s)=\frac{A}{s^2}$

W(s) arrapala

$$V_{ss}=\lim_{s\to 0} s\cdot V(s)$$

$$V(s)=\frac{ms}{s^2+K_3ms+mK_1K_2}\cdot W(s) = \frac{msA}{s^2(s^2+K_3ms+mK_1K_2)}$$

$$V_{ss}=\lim_{s\to 0}\cancel{s}\cdot\frac{m\cancel{s}A}{\cancel{s^2}(s^2+K_3ms+mK_1K_2)} = \frac{mA}{mK_1K_2} = \frac{A}{K_1K_2}$$

d)

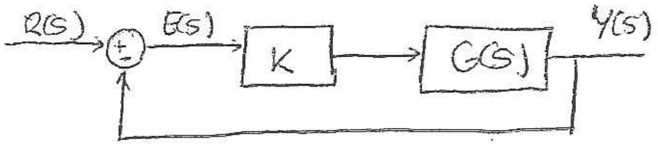
$$G_{BA}(s)=\frac{K_2\cdot m\cdot K_1}{s(s+mK_3)}$$ → Agindu sarrerarekiko 1. MOTAkoa.

$$K_v=\lim_{s\to 0}\cancel{s}\cdot\frac{K_1K_2m}{\cancel{s}(s+mK_3)} = \frac{K_1K_2\cancel{m}}{K_3\cancel{m}} \Rightarrow e_{ss}=\frac{1}{K_v}=\frac{K_3}{K_1K_2}$$

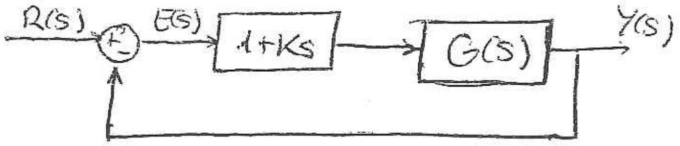
e) $G_{BA}(s)=\frac{m}{K_3s+m}$ ⇒ 0. MOTAKOA perturbazioarekiko $e_{ss}=1/2$ $(K_p=1)$

⑮ $G(s) = \frac{25}{s(s+2)}$ (BA) $\delta(G_{BC}) = 0,5$

Ⓐ P



Ⓑ PD



Ⓐ $G_{BA}(s) = \frac{25K}{s(s+2)}$; $G_{BC}^{A}(s) = \frac{25K}{s^2+2s+25K}$

$2 = \cancel{2}\delta\omega_n \Rightarrow \frac{1}{0,5} = \omega_n \Rightarrow \omega_n = 2 \text{ rad/s}$

$\omega_n^2 = 25K \rightarrow \underline{K = 0,16}$ $\left[t_r = \frac{\pi - \arctan\left(\frac{\sqrt{1-\delta^2}}{\delta}\right)}{\omega_n\sqrt{1-\delta^2}} = 1,21s\right]$

$\left[t_s(\%5) = \frac{3}{\delta\omega_n} = 3s\right]$ $\left[t_p = \frac{\pi}{\omega_n\sqrt{1-\delta^2}} = 1,81s\right]$

$\left[M_p = e^{-\pi\delta/\sqrt{1-\delta^2}} \cdot 100 = \%16\right]$

Ⓑ $G_{BA}(s) = \frac{25\overbrace{(1+Ks)}^{\rightarrow\ (s+1/k)}}{s(s+2)}$; $G_{BC}^{B}(s) = \frac{25(1+Ks)}{s^2+(2+25K)s+25}$

$\omega_n^2 = 25 \longrightarrow \omega_n = 5 \text{ rad/s}$

$2\delta\omega_n = 2+25k \rightarrow \cancel{2}\cdot\frac{1}{\cancel{2}}\cdot 5 = 2+25k \rightarrow \frac{3}{25} = K = 0,12$

$t_r = \frac{\pi - \arctan\left(\frac{\sqrt{1-\delta^2}}{\delta}\right)}{5\sqrt{1-\delta^2}} = \frac{\frac{2}{3}\pi}{5\frac{\sqrt{3}}{2}} = 0,48\,s???$ $t_r = 0,39s$

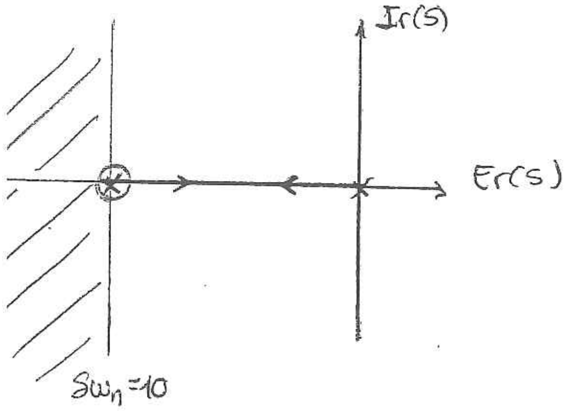
$t_p = 0,577s$

$\left[t_s(\%5) = \frac{3}{\delta\omega_n} = \frac{3}{0,5\cdot 5} = 1,2s\right]$

16) (2013/01/23)

- $G_1(s) = \dfrac{1}{\underbrace{s}_{\text{integradorea}}(s+10)}$ $\left\{ \begin{array}{l} e_{ss}=0 \text{ espalci unitario sarreran} \\ t_s(\%2) \leq 4/10 \text{ s} \end{array} \right.$ $\Rightarrow$ P kontrolagailua ez da nahikoa, beraz PD, PI edo PID bat bilatu behar dugu.

$$t_s = \frac{4}{\delta\omega_n} \leq \frac{4}{10} s \Rightarrow \delta\omega_n \geq 10$$



Jadanik integradore bat daukagu. Iraunkorreko posizio-errorea nulua daleko, beraz ez dugu I akzio bat sartu behar.

PD batekin frogatuko dugu, intersekziorik ez dagoelako.

zero bat sartu, $s = -\dfrac{1}{T_d} = -10$

$\hookrightarrow \boxed{T_d = 0,1}$

$$G_{PD}(s) = K_c\left(s + \frac{1}{T_d}\right) = K_c(s+10)$$

$$G_{BA}(s) = K_c\,\cancel{(s+10)}\,\frac{1 \cdot T_d}{s\cancel{(s+10)}} = \frac{K_c \cdot 0{,}1}{s} = \frac{K_c}{10s}$$

$\boxed{\text{PD KONTROLA}}$

$$G_{BC}(s) = \frac{K_c}{10s + K_c} = \frac{K_c}{1 + \underbrace{\frac{10}{K_c}}_{\tau} s}$$

$$t_s(\%2) = 4\tau \leq \frac{4}{10}$$

$$4 \cdot \frac{10}{K_c} \leq \frac{4}{10} \Rightarrow \boxed{K_c \geq 100}$$

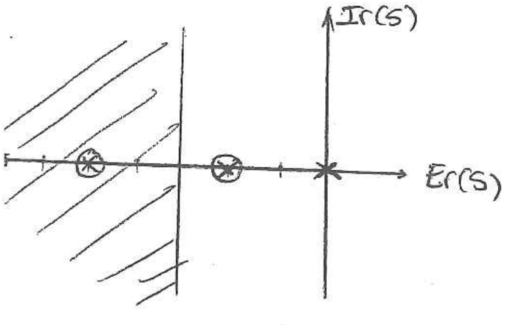
- $G_2(s) = \dfrac{(s+2)(s+5)}{4\underbrace{s}_{\text{integradorea}}(s^2+7s+10)}$ $\rightarrow$ Poloak: $s=0$; $s=-2$; $s=-5$ $n=3$

Zeroak: $s=-2$; $s=-5$

$$G_2(s) = \frac{\cancel{(s+2)}\cancel{(s+5)}}{4s\cancel{(s+2)}\cancel{(s+5)}} = \frac{1}{4s}$$

$e_{ss}=0$ espalci unitario-sarreran

$t_s(\%5) = 1s \rightarrow \dfrac{3}{\delta\omega_n} = 1 \rightarrow \delta\omega_n \geq 3$



Intersekzioa dago, eta integradore bat dago $e_{ss}=0$ egiten duena, beraz P kontroladorearekin nahikoa zaigu.

$$G_{BC}(s) = \frac{K_c \frac{1}{4s}}{1 + \frac{K_c}{4s}} = \frac{K_c}{4s + K_c} = \frac{K_c}{1 + \underbrace{\frac{4}{K_c}}_{\tau} s} \Rightarrow t_s(\%5) = 3\tau = 3 \cdot \frac{4}{K_c} =$$

$\boxed{K_c = 12}$

∘ $G_3(S) = \dfrac{40}{(S+1)(S+2)}$ $\Bigg\}$ $e_{ss}=0$

$\%12{,}25 < M < \%25$

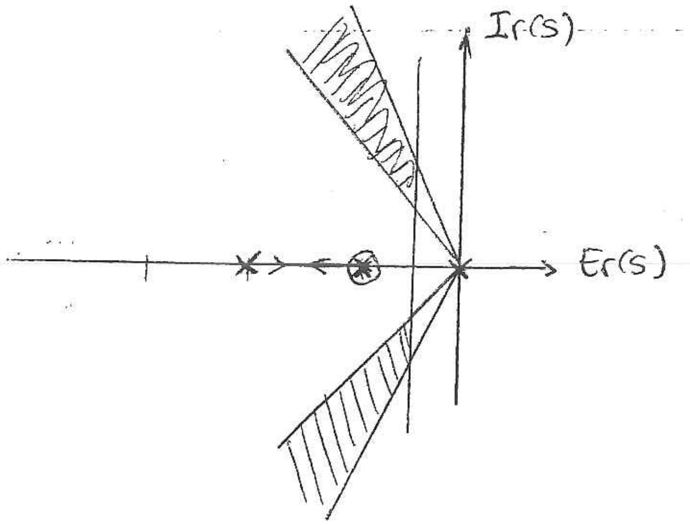
$t_s(\%5) \leq 6s \longrightarrow \dfrac{3}{\delta\omega_n} \leq 6s \Rightarrow 0{,}5 \leq \delta\omega_n$

$\%12{,}25 \rightarrow M_p = e^{-\frac{\pi\delta}{\sqrt{1-\delta^2}}} \cdot 100 \rightarrow \delta = 0{,}556$

$\%25 \Rightarrow \delta = 0{,}4037 \longrightarrow 0{,}4037 < \delta < 0{,}556$

$56{,}22^\circ < \theta < 66{,}2^\circ$

56,22



Poloak: $s=-1$
$s=-2$

$e_{ss}=0$ egiteko integradore bat behar dugu ⇒ I akzioa sartu behar dugu.

PI kontroladorea.

$S = -\dfrac{1}{T_i} = -1 \rightarrow \boxed{T_i = 1}$

$G_{PI}(s) = \dfrac{K\left(s+\frac{1}{T_i}\right)}{S}$

$G_{BA}(s) = \dfrac{K_c\cancel{(s+1)}40}{s\cancel{(s+1)}(s+2)} = \dfrac{40K_c}{s(s+2)} \Rightarrow G_{BC}(s) = \dfrac{40K_c}{s^2+2s+40K_c}$

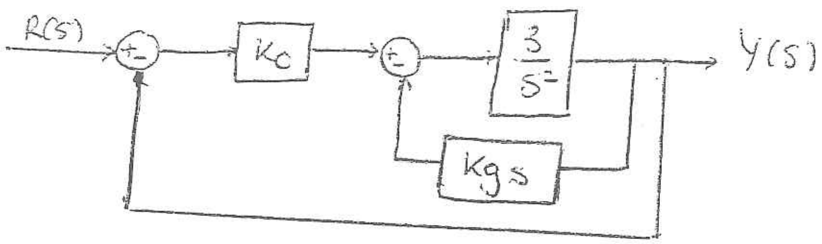
$2 = 2\delta\omega_n$ $\longrightarrow$ $\delta = 0{,}556 \longrightarrow \omega_n = 1{,}798 \simeq 1{,}8 \text{ rad/s}$

$\longrightarrow$ $\delta = 0{,}4037 \longrightarrow \omega_n = 2{,}478 \simeq 2{,}5 \text{ rad/s}$

$40K_c = \omega_n^2$ $\nearrow K_c = 0{,}081$

$\searrow K_c = 0{,}15625$ $\Rightarrow \boxed{0{,}081 < K_c < 0{,}15625}$

⑰ (2012-13 Ekaina)

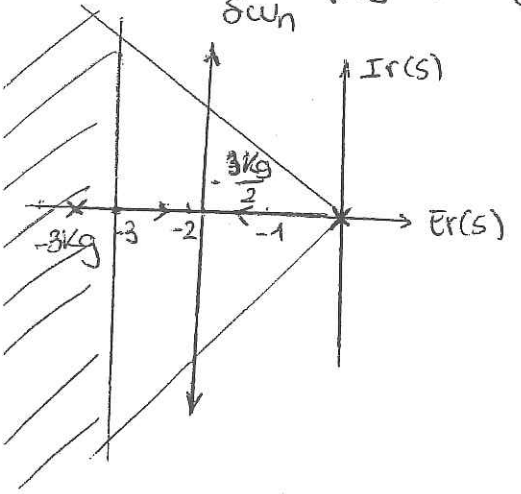


Abiaduraren berrelikadura erabiltzen da, egonkortasuna Kg terminoarekin hobetzeko. Honek baimentzen du Kg iragankorreko eskakizun bat betetzeko erabiltzea.

$M_p \leq \%10$; $t_s(\%5) \leq 1s$

$M_p = e^{\frac{-\pi\delta}{\sqrt{1-\delta^2}}} \rightarrow \delta \geq 0,59 \Rightarrow \theta \leq 53,8°$

$t_s(\%) = \frac{3}{\delta\omega_n} \leq 1s \Rightarrow \delta\omega_n \geq 3$



$$G(s) = \frac{3/s^2}{1+Kg\,s\,\frac{3}{s^2}} = \frac{3}{s^2+3Kg\,s}$$

$$G(s) = \frac{3}{s(s+3Kg)}$$

$$G_{BA}(s) = \frac{3Kc}{\underbrace{\textcircled{s}}_{\text{integradore (1.MOTA)}}(s+3Kg)} \longrightarrow$$

poloak:
$s=0$
$s=-3Kg$

$$G_{BC}(s) = \frac{3Kc}{s^2+3Kg\,s+3Kc}$$

Begizta itxiko poloak begizta irekiko poloetatik abiatuko dira. Kg handitzen den heinean, bi poloak elkartuz doaz polo bikoitza egin arte $s=-\frac{3Kg}{2}$ puntuan. Puntu honetatik aurrera poloak konplexu konjokatu egiten dira. $\delta=0,59$ eta $\delta\omega_n \geq 3$ eginez poloak konjokatuak dira.

$$3Kg = 2\delta\omega_n \longrightarrow \frac{3}{2}Kg = \delta\omega_n \geq 3 \Rightarrow \boxed{Kg \geq 2}$$

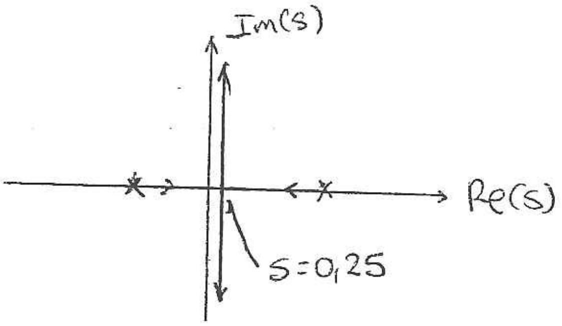
$$3K_c = \omega_n^2 \;;\quad \underbrace{\delta}_{0,59 \simeq 0,6}\omega_n = 3 \Rightarrow \omega_n = 5\ rad/s \Rightarrow \boxed{K_c = \frac{25}{3}}$$

Sistemak errorerik gabe jarraituko dio eskaloi-sarrerari, 1. motako sistema berrelikatua delako.

⑱ (2013-2014 Urtarrila)

Grafikotik : Begizta irekiko poloak $\begin{cases} s = -1 \\ s = 1,5 \end{cases}$

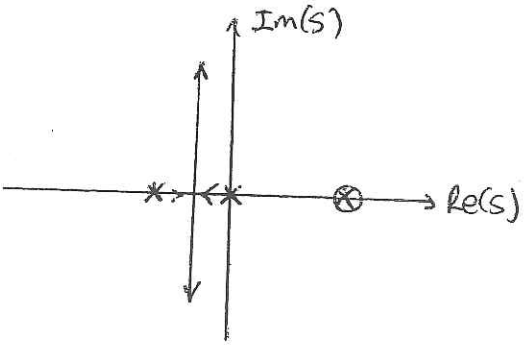
a)



$\boxed{\text{Faltsua}}$, sistema ezegonkorra izango da edozein k-ren balioarentzat, poloek ez dutelako zati erreal negatiboa nahiz eta k handitu.

b)

PI kontrolagailu baten bidez zero bat sartzen dugu : s = 1,5 eta polo bat.

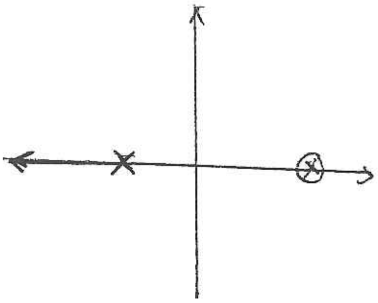


$$G_{PI} = \frac{K_c\left(s - \textcircled{\tfrac{1}{T_i}}\right)}{s} \quad \text{non} \quad T_i = 2/3$$

$$G_{BA}(s) = \frac{K_c\,\cancel{(s-1,5)}\,K\cdot\frac{2}{3}}{s(s+1)\cancel{(s-1,5)}} = \frac{K_c\cdot K\cdot 2/3}{s(s+1)}$$

c)

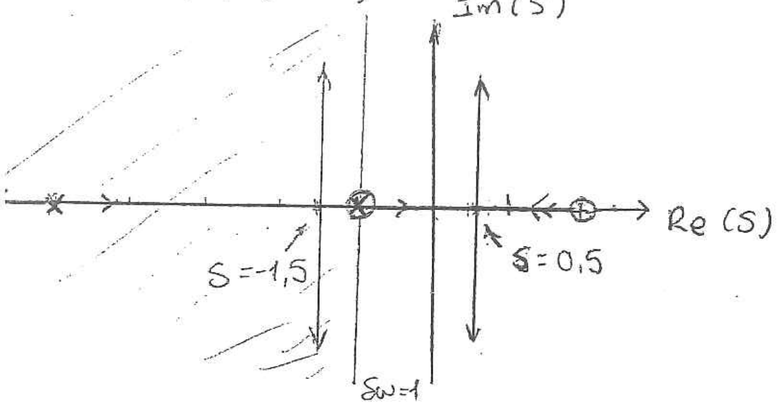
PD kontrolagailu baten bidez zero bat sartzen dugu s = 1,5



Sistema egonkorra da. ⇒ $\boxed{\text{Egia}}$

㉒

$$G(s) = \frac{s-2}{(s+1)(s+5)}$$



a) kasuan, P kontrolagailu batekin nahikoa dugu, (-1,0) tartean baitaudelako k_c-ren balioak sistema egonkorten dutenak.

Bestalde, b) kasuan PD behar dugu sistemak eskakizunak betetzeko. (PI ez dugu erabiliko ETG erdiplano positiboan geratzen delako).

$$G_{PD}(s) = K_c\left(s+\frac{1}{T_d}\right) \quad \text{non} \quad \frac{1}{T_d} = 1 \Rightarrow \boxed{T_d = 1}$$

$$G_p(s) = Kc$$

$$G_{BA}(s) = \frac{Kc(s-2)}{(s+1)(s+5)} \Rightarrow G_{BC}(s) = \frac{Kc(s-2)}{s^2+6s+5+Kc(s-2)}$$

$$\hookrightarrow s^2 + s(6+Kc) + (5-2Kc)$$

<u>R-H</u>

$$B.B: \begin{cases} 6+K_c > 0 \rightarrow Kc > -6 \rightarrow Kc > 0 \\ 5-2Kc > 0 \rightarrow \quad Kc < 2,5 \end{cases}$$

$$\boxed{G_p(s) = k_c \quad \text{non} \quad Kc \in (0, 2'5)}$$

b) $t(\%) \leq 3s \Rightarrow \frac{3}{\delta\omega_n} \leq 3 \Rightarrow 1 \leq \delta\omega_n \Rightarrow$ Intersekzioa dago PD-rekin nahikoa dugu.

$$G_{BC}(s) = \frac{K_c(s-2)}{s(1+Kc)+(5-2Kc)} = \frac{Kc(s-2)}{1+\underbrace{\left(\frac{1+Kc}{5-2Kc}\right)}_{\tau}s}$$

$$t(\%5) = 3\tau \leq 3$$

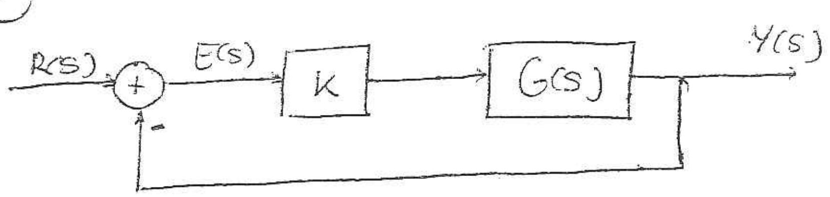
$$\cancel{3}\frac{1+Kc}{5-2Kc} \leq \cancel{3} \Rightarrow 1+Kc \leq 5-2Kc \Rightarrow 3Kc \leq 4 \rightarrow \underline{\underline{Kc \leq 4/3}}$$

balioko tartean dago.

$$0 < Kc < 4/3$$

$$G_c(s) = Kc(1+s)$$

⑤



$$G(s) = \frac{1}{(s+1)(s+4)}$$

$$G_{BA}(s) = \frac{K}{(s+1)(s+4)}$$

$$G_{BC}(s) = \frac{K}{(s+1)(s+4)+K}$$

polo dominatzailea: $s = -1 = -\frac{1}{\tau}$

$\underline{\underline{\tau = 1s}}$

b)

$\tau = \frac{1}{2}s \Rightarrow$ poloa: $s = -\frac{1}{1/2} = -2$

Ek. karakteristikoa: $s^2 + 5s + 4 + K = 0 = s^2 + 2\delta\omega_n s + \omega_n^2$

$$s = \frac{-5 \pm \sqrt{25-4(4+k)}}{2} = -2 \Rightarrow -5 \pm \sqrt{25-16-4K} = -4$$

- $+\sqrt{9-4K} = 1 \rightarrow 9-4K = 1 \rightarrow \underline{\underline{K=2}}$
- $-\sqrt{9-4k} = 1 \rightarrow -9+4K = 1 \rightarrow \underline{\underline{k = \frac{5}{2}}}$

c)

Erroreak:

- $K=2 \rightarrow G_{BA}(s) = \frac{2}{(s+1)(s+4)}$

$$E(s) = R(s) - Y(s)\cdot H(s) = \frac{1}{s} - Y(s) \rightarrow E(s) = \frac{1/s}{1+\frac{K}{(s+1)(s+4)}} = \frac{1}{s}\cdot\frac{1}{(s+1)(s+4)}$$

$$e_{ss} = \lim_{s\to 0} s\cdot E(s) = \cancel{s}\cdot\frac{1}{\cancel{s}}\frac{1}{(s+1)(s+4)+2} = \frac{1}{2}$$

- $k = \frac{5}{2}$ $\quad e_{ss} = \frac{1}{5/2} = \frac{2}{5} = 0{,}4$

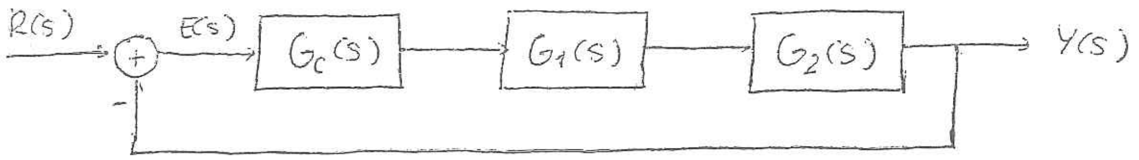
d) Asebsunak mugatuko du irabazpenaren balioa, baita egonkortasunak

e)

$$\begin{cases} 2\delta\omega_n = 5 \rightarrow 2\cdot 0{,}7\,\omega_n = 5 \rightarrow \omega_n = 3{,}57 \text{ rad/s} \\ 4+K = \omega_n^2 \rightarrow 4+K = 3{,}57^2 \Rightarrow \underline{\underline{K = 8{,}755}} \end{cases}$$

$e_{ss} = \frac{1}{8{,}755} = 0{,}11$

Adibidea (51. diapositiba) (2014 EKAINA)



$$G_1(s) = \frac{0{,}5}{s+10}$$

$$G_2(s) = \frac{2}{s^2+s+1}$$

Eskakizunak: $e_{ss}=0$

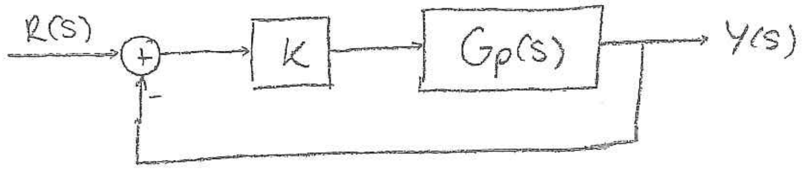
Moteldura erlazioa 1/4

$$G_{BA}(s) = \frac{K_c \cdot 0{,}5 \cdot 2}{(s+10)(s^2+s+1)} = \frac{K_c}{(s+10)(s^2+s+1)}$$

$e_{ss}=0$ izateko integradore bat sartu behar diogu, daukagun 0 motako sistema 1 motako bihurtzeko.

Gainera moteldura erlazioa $\frac{1}{4}$ izan behar denez, sintonizaziorako esperimentuen bidezko metodo bat erabili behar dugu; Ziegler - Nichols.

Lehenengo Ku eta Tu kritikoak kalkulatu behar ditugu:



Ek karakteristikoa

$s^3+s^2+s+10s^2+10s+10+K$

$s^3+11s^2+11s+10+K=0$

B.B: $10+K>0 \longrightarrow K>-10 — K>0$

B.N: (R-H)

$$\begin{array}{c|ccc} s^3 & 1 & 11 & 0 \\ s^2 & 11 & 10+K & 0 \\ \hline s & b_1 & 0 & \\ s^0 & c_1 & & \end{array}$$

$$b_1 = \frac{11^2-10-K}{11} > 0 \longrightarrow 11^2-10>K$$

$$111>K \Longrightarrow \underline{K_u=111}$$

K_u erabiliz:

$P(s)=11s^2+121 \longrightarrow 11s^2+121=0$

$s^2=-11 \Rightarrow s=\pm\sqrt{11}\,j$

$\omega_u=\sqrt{11}$

$\hookrightarrow T_u=\frac{2\pi}{\omega_u}=1{,}894\ s$

Balio hauekin taulara joan behar gara:

PI + Begizta itxia: $K_c=0{,}4K_u$

$T_i=0{,}8T_u$ $\Longrightarrow$ $\boxed{\begin{array}{l} K_c=44{,}4 \\ T_i=\cancel{0}1{,}5\ s \end{array}}$

7. MAIZTASUNAREN EREMUKO AZTERKETA

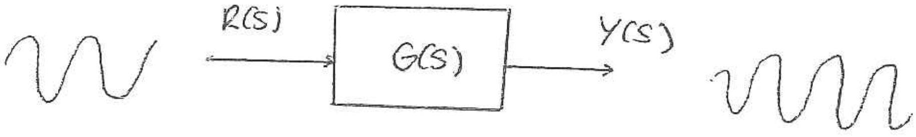
→ HELBURUAK:

- Edozein sarreraren aurrean, sistema dinamikoak zelan erantzuten duen azterketa.
- Begizta itxiko sistema batek noraino egonkortasuna duen kuantifikazioa.

→ SEINALE SENOIDALAK:

$u = A\,\mathrm{sen}(\omega t)$

$\omega = \dfrac{2\pi}{T}$



* Egoera iraunkorra: $Y(s) = G(s)\cdot R(s)$; $R(s) = \dfrac{\omega A}{s^2+\omega^2}$

$$G(s) = \frac{N(s)}{D(s)}$$

→ SISTEMA EGONKOR BATEN ERANTZUNA:

$$Y(s) = G(s)\cdot R(s) = \frac{G(s)\cdot A\omega}{s^2+\omega^2} = \underbrace{\frac{R_1}{s+j\omega} + \frac{R_2}{s-j\omega}}_{\text{Egoera iraunkorra}} + \underbrace{\text{Beste termino batzuk (poloei dagozkie)}}_{\text{Egoera iragankorra}}$$

$$R_1 = -\frac{AG(-j\omega)}{2j}$$

$$R_2 = \frac{AG(j\omega)}{2j}$$

$\downarrow \mathcal{L}^{-1}$

$$y_{ss}(t) = \frac{-AG(-j\omega)}{2j}e^{-j\omega t} + \frac{AG(j\omega)}{2j}e^{j\omega t}$$

* $G(j\omega)$ funtzio konplexua denez:

- **Modulua**: $|G(j\omega)| = \sqrt{X^2(\omega)+Y^2(\omega)} = |G(-j$
- **Argumentua**: $\mathrm{Arg}\,G(j\omega) = \mathrm{arctg}\dfrac{Y(\omega)}{X(\omega)}$ (u

$$y_{ss}(t) = \frac{-A}{2j}\left(X(\omega) - jY(\omega)\right)e^{-j\omega t} + \frac{A}{2j}\left(X(\omega)+jY(\omega)\right)e^{j\omega t}$$

$$\boxed{y_{ss}(t) = A\,\underbrace{|G(j\omega)|}\,\mathrm{sen}(\omega t + \Phi)}$$

Sarreraren ANPLITUDEA

Sistemaren modulua ANPLIFIKAZIOA

Sarreraren MAIZTASUNA

Sistemaren Argumentua

SARRERAREKIKO FASE-ATZERAPENA

Irteera-seinalearen anplitudea eta desfasea kitzikadura maiztasunaren menpeko dira.

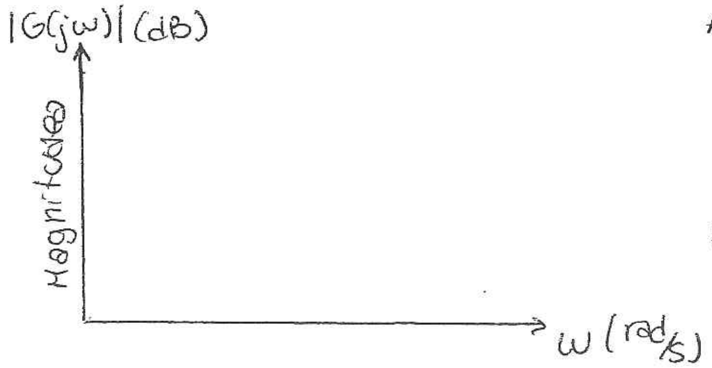
→ MAIZTASUN ERANTZUNAREN ADIERAZPIDE GRAFIKOAK

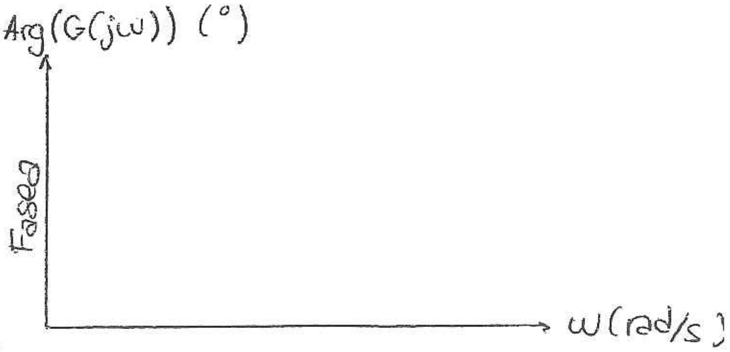
Sistema lineal batek eragiten duen atenuazioa $|G(j\omega)|$ eta desfasea $\phi = \mathrm{Arg}(G(j\omega))$ $G(s)$-ren menpeko dira bakarrik, eta ω maiztasunaren funtzio bezala adieraz daitezke hainbat diagrama erabiliz. $G(s)$-n, s-ren ordez $j\omega$ ipintzen da eta $G(j\omega)$-ren modulu eta argumentuak kalkulatzen dira.

• BODE DIAGRAMAK

$$\left.\begin{array}{ll} 20\log|G(j\omega)| & (dB) \\ \mathrm{Arg}\ G(j\omega) & (^\circ) \end{array}\right\}$$ adierazi ω (rad/s) maiztasunaren menpe.

Dezibelioa $(dB) = 20\log|G(j\omega)|$





↳ ω (rad/s) abzisetan, eskala logaritmikoan adierazten da.

Diagrama logaritmikoak erabiltzen dira moduluen biderketa batuketa bihurtzen delako logaritmoak aplikatzean.

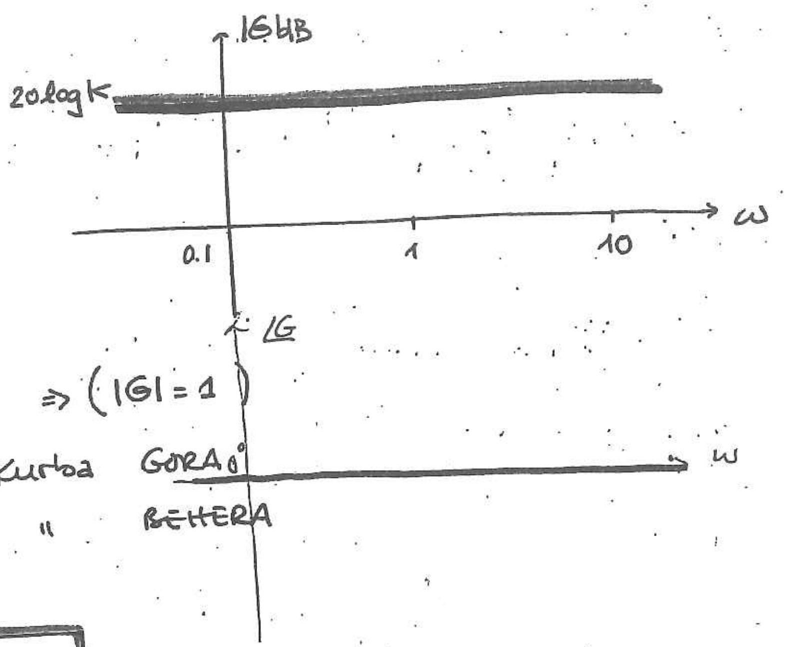
$G(j\omega)$-ren diagrama, $G(s)$-ren dinamiko osagaien diagramak gainezarriz lor daiteke.

K Irabazpena

$G(s) = K$

$20 \log K = Kte$

0° ↳



K=1 ⇒ |G| = 0dB ⇒ (|G| = 1)

K↑ moduluaren kurba GORA

K↓ " " BEHERA

$\frac{1}{j\omega}$ · INTEGRATZAILEA POLOA JATORRIAN

$G(s) = \frac{1}{s}$

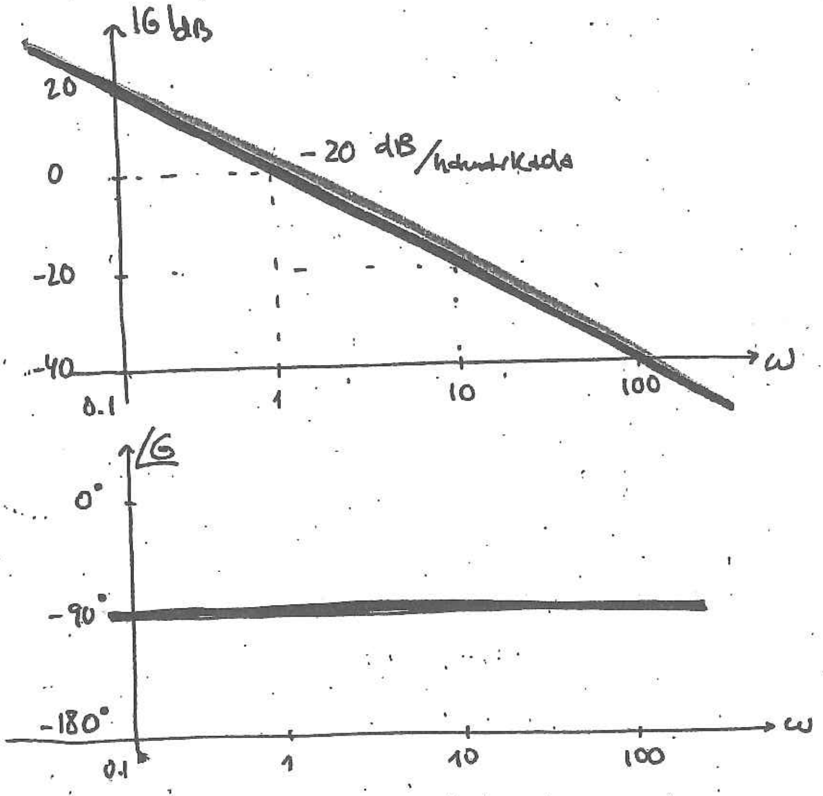
Modulua: -20 dB/h maldadun lerro zuzena.

$20 \log \left| \frac{1}{j\omega} \right| = 20 \log \left| \frac{1}{\omega} \right| = -20 \log \omega \ dB$

-90° ∀ω , Argumentua.

ω=0.1	|G| = 20 dB
ω=1	|G| = 0 dB
ω=10	|G| = -20 dB
ω=100	|G| = -40 dB

hamarkada bakoitzean -20dB gutxiago ematen du.



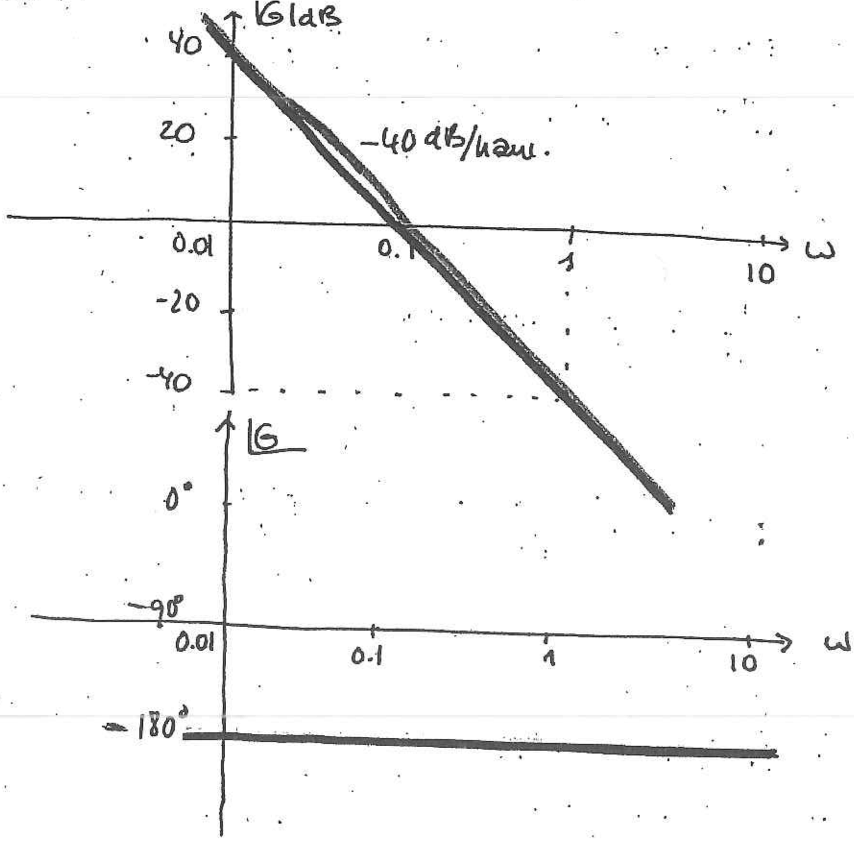
$\left(\frac{1}{j\omega}\right)^n$. n Integratzaile
POLOA JATORRIAN

$G(s)=\frac{1}{s^n}$

$$\begin{cases} 20\log\left|\frac{1}{(j\omega)^n}\right| = 20\log\frac{1}{\omega^n} = -20n\log\omega \text{ dB} & \text{Modulua} \\ -90n^\circ & \text{Argumentua} \end{cases}$$

$-20n$ dB/hamarkadako malda du.

n=2 denean:



$(j\omega)$. Deribatzailea
ZEROA JATORRIAN

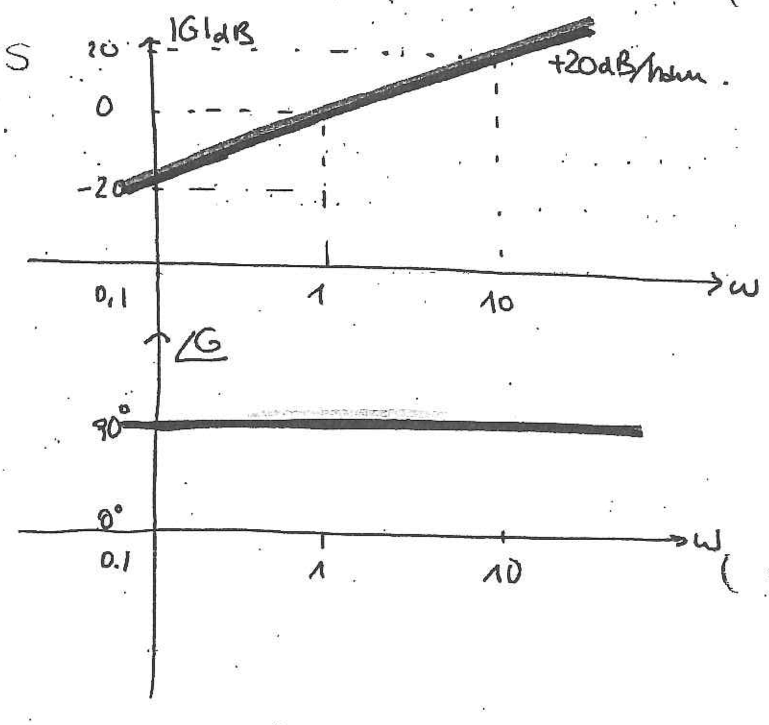
$G(s)=S$

$$\begin{cases} 20\log\omega \\ 90^\circ \end{cases}$$

w=0.1 |G|=-20dB

w=1 |G|= 0dB

w=10 |G|= +20dB



$(j\omega)^n \Rightarrow$ 20n dB/h. malda.

$\boxed{\frac{1}{1+j\omega T} \quad \text{Poloa}}$ $G(s) = \frac{1}{1+Ts}$. (FILTRO PASABAJOS)

Modulua:

$$20 \log \left| \frac{1}{1+j\omega T} \right| = 20 \log \frac{1}{\sqrt{1+\omega^2 T^2}} = -20 \log \sqrt{1+\omega^2 T^2}$$

asintotak

ω txikiak : $(\omega \to 0)$

aprox : $-20 \log \sqrt{1+\omega^2 T^2} \simeq -20 \log 1 = 0\,dB \Rightarrow Arg \to 0^\circ$

ω handiak : $(\omega \to \infty)$

aprox : $-20 \log \sqrt{1+\omega^2 T^2} \simeq -20 \log \omega T \Rightarrow Arg \to -90^\circ$

$\boxed{\omega = \frac{1}{T}}$ **TRANSIZIO-MAIZTASUNEAN:**

erreala : $-20 \log \sqrt{1+\omega^2 T^2} = -20 \log \sqrt{2} = \boxed{-3\,dB}$

asint : $-20 \log \omega T = 0\,dB \Rightarrow$ Asintota bien arteko ebaki-puntua.

$\omega = \frac{1}{T}$ maiztasunetik aurrera $(\omega > \frac{1}{T})$

$$-20 \log \omega T = -20 \log T - 20 \log \boxed{\omega}$$

$\searrow$ $-20\,dB/h$ malda du.

Argumentua: $\angle G(j\omega) = -\text{arctg}\, \omega T$

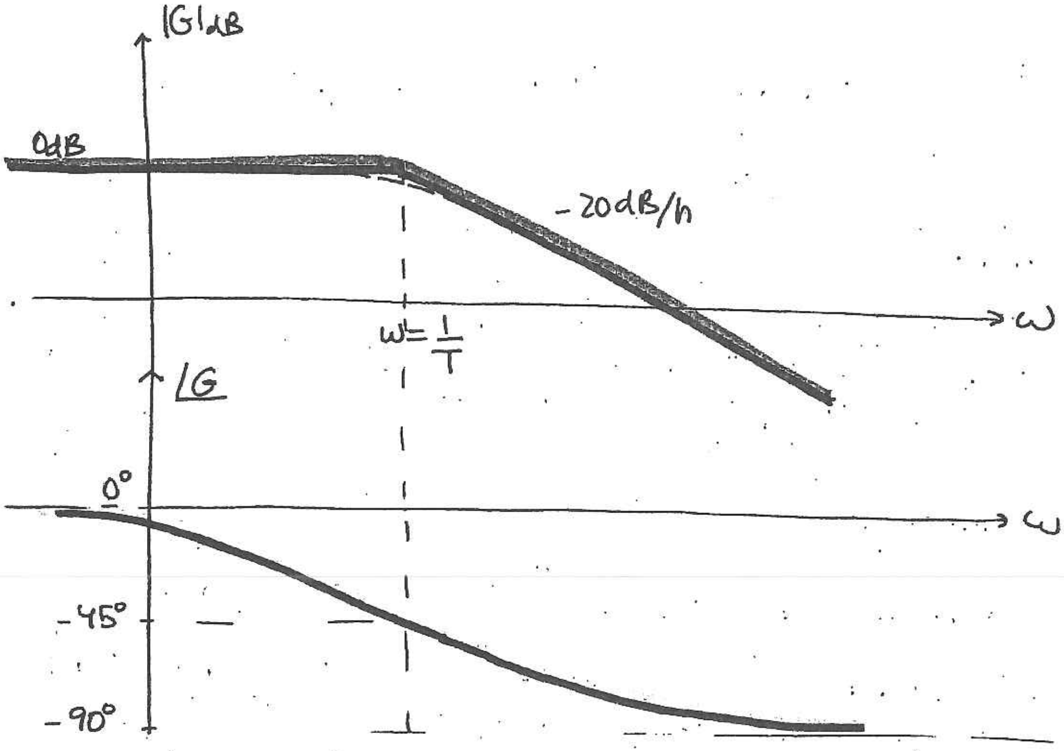
$\omega = 0$ $\angle G(j\omega) = 0^\circ$

$\omega = \frac{1}{T}$ $\angle G(j\omega) = -45^\circ$

$\omega = \infty$ $\angle G(j\omega) = -90^\circ$

→ LABURPENA :

- Atenuazioa txikia da $\underbrace{\omega = \frac{1}{\tau}}_{}$ → hauste-maiztasuna. arte. Hortik aurrera progresiboki ↑
- Sistema geldoak → τ handia → poloa ardatzetik gertu → atenuazioa ω txikietan has
- Sistema azkarrak → τ txikia → poloa ardatzetik urrun → atenuazioa ω handietan.

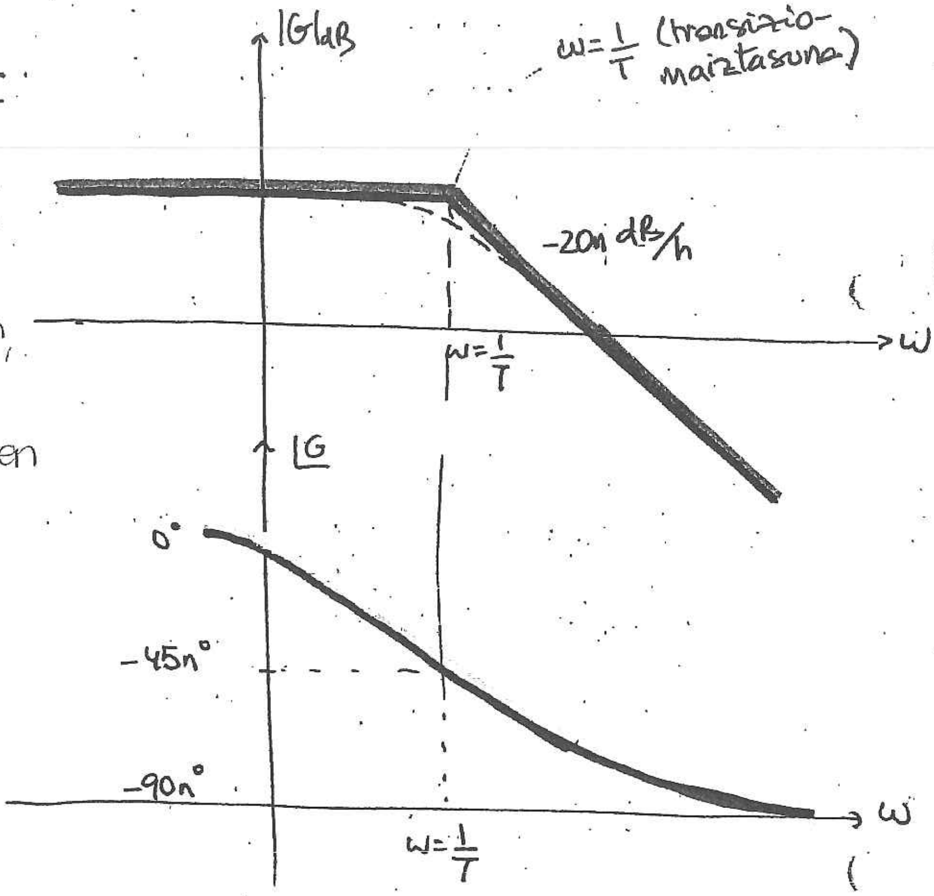


<u>transizio-maiztasunak modulu-kurba bitan banatzen du</u>: maiztasun txikiak eta maiztasun handiak.

$$\boxed{\frac{1}{(1+j\omega T)^n} \text{ gaiak}}$$

$$\begin{cases} -20n \log\sqrt{1+\omega^2T^2} \\ -n.\ \mathrm{arctg}\,\omega T \end{cases}$$

Polo anizkoitzak (n) daudenean, modularen eta argumentuaren kurbak n aldiz biderkatzen dira.

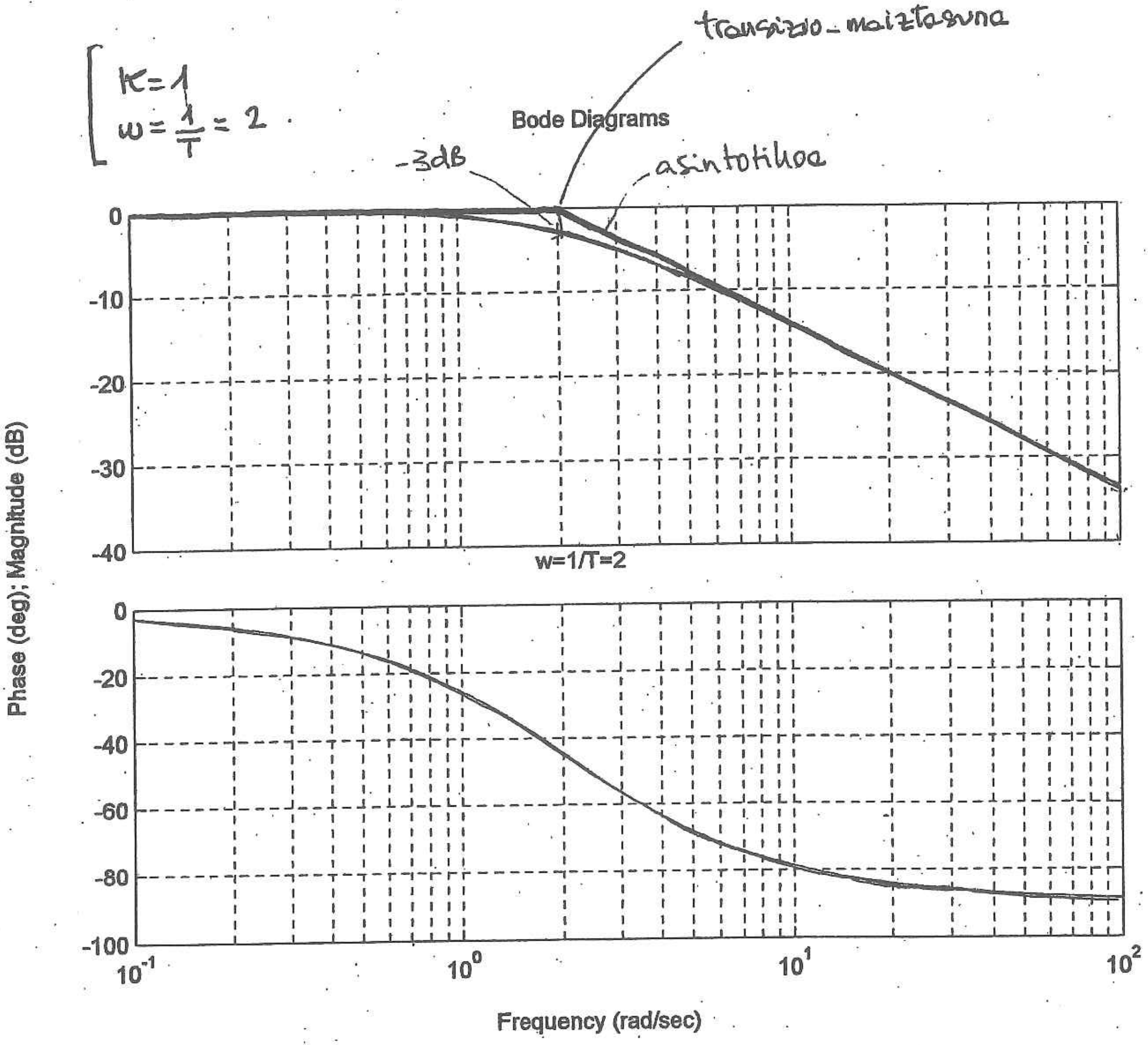


MATLAB

```
>> K=1 ; T=0.5;
>> num=[K];
>> den=[T 1];
>> bode(num,den)
```

$$\boxed{G(s)=\frac{K}{1+Ts}}$$

$K=1$
$\omega=\frac{1}{T}=2$



$\boxed{1+j\omega T \quad \text{Zeroa}}$ (FILTRO PASAALTOS)

$G(s) = 1 + \tau s$

Modulua:

$$20\log|1+j\omega T| = 20\log\sqrt{1+\omega^2T^2}$$

asintotak:

ω txikiak : $(\omega \to 0)$

aprox: $20\log 1 = 0\,dB$

ω handiak : $(\omega \to \infty)$

aprox: $20\log \omega T$

$\omega = \frac{1}{T}$ **TRANSIZIO-MAIZTASUNEAN:**

eredua : $20\log\sqrt{2} = 3\,dB$

asintot : $20\log 1 = 0\,dB$

$\omega = \frac{1}{T}$ tik aurrera $(\omega > \frac{1}{T})$

$$20\log\omega T = 20\log T + 20\log\omega \searrow$$

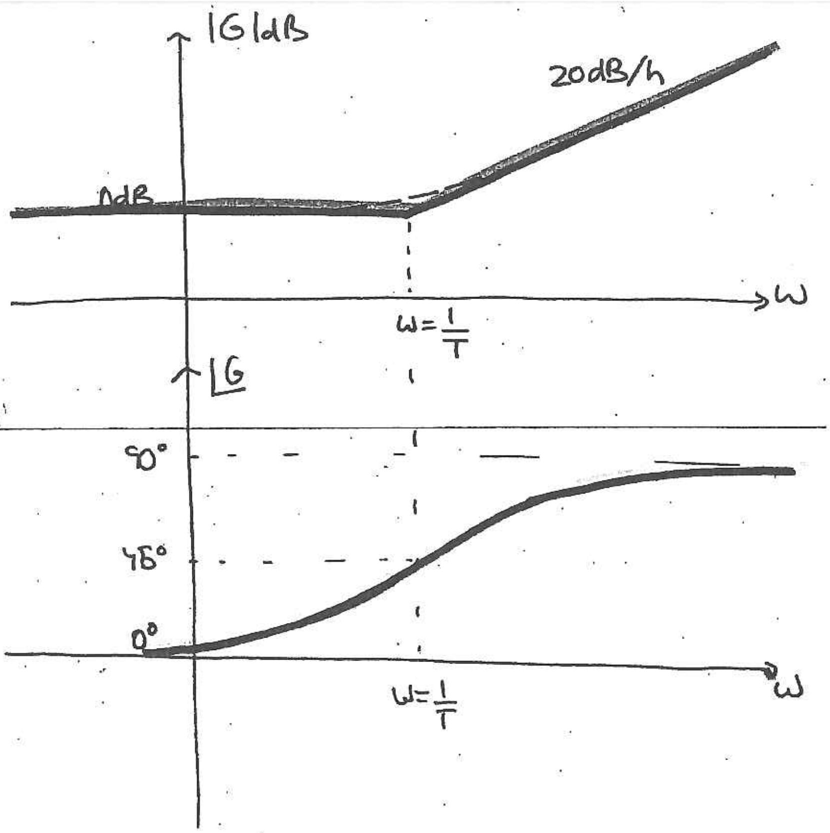
20 dB/h malda du.

Argumentua: $\angle G(j\omega) = \text{arctg}\,\omega T$

$\omega = 0$ $\quad |G| = 0°$

$\omega = \frac{1}{T}$ $\quad \angle G = 45°$

$\omega = \infty$ $\quad \angle G = 90°$

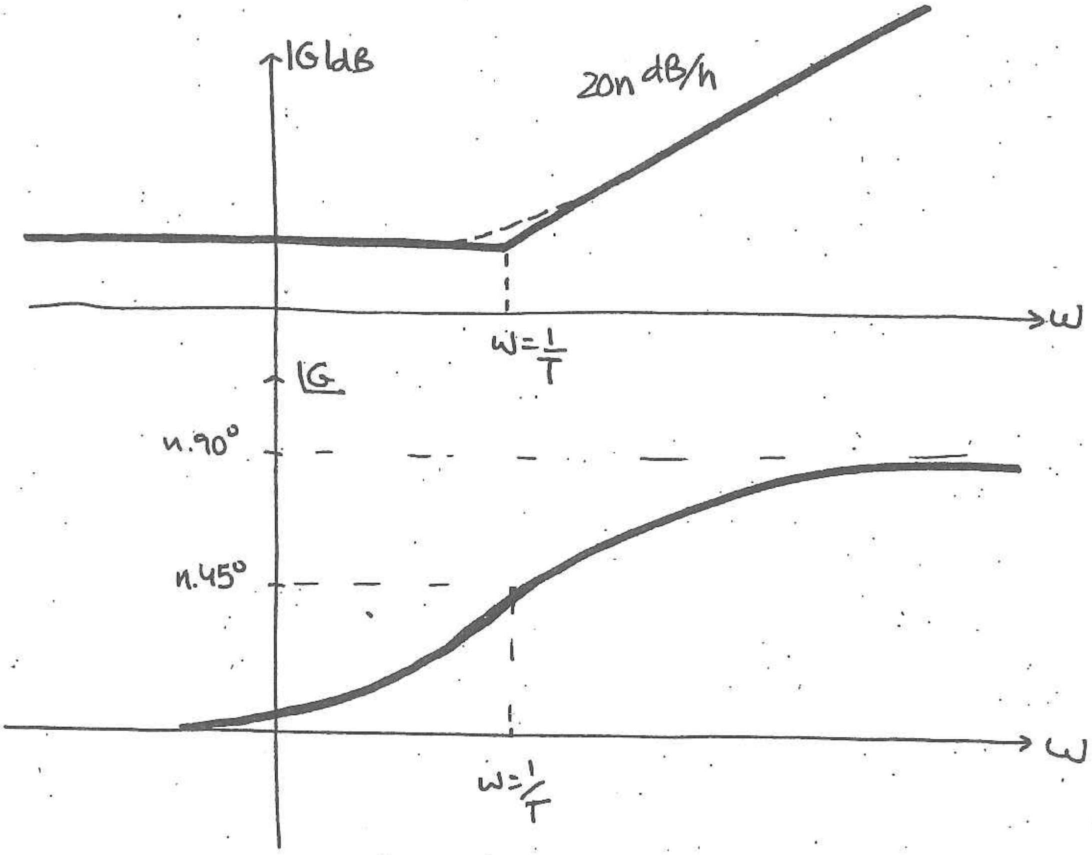


→ LABURPENA:

Anplifikazioa txikia da $\omega = \frac{1}{\tau}$ arte. Hortik aurrera progresiboki handituz doa.

$\boxed{(1+j\omega T)^{n} \text{ gdzie k.}}$

$$\begin{cases} 20n \log\sqrt{1+\omega^2 T^2} \\ n . \mathrm{arctg}\, \omega T \end{cases}$$



2. ORDENAKO SISTEMA

$$\boxed{\frac{1}{1+2\delta\left(j\frac{\omega}{\omega_n}\right)+\left(j\frac{\omega}{\omega_n}\right)^2} \quad \text{gaiak}}$$

$(0<\delta<1)$

- $\delta>1$ 1. ordenako 2 faktore (ikusi dituzu bakarka)
- $0<\delta<1$ Konplexu Konjokatuak
- Asintoten bidezko aproximazioa ez da zehatza δ txikia denean, modulua, δ eta ω_n-ren funtzio delako.

Modulua:

$$20\log\left|\frac{1}{1+2\delta\left(j\frac{\omega}{\omega_n}\right)+\left(j\frac{\omega}{\omega_n}\right)^2}\right| = -20\log\sqrt{\left(1-\frac{\omega^2}{\omega_n^2}\right)^2+\left(2\delta\frac{\omega}{\omega_n}\right)^2}$$

Asintotak:

- ω txikietan $(\omega\to0)$:

$$\lim_{\substack{\omega\to0\\ \omega\ll\omega_n}} |G|_{dB} \simeq -20\log 1 = 0\,dB$$

(zero horizontala 0 dB-tan)

- ω handietan $(\omega\to\infty)$:

$$\lim_{\substack{\omega\to\infty\\ \omega\gg\omega_n}} |G|_{dB} \simeq -20\log\frac{\omega^2}{\omega_n^2} = -40\log\frac{\omega}{\omega_n}$$

- maiztasun handiko asintota $-40\,dB/h$ malda duen lerroa da:

$$-40\log\frac{10\omega}{\omega_n} = -40\log 10 - 40\log\frac{\omega}{\omega_n} =$$
$$= -40 - 40\log\frac{\omega}{\omega_n}$$

- $\omega=\omega_n$ denean: (MAIZTASUN NATURALA)

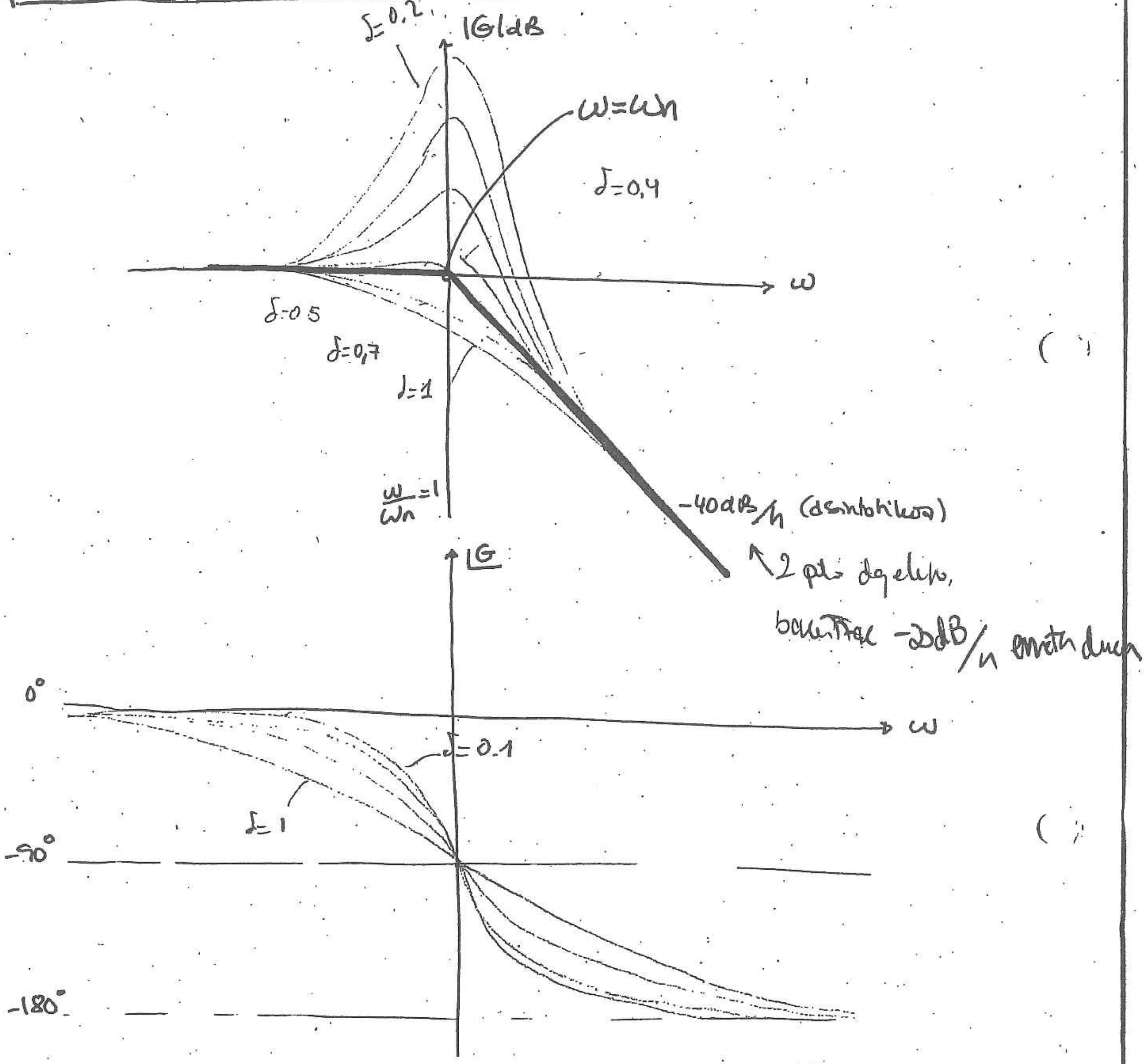
$$-40\log\frac{\omega_n}{\omega_n} = 0\,dB$$ (asintotikoa)

$$-20\log 2\delta$$ (erreala)

Asintotikoa eta errealaren arteko aldea (ERROREA)

$\omega=\omega_n$: maiztasun handi eta txikiko asintoten arteko ebaki-puntua.

- Asintota biek ez dute δ-ren eraginik.
- Baina $\omega=\omega_n$ maiztasunaren inguruan, erresonantzia-maximo bat ematen da.
- Asintoten aldeko hurbilketak erroreak ematen ditu, bereziki δ txikia denean.

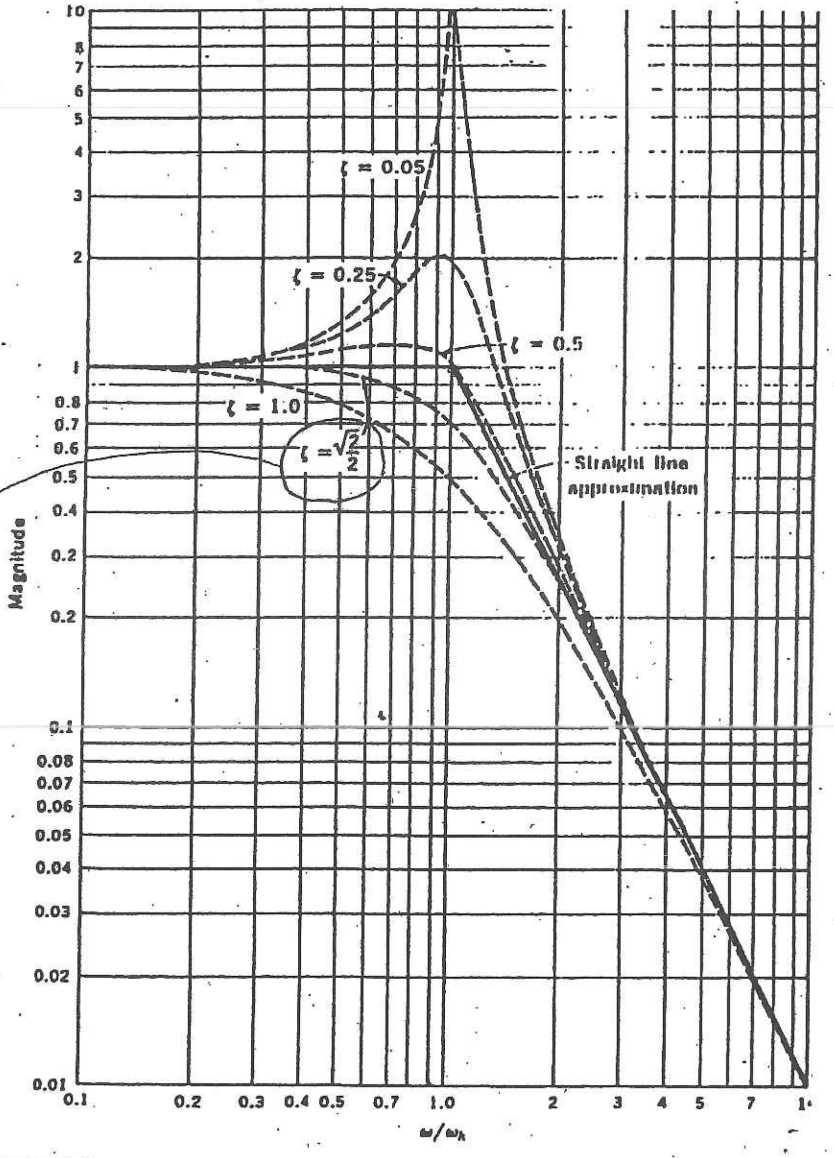


Argumentua:

$$\angle G(j\omega) = -\mathrm{arctg}\,\frac{2\delta\frac{\omega}{\omega_n}}{1-\frac{\omega^2}{\omega_n^2}} \qquad \left\{ \begin{array}{ll} \omega=0 & \angle G = 0^\circ \\ \omega=\omega_n & \angle G = -90^\circ = -\mathrm{arctg}\,\frac{2\delta}{0} \\ \omega=\infty & \angle G = -180 \end{array} \right.$$

$$\frac{1}{1+2j\delta\frac{\omega}{\omega_n}-\frac{\omega^2}{\omega_n^2}}$$ erako gaien modulo-diagrama

$\zeta=\delta$



$\delta=0{,}707$

$\delta<0{,}707$ denean : Anplitudearen kurbak (errealak) **MAXIMOA** M_r ageri du ω_r maiztasunean. Moduluaren kurba deribatuz kalkula daiteke:

$$\frac{d|G(j\omega)|}{d\omega}=0 \rightarrow \omega_r$$

ERRESONANTZIA-MAIZTASUNA: $\omega_r=\omega_n\sqrt{1-\delta^2}$

ERRESONANTZIA-MAKIMOA: $M_r=\frac{1}{2\delta\sqrt{1-\delta^2}}$

$$M_r=|G(j\omega)|_{\omega=\omega_r}$$

$\delta\geq0{,}707$ denean Anplitudearen kurba MONOTONOKI BEHERAKORRA da

$\boxed{e^{-j\omega t_m} \text{ gaiak}}$

ATZERAPEN TERMINOAK

$G(s)=e^{-t_m s}$

$$e^{j\omega t_m} = \cos\omega t_m + j\,\text{sen}\,\omega t_m$$
$$e^{-j\omega t_m} = \cos i\omega t_m - j\,\text{sen}\,\omega t_m$$

Modulua:

$|\cos j\omega t_m - j\,\text{sen}\,\omega t_m| = 1$

$20\log|\cos\omega t_m - j\,\text{sen}\,\omega t_m| = 0\,\text{dB}$

Argumentua:

$$\angle G(j\omega) = -\omega t_m\ (\text{rad}) = \underset{\frac{360^\circ}{2\pi}}{-57{,}3}\,\omega t_m\ (^\circ)$$

Argumentua gero eta negatiboagoa da.

Fasearen kurba ω-rekin jeisten da (esponentzialki eskala logaritmikoan)

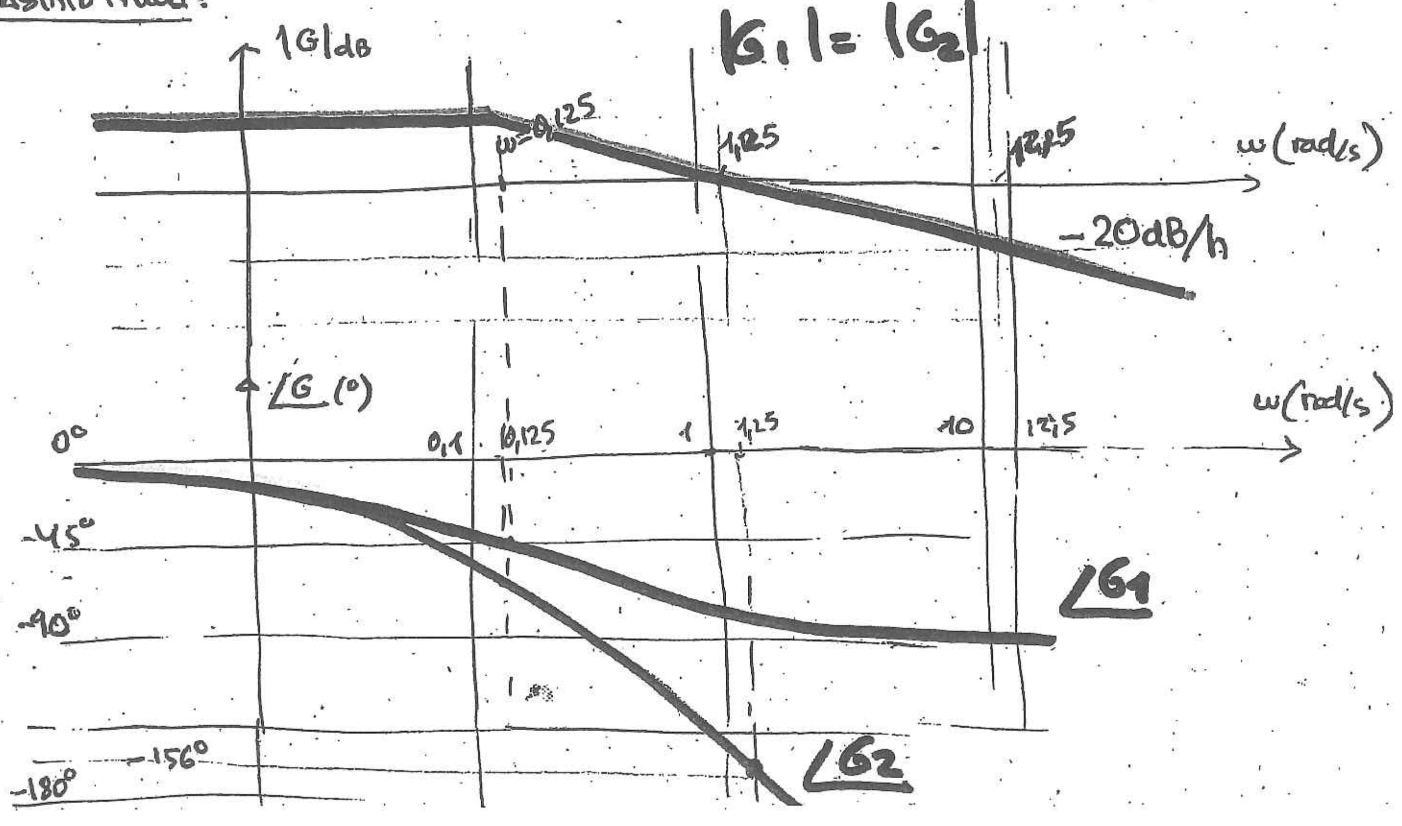
ADIBIDEA: $G_1(s) = \dfrac{10}{8s+1}$ $\qquad G_2(s) = \dfrac{10}{8s+1}\cdot e^{-s}$ $\quad (t_m = 1)$

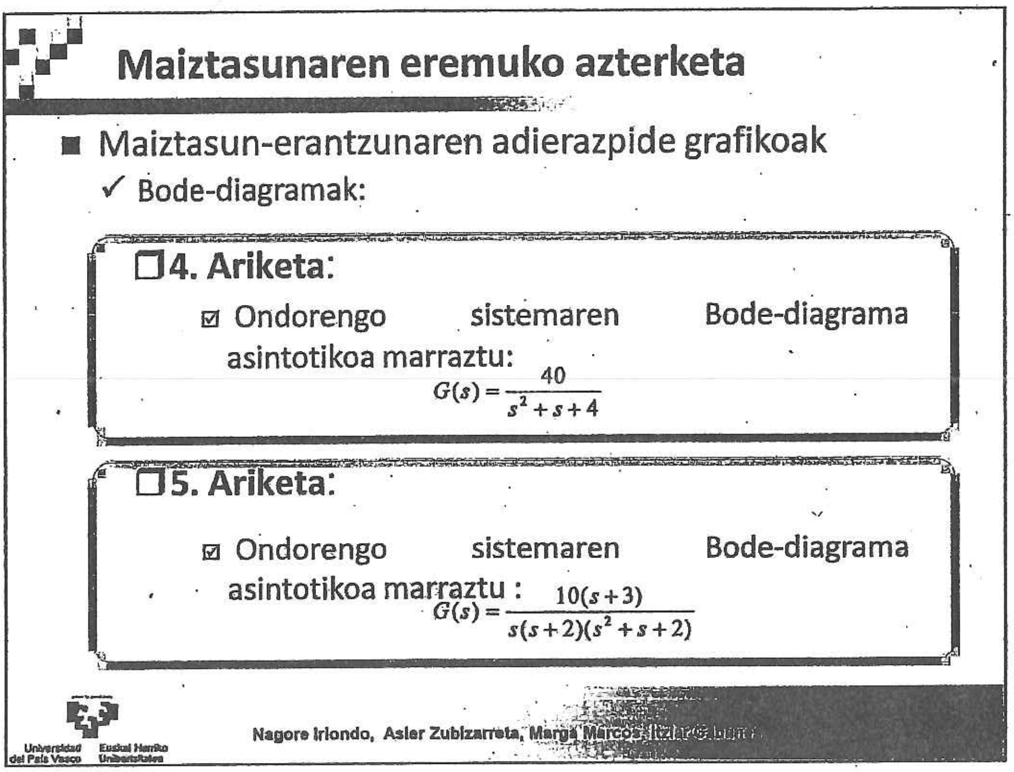
Modulua: $|G_1(j\omega)| = |G_2(j\omega)| = \dfrac{10}{\sqrt{1+64\omega^2}}$

Argumentua: $\angle G_1(j\omega) = -\text{arctg}\,8\omega$

$\angle G_2(j\omega) = -\text{arctg}\,8\omega - \omega\cdot 1$

Bode Asintotikoa:





Maiztasunaren eremuko azterketa

- Maiztasun-erantzunaren adierazpide grafikoak
  - ✓ Bode-diagramak:

☐4. Ariketa:

☑ Ondorengo sistemaren Bode-diagrama asintotikoa marraztu:

$$G(s)=\frac{40}{s^2+s+4}$$

☐5. Ariketa:

☑ Ondorengo sistemaren Bode-diagrama asintotikoa marraztu :

$$G(s)=\frac{10(s+3)}{s(s+2)(s^2+s+2)}$$

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$$G(s)=\frac{40}{s^2+s+4}=\frac{10}{s^2/4+s/4+1}=\frac{1}{\frac{s^2}{\omega_n^2}+2\delta\left(\frac{s}{\omega_n}\right)+1}$$

$$G(j\omega)=\frac{10}{1-\omega^2/4+j\,\omega/4}\qquad \omega_n^2=4\Rightarrow\omega_n=2$$

$$|G(j\omega)|=\frac{10}{\sqrt{\left(1-\omega^2/4\right)^2+\frac{\omega^2}{16}}}\qquad \underline{/G(j\omega)}=-\mathrm{arctg}\,\frac{\omega/4}{1-\omega^2/4}$$

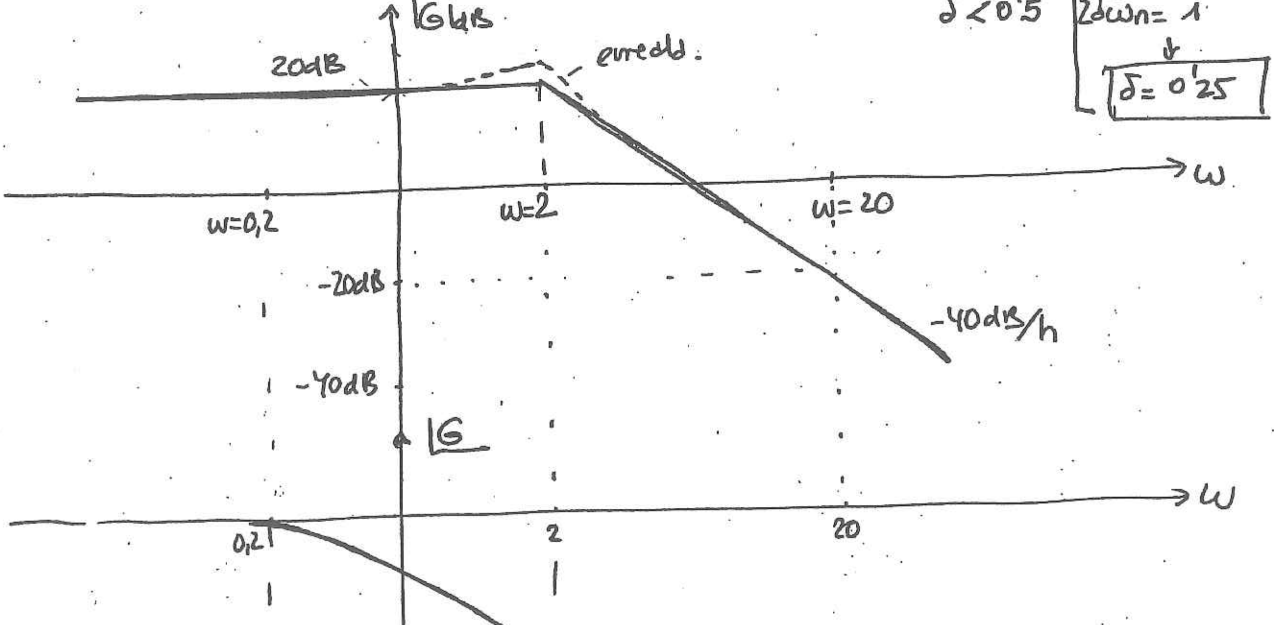
Asintotak:

- $\omega\to 0$: $+20\log 10 = +20\,\mathrm{dB}$
- $\omega\to\infty$: $+20\log 10-20\log\frac{\omega^2}{4}=20\log 10-40\log\frac{\omega}{2}$
- $\underline{\omega=\omega_n=2}$: $+\underline{20\log 10}=20\,\mathrm{dB}$ (asintotikoa)

$$20\log\frac{10}{\sqrt{\left(1-\frac{\omega^2}{4}\right)^2+\frac{\omega^2}{16}}}=26{,}02\,\mathrm{dB}\quad\text{(erreala)}$$

6,02 dB-tako errorea egiten da $(\omega=\omega_n)$ asintotak erabiltzean.

$$\Downarrow\qquad \delta<0'5\qquad \left[\begin{array}{l}\omega_n^2=4\rightarrow\omega_n=2\\ 2\delta\omega_n=1\\ \quad\downarrow\\ \boxed{\delta=0'25}\end{array}\right.$$



Argumentul:

$w=0$, $\angle G = 0^\circ$

$w = 0.2$ $\angle G = -3{,}2^\circ$

$w = 2$ $\angle G = -90^\circ$

$w = 20$ $\angle G = -176{,}78^\circ$

$w = \infty$ $\angle G = -180^\circ$

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$$G(s) = \frac{10(s+3)}{s(s+2)(s^2+s+2)}$$

Bodea egitean akatsik ez egiteko , $G(j\omega)$ era normalizatuan idaztea komeni da .

$$G(j\omega) = \frac{7{,}5\left(1+\frac{j\omega}{3}\right)}{j\omega\left(1+\frac{j\omega}{2}\right)\left(1+\frac{j\omega}{2}+\frac{(j\omega)^2}{2}\right)}$$

Polo eta zeroen trantsizio-maiztasunak : $\left(\underset{\text{erreala}}{\omega = \frac{1}{\tau}}\right.$ eta $\left.\underset{\text{konplexua}}{\omega = \omega_n}\right)$

$\omega = 0$

3 • zeroa : $\omega = 3$

2 • polo erreala : $\omega = 2$

1 • polo konplexuak $\omega_n = \omega = \sqrt{2}$ $(\delta = 0{,}3536)$

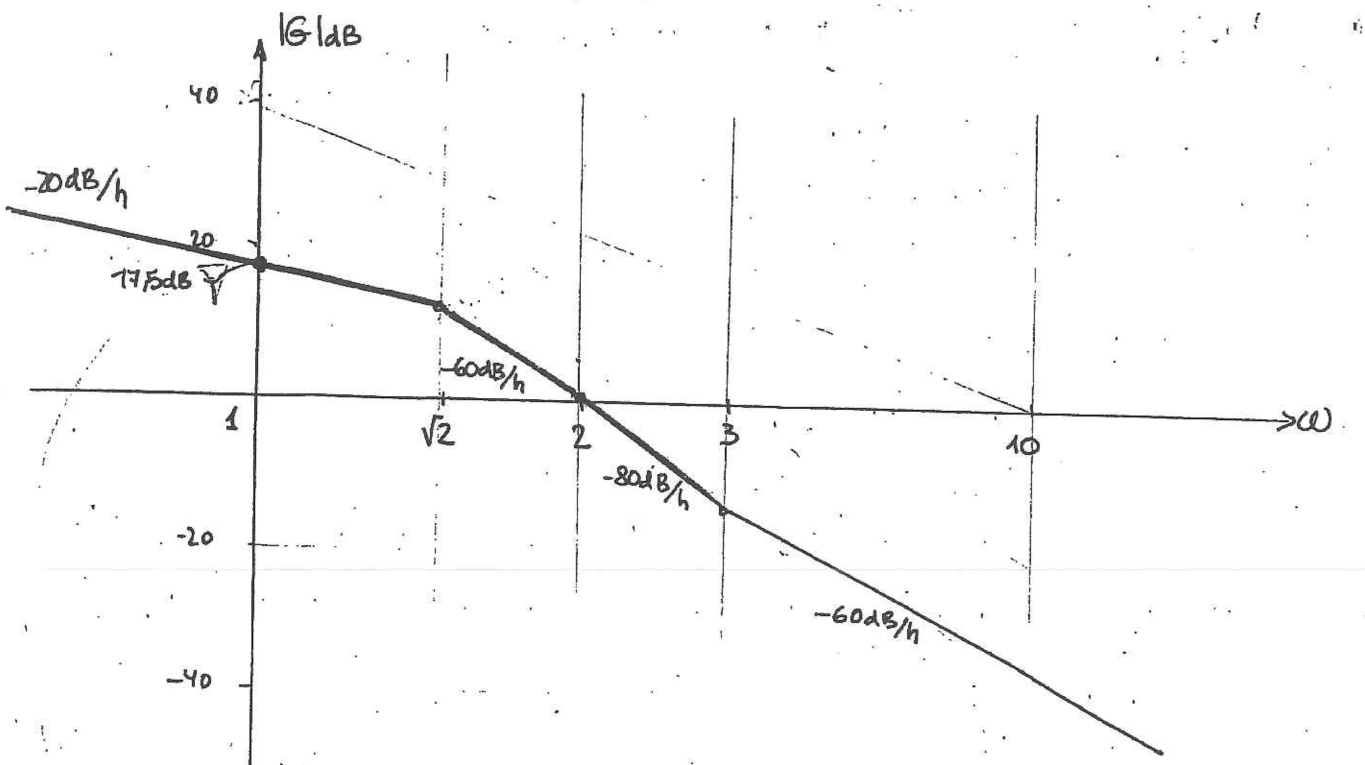
$\omega = \infty$

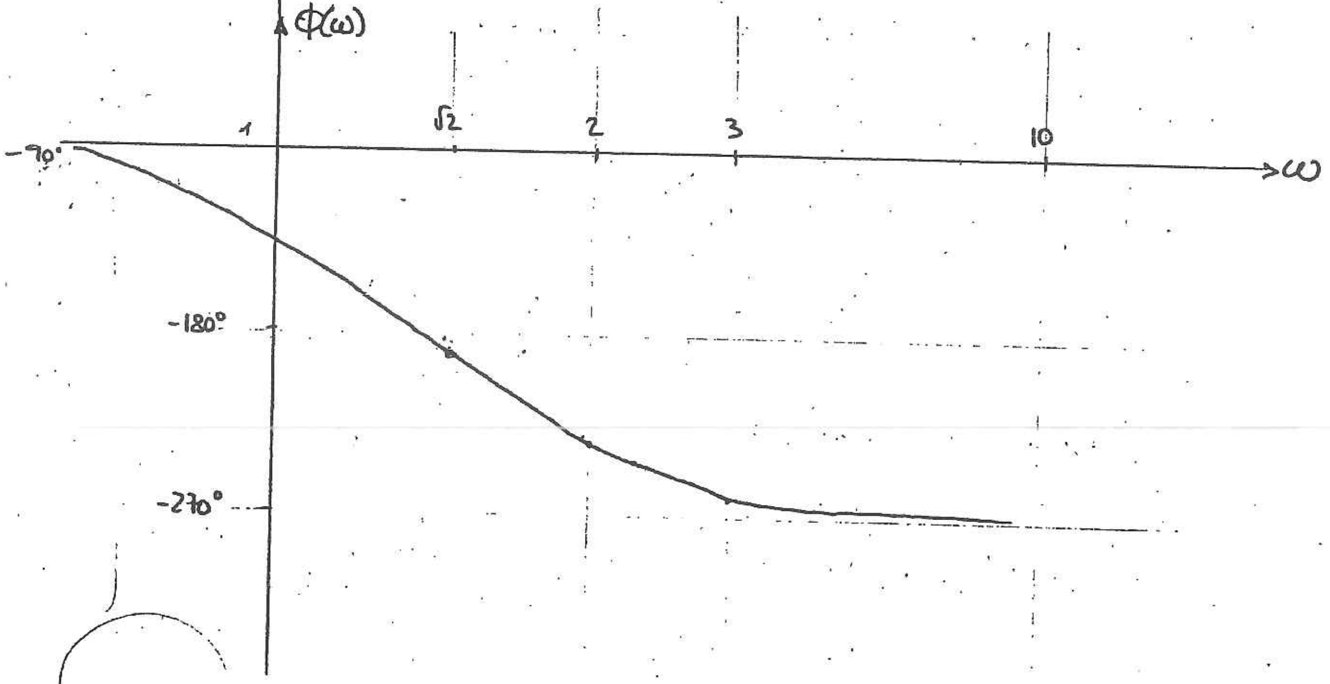
$$\left|G(j\omega)\right|_{dB} = 20\log 7{,}5 + 20\log\left|1+\frac{j\omega}{3}\right| - 20\log\omega - 20\log\left|1+\frac{j\omega}{2}\right| -$$

$$-20\log\left|1+\frac{j\omega}{2}+\frac{(j\omega)^2}{2}\right| \Rightarrow \text{modulu erreala}$$

Bode asintotikoaren maldak:

	$7{,}5$	$\frac{1}{j\omega}$	$\frac{1}{1-\frac{\omega^2}{2}+j\frac{\omega}{2}}$	$\frac{1}{1+j\frac{\omega}{2}}$	$1+j\frac{\omega}{3}$	GUZTIRA
$\omega=0 \to \omega=\sqrt{2}$	0	-20	0	0	0	-20 dB/h
$\omega=\sqrt{2} \to \omega=2$	0	-20	-40	0	0	-60 dB/h
$\omega=2 \to \omega=3$	0	-20	-40	-20	0	-80 dB/h
$\omega=3 \to \omega=\infty$	0	-20	-40	-20	+20	-60 dB/h
			$\omega > \omega_n$ denean, malda -40 koa da		$\omega > \frac{1}{\tau}$ bada malda +20 -koa da	





$\omega = 1 \Rightarrow \quad |G|_{dB} = 20 \log 7{,}5 = 17{,}5\,dB$

Argumentul:

$$\Phi(\omega) = \mathrm{arctg}\,\frac{\omega}{3} - 90^\circ - \mathrm{arctg}\,\frac{\omega}{2} - \mathrm{arctg}\,\frac{\omega/2}{1-\omega^2/2}$$

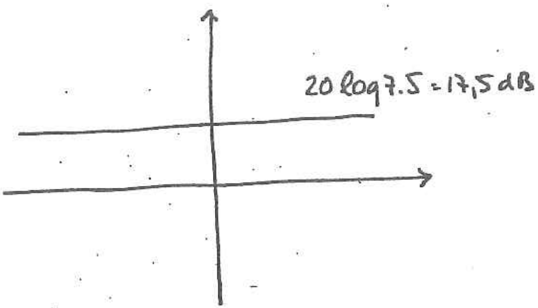
$\omega = 0 \quad : \quad \Phi(\omega) = 0^\circ - 90^\circ$

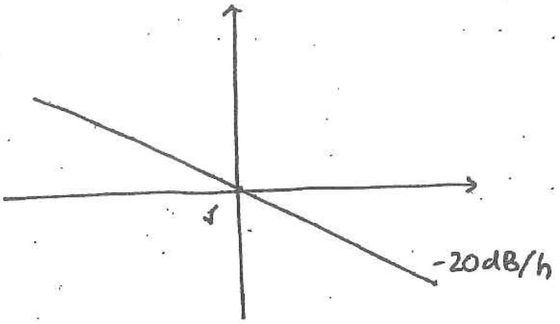
$\omega = \sqrt{2} \quad : \quad \Phi(\omega) = +25{,}24^\circ - 90^\circ - 35{,}26^\circ - 90^\circ = -190^\circ$

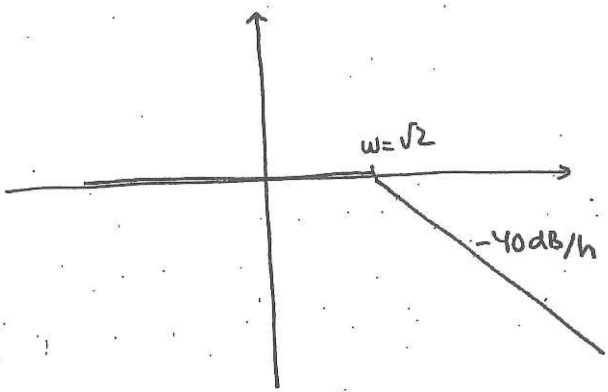
$\omega = 2 \quad : \quad \Phi(\omega) = 33{,}7^\circ - 90^\circ - 45^\circ - 135^\circ = -236{,}3^\circ$

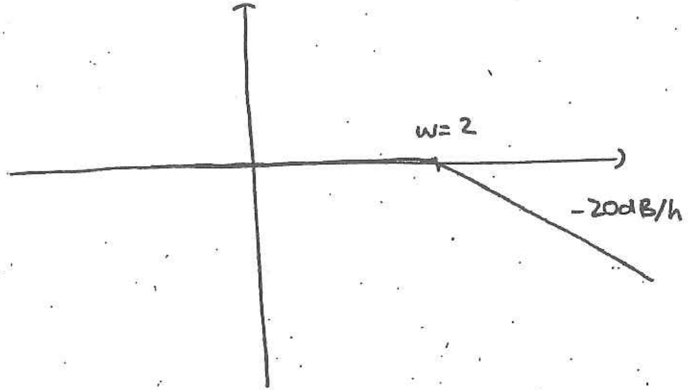
$\omega = 3 \quad : \quad \Phi(\omega) = 45^\circ - 90^\circ - 56{,}3^\circ - 156{,}8^\circ = -258^\circ$

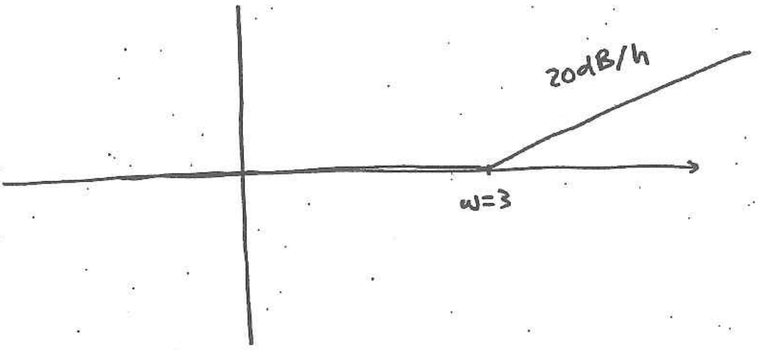
$\omega = \infty \quad : \quad \Phi(\omega) = -270^\circ$



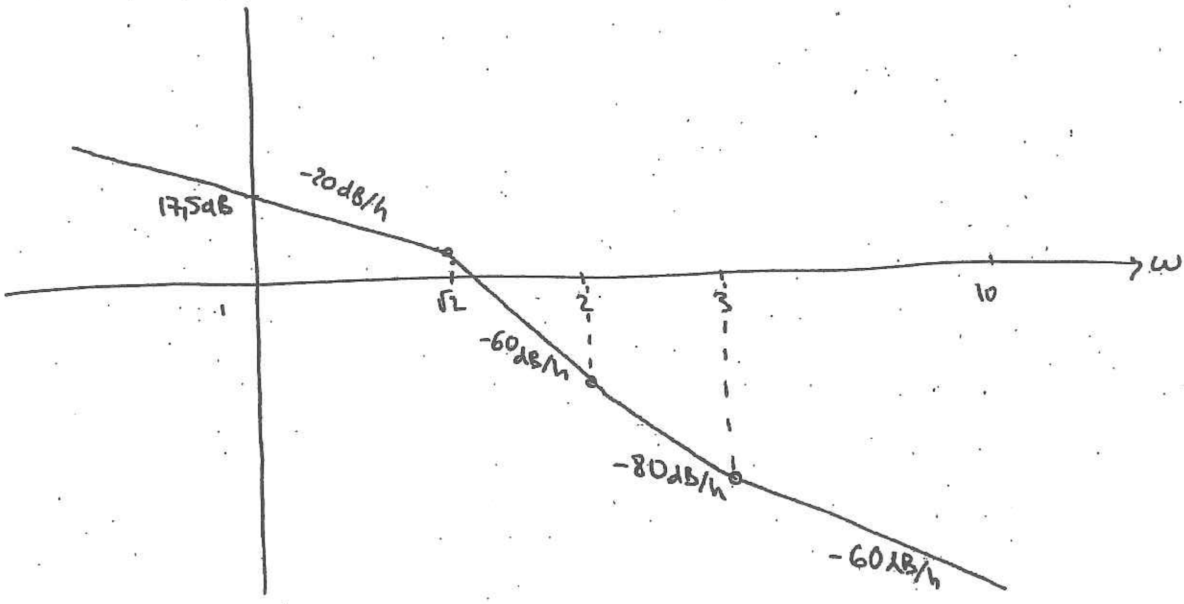








Guztien batuketak modulu-kurba emango digu:



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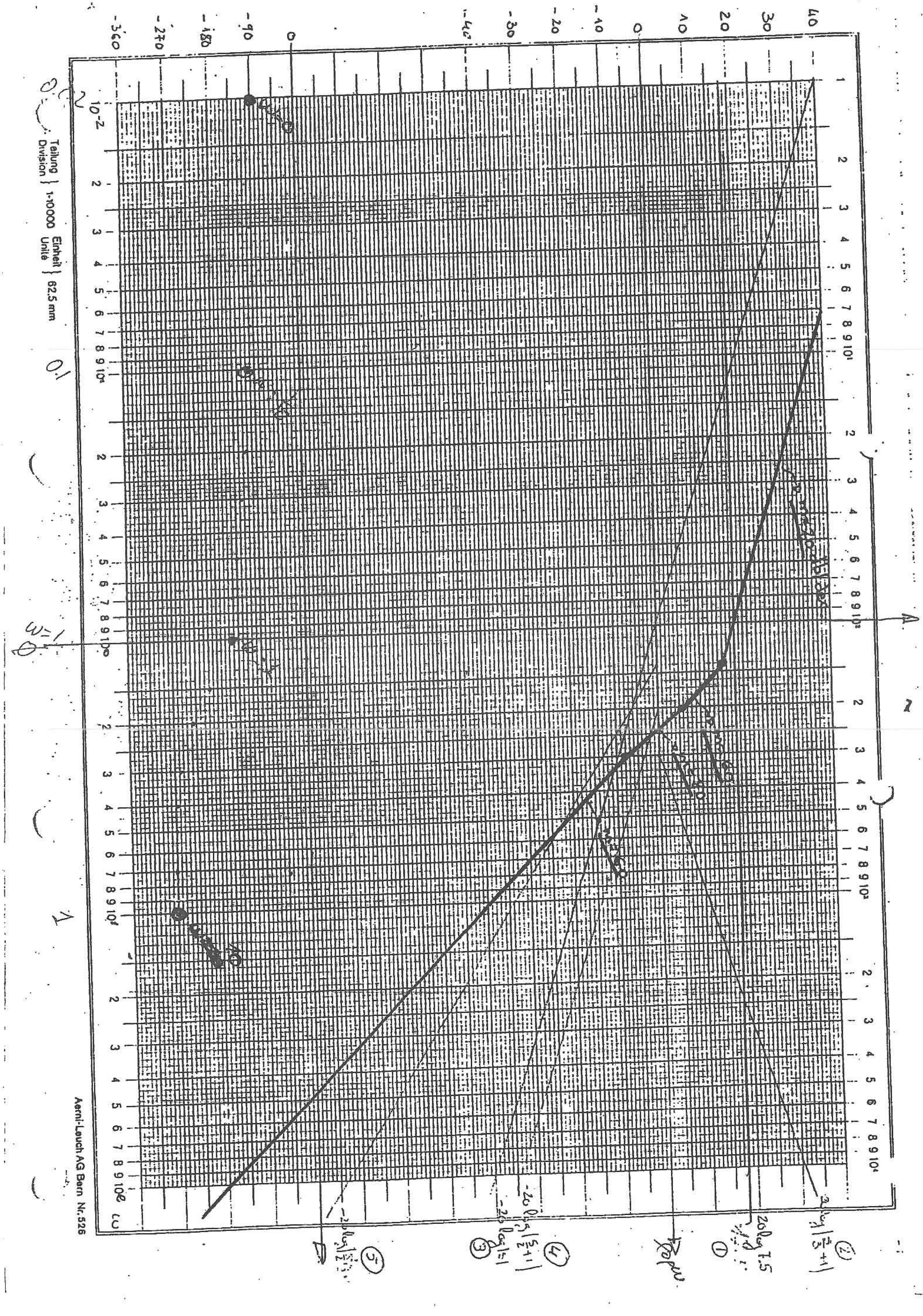
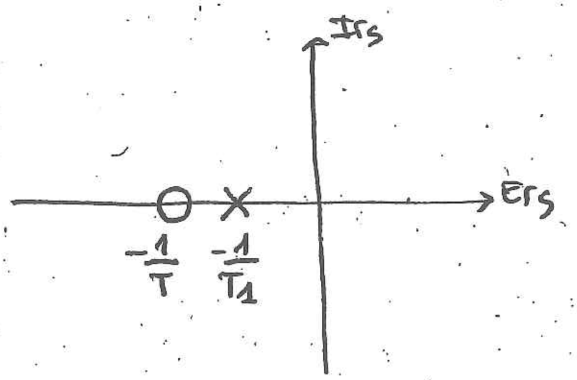

40
30
20
10
0
-10
-20
-30
-40
0
-90
-180
-270
-360
10^-2
0.01
0.1
ω=1
1
10
Teilung | 1-10000 Einheit | 62.5 mm
Division | Unité
Aerni-Leuch AG Bern Nr. 526
20 log 7.5
①
20 log $|\frac{s}{3}+1|$
②
-20 log $|s|$
③
-20 log $|\frac{s}{2}+1|$
④
-20 log $|\frac{s}{2}+1|$
⑤
$F_0(j\omega)$
ω=0
ω=1
ω=10

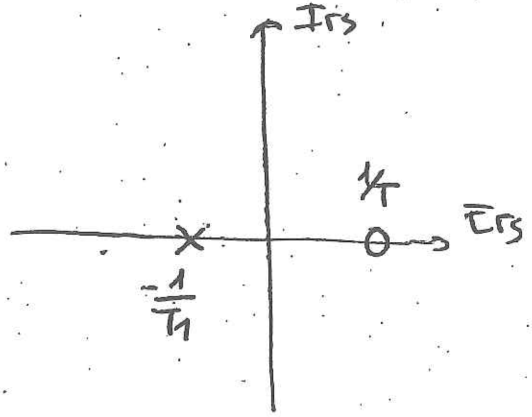
FASE MINIMODUN SISTEMAK

Begiratu 7TR-47 eta TR48 transparentzietan

s-planoaren eskuin planoerdian EZ polorik EZTA zerorik ez duen sistemari FASE MINIMOA duen sistema dela esaten zaio.

$$G_1(j\omega) = \frac{1+j\omega T}{1+j\omega T_1} \qquad G_2(j\omega) = \frac{1-j\omega T}{1+j\omega T_1}$$



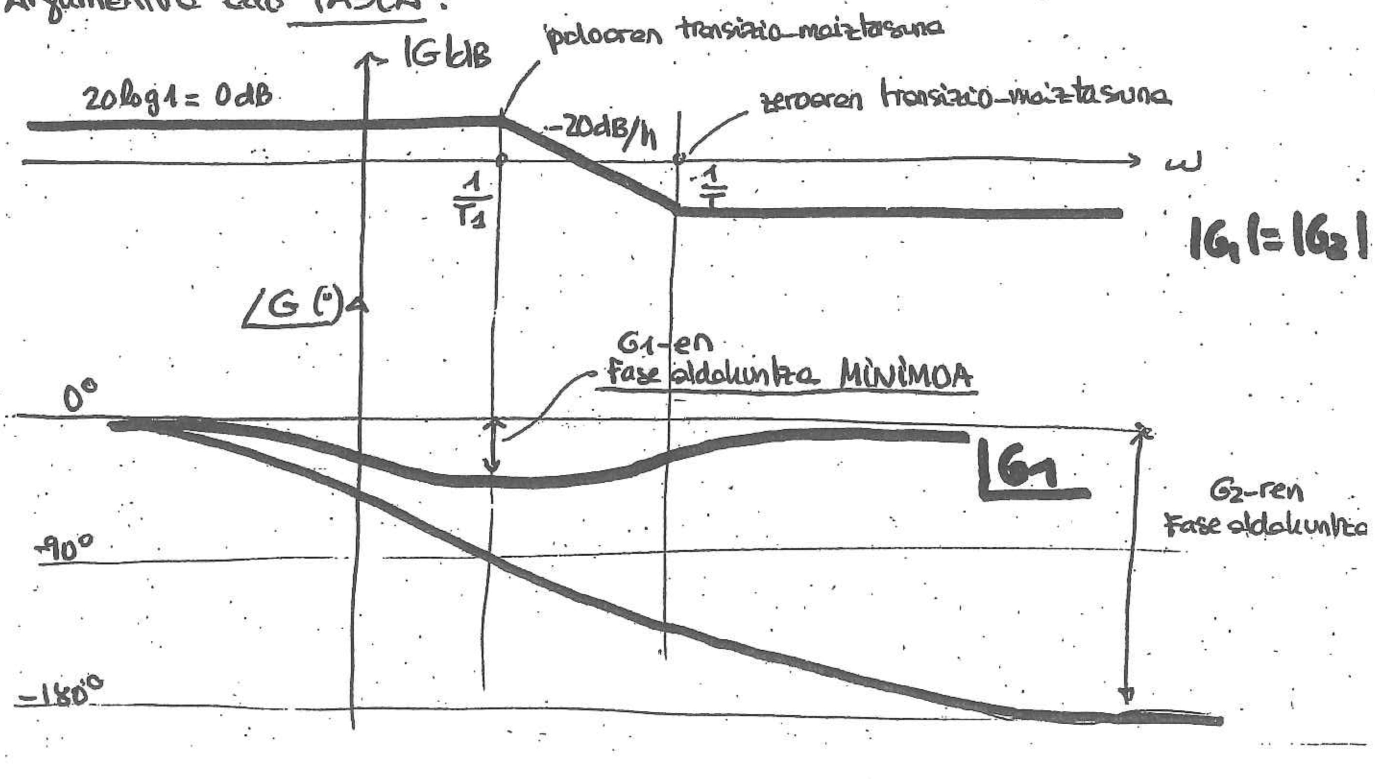


FASE MINIMOA du

Modulua:

$$|G_1| = |G_2| = \frac{\sqrt{1+\omega^2T^2}}{\sqrt{1+\omega^2T_1^2}}$$ modulu berbera dute

Argumentua edo FASEA:



SISTEMEN IDENTIFIKAZIOA

Fase minimoa duen sistema baten BODE diagrama ezagututa, transferentzi funtzioa lor dezakegu.

Gai indibidualen modulu eta fase aportazioa ezagutzen dugunez, posible da zein eratako terminoak euki ditzakeen asmatzea.

Honetarako asintoten bidezko aproximazioaz baliatuko gara:

- Modulu-kurbaren tangente ezberdinak marraztu
- Tangenteen arteko ebaki-puntuak faktore ezberdinen transizio-maiztasunak izango dira
- Asintoten maldak aztertuta, poloei edo zeroei dagozkien transizio-maiztasunak zein diren jakin daiteke.
- Maiztasun baxuetan begizta irekiko moduluaren malda zein den aztertuta, sistema berrelikatuaren mota identifika dezakegu (integratzaile kopurua), baita errore-koefizienteen balioa (Kp, Kv edo Ka).
- Irabazpenaren kalkulua maiztasun baxuetan erraz kalkula daiteke

GH-ren BODEAK

$$* \ \angle GH_{w\to\infty} = -90(n-m)$$

* w baxuan

→ Moduluaren maldetik → sistema mota ⇒ ess (Integratzaile kop)

Fasearen kurbatik ↗

Irabazpena   Kp Kv Ka

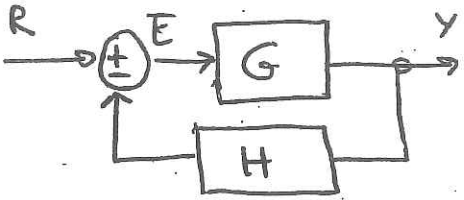
* w ertainetan → Dinamika (poloak, zeroak)

Esmortasuna

* w handietan → polo eta zero kopuru

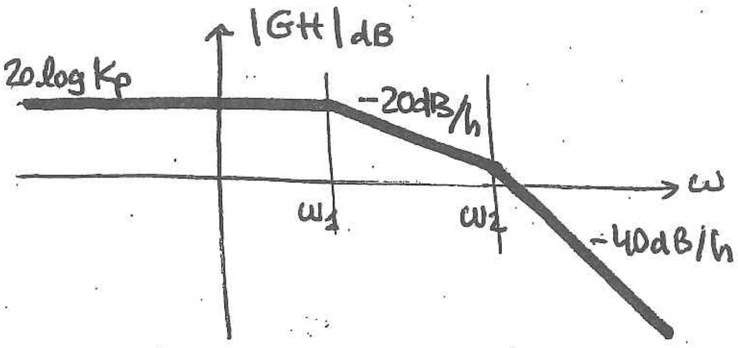
n ↑ m ↑ $n \geq m$

Suposatu:



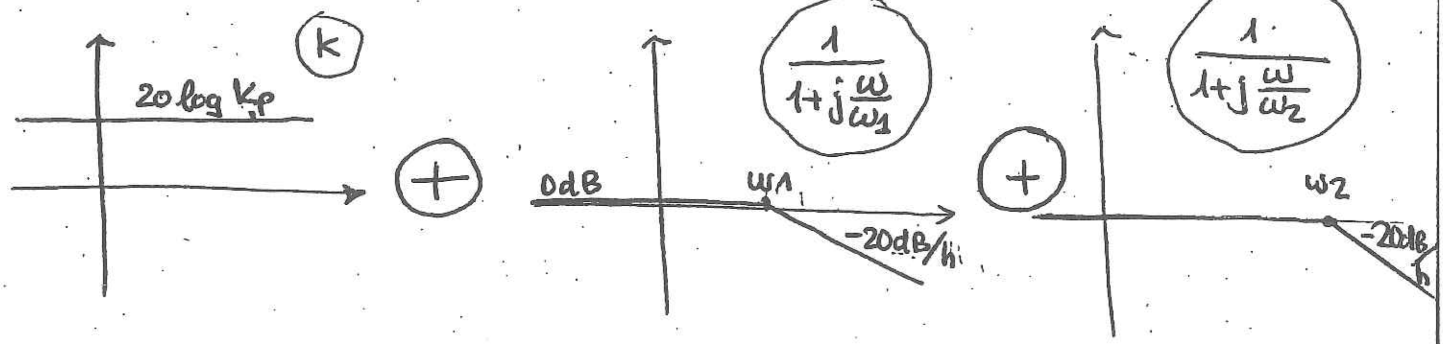
$s \to 0 \Rightarrow \omega \to 0$

①



$K_p = \lim_{\omega \to 0} G(j\omega)H(j\omega) =$

0 motakoa da, maiztasun baxuetan 0dB-tako malda duelako (integratzailerik daudenean, maiztasun baxuetako malda ≠ 0dB/h).



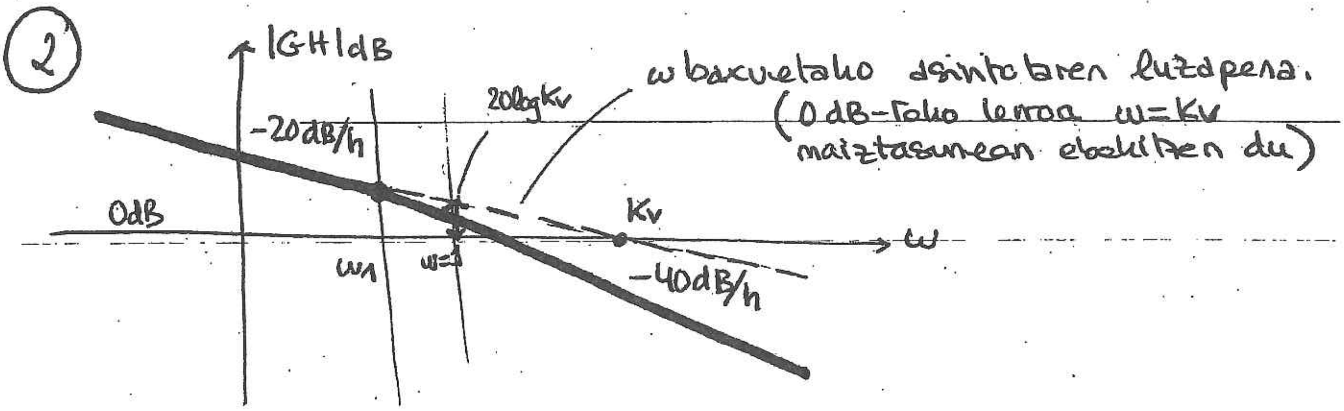
ω_1 : moduluaren kurbak duen malda 0dB-tik -20dB/h -ra igarotzen da ⟶ POLOA $T_1 = \frac{1}{\omega_1}$

ω_2 : malda -20dB/h ⟶ -40dB/h ⟶ BESTE POLO BAT $T_2 = \frac{1}{\omega_2}$

$$G(j\omega)H(j\omega) = \frac{K}{(1+j\omega T_1)(1+j\omega T_2)}$$

Argumentuaren kurbari begiratuta, 0°-tik -180°-ra jeitsiko dela ikusiko genduke.

②



ω baxuetako asintotaren luzapena. (0 dB-rako lerroa ω=Kv maiztasunean ebakitzen du)

ω baxuetan : $\omega \ll \omega_1$

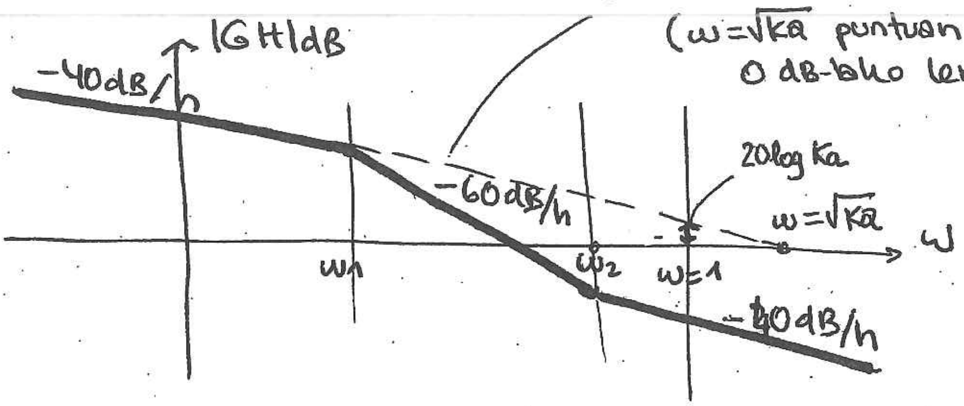
-20 dB/h malda duenez, integratzaile bat duela esan nahi du. Beraz, 1 motako stma da.

$$|G(j\omega)| \simeq \left|\frac{K_v}{j\omega}\right| \Rightarrow \left\{ \begin{array}{l} 20\log\left|\frac{K_v}{j\omega}\right| = 20\log K_v - 20\log\omega \\ (\omega = K_v \text{ bada} \rightarrow 0\,\text{dB}) \end{array} \right.$$

$0 < \omega < \omega_1$ bitartean: erraz kalkula daiteke Kv-ren balioa

Kv sistemaren irabazpena da (begizta irekiko stmarena)

③



ω baxuetako asintotaren luzapena. (ω=√Ka puntuan ebakitzen du 0 dB-tako lerroa)

ω baxuetan -40 dB/h malda ⟶ Bi integratzaile → 2 mota (-20 dB/h aportatzen du bakoitzak)

Maiztasun txikietako asintota: $20\log\left|\frac{K_a}{(j\omega)^2}\right|$

$$20\log K_a - 40\log\omega = 0\,\text{dB} \Rightarrow \omega = \sqrt{K_a}$$

$$20\log\left|\frac{K_a}{(j\omega)^2}\right|_{\omega=1} = 20\log K_a$$

EGONKORTASUN ERLATIBOA

Egonkortasun kritikoaren baldintza:

$$\left|G(j\omega)H(j\omega)\right| = 1$$
$$\angle G(j\omega)H(j\omega) = -180^\circ$$

} Aldi berean gertatzen direnean, egonkortasuna kritikoa da.

Lehenengo, bi baldintza horiek zein maiztasunetan gertatzen diren definituko dugu:

ω_g: IRABAZPEN KRITIKOAREN MAIZTASUNA

$$\left|G(j\omega)H(j\omega)\right|_{\omega=\omega_g} = 1 \quad (0dB)$$

$\text{Arg}\, G(j\omega_g)H(j\omega_g) < -180^\circ \Rightarrow$ egonk

$\text{Arg}\, G(j\omega_g)H(j\omega_g) > -180 \rightarrow$ ezegonkor

ω_f: FASE KRITIKOAREN MAIZTASUNA

$$\angle G(j\omega)H(j\omega)_{\omega=\omega_f} = -180^\circ$$

$|G(j\omega_f)H(j\omega_f)| < 1 \Rightarrow$ egonkora

$|G(j\omega_f)H(j\omega_g)| > 1 \Rightarrow$ ezegonkora

Egonkortasuna neurtzeko, bi parametro erabiltzen dira maiztasunaren eremuan:

- MG : IRABAZPENAREN TARTEA (dB)
- MF : FASEAREN TARTEA (°)

IRABAZPENAREN TARTEA: MG

Fasea $-180°$ denean (ω_f maiztasunean), zenbat handi daitekeen GH-ren irabazpena modulua 1 (0 dB) egin arte

$$MG(dB) = 20\log 1 - 20\log GH(j\omega_f) =$$
$$= 20\log\frac{1}{GH(j\omega_f)} = -20\log GH(j\omega_f)$$

$$\boxed{MG(dB) = -20\log GH(j\omega_f)}$$

Irabazpena kontrolagailuaren bidez handi daiteke.

$$\boxed{MG(dB) = 20\log K_{cr}}$$ $K\uparrow \quad MG\downarrow$

FASEAREN TARTEA: MF

Modulua 1 (0 dB) denean (ω_g maiztasunean gertatzen da), zenbat atzeratu daitekeen GH-ren fasea $-180°$ izan arte

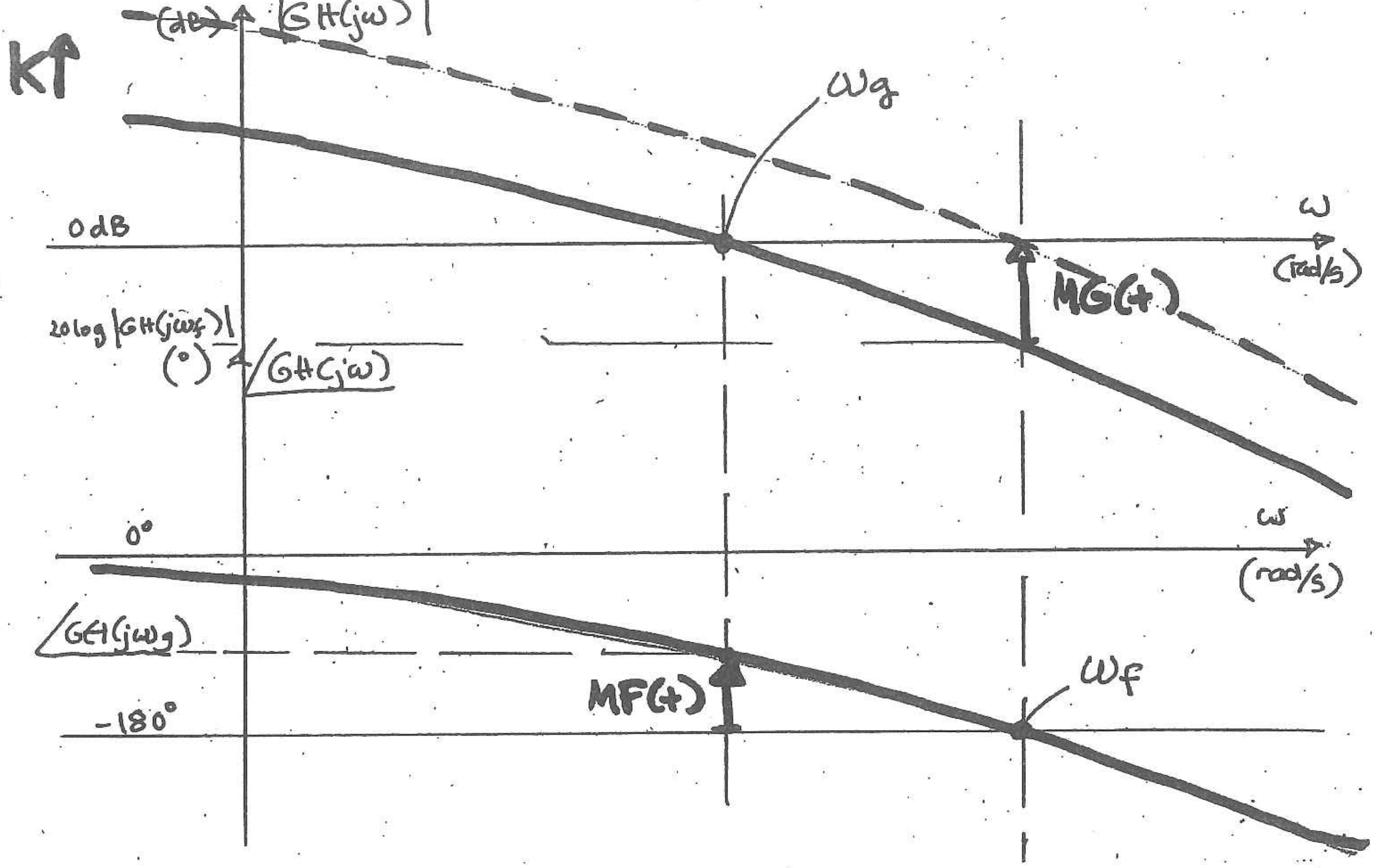
$$\boxed{MF = 180° + \angle GH(j\omega_g)}$$

EGONKORTASUNAREN IRIZPIDEA

* MF (+) eta MG (+) izan behar dira sistema egonkorra izateko.

* MF = 0 eta MG = 0 $\longrightarrow$ Egonkortasun kritikoa

GRAFIKOKI BODE diagraman:



K handitzean (kontrolagailuaren irabazpena), modulu kurba gorantz mugituko da, MG txikiagoa eginez.

K↑ ω_g↑ MF⇓ MG⇓

$\omega_g = \omega_f$ denean → KRITIKOA

$\omega_g < \omega_f$ → Egonkorra

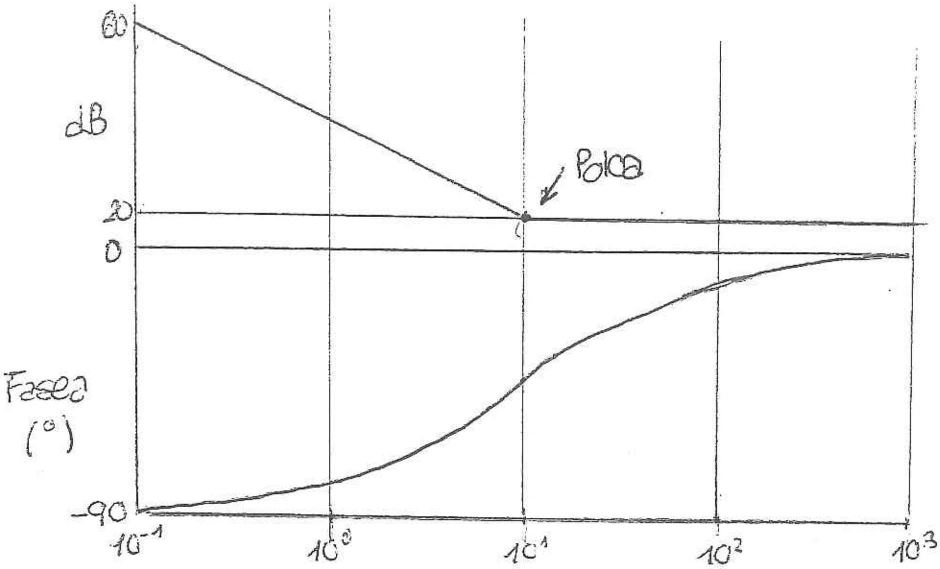
$\omega_g > \omega_f$ → Ezegonkorra

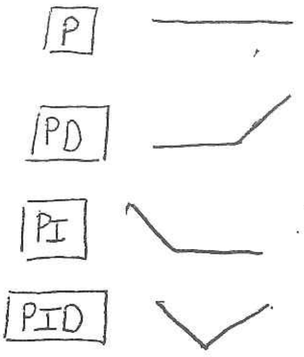
Egonkortasunaren mugan:

$$\omega_g = \omega_f = \omega_{cr} = \omega_u = \omega_n$$

7. GAIKO ARIKETAK

④





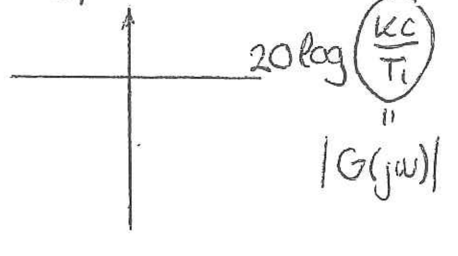
PI ekintzek osatzen dute kontrolagailua integradore bat dagoelako egiten duenak malda behera joatea.

$$G_{PI}(s) = \frac{K_c(1+T_i s)}{T_i\, s} \longrightarrow \frac{K_c(1+j\omega T_i)}{T_i \cdot j\omega}$$

$\boxed{T_i = 0{,}1 s}$

↳ Poloa nan behar du: $\left(1+s\cdot\frac{1}{10}\right) = (1+s\cdot T_i) \Rightarrow T_i = \frac{1}{10}$

Irabazpena:

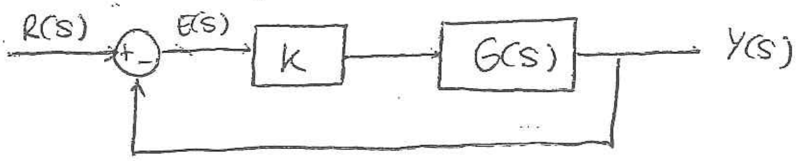


$$|G|_{\omega=1} = 60\,dB = \overbrace{20\log\frac{K_c}{T_i}}^{\text{irabazpena}} + \overbrace{20\log\frac{1}{\omega}}^{\text{integradorea}} = 20\log\frac{K_c}{T_i\omega}$$

$$60\,dB = 20\log_{10}\frac{K_c}{0{,}1\cdot 0{,}1} \Rightarrow \log\frac{K_c}{0{,}01} = 3$$

$$\frac{K_c}{0{,}01} = 10^3 \Rightarrow \boxed{K_c = 10,}$$

⑨



Grafikotik:

↳ Fasea -180° → $\omega_{fc} = 4{,}5$

MG = 15

aurretik. ↳ 0 dB → $\omega_{gc} = 0{,}4$ rad/s

MF = 90

15 dB igo ditzakegu kritikora

$15\,dB = 20\log K_{cr} \Rightarrow K_{cr} < 10^{15/20}$

①

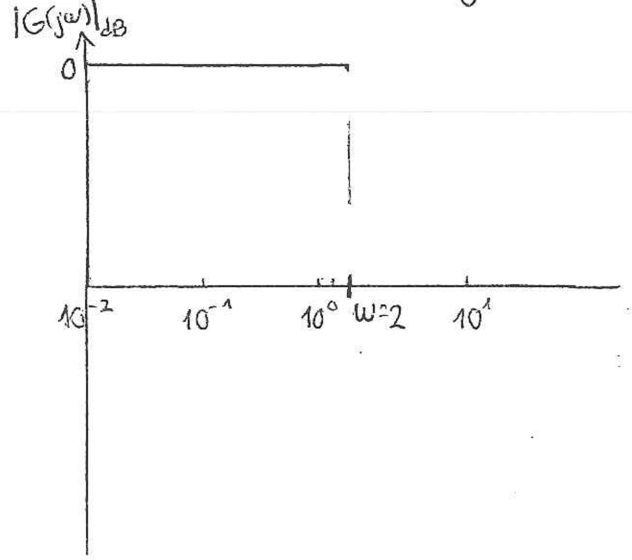
a) $G(s) = \frac{4}{s+2}$ Poloa: $G(s) = \frac{1}{1+\tau s}$ $\Rightarrow$ $G(s) = \frac{4/2}{2(1+\frac{s}{2})} = \frac{2}{1+s/2}$

$\tau = \frac{1}{2}$; hausle maiztasuna: $\omega_n = 2\ rad/s$

$K=2 \Rightarrow K_{DB} = -20 \log \sqrt{1+\omega^2\tau^2}$

$\omega \to 0$: $G(j\omega) \to 2$, $|G(j\omega)| = 20 \log |G(j\omega)| \to 0$ // $\angle G(j\omega) = 0°$

$\omega \to \infty$: $|G(j\omega)| \simeq -20 \log(\omega\tau) = \ldots$ // $\angle G(j\omega) = -90°$



b) $G(s) = \frac{40}{s^2+s+4} = \frac{10}{0{,}25s^2+0{,}25s+1}$

$K_{dB} = 20 \log 10 = 20$

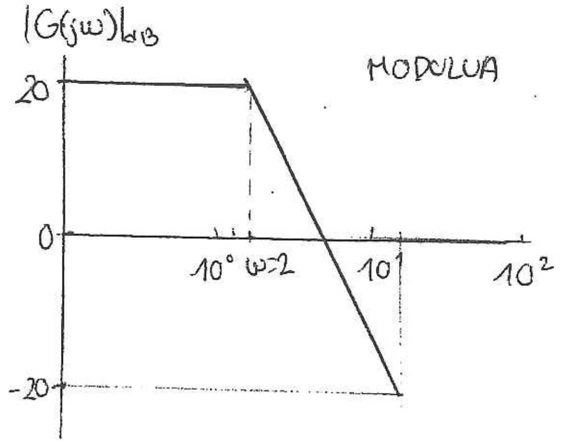
$k = 10$

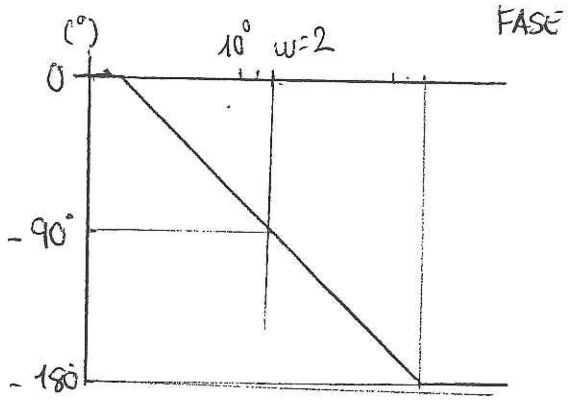
$s^2+s+4 = s^2+2\delta\omega_n+\omega_n^2 \rightarrow \begin{cases} \omega_n = 2\ rad/s \\ 2\delta \cdot 2 = 1 \rightarrow \delta = 0{,}25 \end{cases}$

Asintotak:

$\omega \to 0$: $|G(j\omega)|_{dB} = 0$;

$\omega \to \infty$: $-40\ dB/h$ malda duen asintota





d) $G(s) = \frac{8}{s(1{,}25s+1)\underbrace{(s+2)}_{2(\frac{1}{2}s+1)}} = \frac{4}{\boxed{s}(1{,}25s+1)(0{,}5s+1)}$ ↳ integratzailea ; diagama hasiko da -20 dB/dec -eko maldarekin.

$k=4$

$K_{dB} = 20\log K = 12{,}05 \longrightarrow \omega = 1$

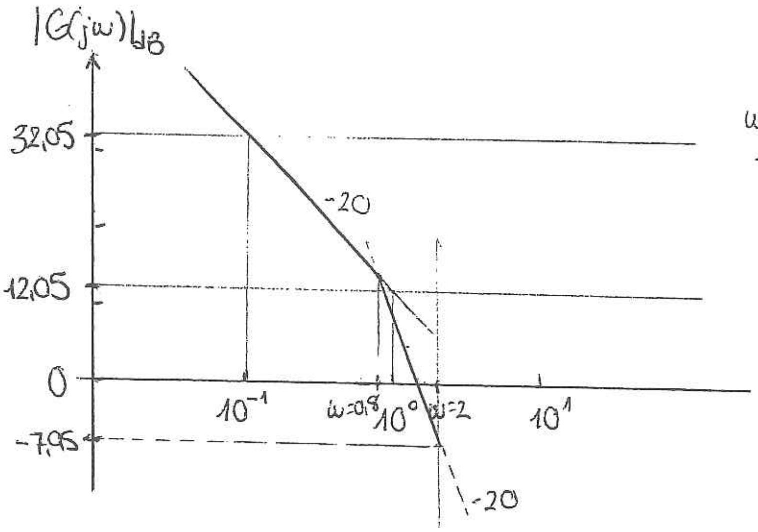
$s(0{,}625s^2 + 1{,}75s + 1)$

Poloak: $s=0 \longrightarrow \omega = 0$

$s = -\frac{1}{1{,}25} \longrightarrow \omega = \frac{1}{\tau} = 0{,}8$ rad/s ⇒ puntu honetatik -20-ko malda

$s = -2 \longrightarrow \omega = \frac{1}{\tau} = 2$ rad/s

	4	$\frac{1}{s}$	$\frac{1}{1+0{,}5s}$	$\frac{1}{1+1{,}25s}$	
$\omega=0 \rightarrow \omega=0{,}8$	0	-20	0	0	-2
$\omega=0{,}8 \rightarrow \omega=2$	0	-20	0	-20	-4
$\omega=2 \rightarrow \omega=\infty$	0	-20	-20	-20	-6



② $G(s) = \frac{1}{(s+1)(0{,}1s+1)}$ ⇒

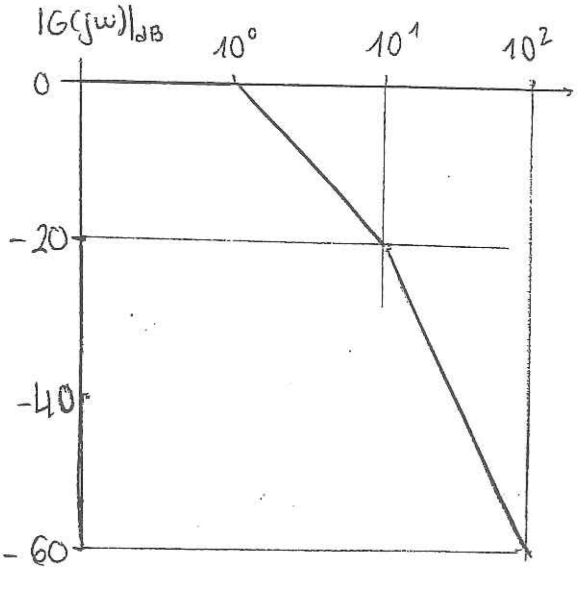
	1	$\frac{1}{s+1}$	$\frac{1}{1+0{,}1s}$
$0 \rightarrow 1$	0	0	0
$1 \rightarrow 10$	0	-20	0
$10 \rightarrow \infty$	0	-20	-20

$r(t) = 2\,\text{sen}(0{,}5t)$ (1)

$r(t) = 2\,\text{sen}(5t)$

Poloak: $s = -1 \rightarrow \omega = 1$ rad/s

$s = -0{,}1 \rightarrow \omega = 10$ rad/s



$Y(s) = G(s)R(s)$

(1) $R(s) = \frac{\omega A}{s^2+\omega^2} = \frac{1}{s^2+0{,}25}$

$Y(s) = \frac{1}{(s^2+0{,}25)(s+1)(0{,}1s+1)}$

$y_{ss} = \frac{-2G(-j0{,}5)}{2j}e^{-j0{,}5t} + \frac{2 \cdot G(j0{,}5)}{2j}e^{+j0{,}5t}$

(2) $R(s) = \frac{10}{s^2+25}$

$y_{ss} = \frac{10\,G(5j)}{2j}e^{j5t} - \frac{10\,G(-j5)}{2j}e^{-j5t}$

⑤

Behe maiztasuneko asintotak jatorrian dauden polo/zero kopurua adierazten du: ± 20×n dB/h

Asintotak maldarik ez duenez, bere altueratik K atera daiteke:

$$20\log K = 20 \Rightarrow K = 10$$

Malda aldaketa bakoitzeko polo bat edukiko dugu, malda aldaketa guztiak negatiboak direlako.

- Hauste maiztasunak:
  - $\omega = 10$ rad/s $\Rightarrow (1+s\frac{1}{10})$
  - $\omega = 30$ rad/s $\Rightarrow (1+s\frac{1}{30})$
  - $\omega = 100$ rad/s $\rightarrow (1+s\,1/100)$
  - $\omega = 300$ rad/s $\Rightarrow (1+s\,1/300)$

$$G(s) = \frac{10}{(1+\frac{s}{10})(1+\frac{s}{30})(1+\frac{s}{100})(1+}$$

⑥

Aurreko ariketako prozedura berdina:

$K = 10$

$\omega = 0{,}4$ rad/s $\rightarrow (1+\frac{s}{0{,}4})^{②} = (1+2{,}5s)^2$ $\rightarrow$ -40-ko malda dauka $\Rightarrow$ bi polo $\rightarrow$ polo biko

$\omega = 10$ rad/s $\rightarrow (1+0{,}1s)$

$$\Rightarrow \boxed{G_{BA}(s) = \frac{10}{(1+2{,}5s)^2(1+0{,}1s)}}$$

$$e_{ss} = \frac{1}{1+K_p} \quad \text{non} \quad K_p = \lim_{s\to 0} \frac{10}{(1+2{,}5s)^2(1+0{,}1s)} = 10$$

$$\boxed{e_{ss} = \frac{1}{11} = 0{,}09}$$

⑦ Berelikadua unitarioa ; $MF = 45°$; $G(s) = \frac{as+1}{s^2}$

$$MF = 180 + Arg(G_{BA}(j\omega_g))$$

ω_g lortzeko $|G(j\omega_g)|_{BA} = 1$ dela esango dugu:

$$|G(j\omega_g)| = \frac{\sqrt{1+(a\omega_g)^2}}{\omega_g^2} = 1 \Rightarrow 1+(a\omega_g)^2 = \omega_g^4$$

$$\omega_g^2 = \frac{-a^2 \pm \sqrt{a^4+4}}{-2} = \frac{a^2}{2} \pm \frac{\sqrt{a^4+4}}{2} \Rightarrow \omega_g^2 > 0$$

$$\omega_g = \sqrt{\frac{a^2+\sqrt{a^4+4}}{2}}$$

$$Arg(G_{BA}(j\omega_g)) = \arctan(a\omega_g) - 180°$$

$$MF = \cancel{180°} + \arctan(a\omega_g) - \cancel{180°} \Rightarrow 45° = \arctan(a\omega_g)$$

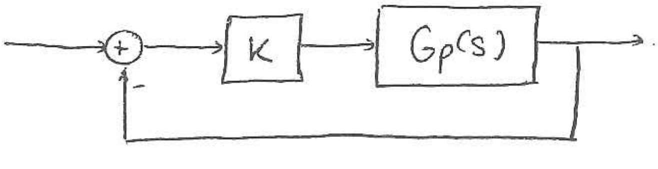
$$1 = a\omega_g$$

$$1 = a\sqrt{\frac{a^2+\sqrt{a^4+4}}{2}} \Rightarrow 1 = a^2\left(\frac{a^2+\sqrt{a^4+4}}{2}\right)$$

$$\Rightarrow \boxed{a = 0{,}84}$$

⑧ $G_{BA}(s) = \frac{ke^{-s}}{s}$; K_{max} egonkortasuna bermatzeko?

$$G_p(s) = \frac{e^{-s}}{s}$$



$$G_p(j\omega) = \frac{1}{j\omega}e^{-j\omega}$$

$$|G_p(j\omega)| = \frac{1}{j\omega} \quad ; \quad Arg(G_p(j\omega)) = Arg\left(\frac{1}{j\omega}\right) = -90° = -\pi/2$$

$$\hookrightarrow |G(j\omega)|_{dB} = -20\log\omega$$

$$Arg(G(s)H(s)) = -180 = -\pi = Arg(G_p(j\omega_f)) = -\omega_f - Arg(G_p(j\omega))$$

egonkorra izateko

$$\hookrightarrow \omega_f = \pi/2 \text{ rad/s}$$

Maiztasuna ω_f denean: $|G_{BA}(j\omega_f)| = 1$

$$G_{BA}(j\omega) = \frac{1}{j\omega}Ke^{-j\omega}$$

$$|G_{BA}(j\omega_f)| = 1 = \frac{K}{\omega_f}$$

$$\left[K = \omega_f = \frac{\pi}{2}\right]$$

⑩

a)

Grafikotik lortzen dugu fasearen kurbak ez duenez -180°-tako zuzena mozten ⇒ $\boxed{\begin{array}{l} MG=\infty \\ MF \simeq 74° \end{array}}$

b)

Hasieran grafikoak beherantz egiten duenez, badakigu integradore bat duela → 1. MOTA → $\boxed{e_{ss}=0}$

c)

Arrapala sarrerarekin:

$$\left[e_{ss} = \frac{0{,}5}{K_v} = \frac{0{,}5}{\omega_g} = \frac{1}{1} \cdot 0{,}5 = 0{,}5 \right]$$

d)

$r(t) = 3sen(5t)$

↳ $\omega = 5$ rad/s $\xrightarrow{grafikoa}$ $|G(j\omega)| = -20$ dB atenuazio handia dauka maiztasun horretarako

⇒ ez da gai sarrerari jarraitzeko.

e)

Bai, nahi duguna handitu dezakegu, $MG=\infty$ delako.

⑪

a) MG? MF?

Grafikotik: $MF \simeq -50°$; $MG \simeq -40$ dB

b) K zenbat handitu?

-40 dB igo ditzakegu kritikora ⇒ $-40 \text{ dB} = 20\log\frac{K_{cr}}{T_i} + 20\log\frac{K_{cr}}{\omega}$

$$-40 = 20\log\frac{K_{cr}}{T_i\omega}$$

$T_i = \frac{1}{1}$ ⇒ Poloa izan behar du: $(1+sT_i)$

$(1+s)$

$$G_{PI}(s) = \frac{K_c(1+T_is)}{T_i s} = \frac{K_c(1+s)10}{s}$$

$$-40 = 20\log\frac{10K_{cr}}{T_i} + 20\log\frac{10K_{cr}}{\omega} + 60\log\frac{K_{cr}}{10^2}$$

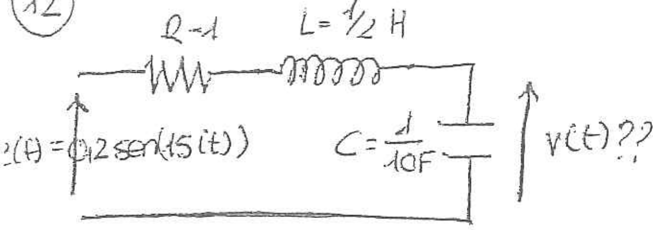
• Beste poloak:

↳ $\omega = 10^2 \rightarrow s = -\frac{1}{10^2} \rightarrow \left(1+\frac{s}{10^2}\right)$

$$10^{-40} = 20K_{cr} + 20K_{cr} + 0.6K_c$$

$20\log K_{cr}-\omega_1 + 20\log 10^2 K_{cr}$

⑫



$$e(t) = V_R + V_L + v(t)$$

$$\begin{cases} V_R = R \cdot i(t) \\ V_L = L \cdot \dfrac{di(t)}{dt} \\ v(t) = \dfrac{1}{C}\displaystyle\int_0^t i(t)\,dt \Rightarrow \dfrac{dv(t)}{dt} = \dfrac{1}{C}\, i(t) \end{cases}$$

$$i(t) = \frac{dv(t)}{dt} \cdot C$$

$$V_R = R \cdot C \cdot \frac{dv(t)}{dt}$$

$$V_L = L \cdot C \cdot \frac{d^2v(t)}{dt}$$

$$\rightarrow e(t) = RC\,\frac{dv(t)}{dt} + LC\,\frac{d^2v(t)}{dt} + v(t) \overset{\mathcal{L}}{\Rightarrow} E(s) = \left(\frac{1}{10}s + \frac{1}{20}s^2 + 1\right)V(s)$$

$$\frac{V(s)}{E(s)} = \frac{1}{\frac{1}{10}s + \frac{1}{20}s^2 + 1} = G(s) \;; \quad e(t) = 0{,}2\,\mathrm{sen}(15t) \begin{cases} A = 0{,}2\ v \\ \omega = 15\ \mathrm{rad/s} \end{cases}$$

$$G(j\omega) = \frac{1}{-\frac{1}{20}\omega^2 + \frac{1}{10}j\omega + 1} \longrightarrow |G(j\omega)| = \frac{1}{\sqrt{\left(1 - \frac{1}{20}\omega^2\right)^2 + \left(\frac{1}{10}\omega\right)^2}}$$

$$\mathrm{Arg}(G(j\omega)) = 0 - \arctan\left(\frac{1/10\,\omega}{1 - \frac{1}{20}\omega^2}\right)$$

$$\omega = 15\ \mathrm{rad/s} \Longrightarrow \begin{cases} |G(j\omega)| = 0{,}0965 \\ \mathrm{Arg}(G(j\omega)) = 0 - (180^\circ - 8{,}32^\circ) = -171{,}67^\circ \end{cases}$$

$$\left[v(t) = (0{,}2 \cdot 0{,}0965)\sin(15t - \underbrace{171{,}67^\circ}_{\text{radianetan jarri}}) \doteq 0{,}0193 \sin(15t - 171{,}67^\circ)\right]$$

⑬ $r(t) = 2\sin(3t) \begin{cases} A = 2 \\ \omega = 3\ \mathrm{rad/s} \end{cases}$

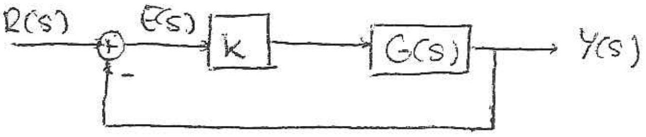
$$G(s) = \frac{10(s+1)}{s^2(s+5)} \;; \quad G(j\omega) = \frac{2\,(j\omega + 1)}{-\omega^2\left(1 + \frac{1}{5}j\omega\right)} =$$

$$|G(j\omega)| = \frac{2}{\omega^2\sqrt{1 + \omega^2\left(\frac{1}{5}\right)^2}\,\sqrt{1 + \omega^2 \cdot 1}} \cdot 10 \overset{\omega = 3\ \mathrm{rad/s}}{\doteq} 0{,}6$$

$$\mathrm{Arg}(G(j\omega)) = \underbrace{\arctan(\omega)}_{\text{zeroa}} - \underbrace{\arctan\left(\frac{\omega}{5}\right)}_{\text{poloa}} - \underbrace{90^\circ}_{\text{integradorea}} \overset{\omega = 3\ \mathrm{rad/s}}{\doteq} -139{,}39 = -139{,}4^\circ$$

$$\left[y(t) = 2 \cdot 0{,}6 \sin(3t - 139{,}4^\circ) = 1{,}2 \sin(3t - 139{,}4^\circ)\right]$$

⑭



$$e_{ss} = \frac{1}{K_v} = \frac{1}{50} \Rightarrow K_v = 50$$

↳ $e_{ss} \neq 0$ arrapala unitarioan izateko, sistema 1. motakoa izan behar da, beraz, integradore bat dauka.

- Arrapala unitarioan 1/50 errore iraunkorra
- irabazpen tartea 6 dB
- G(s)·K polo bat du jatorritik kanpo

$$K_v = \omega = 50_f \Rightarrow \tau = \frac{1}{\omega} = \frac{1}{50}$$

↳ Poloa: $\left(s\frac{1}{50}+1\right)^2$ (bikoitza)

$\cancel{}$ MG = 6 dB

$$MG = 20\log\left(\frac{1}{G_{BA}(j\omega_f)}\right)$$

$$\frac{6}{20} = \log\left(\frac{1}{G_{BA}(j\omega_f)}\right) \Rightarrow -\frac{6}{20} = \log(G_{BA}(j\omega_f))$$

$$10^{-6/20} = G_{BA}(j\omega_f)$$

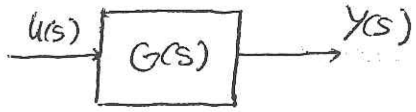
MG: Marjina ω_f denean $|G_{BA}(j\omega)| = 1$ izateko biderkatu beharko litzatekeen faktorea

$$|G_{BA}(j\omega_f)| = 1 \Rightarrow K = 1$$

$$\left[G(s) = \frac{1}{s(1+s/50)^2} \right]$$

3) (2012/2013 URTARRILA)

$$G(s) = \frac{-0{,}85}{1+0{,}25s}$$



$K = 0{,}85$

Transferentzi-funtzioak daukan zeinu negatiboak esan nahi du u handitzen denean y txikituko dela, hau da, alderantzizko proportzionaltasuna dagoela. Horregatik kontutan hartu behar ditugun OP-ak u2-tik aurrera daudenak dira, u2-koa barne.

- u4-ko puntuak ez du balio, hortik aurrera y handitzen hasiko delako berriro.
- $K = \frac{\bar{y}}{\bar{u}} \rightarrow \begin{cases} u_2: K = 1/2{,}5 = 0{,}4 \\ u_3: K = \frac{0{,}56}{3{,}25} = 0{,}17 \end{cases}$??

⑮ (2013-2014 URTARRILA)

a) $G(s)$?

$\omega_1 = 0{,}1$ rad/s
$\omega_2 = 1$ rad/s ‖ Grafikotik
$\omega_3 = 10$ rad/s

↳ hemendik aurrera malda -60 → zero

→ -40 dB/h -ko malda →

→ -20 dB/h · 2 ⇒ 2 polo ⊛

Hasiean beheranzko -20 dB/h -ko malda bat dauka, beraz, polo bat dauka jatorrian. Transferentzia funtzioak hurrengo itxura izango du:

$$G(j\omega) = \frac{K(1+10j\omega)}{j\omega(1+0'1j\omega)(1+j\omega)^{2⊛}}$$

$$T_1 = \frac{1}{0{,}1} = 10s \; ; \; T_2 = \frac{1}{1} = 1s \; ; \; T_3 = \frac{1}{10} = 0{,}1s$$

ω baxuetan: $\omega < \omega_1$ ⇒ -20dB/h -ko malda → integradorea →

$$\Rightarrow |G(j\omega)| \simeq \left|\frac{K_v}{j\omega}\right| \Rightarrow 20 \log \frac{K}{\omega_1} = 80 \Rightarrow k = 1000$$

$$G(s) = \frac{1000(1+0{,}1s)}{s(1+10s)(1+s)^2}$$

b) Egonkortasuna?

Bi eratara aztertu dezakegu:

① MF eta MG-ren bidez ⇒ Negatiboak badira ⇒ ezegonkorra

MG = - 60 dB
MF = - 112,5° ‖ Grafikotik ⇒ EZEGONKORRA

② R-H :

Ek. karakteristikoa : $1 + G(s)H(s) = 0 \rightarrow 1 + \dfrac{1000(1+0{,}1s)}{s(1+10s)(1+s)^2} = 0$

$\Rightarrow s(1+10s)(1+s)^2 + 1000(1+0{,}1s) = 0 \rightarrow 10s^4 + 21s^3 + 12s^2 + 101s + 1000 = 0$

B.B: koefizienteak guztiak dira eta zeinu berekoak.

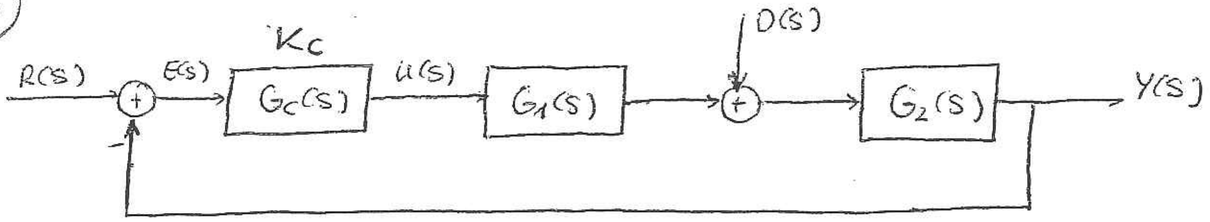
BN:

$$\begin{array}{c|ccc} s^4 & 10 & 12 & 1000 \\ s^3 & 21 & 101 & 0 \\ \hline s^2 & b_1 & b_2 & \\ s & & & \\ s^0 & & & \end{array}$$

$$b_1 = \frac{21 \cdot 12 - 10 \cdot 101}{21} = -36{,}09 < 0$$

↳ lehenengo zutabeko elementu guztiak ez dira positiboak ⇒ EZEGONKORRA

⑯



r(t) ereferentzian : 2 anplitudeko maila : $R(s) = 2/s$

d(t) perturbazioan : 0,5 anplitudeko maila, $D(s) = \frac{0,5}{s}$

$G_1(s)$ eta $G_2(s)$ grafikoetatik lortuko ditugu:

- $G_1(s)$: Lehenengo ordeneko sistema. ($e_{ss_{G_1}} = 0$)

$$K = 1 \left\{ \begin{array}{l} \Delta y = 1 \\ \Delta u = 1 \end{array} \right\} \qquad G_1(s) = \frac{1}{s+1}$$

$$y_{63} = \bar{y} + 0,63 \cdot \Delta y = 0,63 \rightarrow t_{63} = 1s = \tau$$

- $G_2(s)$ (BODE diagramatik)
  - Diagramak maiztasun txikietan -20 dB/h-ko malda dauka beraz polo bat egongo da jatorrian.
  - $\omega = 10$ rad/s-tan beste polo bat $\longrightarrow T = \frac{1}{\omega} = \frac{1}{10} = 0,1s \rightarrow (1+0,1s)$
  - $\underbrace{20 \log\left(\frac{K}{\omega}\right)}_{\text{hasierakoa}:\ \omega = 10^{-1}} = 20 \Rightarrow \log \frac{K}{\omega} = 1 \rightarrow \frac{K}{\omega} = 10 \Rightarrow \underline{K = 1}$

$$G_2(s) = \frac{1}{s(1+0,1s)} \quad 1.\ \text{motakoa} \rightarrow e_{ss_p} = 0 \longrightarrow e_{ss_{G_2}} = 0$$

$$- e_{ss} = e_{ss}\Big|_{\substack{D(s) \\ R(s)=0}} + e_{ss}\Big|_{\substack{R(s) \\ D(s)=0}} = e_{ss_D} + e_{ss_R}$$

$e_{ss_D} \neq 0$

- $e_{ss_R} \Rightarrow$ sistema 1 motakoa da $\rightarrow e_{ss_R} = 0$
- $e_{ss_D} \rightarrow$ Integradorea daukagunez $G_2(s)$-n eta perturbazioa eta gero dagoenez

$$E(s) = - \frac{D(s) \cdot G_2(s)}{1 + G_c(s) \cdot G_1(s) G_2(s)} = - \frac{0,5}{s} \cdot \frac{1}{\frac{1}{G_2(s)} + \frac{K_c}{s+1}}$$

$$e_{ss_D} = \lim_{s \to 0} \cancel{s} \cdot \left(-\frac{0,5}{\cancel{s}}\right) \cdot \frac{1}{\cancel{\frac{1}{G_2(s)}}_{0} + \frac{K_c}{s+1}} = \frac{1}{K_c} \cdot (-0,5) ?$$

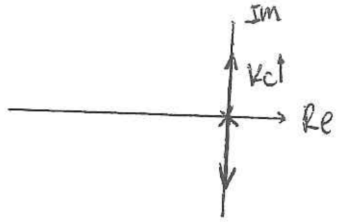
$e_{ss} = 0 + \underline{\underline{\qquad}}$?

⑰ (2013 / 2014 URTARRILA)

- Integatzaile bikoitza:

a) Posible da sistema egonkortzeko kontrolagailu proportzionala diseinatzea?

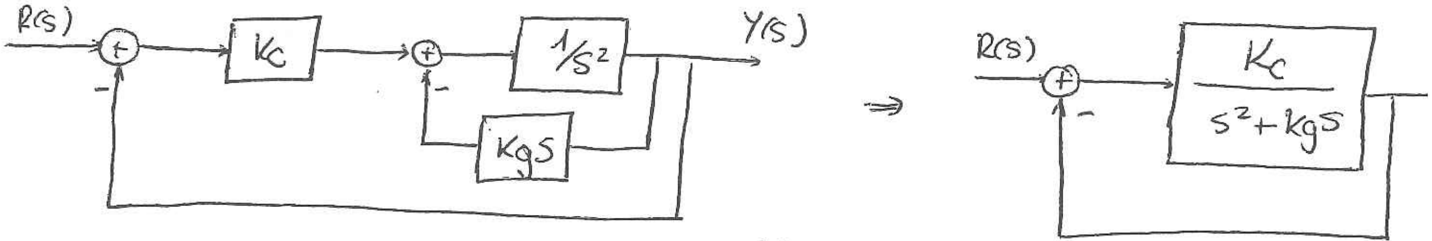
$G(s) = \frac{K_c}{s^2}$ ⇒ Polo bikoitza jatorrian



K_c handitzean ETG-k ardatz irudikarian gora eta behera egingo du, beti egonkortasuna mantenduz, beraz, P batekin posible da egonkor mantentzea $\forall K_c$-rako.

$\underset{\downarrow}{\text{egonkor}}$ Kritikoki egonk

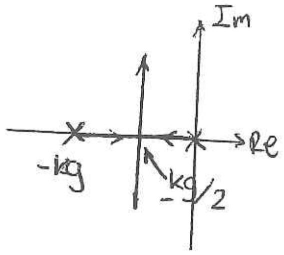
b) P kontrola + abiaduraren berrelikadura.



$G_{BA}(s) = \frac{K_c}{s^2+k_g s}$; $G_{BC}(s) = \frac{K_c}{s^2+k_g s+K_c}$

Poloak:

$s(s+k_g)$ ⇒ $s=0$, $s=-k_g$



c) HF = 45° ; tss (2%) = 4 s ; kg? Kc?

$$t_s(2\%) = \frac{4}{\delta\omega_n} = 4 \rightarrow \delta\omega_n = 1 \rightarrow -\frac{k_g}{2} = \delta\omega_n$$

$$+\frac{k_g}{2} = +1 \rightarrow \boxed{k_g = 2}$$

$$s^2 + \underbrace{k_g s}_{2s} + K_c = s^2 + 2\delta\omega_n s + \omega_n^2 \quad \begin{cases} 2\delta\omega_n = \cancel{2} \rightarrow \delta\omega_n = 1 \\ \omega_n^2 = K_c \end{cases}$$

$$HF = 180 + \text{Arg}\, G_{BA}(j\omega_g) \qquad G_{BA}(j\omega) = \frac{K_c}{(j\omega)^2 + 2j\omega} = \frac{K_c}{2\omega j - \omega^2} = \frac{K_c}{j\omega(j\omega+K}$$

$$G_{BA}(j\omega_g) = \frac{K_c}{j\omega_g(j\omega_g+k_g)} \longrightarrow \arg[G_{BA}(j\omega)] = -\left[90 + \arctan\left(\frac{\omega_g}{k_g}\right)\right]$$

↳ $\underbrace{\arg(\text{polo jatorrian})}_{-90} + \underbrace{\arg(\text{polo})}_{-\arctan(\omega T)}$

$T = 1/k_g$

$$45 - 180 = -\left[90 + \arctan\left(\frac{\omega_g}{k_g}\right)\right]$$

$$45 = \arctan\left(\frac{\omega_g}{k_g}\right) \Rightarrow \frac{\omega_g}{k_g} = 1 \longrightarrow \omega_g = k_g = 2$$

$$|G_{BA}(j\omega_g)| = 1$$

$$\frac{K_c}{\omega_g\sqrt{\omega_g^2+K_g^2}} = 1 \longrightarrow K_c = 2\sqrt{2^2+2^2} \longrightarrow \boxed{K_c = 4\sqrt{2}}$$

d)

Ek. Karakteristikoa: $s^2 + K_g s + K_c = 0$

$$s^2 + 2s + 4\sqrt{2} = 0$$

$$\underline{\underline{s_{1,2}}} = \frac{-2 \pm \sqrt{4 - 16\sqrt{2}}}{2} = \underline{\underline{-1 \pm 4{,}32j}}$$

30/Noviembre/2013 - AUTOMATIKAKO PARTZIALA

①

sarrera : inpultsu-sarrera

a) $G(s) = \frac{-3,5}{4s+1} e^{-1,25s}$ [✓ b)] $G(s) = \frac{-10}{1,8s+1} e^{-1,2s}$

c) $G(s) = \frac{-10}{4s+1} e^{-2,25s}$ d) $G(s) = \frac{3,5}{1,8s+1} e^{-2,25s}$

$\Delta u = 2,75 - 2,5 = 0,25$

$\Delta y = -7 - (-4,5) = -2,5$ || $K = \frac{\Delta y}{\Delta u} = \frac{-2,5}{0,25} = -10$ ⇒ a) eta d) ezinezkoak.

$y(t_{63}) = -4,5 + 0,63(0,25)(-10) = -6,08 \Rightarrow t_{63} \simeq 4-1 = 3 \text{ min}$ ||

$y(t_{28}) = -4,5 + 0,28(-2,5) = -5,2 \rightarrow t_{28} \simeq 2,8-1 = 1,8 \text{ min}$ || ⇒

$$\Rightarrow \left\{ \begin{array}{l} t_{63} = t_m + \tau \rightarrow 3 - t_m = \tau \\ t_{28} = t_m + \frac{\tau}{3} \rightarrow 1,8 \cdot 3 = 3t_m + \tau \end{array} \right\} \begin{array}{l} 5,4 = 3t_m + 3 - t_m \\ t_m = 1,2 \end{array}$$

$G(s) = \frac{K}{\tau s+1} e^{-t_m s}$ → $\boxed{b)}$ da aukera

2)

$a_1 = 1,3696$

$a_2 = 0,5109$

$a_3 = 0,1904$

$a_4 = 0,0709$

$a_5 = 0,0264$

a) $G(s) = \frac{10}{0,1s^2+0,6s+10}$ [✓ b)] $G(s) = \frac{10s}{0,1s^2+0,6s+10}$

c) $G(s) = \frac{10s}{s^2+6s+10}$ d) $G(s) = \frac{10s}{s^2+6s+100}$

$y(t_p) = a_1 = 1,3696$

$y_{ss} = a_1 - a_2 + a_3 - a_4 + a_5 = 1,0046$

$$M_p = \frac{y(t_p) - y_{ss}}{y_{ss}} = 0,363 = e^{\frac{-\pi\delta}{\sqrt{1-\delta^2}}} \Rightarrow \delta = \sqrt{\frac{(\ln 0,363)^2}{(\ln 0,363)^2 + \pi^2}} = 0,307$$

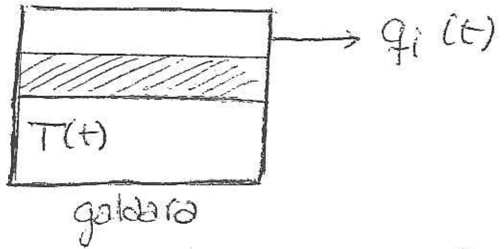
$$t_p = 0,325 \text{ (grafikotik)} = \frac{\pi}{\omega_n\sqrt{1-\delta^2}} \Rightarrow \omega_n = \frac{\pi}{0,325\sqrt{1-0,307^2}} = 10$$

$$K = \frac{y_{ss}}{\underset{\overset{\shortparallel}{1}}{u}} = 1,0046$$

$$G(s) = \frac{K\omega_n^2}{s^2 + 2\delta\omega_n s + \omega_n^2} = \frac{1,0046 \cdot 100}{s^2+6s+100} = \frac{100}{s^2+6s+100} = G(s)_{maila}$$

$$G(s)_{inpultsu} = s \cdot \frac{\cancel{10}}{\cancel{100}} \cdot \frac{10}{0,1s^2+0,6s+10} = \frac{10s}{0,1s^2+0,6s+10} \quad \boxed{b)}$$

③



$q_{io} = 1{,}25 \ \ell/min$

$$-45 q_i(t)\frac{dT(t)}{dt} + 400 q_i(t) = -\frac{d^2 q_i(t)}{dt^2} + \frac{T^2(t)}{20} - 2 q_i(t)\frac{dq_i(t)}{dt}$$

Linealizatu:

$$f\left(q_i(t), \dot{q}_i(t), \ddot{q}_i(t), T(t), \dot{T}(t)\right) = 0$$

$$-45 q_i(t)\frac{dT(t)}{dt} + 400 q_i(t) + \frac{d^2 q_i(t)}{dt^2} - \frac{T^2(t)}{20} + 2 q_i(t)\frac{dq_i(t)}{dt} = 0$$

OP: $\bar{q}_{io} = 1{,}25$

$\bar{\dot{q}}_{io} = \bar{\ddot{q}}_{io} = 0$

$\bar{\dot{T}}_0 = 0$

Ekuazio estatikoa OP-n:

$400 \cdot 1{,}25 - \frac{\bar{T}_0^2}{20} = 0 \Rightarrow T_0 = 100$

Taylor: $\left.\frac{\partial f}{\partial q_i}\right|_{op} \Delta q_i + \left.\frac{\partial f}{\partial \dot{q}_i}\right|_{op} \Delta \dot{q}_i + \left.\frac{\partial f}{\partial \ddot{q}_i}\right|_{op} \Delta \ddot{q}_i + \left.\frac{\partial f}{\partial T}\right|_{op} \Delta T + \left.\frac{\partial f}{\partial \dot{T}}\right|_{op} \Delta \dot{T} = 0$

$\left.\frac{\partial f}{\partial q_i}\right|_{op} = 400 \quad ; \quad \left.\frac{\partial f}{\partial \dot{q}_i}\right|_{op} = 2 \cdot 1{,}25 = 2{,}5 \quad ; \quad \left.\frac{\partial f}{\partial \ddot{q}_i}\right|_{op} = 1$

$\left.\frac{\partial f}{\partial T}\right|_{op} = -\frac{2T_0}{20} = -\frac{100}{10} = -10 \quad ; \quad \left.\frac{\partial f}{\partial \dot{T}}\right|_{op} = -45 \cdot 1{,}25 = -56{,}25$

$$400 \Delta q_i + 2{,}5 \Delta \dot{q}_i + \Delta \ddot{q}_i - 10 \Delta T - 56{,}25 \Delta \dot{T} = 0$$

Laplace

$$400 Q_i(s) + 2{,}5 s Q_i(s) + s^2 Q_i(s) = 10 T(s) + 56{,}25 s T(s)$$

$$Q_i(s)\left(s^2 + 2{,}5 s + 400\right) = T(s)\left(56{,}25 s + 10\right)$$

$$\frac{Q_i(s)}{T(s)} = \boxed{G(s) = \frac{56{,}25 s + 10}{s^2 + 2{,}5 s + 400} \quad c)}$$

④

$$20\frac{d^2f(t)}{dt^2} + 80\frac{df(t)}{dt} - 6\frac{dx(t)}{dt} + 20f(t) = \frac{2d^3x(t)}{dt^3} + 10\frac{d^2x(t)}{dt^2} + 6\frac{dx(t)}{dt} - 40f(t$$

Laplace: (Hasierako baldintzak nuluak)

$$20s^2 F(s) + 80s F(s) + 20F(s) + 40F(s) = 6s X(s) + s^3 \cdot 2 X(s) + 10s^2 X(s) + \cdot \overset{s}{\cancel{X}}$$

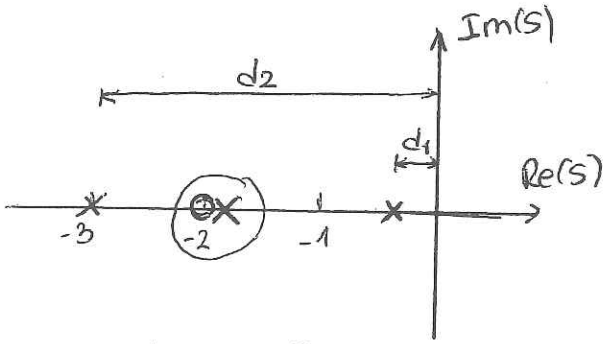
$$F(s)\left(20s^2 + 80s + 60\right) = X(s)\left(2s^3 + 10s^2 + 12s\right)$$

$$\frac{X(s)}{F(s)} = \left[G(s) = \frac{20s^2+80s+60}{2s^3+10s^2+12s} = \overset{1}{\cancel{20}} \cdot \frac{1}{\cancel{2}} \cdot \frac{s^2+4s+3}{s^3+5s^2+6s} \right. = \boxed{10\,\frac{s^2+4s+3}{s^3+5s^2+6s}}\ \boxed{a)}$$

⑤

$$G(s) = \frac{0{,}05(s+2)}{(s^2+2{,}4s+0{,}85)(s+3)} = \frac{0{,}05(\cancel{s}+2)}{(s+3)(s+0{,}43)(\cancel{s}+1{,}97)}$$

espaloi-erantzunetatik zein dagokio?



zeroa: $s+2=0 \rightarrow s=-2 \Rightarrow$ zero negatiboa

Poloak: $s+3=0 \longrightarrow s=-3$

$s^2+2{,}4s+0{,}85=0 < \begin{matrix} s=-0{,}43 \\ s=-1{,}97 \end{matrix}$

- zeroa ez da dominatzailea ⟶ ez dago gaindipenarik ⟶ G_2 ez da.

$$G(s) = \frac{0{,}1}{(s+3)(s+0{,}43)1{,}97}$$

$\frac{d_2}{d_1} = \frac{-3}{-0{,}43} = 6{,}97 > 5 \Rightarrow s=-3$ mespreza daiteke.

$G(s) = \frac{0{,}1}{5{,}91(s+0{,}43)} \Rightarrow$ polo bakarrarekin geratu garenez, aurreratuago dagoen grafikoa hartuko dugu.

$\boxed{d)\ G_4(s)}$

⑥ sarrera: espaloi unitarioa.

grafikoak → azpimoteldua $0<\delta<1$

$t_p^A = t_p^B \Rightarrow t_p = \frac{\pi}{\omega_d} \rightarrow \omega_d^A = \omega_d^B \rightarrow$ zati irudikari berdina $\Rightarrow G_1$ eta G_2

G_2 ardatzetik gertuago $\Rightarrow$ motelago $\Rightarrow t_s \uparrow \longrightarrow t_s^A > t_s^B$

$\boxed{G_2 \longrightarrow A\ ;\ G_1 \longrightarrow B}$ a)

⑦ sarrera: espaloi - unitarioa

$t_p^A > t_p^B \Rightarrow \omega_d^A < \omega_d^B \rightarrow$ A-K zati irudikari txikiagoa

$M_p^B > M_p^A \Rightarrow \theta^B > \theta^A \xRightarrow{} \theta_2 > \theta_3$ $\quad \boxed{\begin{matrix} G_3 \longrightarrow A \\ G_2 \longrightarrow B \end{matrix}}$ d)

$\Uparrow$

$t_s^A = t_s^B \Rightarrow$ zati erreal berdina $\rightarrow G_3$ eta G_2

⑧ sarrera: maila/espaloi - unitario

$t_s^A > t_s^B \Rightarrow$ zati erreala (B) > zati erreala (A)

$t_p^B < t_p^A \Rightarrow \omega_d^B > \omega_d^A \xrightarrow{} \omega_d^1 > \omega_d^4$ $\quad \boxed{\begin{matrix} G_1 \longrightarrow B \\ G_4 \longrightarrow A \end{matrix}}$ a)

$\uparrow$

$M_p^A = M_p^B \Rightarrow \delta^A = \delta^B \Rightarrow G_1$ eta G_4

⑨

$\delta = 1$ edo $\delta > 1 \Rightarrow$ zergatik ez dute osagai irudikaririk. $\Rightarrow G_5, G_6, G_7$

A grafiko azkarragoa $\longrightarrow$ Bere poloek zati erreal handiagoa

$\delta\omega_n^A > \delta\omega_n^B \Rightarrow t_s^A < t_s^B$

A grafikoa $\longrightarrow$ 1. ordeneko sistema $\longrightarrow$ polo bakarra $\Rightarrow G_6$ edo G_7

$\boxed{B \longrightarrow G_5}$

Aldaketa handirik ez dagoenez A eta B grafikoen artean, poloak gertu egongo dira.

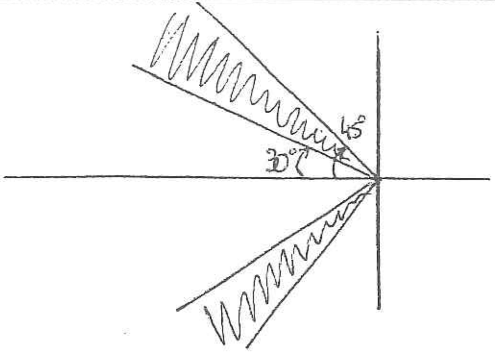
$\boxed{A \longrightarrow G_7}$

$\boxed{c)}$

? ⑩ $G(s) = \dfrac{10s(s+1)(s+2)}{s^4+1}$ berrelikadura unitarioa eta K_c irabazpena.

Ekuazio karakteristikoa: $1 + K_c G(s) = 0 \Rightarrow s^4 + K_c(10s^3 + 30s^2 + 20s) + 1 =$

(11) Bigarren ordenako sistema.



Gaininpultsua: %4,3 ≤ Mp ≤ %16,3

Non daude kokatuak bere poloak?

$$0{,}043 \leq e^{-\frac{\pi\delta}{\sqrt{1-\delta^2}}}$$

$$\ln 0{,}043 \leq -\frac{\pi\delta}{\sqrt{1-\delta^2}} \Rightarrow \delta \leq \sqrt{\frac{(\ln(0{,}043))^2}{(\ln 0{,}043)^2+\pi^2}} = 0{,}707$$

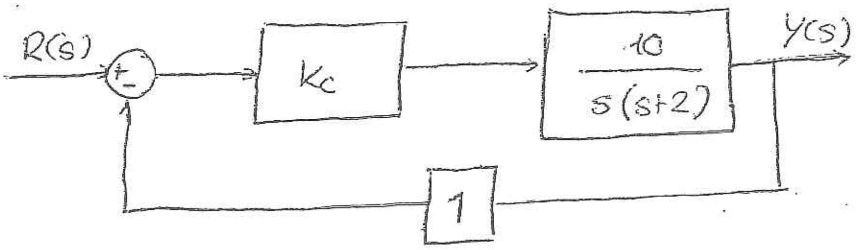
$$0{,}163 \geq e^{\frac{-\pi\delta}{\sqrt{1-\delta^2}}} \Rightarrow \delta \geq \sqrt{\frac{(\ln 0{,}163)^2}{(\ln 0{,}163)^2+\pi^2}} = 0{,}5$$

$$0{,}5 \leq \delta \leq 0{,}707$$

$$\theta = \arccos\delta \begin{cases} \delta = 0{,}5 \rightarrow \theta = 60^\circ \\ \delta = 0{,}707 \longrightarrow \theta = 45^\circ \end{cases} \Rightarrow 45^\circ < \theta < 60^\circ$$

$\boxed{C)}$

(12) Berrelikatutako sistema. %4,3 ≤ Mp ≤ %16,3



$$G_{BC} = \frac{\frac{10K_c}{s^2+2s}}{1+\frac{10K_c}{s(s+2)}} = \frac{10K_c}{s^2+2s+10K_c}$$

$$\begin{cases} \omega_n^2 = 10K_c \\ \cancel{2}\delta\omega_n = \cancel{2} \rightarrow \omega_n = \frac{1}{\delta} \end{cases}$$

$$K \cdot \omega_n^2 = 10K_c \Rightarrow \underline{K=1}$$

$$\frac{1}{\delta^2} = 10K_c \longrightarrow \frac{1}{\delta} = \sqrt{10K_c} \longrightarrow K_c = \frac{1}{\delta^2 \cdot 10} = \begin{cases} 0{,}2 \text{ baldin } \delta = 0{,}707 \\ 0{,}4 \text{ baldin } \delta = 0{,}5 \end{cases}$$

$$\boxed{0{,}2 < K_c < 0{,}4} \quad \boxed{d)}$$

(13) Sarrera: eskaloi unitarioa

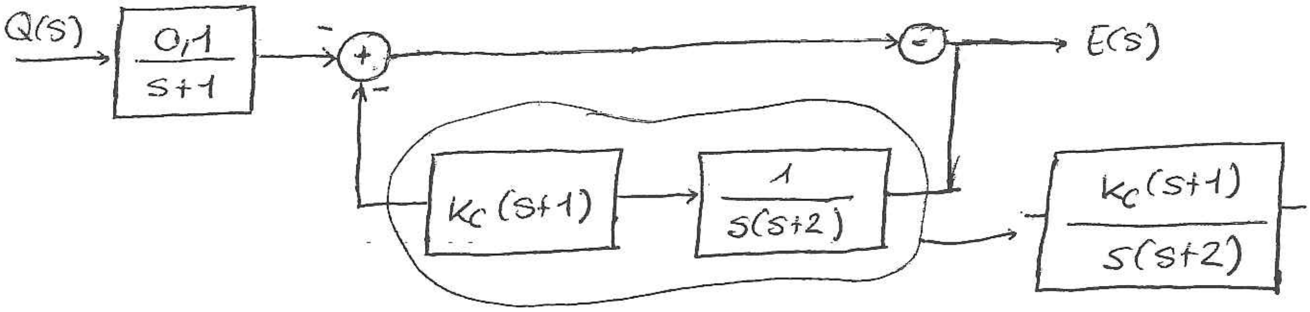
$0{,}5 \leq \delta \leq 0{,}707 \Rightarrow 0 < \delta < 1 \rightarrow$ sistema azpimoteldua $\Rightarrow$ d ezinezkoa.

Guztiek daukate y_{ss} berdina $\Rightarrow y_{ss} = 1$

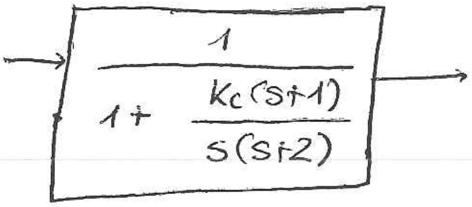
$$M_p = \frac{y(t_p)-1}{1} = y(t_p)-1 \Rightarrow \begin{cases} 0{,}043+1 = y(t_p) \rightarrow y(t_p) = 1{,}043 \\ 0{,}163 = y(t_p)-1 \rightarrow y(t_p) = 1{,}163 \end{cases}$$

$1{,}043 \leq y(t_p) \leq 1{,}163 \Rightarrow$ Baldintza hau betetzen duen bakarra $\Rightarrow \boxed{b)}$

⑬ $\frac{E(s)}{Q(s)}$?



Berrelikadura :



$$\frac{E(s)}{Q(s)} = \frac{0{,}1}{s+1} \cdot \frac{s(s+2)}{s^2+2s+k_c s+K_c} = \frac{-0{,}1s(s+2)}{s(s+1)(s+2)+K_c(s+1)^2} \quad \boxed{d)}$$

⑭ u=10 anplitudeko espalai-sarrera // Q(s): espalai-sarrera 0,5 anplitud

??

$$R(s) = \frac{10}{s} \quad ; \quad Q(s) = \frac{0{,}5}{s}$$

$$E(s) = R(s) - H(s) = \frac{10}{s} - \left[\frac{k_c(s+1)}{s(s+2)} E(s) + \frac{0{,}1}{s+1} \cdot \overset{0{,}5/s}{Q(s)}\right] \Rightarrow$$

$$\Rightarrow E(s)\left[1+\frac{k_c(s+1)}{s(s+2)}\right] = \frac{10}{s} - \frac{0{,}05}{s(s+1)} \Longrightarrow E(s) = \frac{\frac{10}{s} - \frac{0{,}05}{s(s+1)}}{1+\frac{k_c(s+1)}{s(s+2)}}$$

$$e_{ss} = \lim_{s\to 0} s \cdot E(s) = \frac{10-0{,}05}{1+\infty} = 0 \quad \boxed{a}$$

⑮ arrapala-sarrera : 0,5 (Q(s))

$$R(s) = \frac{10}{s} \quad ; \quad Q(s) = \frac{0{,}5}{s^2}$$

$$E(s) = \frac{\frac{10}{s} + \frac{0{,}1}{s+1} \cdot \frac{0{,}5}{s^2}}{1+\frac{k_c(s+1)}{s(s+2)}} = \frac{1}{s} \cdot \frac{(10(s+1)s - 0{,}05)(s+2)}{(s+2)s(s+1)+K_c(s+1)^2}$$

$$e_{ss} = \lim_{s\to 0} s \cdot E(s) = \frac{-0{,}1}{k_c} \quad \boxed{d)}$$

⑯ $G(s)=\frac{10}{(s+1)(s+10)}$ Berrelikadura unitarioa eta $K_c = 37{,}6$

M_p?

$$G_{BC} = \frac{\frac{10\cdot 37{,}6}{(s+1)(s+10)}}{1+\frac{376}{(s+1)(s+10)}} = \frac{376}{(s^2+11s+\underbrace{10+376}_{386})} \quad \left\{\begin{array}{l} \omega_n^2 = 386 \\ 2\delta\omega_n = 11 \\ K\cdot\omega_n^2 = 376 \end{array}\right.$$

$$\delta = \frac{11}{2\cdot\sqrt{386}} = 0{,}28 \Rightarrow M_p = e^{\frac{-\pi\cdot 0{,}28}{\sqrt{1-0{,}28^2}}}\cdot 100 = \%40 \ \boxed{b)}$$

⑰ Aurreko sistemarako $t_s(\%2)$?

$$t_s(\%2) = \frac{4}{\delta\omega_n} = \frac{4}{0{,}28\cdot\sqrt{386}} = 0{,}72\,s \quad \boxed{d)}$$

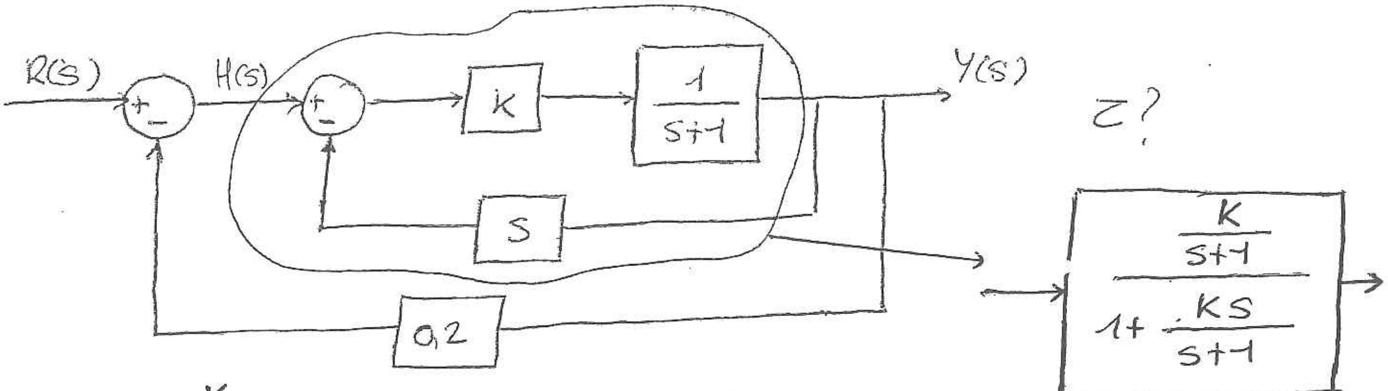
⑱ $M_p \leq \%20$, $t_s =$ 17. galderakoa, K_c?

$$0{,}20 = e^{\frac{-\pi\delta}{\sqrt{1-\delta^2}}} \Longrightarrow \delta = \sqrt{\frac{(\ln 0{,}20)^2}{(\ln 0{,}20)^2+\pi^2}} = 0{,}46$$

$$0{,}72 = \frac{4}{0{,}46\,\omega_n} \Rightarrow \delta\omega_n = 4/0{,}72 = 5{,}56 \rightarrow \omega_n = 12{,}077 \text{ rad/s}$$

$$G = \frac{10\cdot K_c}{s^2+11s+10+10\cdot K_c} \Rightarrow \left\{\begin{array}{l} 10+10K_c = \omega_n^2 \\ K_c = \frac{12{,}077^2-10}{10} = 13{,}6 \quad \boxed{a)} \end{array}\right.$$

(19)



Z?

$$\frac{Y(s)}{R(s)} = \frac{\frac{K}{s+1+Ks}}{1+0{,}2\cdot\frac{K}{1+(1+K)s}} = \frac{\frac{K}{1+s(1+K)}}{\frac{1+(1+K)s+0{,}2K}{1+s(1+K)}} = \frac{K}{(1+0{,}2K)+(1+K)s} =$$

$$= \frac{K/(1+0{,}2K)}{1+\frac{1+K}{1+0{,}2K}s} \Longrightarrow Z = \frac{1+K}{1+0{,}2K} \quad \boxed{a)}$$

⑳ Sorrera espaloi unitarioa. Aurreko sistemako egoera egonkorreko

??? errorea? $R(s) = 1/s$

$$H(s) = R(s) - 0{,}2\, Y(s) = \frac{1}{s} - 0{,}2 \left[\frac{K}{(1+0{,}2K) + (1+K)s} \cdot \frac{1}{s} \right]$$

$$e_{ss} = \lim_{s \to 0} s\, H(s) = \lim_{s \to 0} 1 - \frac{0{,}2}{(1+0{,}2K) + (1+K)s} = 1 - \frac{0{,}2}{1+0{,}2K} =$$

$$= \frac{1+0{,}2K - 0{,}2}{1+0{,}2K} =$$

2013/2014 EKAINA

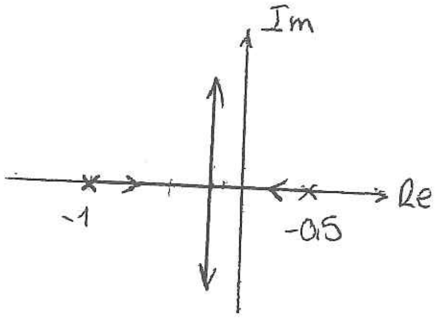
1. PROBLEMA

a) EGIA

~~Kontrol proportzionala erabiliz sistemak polo positibo bat izaten jarraituko du eta beraz desegonkorra izango da.~~

$$G_{BA} = G_c(s)G_p(s) = k_c \frac{K}{\underbrace{(s+1)(s-0{,}5)}_{s^2+0{,}5s+(-0{,}5)}} = k_c \frac{0{,}5}{(s+1)(s-0{,}5)} \quad \begin{cases} n=2 \\ m=0 \end{cases}$$

$n=2 \rightarrow 2$ adar



$$\sigma = \frac{+0{,}5-1}{2} = -0{,}25$$

$$\theta_{1/2} = \frac{(2k+1)\pi}{2} \rightarrow k=0 \longrightarrow \theta_{1/2} = \pm 90$$

Ikusten denez, k_c balietik aurrera begizta itxiko poloak erdiplano negatiboan kokatzen dira, sistema egonkortuz.

b) EGIA

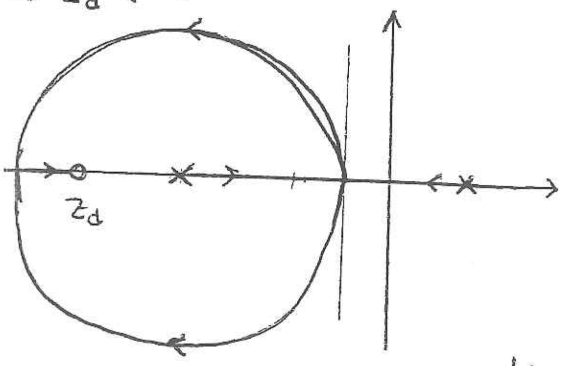
PD kontroladorea: $G_c(s) = k_c(1+T_d s)$

$$G_{BA} = \frac{k_c(1+T_d s)\cdot K}{(s+1)(s-0{,}5)} \quad \begin{cases} n=2 \\ m=1 \end{cases} \Rightarrow$$

2 adar

↳ Bata zerora eta bestea infinitura $(n-m)$

→ $z_d < -1$



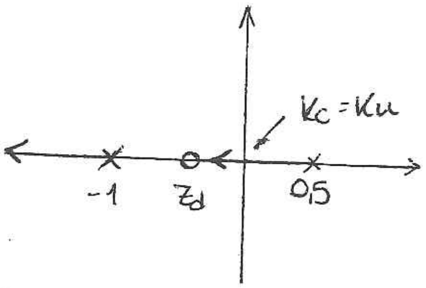
Adarrak: $(-\infty, z_d) \cup (-1, 0'5)$

$$\sigma = \frac{z_d - 1 + 0{,}5}{2} = \frac{z_d - 0{,}25}{2}$$

$$\theta_1 = \frac{(2k+1)\pi}{1} = \pm 180°$$

k_c balietik aurrera begizta itxiko poloak erdiplano negatiboan kokatzen dira, sistema egonkortuz.

- $z_d \in (-1,0)$



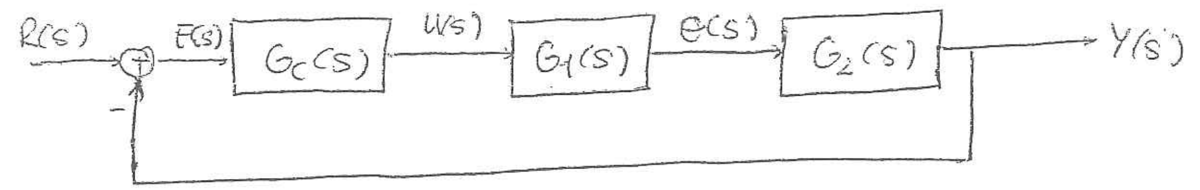
Adarrak: $(-\infty, -1) \cup (z_d, 0'5)$

$$\sigma = \frac{z_d - 0{,}25}{2}$$

$$\theta_1 = \pm 180°$$

$\Rightarrow$ Egonkortzen da sistema.

2. PROBLEMA



G_1: $2\dot{\theta}(t) + 20\theta(t) = u(t)$

G_2: grafikoa

→ G_1 lortzeko: $G_1(s) = \frac{\theta(s)}{u(s)}$

$2\theta(s)\cdot s + 20\theta(s) = u(s) \longrightarrow \theta(s)(2s+20) = u(s)$ $\Rightarrow G_1(s) = \frac{1}{2s+20} = \frac{0.5}{s+10}$

→ G_2 grafikotik:

$y_{ss} = a_1 + a_2 + a_3 = 2,326 - 0,381 + 0,055 = 2$; $K = \frac{y_{ss}}{\Delta u} = 2$

$t_p = \frac{2\pi}{\sqrt{3}} = \frac{\pi}{\omega_n\sqrt{1-\delta^2}} \Rightarrow 2\omega_n\sqrt{1-\delta^2} = \sqrt{3} \longrightarrow \omega_n = 1 \text{ rad/s}$

$y(t_p) = a_1 = 2,326$

↳ $M_p = \frac{y(t_p) - y_{ss}}{y_{ss}} = \frac{2,326-2}{2} = 0,163 \longrightarrow \delta = 0,5$

$$G_2(s) = \frac{2}{s^2+s+1}$$

→ 1/4 moteldura erlazioa dagoenez K_u eta ω_u lortu behar ditugu taulak erabili ahal izateko. Horretarako R-H erabiliko dugu.

Errorea nulua izan behar denez, integradore bat behar dugu, beraz PI kontrolagailu bat erabiliko dugu. Lehenengo P kontrolagailu modura ipiniko dugu eta irabazpen eta periodo kritikoak kalkulatuko ditugu.

$$G_{BA}(s) = G_c(s)\cdot G_1(s)\cdot G_2(s) = \frac{K_c\cdot 0,5\cdot 2}{(s+10)(s^2+s+1)} = \frac{K_c}{s^3+11s^2+11s+10}$$

Ek. karakteristikoa: $s^3+11s^2+11s+10+K=0$

R-H   B.B ≡ $K+10>0 \rightarrow K>-10 \Rightarrow K>0$ (K ezin da negatiboa izan

BN:

s^3	1	11	0
s^2	11	10+K	0
s	b_1	0	
s^0	c_1		

$$b_1 = \frac{11^2-10-K}{11} > 0 \rightarrow 11^2-10 > K$$

$$K < 111$$

$$\underline{\underline{0 < K < 11}} \Rightarrow k_u = 111$$

→ Ku erabilita taulako 3. lerroa zeroz osatuta dago.

$P(s) = 11s^2 + 121$ (polinomio laguntzaile)

↳ $11s^2+121=0 \rightarrow s^2=-11 \rightarrow s_{1,2} = \pm\sqrt{11}\,j$ non $w_u = \sqrt{11}$

$$T_u = \frac{2\pi}{w_u} = 1{,}894\,s$$

→ Taulara goaz: PI + BC : $\begin{cases} K_c = 0{,}4K_u \\ T_i = 0{,}8T_u \end{cases} \rightarrow \begin{matrix} K_c = 44{,}4 \\ T_i = 1{,}5 \end{matrix}$

$$\boxed{G_c(s) = 4{,}44\left(1 + \frac{1}{1{,}5s}\right)}$$

$\boxed{\text{2014/2015 URTARRILA}}$

1. ARIKETA

1)

Grafikotik:

Poloak $\longrightarrow s=-1$
$s=-5$
$s=-10$

$$G(s)=\frac{k}{\underbrace{(s+1)(s+5)}_{s^2+6s+5}(s+10)}=\frac{k}{s^3+6s^2+5s+10s^2+60s+50}$$

$$G(s)=\frac{50}{s^3+16s^2+65s+50}=\frac{50}{(s+1)(s+5)(s+10)}$$

2)

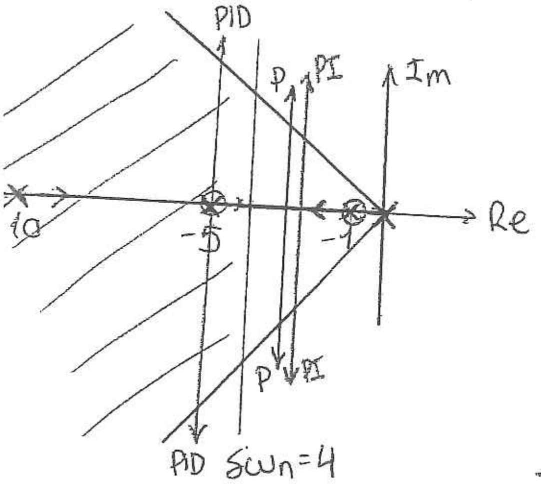
$e_{ss}=0$ (maila arean) → Plantaren transferentzi-funtzioak integradorerik ez duenez, integradore bat behar dugu $e_{ss}=0$ izateko. → PI kontrolagailua.

$t_s(\%2)\leq 1s$

$M_p\leq \%4{,}3$

$$G_c(s)=\frac{K(s+z_i)}{s}$$; Eskakizunak betetzeko: $t_s(\%2)=\frac{4}{\delta\omega_n}\leq 1 \rightarrow 4\leq\delta\omega_n$

$M_p=0{,}043 \Rightarrow \delta=0{,}707 \longrightarrow \theta\leq 45°$



Zero bat sartuko dugu polo dominatzailea baliogabetzeko: $z_i=-1 \rightarrow \frac{1}{T_i}=1 \rightarrow T_i=1s$

$$G_c(s)=\frac{K_c(T_is+1)}{s}=\frac{K_cT_i(s+1/T_i)}{s}$$

$$G_{BA}(s)=\frac{K_c\cancel{(s+1)}50}{s\cancel{(s+1)}(s+5)(s+10)}\quad\begin{cases}n=3\\ m=0\end{cases}$$

→ Ikus dezakegu ez dagoela intersekziorik, beraz, PID bat behar dugu: $G_c(s)=K_c\left(1+\frac{1}{T_is}+T_ds\right)=\frac{K(s+z_i)(s+z_d)}{s}$

$(s+z_d)=(s+5)$ izan behar da beste poloa baliogabetzeko.

$$G_{BA}(s)=\frac{50\cdot K\cancel{(s+1)}\cancel{(s+5)}}{s\cancel{(s+1)}\cancel{(s+5)}(s+10)}=\frac{50K}{s(s+10)}$$

$$G_{BC}(s)=\frac{50K}{s^2+10s+50K}\Longrightarrow s^2+10s+50K=s^2+2\delta\omega_n s+\omega_n^2\quad\begin{cases}2\delta\omega_n=10\\ \omega_n^2=50K\end{cases}$$

$$\delta=1\Rightarrow\begin{cases}\omega_n=5\text{ rad/s}\\ K=0{,}5\end{cases}\qquad\delta=0{,}707\Longrightarrow\begin{cases}\omega_n=7{,}07\text{ rad/s}\\ K=1\end{cases}$$

$$\frac{K(s+1)(s+5)}{\cancel{s}} = K_c T_d \frac{s^2 + \frac{1}{T_d}s + \frac{1}{T_d T_i}}{\cancel{s}}$$

$$k(s^2+6s+5) = K_c T_d \left(s^2 + \frac{1}{T_d}s + \frac{1}{T_d T_i}\right) \Rightarrow \begin{cases} k = K_c T_d \\ 6 = \frac{1}{T_d} \Rightarrow \underline{\underline{T_d = 1/6\ s}} \\ 5 = \frac{1}{T_d T_i} \Rightarrow \frac{5}{6} = \frac{1}{T_i} \end{cases}$$

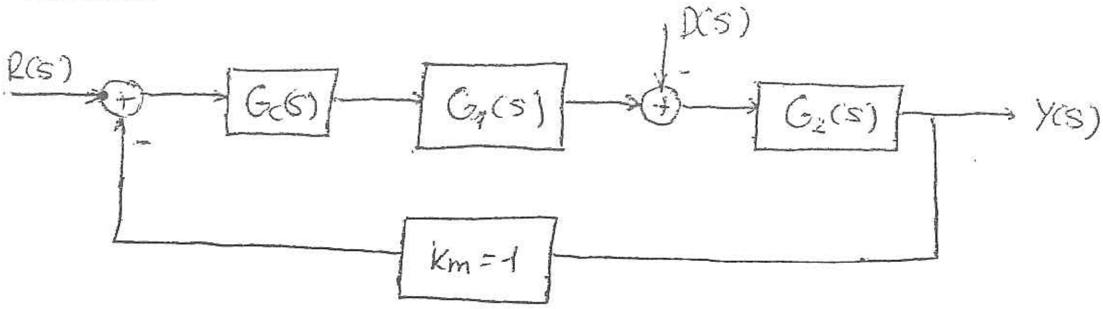
$$\hookrightarrow \underline{\underline{T_i}} = \frac{6}{5} = \underline{\underline{1{,}2\ s}}$$

- $k = 0{,}5$: $0{,}5 = K_c \frac{1}{6} \rightarrow K_c = 3$
- $k = 1$: $1 = K_c \cdot \frac{1}{6} \Rightarrow K_c = 6$

$$\left[G_{PID}(s) = K_c\left(1 + \frac{1}{6}s + \frac{1}{1{,}2s}\right) \quad \text{non} \quad 3 \leq K_c \leq 6 \right]$$

2015 URTARRILA

2. ARIKETA



1)

$G_2(s)$: arrapala sarrera, 1 anplitudekoa $\rightarrow \boxed{G_2(s) = \frac{1}{s}}$

$G_1(s)$: Bode diagramatik.

$\omega_1 = 10$ rad/s $\Rightarrow (1+0{,}1s)$ $\qquad \rightarrow \quad G_1(s) = \frac{1}{(1+0{,}1s)} = \frac{10}{s+10}$

K aurkitzeko: $\quad 0 = 20\log\left(\frac{K}{0{,}1}\right) \Rightarrow K = 1$

2)

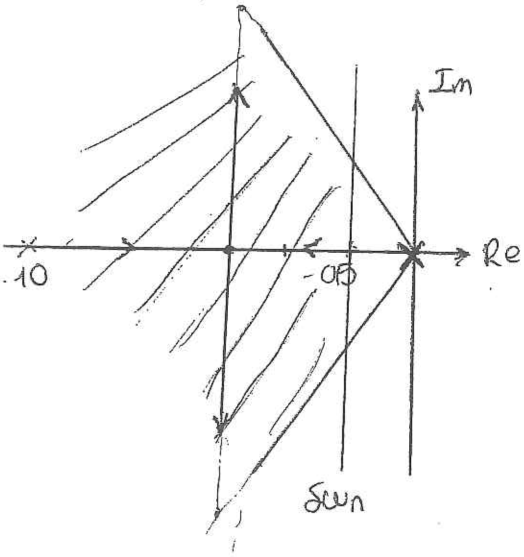
$M_p \leq \%10 \rightarrow M_p = 0{,}1 \Rightarrow \delta = 0{,}59 \Rightarrow \theta \leq 53{,}76°$

$t_s(\%5) = 6s \Rightarrow \frac{3}{\delta\omega_n} = 6 \Rightarrow \delta\omega_n \geq 0{,}5$

$e_{ss} = \%20$

↳ D(s) maila sarrera

$$G_1(s) \cdot G_2(s) = \frac{10}{s(s+10)}$$



Intersekzioa dago → P kontrolagailua.

$G_c(s) = K_c$

↳ $G_{BA}(s) = \frac{10K_c}{s(s+10)}$

$G_{BC}(s) = \frac{10K_c}{s^2+10s+10K_c}$

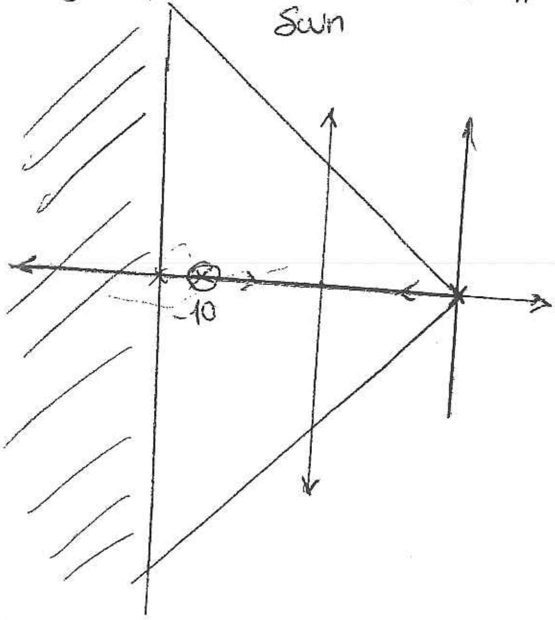
$$s^2+10s+10K_c = s^2+2\delta\omega_n s+\omega_n^2 \Rightarrow \begin{cases} 2\delta\omega_n = 10 \longrightarrow \begin{cases} \delta = 1 \rightarrow \omega_n = 5 \\ \delta = 0{,}59 \rightarrow \omega_n = 8{,}47 \end{cases} \\ 10K_c = \omega_n^2 \rightarrow \begin{cases} \omega_n = 5 \rightarrow 2{,}5 = K_c \\ \omega_n = 8{,}47 \rightarrow K_{c_{max}} = 7{,}18 \end{cases} \end{cases}$$

Bilatuko dugu $K_{c\,min}$ zehatzago bat;

~~$e_{ss}| = 0{,}2 = \frac{1}{1+K_p} \Rightarrow K_p = 4$~~

3)

$t_s \leq 0{,}25 = \frac{3}{\delta\omega_n} \rightarrow \delta\omega_n = 12$



$3\tau = t_s(\%5) = 0{,}25$

$\tau = \frac{1}{12} \Rightarrow \frac{1}{K_c} = \frac{1}{12} \Rightarrow 12 = K_c$

$\boxed{K_c = 12}$

$\boxed{G_{PD}(s) = 12(1+0{,}1s)}$

~~$G_p(s)\big|_{R(s)=0} = \frac{G_2}{1+G_1 \cdot G_c} = \frac{1}{s} \cdot \frac{1}{1+K_c \cdot \frac{10}{s+10}} = \frac{1}{s} \cdot \frac{s+10}{(s+10+10K_c)}$~~

P kontrolagailuek ez ditu eskakizunak betetzen.

↳ Hau konpontzeko sartuko dugu zero bat $s=-10$-en. $G_{PD} = K_c(T_d s + 1) = K_c T_d (s+$

$G_{BA}(s) = \frac{10 \cdot K_c \cdot T_d (s + 1/T_d)}{(s+10)s}$ $\quad \overset{1/T_d}{\overset{\shortparallel}{z_d}} = 10 \rightarrow \boxed{T_d = 0{,}1\,s}$

$G_{BA}(s) = \frac{0{,}1 \cdot K_c \cdot 10}{s} \rightarrow G_{BC}(s) = \frac{10 \cdot 0{,}1 K_c}{s + K_c} = \frac{1/K_c}{\underbrace{\frac{1}{K_c}}_{\tau} s + 1}$

3. ARIKETA

1)

ω txikietan → -20 dB/h -ko malda ⇒ polo bat dago jatorrian

(polo bikoitza)

$\omega_1 = 10$ rad/s → $T_1 = 0{,}1s$ → -60 dB/h -ko malda $G(s) = \frac{\cdot}{s(1+0{,}1s)^2}$

$0 = 20\log\left(\frac{K}{20}\right) \rightarrow \quad 1 = \frac{K}{20} \rightarrow K =$

$$\left[G(s) = \frac{200}{s(s+10)^2} \right]$$

2)

$R(s) = \frac{2}{s}$ (sarrera) $G_{BC}(s) = \frac{200}{s^3(s+10)^2 + 200}$

$G_s(s) = 1$ (sentsorea)

$$e_{ss_v} = \frac{2}{K_v} \Rightarrow K_v = \lim_{s\to 0} \cancel{s}\frac{200}{\cancel{s}(s+10)^2} \cdot 1 = \frac{20\cancel{0}}{10\cancel{0}} = 2 \Rightarrow \boxed{e_{ss_v} = 1}$$

3)

$\omega_f = 10$ rad/s ⇒ $MG = 15$ dB >0

$\omega_g = 2$ rad/s ⇒ $MF = 68°$ >0

|| Biak positibo ⇒ sistema egonkorra.

4)

15 dB handitu daiteke: $15 = 20\log K_{cr} \Rightarrow \boxed{K_{cr} = 5{,}623}$

2014/2015 EKAINA

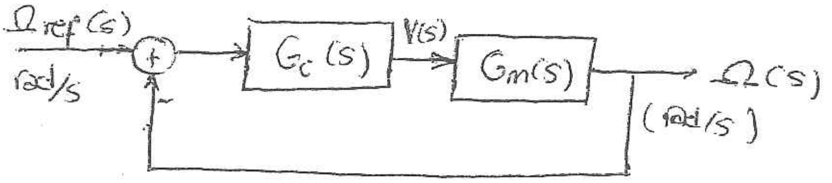
1. ARIKETA

1)

$$G_m(s) = \frac{\Omega(s)}{V(s)} \qquad y_{63} = 0{,}63 \cdot 9 = 5{,}67 \rightarrow \tau = 0{,}05\,s$$

$$\left.\begin{array}{l}\Delta u = 1 \\ \Delta y = 9\end{array}\right| K = 9 \quad ; \qquad G_m(s) = \frac{9}{0{,}05s+1} = \frac{180}{s+20}$$

2) $t_s(\%2)$ = Kalkulatu



Grafikotik → arrapala sarrera

$e_{ss\,v} = 0{,}1 = \%10$

$$\frac{1}{K_v} = 1 \Rightarrow K_v = 1 \qquad K_v = \lim_{s\to 0} s \cdot G_{BA}(s)$$

→ PI bat behar dugu $K_v \neq 0$ izateko

$t_s(\%2) = 4\tau =$ [illegible]

$$G_c(s) = \frac{K_c\,(s+z_i)}{s}$$

Polo dominatzailea anulatzeko: $z_i = 20 \rightarrow \underline{\underline{T_i = 0{,}05\,s}}$

$$G_{BA}(s) = \frac{K_c\cancel{(s+20)} \cdot 180}{s\cancel{(s+20)}} = \frac{180K_c}{s} \Rightarrow K_v = \lim_{s\to 0} \cancel{s} \cdot \frac{180K_c}{\cancel{s}} = 1 \rightarrow \underline{\underline{K_c = \frac{1}{1}}}$$

$$\left[G_c(s) = \frac{1}{180} \cdot \frac{(s+20)}{s} \right] \quad ; \quad G_{BC}(s) = \frac{180K_c}{s+180K_c} = \frac{1}{s+1} \rightarrow \tau = 1\,s$$

$$\underline{\underline{t_s(\%2)}} = 4 \cdot 1 = \underline{\underline{4\,s}}$$

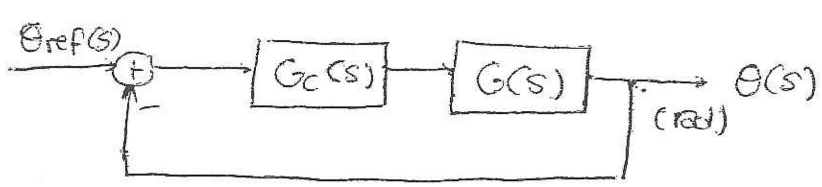
3) $\frac{N_1}{N_2} = 1/10 \qquad w_r(t) = \frac{N_1}{N_2} w(t) \quad ; \quad \theta(t) = \int w_r(t)\,dt \quad ; \quad G(s) = \frac{\theta(s)}{V(s)}\,?$

$$\Omega_r(s) = \frac{N_1}{N_2}\,\Omega(s)$$

$$\theta(s) = \frac{\Omega_r(s)}{s} = \frac{1/10\ \Omega(s)}{s} \Rightarrow \frac{\theta(s)}{\Omega(s)} = \frac{1}{10s}$$

$$G(s) = \frac{\theta(s)}{\Omega(s)} \cdot \underbrace{\frac{\Omega(s)}{V(s)}}_{G_m(s)} = \frac{1}{10s} \cdot \frac{9}{0{,}05s+1} = \frac{0{,}9}{s\,(0{,}05s+1)}$$

4)



$$G_{BA}(S) = G_C(S)\cdot\frac{0{,}9}{S(0{,}05S+1)} = G_C(S)\cdot\frac{18}{S(S+20)}$$

$$t_p = \frac{\pi}{\omega_d} = \frac{\pi}{\omega_n\sqrt{1-\delta^2}} = 0{,}16 \Rightarrow \pi = 0{,}16\cdot\omega_n\sqrt{1-0{,}46^2} \Rightarrow \omega_n = 22{,}11 \text{ rad/s}$$

$$M_p = \frac{1}{5} = 0{,}2 = e^{\frac{-\pi\delta}{\sqrt{1-\delta^2}}} \rightarrow \delta = 0{,}46$$

→ P batekin nahikoa dugu iraunkorreko eskakizunak betetzeko.

$$G_C(S) = K_C$$

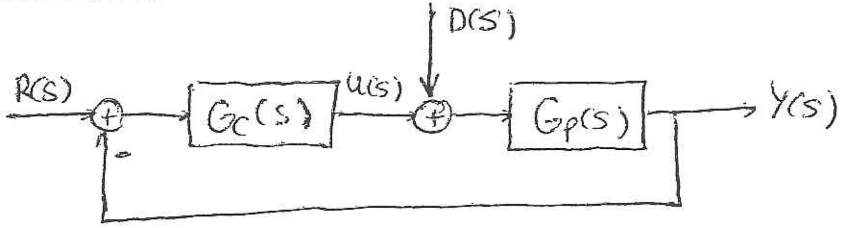
$$G_{BC}(S) = \frac{0{,}9K_C}{S^2 0{,}05 + S + 0{,}9K_C} = \frac{\overset{18}{\cancel{0{,}9}}K_C}{S^2 + 20S + 18K_C}$$

$$\begin{cases} 20 = 2\delta\omega_n \rightarrow \omega_n = 21{,}7 \\ \omega_n^2 = 18K_C \Rightarrow \boxed{K_C = 27{,}1} \end{cases} \qquad \boxed{G_C(S) = 27{,}1}$$

$$e_{ss} = \frac{5}{K_V} \quad \text{non} \quad K_V = \lim_{S\to 0} \cancel{S}\,\frac{18\cdot 27{,}1}{\cancel{S}(S+20)} = \frac{18\cdot 27{,}1}{20} = 24{,}44$$

$$\boxed{e_{ss} = 0{,}205}$$

2. ARIKETA



K=0,5

1) $R(S) = \frac{1}{S}$; $D(S) = \frac{e^{-0{,}9S}}{S}$

maiztasun txikietan : −20 dB/h -ko malda → polo bat jatorrian

$\omega_1 = 0{,}9$ rad/s (zeroa) → $T_1 = \frac{1}{0{,}9}$

$\omega_2 = 1$ rad/s → $T_2 = \frac{1}{1} = 1$s

$\omega_3 = 4$ rad/s → $T_3 = \frac{1}{4} = 0{,}25$ s

$\omega_4 = 7$ rad/s → $T_4 = \frac{1}{\ } = 0{,}14$ s

$$G_{BA}(S) = \frac{K(1+1{,}1S)}{S(1+S)(1+0{,}25S)(1+0{,}14S)}$$

$$32{,}04 = 20\log\left(\frac{K}{0{,}1}\right) \Rightarrow K = 3{,}99 \simeq 4$$

$$G_{BA}(s) = \frac{4(1+1{,}1s)}{s(1+s)(1+0{,}25s)(1+0{,}14s)}$$

$$K_p = \lim_{s\to 0} \cancel{s} \frac{4(1+1{,}1s)}{s(1+s)(1+0{,}25s)(1+0{,}14s)} = \frac{4}{0} = \infty \Rightarrow e_{ss_R} = \frac{1}{1+K_p} = 0$$

$$G_c(s) = K_c T_d \frac{s^2 + s\frac{1}{T_d} + \frac{1}{T_d \cdot T_i}}{s} \qquad e_{ss_D} = 0$$

$$\boxed{e_{ss} = e_{ss_R} + e_{ss_D} = 0}$$

2)

$$\cancel{G_B} \quad K_v = \lim_{s\to 0} \cancel{s} \cdot \frac{4(1+1{,}1s)}{\cancel{s}(\ )(\ )(\ )} = 4 \Rightarrow \boxed{e_{ss_v} = \frac{1}{K_v} = 0{,}25}$$

3)

$$G_{BA}(s) = \frac{4(s \cdot 1{,}1s)}{s(1+s)(1+0{,}25s)(1+0{,}14s)} = \frac{124{,}4(s+0{,}9)}{s(s+1)(s+4)(s+7)}$$

4)

$$t_p = 2 = \frac{\pi}{\omega_n\sqrt{1-\delta^2}} \Rightarrow \omega_n = 1{,}64 \text{ rad/s}$$

$$M_p = \frac{0{,}4}{1} = 0{,}4 \Rightarrow \delta = \sqrt{\frac{\ln(M_p)^2}{\ln(M_p)^2 + \pi^2}} = 0{,}28$$

$$K = \frac{\Delta y}{\Delta u} = 1$$

$$G(s) = \frac{K\omega_n^2}{s^2 + 2\delta\omega_n s + \omega_n^2}$$

$$Y(s) = G(s) = \frac{2{,}6896}{s^2 + 0{,}9184s + 2{,}6896}$$

$$Y(s) = \frac{G_{BA}(s)}{1+G_{BA}(s)} R(s) = \frac{124{,}4(s+0{,}9)}{s(s+1)(s+4)(s+7) + 124{,}4(s+0{,}9)} \cdot R(s)$$

~~GBA(s)-a~~ Sistemaren portaera Gp-K adierazten du, eta grafikoan ikus dezakegu sistemak ez duela poloaren eraginik sufritzen ⇒ poloa kontrolagailuarena da. $K_c = 4 \cdot K / 0{,}9 \rightarrow K_c = 8{,}88$

$$G_c(s) = \frac{8{,}88(s+0{,}9)}{s}$$

5)

~~WM~~ MG = 4dB

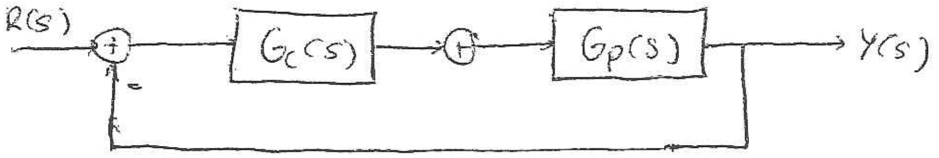
MF = 30°

6)

$$4 = 20\log K_{cr} \Rightarrow K_{cr} = 1{,}58 \text{ (BA)}$$

$$\text{(BC)}: \underline{K_{cr}} = 8{,}8 \cdot 1{,}58 = \underline{\underline{14}}$$

3. ARIKETA



1)

Grafikotik: poloak $\rightarrow s = 1$

$\qquad\qquad\qquad\quad \rightarrow s = \cancel{2} -2 \pm 2j \quad \rightarrow \quad G_p(s) = \dfrac{K}{(s-1)(s+2-2j)(s+2+2j)} =$

$$= \frac{k}{(s-1)(s^2+2s-\cancel{2j}s+2s+4-\cancel{4j}+\cancel{2sj}+\cancel{4j}+4)} = \frac{K}{(s-1)(s^2+4s+4+4)} =$$

$$= \frac{K}{(s-1)((s+2^2)+4)} = \frac{k}{(s-1)(s^2+4s+8)} \Rightarrow$$

s=0 denean $G_p(s) = 1$
izateko $\rightarrow K = 8$

$$\boxed{G_p(s) = \frac{8}{s^3+3s^2+4s-8}}$$

2)

$$G_{BC}(s) = \frac{8K_c}{s^3+3s^2+4s-8+8K_c}$$

R-H :

BB : $8K_c - 8 > 0 \rightarrow \underline{\underline{K_c > 1}}$

BN :

$$\begin{array}{c|ccc} s^3 & 1 & 4 & 0 \\ s^2 & 3 & -8+8K_c & 0 \\ \hline s & b_1 & 0 & \\ s^0 & -8+8K_c & & \end{array}$$

$$b_1 = \frac{4 \cdot \cancel{3} - (-8+8K_c)}{\cancel{3}} = 4 + \frac{8}{3} - \frac{8}{3}K_c$$

$b_2 = 0$

$$4 + \frac{8}{3} - \frac{8}{3}K_c > 0 \rightarrow \frac{20}{\cancel{3}} > \frac{8}{\cancel{3}}K_c$$

$K_c < 2{,}5$

$$\boxed{1 < K_c < 2{,}5}$$

3)

Sistemaren egonkortze tartea handitzeko ⟹ adarrak ezkerrerantz mugitu.

Horretarako kontrolagailuak zero bat txertatu behar du ⟹ kontrolagailurik errazena PD bat.

$n=3$ ┌ sartzen dugun zeroa

$n-m = 3-1 = 2$

$$\theta_k = \frac{2k+1}{n-m} \cdot 180^\circ \rightarrow k=0: \theta = 90^\circ \ ; \ k=1: \theta = 270^\circ = -90^\circ$$

$$\sigma = \frac{\Sigma z - \Sigma p}{n-m} = \frac{z_i - 2 - 2 + 1}{2} = \frac{z_i - 3}{2} \Rightarrow z_i > -3 \Rightarrow$$ erdiplano negatiboan egoteko $\nexists \sigma$.

Asintotak 90° eta -90° direnez, polo guztien kokapena erdiplano negatiboan kokatuko da $\forall K_c$-rentzako.

Zero egonkorra nahi dugunez: $-3 < z_0 < 0$ $\qquad z_0 = \frac{1}{T_d}$

Ek. karakteristikoa:

$$1 + K_c \frac{8K_c(1+T_d s)}{(s-1)(s^2+4s+8)} = 0$$

⟹ RH)

↳ $K_c > 2{,}5$ ($T_d > 0$ izateko)

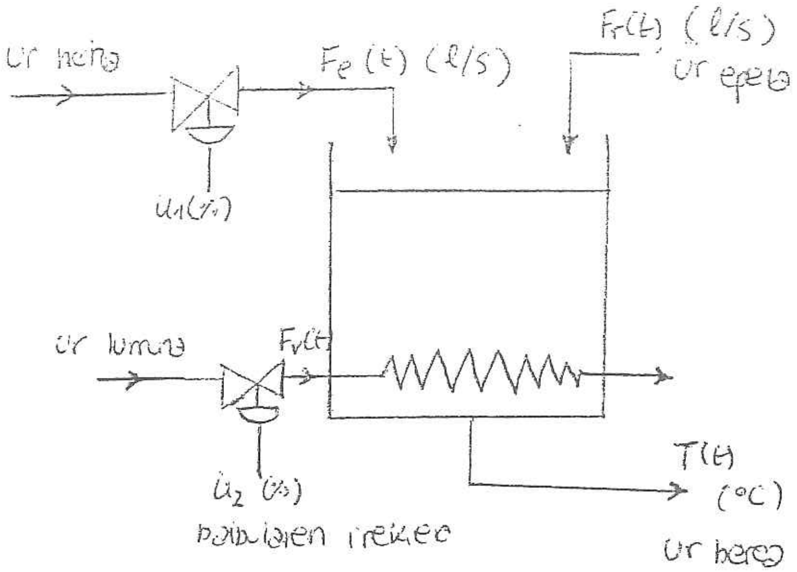
2012/2013 EKAINA

1. GALDERA

h eta T kontrolatu nahi dira.

Ur beroaren emanak aldaketa sufri ditzake baina ezin daiteke manipulatu.

Manipula daitezke ur hotzaren emana eta ur lurrin emana.



1)

Kontrolatuko aldagaiak: h (m), T (°C)

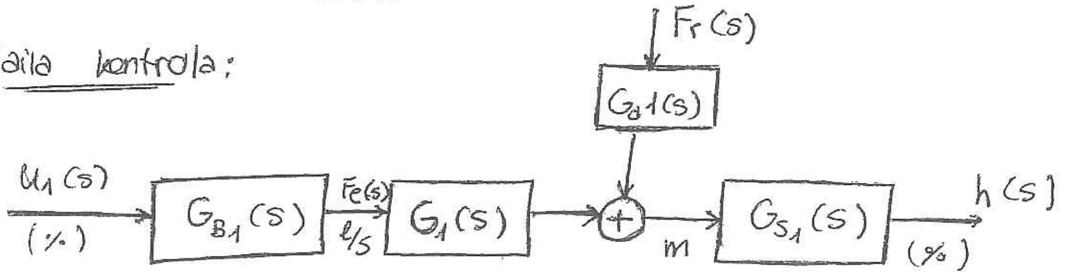
Manipulatuko aldagaiak: u_1 (%), u_2 (%)

Perturbazioak: $F_r(t)$ (l/s)

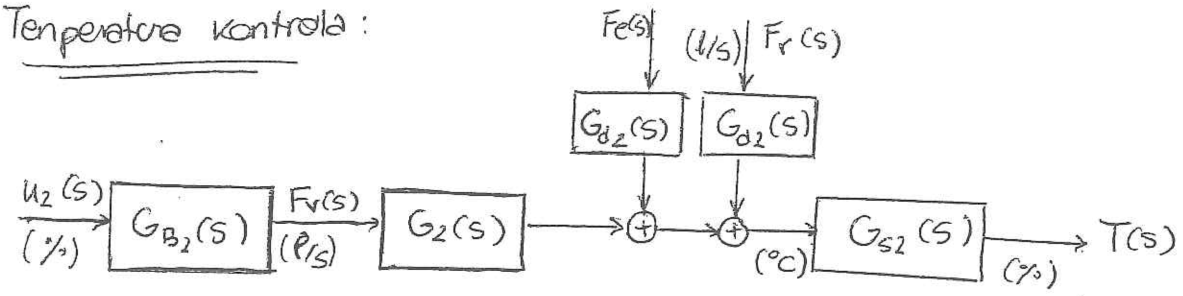
Prozesuan manipulatuko aldagaiak: $F_e(t)$, $F_v(t)$ (l/s)

Bi bloke diagrama ezberdin beregitu behar ditugu, bata h kontrolatzeko eta bestea T kontrolatzeko.

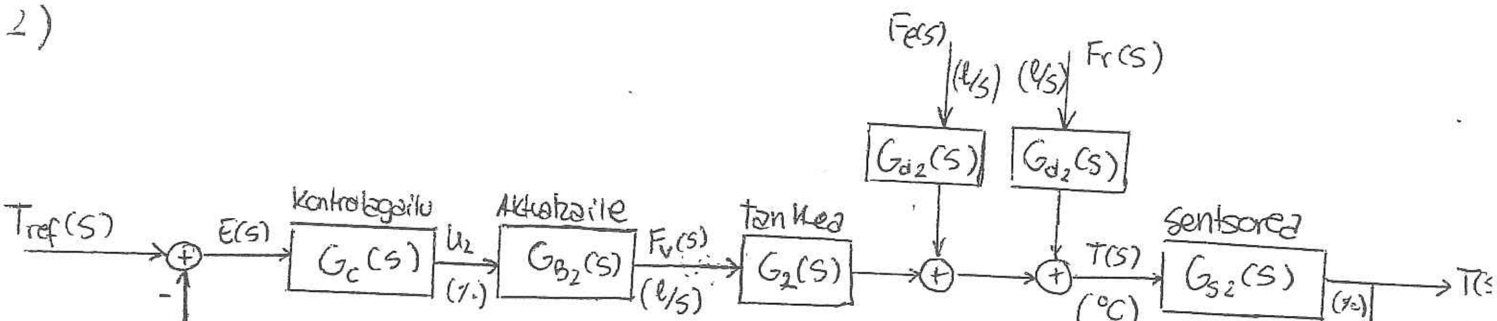
Maila kontrola:



Tenperatura kontrola:



2)



2. GALDERA

$\Delta y = 40-20=20$ || $K=\frac{20}{10}=2$

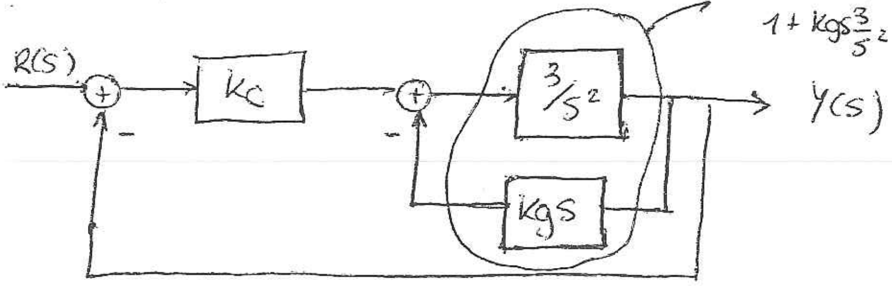
$\Delta u = 10$

$$\Rightarrow \quad G(s) = \frac{2}{2s+1}$$

$y_{63} = 20+0{,}63\cdot 20 = 32{,}6 \Rightarrow \tau = 2s$

3. GALDERA

$$\frac{3/s^2}{1+kg s\frac{3}{s^2}} = \frac{3}{s^2+3kgs} = \frac{3}{s(s+3kg)}$$



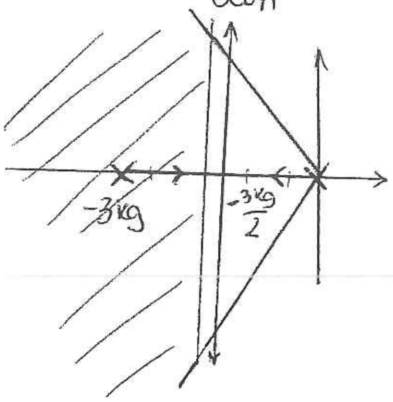
1) kc? kg?

$M_p \leq \%10$

$t_s \leq 1s\ (\%5)$

$$M_p \leqslant 0{,}1 \Rightarrow e^{-\frac{\pi\delta}{\sqrt{1-\delta^2}}} \leqslant 0{,}1 \rightarrow \delta \geqslant 0{,}591 \rightarrow \theta = 53{,}76°$$

$$t_s(\%) = \frac{3}{\delta\omega_n} \leqslant 1s \Rightarrow \delta\omega_n \geqslant 3$$



$$G_{BA}(s) = \frac{3Kc}{s(3kg+s)} \rightarrow \text{Poloak} \begin{cases} s=0 \\ s=-3kg \end{cases}$$

$$G_{BC}(s) = \frac{3Kc}{s(s+3kg)+3Kc} = \frac{3Kc}{s^2+3kgs+3Kc}$$

$$s^2+3kgs+3kc = s^2+2\delta\omega_n s+\omega_n^2$$

$$\left.\begin{array}{l} 2\delta\omega_n = 3kg \\ \downarrow \\ \text{Eskakizuna: } 3 \leqslant \delta\omega_n \end{array}\right\} \quad 2\cdot\cancel{3} \leqslant \cancel{3}\,kg \Rightarrow \boxed{kg \geqslant 2}$$

$\delta \geqslant 0{,}591 \rightarrow \delta\omega_n = 0{,}591\omega_n = 3 \Rightarrow \omega_n = 5{,}076$ rad/s

$$\underbrace{\omega_n^2}_{25{,}77} = 3Kc \rightarrow \boxed{Kc \geqslant 8{,}59} \simeq \frac{25}{3}$$

2)

$MF = 180+Arg[G_{BA}(j\omega_g)]$

$MG = 20\log\left(\frac{1}{|G_{BA}(j\omega_f)|}\right) \longrightarrow G_{BA} = \frac{25}{s(s+6)} = \frac{25}{s^2+6s} \Rightarrow G_{BA}(j\omega) = \frac{25}{\underbrace{j\omega(j\omega+6}_{-\omega^2+6j\omega}}$

ω_g eta ω_f lortu behar ditugu:

- $\underline{\underline{\omega_f}}$: $\mathrm{Arg}[G_{BA}(j\omega_f)] = -180^\circ$

$$-90 - \arctan\left(\omega_f \frac{1}{6}\right) = -180$$

$$\arctan\left(\omega_f/6\right) = -90^\circ \rightarrow \tan 90 = \nexists \Rightarrow \omega_f \nexists \rightarrow |G_{BA}(j\omega_f)| =$$

$$\boxed{MG = \infty}$$

- $\underline{\underline{\omega_g}}$: $|G_{BA}(j\omega_g)| = 1 \longrightarrow \cancel{\frac{25}{\omega_g\sqrt{1+\frac{1}{36}\omega_g^2}} = 1 \Rightarrow 25 = \omega_g\sqrt{1+\frac{\omega_g^2}{36}}}$

$$\cancel{25^2 = \omega_g^2\left(1+\frac{\omega_g^2}{36}\right) \rightarrow \omega_g^2 + \frac{\omega_g^4}{36} - 25^2 = 0}$$

$$|G_{BA}(j\omega_g)| = 1 \longrightarrow \frac{25}{\sqrt{(\omega_g^2)^2 + (6\omega_g)^2}} = 1 \Longrightarrow \omega_g = 3{,}58 \text{ rad/s}$$

$$\hookrightarrow \boxed{MF = 180 + \left(-90 - \arctan\left(3{,}58 \cdot \frac{1}{6}\right)\right) = 59{,}177 \simeq 60^\circ}$$

4 GALDERA

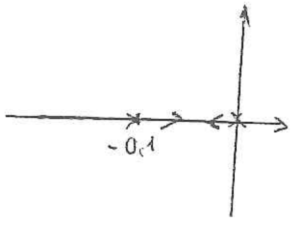
$$G_1(s) = \frac{5}{s(1+10s)} \quad ; \quad G_2(s) = \frac{0{,}5}{(0{,}1+s)} \quad ; \quad G_3(s) = \frac{10}{(1+10s)(1+0{,}1s)(1+0{,}0}$$

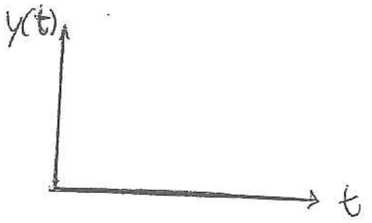
$$G_4(s) = \frac{(1-s)}{(1+10s)(1+s)}$$

- $G_1(s)$: Polo/zero diagrama eta denbora erantzun hurbildua.

Poloak: $s = 0$; $s = -0{,}1$

$$G_1(s) = \frac{0{,}5}{\underbrace{s(0{,}1+s)}_{0{,}1s + s^2 \Rightarrow \omega_n = 0?}}$$





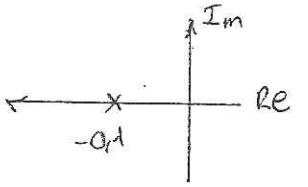
$\rightarrow$ k? t(%5)?

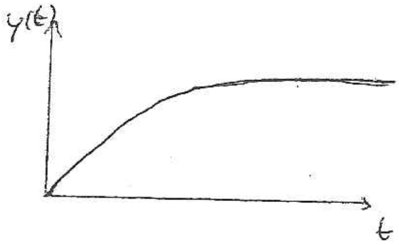
k = ez dago

t_S = ez dago

• $G_2(s)$:

$$G_2(s) = \frac{0,5}{s+0,1} \quad (1.\text{ordenekoa})$$





poloa: $s = -0,1 = -\frac{1}{\tau} \rightarrow \underline{\tau = 10}$

→ K? $t_s(\%5)$?

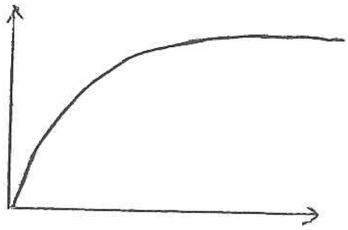
$\underline{K = 0,5}$

$t_s(\%5) = 3\tau \Rightarrow \underline{t_s = 30\,s}$

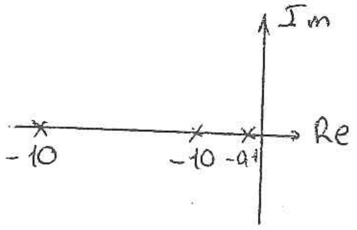
• $G_3(s)$:

$$G_3(s) = \frac{10}{(1+10s)(1+0,1s)(1+0,01s)} = \frac{10}{10(0,1+s)\,0,1(10+s)\,0,01(100+s)}$$

$$G_3(s) = \frac{1000}{(s+0,1)(s+10)(s+100)} = \cancel{\frac{\ldots}{(\ldots 0,1)\,10\cdot 100}}$$



Poloak: $s = -0,1$
$s = -10$
$s = -100$



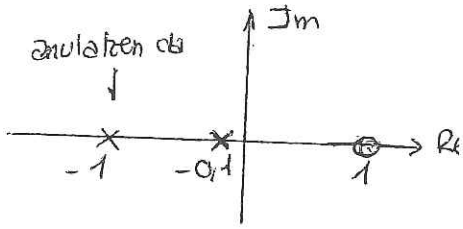
$$G_3(s) = \frac{10}{(1+10s)\,10\cdot 100} \rightarrow 1.\text{ ordeneko sistema}$$

↳ irabazpen estatikoa ez da aldatzen

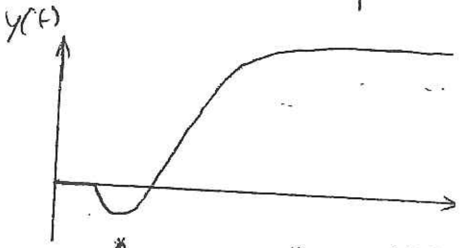
$\underline{K = 10}$; $\underline{\tau = 10} \rightarrow \underline{t_s(\%5) = 30\,s}$

• $G_4(s)$:

$$G_4(s) = \frac{(1-s)\cdot 1}{(1+10s)(1+s)} = \frac{0,1(1-s)}{(0,1+s)(1+s)}$$



poloak: $s = -0,1$ zeroak: $s = 1$
$s = -1$



↑ zero positiboagatik.

$\underline{K = 1}$

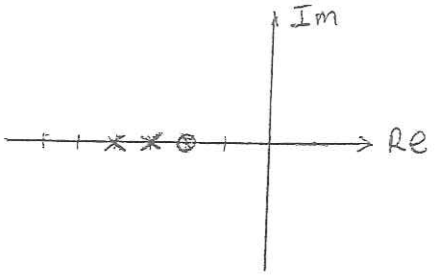
$$G_4(s) = \frac{0,1(1-s)}{\underbrace{(0,1+s)}_{(1+10s)}\,1}$$

↳ $\tau = 10$

$\underline{t_s(\%5) = 30\,s}$

5. GALDERA

$$G_p(s) = \frac{10(s+2)}{(s+3)(s+4)}$$



1) $\boxed{a}$

$$Y(s) = G_p(s) \cdot R(s) = \frac{1}{s} \cdot \frac{10(s+2)}{(s+3)(s+4)} = \frac{10s+20}{s(s+3)(s+4)} = \frac{A}{s} + \frac{B}{s+3} + \frac{C}{s+4}$$

$$10s+20 = A(s^2+7s+12) + Bs^2 + 4Bs + Cs^2 + 3Cs$$

$s^2 : 0 = A + B + C$

$s : 10 = 7A + 4B + 3C$

$(\cdot) : 20 = 12A \rightarrow A = \frac{5}{3}$

$B = -50/3$

$C = 15$

$$\Rightarrow Y(s) = \frac{5}{3} \cdot \frac{1}{s} + \left(-\frac{50}{3}\right)\frac{1}{s+3} + 15 \cdot \frac{1}{s+4}$$

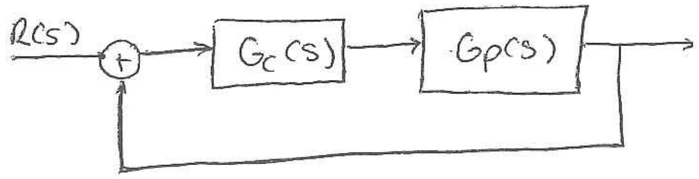
$\mathcal{L}^{-1}$

$$y(t) = \frac{5}{3} + \left(-\frac{50}{3}\right)e^{-3t} + 15e^{-4t}$$

2) Maila erantzunak gainditzea izango du?

Bai, zeroa dominantea denez erantzunaren gainditzea handitzen delako.

3)



sarrera maila unitarioa denez, errorea ezberdin zero emateko sisteman e dogu integradorerik behar, sistema 0 motakoa delako.

P kontrolagailua

$$e_{ss} = \frac{1}{1+K_p} \quad ; \quad G_{BA} = \frac{K_c \cdot 10(s+2)}{(s+3)(s+4)}$$

$$K_p = \lim_{s \to 0} G_{BA}(s) = \lim_{s \to 0} \frac{10K_c(s+2)}{(s+3)(s+4)} = \frac{20K_c}{12}$$

$$0{,}166 = \frac{1}{1+K_p} \Rightarrow K_p = 5{,}02 \simeq 5$$

$$\Rightarrow \frac{20K_c}{12} = 5 \longrightarrow K_c = 3$$

$$\boxed{G_c(s) = 3}$$

6. GALDERA

Grafikotik:

$\omega_f = 1$ rad/s $\Rightarrow$ MG = −40 dB

$\omega_g = 7$ rad/s $\rightarrow$ MF = −50°

MG<0 eta MF<0 direnez, sistema ezegonkor da. Kontrolagailu bat erabili daiteke egonkortu arte.

7. GALDERA

$\Delta u = 4$
$\Delta y = 2$ $\Bigg\|$ $k = \frac{2}{4} = 0{,}5$

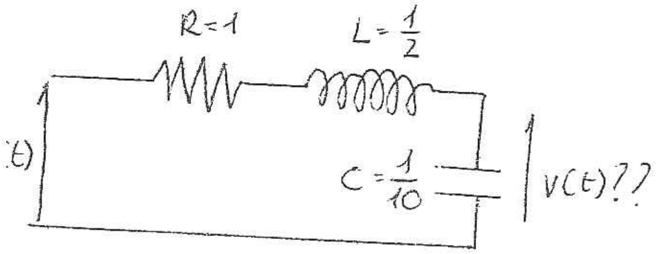
c) aukera ezin da izan, erantzuna bigarren ordeneko baitena delako.

$\hookrightarrow \omega_n^2 = 9 \rightarrow \omega_n = 3 \quad \wedge \quad 2\delta\omega_n = 2{,}4 \Rightarrow \delta = \frac{2{,}4}{2\cdot 3} = 2/5 = 0{,}4$

~~$Mp \frac{A}{B_1}$~~ $K\omega_n^2 = 0{,}5 \cdot 9 = 4{,}5$

$$G(s) = \frac{4{,}5}{s^2 + 2{,}4 + 9} \quad \boxed{a}$$

8. GALDERA



1) $G(s) = \frac{V(S)}{E(S)}$?

$e(t) = v_R(t) + v_L(t) + v(t)$

$v_R(t) = R\,i(t)$

$v_L(t) = L\,\frac{di(t)}{dt} \Longrightarrow$

$v_c = \frac{1}{C}\int i\,dt$

$$\Rightarrow e(t) = RC\,\frac{dv(t)}{dt} + LC\,\frac{d^2v(t)}{dt} + v(t)$$

$\downarrow \mathcal{L}$

$$E(s) = RC\,V(s)\cdot s + LC\,V(s)s^2 + V(s)$$

$$E(s) = V(s)\left(1\cdot\frac{1}{10}\cdot s + \frac{1}{2}\cdot\frac{1}{10}\cdot s^2 + 1\right) \Rightarrow G(s) = \frac{V(s)}{E(s)} = \frac{1}{\underbrace{\frac{1}{20}s^2 + \frac{1}{10}s + 1}_{-\omega^2 1/20 + \frac{1}{10}j\omega + 1}}$$

2) $e(t) = 0{,}2\,\text{sen}(15t)$

$$\downarrow E(s) = 0{,}2\,\frac{15}{s^2 + 15^2} = \frac{3}{s^2 + 225}$$

$$V(t) = A\cdot|G(j\omega)|_{\omega=3}\,\text{sen}(3t + \varphi)$$

$$V(s) = \frac{3}{(s^2 + 225)}\cdot\frac{1}{\left(\frac{1}{20}s^2 + \frac{1}{10}s^2 + 1\right)}$$

$$\bullet\ |G(15j)| = \frac{1}{\sqrt{\left(-\frac{15^2}{20} + 1\right)^2 + \left(\frac{15}{10}\right)^2}} = 0{,}096$$

12
0,1

$$\bullet\ \varphi = \text{Arg}[G(j15)] = -\arctan\left(\frac{1/10\cdot 15}{1 - 15^2/20}\right) = -(-8{,}32^\circ)\underbrace{-180^\circ}_{} = -171{,}68^\circ$$

$\hookrightarrow$ irteera ezin da sarrer
baino aurreratuago ego

$$\boxed{v(t) = 0{,}02\,\text{sen}(15t - 171{,}68^\circ)}$$

eman ta zabal zazu

Universidad del país vasco
Euskal herriko unibertsitatea

INGENIERÍA DE SISTEMAS I

Nombre
Izena ______________________________
1°Apellido ______________________________
1 Deitura
2° Apellido ______________________________
2 Deitura

Curso: 2011/2012

10 de Enero de 2012

Tiempo:

1,5 Horas

Grupo:

PROBLEMA 1 (5 PUNTOS)

El sistema de control representado en la *Figura 1* se corresponde con el de un sistema de posicionamiento para paneles solares. Inicialmente, el controlador ($G_c(s)$) es un controlador proporcional de ganancia unidad. El motor es un motor de corriente continua controlado por inducido cuya constante para la fuerza contraelectromotric tiene un valor (K_b) de 1.5 v/rad seg^{-1}. La reductora presenta un factor de reducción en la velocidad de rotación (K_r) de 1/100. La función de transferencia del conjunto motor/reductora ($\Theta_r(s)/ V_a(s)$) presenta un diagrama de Bode tal y como el representado en la *Figura 2*. El panel solar tiene una respuesta ante una entrada escalón unitario tal como la representada en la *Figura 3*. El sensor se puede modelar por una ganancia (K_s) de valor de 1.6 v/rad.

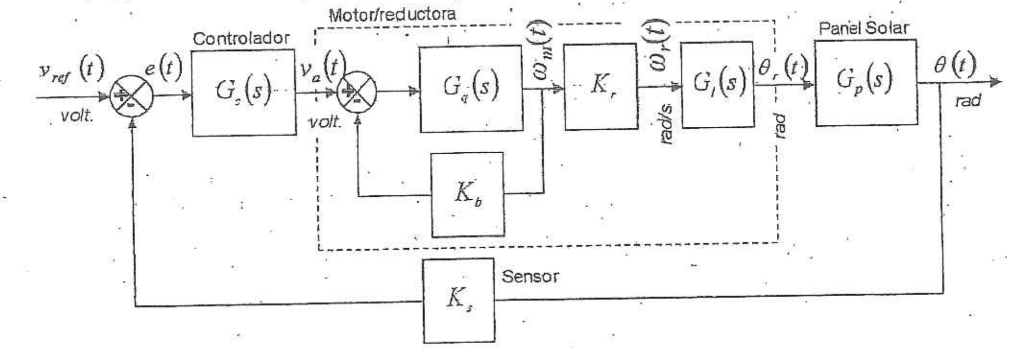


Figura 1

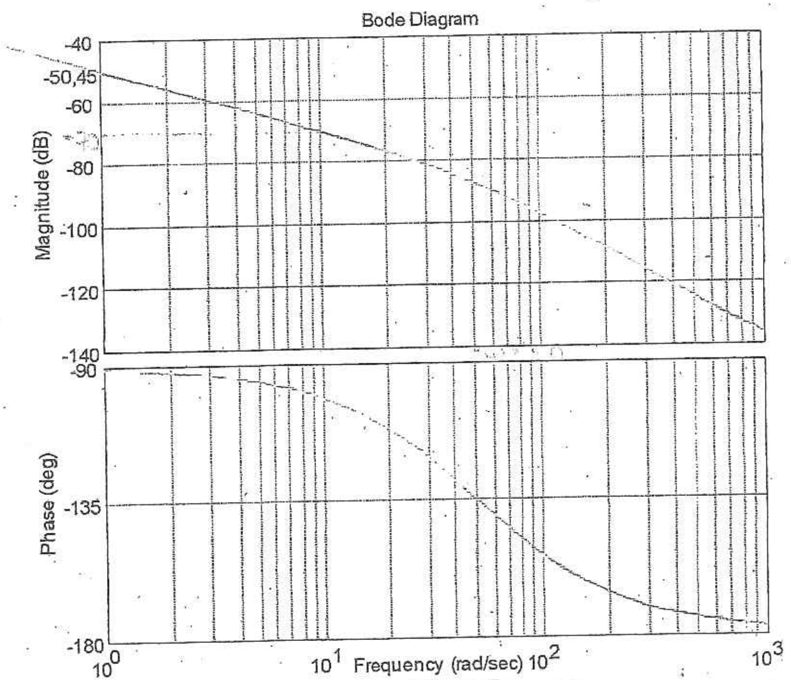


Figura 2

Modelo matemático del motor

$$e_b(t) = K_b\omega_m(t)$$

$$T_m(t) = K_m i(t)$$

$$v_a(t) = Ri(t) + e_b(t)$$

$$J\frac{d\omega_m(t)}{dt} + B\omega_m(t) = T_m(t)$$

Modelo matemático de la reductora

$$\omega_r(t) = K_r\omega_m(t)$$

Salida en posición

$$\frac{d\theta_r(t)}{dt} = \omega_r(t)$$

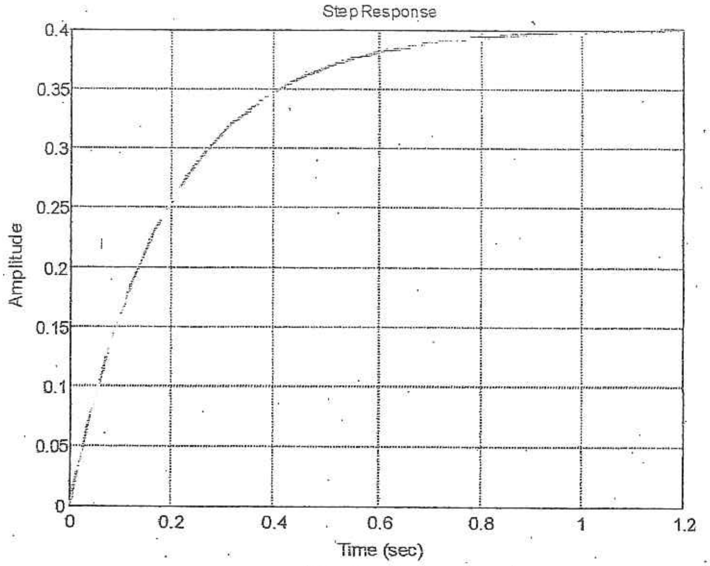


Figura 3

Se pide:

1) Identificar las funciones de transferencia correspondientes a los bloques del elemento motor/reductora.
2) Obtener la función de transferencia en lazo abierto correspondiente al sistema de control.
3) Se observa una perturbación que se opone a la velocidad de giro ofrecida por la reductora:
   a) Dibujar el diagrama de bloques del sistema de control con la perturbación y obtener la expresión de la salida en función de la entrada de referencia y la perturbación.
   b) Obtener el valor del error en estado estacionario en unidades de salida ante una entrada de referencia constante de 5 voltios y una perturbación también constante de 0,001 rad/s.
4) Sin tener en cuenta el efecto de la perturbación, diseñar en el lugar de las raíces el controlador más sencillo que tenga como respuesta ante una entrada escalón unitario la representada en la figura 4 (respuesta con comportamiento subamortiguado).

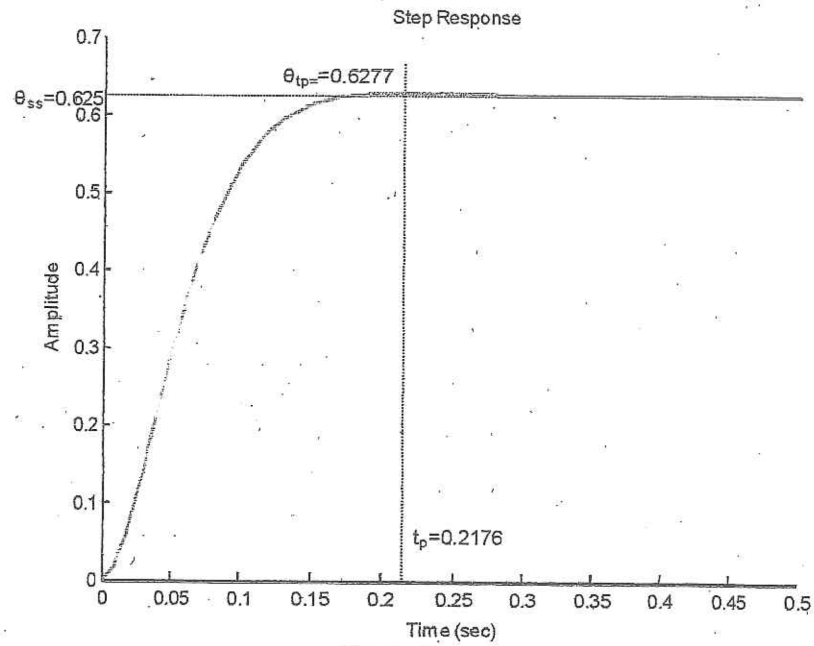


Figura 4

SOLUCIÓN

1. Identificar las funciones de transferencia correspondientes a los bloques del elemento motor/reductora.

Las ecuaciones del modelo matemático correspondientes al motor son:

$$e_b(t)=K_b\omega_m(t)$$
$$T_m(t)=K_m i(t)$$
$$v_a(t)=Ri(t)+e_b(t)$$
$$J\frac{d\omega_m(t)}{dt}+B\omega_m(t)=T_m(t)$$

Aplicando transformada de Laplace con condiciones iniciales nulas y desarrollando,

$$\left.\begin{array}{l} E_b(s)=K_b\Omega_m(s)\\ \left.\begin{array}{l}T_m(s)=K_mI(s)\\ V_a(s)-E_b(s)=RI(s)\end{array}\right\} \; T_m(s)=\frac{K_m}{R}\left[V_a(s)-E_b(s)\right] \\ \Omega_m(s)=T_m(s)\frac{1}{Js+B}\end{array}\right\} \; \Omega_m(s)=\frac{K_m}{R}\frac{1}{Js+B}\left[V_a(s)-E_b(s)\right]$$

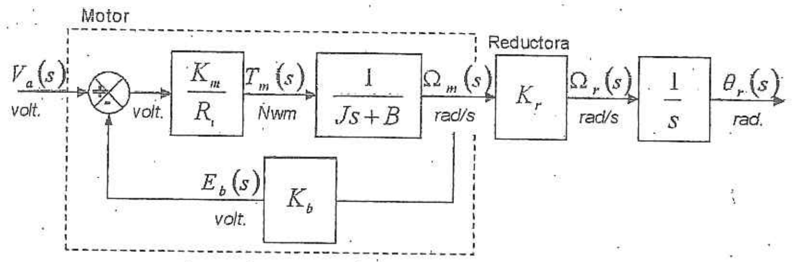
Para la reductora,

$$\omega_r(t)=K_r\omega_m(t)\xrightarrow{L}\Omega_r(s)=K_r\Omega_m(s)$$

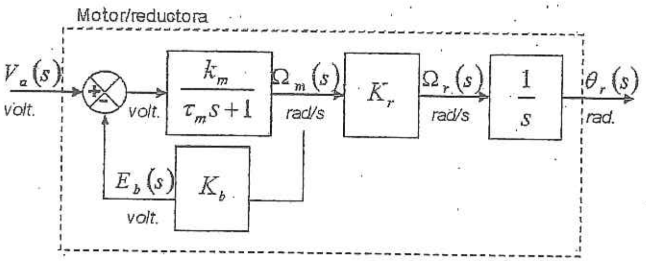
Y dado que se trata de una salida en posición,

$$\frac{d\theta_r(t)}{dt}=\omega_r(t)\xrightarrow{L}\theta_r(s)=\frac{1}{s}\Omega_r(s)$$

Así pues, el diagrama de bloques correspondiente al conjunto motor/reductora es el siguiente:



O lo que es lo mismo:



Comparando con el diagrama de bloques del enunciado:

$$\boxed{G_q(s)=\frac{k_m}{\tau_m s+1}} \quad ; \quad \boxed{G_i(s)=\frac{1}{s}}$$

La función de transferencia correspondiente al conjunto motor/reductora sería:

$$G_{mr}(s)=\frac{k_m}{(\tau_m s+1+k_mK_b)}\frac{K_r}{s}=\frac{k_mK_r}{1+k_mK_b}\frac{1}{s\left(\frac{\tau_m}{1+k_mK_b}s+1\right)}$$

La función de transferencia correspondiente al conjunto motor/reductora puede obtenerse a partir de del diagrama de Bode de la figura 2. Analizando dicho diagrama se puede observar que se trata de la función de transferencia correspondiente a un integrador y un factor de primer orden con una frecuencia de cruce de *50 rad/s* con una ganancia k_v=*-50,45dBs*. Por lo tanto:

$$G_{mr}(s)=\frac{k_v}{s(\tau s+1)}=\frac{0{,}003}{s(0.02s+1)}=\frac{k_m K_r}{1+k_m K_b}\frac{1}{s\left(\frac{\tau_m}{1+k_m K_b}s+1\right)}\rightarrow\begin{cases}k_m=0{,}54\\ \tau_m=0{,}036\end{cases}$$

Así pues:

$$G_q(s)=\frac{k_m}{\tau_m s+1}\quad\rightarrow\quad G_q(s)=\frac{0{,}54}{0.036s+1}\quad\rightarrow\quad\boxed{G_q(s)=\frac{15}{s+27.5}}$$

2. Obtener la función de transferencia en lazo abierto correspondiente al sistema de control.

La función de transferencia correspondiente al sistema de control en lazo abierto viene dada por la expresión:

$$G_{LA}(s)=G_c(s)G_{mr}(s)G_p(s)K_s$$

La función de transferencia del panel solar $G_p(s)$ puede obtenerse a partir de respuesta a escalón unitario representada en la figura 3 del enunciado. Observando la gráfica, se corresponde con la de un sistema de primer orden de ganancia 0,4 y constante de tiempo 0,2:

$$G_p(s)=\frac{0{,}4}{0.2s+1}\quad\rightarrow\quad\boxed{G_p(s)=\frac{2}{s+5}}$$

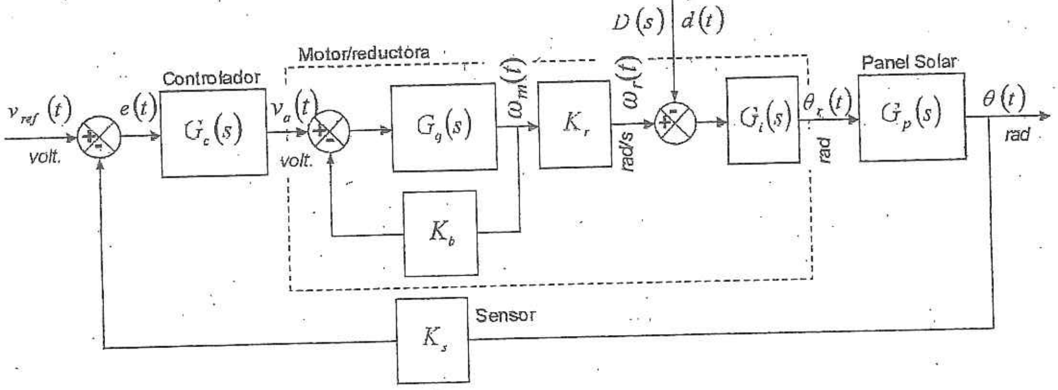
Así pues, la función de transferencia en lazo abierto es:

$$G_{LA}(s)=G_c(s)G_{mr}(s)G_p(s)K_s=\frac{0{,}003}{s(0.02s+1)}\frac{0{,}4}{0.2s+1}1.6=\frac{0{,}48}{s(s+50)(s+5)}\quad\rightarrow\quad\boxed{G_{LA}(s)=\frac{0{,}48}{s(s+50)(s+5)}}$$

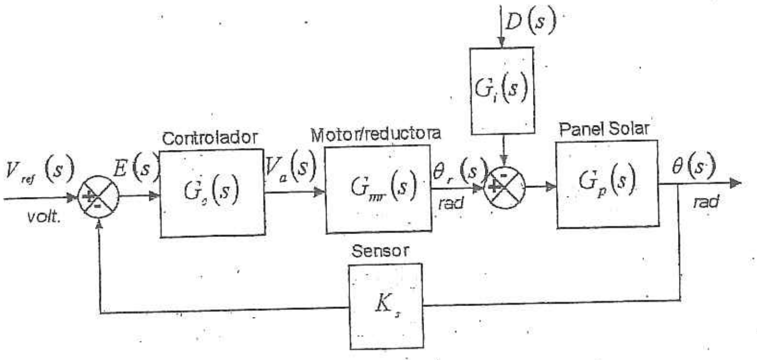
3. Se observa una perturbación que se opone a la velocidad de giro ofrecida por la reductora.

3.a Dibujar el diagrama de bloques del sistema de control con la perturbación y obtener la expresión de la salida en función de la entrada de referencia y la perturbación.

El diagrama de bloques buscado será el siguiente:



O lo que es lo mismo:



Aplicando el principio de superposición:

$$\theta(s) = M_V(s)V(s) + M_D(s)D(s)$$

$$M_V(s) = \left.\frac{\theta(s)}{V(s)}\right|_{D(s)=0} = \frac{G_c(s)G_{mr}(s)G_p(s)}{1 + K_s G_c(s)G_{mr}(s)G_p(s)}$$

$$M_D(s) = \left.\frac{\theta(s)}{D(s)}\right|_{V(s)=0} \quad \left\{ \begin{array}{l} [-\theta(s)K_s G_c(s)G_{mr}(s) - D(s)G_i(s)]G_p(s) = \theta(s) \\ \theta(s)[1 + K_s G_c(s)G_{mr}(s)G_p(s)] = -G_p(s)D(s) \end{array} \right.$$

$$M_D(s) = \frac{-G_i(s)G_p(s)}{1 + K_s G_c(s)G_{mr}(s)G_p(s)}$$

Sustituyendo valores:

$$M_V(s) = \frac{\frac{0,003}{s(0.02s+1)}\frac{0,4}{0.2s+1}}{1+1.6\frac{0,003}{s(0.02s+1)}\frac{0.4}{0.2s+1}} = \frac{0,0012}{s(0,02s+1)(0.2s+1)+1.6\cdot 0.0012} = \frac{0,0012}{0.004s^3+0.22s^2+s+1.6\cdot 0.0012}$$

$$M_V(s) = \frac{0,3}{s^3+55s^2+250s+0,48}$$

$$M_D(s) = -\frac{\frac{1}{s}\frac{0,4}{0,2s+1}}{1+1.6\frac{0.003}{s(0,02s+1)}\frac{0.4}{0.2s+1}} = -\frac{0.4(0.02s+1)}{0.004s^3+0.22s^2+s+1.6\cdot 0.0012} = -\frac{2(s+50)}{s^3+55s^2+250s+0,48}$$

Así pues,

$$\boxed{\theta(s) = \frac{0,3}{s^3+55s^2+250s+0,48}V(s) - \frac{2(s+50)}{s^3+55s^2+250s+0.48}D(s)}$$

3.b Obtener el valor del error en estado estacionario en unidades de salida ante una entrada de referencia constante de 5 voltios y una perturbación también constante de 0,001 rad/s.

Dado que se trata de un sistema tipo 1, el error estacionario debido a una entrada en posición (como es el caso) es cero. El único error observable es el debido a la perturbación. Dado que se pide en unidades de salida, el error estacionario es la propia salida estacionaria correspondiente a la perturbación ($e_{ss\,d} = \theta_{ss\,d}$):

$$\left. \begin{array}{l} \theta_{ssd} = \lim\limits_{s\to 0} s\theta_D(s) \\ \theta_D(s) = M_D(s)D(s) \\ D(s) = \frac{0,01}{s} \end{array} \right\} \quad \theta_{ssd} = \lim_{s\to 0} s \cdot -\frac{2(s+50)}{s^3+55s^2+250s+0,48}\cdot\frac{0,001}{s} \rightarrow \boxed{e_{ssd} = 0.208\ rad.}$$

4. Diseñar en el lugar de las raíces el controlador más sencillo que tenga como respuesta ante una entrada escalón unitario la representada en la figura.

Se puede observar que la respuesta ante una entrada escalón unitario tiene un comportamiento subamortiguado en donde el valor en el estacionario es $\theta_{ss} = 0.625$, el tiempo de pico $t_p = 0,2176$, y el valor de la salida en el tiempo de pico es $\theta_{t_p} = 0,6277$. Aproximando a un sistema de segundo orden subamortiguado,

$$\left. \begin{array}{l} M_p = \frac{\theta_{t_p} - \theta_{ss}}{\theta_{ss}} = \frac{0.6277-0.625}{0,625} = 0.00432 \\ M_p = e^{\frac{-\delta\pi}{\sqrt{1-\delta^2}}} \end{array} \right\} \begin{array}{l} \delta = 0.866 \\ \delta = \cos\vartheta \end{array} \rightarrow \vartheta = 30° \ : \ t_p = \frac{\pi}{\omega_n\sqrt{1-\delta^2}} = 0.2176 \rightarrow \omega_n = 28,87$$

Por lo tanto, una valor absoluto en la parte real correspondiente a los polos complejos conjugados de $\delta\omega_n = 25$.

La función de transferencia en lazo abierto de la que partimos es la siguiente,

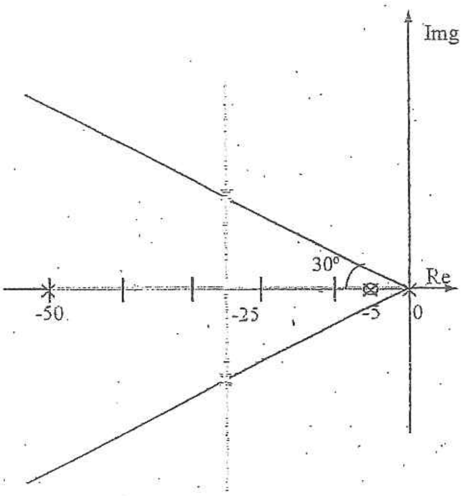
$$G_{LA}(s) = \frac{0.48 G_c(s)}{s(s+50)(s+5)}$$

Para que se cumplan las condiciones de la respuesta es necesario introducir un controlador que elimine la acción del polo en $s = -5$. Es decir que partimos de un controlador PD ideal con $T_d = 1/5 = 0,2$.

$$G_c(s) = K_c T_d (s + 1/T_d) = 0.2K_c(s+5)$$

$$G_{LA}(s) = \frac{0.096 K_c}{s(s+50)}$$

$$G_{LC}(s) = \frac{0.096 K_c / 1.6}{s^2 + 50s + 0.096 K_c} = \frac{0.06 K_c}{s^2 + 50s + 0.096 K_c}$$



Comparando la función de transferencia en lazo cerrado con la función de transferencia de un sistema de segundo orden,

$$\left.\begin{array}{l} G_{2^\circ}(s) = \dfrac{K\omega_n^{\,2}}{s^2 + 2\delta\omega_n s + \omega_n^{\,2}} \\ G_{LC}(s) = \dfrac{0.06 K_c}{s^2 + 50s + 0.096 K_c} \end{array}\right\}_{\substack{\delta=0,866 \\ \omega_n=28,87}} \quad \begin{array}{l} K = 0,625 \\ K_c = 8682 \end{array}$$

Así pues el controlador buscado será,

$$\boxed{G_c(s) = 1736.4(s+5)}$$

eman ta zabal zazu
Universidad del país vasco
Euskal herriko unibertsitatea

INGENIERÍA DE SISTEMAS I

Curso: 2011/2012
10 de Enero de 2012

Nombre
Izena

1°Apellido
1 Deitura

2° Apellido
2 Deitura

Tiempo:
1,5 Horas

Grupo:

PROBLEMA 2 (4 PUNTOS)

El diagrama de bloques de la *Figura 1* representa el sistema de control de un proceso *G(s)*, en el que *$G_c(s)$* es un controlador proporcional.

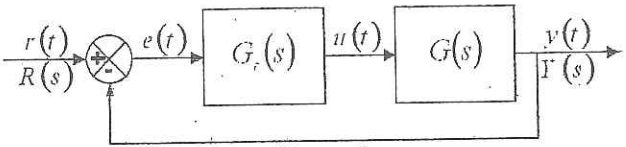


Figura 1

En la *Figura 2* se muestra el diagrama de Bode de *G(s)*, y la *Figura 3* representa el diagrama de Bode del sistema en bucle cerrado *Y(s)/R(s)* con el controlador sintonizado para un valor de $K_c = 5$.

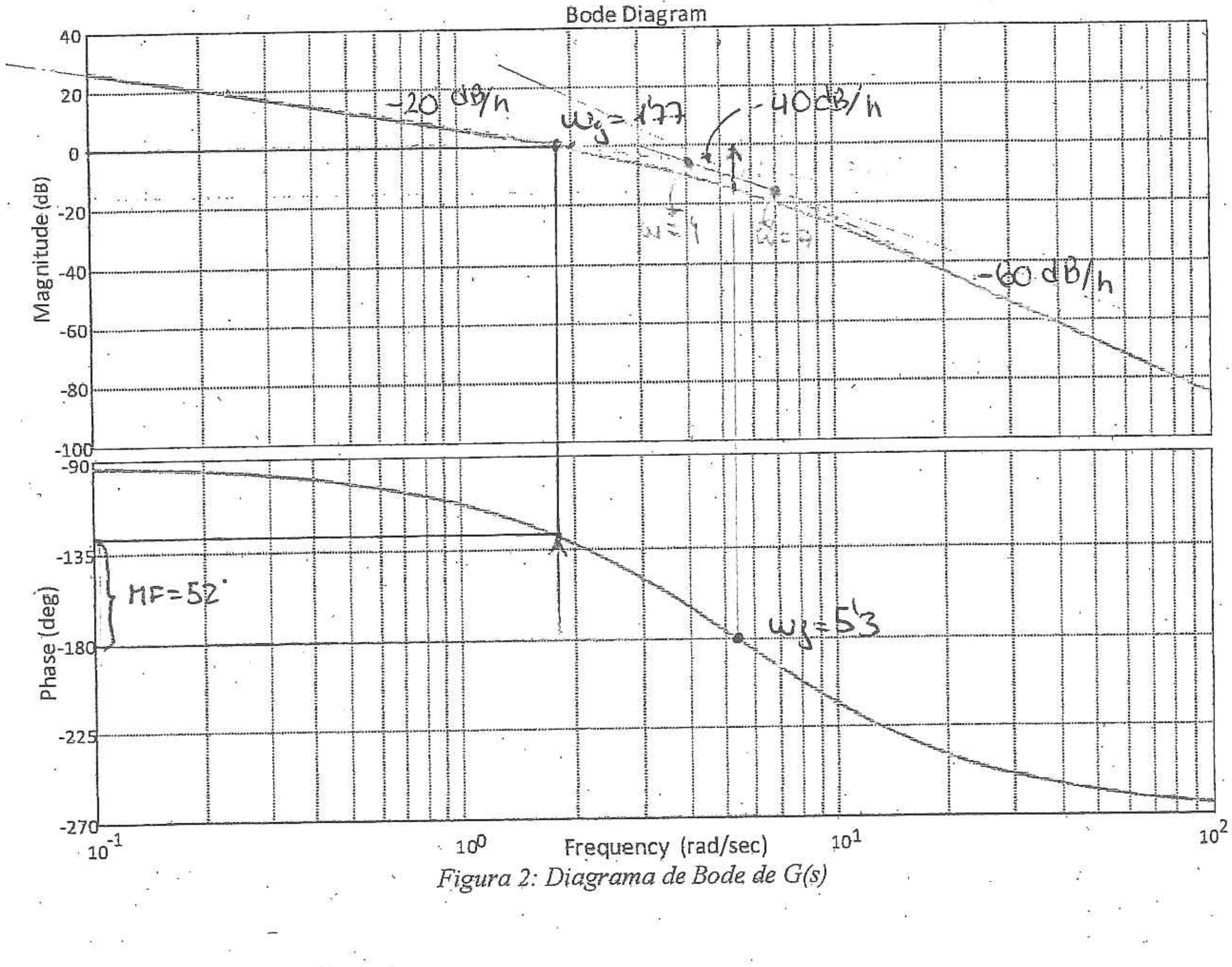


Figura 2: Diagrama de Bode de G(s)

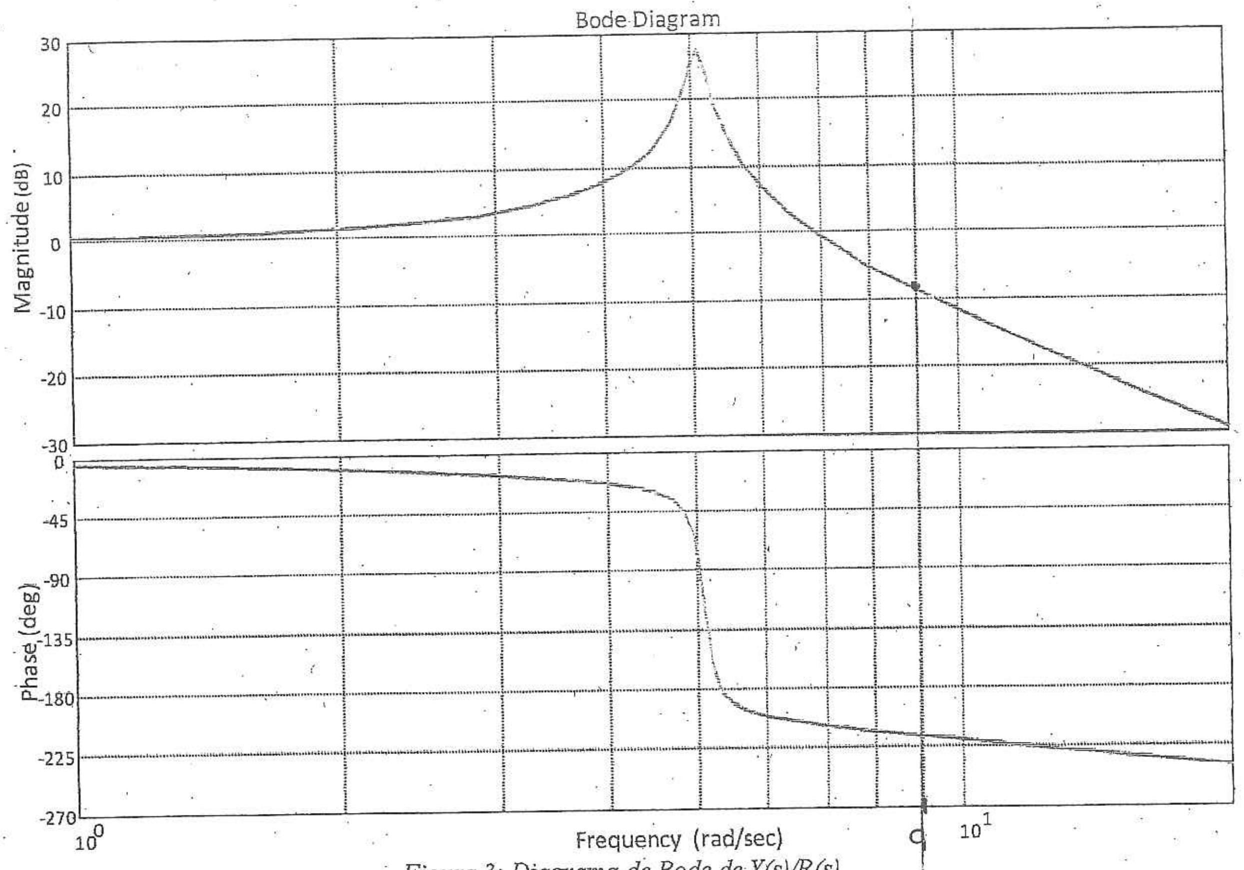


Figura 3: Diagrama de Bode de Y(s)/R(s)

Se pide:

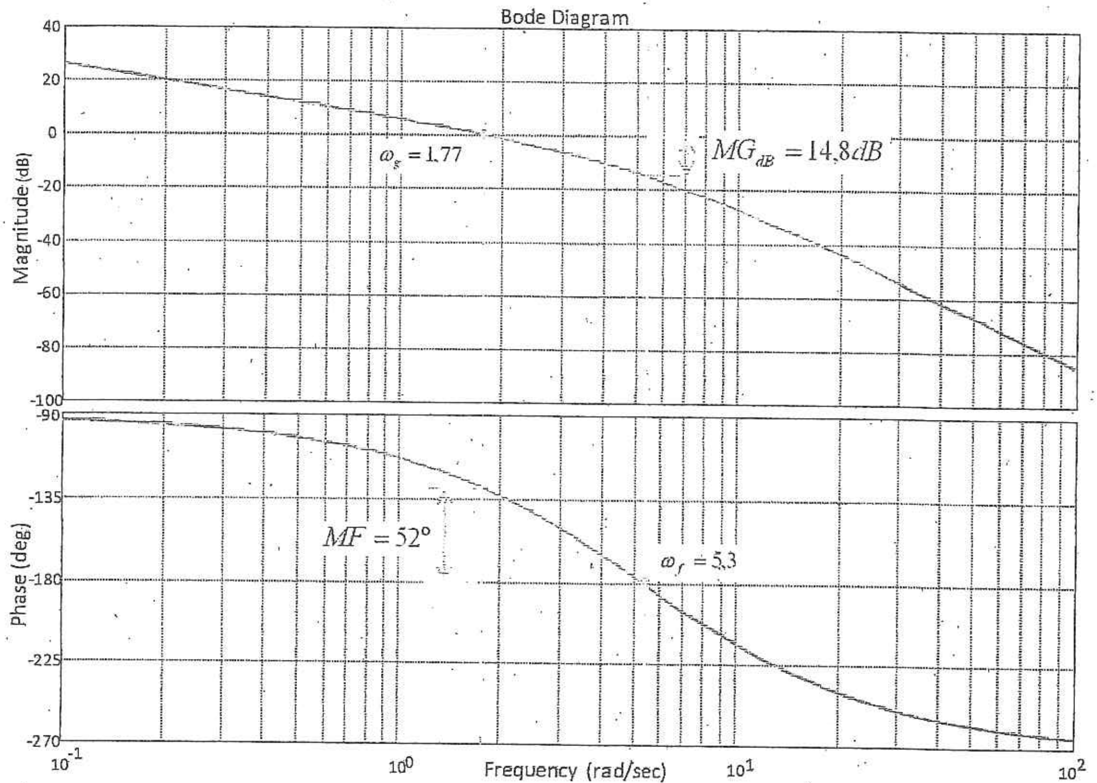
1) Calcular el margen de ganancia y el margen de fase del sistema, para un valor de $K_c = 1$. Realizar el cálculo aproximado a partir de las gráficas proporcionadas e indicándolo sobre ellas y, posteriormente, comprobar los resultados de forma analítica.
2) ¿Cuál es el valor máximo que puede darse a la ganancia de controlador sin desestabilizar el sistema?
3) ¿Qué valor hay que darle a la ganancia del controlador para que $e_{ss} = 10\%$ ante una entrada rampa unitaria en la referencia?
4) ¿Cuál será el nuevo valor del margen de ganancia para el valor de K_c obtenido en el apartado 3?
5) Calcular la respuesta en estado estacionario del sistema ante una entrada $r(t) = 2\ sen\ 9t$ para el valor de K_c obtenido en el apartado 3. Realizar el cálculo aproximado a partir de las gráficas proporcionadas e indicándolo sobre ellas y, posteriormente, comprobar los resultados de forma analítica.

SOLUCIÓN

1. Calcular analíticamente el margen de ganancia y el margen de fase del sistema, para un valor de $K_c = 1$.

La función de transferencia correspondiente al sistema en lazo abierto viene dada por la expresión $G_{LA}(s) = G_c(s)G(s)$. Dado que, inicialmente se trabaja de un controlador proporcional de ganancia unidad $G_{LA}(s) = G(s)$.

La función de transferencia de la planta $G(s)$ puede identificarse desde el diagrama de Bode de la figura 2. Se puede observar que el sistema es de fase mínima. A partir de dicha gráfica es posible encontrar los márgenes de fase y ganancia a partir de la identificación de las frecuencias de fase crítica ω_f y ganancia crítica (ω_g), tal y como se indica en la siguiente figura,



Para realizar el cálculo analítico es necesario identificar $G(s)$. Se aprecia una pendiente de -20dBs en la asíntota de bajas frecuencias, por lo que se trata de un sistema con un integrador puro. Por otra parte, se pueden identificar dos cambios de pendiente, cada uno de -20dBs/dec, es decir dos factores de primer orden en el denominador con frecuencias de cruce $\omega_{c1} = 4$ y $\omega_{c2} = 7$. En cuanto a la ganancia, se puede observar que la asíntota de bajas frecuencias cruza por 0dBs en $\omega_v = 2$, por lo que $K_v = 2$. Así pues, la función de transferencia de la planta viene dada por:

$$G(s) = \frac{K_v}{s(T_1 s + 1)(T_2 s + 1)} \cdot \left\{ \begin{array}{l} T_1 = 1/\omega_{c1} = 0{,}25 \\ T_2 = 1/\omega_{c2} = 0{,}14 \\ K_v = 2 \end{array} \right\} \quad G(s) = \frac{2}{s(0.25s + 1)(0.14s + 1)} \rightarrow G(s) = \frac{56}{s(s+4)(s+7)}$$

En el dominio de la frecuencia:

$$G(j\omega) = \frac{2}{j\omega(1 + j0.25\omega)(1 + j0.14\omega)} \quad \begin{cases} |G(j\omega)| = \dfrac{2}{\omega\sqrt{1 + 0.25^2\omega^2}\sqrt{1 + 0.14^2\omega^2}} \\ Arg[G(j\omega)] = -90 - \arctan 0.25\omega - \arctan 0.14\omega \end{cases}$$

Para calcular el margen de ganancia ($MG_{dB} = -20\log|G(j\omega_f)|$) es necesario calcular la frecuencia de fase crítica (ω_f).

$$Arg\left[G(j\omega_f)\right] = -180 = -90 - \arctan 0.25\omega_f - \arctan 0.14\omega_f \rightarrow \arctan 0.25\omega_f + \arctan 0.14\omega_f = 90 \quad \rightarrow \quad \omega_f = 5{,}3 rad/s$$

$$MG_{dB} = -20\log\left|G(j\omega_f)\right| = -20\log\frac{2}{\omega_f\sqrt{1+0.25^2\omega_f^2}\sqrt{1+0.14^2\omega_f^2}} \quad \rightarrow \quad \boxed{MG_{dB} = 14.8dB}$$

Para calcular el margen de fase ($MF = 180 + Arg\left[G(j\omega_g)\right]$) es necesario calcular la frecuencia de ganancia crítica (ω_g).

$$\left|G(j\omega_g)\right| = 1 = \frac{2}{\omega_g\sqrt{1+0.25^2\omega_g^2}\sqrt{1+0.14^2\omega_g^2}} \quad \rightarrow \quad \omega_g = 1{,}77 rad/s$$

$$MF = 180 + Arg\left[G(j\omega_g)\right] = 180 - 90 - \arctan(0.25\cdot 1.77) - \arctan(0.14\cdot 1.77) \quad \rightarrow \quad \boxed{MF = 52^\circ}$$

2. ¿Cuál es el valor máximo que puede darse a la ganancia de controlador sin desestabilizar el sistema?

El valor máximo que se le puede tomar la ganancia del controlador antes de desestabilizarse el sistema (K_{cr}) viene dado por la el margen de ganancia. Así pues,

$$MG = 20\log K_{cr} \quad \rightarrow \quad 14.8 = 20\log K_{cr} \quad \rightarrow \quad \boxed{K_{cr} = 5.5}$$

3. ¿Qué valor hay que darle a la ganancia del controlador para que *ess = 10%* ante una entrada rampa unitaria en la referencia?

A partir del coeficiente estático de error en velocidad y suponiendo una ganancia del controlador K_c,

$$\left.\begin{array}{l} e_{ss\,v} = \dfrac{1}{K_v} = 0{,}1 \\ K_v = \lim_{s\to 0} sG_{LA}(s) = \lim_{s\to 0} sK_cG(s) = \lim_{s\to 0} s\dfrac{2K_c}{s(0.25s+1)(0.14s+1)} = 2K_c \end{array}\right\} \quad \frac{1}{2K_c} = 0{,}1 \quad \rightarrow \quad \boxed{K_c = 5}$$

Dado que el resultado obtenido es menor que el valor máximo, las suposiciones necesarias para aplicar las fórmulas empleadas se cumplen.

4. ¿Cuál será el nuevo valor del margen de ganancia para el valor de K_c obtenido en el apartado 3?

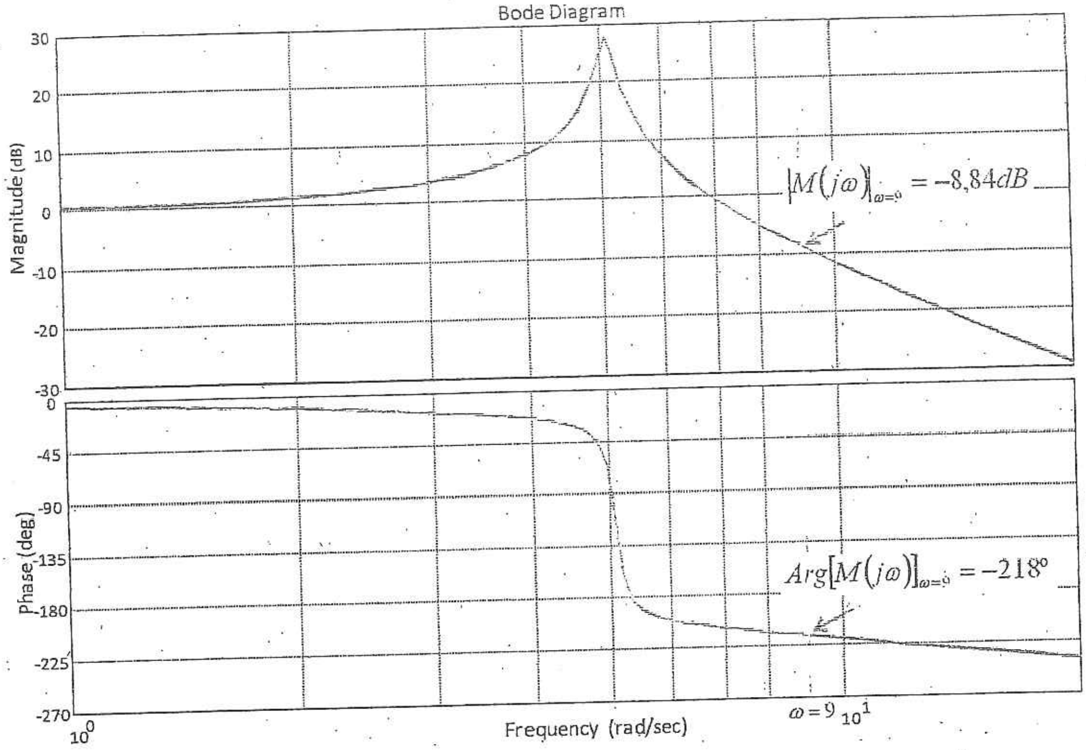
$$MG = 14.8 - 20\log K_c \quad \rightarrow \quad \boxed{MG = 0.83dB}$$

5. Calcular la respuesta en estado estacionario del sistema ante una entrada *r(t) = 2 sen 9t* para el valor de K_c obtenido en el apartado 3, explicando cómo se ha obtenido.

La respuesta de un sistema de función de transferencia en lazo cerrado *M(s)* ante una entrada sinusoidal de amplitud 2 y frecuencia 9 radianes como la dada es la siguiente:

$$y_{ss}(t) = 2\cdot\left|M(j\omega)\right|_{\omega=9} sen\left(9t + Arg\left[M(j\omega)\right]_{\omega=9}\right)$$

Se puede obtener los valores de módulo y fase a partir de la gráfica correspondiente a la función de transferencia en lazo cerrado para $K_c = 5$ proporcionada en el enunciado. En este caso:



Se puede observar, para $\omega = 9$:

$$|M(j\omega)|_{dB} = -8.84dB \rightarrow |M(j\omega)| = 0{,}36$$

$$Arg[M(j\omega)] = -218° \rightarrow Arg[M(j\omega)] = -3.8rad$$

Así pues, la respuesta en estado estacionario del sistema ante una entrada *r(t) = 2 sen 9t* para el valor de $K_c = 5$ será la siguiente:

$$y_{ss}(t) = 2 \cdot 0.36\, sen(9t - 3.8) \rightarrow \boxed{y_{ss}(t) = 0{,}72\, sen(9t - 3{,}8)}$$

Realizando el cálculo analítico, la función de transferencia del sistema en lazo cerrado para $K_c = 5$ será la siguiente:

$$M(s) = \frac{K_c G(s)}{1 + K_c G(s)} = \frac{5 \cdot 56}{s(s+4)(s+7) + 5 \cdot 56} \rightarrow M(s) = \frac{280}{s^3 + 11s^2 + 28s + 280}$$

Por lo que la función de transferencia sinusoidal será:

$$M(j\omega) = \frac{280}{280 - 11\omega^2 + j\omega(28 - \omega^2)}$$

Calculando módulo y argumento:

$$|M(j\omega)| = \frac{280}{\sqrt{(280 - 11\omega^2)^2 + (\omega(28 - \omega^2))^2}}$$

$$Arg[M(j\omega)] = -\arctan\frac{\omega\,(28 - \omega^2)}{280 - 11\omega^2}$$

Para $\omega = 9$:

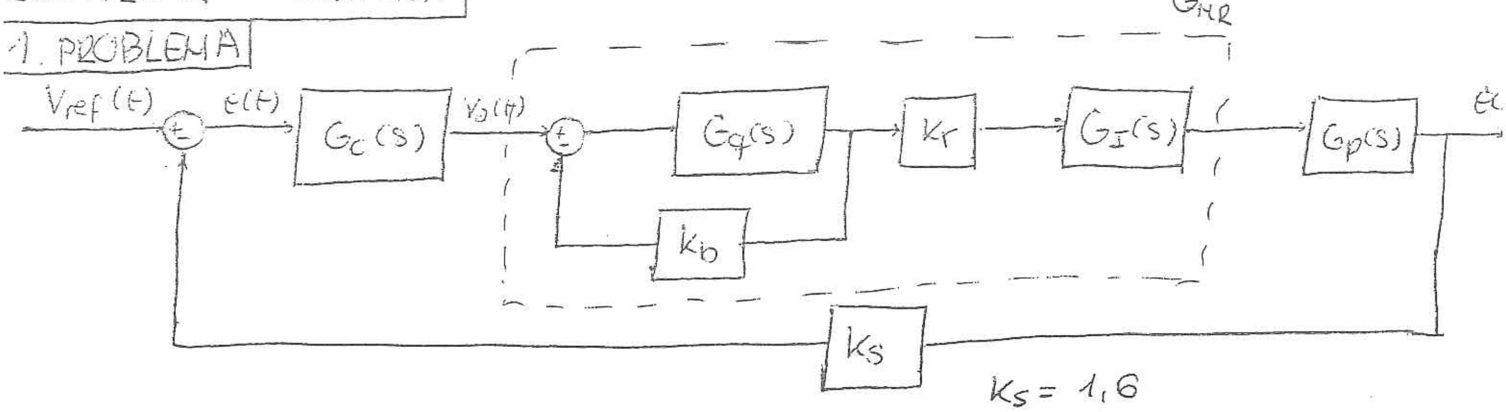
$$|M(j\omega)| = 0{,}36 \qquad Arg[M(j\omega)] = -218° \rightarrow Arg[M(j\omega)] = -3{,}8rad$$

Así pues, la respuesta en estado estacionario del sistema ante una entrada *r(t) = 2 sen 9t* para el valor de $K_c = 5$ será la siguiente:

$$y_{ss}(t) = 2 \cdot 0.36\, sen(9t - 3.8) \rightarrow \boxed{y_{ss}(t) = 0.72\, sen(9t - 3.8)}$$

2011/2012 URTARRILA

1. PROBLEMA



$K_s = 1,6$

1)

→ Bode diagramatik motor osoaren transferentzi funtzioa atera dezakegu.

- $\omega_1 \simeq 50\ rad/s \longrightarrow T_1 = \frac{1}{50} = 0,02s$
- $-20\ dB/h$ -ko malda dauka maiztasun txikietan → integradore bat.
- $-50,45 = 20 \log\left(\frac{K}{1}\right) \Rightarrow \log K = -2,5225 \Rightarrow \underline{\underline{K = 3 \cdot 10^{-3}}}$

$$G_{HR}(s) = \frac{0,003}{s(1+0,02s)}$$

→ Bloke-diagramatik ere $G_{HR}(s)$ lortuko dugu:

$$G_{HR} = K_r \cdot G_I(s) \cdot \frac{G_q(s)}{1+K_b G_q(s)} = \frac{K_r \cdot G_I(s) \cdot G_q(s)}{1+K_b \cdot G_q(s)}$$

$K_b = 1,5$; $K_r = \frac{1}{100}$

→ Eredu matematikotik $G_I(s)$, $G_q(s)$ lor ditzakegu:

$$G_I(s) = \frac{\theta_r(s)}{\omega_r(s)} \quad ; \quad G_q(s) = \frac{\omega_m(s)}{V_a(s) - K_b \omega_m(s)}$$

$$\left\{\begin{array}{l}
T_m(t) = K_m \cdot i(t) \longrightarrow I(s) = \frac{T_m(s)}{K_m} \\
e_b(t) = K_b \cdot \omega_m(t) \longrightarrow E_b(s) = K_b \cdot \omega_m(s) \\
V_a(t) = Ri(t) + E_b(t) \longrightarrow V_a(s) = R \cdot \frac{T_m(s)}{K_m} + K_b \omega_m(s) \\
J \frac{d\omega_m(t)}{dt} + B\omega_m(t) - T_m(t) \longrightarrow \omega_m(s) = \frac{T_m(s)}{Js+B} \longrightarrow T_m(s) = \omega_m(Js+
\end{array}\right.$$

$$V_a(s) - K_b \cdot \omega_m(s) = R\ \frac{T_m(s}{K_m}$$

$$\omega_r(t) = K_r\, \omega_m(t) \longrightarrow \omega_r(s) = K_r\, \omega_m(s) \longrightarrow K_r = \frac{\omega_r(s)}{\omega_m(s)}$$

$$\frac{d\theta_r(t)}{dt} = \omega_r(t) \longrightarrow s\theta_r(s) = \omega_r(s) \longrightarrow \frac{\theta_r(s)}{\omega_r(s)} = \frac{1}{s} \Longrightarrow \boxed{G_I(s) = \frac{1}{s}}$$

$$V_A(s) - K_b\,\omega_m(s) = \frac{R}{K_m}\,(\omega_m(s)\,(Js+B)$$

$$\frac{V_A(s) - K_b\,\omega_m(s)}{\omega_m(s)} = \frac{(Js+B)R}{K_m} \Rightarrow \frac{\omega_m(s)}{V_A(s) - K_b\,\omega_m(s)} = \frac{K_m}{R(sJ+B)} = G_q(s)$$

$$G_q(s) = \frac{K'_m}{\tau_m s+1}$$ ⇒ K'_m eta τ_m-ren balioak lortu behar ditugu

Horrebiako:

$$\frac{0{,}003}{\cancel{s}(1+0{,}02s)} = \frac{0{,}01 \cdot 1/\cancel{s} \cdot G_q(s)}{1+1{,}5\,G_q(s)}$$

$$\frac{0{,}003}{1+0{,}02s} = \frac{0{,}01\,\frac{K'_m}{\cancel{\tau_m s+1}}}{\frac{\tau_m s+1+1{,}5K'_m}{\cancel{\tau_m s+1}}} \Rightarrow \frac{0{,}003}{1+0{,}02s} = \frac{0{,}01\,K'_m}{\underbrace{\tau_m s + (1+1{,}5K'_m)}_{\frac{\tau_m}{1+1{,}5K'_m}+1}}$$

$$\frac{0{,}003}{1+0{,}02s} = \frac{\frac{0{,}01\,K'_m}{1+1{,}5K'_m}}{\frac{\tau_m}{1+1{,}5K'_m}+1} \Rightarrow \begin{cases} 0{,}003 = \dfrac{0{,}01\,K'_m}{1+1{,}5K'_m} \\ 0{,}02 = \dfrac{\tau_m}{1+1{,}5K'_m} \end{cases}$$

$$0{,}003 + 4{,}5\cdot 10^{-3} K'_m = 0{,}01\,K'_m \Rightarrow K'_m = 0{,}545 \longrightarrow \tau_m = 0{,}036$$

$$\boxed{G_q(s) = \frac{0{,}545}{0{,}036s+1}}$$

2) G_{BA}?

$$G_{BA}(s) = \underset{\underset{K_c=1}{\parallel}}{G_c}(s) \cdot \overset{\frac{0{,}003}{s(1+0{,}02s)}}{\overset{\parallel}{G_{MR}}}(s) \cdot G_p(s) \cdot \overset{1{,}6}{\overset{\parallel}{K_s}} \qquad (\;G(s)\cdot H(s) = G_{BA}(s))$$

cenuntzaten esaten digute)

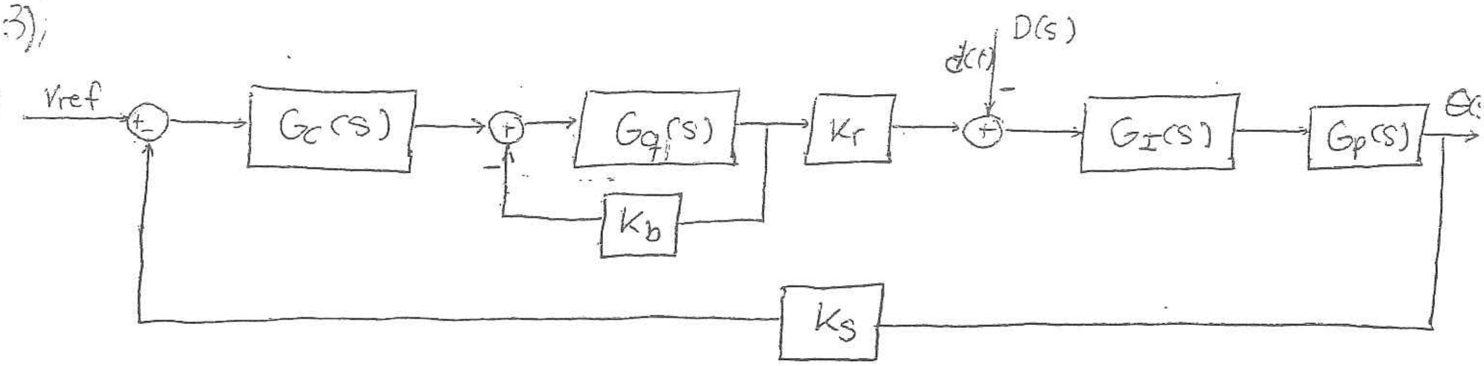
→ $G_p(s)$ 3. grafikotik lortuko dugu:

$$\left.\begin{aligned} K &= \frac{\Delta y}{\Delta u} = \frac{0{,}4}{1} = 0{,}4 \\ y_{63} &= 0{,}63\cdot 0{,}4 = 0{,}252 \Rightarrow \tau = 0{,}2 \end{aligned}\right\} \; G_p(s) = \frac{0{,}4}{0{,}2s+1}$$

$$G_{BA}(s) = \frac{0{,}003\cdot 0{,}4\cdot 1{,}6}{s(1+0{,}02s)(0{,}2s+1)} = \frac{0{,}48}{s(s+50)(s+5)}$$ (1. motakoa)

3);

)



$$\left.\frac{\theta(s)}{V(s)}\right|_{D(s)=0} = \frac{G_C(s)\,G_{HR}(s)\,G_P(s)}{1+G_C(s)\,G_{HR}(s)\,G_P(s)\cdot K_S} = \frac{0{,}0012}{1{,}92\cdot10^{-3}+4\cdot10^{-3}s^3+0{,}22s^2+s}$$

$$= \frac{0{,}3}{s^3+55s^2+250s+0{,}48}$$

$$\left.\frac{\theta(s)}{D(s)}\right| = -\frac{G_I\,G_p}{1+G_p\cdot G_c\cdot G_{HR}\cdot K_S} = \frac{-2(s+50)}{s^3+55s^2+250s+0{,}48}$$

b) $V(s)=\frac{5}{s}$; $D(s)=\frac{0{,}001}{s}$

$$\boxed{e_{ss}=e_{ss_R}+e_{ss_D}=0{,}208}$$

- $e_{ss_R}=0$ → Lehen ondorioztatu dugu 1. motakoa zela

- e_{ss_D}

$$E(s)|_{R(s)=0} = -\frac{2(s+50)}{s^3+55s^2+250s+0{,}48}\cdot\frac{0{,}001}{s}$$

$$e_{ss_D}=\lim_{s\to0}\cancel{s}\cdot\frac{0{,}001}{\cancel{s}}\cdot\left(-\frac{2(s+50)}{s^3+55s^2+250s+0{,}48}\right) = -\frac{0{,}001\cdot2\cdot50}{0{,}48}$$

$$e_{ss_D}=0{,}208$$

4)

→ 4. grafikotik lortuko ditugu M_p eta t_p-ren balioak :

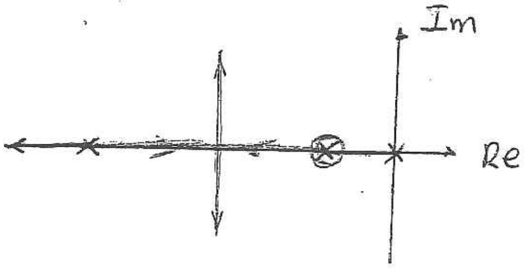
$$M_p=\frac{\theta_{tp}-\theta_{ss}}{\theta_{ss}}=\frac{0{,}6277-0{,}625}{0{,}625}=0{,}00432 \Rightarrow \zeta=0{,}866 \quad (\theta=30^\circ)$$

$$t_p=\frac{\pi}{\omega_d}=\frac{\pi}{\omega_n\sqrt{1-\zeta^2}}=0{,}2176 \longrightarrow \omega_n=28{,}87 \text{ rad/s} \longrightarrow \zeta\omega_n=25$$

$$G_{BA}(S) = \frac{G_c(s)\cdot 0{,}48}{S(S+50)(S+5)}$$

$n=3 \rightarrow 3$ polo $\begin{cases} s=0 \\ s=-50 \\ s=-5 \end{cases}$

$m=0$



PD bat erabiliko degu, zero bat sartuz polo dominantea degen lekuan, anulatzeko.

$$z_D = -5 \rightarrow \frac{1}{T_d} = 5 \longrightarrow T_d = 0{,}2S$$

$$G_c(s) = G_{PD}(S) = K_c(s+z_D) = K_c(S+5)$$

$$G_{BA}(S) = \frac{T_d \cdot K_c\,\cancel{(S+5)}\cdot 0{,}48}{S(S+50)\cancel{(S+5)}} = \frac{0{,}48/5\ K_c}{S(S+50)} = \frac{0{,}096\,K_c}{S(S+50)}$$

$$G_{BC}(S) = \frac{0{,}096K_c/1{,}6}{S^2+50S+0{,}096K_c} = \frac{0{,}06K_c}{S^2+50S+0{,}096K_c}$$

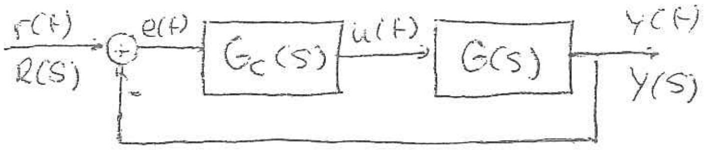
$$S^2+50S+0{,}096K_c = S^2+2\delta\omega_n+\omega_n^2$$

$$\begin{cases} 50 = 2\delta\omega_n \longrightarrow \omega_n = \dfrac{50}{2\cdot 0{,}866} = \\ \omega_n^2 K = 0{,}06K_c \qquad\qquad\qquad = 28{,}87 \\ 0{,}096K_c = \omega_n^2 \end{cases}$$

$$\boxed{K_c = \frac{28{,}87^2}{0{,}096} = 8681{,}1}$$

$$\boxed{K = \frac{0{,}06\cdot 8681{,}1}{28{,}87^2} = 0{,}625}$$

2. PROBLEMA



1) $K_c = 1 \rightarrow$ MF? MG?

→ Grafikoetatik lortzen dugu:

$$G(s) \begin{cases} \boxed{MG = 14{,}8 \text{ dB}} \\ \boxed{MF = 52^\circ} \end{cases}$$

$K_c = 5$

$G(S) \rightarrow$ 2. irudia

$\frac{Y(S)}{R(S)} \rightarrow$ 3. irudia

$$G_{BA}(s) = G_c(s) G(s) = 1 \cdot G(s) \Rightarrow G_{BA}(S) = G(S)$$

Sistemaren MF eta MG, G(s) funtzioarenak rango dira.

→ Analitikoki egiteko $G_{BA}(s)$ lortu behar dugu, G(s) rango dena.

Bode diagramatik lortuko dugu:

- $\omega_1 = 4$ rad/s $\longrightarrow T_1 = \frac{1}{\omega_1} = \frac{1}{4} = 0{,}25$ s : Poloa
- $\omega_2 = 7$ rad/s $\longrightarrow T_2 = \frac{1}{\omega_2} = \frac{1}{7}$ s : Poloa

$$G(s) = \frac{K}{s(s0{,}25+1)(1+\frac{1}{7}s)}$$

ω txikietan −20 dB/h-ko malda dauka ⇒ integradore bat.

$$20 \log\left(\frac{K}{0{,}2}\right) = 20 \rightarrow \log\left(\frac{K}{0{,}2}\right) = 1 \Rightarrow \frac{K}{0{,}2} = 10 \rightarrow \underline{\underline{K = 2}}$$

$$G(s) = G_{BA}(s) = \frac{2}{s(1+\frac{1}{4}s)(1+\frac{1}{7}s)} \qquad |G(j\omega)| = \frac{2}{\omega\sqrt{1+0{,}25^2\omega^2}\sqrt{1+(\frac{1}{7})^2\omega^2}}$$

$$MG = 20 \log\left(\frac{1}{|G_{BA}(j\omega_f)|}\right)$$

$$MF = 180 + \text{Arg}\, G_{BA}(j\omega_g)$$

⟹ ω_f eta ω_g kalkulatu behar ditugu

$$\text{Arg}\, G_{BA}(j\omega_g) = -90 - \arctan(\omega_g \overset{0{,}25}{\widehat{T_1}}) - \arctan(\omega_g \overset{1/7}{\widehat{T_2}})$$

- $\underline{\underline{\omega_f}}$:

$$\text{Arg}\, G_{BA}(j\omega_f) = -180^\circ = -90 - \arctan(\omega_f \cdot 0{,}25) - \arctan(\omega_f \cdot 0{,}14)$$

$$+90 = +\arctan(\omega_f 0{,}25) + \arctan(\omega_f 0{,}14) \Rightarrow \omega_f = 5$$

$$\hookrightarrow MG = 20 \log\left(\frac{5{,}3\sqrt{1+0{,}25^2 \cdot 5{,}3^2} \cdot \sqrt{1+(1/7)^2 \cdot 5{,}3^2}}{2}\right) \Rightarrow \boxed{MG = 14{,}84 \text{ dB}}$$

- $\underline{\underline{\omega_g}}$:

$$|G_{BA}(j\omega_g)| = 1 = \frac{2}{\omega_g\sqrt{1+0{,}25^2\omega_g^2}\sqrt{1+(\frac{1}{7})^2\omega_g^2}} \rightarrow \omega_g = 1{,}77 \text{ rad/s}$$

$$\hookrightarrow MF = 180 + (-90 - \arctan(1{,}77 \cdot 0{,}25) - \arctan(1{,}77 \cdot 1/7)) \Rightarrow \boxed{MF = 51{,}94^\circ}$$

2) K_{max} (desegonkortu gabe)

~~(Egonkortasunaren muga ; $\omega_g = \omega_f = \omega_r$)~~

MG-k adierazten du zenbat handitu daitekeen K_c :

$MG = 20 \log K_{cr} = 14{,}8\,dB \rightarrow \log K_{cr} = 0{,}74 \rightarrow \boxed{K_{cr} = 10^{0{,}74} = 5{,}5}$

3) $e_{ss} = \%\,10$

sarrera arrapala unitarioa.

$e_{ss} = \frac{1}{K} = 0{,}1 \rightarrow K = 10$

↳ arrapala unitarioa

$$K = \lim_{s\to 0} \cancel{s} \frac{K_c \cdot 2}{\cancel{s}\,(s\cdot 0{,}25+1)(s\cdot 0{,}14+1)} = \frac{2K_c}{1} = 10 \rightarrow \boxed{K_c = 5}$$

4) MG? $K_c = 3$. ablekua

$$\boxed{MG = 14{,}8 - 20\log K_c = 14{,}8 - 20\log 5 = 0{,}82}$$

5) $r(t) = 2 sen\, 9t$, $K_c = 5$

$r(t) = A sen(\omega t) \rightarrow \begin{cases} A = 2 \\ \omega = 9\ rad/s \end{cases}$

$y_{ss}(t) = A\,|G(j\omega)|\,sen(\omega t + \varphi)$ ↳ $arg(G(j\omega))$

$\Rightarrow y_{ss}(t) = 2\,|G_{BC}(j9)|\,sen(9t + \varphi)$ ↑ $arg[G_{BC}(9j)]$

→ $\omega = 9$ rad/s denean, grafikotik: $|G(j\omega)|_{dB} = -8{,}84\ dB$; $Arg[G_{BC}(j\omega)] = -218$

$-8{,}84 = 20\log(|G_{BC}(j\omega)|) \Rightarrow |G_{BC}(j\omega)| = 0{,}36$

$218° \longrightarrow Arg[G_{BC}(j\omega)] = -\frac{218\cdot 2\pi}{360} = -3{,}8\ rad$

$\rightarrow y_{ss}(t) = 2\cdot 0{,}36\ sen(9t$

$$\boxed{y_{ss}(t) = 0{,}72 sen(9t - 3{,}8)}$$

→ Analitikoki $|G_{BC}(j\omega)|$ eta φ lortzeko:

$$G_{BC}(s) = \frac{K_c\cdot G(s)}{1 + K_c\,G(s)} = \frac{5\cdot 56}{s(s+4)(s+7) + 5\cdot 56} = \frac{280}{s^3+11s^2+28s+280}$$

$$G_{BC}(j\omega) = \frac{280}{-j\omega^3 - 11\omega^2 + 28j\omega + 280} \Rightarrow |G_{BC}(j\omega)| = \frac{280}{\sqrt{(280-11\omega 3)^2 + (\omega\cdot 28 - \omega}}$$

$\omega = 9 \longrightarrow |G_{BC}(j\omega)| = 0{,}36$

$$\text{Arg}\left[G_{BC}(j\omega)\right] = -\text{arctan}\,\frac{(28\omega-\omega^2)}{(280-11\omega^2)} = -280^\circ \longrightarrow -3{,}8\ \text{rad}$$

$$\boxed{y_{ss}(t) = 0{,}72\ \text{sen}(9t-3{,}8)}$$

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INGENIERÍA DE SISTEMAS I

Nombre ______________________________
Izena
1ºApellido ______________________________
1 Deitura
2º Apellido ______________________________
2 Deitura

Curso: 2010/2011
[illegible] de Enero de 2011

Tiempo:
1.5 Horas

Grupo:

PROBLEMA 1 (5 PUNTOS)

El sistema de control representado en la *Figura 1* se corresponde con el de un banco de pruebas en vacío para la multiplicadora de un aerogenerador. Inicialmente, el controlador ($G_c(s)$) es un controlador proporcional de ganancia unidad. El amplificador (K_a) es una ganancia de amplificación de tensión de valor 10. La constante del motor (K_t) tiene un valor de 10 Nwm/v. La carga en el motor ($G_q(s)$) presenta una respuesta ante una entrada escalón unitario tal como la representada en la *Figura 2*. La constante contraelectromotriz del motor (K_b) tiene un valor de 0.1 v/rpm. La reductora presenta un factor de reducción en la velocidad de rotación (K_r) de 1/50. La multiplicadora tiene una función de transferencia que presenta el diagrama de Bode de la *Figura 3*. El sensor se puede modelar por una ganancia (K_s) de valor de 0.005 v/rpm.

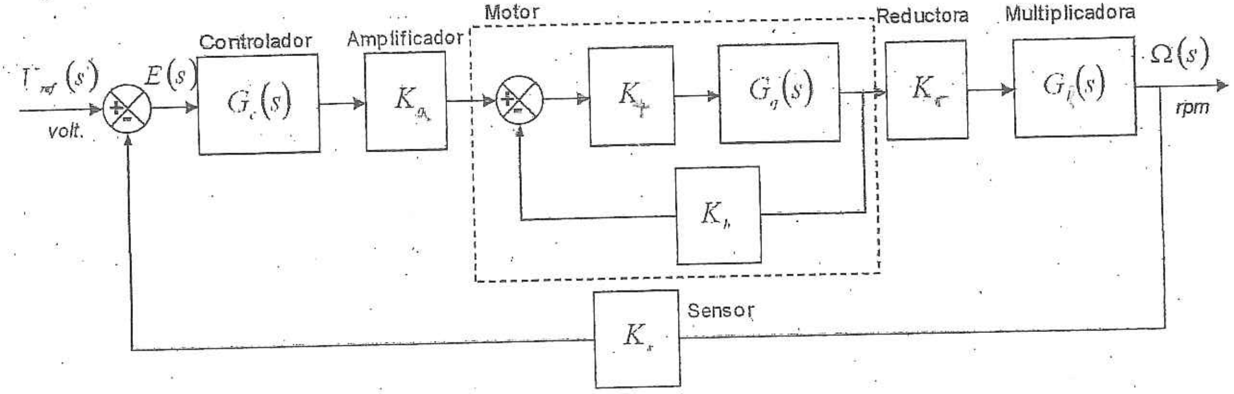


Figura 1

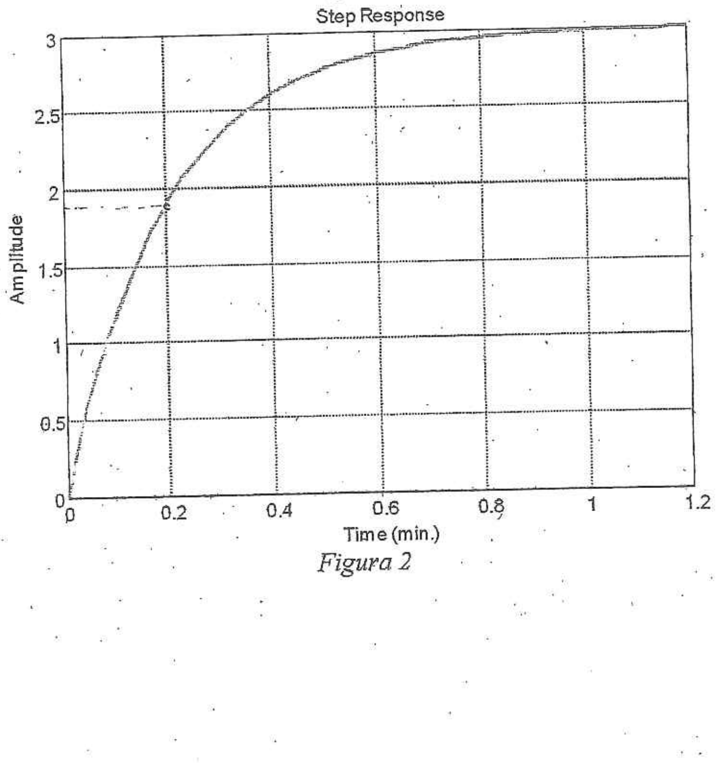


Figura 2

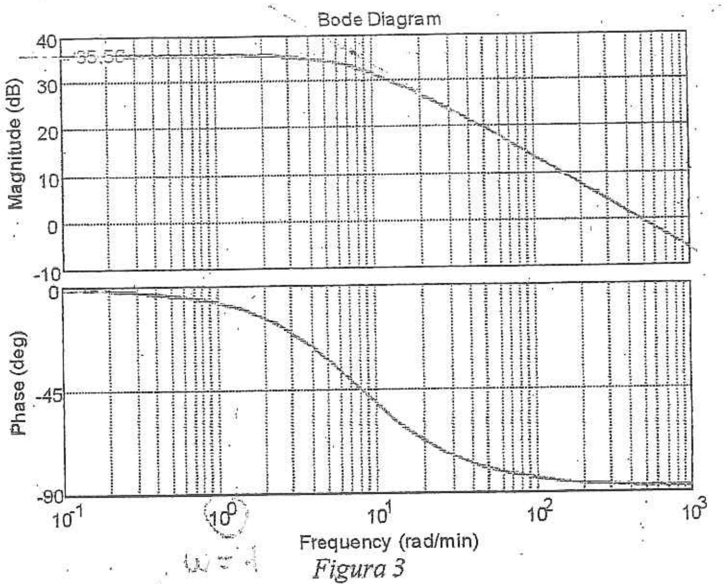


Figura 3

Se pide:

1) Obtener la función de transferencia en lazo abierto correspondiente al sistema de control.
2) Se observa una perturbación que se opone al par del eje del motor:
   a) Dibujar el diagrama de bloques del sistema de control con la perturbación y obtener la expresión de la salida en función de la entrada de referencia y la perturbación.
   b) Obtener el valor del error en estado estacionario en unidades de salida ante una entrada de referencia constante de 10 voltios y una perturbación también constante de 100 Nwm.
3) Sin tener en cuenta el efecto de la perturbación, diseñar un nuevo controlador en el lugar de las raíces de forma que se trate del controlador más sencillo y de mayor ganancia para que el control cumpla las siguientes especificaciones: error estacionario en posición nulo, sobreimpulso menor o igual que el 4,3% y tiempo de establecimiento (5%) menor o igual que 24 segundos.

SOLUCIÓN

1. Obtener la función de transferencia en lazo abierto correspondiente al sistema de control

La función de transferencia correspondiente al sistema en lazo abierto será:

$$G_{LA}(s)=G_c(s)K_aG_m(s)K_rG_l(s)K_s$$

La función de transferencia $G_m(s)$ se corresponde con la función de transferencia del bloque motor:

$$G_m(s)=\frac{K_tG_q(s)}{1+K_tG_q(s)K_b}$$

La función de transferencia correspondiente a la carga del motor puede obtenerse a partir su respuesta escalón unitario. Observando dicha respuesta se puede observar que se trata de un sistema de primer orden con una ganancia estática de 3 y constante de tiempo de 0,2 minutos. Por lo tanto:

$$G_q(s)=\frac{K_q}{T_qs+1}\quad\left.\right\}\quad \begin{matrix}K_q=3\\ T_q\big|_{63\%K_q}=0{,}2\end{matrix}\quad\left.\right\}\quad G_q(s)=\frac{3}{0.2s+1}\;\rightarrow\;G_q(s)=\frac{15}{s+5}$$

Por tanto, sustituyendo valores, la función de transferencia del motor será:

$$G_m(s)=\frac{10\cdot\dfrac{15}{s+5}}{1+10\cdot\dfrac{15}{s+5}\cdot0{,}1}\;\rightarrow\;G_m(s)=\frac{150}{s+20}$$

La función de transferencia correspondiente a la multiplicadora en vacío puede obtenerse a partir de del diagrama de bode. Observando dicho diagrama se puede observar que se trata de un sistema de primer orden con una ganancia estática de 35,56dBs y una frecuencia de cruce de 8 rad/min. Por lo tanto:

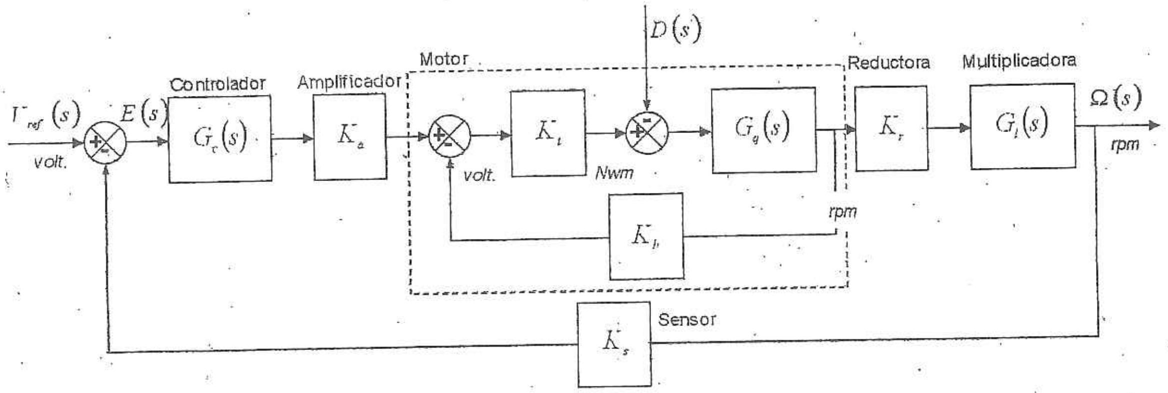
$$G_l(s)=\frac{K_l}{T_ls+1}\quad\left.\right\}\quad \begin{matrix}20\log K_l=35.56\rightarrow K_l=60\\ T_l=\dfrac{1}{\omega_c}=\dfrac{1}{8}=0{,}125\end{matrix}\quad\left.\right\}\quad G_l(s)=\frac{60}{0.125s+1}\;\rightarrow\;G_l(s)=\frac{480}{s+8}$$

Sustituyendo todos los valores sobre la función de transferencia en lazo abierto:

$$G_{LA}(s)=1\cdot10\cdot\frac{150}{s+20}\cdot\frac{1}{50}\cdot\frac{480}{s+8}\cdot0{,}005\;\rightarrow\;\boxed{G_{LA}(s)=\frac{72}{(s+8)(s+20)}}\;\rightarrow\;\boxed{G_{LA}(s)=\frac{72}{s^2+28s+160}}$$

2.a Obtener la expresión de la salida en función de la entrada y la perturbación.

El sistema de control con la perturbación puede representarse según el siguiente diagrama de bloques:



Aplicando el principio de superposición:

$$\Omega(s)=M_V(s)V(s)+M_D(s)D(s)$$

$$M_V(s)=\left.\frac{\Omega(s)}{V(s)}\right|_{D(s)=0}=\frac{G_c(s)K_aG_m(s)K_rG_l(s)}{1+G_c(s)K_aG_m(s)K_rG_l(s)K_s}\;\rightarrow\;M_V(s)=\frac{G_c(s)K_aK_tG_q(s)K_rG_l(s)}{1+K_tG_q(s)K_b+G_c(s)K_aK_tG_q(s)K_rG_l(s)K_s}$$

$$M_D(s)=\left.\frac{\Omega(s)}{D(s)}\right|_{V(s)=0} \quad \left\{ \begin{array}{l} \left[\left[-\Omega(s)K_sG_c(s)K_a-\Omega(s)\frac{1}{K_rG_l(s)}K_b\right]K_t-D(s)\right]G_q(s)K_rG_l(s)=\Omega(s) \\ \Omega(s)\left[1+G_c(s)K_aK_tG_q(s)K_rG_l(s)K_s+K_tG_q(s)K_b\right]=-G_q(s)K_rG_l(s)D(s) \end{array}\right.$$

$$M_D(s)=\frac{-G_q(s)K_rG_l(s)}{1+K_tG_q(s)K_b+G_c(s)K_aK_tG_q(s)K_rG_l(s)K_s}$$

Sustituyendo valores:

$$M_V(s)=\frac{14400}{s^2+28s+232}$$

$$M_D(s)=\frac{-144}{s^2+28s+232}$$

Así pues,

$$\boxed{\Omega(s)=\frac{14400}{s^2+28s+232}V(s)-\frac{144}{s^2+28s+232}D(s)}$$

2.b Obtener el valor del error en estado estacionario en unidades de salida ante una entrada constante de 10 voltios y una perturbación también constante de 100 Nwm.

Aplicando el principio de superposición para el error en unidades de entrada:

$$e_{sse}=e_{ssv}+e_{ssd}$$

El error correspondiente a la entrada es:

$$e_{ssv}=\frac{1}{1+K_p}\cdot 10;\quad K_p=\lim_{s\to 0}G_{LA}(s)=\lim_{s\to 0}\frac{72}{s^2+28s+160}=2 \quad\rightarrow\quad e_{ssv}=\frac{1}{1+0{,}45}\cdot 10 \quad\rightarrow\quad e_{ssv}=6{,}89\ volt.$$

El error correspondiente a la perturbación es:

$$\left.\begin{array}{l} e_{ssd}=\lim_{s\to 0}sE_D(s) \\ E_D(s)=V(s)-\Omega(s)K_s\big|_{V(s)=0}=-M_D(s)K_sD(s) \\ D(s)=\frac{100}{s} \end{array}\right\} \quad e_{ssd}=\lim_{s\to 0}s\cdot\frac{144}{s^2+28s+232}\cdot 0.005\cdot\frac{100}{s} \quad\rightarrow\quad e_{ssd}=0{,}31\ volt.$$

Por lo tanto, el error en unidades de entrada es:

$$e_{ss}=e_{ssv}+e_{ssd}=6.89+0.31=7{,}2\ volt.$$

Y el error en unidades de salida es:

$$e_{sss}=\frac{1}{K_s}e_{sse}=\frac{7{,}2}{0{,}005}\quad\rightarrow\quad \boxed{e_{sss}=1440\ rpm}$$

3. Diseñar en el lugar de las raíces el controlador de posición más sencillo y mayor ganancia que cumpla las siguientes especificaciones: error en posición nulo, sobreimpulso menor que el 4,3% y tiempo de establecimiento (5%) menor que 24 segundos.

Partiendo de la función de transferencia en lazo abierto y, dado que se trata de un sistema tipo 0, para conseguir un error de posición nulo el sistema en lazo abierto debería tener al menos un integrador (sistema tipo 1 o superior). Así pues, es necesario partir de un controlador PI:

$$G_c(s)=K_c\frac{s+1/T_i}{s}$$

El cero del PI lo colocamos por simplicidad sobre el polo más dominante $s=-8$ ($T_i=0{,}125$). En estas circunstancias, la función de transferencia en lazo abierto sería,

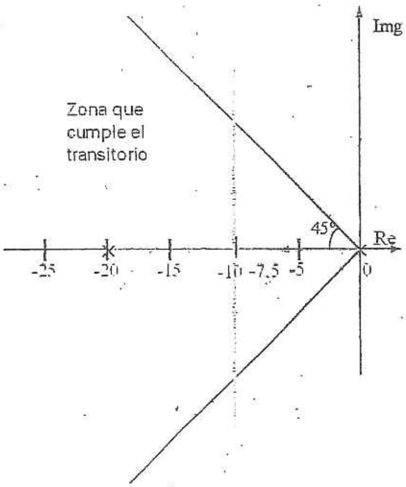
$$G_{LA}(s)=\frac{72K_c}{s(s+20)}$$

Para que se cumplan las especificaciones del transitorio: sobreimpulso menor que el 4,3% y tiempo de establecimiento (5%) menor que 24 segundos (0,4 minutos),

$$M_p = e^{-\frac{\delta\pi}{\sqrt{1-\delta^2}}} \leq 0{,}043 \rightarrow \delta \geq 0{,}707 \rightarrow \cos\theta \geq 0{,}707 \rightarrow \theta \leq 45^\circ$$

$$t_s\big|_{5\%} = \frac{3}{\delta\omega_n} \leq 0{,}4 \rightarrow \delta\omega_n \geq 7{,}5$$

Dibujando el lugar de las raíces junto con las especificaciones del transitorio,



Como se puede observar existe intersección entre el lugar de las raíces y las condiciones de transitorio, por lo que parece que la elección del tipo de controlador es correcta.

Para calcular el valor de K_c empleamos la condición del módulo y, dado que buscamos la mayor ganancia, calculamos el valor de K_c que hace que los polos del sistema en lazo cerrado se encuentren en $s = -10 \pm 10j$,

$$K = |s||s+20|\big|_{s=-10+10j} = |-10+10j||10+10j| = 200 \rightarrow K = 72K_c \rightarrow K_c = 2{,}77$$

Así pues el controlador más sencillo que cumple las especificaciones es,

$$\boxed{G_c(s) = 2.77\frac{s+8}{s}}$$

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INGENIERÍA DE SISTEMAS I

Nombre ______________________
Izena

1°Apellido______________________
1 Deitura

2° Apellido______________________
2 Deitura

Curso: 2010/2011

26 de Enero de 2011

Tiempo:

1.5 Horas

Grupo:

PROBLEMA 2 (5 PUNTOS)

La *Figura 1* representa el diagrama de bloques de un sistema de control donde *R(s)* es la entrada de referencia, $G_c(s)$ es un controlador, $G_p(s)=\dfrac{50}{s(s+5)(s+10)}$ y *D(s)* es una perturbación.

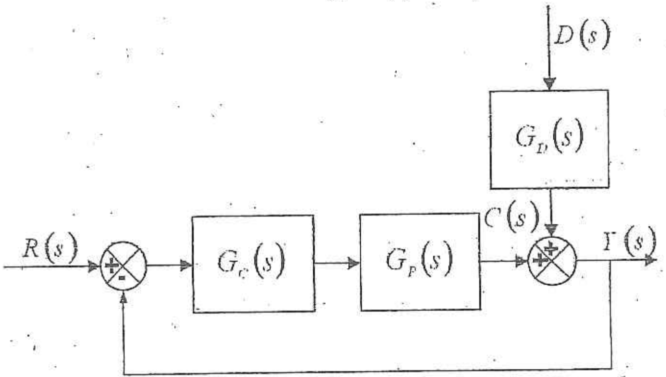


Figura 1: Diagrama de bloques del sistema de control

El diagrama de bloques del subsistema $G_D(s)$ es el mostrado en la *Figura 2*.

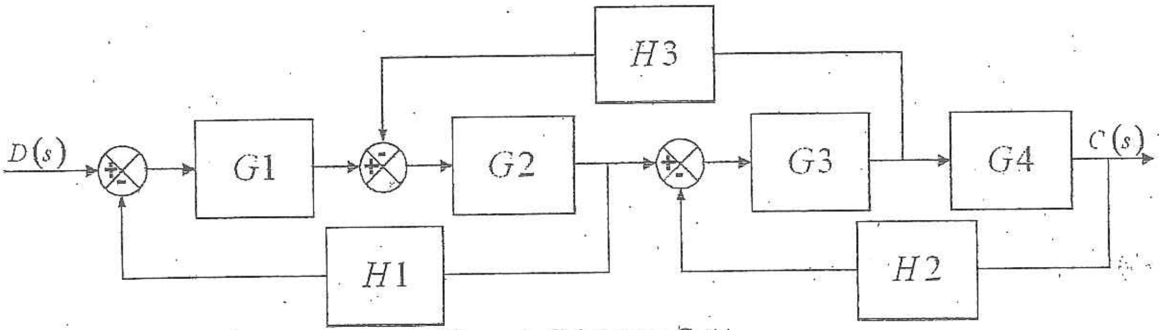


Figura 2: Subsistema $G_D(s)$

En la *Figura 3* se muestra el diagrama de módulo de Bode del sistema sin perturbación en lazo abierto para un controlador proporcional.

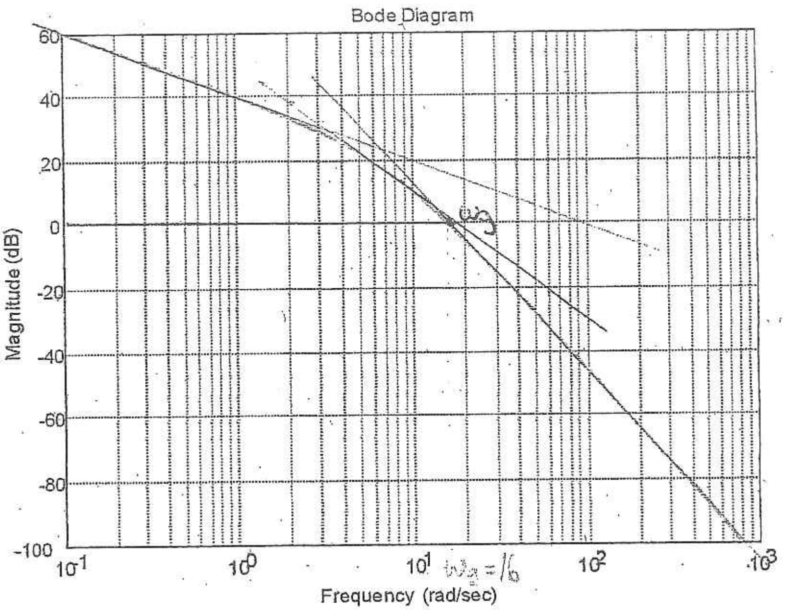


Figura 3: Diagrama de Bode de módulo en lazo abierto

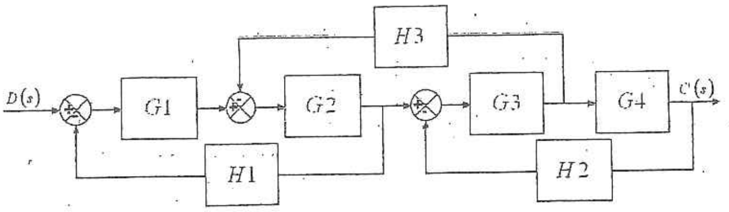
Se pide:

1) Aplicando las leyes del álgebra de bloques obtener la función de transferencia del subsistema $G_D(s)$ mostrado en la *Figura 2*. Particularizar la función de transferencia del subsistema para los siguientes valores: $G1 = 1/2$; $G2 = 1/s$; $G3 = 1$; $G4 = 1/s$; $H1 = -s$; $H2 = s$; $H3 = 10$.
2) Para una perturbación nula, determinar analíticamente el margen de fase del sistema e indicar si en esas condiciones el sistema es estable.
3) En las mismas condiciones del apartado anterior, se pretende mejorar la estabilidad sustituyendo el controlador proporcional por otro proporcional derivativo. Calcular el valor de la ganancia del nuevo controlador (K_c) para garantizar un error de velocidad del 10%.
4) Siguiendo en las mismas condiciones y utilizando el valor de K_c obtenido en el apartado anterior y una constante de tiempo derivativo $T_d = 0,2$, calcular analíticamente la salida del sistema $y(t)$ cuando se le somete a una entrada de referencia impulso unitario.
5) Empleando el mismo controlador del apartado anterior, calcular la salida en estado estacionario del sistema $y_{ss}(t)$ cuando se le somete a una entrada de referencia escalón de amplitud 5 y en presencia de una perturbación $d(t) = 0,25\,sen\,20t$.

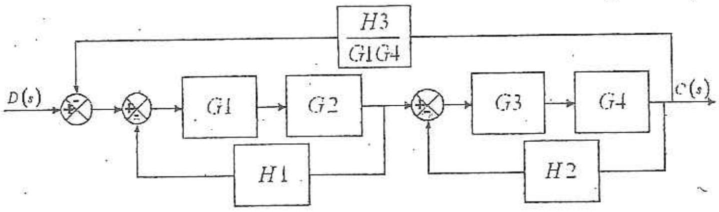
SOLUCIÓN

1. Obtener la función de transferencia del subsistema $G_D(s)$.

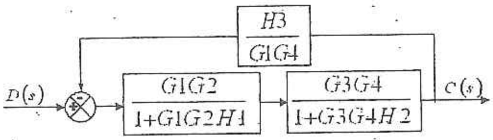
Aplicando algebra de bloques,



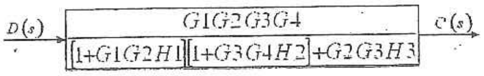
Moviendo el sumador a la izquierda del bloque G1 y el punto de bifurcación a la derecha de G4,



Realizando las dos realimentaciones de la cadena directa,



Resolviendo la realimentación,



Desarrollando,

$$\frac{C(s)}{D(s)}=G_D(s)=\frac{G1\,G2\,G3\,G4}{1+G1\,G2\,H1+G3\,G4\,H2+G2\,G3\,H3+G1\,G2\,G3\,G4\,H1\,H2}$$

Particularizando para los valores aportados G1 = 1/2; G2 = 1/s; G3 = 1; G4 = 1/s; H1 = -s; H2 = s; H3 = 10:

$$\boxed{G_D(s)=\frac{0{,}5}{s(s+10)}}$$

2. Determinar analíticamente el margen de fase del sistema

La función de transferencia del sistema en lazo abierto es:

$$G_{LA}(s)=\frac{50K_c}{s(s+5)(s+10)} \rightarrow G_{LA}(j\omega)=\frac{50K_c}{j\omega(5+j\omega)(10+j\omega)} \quad \left\{ \begin{array}{l} |G_{LA}(j\omega)|=\dfrac{50K_c}{\omega\sqrt{5^2+\omega^2}\sqrt{10^2+\omega^2}} \\ Arg[G_{LA}(j\omega)]=-90-\arctan\dfrac{\omega}{5}-\arctan\dfrac{\omega}{10} \end{array} \right.$$

De la inspección de la gráfica se observa que la frecuencia de cruce de ganancia es $\omega_g \approx 16\ rad/s$. Así pues,

$$MF=180+Arg[G_{LA}(j\omega)]_{\omega_g} \rightarrow MF=90-\arctan\frac{\omega_g}{5}-\arctan\frac{\omega_g}{10} \xrightarrow{\omega_g=16} \boxed{MF=-40.64°}$$

Dado que el margen de fase es negativo, el sistema es inestable.

3. Calcular el valor de K_c en un control proporcional derivativo para garantizar un error de velocidad del 10%.

El nuevo controlador tiene una función de transferencia $G_c(s)=K_c(1+T_d s)$, por lo tanto,

$$\left.\begin{array}{l} e_{ssv} = \dfrac{1}{k_v} = 0{,}1 \quad \rightarrow \quad k_v = 10 \\ k_v = \lim\limits_{s\to 0} sG_{LA}(s) = \lim\limits_{s\to 0} sG_c(s)G_p(s) = \lim\limits_{s\to 0} sK_c(1+T_d s)\dfrac{50}{s(s+5)(s+10)} = K_c \end{array}\right\} \boxed{K_c = 10}$$

4. Para el valor de K_c obtenido en el apartado anterior y $T_d = 0{,}2$, calcular analíticamente la salida del sistema *y(t)* cuando se le somete a una entrada de referencia impulso unitario.

El controlador tendrá una función de transferencia $G_c(s)$:

$$G_c(s) = K_c(T_d s + 1) = 10(1 + 0{,}2s) \quad \rightarrow \quad G_c(s) = 2(s+5)$$

De esta forma, la función de transferencia en lazo abierto será:

$$M_{RLA}(s) = G_c(s)G_p(s) = 2(s+5)\frac{50}{s(s+5)(s+10)} = \frac{100}{s(s+10)}$$

Como se puede observar, se trata de un sistema tipo 1. La función de transferencia en lazo cerrado debida a la entrada de referencia (en ausencia de perturbación) se puede calcular como:

$$M_R(s) = \left.\frac{Y(s)}{R(s)}\right|_{D(s)=0} = \frac{G_c(s)G_p(s)}{1+G_c(s)G_p(s)} \quad \rightarrow \quad M_R(s) = \frac{100}{s^2+10s+100}$$

Podemos calcular la salida del sistema debida a la entrada de referencia impulso unitario como:

$$\left.\begin{array}{l} Y_R(s) = M_R(s)R(s) \\ r(t) = \delta(t) \rightarrow R(s) = 1 \end{array}\right\} \; Y_R(s) = \frac{100}{s^2+10s+100}\cdot 1 = \frac{11.55\cdot 8.66}{(s+5)^2+8{,}66^2} \quad \rightarrow \quad \boxed{y_r(t) = 11.55e^{-5t}\,sen 8.66t}$$

5. Empleando el mismo controlador del apartado anterior, calcular la salida en estado estacionario del sistema $y_{ss}(t)$ cuando se le somete a una entrada de referencia escalón de amplitud 5 y una perturbación $d(t) = 0{,}25 sen 20t$.

Dado que el sistema es lineal es posible aplicar el principio de superposición:

$$Y(s) = M_R(s)R(s) + M_D(s)D(s) \quad \rightarrow \quad y(t) = L^{-1}\{Y(s)\} = L^{-1}\{M_R(s)R(s)\} + L^{-1}\{M_D(s)D(s)\} = y_r(t) + y_d(t)$$

$$y_{ss}(t) = \lim_{t\to\infty} y(t) = y_{ssr}(t) + y_{ssd}(t)$$

Dado que se trata de un sistema estable tipo 1, el sistema es capaz de seguir a una entrada escalón sin error en el estacionario. La entrada de referencia es un escalón de amplitud 5 y tenemos realimentación unitaria; por lo tanto, la componente de la salida en estado estacionario correspondiente a la entrada de referencia será:

$$y_{ssr}(t) = 5$$

Por otra parte, dado que la entrada de perturbación es una señal sinusoidal $d(t) = 0{,}25 sen 20t$, la componente de la salida del sistema en estado estacionario debida a la perturbación puede calcularse como:

$$y_{ssd}(t) = 0.25\left|M_D(j\omega)\right|_{\omega=40} sen\left(t + Arg\left[M_D(j\omega)\right]_{\omega=40}\right)$$

La función de transferencia en lazo cerrado debida a la perturbación (en ausencia de referencia) será:

$$M_D(s) = \left.\frac{Y(s)}{D(s)}\right|_{R(s)=0} = G_D(s)\frac{1}{1+G_c(s)G_p(s)} \quad \rightarrow \quad M_D(s) = \frac{0{,}5}{s(s+10)}\frac{s(s+10)}{s^2+10s+100} \quad \rightarrow \quad M_D(s) = \frac{0{,}5}{s^2+10s+100}$$

Se puede observar que el sistema sigue siendo estable. Por lo tanto, desarrollando la función de transferencia sinusoidal y calculando el módulo y argumento para $\omega = 20 rad/s$:

$$M_D(j\omega) = \frac{0{,}5}{100-\omega^2+j10\omega} \left\{\begin{array}{l} \left|M_D(j\omega)\right| = \dfrac{0{,}5}{\sqrt{(100-\omega^2)^2+(10\omega)^2}} \\ Arg\left[M_D(j\omega)\right] = -\arctan\dfrac{10\omega}{100-\omega^2} \end{array}\right|_{\omega=40} \left\{\begin{array}{l} \left|M_D(j\omega)\right| = 1{,}38\cdot 10^{-3} \\ Arg\left[M_D(j\omega)\right] = -146{,}31° = -2{,}55 rad \end{array}\right.$$

De esta forma, la componente de la salida en estado estacionario del sistema correspondiente a la perturbación será:

$$y'_{sen}(t) = 3.47 \cdot 10^{-4}\, sen(20t - 2.55)$$

Así pues, la salida en estado estacionario del sistema en las condiciones especificadas será:

$$\boxed{y'_{ss}(t) = 5 + 3.47 \cdot 10^{-4}\, sen(20t - 2.55)}$$

2010/2011 URTARRILA

1. PROBLEMA

$K_c = 1$ (hasieran)

$K_a = 10$

$K_t = 10$

$G_q(s) \rightarrow$ 2. irudia

$k_b = 0{,}1$

$K_r = \frac{1}{50}$

$G_\ell(s) \longrightarrow$ 3 irudia

$K_s = 0{,}005$

1) $G_{BA}(s)$?

→ $G_q(s)$ 2. iruditik lortuko dugu.

$$\left.\begin{matrix}\Delta u = 1 \\ \Delta y = 3\end{matrix}\right\| k = 3 \qquad G_q(s) = \frac{3}{0{,}2s+1} = \frac{15}{(s+5)}$$

$$y_{63} = 0{,}63 \cdot 3 = 1{,}89 \longrightarrow \tau = 0{,}2s$$

→ $G_\ell(s)$ 3. iruditik lortuko dugu: Bode diagrama

ω txikietan ez dauka maldarik → ez dago integradorerik

- $\omega_1 = 8$ rad/s $\longrightarrow T_1 = \frac{1}{8} = 0{,}125s$
- $35{,}56 = 20\log\left(\frac{k}{0{,}1}\right) \Rightarrow k = 5{,}99 \simeq 60$

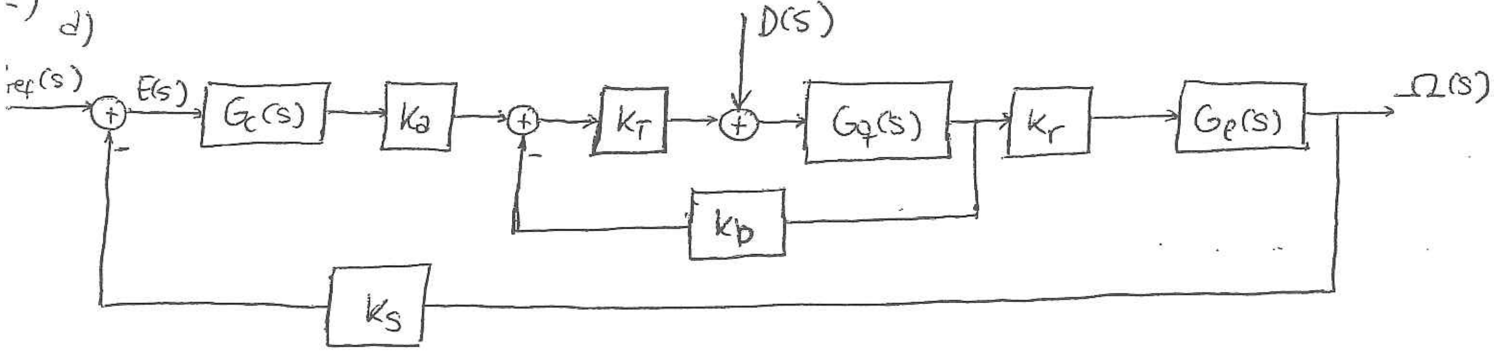
$$\Rightarrow G_\ell(s) = \frac{60}{0{,}125s+1} = \frac{480}{(s+8)}$$

$$G_{BA}(s) = G(s) \cdot H(s)$$

$$G_M(s) = \frac{k_T G_q(s)}{1 + k_b k_T G_q(s)} = \frac{150}{(s+5)+15} = \frac{150}{s+20}$$

$$G_{BA}(s) = G_c(s) \cdot k_a \cdot G_M(s) \cdot K_r \cdot G_\ell(s) \checkmark \overset{\cdot k_s}{=} \frac{72}{(s+8)(s+20)} = \frac{72}{s^2+28s+160}$$

2) d)



$$\frac{\Omega(s)}{V_{ref}(s)} = G_{BC}(s)\Big|_{D(s)=0} = \frac{\frac{72/k_s}{s^2+28s+160}}{1+\frac{72}{s^2+28s+160}} = \frac{14400}{s^2+28s+232}$$

$$\frac{\Omega(s)}{D(s)} = G_{BC}(s)\Big|_{V(s)=0} = \frac{-G_q k_r G_\ell(s)}{1 + K_T \cdot K_b \cdot G_q(s) + G_c(s) K_a k_T G_q(s) K_r G_\ell K_s}$$

$$G_{BC}\Big|_{V(s)=0} = \frac{-144}{s^2+28s+232}$$

$$\Omega(s) = \frac{14400\,V(s)}{s^2+28s+232} - \frac{144}{s^2+28s+232}$$

b)

$V(s)=10$

$D(s)=100/s$

$$\rightarrow\ e_{ss}\Big|_{D(s)=0} = e_{ss_V} = \frac{10}{1+K_p}$$

$$K_p = \lim_{s\to 0} G_{BA}(s) = \lim_{s\to 0} \frac{72}{s^2+28s+160} = \frac{72}{160} = 0{,}45 \Rightarrow e_{ss_V} = 0{,}689\cdot 10$$

$$e_{ss_V} = 6{,}89$$

$$\rightarrow e_{ss}\Big|_{V(s)=0} = \lim_{s\to 0} s\cdot E_D(s) = e_{ss_D}$$

$$E_D(s) = G_{BC}\Big|_{V(s)=0}\cdot K_s\cdot D(s) = \frac{-0{,}72\cdot 100}{s(s^2+28s+232)} \Rightarrow$$

$$\Rightarrow \lim_{s\to 0} \cancel{s}\cdot\frac{100}{\cancel{s}}\cdot\frac{-0{,}72}{(s^2+28s+232)} = 0{,}31$$

$e_{ss} = 6{,}89+0{,}31 = 7{,}2$ volt $\Rightarrow$ irteera unitateetan: $e'_{ss} = \frac{1}{K_s}\cdot e_{ss} = 1440$ rpm

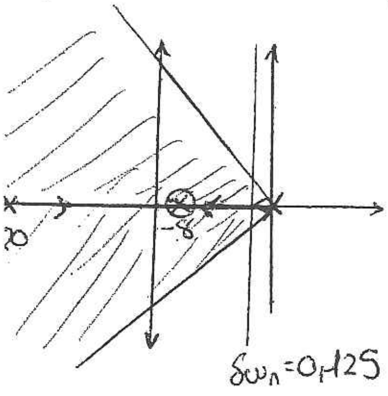
3) $e_{ss}=0$

$M_p \leq \%4{,}3$

$t_s(\%5) \leq 24s$

$$M_p = 0{,}043 = e^{-\frac{\delta\pi}{\sqrt{1-\delta^2}}} \rightarrow \delta \geq 0{,}707 \rightarrow \theta = 45°$$

$$t_s(\%5) = \frac{3}{\delta\omega_n} \leq 24 \rightarrow \delta\omega_n \geq 0{,}125$$



$$G_{BA}(s) = \frac{72}{(s+8)(s+20)}$$

↳ ez denez 1. motakoa, eta $e_{ss}=0$ izan behar denez, integradore bat sartu behar dugu ⇒ PI → PI batekin intersekzioa dago eta iraunkorreko eskakizunak betetzen dira.

$$\frac{1}{T_i} = 8$$

$$G_{PI}(s) = G_c(s) = \frac{k_c(s+\boxed{z_i})}{s} \longrightarrow T_i = 0{,}125\,s$$

$$G_{BA}(s) = \frac{k_c\cdot 72\cdot\cancel{(s+8)}}{s\cancel{(s+8)}(s+20)} = \frac{72K_c}{s(s+20)} \longrightarrow G_{BC}(s) = \frac{72k_c}{s^2+20s+72k_c} = \frac{k\omega_n^2}{s^2+2\delta\omega_n s+\omega}$$

$72k_c = k\omega_n^2 \rightarrow k=1$

$2\delta\omega_n = 20 \rightarrow \delta = 0{,}707 \rightarrow \omega_n = \frac{20}{2\cdot 0{,}707} = 14{,}144 \rightarrow k_c = 2{,}778$

$72k_c = \omega_n^2 \rightarrow k_c = \omega_n^2/72$

$$\boxed{G_c(s) = \frac{2{,}77(s}{s}}$$

PROBLEMA

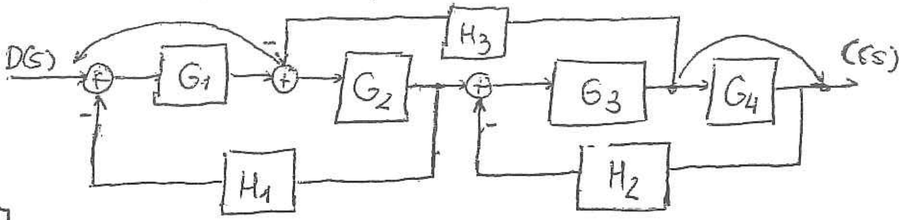
$$G_p(s) = \frac{50}{s(s+5)(s+10)}$$

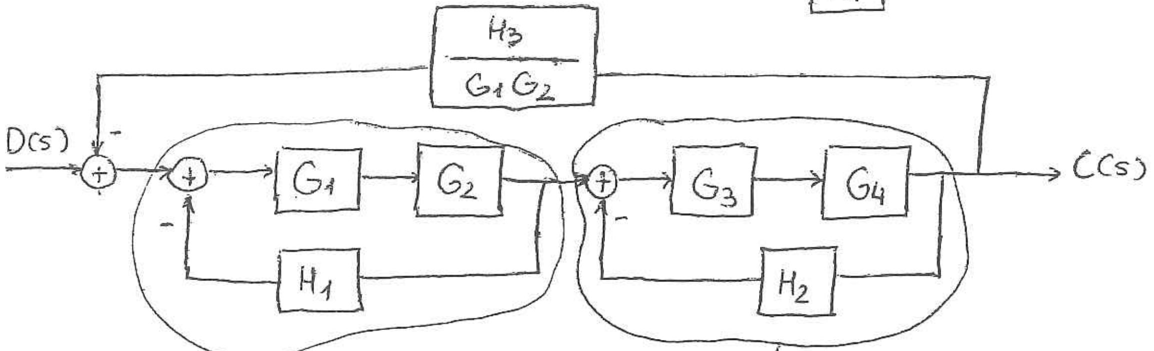
$G_D \longrightarrow$ 2. iruditik

$G_{BA}(s)\Big|_{D(s)=0} \longrightarrow$ 3. irudtik.

1) $G_1 = \frac{1}{2}$; $G_2 = \frac{1}{5}$; $G_3 = 1$; $G_4 = \frac{1}{5}$

$H_1 = -5$; $H_2 = 5$; $H_3 = 10$





$$\frac{G_1G_2}{1+H_1G_1G_2} \qquad \frac{G_3G_4}{1+H_2G_3G_4}$$

$$\frac{C(s)}{D(s)} = \frac{\dfrac{G_1G_2G_3G_4}{(1+H_1G_1G_2)(1+H_2G_3G_4)}}{1+\dfrac{H_3}{\cancel{G_1}\cancel{G_4}}\cdot\dfrac{\cancel{G_1}G_2G_3\cancel{G_4}}{(1+H_1G_1G_2)(1+H_2G_3G_4)}} = \frac{G_1G_2G_3G_4}{(1+H_1G_1G_2)(1+H_2G_3G_4)+H_3G_2G_3}$$

$$= \frac{\frac{1}{2}\cdot\frac{1}{s}\cdot 1\cdot\frac{1}{s}}{(1-\cancel{5}\cdot\frac{1}{2}\cdot\frac{1}{\cancel{5}})(1+5\cdot 1\cdot\frac{1}{\cancel{s}})+10\cdot\frac{1}{s}\cdot 1} = \frac{1/2s^2}{\frac{1}{\cancel{2}}\cdot 2+10/s} = \frac{0{,}5}{s^2+10s}$$

$$\frac{C(s)}{D(s)} = G_D(s) \longrightarrow \boxed{G_D(s) = \frac{0{,}5}{s(s+10)}}$$

2)

Bode diagrematik: $\omega_g = 16$ rad/s

$G_{BA}(s)$ lortu behar dugu : $G_{BA}(s) = \frac{50K_c}{s(s+5)(s+10)}$ → esaten digute $G_c(s)$ proporb dela.

$MF = 180 + Arg[G_{BA}(j\omega_g)] = 180 - 220{,}64 = -40{,}64 = -0{,}709$ rad $\Rightarrow \boxed{MF = -40{,}64°}$

$$|G_{BA}(j\omega)| = \frac{50K_c}{j\omega\sqrt{5^2+\omega^2}\sqrt{10^2+\omega^2}}$$

MF<0 ⇒ ezegonkorra

$$Arg[G_{BA}(j\omega)] = -90 - \arctan(\omega\tfrac{1}{5}) - \arctan(\omega\tfrac{1}{10}) \xRightarrow{\omega_g=16} Arg[G_{BA}] = -220{,}64°$$

3) $e_{ss_v} = \%10$ (PD dotekin)

$$e_{ss_v} = \frac{1}{K_v} \rightarrow \frac{1}{K_v} = 0{,}1 \; ; \; K_v = 10$$

$$K_v = \lim_{s\to 0} s \cdot G_{BA}(s) = \lim_{s\to 0} \cancel{s} \frac{50 K_c(1+T_d s)}{\cancel{s}(s+5)(s+10)} = \frac{\cancel{50} K_c}{\cancel{50}} = K_v \Rightarrow \boxed{K_c = 10}$$

4) $T_d = 0{,}2$

Sarrea inpultsu unitarioa

$$G_{BA}(s) = \frac{500\,(1+0{,}2s)}{s(s+5)(s+10)} = \frac{100\,\cancel{(s+5)}}{s\cancel{(s+5)}(s+10)}$$

$$G_{BC}(s) = \frac{\dfrac{100}{s(s+10)}}{1+\dfrac{100}{s(s+10)}} = \frac{100}{s^2+10s+100} = \frac{Y(s)}{\underbrace{R(s)}_{1}} \Rightarrow Y(s) = \frac{100}{s^2+10s+100}$$

$$\mathcal{L}^{-1}\downarrow \quad Y(s) = \frac{100}{(s^2+10s+25)-25+100} = \frac{100}{(s+5)^2+75} = \frac{\frac{100}{8{,}66}\cdot 8{,}66}{(s+5)^2+8{,}66^2} = \frac{11{,}55\cdot 8{,}6}{(s+5)^2+8{,}6}$$

$\hookrightarrow \sqrt{75} = 8{,}66$

$$\boxed{y(t) = 11{,}55\, e^{-5t} \cdot \operatorname{sen}(8{,}66t)}$$

5)

$$R(s) = \frac{5}{s} \qquad \longrightarrow d(t) = A \operatorname{sen}(\omega t) \longrightarrow \begin{cases} A = 0{,}25 \\ \omega = 20 \text{ rad/s} \end{cases}$$

$$d(t) = 0{,}25 \operatorname{sen}(20t) \qquad y_{ss_d}(t) = 0{,}25\, |G_d(j\omega)|_{\omega=20} \cdot \operatorname{sen}(20t+\varphi)$$

$$\cancel{G_d(s) = \frac{0{,}5}{s(s+10)}} \cancel{\rightarrow G_d(j\omega) = \frac{0{,}5}{j\omega(j\omega+}}$$

$$\frac{Y(s)}{D(s)} = G_D(s) = \frac{G_d(s)}{1+G_c(s)G_p(s)} \rightarrow G_D(s) = \frac{0{,}5}{s^2+10s+100}$$

$$G_D(j\omega) = \frac{0{,}5}{100-\omega^2+j10\omega} \qquad \begin{cases} |G_D(j\omega)| = \dfrac{0{,}5}{\sqrt{(100-\omega^2)^2+(10\omega)^2}} \\ \operatorname{Arg}[G_D(j\omega)] \underset{\uparrow}{=} \dfrac{10\omega}{100-\omega^2} \end{cases}$$

-arctan( )

$$\omega = 20 \text{ rad/s} \Rightarrow \begin{cases} |G_D(j\omega)| = 1{,}38\cdot 10^{-3} \\ \operatorname{Arg}[G_D(j\omega)] = -146{,}31^\circ = -2{,}55 \text{ rad} = \varphi \end{cases}$$

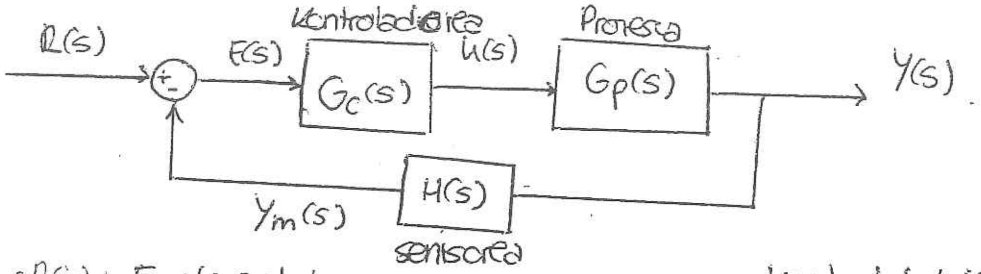
$$y_{ss_d}(t) = 0{,}25 \cdot 1{,}38\cdot 10^{-3} \operatorname{sen}(20t-2{,}55) = 3{,}47\cdot 10^{-4} \operatorname{sen}(20t-2{,}55)$$

$\longrightarrow \; y_{ss}(t) = \lim_{s \to 0} y(t) = y_{ss_d} + y_{ss_r}$ (y_{ss_d} lortuta daukagu)

$G_{BA}(s) \longrightarrow$ 1 motakoa $\Rightarrow$ Honek esan nahi du espaloi sarrera baten aurrean sistemak errorerik jarraitzeko gai dela, beraz, irteerako elementua egoera iraunkorrean ; $y_{ss_r}(t) = \underset{\uparrow}{5}$ anplitudea.

$$\boxed{y_{ss}(t) = 5 + 3{,}74 \cdot 10^{-4} \operatorname{sen}(20t - 2{,}55)}$$

5. SISTEMA BERRELIKATUAK



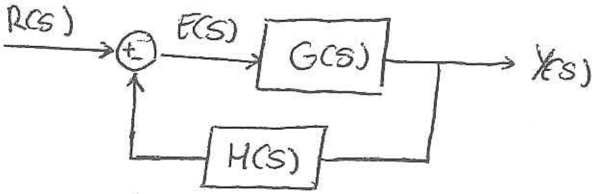
- R(s): Erreferentzi seinalea
- E(s): Errorea
- u(s): Kontrol seinalea
- y(s): Sistemaren irteera
- $Y_m(s)$: Kontrol unibleak

kontrolatuko aldagaia neurtu eta erreferentziarekin alderatuko da. Bien arteko errorea erabiltzen da kontrol-seinalea sortzeko.

$E(s) = R(s) - Y(s) \cdot H(s)$

↳ $H(s)=1 \longrightarrow E(s) = R(s) - Y(s)$

→ Sinplifikazioa:



Non $G(s) = G_c(s) \cdot G_p(s)$

- Begizta irekian:

$$G_{BA}(s) = G_c(s) \cdot G_p(s) \cdot H(s) = G(s)H(s)$$

- Begizta itxian:

$$G_{BC}(s) = \frac{Y(s)}{R(s)} = \frac{G_c(s) \cdot G_p(s)}{1 + G_c(s) G_p(s) H(s)}$$

Ek. karakteristikoa

↳ $1 + G_c(s) G_p(s) H(s) = 0$

(begizta itxiko poloak)

→ Berrelikaduaren abantailak: perturbazioen eragina murrizten du

Berrelikaduaren desabantailak: kontroladoreak eragiteko, errorea egon behar da (atzerapena).

• SISTEMA BERRELIKATUEN EGONKORTASUNA

Bi era daude aztertzeko:
- ABSOLUTUA: egonkorra den edo ez aztertzen du
- ERLATIBOA: egonkortasun gradaren berri ematen du.

$\boxed{\text{E. ABSOLUTUA}}$ $\boxed{\text{E. ERLATIBOA}}$ $\boxed{\text{DENBORA - ERANTZUNA}}$

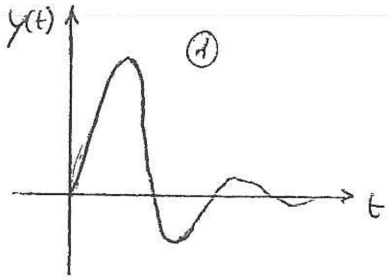
$$\ominus \xrightarrow[\text{Eskeinitako informazioa}]{\text{Kalkuluaren konplexutasuna}} \oplus$$

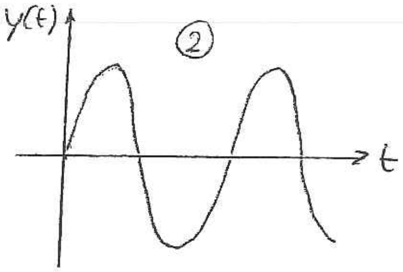
→ Egonkortasunaren definizioak:

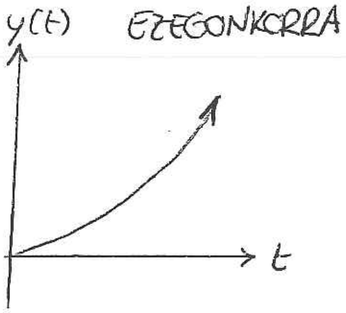
1) BIBO-EGONKORTASUNA: Sistema lineal egonkor baten irteera mugatua dago bere sarrera mugatuta egotekotan. (Aurreko orekara itzultzen da)

2) EGONKORTASUN ASISTOTIKOA: Sistema lineal bat egonkorra da, honen inpultso-erantzuna g(t) integra ahal badaiteke tarte infinituan. $\int_0^\infty g(t)dt = Kte$ (oreka-egoera berri bat lortzen du)

3) Sistema lineal bat egonkorra da bere begizta itxiko polo guztiek zati erreal negatiboa badute.







→ ROUTH HURWITZ-en irizpidea

① Hurwitz-polinomioa: bere erro guztiek zati erreal negatiboa badute.

$$\boxed{a_0 S^n + a_1 S^{n-1} + a_2 S^{n-2} + \ldots + a_n S^0 = 0}$$

↳ BEHARREZKO BALDINTZA: koefizienteak guztiak izan behar ditu, eta zeinu berekoak izan behar dute.

② BALDINTZA NAHIKOA: Polinomio baten erro guztien zati erreala negatiboa izateko, ROUTH-en taulako lehenengo zutabeko koefiziente guztiak positiboak izan behar dute.

③ ROUTH-en taula

$$a_0 S^n + a_1 S^{n-1} + a_2 S^{n-2} + a_3 S^{n-3} + \ldots + a_n S^0 = 0$$

$$\begin{array}{c|cccc}
S^n & a_0 & a_2 & a_4 & \cdots \\
S^{n-1} & a_1 & a_3 & a_5 & \cdots \\
\hline
S^{n-2} & b_1 & b_2 & b_3 & \cdots \\
S^{n-3} & C_1 & C_2 & \cdots & \cdots \\
\vdots & \vdots & \vdots & & \\
S^{n-n} & K_1 & & &
\end{array}$$

} Polinomioarekin

Denak ⊕

$$b_1 = \frac{a_1 a_2 - a_0 a_3}{a_1} \; ; \; b_2 = \frac{a_1 a_4 - a_0 a_5}{a_1} \; ; \; b_3 = \frac{a_1 \cdot a_6 - a_0 a_7}{a_1}$$

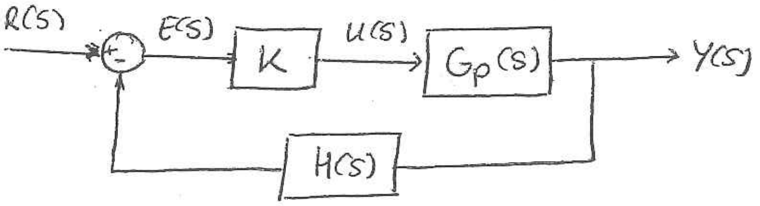
$$C_1 = \frac{b_1 a_3 - a_1 b_2}{b_1} \; ; \; C_2 = \frac{b_1 a_5 - a_1 b_3}{b_1}$$

④ Routh Hurwitz-en irizpidea

- Lehenengo zutabeko elementu guztiak positiboak badira, polinomioa Hurwitz da, eta bere erro guztiak zati erreal negatiboa dute.
- Positiboak ez dalekotan, zeinu-aldaketa kopuruak adierazten du zenbat errok duten zati erreal positiboa.
- Aplikazioa: egonkortasuna kalkulatzeko → transferentzi funtzioak adierazten duen sistema egonkorra da bere polinomio izendatzailea Hurwitz bada.

→ K aldakorra:

$$G_{BC}(s) = \frac{K\,G_p(s)}{1+K\,G_p(s)H(s)}$$



K-ren zein baliotarako da sistema egonkorra?

K-ren mugak { - Egonkortasuna
- Eragingailuaren asetasuna.

⇒ Bietatik zeinetara iristen den lehenengo, horixe izango da K-ren muga.

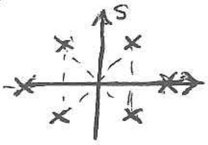
→ Routh Hurwitz-en egonkortasunerako irizpidea: KASU BEREZIAK

1) Lehenengo zutabeko elementuren bat zero da.

Soluzioa: · zeroa ε koefiziente batez ordezkatzen dugu, non $\varepsilon \to 0$

· Polinomioa $(s+a)$ koefizienteaz biderkatu non $a>0$, eta taula berriro kalkulatu.

2) Lerro bateko elementu guztiak zero dira. Erroak jatorritik simetrikoki kokatuta daudenean gertatzen da.



Soluzioa: Polinomio laguntzailea erabiltzen da zeroz osatutako lerroa ordezkatzeko. Polinomio hau aurreko lerroko koefizienteek osatzen duten polinomioa deribatuz lortzen da.

$\delta=0$

→ Sistema kritikoki egonkorra : oszilazio-maiztasunaren kalkulua (ω_u)

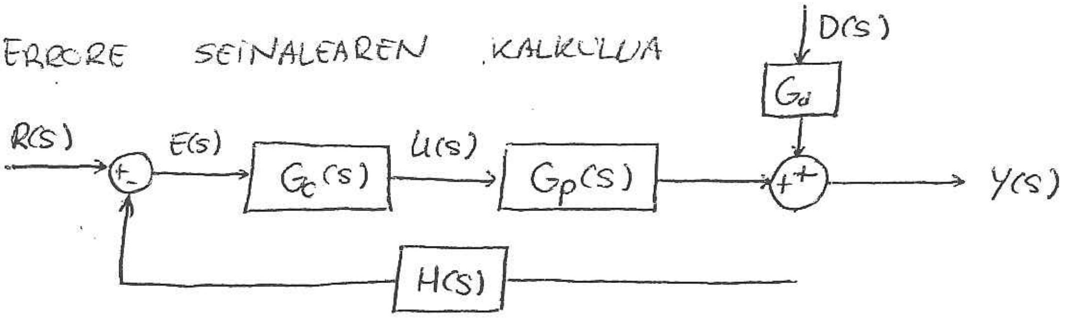
1) Ekuazio laguntzailea kalkulatuz. (Polinomio laguntzailearen erroekin) $s=\pm j\omega_u$

2) $T_u = \frac{2\pi}{\omega_u}$ → oszilazio-periodoa

• EGOERA IRAUNKORRA

↳ Poloen eragina desagertu denean lortzen dugun erantzuna; egoera iragankora desagertu denean (sistema egonkora denean).

* ERRORE SEINALEAREN KALKULUA



→ Bide Luzea:

$$E(s) = R(s) - Y(s)H(s) = R(s) - H(s)\left[G(s)E(s) + G_d \cdot D(s)\right] =$$

$$= R(s) - G(s)H(s)E(s) - H(s)G_d(s)D(s)$$

$$E(s)\left[1 + G(s)H(s)\right] = R(s) - H(s)G_d(s)D(s)$$

Egoera iraunkorreko errorea:

$$\left[E(s) = \frac{R(s)}{1+G(s)H(s)} - \frac{H(s)\,G_d(s)}{1+G(s)H(s)}D(s)\right] \Longrightarrow \boxed{e_{ss} = \lim_{s\to 0} s\cdot E(s)} = \lim_{t\to\infty}$$

⚠ Bakarrik sistema EGONKORRETAN

→ Bide laburra: ERRORE KOEFIZIENTE ESTATIKOAK

- R(s) erreferentzia - sarreren aurrean errorea kalkulatzeko erabiltzen dira. Zenbat eta handiagoa izan → egoera iraunkorreko errorea txikiagoa

⊛ $\boxed{K_p}$ Posizioaren errore-koefizientea. ESPALOI SARRERA

$$\boxed{K_p = \lim_{s\to 0} G(s)H(s)} \quad ; \quad \boxed{e_{ss} = \frac{1}{1+K_p}}$$

⊛ $\boxed{K_v}$ Abiaduraren errore-koefizientea. ARRAPALA SARRERA

$$\boxed{K_v = \lim_{s\to 0} s\cdot G(s)H(s)} \quad ; \quad \boxed{e_{ss} = \frac{1}{K_v}}$$

⊛ $\boxed{K_a}$ Azelerazioaren errore-koefizientea. PARABOLA SARRERA

$$\boxed{K_a = \lim_{s\to 0} s^2 G(s)H(s)} \quad ; \quad \boxed{e_{ss} = \frac{1}{K_a}}$$

→ Sistema mota:

Begizta irekiko transferentzi funtzioan dauden integratzaile-kopuruak egoera egonkorreko errorea egongo den edo ez adierazteko ahalbidetzen digu. Hortaz, sistemak sailkatzen dira **begizta irekian** dituen integratzaile-kopuruaren arabera.

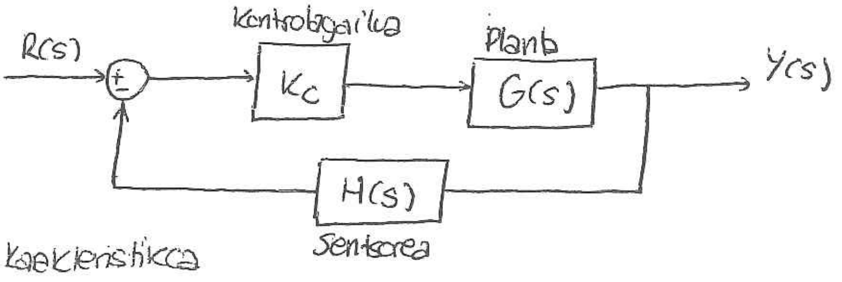
	ESPALOIA	ARRAPALA	PARABOLA
0 MOTA	$\frac{1}{1+K_p}$	∞ $K_v=0$	∞ $K_a=0$
1 MOTA	0	$\frac{1}{K_v}$	∞ $K_a=0$
2 MOTA	0	0	$\frac{1}{K_a}$
3 MOTA	0	0	0

Laburpena: Egoera iraunkorreko errorea kentzeko, kontrolagailuaren bitartez integratzaileak gehitu daitezke. Hala ere, honek egonkortasun arazoak sortu ditzake begizta itxian!

• ERROEN TOKI GEOMETRIKOA

→ DEFINIZIOA:

K_c irabazpenaren bidez berrelikatutako sistema batean:

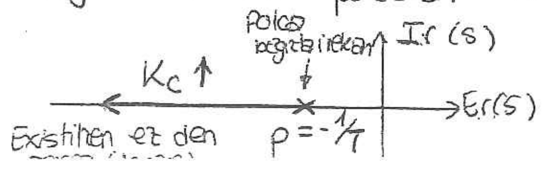


$$G_{BC}=\frac{Y(s)}{R(s)}=\frac{K_c G(s)}{1+K_c G(s)H(s)}$$

EK. karakteristikoa
Begizta itxian: $1+K_c G(s)H(s) = 1+G_{BA}(s)$

↳ Erroen toki geometrikoa esaten zaio, begizta itxiko sistemaren poloek K_c irabazpena 0-tik ∞-ra aldatzean osatzen duten toki geometrikoari.

Begizta itxiko poloaren kokapena: $\boxed{P_{BC}=-(1+K_c K)/\tau}$



→ Erroen Toki Geometrikoko poloen baldintzak ($K_c G(s) H(s) = -1$)

- Moduluaren irizpidea

$$\| K_c G(s) H(s) \| = 1 \longrightarrow K_c \frac{\prod_{i=1}^{m} \|s+z_i\|}{\prod_{i=1}^{n} \|s+p_i\|} = 1$$

- Argumentuaren irizpidea

$$Arg(K G(s) H(s)) = Arg(K) + \sum_{j=1}^{m} Arg(s+p_j) = (2q+1)\pi$$

$q = 0, \pm1, \pm2 \ldots$

↳ TF: Begizta irekia: $G_{BA} = K_c \cdot G(s)$

Begizta itxia: $G_{BC} = \dfrac{K_c G(s)}{1 + K_c G(s)}$

→ Erroen toki geometrikoaren eraikuntza.

① Adar-kopurua: Erroen tokiaren adar kopurua n da, $G_{BA}(s)$-ren polo-kopurua.

② Hasiera eta amaiera-puntuak: Adar bakoitza begizta irekiko sistemaren polo baten hasi ($K_c = 0$) eta begizta irekiko sistemaren zero batean amaitzen da ($K_c = \infty$).

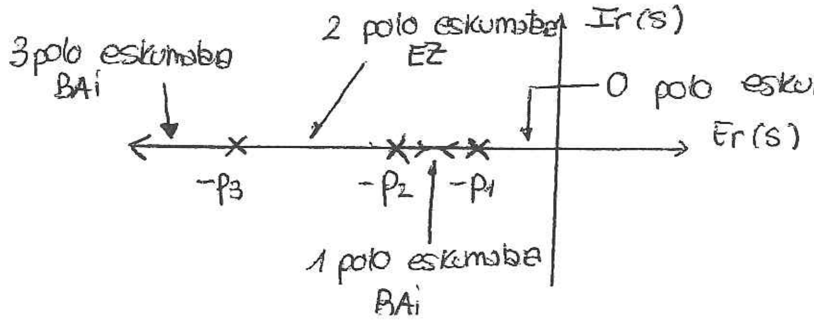
$n > m$ bada → $(n-m)$ adar infinituan dauden zeroetan amaituko dira.

Adib: $G(s) = \dfrac{s+1}{(s+2)(s+4)}$ $\left\{ \begin{array}{l} m=1 \\ n=2 \end{array} \right.$

- Adar-kopurua = $G_{BA}(s)$-ren polo kopurua=2
- $n > m \rightarrow n-m=1$ adar bat infinituan amaituko da eta bestea $s=-1$ zeroan.

③ Erroen toki geometrikoak diren ardatz errealeko puntuak: Ardatz errealeko puntu bat ETG-ko puntu bat da, bere eskumaldea dauden polo eta zeroen batuketa bakoitia denean:

Adib: $(-\infty, -4)$ eta $(-2, -1)$ tarteak erroen toki geometrikoan daude.

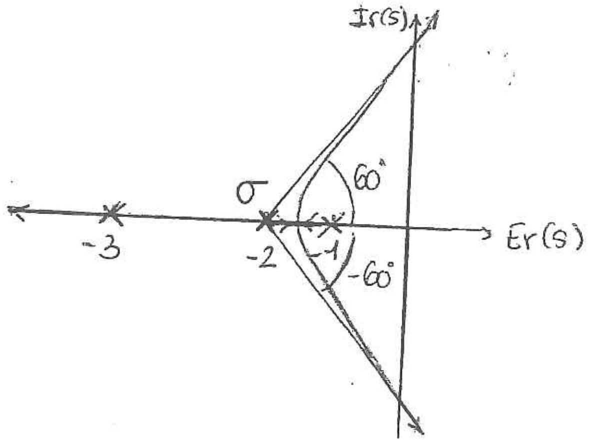


④ Erroen toki geometrikoaren simetria: ETG ardatz errealarekiko simetrikoa

⑤ Erroen toki geometrikoaren asintotak: Infinituko zeroetan amaitzen diren (n−m) adarrak, ardatz errealarekin ondorengo angelua osatzen duten lerroekiko asintotikoak dira. $\theta = \pm \frac{(2k+1)\pi}{n-m}$ $k=0,\pm1,\pm2..$

⑥ Asintoten eta ardatz errealaren arteko ebaki-puntua

$$\sigma = \frac{\sum_{i=1}^{m} z_i - \sum_{j=1}^{n} p_j}{n-m} = \frac{\overset{\text{zeroak}}{0}-3-2-1}{3-0} = -2$$



$$\theta_{1,2} = \pm\frac{(2\cdot0+1)\pi}{3} = \frac{\pi}{3}\text{rad} = \pm60°$$

$$\theta_3 = \pm\frac{(2\cdot1+1)\pi}{3} = \pi\,\text{rad} = \pm180°$$

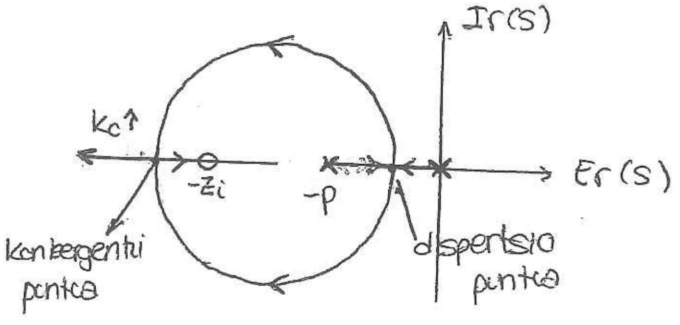
⑦ Asintoten angeluak poloetatik irten eta zeroetara iristean. Irteera eta iriste-puntuetan, adarren tangenteek ardatz errealarekin osatzen dituzten angeluak dira:

$$\phi_{P_k} = \sum_{i=1}^{m} \phi_{z_i} - \sum_{j=1, j\neq k}^{n} \phi_{p_j} - (2r+1)\pi \qquad r=1,2,3...$$

$$\phi_{z_k} = \sum_{i=1, i\neq k}^{m} \phi_{z_i} - \sum_{j=1}^{n} \phi_{p_j} - (2r+1)\pi \qquad r=1,2,..$$

ϕ_{z_i} eta ϕ_{p_j} ≡ dagokion polo eta zeroan tangenteak ardatz errealarekin duen angelua.

⑧ Adarren dispertsio eta konfluentzia-puntuak: Dispertsio puntua da ETG bi adar konplexutan banatzen den ardatz errealeko puntua. Konfluentzia-puntua aldiz, bi adar konplexu ardatz errealean biltzen diren puntua da. Hurrengo funtzioaren minimoak dira: $K_c = -\frac{1}{G(s)H(s)}$



⑨ Ardatz irudikariarekiko ebaki-puntuak: Sistema kritikoki egonkor egiten duten K_c-ren balioei dagozkien poloak dira. Routh-Hurwitz-en bitartez kalkula daitezke.

→ ETG hurbilduaren eraikuntza:

1) $G_{BA}(s)$-ren n poloak eta m zeroak s planoan kokatu.

2) Adar-kopurua = n

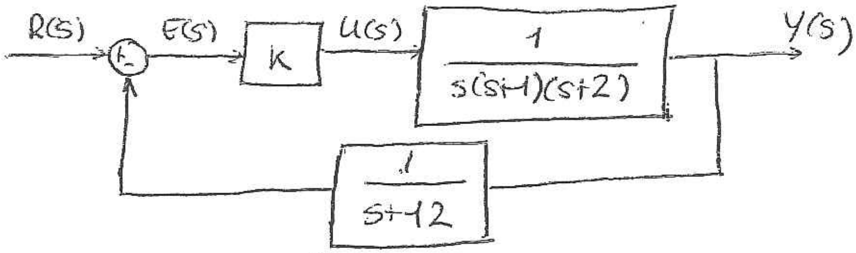
3) Ardatz errealeko zatiak: Eskumaldean dauden polo eta zeroen batuketa bakoitia

4) $G_{BA}(s)$-ren n poloen adarren hasierak. Adarrak $G_{BA}(s)$-ren m zeroetara iritsiko dira. $n>m$ bada; $n-m$ adar asintotikoki doaz infinitura.

5) Asintotak definitu: $\sigma = \frac{\sum_{i=1}^{m} z_i - \sum_{j=1}^{n} p_j}{n-m}$; $\theta = \pm \frac{(2k+1)\pi}{n-m}$

6) K_u eta ω_u kalkulatu; ardatz irudikariarekin ebaki-puntuak $\pm j\omega_u$ dira.

4. Ariketa (37. diap)



$$G_{BC}(s) = \frac{\frac{K}{s(s+1)(s+2)}}{1+\frac{1}{(s+12)}\cdot\frac{K}{s(s+1)(s+2)}} = \frac{K(s+12)}{s(s+12)(s+1)(s+2)+K} = \frac{K(s+12)}{s^4+15s^3+38s^2+24s+}$$

→ $K>0$ bada

B.B = koefiziente guztiak ditu eta zeinu berekoak ⇒ BETETZEN DA

B.B = R-H

s^4	1	38	K
s^3	15	24	0
s^2	36,4	K	0
s	c_1	0	
s^0	0		

→ Denak positibo izan behar dira.

$b_1 = \frac{38\cdot15-24}{15} = 36,4$; $b_2 = \frac{15K-0}{15} = K$

$c_1 = \frac{36,4\cdot24-15K}{36,4} = 24-0,412K$; $c_2 = 0$

$24-0,412K > 0 \Rightarrow K < 58,24$

$\boxed{0 < K < 58,24}$

Egonkortasun kritikoa : $K = 58{,}24 \longrightarrow \boxed{K_u = 58{,}24}$

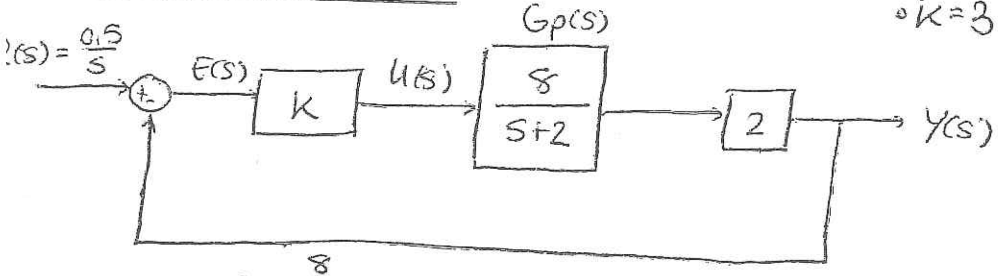
s^4	1	38	58,24
s^3	15	24	0
s^2	36,4	58,24	0
s	0	0	
s^0	d_1		

$\longrightarrow P(s) = 36{,}4s^2 + 58{,}24$

↳ Poloak : $s = \pm\sqrt{1{,}6}\,j \longrightarrow \omega_u = \sqrt{1{,}6}$ rad/s

$$\boxed{T_u = \frac{2\pi}{\omega_u} = \frac{2\pi}{\sqrt{1{,}6}} = 4{,}97\,s}$$

10. Adibidea (50. diap)



• K=3

$$E(s) = R(s) - Y(s) = R(s) - \frac{2\cdot K\,G_p(s)\cdot R(s)}{1+2KG_p(s)\cdot 1} =$$

$$= R(s) - \frac{6\cdot\frac{8}{s+2}}{1+6\,\frac{8}{s+2}}R(s) = \frac{s+2}{s+50}R(s) = \frac{0{,}5(s+2)}{s(s+50)}$$

$$\underline{\underline{e_{ss}}} = \lim_{s\to 0} s\cdot E(s) = \frac{0{,}5\cdot 2}{50} = \underline{\underline{0{,}02}}$$

• K=0,25

$$E(s) = R(s) - \frac{2\cdot 0{,}25\cdot\frac{8}{s+2}}{1+2\cdot 0{,}25\,\frac{8}{s+2}}R(s) = \frac{s(s+2)}{s^2+2s+4}R(s) = \frac{\cancel{s}(s+2)}{s^2+2s+4}\cdot\frac{0{,}5}{\cancel{s}}$$

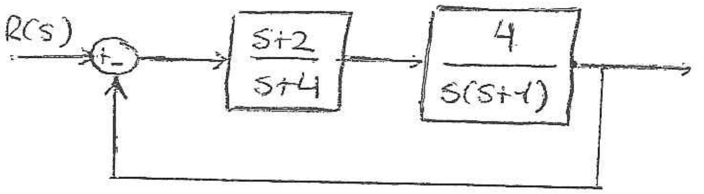
$e_{ss} = \lim_{s\to 0} sE(s) = 0 \longrightarrow$ Integratzailea gehitzean errorea desagertuko da.

↳ Sarrera aldatu : $R(s) = \frac{0{,}1}{s^2}$

$$E(s) = \frac{\cancel{s}(s+2)}{s^2+2s+4}\cdot\frac{0{,}1}{s^{\cancel{2}}} \Rightarrow e_{ss} = \lim_{s\to 0}\cancel{s}\cdot\frac{(s+2)0{,}1}{\cancel{s}(s^2+2s+4)} = 0{,}05$$

Sarrera - mota aldatzean errorea agertuko da.

8, Ariketa (5'4 diapositiba)



$$G_{BC}(s) = \frac{4(s+2)}{s(s+4)(s+1)+4(s+2)} = \frac{4(s+2)}{s^3+5s^2+8s+8}$$

Konprobatu behar dugu sistema egonkorra dela:

R-H

$$\begin{array}{c|ccc} s^3 & 1 & 8 & 0 \\ s^2 & 5 & 8 & 0 \\ \hline s & 6{,}4 & 0 & 0 \\ s^0 & 8 & 0 & 0 \end{array}$$

B.B : koefiziente guztiak dira eta zeinu berekoak.

$$b_1 = \frac{40-8}{5} = 6{,}4 \qquad b_2 = 0$$

$$c_1 = \frac{6{,}4 \cdot 8 - 0}{6{,}4} = 8 \implies \text{EGONKORRA}$$

→ Denak positibo ⟹ B.N betetzen da

$$G_{BA}(s) = \frac{4(s+2)}{s(s+1)(s+4)}$$ ⟹ Integradore bat dauka ⟹ 1 motako sist.

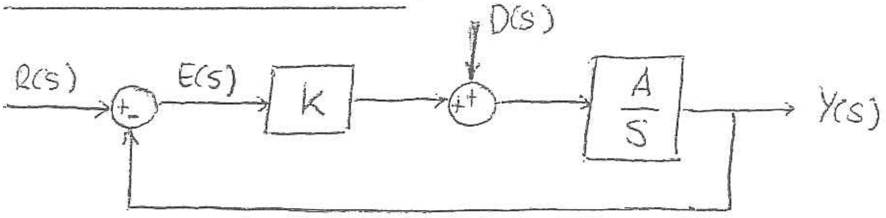
1. motako sistema ⟹
- Espaloi : $e_{ss} = 0$
- Arapala : $e_{ss} = 1/K_v$
- Parabola : $e_{ss} = \infty$

$$K_p = \lim_{s\to 0} G(s)H(s) = \infty$$

$$K_v = \lim_{s\to 0} G(s)H(s)\cdot s = 2$$

$$K_a = \lim_{s\to 0} s^2 G(s)H(s) = 0$$

10. Anketa (56. diapositiba)



$$E(s) = R(s) - \underbrace{H(s)}_{1} \cdot Y(s) = R(s) - \left[E(s) \cdot K + D(s)\right] \frac{A}{s}$$

$$E(s) = R(s) - E(s)\frac{KA}{s} + \frac{A \cdot D(s)}{s} \Rightarrow E(s)\left[1 + \frac{KA}{s}\right] = R(s) - \frac{A}{s} D(s)$$

$$E(s) = \frac{1}{1+\frac{KA}{s}} R(s) - \frac{A}{s(1+K)} D(s)$$

sarrera: espaloi unitario $\Longrightarrow E(s) = \frac{1}{s}$; $D(s) = \frac{1}{s}$

$$E(s) = \frac{\cancel{s}}{s+KA} \cdot \frac{1}{\cancel{s}} - \frac{A}{s+KA} \cdot \frac{1}{s} = \frac{1}{s+KA} - \frac{A}{s(s+KA)} \Rightarrow \text{Integradore } 1$$

1. mailako sistema:

espaloi errorea: $e_{ss} = 0$

arapala (egoera iraunkorra): $\underline{\underline{e_{ss}}} = \lim_{s \to 0} s E(s) = 0 - \frac{A}{KA} = \underline{\underline{-\frac{1}{K}}}$

Parabola: $e_{ss} = \infty$

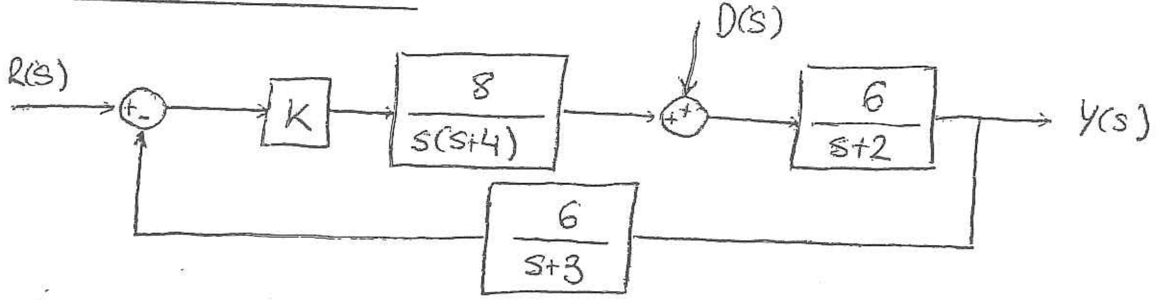
$\Rightarrow$ 2. modua:

$$\left.e_{ss}\right|_{\substack{D(s)\\R(s)=0}} = \lim_{s \to 0} s \cdot \left.E(s)\right|_{\substack{D(s)\\R(s)=0}} = -\frac{1}{K}$$

$$\left.e_{ss}\right|_{\substack{D(s)=0\\R(s)}} = \lim_{s \to 0} s \cdot \left.E(s)\right|_{\substack{D(s)=0\\R(s)}} = 0$$

$$\left.\right\} \underline{\underline{e_{ss}}} = \left.e_{ss}\right|_{\substack{D(s)\\R(s)=0}} + \left.e_{ss}\right|_{\substack{D(s)=0\\R(s)}} = \underline{\underline{-\frac{1}{K}}}$$

11. Anketa (58. diapositiba)



a) $\frac{Y(s)}{R(s)}$? $\frac{Y(s)}{D(s)}$?

$$\left.Y(s)\right|_{\substack{R(s)\\D(s)=0}} = \frac{48K(s+3)}{s(s+4)(s+2)(s+3) + 288K} R(s) \Rightarrow \frac{Y(s)}{R(s)} = \frac{48K(s+3)}{s^4+9s^3+26s^2+24s+88K}$$

$$Y(s)\Big|_{\substack{R(s)=0\\D(s)}} = \frac{\frac{6}{s+2}}{\frac{8\cdot6\cdot k}{s(s+4)(s+3)}+\frac{6}{(s+2)}}D(s) = \frac{6s(s+4)(s+3)}{s^4+9s^3+26s^2+24s+288K}D(s)$$

$$\frac{Y(s)}{D(s)} = \frac{6s(s+4)(s+3)}{s^4+9s^3+26s^2+24s+288K}$$

b) Egonkortasuna:

B.B: $K>0$

$$b_1 = \frac{26\cdot9-24}{9} = 23{,}33 \quad ; \quad b_2 = \frac{9\cdot288K-0}{9} = 288K$$

R-H

$$\begin{array}{c|ccc} s^4 & 1 & 26 & 288K \\ s^3 & 9 & 24 & 0 \\ \hline s^2 & 23{,}3 & 288K & 0 \\ s & c_1 & 0 & 0 \\ s^0 & d_1 & 0 & 0 \end{array}$$

$$c_1 = \frac{24\cdot23{,}33-288K\cdot9}{23{,}33}$$

$$c_2 = 0$$

BN: $24\cdot23{,}33-288\cdot9K>0 \Rightarrow K<2{,}1$

$$\boxed{0<K<0{,}216}$$

c) $T_u?\ (K=K_u)$

$K_u = 0{,}216$

$P(s) = 23{,}3s^2+288\cdot0{,}216 = 23{,}3s^2+62{,}208$

Poloak: $s=\pm1{,}63j \Rightarrow \omega_u = 1{,}63$ rad/s

$$\boxed{T_u = \frac{2\pi}{\omega_u} = 3{,}845s}$$

d) $K=0{,}1$

① $R(s) = 2/s$; $D(s) = 0{,}5/s$

~~$e_{ss}\Big|_{TOT} = e_{ss}\Big|_{\substack{R(s)\\D(s)=0}} + e_{ss}\Big|_{\substack{R(s)=0\\D(s)}}$~~

~~$K_p = \lim_{s\to0}\frac{6s(s+4)(s+3)}{s^4+9s^3+26s^2+24s+288K} = 0$~~ ~~$(R(s)=0) \Rightarrow e_{ss}\Big|_{\substack{R(s)=0\\D(s)-}} = 1$~~

$e_{ss}\Big|_{R(s)} = e_p\cdot2 = 0$

$\hookrightarrow e_p = \frac{1}{1+K_p}$; $K_p = \lim_{s\to0} 0{,}1\cdot\frac{8}{s(s+4)}\cdot\frac{6}{s+2}\cdot\frac{6}{s+3} = \infty \Rightarrow e_p = 0$

$$E(s) = \cancelto{0}{R(s)} - Y(s) \cdot \frac{6}{s+3} = - \frac{36\cancel{8}(s+4)}{s^4+9s^3+26s^2+24s+288 \cdot 0{,}1} \cdot \frac{0{,}5}{\cancel{8}}$$

$$\hookrightarrow e_{ss}\Big|_{D(s)} = \lim_{s \to 0} s \cdot E(s) = 0$$

$$\boxed{e_{ss} = 0}$$

② $R(s) = \frac{2}{s^2}$; $D(s) = \frac{0{,}5}{s} \cdot e^{-3s}$

$$e_{ss}\Big|_{R(s)} = e_v \cdot 2 = 10/6$$

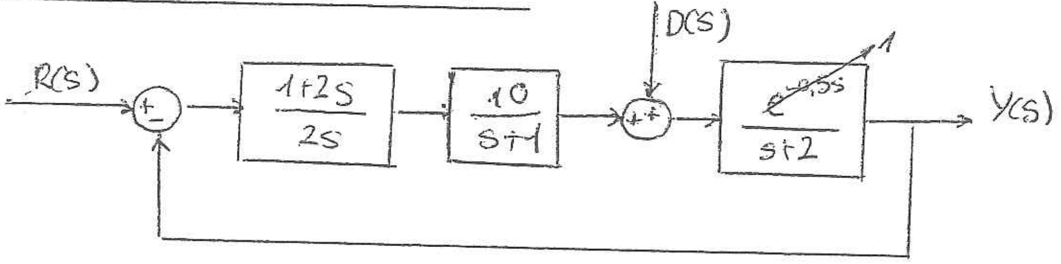
$$\hookrightarrow \frac{1}{K_p} = \frac{1}{6/5} = \frac{5}{6}$$

$$\boxed{e_{ss} = \frac{10}{6}}$$

$$K_p = \lim_{s \to 0} \cancel{s} \cdot 0{,}1 \cdot \frac{8}{\cancel{s}(s+4)} \cdot \frac{6}{s+2} \cdot \frac{6}{s+3} = \frac{6}{5}$$

Aurrekoan bezala: $e_{ss}\Big|_{D(s)} = 0$

12. Ariketa (59. diapositiba)



a) e_{ss} (sarrera bak espaloi unitarioak)

$$e_{ss} = \lim_{s \to 0} s \cdot E(s) \quad \text{edo} \quad e_{ss} = \frac{1}{1+K_p} \quad \text{non} \quad K_p = \lim_{s \to 0} G(s) \cdot H(s)$$

$$e_{ss} = e_{ss}\Big|_{R(s)=0} + e_{ss}\Big|_{D(s)=0}$$

$$E(s) = R(s) - Y(s) \cdot H(s) = \frac{R(s)}{1+G(s)\cancelto{1}{H(s)}} - \frac{\cancelto{1}{H(s)}G_d(s)}{1+G(s)H(s)} D(s)$$

- $R(s) = 0 \rightarrow E(s) = -Y(s)$

$$\frac{Y(s)}{\underbrace{D(s)}_{1/s}} = - \frac{\frac{1}{s+2}}{1 + \frac{10(1+2s)}{2s} \cdot \frac{1}{s+1}} = - \frac{\cancel{2}s(s+1)}{\cancel{2}s(s+1)(s+2) + \cancelto{5}{10}(1+2s)}$$

$$Y(s) = -\frac{1}{\cancel{s}} \cdot \frac{\cancel{s}(s+1)}{s(s+1)(s+2)+5(1+2s)} = - \frac{s+1}{s(s+1)(s+2)+5(1+2s)}$$

$$\lim_{s\to 0} s\cdot E(s) = e_{ss} = \lim_{s\to 0} s\cdot\left(-\frac{s+1}{s(s+1)(s+2)+5(2s+1)}\right) = 0$$

- $D(s)=0 \longrightarrow E(s) = \frac{1/s}{1+G(s)} = \frac{1/s}{1+\frac{10(1+2s)}{2s(s+1)(s+2)}} = \frac{2s(s+1)(s+2)}{2s^2(s+1)(s+2)+10(1+2s}$

$$e_{ss} = \lim_{s\to 0} s\cdot \frac{2s(s+1)(s+2)}{2s^2(s+1)(s+2)+10(1+2s)} = 0$$

$$\boxed{e_{ss}=0}$$

b) sarrerak 2 maldako arrapalak. $\left\{ \begin{array}{l} D(s)=\frac{2}{s^2} \\ R(s) = 2/s^2 \end{array}\right.$

- $R(s)=0$

$$E(s) = -Y(s) = -\frac{2}{\cancel{s^2}}\cdot\frac{\cancel{s}(s+1)}{\underbrace{s(s+1)(s+2)}_{s^2+3s+2}+\underbrace{5(1+2s)}_{5+10s}} = -\frac{2s+2}{s(s^3+3s^2+2s+10s+5)}$$

$$e_{ss} = \lim_{s\to 0} \cancel{s}\left(-\frac{2s+2}{\cancel{s}(s^3+3s^2+2s+10s+5)}\right) = \frac{2}{5}$$

- $D(s)=0$

$$E(s) = \frac{2}{s^2}\cdot\frac{1}{1+\frac{10(1+2s)}{2s(s+1)(s+2)}} = \frac{2}{s^2}\cdot\frac{\cancel{2}\cancel{s}(s+1)(s+2)}{\cancel{2}s(s+1)(s+2)+\overset{}{\underset{5}{\cancel{10}}}(1+2s)} =$$

$$= \frac{2(s^2+3s+2)}{s(s^3+3s^2+2s+10s+5)}$$

$$e_{ss} = \lim_{s\to 0} s\cdot\frac{2(s^2+3s+2)}{s(s^3+3s^2+2s+10s+5)}$$

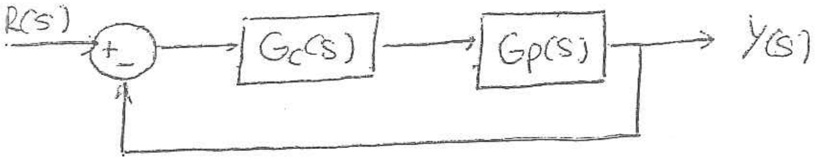
$$E(s) = R(s) - Y(s) = R(s) - \left[\frac{10(1+2s)}{2s(s+1)}E(s) + D(s)\right]\cdot\frac{1}{s+2}$$

$$E(s)\left[1+\frac{10(1+2s)}{2s(s+1)(s+2)}\right] = \frac{2}{s^2} - \frac{2}{s^2(s+2)}$$

$$E(s) = \frac{\frac{2s+2}{s^2(s+2)}}{1+\frac{10(1+2s)}{2s(s+1)(s+2)}} = \frac{2(s+1)(2s+2)}{s\left[2s(s+2)(s+1)+10(2s+1)\right]}$$

$$\boxed{e_{ss} = \lim_{s\to 0} s\cdot E(s) = \frac{2}{5}}$$

2. Ariketa (28. diap - 5)



$$G_p(s) = \frac{1}{(s+1)(s+2)(s+5)}$$

I) Sistema egonkorra?

R-H taula:

$$G_{BC}(s) = \frac{K_c}{\underbrace{(s+1)(s+2)}(s+5) + K_c}$$

$$\underbrace{(s^2+3s+2)\,(s+5)}$$

↳ $s^3 + 8s^2 + 17s + 10$

s^3	1	17
s^2	8	$10+K_c$
s	b_1	0
s^0	c_1	

$$b_1 = -\frac{1}{8}(10 + K_c - 8 \cdot 17) =$$

$$= 15{,}75 - \frac{1}{8}K_c$$

- $10 + K_c > 0 \rightarrow K_c > -10 \rightarrow K_c > 0$
- $15{,}75 - \frac{1}{8}K_c > 0 \rightarrow K_c < 126$ $\boxed{0 < K < 126}$

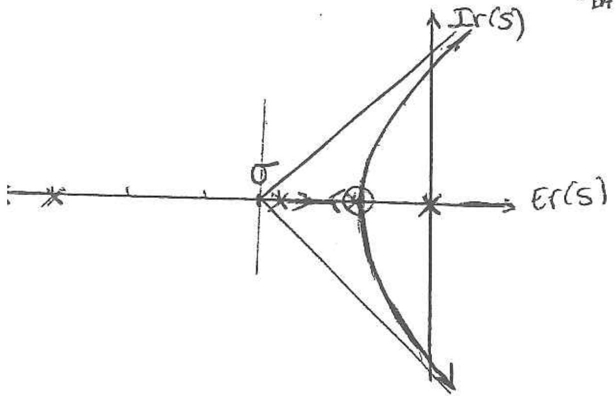
II) $e_{ss_p} = 0$

1) Eskakizun eremua

2) I? → BAi → PI
(ess=0)

3) ET PI-arekin:

$$G_{BA}^{PI} = G(s)H(s) = \frac{K_c\left(\frac{1}{T_i} + s\right)}{s(s+1)(s+2)(s+5)} \; ; \quad \begin{matrix} m=1 \\ n=4 \end{matrix}$$



$$\sigma = \frac{\Sigma z_i - \Sigma p_i}{n-m} = \frac{-(1+2+5)+1}{3} = \frac{-7}{3} = -2{,}33$$

$\boxed{T_i = 1}$ (polo nagusia (s = -1) anulatzeko)

$$\theta_{1,2} = \pm \frac{(2k+1)\pi}{n-m} = \pm \frac{\pi}{3} = \pm 60° \quad (k=0)$$

$$\theta_3 = \pm \frac{3\pi}{3} = \pm \pi = \pi \quad (k=1)$$

ek. karakteristikoa:

$1 + G(s)H(s) = 0$

↳ $1 + \dfrac{K_c\left(\frac{1}{1} + s\right)}{s(s+1)(s+2)(s+5)} = 0 \rightarrow s\cancel{(s+1)}(s+2)(s+5) + K_c\cancel{(s+1)} = 0$

$s(s+2)(s+5) + K_c = 0 \rightarrow s(s^2 + 7s + 10) + K_c = 0 \rightarrow s^3 + 7s^2 + 10s + K_c = 0$

. R-H irizpidea:

↳ BB: $K_c > 0$

↳ BN:

$10 - \frac{1}{7}K_c > 0$

$K_c < 70$

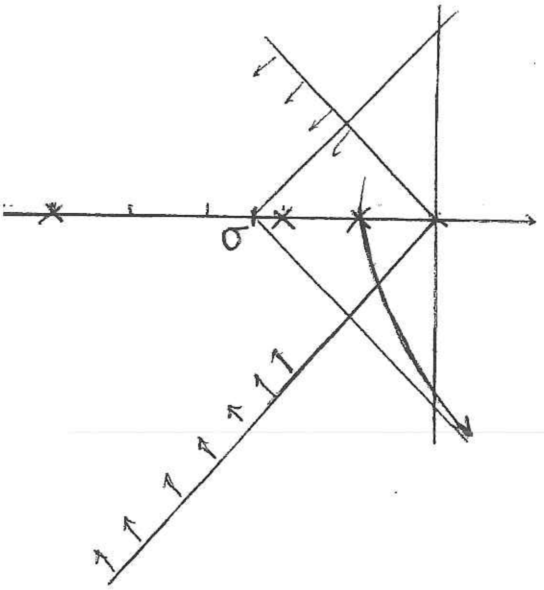
s^3	1	10
s^2	7	K_c
s	b_1	0
s^0	c_1	

$$b_1 = \frac{70 - K_c}{7} = 10 - \frac{1}{7}K_c$$

$$\boxed{0 < K_c < 70}$$

(III) $e_{ss}=0$; $M_p \leq \% 4{,}3$ PI

$$M_p \leq \% 4{,}3 = e^{-\frac{\pi\delta}{\sqrt{1-\delta^2}}} \cdot 100 \rightarrow \delta \geq \sqrt{\frac{(\ln 0{,}043)^2}{(\ln 0{,}043)^2+\pi^2}} = 0{,}707 \rightarrow \theta = 45°$$



θ eta σ aurrekoaren berdina.

$1+G(s)H(s)=0$

↳ $s^3+7s^2+10s+K_c = (s+a)(s^2+2\delta\omega_n+\omega_n^2)$

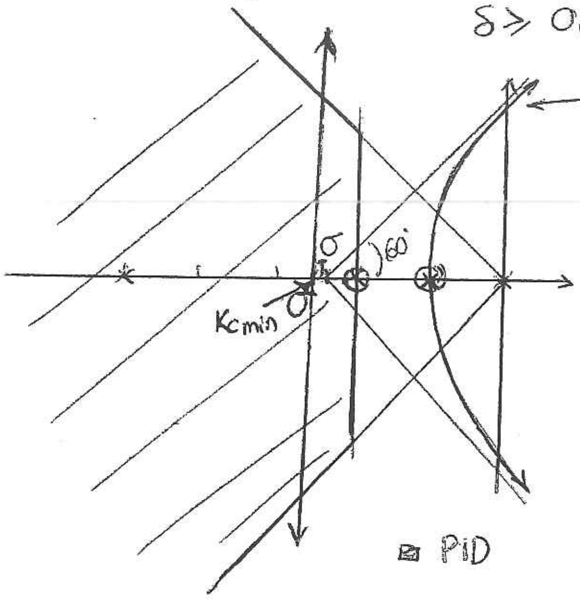
$$s^3+7s^2+10s+K_c = s^3+(a+2\delta\omega_n)s^2+(2\delta\omega_n a+\omega_n^2)s+a\omega$$

$$\left\{\begin{array}{l} 7 = a+2\delta\omega_n \\ 10 = 2\delta\omega_n a+\omega_n^2 \\ K_c = a\omega_n^2 \\ \delta = 0{,}707 \end{array}\right. \Longrightarrow \left\{\begin{array}{l} \omega_n = \\ a = \\ K_c|_{max} = \end{array}\right.$$

(IV) $e_{ss}=0$; $M_p \leq \% 4{,}3$; $t_s \leq 2s$ PI

$$t_s = \frac{4}{\delta\omega_n} \leq 2 \rightarrow 2 \leq \delta\omega_n$$

$$\delta \geq 0{,}707$$



Ez dago intersekziorik eskakizun eremuan beraz D bat gehitu behar dugu. $G_{PID}(s)$ erabili

↳ zero bat $s=-2$-n sartu

$$\sigma = \frac{\cancel{2+1-1-2}-5}{\cancel{3}2} = -2{,}5$$

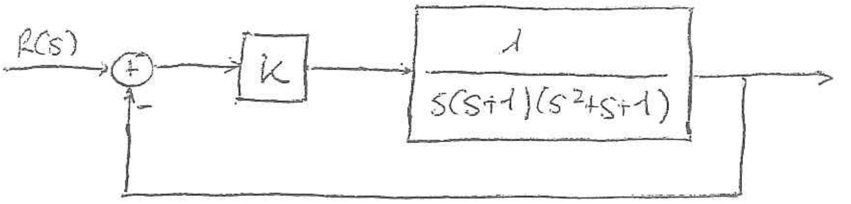
$$\theta = \pm\frac{\pi}{2}$$ ⇒ intersekzioa dago

ek. karakteristikoa: $1+\frac{K_c\cancel{(s+1)}\cancel{(s+2)}}{s}\cdot\frac{1}{\cancel{(s+1)}\cancel{(s+2)}(s+5)} = 0$ →

→ $s^2+5s+K_c = s^2+2\delta\omega_n s+\omega_n^2$ ⇒ sistema kritikoki egonkorra lortu dugu

$$\left\{\begin{array}{l} 5 = 2\delta\omega_n \longrightarrow \omega_n = 5/2 \text{ rad/s} \\ K_c = \omega_n^2 \longrightarrow K_c = \frac{25}{4} = 6{,}25 = K_c|_{min} \\ \delta = 1 \end{array}\right.$$

6. Ariketa (39. diapositiba -5)



K ?
Tu ?

$$G_{BC}(s) = \frac{K}{\underbrace{s(s+1)}_{s^2+s}(s^2+s+1) + K} = \frac{K}{s^4+s^3+s^2+s^3+s^2+s+K} = \frac{K}{s^4+2s^3+2s^2+s+k}$$

B.B: $K>0$

B.N:

$$\begin{array}{c|ccc} s^4 & 1 & 2 & K \\ s^3 & 2 & 1 & 0 \\ \hline s^2 & 3/2 & K & 0 \\ s & 1-\frac{4}{3}K & 0 & \\ s^0 & d_1 & & \end{array}$$

$$b_1 = \frac{4-1}{2} = \frac{3}{2} \quad ; \quad b_2 = \frac{2K-0}{2} = K$$

$$C_1 = \frac{3/2 - 2K}{3/2} = 1 - \frac{4}{3}K$$

$$1 - \frac{4}{3}K > 0$$

$$1 > \frac{4}{3}K \rightarrow K < \frac{3}{4}$$

$$\boxed{0 < K < \frac{3}{4}}$$

$$\boxed{K_u = \frac{3}{4}}$$

$K_u = \frac{3}{4}$ erabiliz R-H taulan 4. lerroan guztiak zero dira, beraz, polinomio laguntzailea:

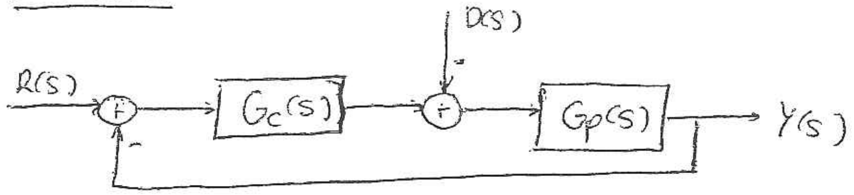
$$\frac{3}{2}s^2 + \frac{3}{4} = 0 \rightarrow 6s^2+3=0 \rightarrow 2s^2+1=0$$

$$s^2 = -\frac{1}{2} \Rightarrow s = \pm\sqrt{0{,}5}\,j \rightarrow \boxed{\omega_u = \sqrt{0{,}5} = \frac{1}{\sqrt{2}} \text{ rad/s}}$$

$$\boxed{T_u = \frac{2\pi}{\omega_u} = \frac{2\pi}{\sqrt{0.5}} = 8{,}886\ s}$$

29/11/2014

8. ARIKETA



21) A

3. eta 4. grafikoetatik dedukitu dezakegu $G_c(s)$ kontrolagailua proportzionala dela, K_c, errorea prozesatzen duelako eta irabazpen batengatik biderkatzen duelako. Beraz ez dago integradorerik kontrolagailuan.

Beste aldetik, R(s) arrapala bat denean, sistemaren irteerak errore txiki bat dauka, konstantea dena, eta horrek esan nahi du sistemak begizta irekian integradore bat duela jatorrian (jatorrian errorea hasi eta kte mantentzen delako).

22) A

Esan dugu sistemak begizta irekian integradore bat daukala, beraz 1 motako sistema berrelikatua daukagu.

errorea = 1 ⇒ $e_{ss} = 1$

R(s)-ren malda = 1

$$\Rightarrow \quad e_{ss} = \frac{1}{K_v} = 1 \longrightarrow K_v = 1$$

EJERCICIO 8

Para el sistema de control realimentado mostrado en la figura 1 se han obtenido las gráficas mostradas en las figuras 2, 3 y 4

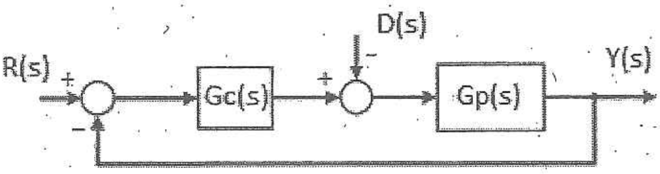
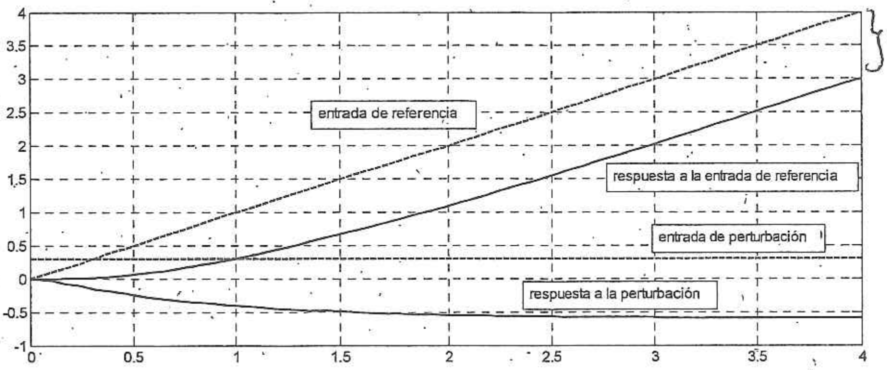


Figura 1: Sistema de control



} errorea = 1

Figura 2: Respuesta del sistema a una entrada de referencia rampa y una perturbación escalón

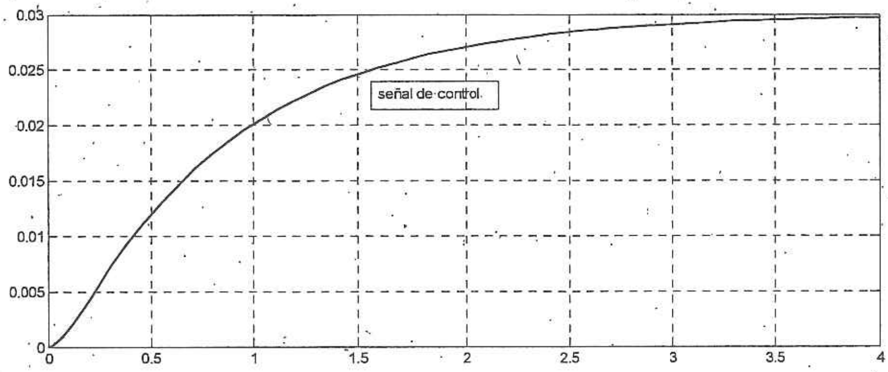


Figura 3: Señal de control

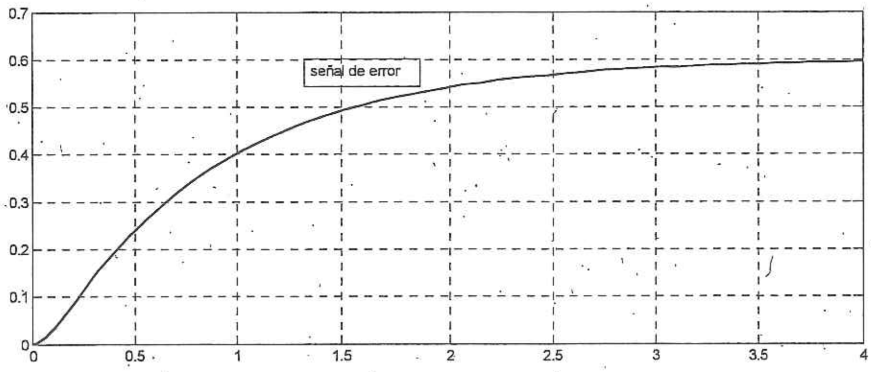
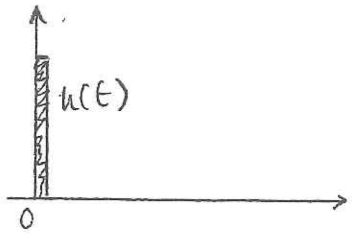


Figura 4: Señal de error

4. GAIA - DENBORAREN EREMUKO AZTERKETA

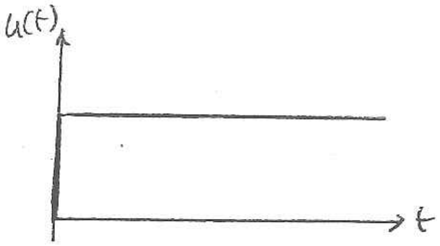
- FROGA SEINALEAK

INPULTSU - SARRERA



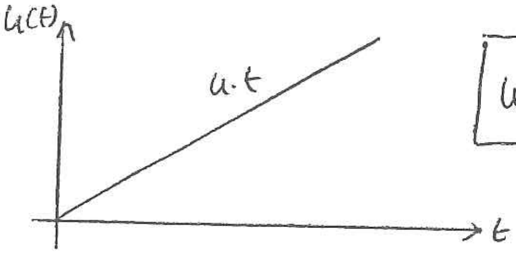
$\boxed{u(s) = u}$

ESPALOI - SARRERA



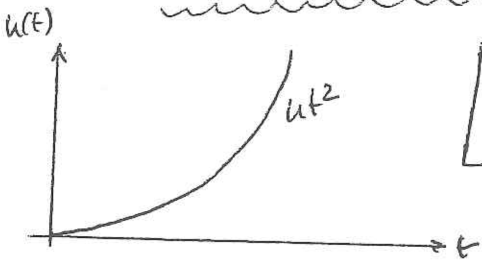
$\boxed{u(s) = \frac{u}{s}}$

ARRAPALA - SARRERA



$\boxed{u(s) = \frac{u}{s^2}}$

PARABOLA - SARRERA



$\boxed{u(s) = \frac{u}{s^3}}$

- **LEHEN ORDENEKO SISTEMAK**

- Eredu matematikoa: $\tau \frac{dy(t)}{dt} + y(t) = k r(t)$

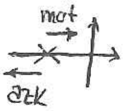
  ↳ sistema OP (operazio-puntu)-tik abiatzen bada, hasierako baldintzak nulak izango dira.

- Transferentzia-funtzioa:

$G(s) = \frac{y(s)}{u(s)} = \frac{K}{\tau s+1}$ non

K : Irabazpen estatikoa $K = \frac{\Delta y}{\Delta u}$
τ : sistemaren denbora-konstantea

⊛ Poloa: $\tau s+1=0$; $s = -\frac{1}{\tau}$



⊛ $K>0$ edo $K<$ izan ahal da:

$K<1$ MOTELDU
$K>1$ ANPLIFIKATU
$K=1$ EZ ALDATU

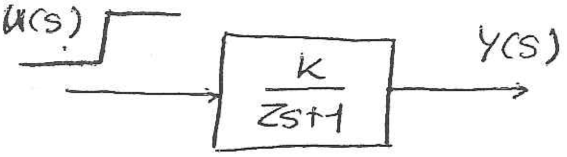
⊛ τ: Irteerak amaierako balioaren %63,2-a lortzeko behar duen denbora (sistema egonkorretan)

$t=\tau \Rightarrow y(\tau) = 0{,}632\,Ku$

$y(\tau) = y_{63} = \bar{y} + 0{,}63 \cdot \Delta y = \underline{\quad}$ ⇒ Lortzen dugun balioa taulan kokatu eta dagokion abzisa izango da: $t_{63} = \tau$

MAILA

- ESKALOI - ERANTZUNA



$$\boxed{y(t)=Ku\left(1-e^{-t/\tau}\right)}$$

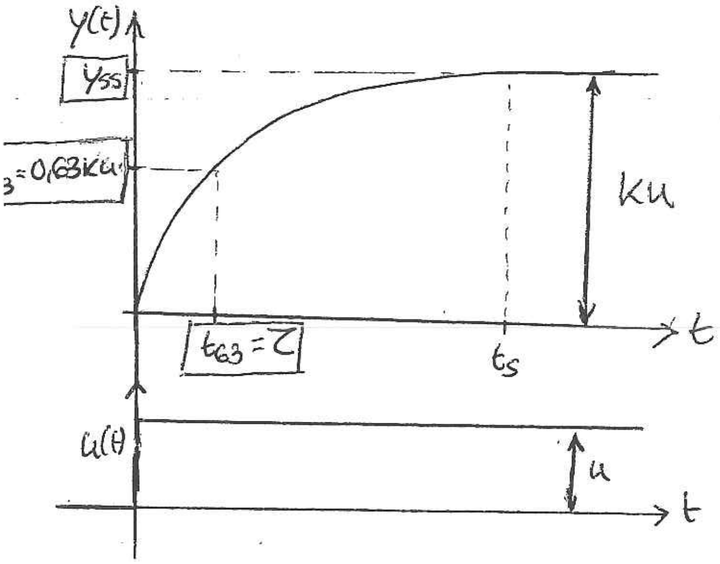
→ zenbat eta handiago ⇒ sistema motelago

$\tau>0$ ⇒ erantzun egonkorra, monotonoki gorakorra.

$$y(t)=\underbrace{Ku}_{y_{ss}}-\underbrace{Ku\,e^{-t/\tau}}_{y_t}$$

$$y(t)=y_{ss}+y_t=y_{egonkor}+y_{iragankor}$$

*Egonkortze denbora (t_s): Irteerak bere amaierako balioa lortzeko behar duen denbora.



%5 irizpidea

$t=3\tau$

$$y(t_{95})=0{,}95\,Ku=Ku\left(1-e^{-\frac{t_{95}}{\tau}}\right)$$

%2 irizpidea

$t=4\tau$

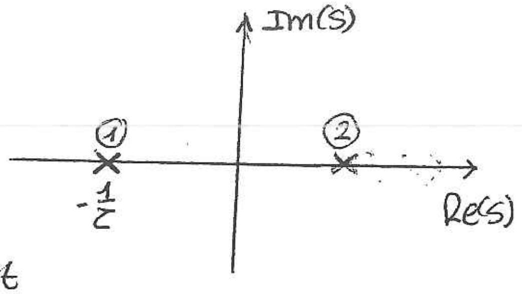
$$y(t_{98})=0{,}98\,Ku=Ku\left(1-e^{-\frac{t_{98}}{\tau}}\right)$$

* Polo erreal negatiboa: ①

$\tau s+1=0 \Rightarrow s=-\frac{1}{\tau}$

$\tau>0$: erantzun egonkorra.

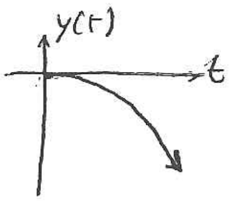
s planoa



* Polo erreal positiboa: ②

$\tau s+1=0 \Rightarrow s=-\frac{1}{\tau}$

$\tau<0$: erantzun ez-egonkorra



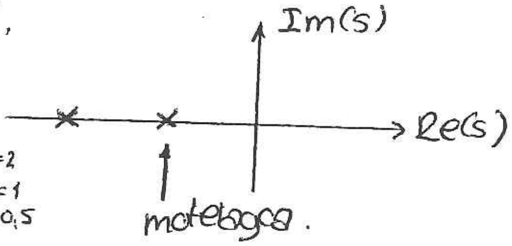
⊛ K berdina duten sistemek ⟶ $y_{ss}=Ku$ berdina (u berdina izanda)

⊛ τ zenbat eta handiagoa izan orduan eta motelagoa izango da sistema.

τ↑ ⇒ motelago

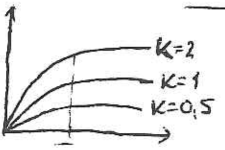
τ↓ ⇒ azkarago

zenbat eta ardatz irudikaritik hurbilago egon, gero eta handiagoa izango da τ.

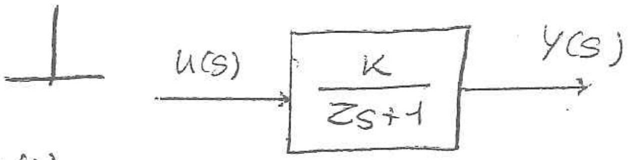


⊛ K↑ ⇒ azkarago

K↓ ⇒ motelago

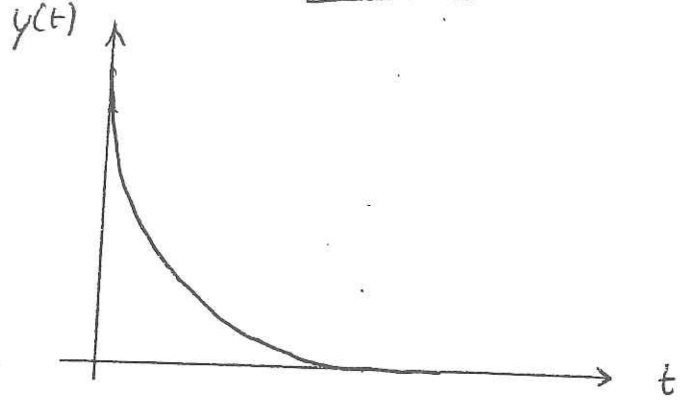


- INPULTSU - ERANTZUNA



$u(s)=u$

$$\boxed{y(t)=\frac{Ku}{\tau}e^{-t/\tau}}$$

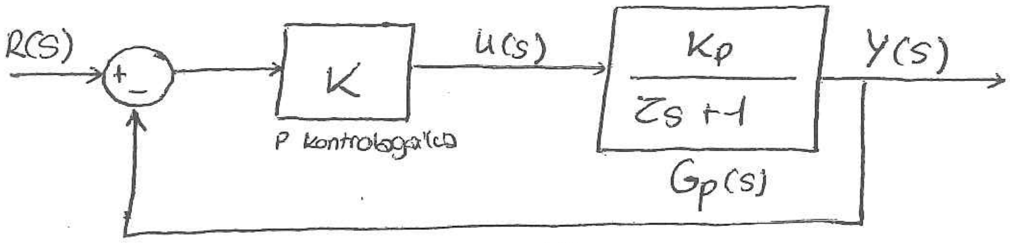


⚠ EGONKORTASUNA

POLOAREN KOKAPENAK DEFINITZEN DU, EZ SARRERA MOTAK!

Hau da, sarrera motak ez du poloaren egonkortasuna definitzen.

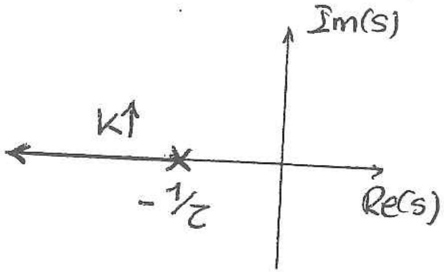
- BERRELIKADURAREN ERAGINA



$$\boxed{G_{BC}(s)=\frac{KG_p(s)}{1+KG_p(s)}}=\frac{\frac{KK_p}{1+KK_p}}{1+\frac{\tau}{1+KK_p}s} \quad \text{non} \begin{cases} K'=\frac{KK_p}{1+KK_p} \\ \tau'=\frac{\tau}{1+KK_p} \end{cases}$$

- Ekuazio karakteristikoa: $1+KG_p(s)=0$

$$1+K\frac{K_p}{1+\tau s}=0 \Rightarrow s=-\frac{1+KK_p}{\tau}$$



* $K'>1$ da eta K-rekin handitzen da.

* τ' K-rekin txikitzen da.

⇒ K handitzean sistema azkarragoa eta doitasun handiagokoa da.

BIGARREN ORDENEKO SISTEMAK

- Eredu matematikoa: $\frac{d^2y(t)}{dt^2} + 2\delta\omega_n \frac{dy(t)}{dt} + \omega_n^2 y(t) = K\omega_n^2 u(t)$

sistema OP-tik abiatzen bada, hasierako baldintzak nulak zango dira.

- Transferentzia - funtzioa:

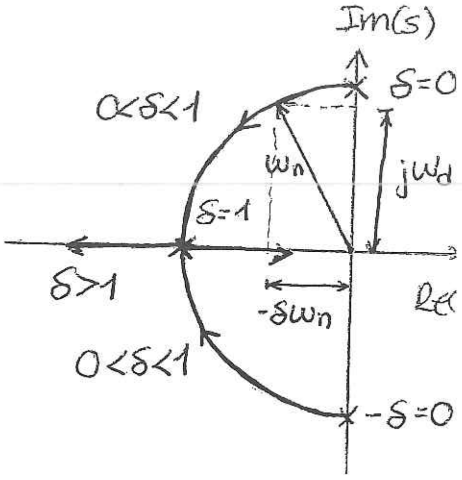
$$G(s) = \frac{Y(s)}{U(s)} = \frac{K\omega_n^2}{s^2+2\delta\omega_n s+\omega_n^2} \quad \text{(zerorik gabe)}$$

non
- K : Irabazpen estatikoa $K = \frac{\Delta y}{\Delta u}$
- δ : Moteldura - koefizientea
- ω_n : Maiztasun naturala (rad/s)
  - ↳ moteldurarik ez duenean, sistemak maiztasun honetan oszilatzen du.

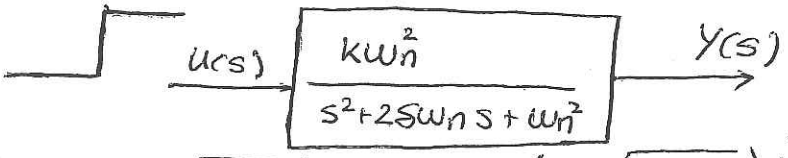
⊛ Poloak: $s^2+2\delta\omega_n s+\omega_n^2=0 \Rightarrow s_{1,2} = -\delta\omega_n \pm \omega_n\sqrt{\delta^2-1}$

- POLOEN KOKAPENA

- $\delta=0$ POLO IRUDIKARIAK
  $s=\pm j\omega_n$
- $\delta=1$ POLO ERREAL BIKOITZA
  $s_{1,2}=-\omega_n$
- $0<\delta<1$ POLO KONPLEXU KONJOKATUAK
  $s_{1,2}=-\delta\omega_n \pm j\omega_n\sqrt{1-\delta^2}$
- $\delta>1$ POLO ERREALAK
  $s_{1,2}=-\delta\omega_n \pm \omega_n\sqrt{\delta^2-1}$
- $\delta<0$ ZATI ERREALA DUTEN POLOAK
  (EZ - EGONKORRA)



MAILA
- ESPALOI ERANTZUNA



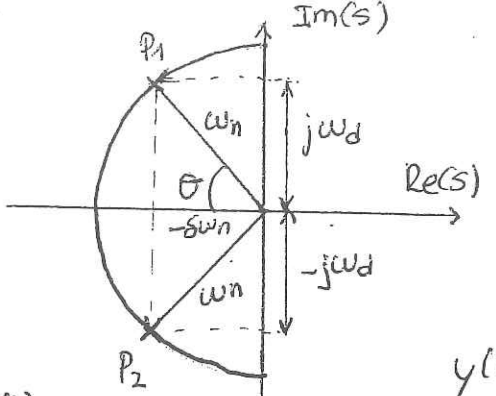
1) $\boxed{0<\delta<1}$

$$y(t) = \underbrace{Ku}_{y_{ss}\ \text{(egonkor)}}\left[1 - \underbrace{\frac{e^{-\delta\omega_n t}}{\sqrt{1-\delta^2}}\left[\sqrt{1-\delta^2}\cos(\omega_n\sqrt{1-\delta^2})t + \delta\sin(\omega_n\sqrt{1-\delta^2})t\right]}_{y_t\ \text{(iegonkor)}}\right]$$

Moteldutako maiztasuna:

$$\omega_d = \omega_n\sqrt{1-\delta^2} \quad \text{(zati irudikaria)}$$

$\boxed{0 < \delta < 1}$ ⇒ AZPIMOTELDUA

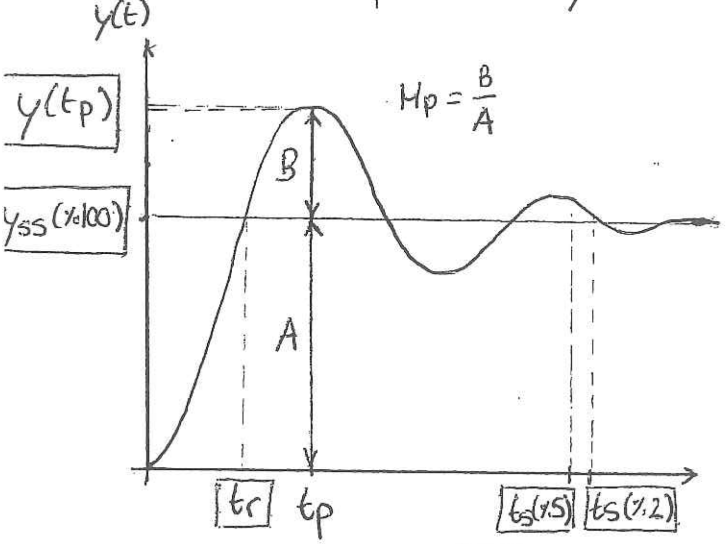


$$\operatorname{sen}\theta = \sqrt{1-\delta^2}$$

$$\cos\theta = \delta \Longrightarrow \theta = \arccos\delta$$

$$tg\,\theta = \frac{\sqrt{1-\delta^2}}{\delta} \Rightarrow \theta = \operatorname{arctg}\frac{\sqrt{1-\delta^2}}{\delta}$$

$$y(t) = ku\left[1 - \frac{e^{-\delta\omega_n t}}{\sqrt{1-\delta^2}}\right]\operatorname{sen}\left[\underbrace{\left(\omega_n\sqrt{1-\delta^2}\right)}_{\omega_d}t + \operatorname{arctg}\frac{\sqrt{1-\delta^2}}{\delta}\right]$$



* Egoera iraunkorreko irteera y_{ss}

* Igoera - denbora t_r

* Punta- denbora t_p

* Gaindiketa $M_p = \frac{B}{A} = \frac{y(t_p) - y_{ss}}{y_{ss}}$

* Egonkortze - denbora t_s

%2 irizpidea: $t_s = 4/\delta\omega_n$

% 5 irizpidea: $t_s = 3/\delta\omega_n$

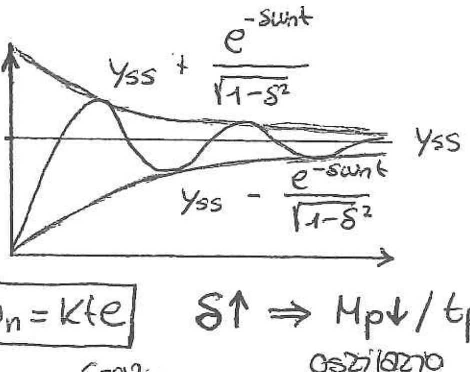
$$t_r = \frac{\pi-\theta}{\omega_d} = \frac{\pi - \operatorname{arctg}\frac{\sqrt{1-\delta^2}}{\delta}}{\omega_n\sqrt{1-\delta^2}} \quad ; \quad t_p = \frac{\pi}{\omega_d} = \frac{\pi}{\omega_n\sqrt{1-\delta^2}} \qquad M_p\uparrow \Longrightarrow \delta\downarrow$$

$M_p = e^{-\frac{\pi\delta}{\sqrt{1-\delta^2}}}$ (x100 egin %-tan lortzeko)

Erantzuna:
- $0{,}2 < \delta < 0{,}8$ ⟶ Erantzun oszilakorra 
- $\delta = 0{,}8$ ⇒ Ez du oszilatzen (gain-inpultsua) 
- $\delta \geqslant 1$ ⇒ Gain-inpultsurik ez (bi polo erreal)

⊛ $\boxed{\delta\uparrow \Rightarrow t_p\uparrow \Rightarrow M_p\downarrow}$

⊛



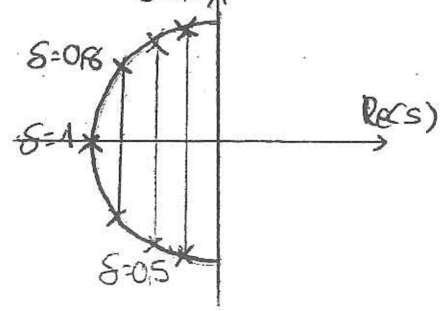
Kurba inguratzaileak ⇒ esponentzialak

$\tau = \frac{1}{\delta\omega_n}$: Kurba inguratzaileen denbora ktea.

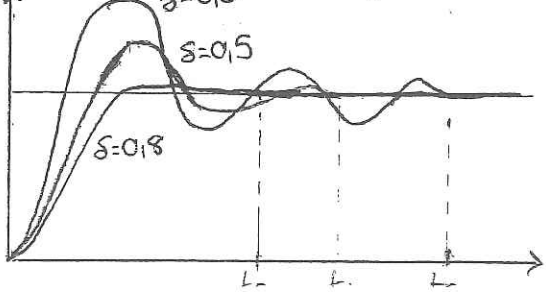
⊛ $\boxed{\omega_n = kte}$ $\delta\uparrow \Rightarrow M_p\downarrow / t_p\uparrow / t_s\downarrow$ (k=kte)

oszilazio maiztasuna handitzen da.

zenbat eta ardatz irudikarritik gertuago egon orduan eta motelagoa egonkortzen ($t_s\uparrow$)

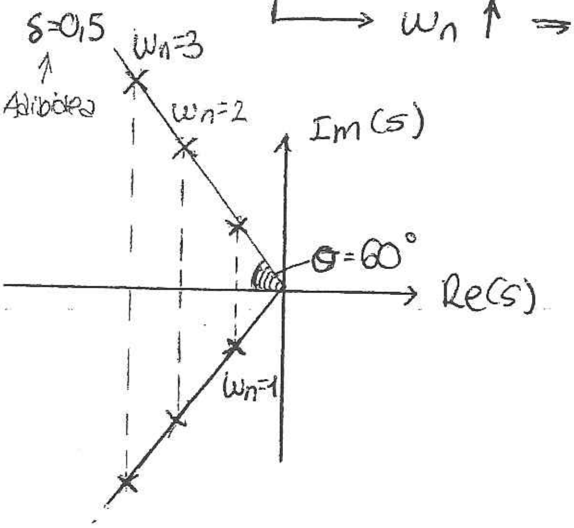


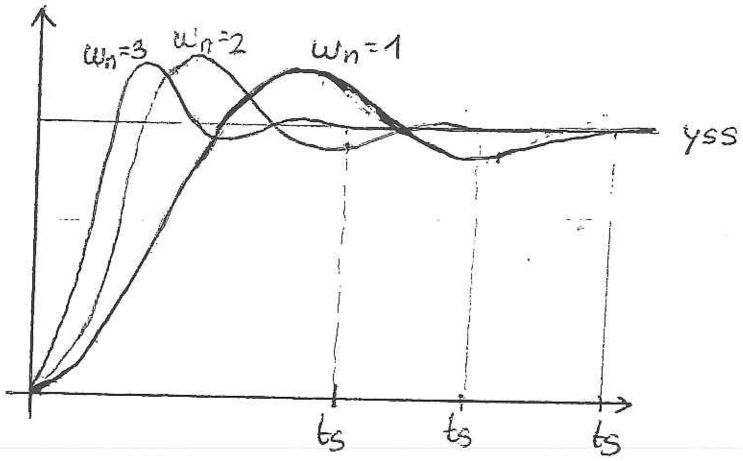
$\Longrightarrow$



⊛ $\boxed{\delta = kte}$ → $M_p = kte$ (azaleren batuketa kte)

→ $\omega_n \uparrow \Rightarrow t_s \downarrow$ / oszilazio maiztasuna ↑

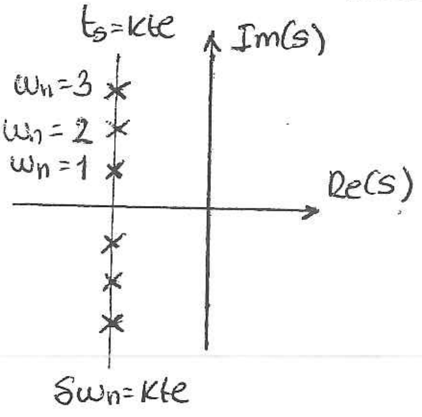


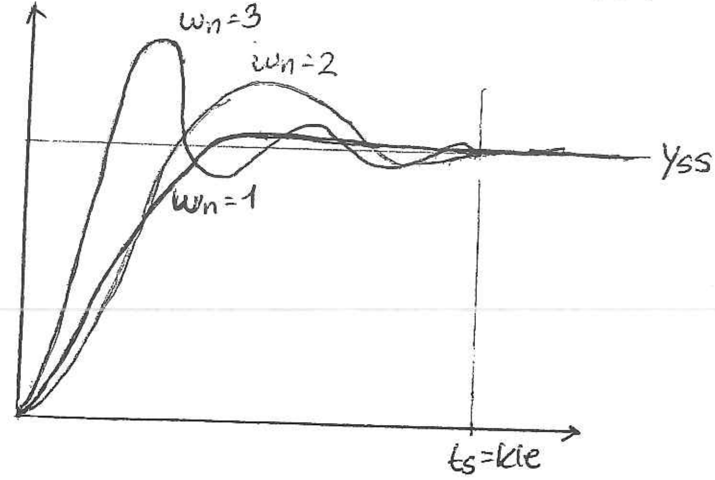


zenbat eta ardatz irudikaritik gertuago egon orduan eta motelagoa rango da sistema; motelagoa egonkortzen (ts ↑)

⊛ $\boxed{\delta\omega_n = kte}$ → $\boxed{t_s = kte}$ $\delta\uparrow \Rightarrow M_p\downarrow // \omega_d\downarrow // t_p\uparrow$ (K=kte)

oszilazio maiztasuna txikitu.

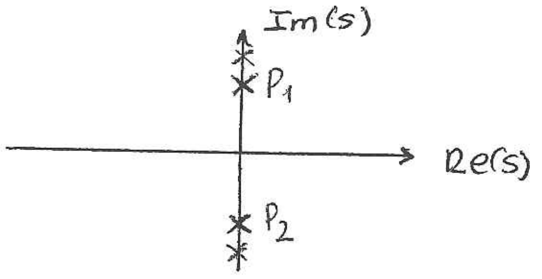


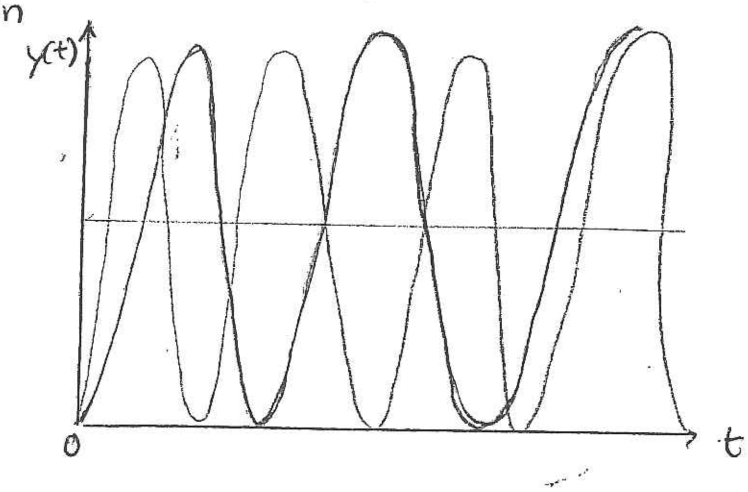


2) $\boxed{\delta = 0}$ ⇒ KRITIKOKI EGONKORRA

$$y(s) = ku\left[\frac{1}{s} - \frac{s}{s^2+\omega_n^2}\right] \longrightarrow y(t) = ku\left[1-\cos(\omega_n t)\right]$$

⊛ Polo irudikariak: $s_{1,2} = \pm j\omega_n$



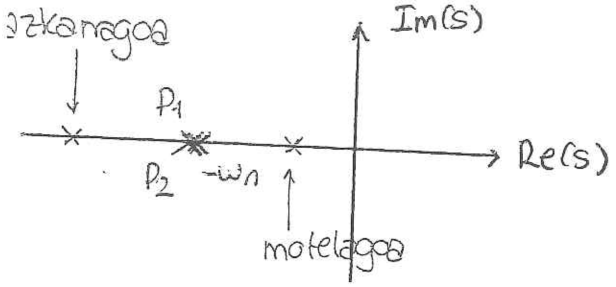


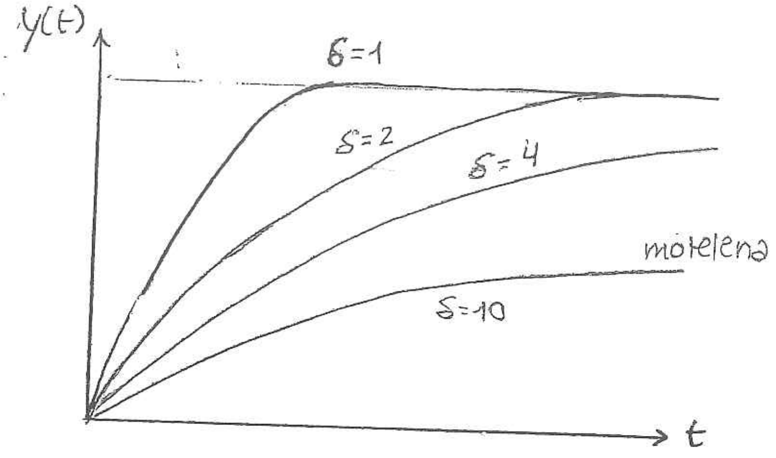
Poloak zenbat eta zati irudikari handiagoa izan orduan eta frekuentzia handiagoarekin oszilatuko du.

3) $\boxed{\delta=1}$ ⇒ KRITIKOKI MOTELDUA

$$y(t)=ku\left(1-e^{\delta\omega_n t}-\omega_n t e^{\delta\omega_n t}\right)$$

⊛ Polo bikoitza: $s_{1,2}=-\omega_n$





$\boxed{\delta\uparrow \Rightarrow \text{motelagoa}}$

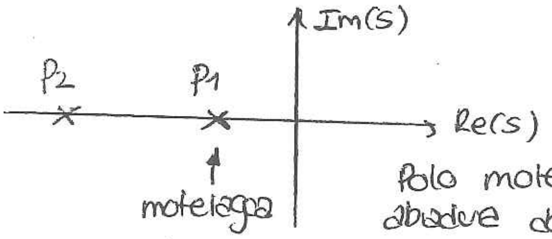
4) $\boxed{\delta>1}$ ⇒ GAINMOTELDUA

$$y(t)=ku\left[1+\frac{1}{2\sqrt{\delta^2-1}}\left(\underbrace{\frac{1}{\delta+\sqrt{\delta^2-1}}e^{-(\delta+\sqrt{\delta^2-1})\omega_n t}}_{\text{Esponentzial azkarra}}-\underbrace{\frac{1}{\delta-\sqrt{\delta^2-1}}e^{-(\delta-\sqrt{\delta^2-1})\omega_n t}}_{\text{Esponentzial motela}}\right)\right]$$

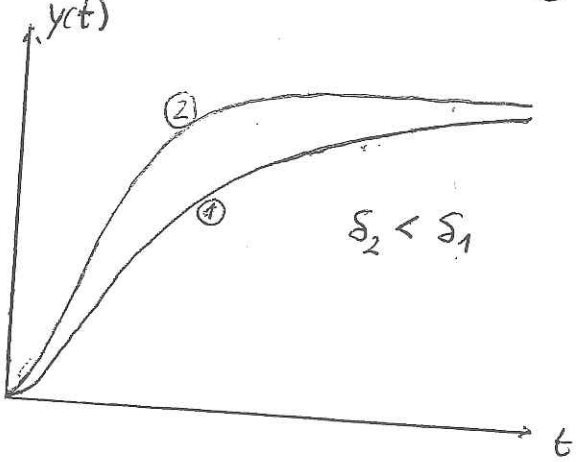
⊛ Polo erreal ezberdinak:

$p_1=-\delta\omega_n+\omega_n\sqrt{\delta^2-1}$

$p_2=-\delta\omega_n-\omega_n\sqrt{\delta^2-1}$



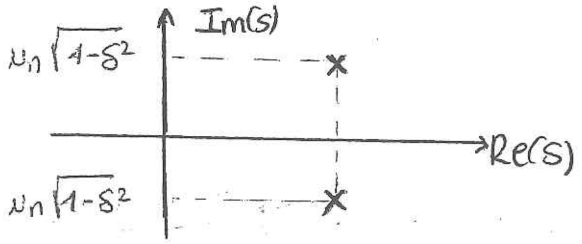
Polo motelak abiadura dominantea du

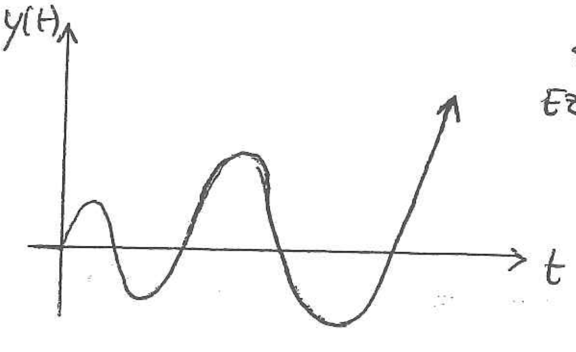


5) $\boxed{\delta<0}$ ⇒ EZEGONKORRA

$$y(t)=ku\left[1-\frac{1}{\sqrt{1-\delta^2}}e^{\delta\omega_n t}\,\text{sen}(\omega_n\sqrt{1-\delta^2}\,t+\phi)\right]$$

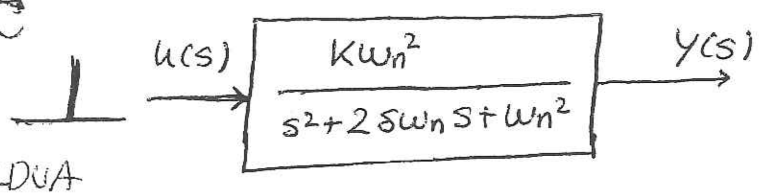
⊛ Poloak: zati erreal positiboa $s=-\delta\omega_n\pm\omega_n\sqrt{\delta^2-1}$





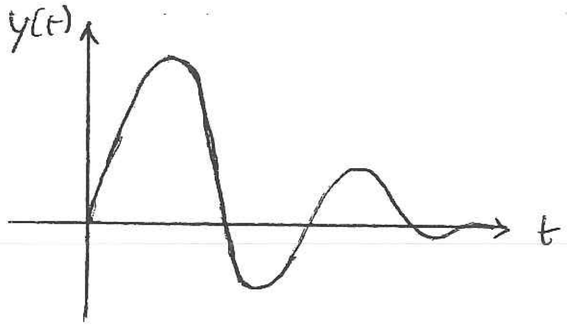
SISTEMA EZ-EGONKORRA

- INPULTSU ERANTZUNA



1) $\boxed{0 < \delta < 1}$ → AZPIMOTELDUA

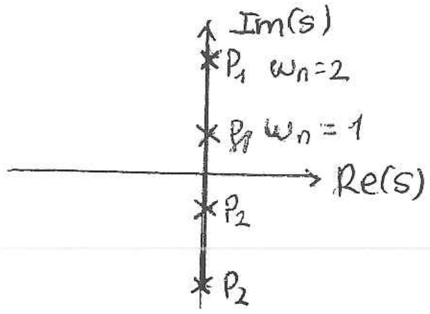
$$y(t) = \frac{Ku\omega_n}{\sqrt{1-\delta^2}} e^{-\delta\omega_n t} \cdot \text{sen}\left(\omega_n\sqrt{1-\delta^2}\right)t$$



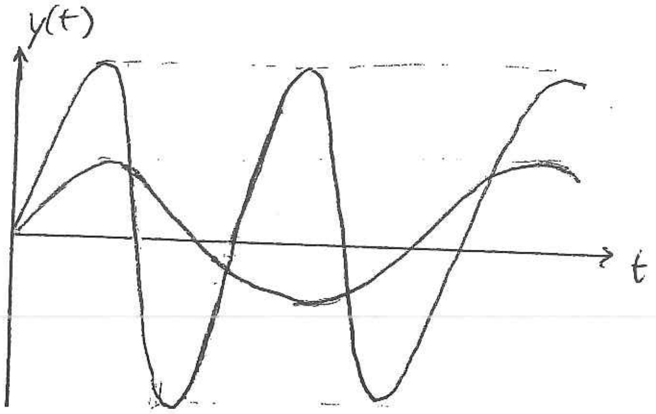
2) $\boxed{\delta = 0}$ → KRITIKOKI EGONKORRA

$$Y(s) = Ku\frac{\omega_n^2}{s^2+\omega_n^2} \quad (u(s)=1) \Longrightarrow y(t) = Ku\omega_n \,\text{sen}(\omega_n t)$$

⊛ Poloa irudikariak: $s_{1,2} = \pm j\omega_n$



⇒

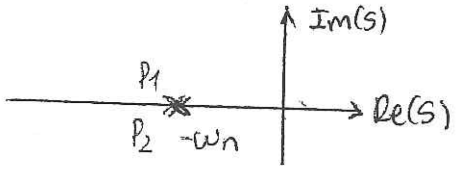


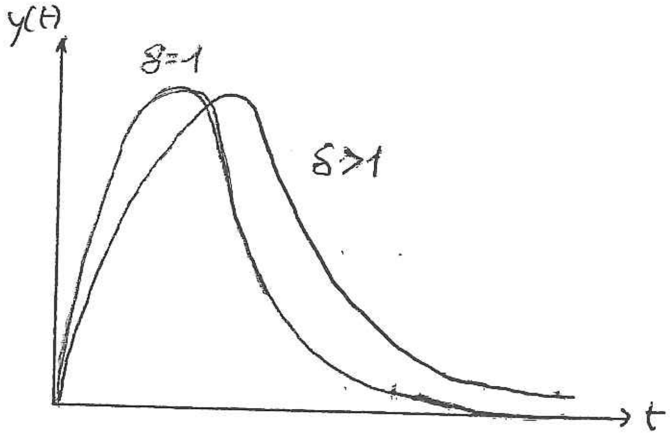
$\omega_n\uparrow \rightarrow$ { Anplitudea ↑
Maiztasuna ↑↓?

3) $\boxed{\delta = 1}$ → KRITIKOKI MOTELDUA

$$y(t) = Ku\omega_n^2 t\, e^{-\omega_n t}$$

⊛ Polo bikoitza: $s_{1,2} = -\omega_n$

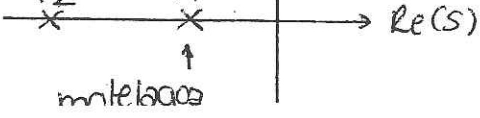




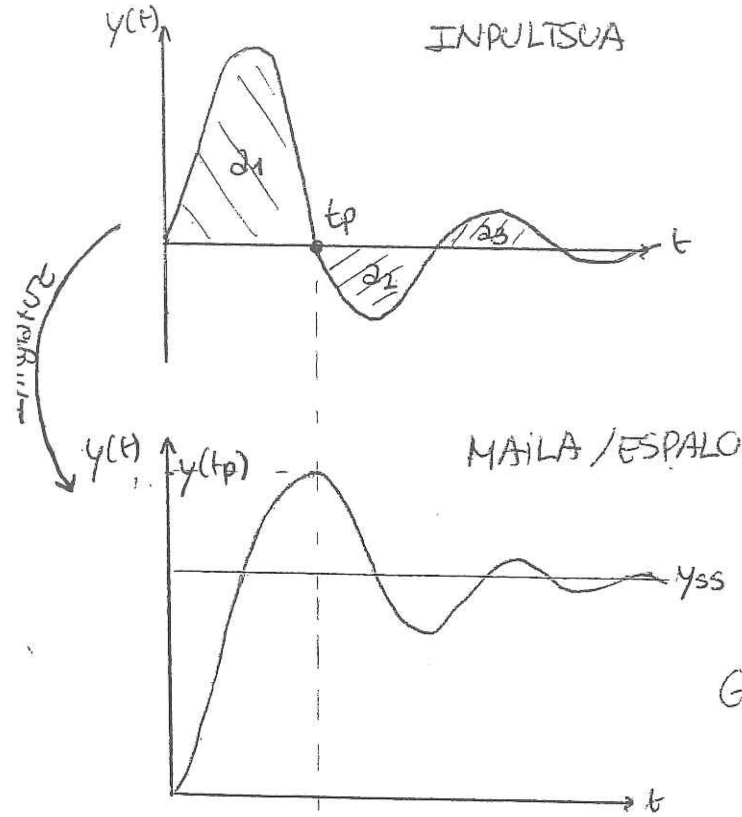
4) $\boxed{\delta > 1}$ → GAINMOTELDUA

$$y(t) = Ku\left(\frac{\omega_n}{2\sqrt{1-\delta^2}} e^{P_1 t} - \frac{\omega_n}{2\sqrt{1-\delta^2}} e^{P_2 t}\right)$$

⊛ Polo erreal bikoitza: $P_1 = -\delta\omega_n + \omega_n\sqrt{\delta^2-1}$
$P_2 = -\delta\omega_n - \omega_n\sqrt{\delta^2-1}$



- IDENTIFIKAZIO ESPERIMENTALA INPULTSU-ERANTZUNEAN OINARRITUTA



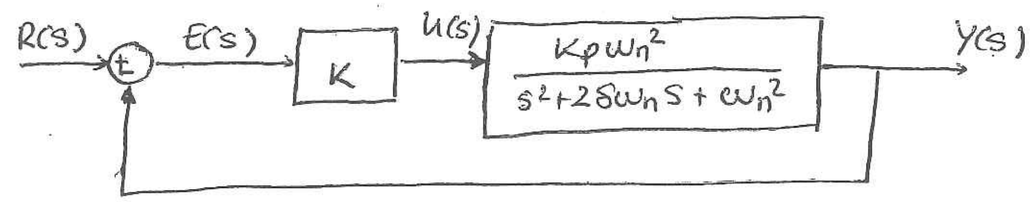
$$\left\| \begin{array}{l} Y_{INPULTSU}(s) = G(s) \\ Y_{MAILA}(s) = \dfrac{G(s)}{s} \end{array} \right.$$

$$\boxed{Y_{INPULTSU}(s) = s\,Y_{MAILA}(s)}$$

$$\left.\begin{array}{l} \underline{\underline{y_{maila}(t_p)}} = \int_0^{t_p} y_{inpultsu}(t)\,dt = \underline{\underline{a_1}} \\ \underline{\underline{y_{maila}(\infty)}} = y_{ss} = \int_0^{\infty} y_{inpultsu}(t)\,dt = \\ \qquad = \underline{\underline{a_1 - a_2 + a_3}} = \underline{\underline{y_{ss}}} \end{array}\right\}$$

$$G(s) \left\| \begin{array}{l} K = \dfrac{\Delta y}{\Delta u} = \dfrac{y_{ss}}{\Delta u} \rightarrow \quad y_{ss} = K \cdot u \\ M_p = \dfrac{y(t_p) - y_{ss}}{y_{ss}} \longrightarrow \delta \\ t_p = \dfrac{\pi}{\omega_n\sqrt{1-\delta^2}} \longrightarrow \omega_n \end{array} \right.$$

- BERRELIKADURAREN ERAGINA

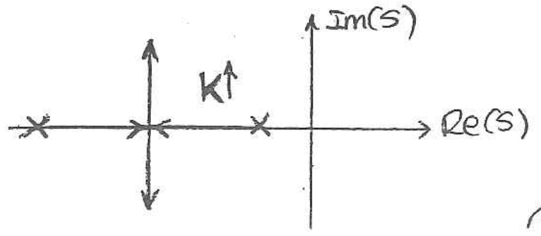


$$G_{BC}(s) = \frac{K K_p \omega_n^2}{s^2 + 2\delta\omega_n s + \omega_n^2 + K K_p \omega_n^2}$$

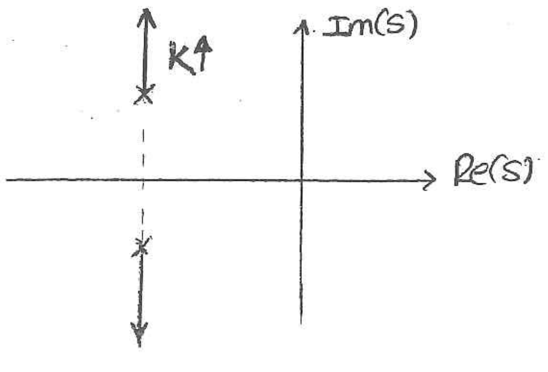
- Ekuazio Karakteristikoa: $1 + K\,G(s) = 0$
- Poloak: $s = -\delta\omega_n \pm \omega_n\sqrt{\delta^2 - 1 - K \cdot K_p}$ poloak mugitu.

Ez dira G(s)-ren poloak aldatzen, $\frac{Y(s)}{R(s)}$-renak baizik.

- Begizta irekiko sistema gainmoteldua: Sistema azkarragoa eta k-ren balio batetik aurrera → azpimoteldua.



- Begizta itxiko sistema gainmoteldua:

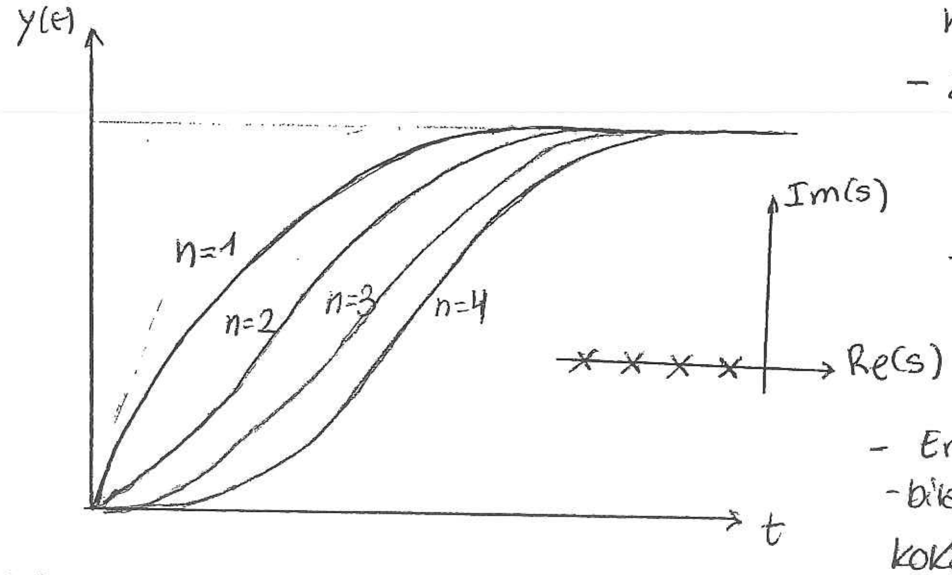


GOI-ORDENEKO SISTEMAK

- ESPALOI/MAILA-ERANTZUNA

Sistema batean, poloek erantzunean eragina dute. Polo erreal bakoitzak erantzun totalean eragin bat du, atzerapen txiki bat gehituz.

Jatorritik urrutiago → zati esponentzial handiagoa → eragin txikiagoa

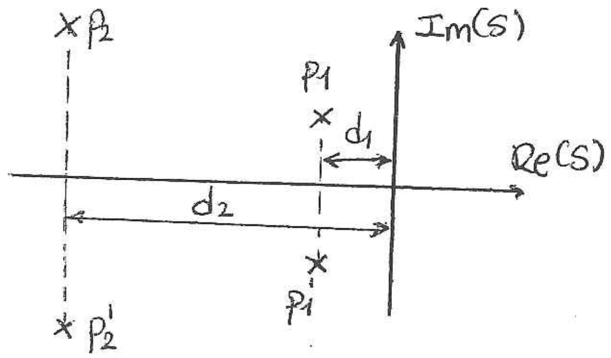


n= polo kopurua

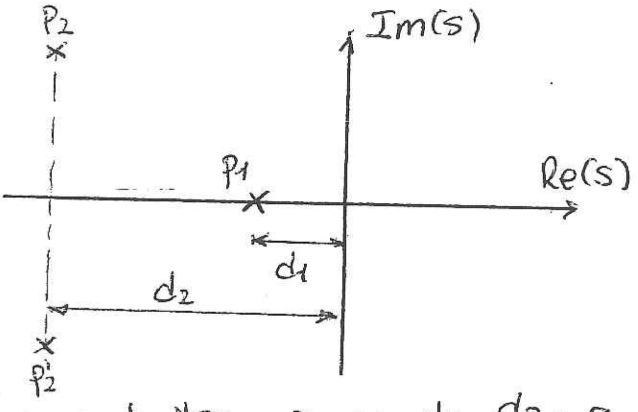
- Zenbat eta polo gehiago izan, motelagoa izango da sistema.
- Jatorritik hurbilago → τ↓
- Erantzunaren denbora-bilakaera polo dominatzaileen kokapenaren araberakoa izango da.

* POLO DOMINATZAILEAK

Polo bat zenbat eta jatorritik hurbilago egon, hainbat eta dominatzaileagoa izango da, betiere beste poloekiko kokapena aztertuz.



p_1, p_1' dominatzaileak izango dira $\frac{d_2}{d_1} > 5$ betetzen bada.



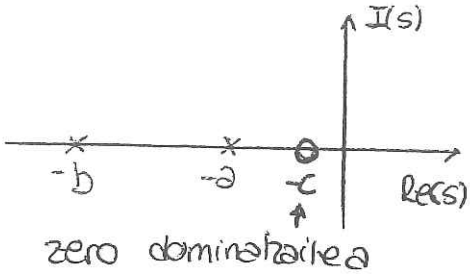
p_1 dominatzailea izango da $\frac{d_2}{d_1} > 5$ betetzen bada.

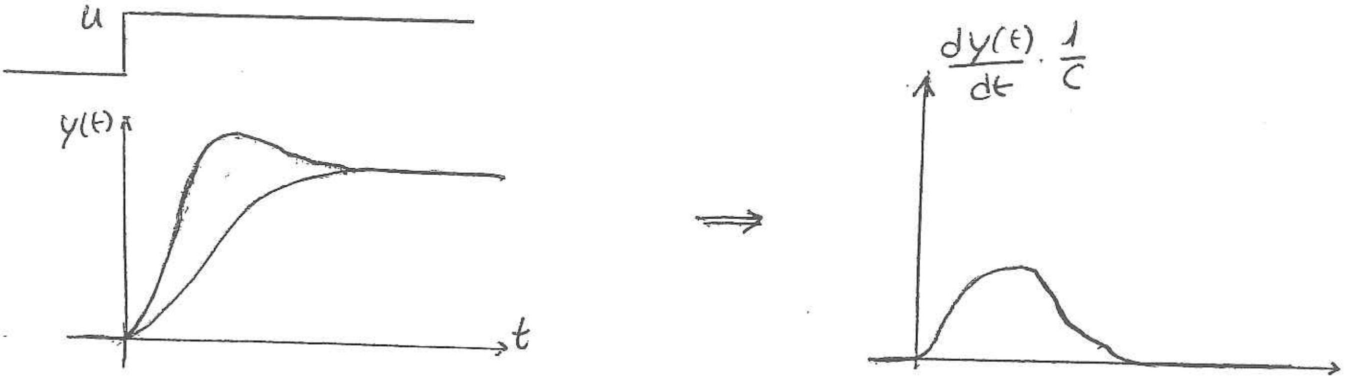
- Goi-ordeneko sistema baten ordena murrizteko dominatzen ez duten poloen eragina mespretxatuko dugu. (polo azkarrenak kendu)
- Poloak kentzerakoan IRABAZPEN ESTATIKOA EZ DA ALDATZEN !!
- Interpretazio fisikoa: Dominatzaileak ez diren poloak kentzerakoan, beraien eragina "bat-batekoa" dela ematen da.

* ZEROEN ERAGINA

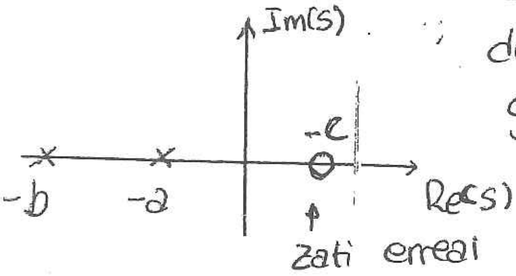
→ ZERO EGONKORRA : $\boxed{c>0}$ bada, erantzuna aurreratuko da. Erantzunak oszilaziorik ez baditu, zeroak ez du oszilaziorik sortuko.

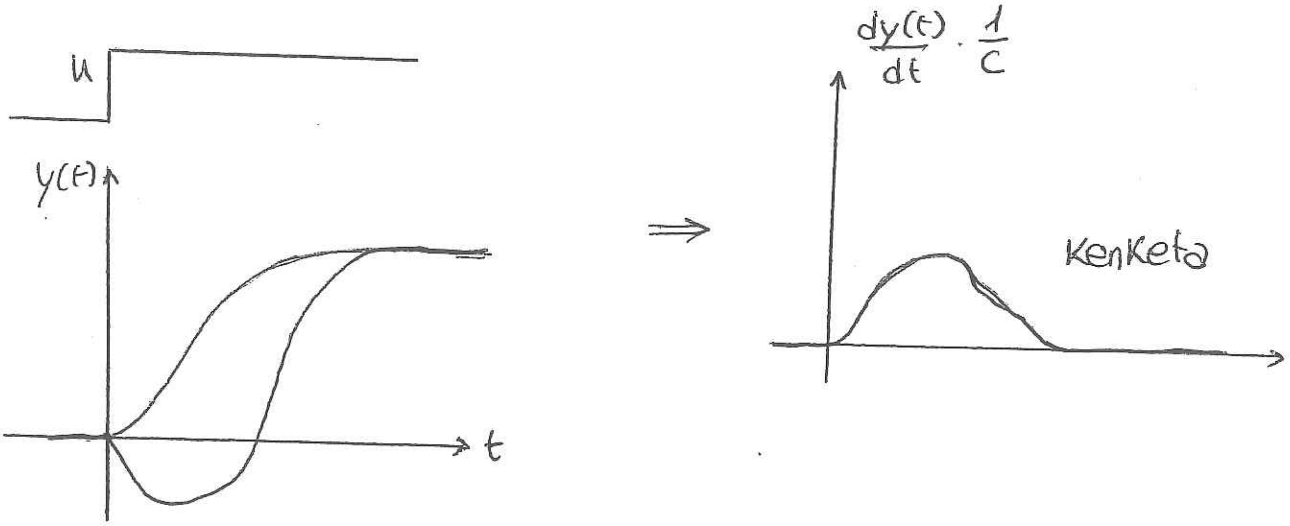
Zeroa dominatzailea bada, gaindiketa sortu dezake.





→ ZERO EZEGONKORRA : $\boxed{c<0}$ dugunean erantzunak alderantziz jokatuko du hasieran (lehenengo beherantz egingo du eta gero gorantz)





→ Eragina

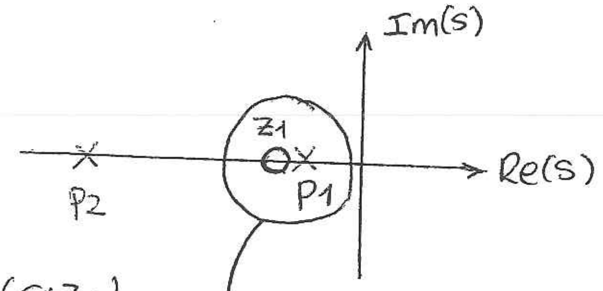
- zero batek erantzuna aurreratzen du.
- zenbat eta jatorritik urrunago egon, eragina txikiagoa izango da.
- Erantzunaren forma polo dominatzaileen araberakoa da, EZ ZEROENA. ZEROEK EZ DUTE FORMA ALDATZEN
- zero bat polo batetik oso hurbil egotean, elkar anulatzen dute.

* SISTEMA BATEN ORDENAREN MURRIKETA

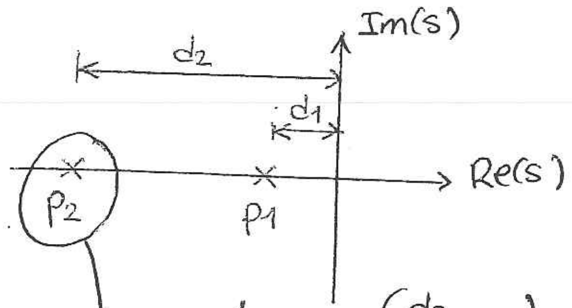
→ Murrizketa egiteko

- Hurbil dauden polo eta zeroak bata bestearekin baliogabetzen dira, irabazpen estatikoa mantenduz.
- Dominatzaileak ez diren poloak, mespreza daitezke.

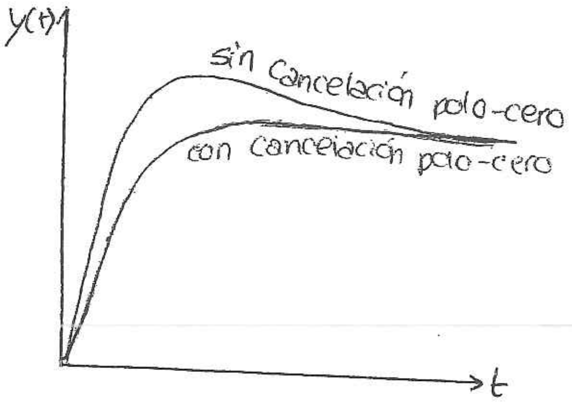
⊛ Lehenengo, polo/zero baliogabeteak aztertzen dira, eta gero nagusitasuna. Bi kasoeta K mantendu behar da.

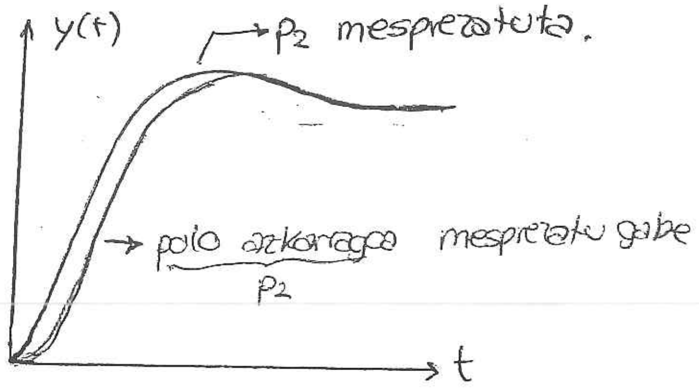


$$\frac{(s+z_1)}{(s+p_1)(s+p_2)} \Rightarrow \frac{z_1}{p_1(s+p_2)}$$



$$\frac{1}{(s+p_1)(s+p_2)} \rightarrow \frac{1}{(s+p_1)p_2} \quad \left(\frac{d_2}{d_1} > 5\right)$$





- IDENTIFIKAZIO ESPERIMENTALA

4.GAIA

FROGA SEINALEAK

- INPULTSU - SARRERA $u(s)=u$
- ESPALOI - SARRERA $u(s)=\frac{u}{s}$
- Abiadura
  ARRAPALA - SARRERA $u(s)=\frac{u}{s^2}$
- Azelerazio
  PARABOLA - SARRERA $u(s)=\frac{u}{s^3}$

⇒ LEHEN ORDENEKO SISTEMAK

Denbora erantzuna

$$G(s)=\frac{y(s)}{u(s)}=\frac{K}{\tau s+1} \quad \text{non} \quad \begin{cases} k=\frac{\Delta y}{\Delta u} \\ \text{Poloa: } \tau s+1=0 \text{ denean} \end{cases}$$

- INPULTSU erantzuna: $y(t)=\frac{K\cdot u}{\tau}e^{-t/\tau}$
- ESPALOI erantzuna: $y(t)=Ku\left(1-e^{-t/\tau}\right)$ $(\tau>0)$

  $y(\tau)=0{,}63Ku=y_{63}=\bar{y}+0{,}63\cdot\Delta y$.

- Egonkortze denbora (t_{ss}):

  $\boxed{\%5 \text{ irizpidea}} \Rightarrow t_{ss}(\%95)=0{,}95Ku \rightarrow t=3\tau$

  $\boxed{\%2 \text{ irizpidea}} \Rightarrow t_{ss}(\%98)=0{,}98Ku \longrightarrow t=4\tau$

2. Ariketa (28. diapositiba)

t = 60s egonkortzeko

%2 irizpidea

$t_{ss}(\%98) = 60 = 4\tau \Rightarrow \tau = 15s$

$K = \frac{\Delta y}{\Delta u} = 1 \rightarrow$ egonkortzen denean gorputzaren eta termometroaren tenperaturak berdinak dira.

$$\left[G(s) = \frac{1}{15s+1} \right]$$

4. Ariketa (④.15)

$y(t) = 5 - 5e^{-t}$ inpultsu erantzuna.

Inpultsu sarrera: $U(s) = u =$

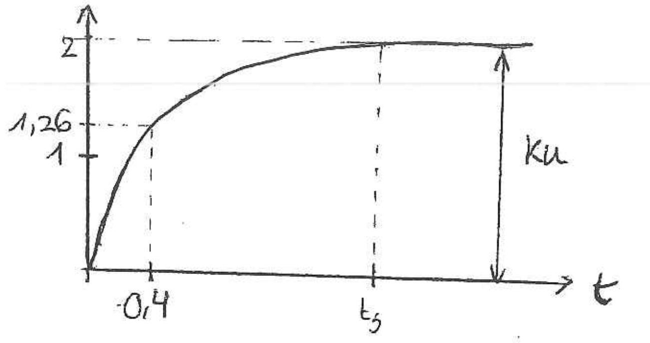
$y(t) = 5 - 5e^{-t}$

$$\downarrow \mathcal{L} \quad Y(s) = \frac{5}{s} - \frac{5}{s+1} = \frac{\cancel{5s} + 5 - \cancel{5s}}{s(s+1)} = \frac{5}{s^2+s}$$

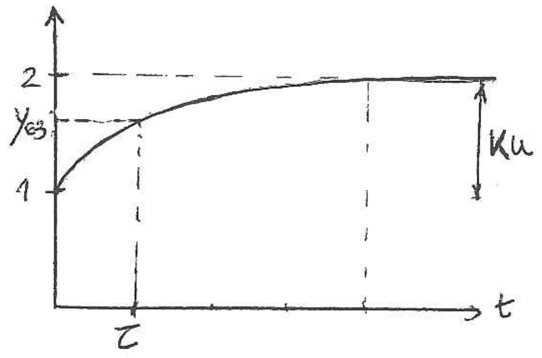
$$G(s) = \frac{Y(s)}{U(s)} = \frac{5}{u(s^2+s)}$$

3. Ariketa

- $$G(s) = \frac{5}{s+2,5} = \frac{5}{2,5\left(\frac{1}{2,5}s+1\right)} = \frac{2}{\frac{2}{5}s+1} \rightarrow \begin{cases} \tau = 2/5 = 0,4 \\ K = 2 \end{cases}$$

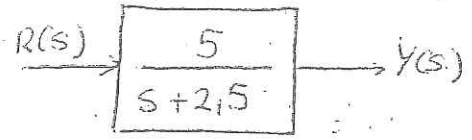


- $$G(s) = \frac{s+2}{s+1} = \frac{s+1}{s+1} + \frac{1}{s+1} = 1 + \frac{1}{s+1} \rightarrow \begin{cases} \tau = 1 \\ K = 1 \end{cases}$$



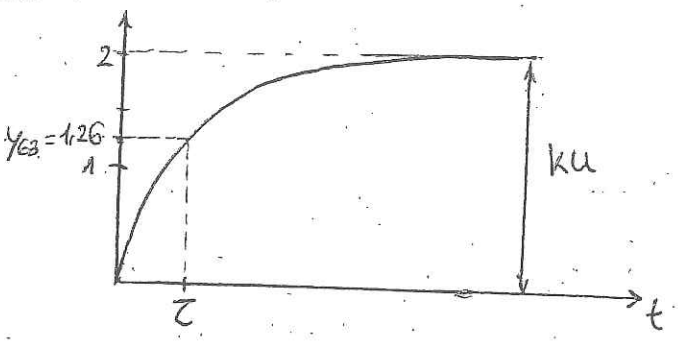
ARIKETAK

①



$$G(s)=\frac{Y(s)}{R(s)}=\frac{5}{s+2,5}=\frac{2}{0,4s+1}$$

$$\left\{\begin{array}{l}K=2\\ \tau=0,4\end{array}\right\}$$



$$y(t)-ku\left(1-e^{-t/\tau}\right)$$

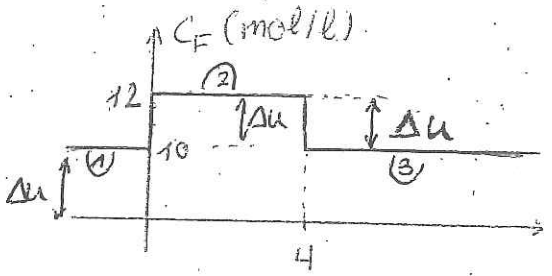
$$y(t)=2\left(1-e^{-2,5t}\right)$$

2)

$$G(s):\frac{C'(s)}{C'_F(s)}=\frac{0,3}{4s+1}$$

irteea - kontzentrazioa

elikadura kontzentrazioa



$c'(t)$?

$0\leq t\leq 4$ $\quad\Delta u=2$

$t>4$ $\quad\Delta u=-2$

$$K=0,3=\frac{\Delta y}{\Delta u}\Rightarrow\Delta y=0,3\cdot 2=0,6$$

$\tau=4$

espaloi erantzuna: $y(t)=ku(1-e^{-t/\tau})$

$$c'(t)=\underbrace{ku\left(1-e^{-t/\tau}\right)}_{①}+\underbrace{ku\left(1-e^{-t/\tau}\right)}_{②}+\underbrace{ku\left(1-e^{-t/\tau}\right)}_{③}=$$

$$=0,3\cdot 10+0,3\cdot 2\left(1-e^{-t/4}\right)+0,3\cdot(-2)\left(1-e^{-(t-4)/4}\right)\delta(t-4)=$$

$$=3+0,6\left(1-e^{-0,25t}\right)-0,6\left(1-e^{-0,25(t-4)}\right)\delta(t-4)\quad\text{non}\quad\delta(t-4)=\left\{\begin{array}{ll}0 & t\ 4\\ 1 & t\geq 4\end{array}\right.$$

3)

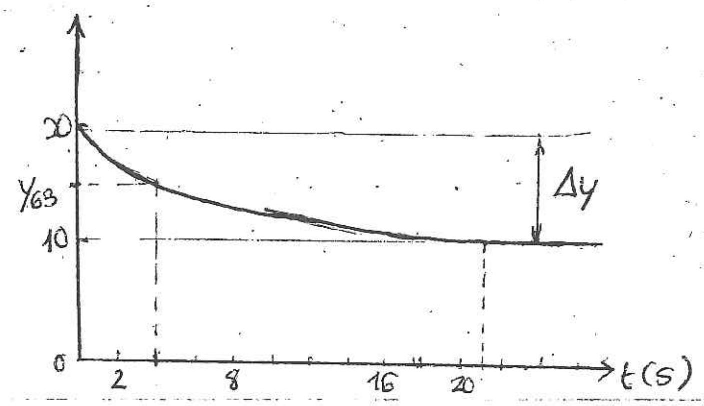
$T_{giro}=20°C$

$T_{ontzi}=10°C$

a) Sarrera-aldagaia? Irteera-aldagaia?

b) Transferentzia funtzioa?

Taulako datuekin grafiko bat irudikatuko dugu, gure erantzuna denborarekiko rango dena.



$\Delta y = -10$

$\Delta u = 10$ $\rightarrow$ $K = \frac{\Delta y}{\Delta u} = 1$

$y_{63} = 20 + 0,63\,(-10) = 13,7$

$T(°C) = 13,7 \rightarrow t = 4s = \tau$

$G(s) = \frac{-1}{4s+1}$

Sarrera : −10°C-ko espoloi-sarrera.

irteera: T(s)

④ $t=6 \rightarrow 3\ kg/m^3$-ko aldaketa dago. $\Rightarrow$ $t=6$ s-tik aurrera 1. mailakoa.

$G(s) = \frac{C_m}{C}$?

$K = \frac{\Delta y}{\Delta u} =$

Iruditik: $\Delta y = 4-1 = 3$

$\Delta u = 3$ $\longrightarrow K = 1$

$\boxed{G(s) = \frac{1}{s+1}}$

$y(t_{63}) = \underbrace{\Delta y}_{3} \cdot 0,63 + 1 = 2,89 \xrightarrow{irudítik} t = 7$

$t=6$ s-tik aurrera daukagunez 1. mailako sistema: $\tau = 7-6 = 1$

⑤

$\Longrightarrow$ sarreran maila unitarioa $\longrightarrow$ B grafikoa da egokiena.

a)

$\Delta y = 3-0 = 3$

$\Delta u = 1$ (unitarioa) $\Big\|$ $\left[K = \frac{\Delta y}{\Delta u} = \frac{3}{1} = 3\right]$

$G(s) = \frac{3}{0,25s+1}$

$y(t_{63}) = 0 + 0,63 \cdot \Delta y = 0,63 \cdot 3 = 1,89$ $\rightarrow \left[t_{63} = \tau\ \ 0,\ \ s\right]$

b)

$$G'(s) = \frac{\frac{K}{\tau s+1}}{1+\frac{1}{2}\cdot\frac{K}{\tau s+1}} = \frac{K}{\tau s+1+0,5K} = \frac{\frac{K}{1+0,5K}}{\frac{\tau}{1+0,5K}s+1} = \frac{K'}{\tau' s+1}$$

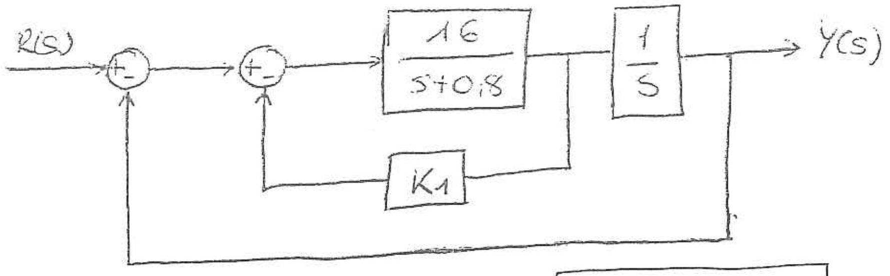
non $\begin{cases} K' = \frac{k}{1+0,} \\ \tau' = \frac{\tau}{1+} \end{cases}$

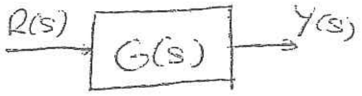
$K' = \frac{3}{1+0,5\cdot 3} = 1,2$; $\tau' = \frac{0,}{1+0,5\cdot 3} = 0,24$ $\qquad G'(s) = \frac{1,2}{0,6s+1}$

$y'(t) = 1,2\left(1-e^{-t/0,24}\right)$ grafikoa falta da

- BIGARREN ORDENEKO SISTEMAK -

Ariketa

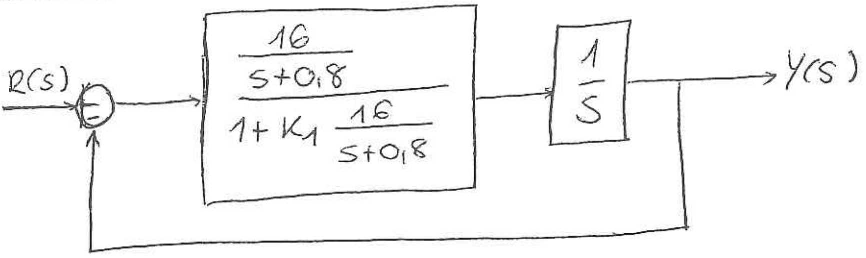


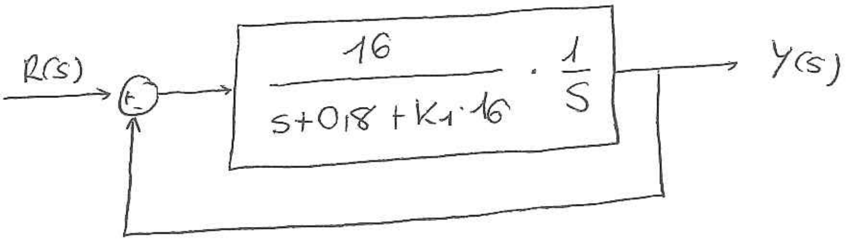


K_1? $\delta=0{,}5$

K_p? t_p?

$R(s)=\frac{1}{s}$





$$G(s)=\frac{Y(s)}{R(s)}$$

$$G(s)=\frac{\frac{16}{s^2+0{,}8s+K_1 16s}}{1+\frac{16}{s^2+0{,}8s+16K_1s}}=\frac{16}{s^2+(0{,}8+16K_1)s+16}=\frac{K\cdot\omega_n^2}{s^2+2\delta\omega_n s+}$$

$\delta=0{,}5$ (Datua)

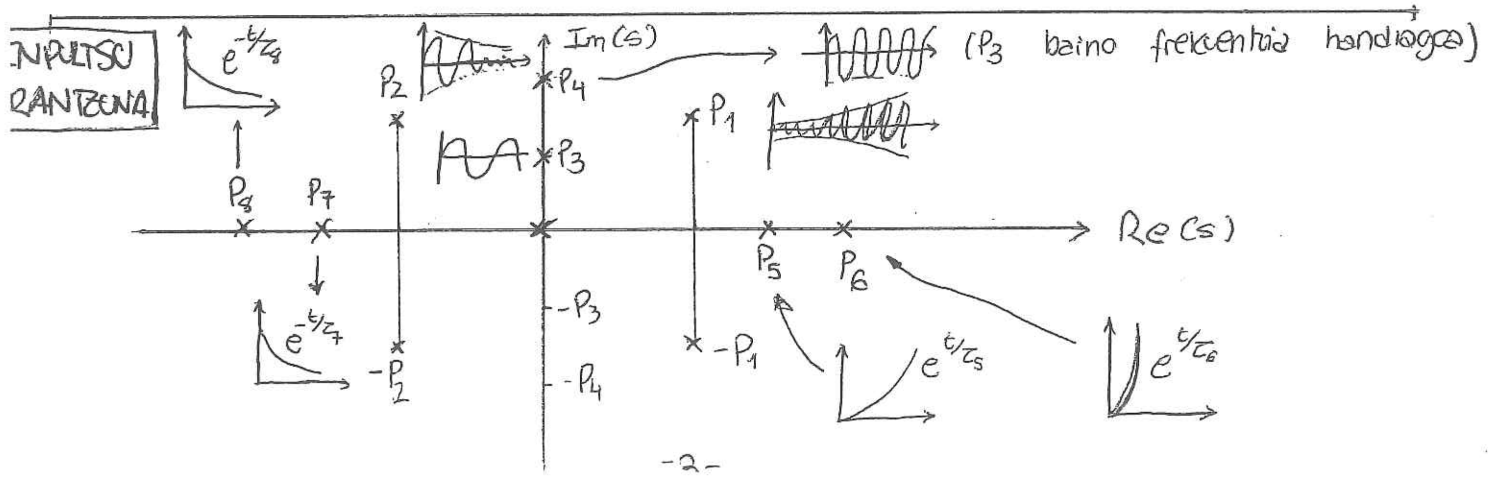
$2\delta\omega_n=0{,}8+16K_1 \rightarrow \overbrace{2\cdot0{,}5\cdot4}^{1}=0{,}8+16K_1 \rightarrow \boxed{K_1=0{,}2}$

$16=\omega_n^2 \longrightarrow \omega_n=4$ rad/s

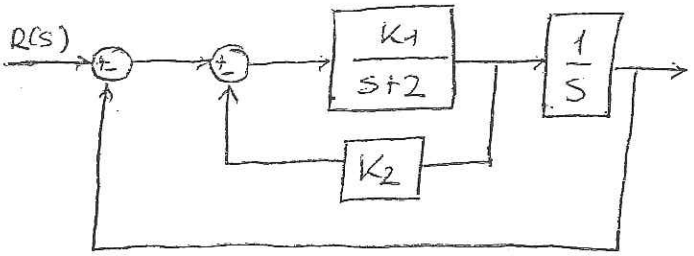
↳ $\boxed{K=1}$ ↱ $\boxed{t_p=0{,}9s}$

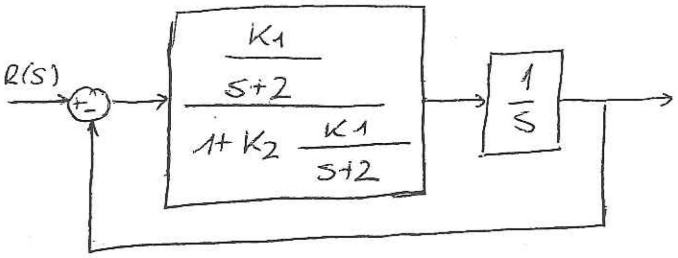
$$M_p=e^{\frac{-\pi\delta}{\sqrt{1-\delta^2}}}=e^{-\frac{\pi\cdot0{,}5}{\sqrt{1-0{,}5^2}}}=0{,}16=\frac{y(t_p)-y_{ss}}{y_{ss}}\ ;\ \boxed{y(t_p)=\frac{\pi}{\underbrace{\omega_n}_{4}\sqrt{1-\underbrace{\delta^2}_{0{,}5}}}}=\boxed{0{,}9s}$$

$$t_s(\%5)=\frac{3}{\delta\omega_n}=1{,}5s\ ,\quad t_s(\%2)=\frac{4}{\delta\omega_n}=2s$$



Ariketa:





K_1 eta K_2? ($\delta=0{,}7$, $\omega_n=4$ rad/s)

$$G(s)=\frac{\frac{K_1}{(s+2+K_2K_1)s}}{1+\frac{K_1}{s^2+2s+K_2K_1s}}=\frac{K_1}{s^2+(2+K_2K_1)s+K_1}$$

$K_1=K\cdot\omega_n$

$K_1=\omega_n^2$

$2+K_2K_1=2\delta\omega_n$

$$2+K_2K_1=1{,}4\,\omega_n \Longrightarrow 2+\omega_n^2K_2-1{,}4\,\omega_n=0$$

$$2+16K_2-1{,}4\cdot4=0 \longrightarrow \boxed{K_2=0{,}0225}$$

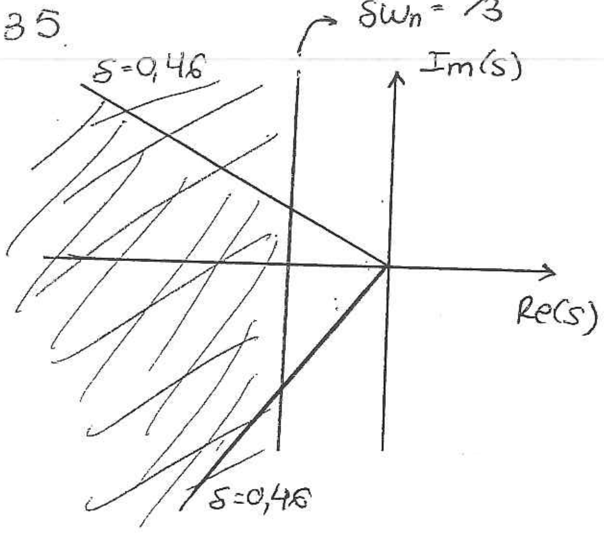
$\boxed{K_1=16}$ $\boxed{K=1}$

Adib: Marraztu s planoko zonaldea, espaloi erantzunaren ezaugarrik haiek noleko. $M_p\leq\%20$, $t_s(\%2)\leq3s$.

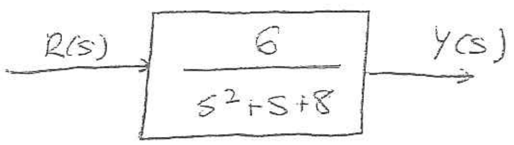
$$M_p\leq0{,}2=e^{-\pi\delta/\sqrt{1-\delta^2}}$$

$$\delta=\sqrt{\frac{\ln^2M_p}{\pi^2+\ln^2M_p}}\geq0{,}46$$

$$t_s(\%2)=\frac{4}{\delta\omega_n}\leq3s \longrightarrow \delta\omega_n\geq\frac{4}{3}$$



⑦



$$G(s) = \frac{6}{s^2+s+8} = \frac{K \cdot \omega_n^2}{s^2+2\delta\omega_n s+\omega_n^2}$$

$$\begin{cases} \omega_n^2 = 8 \Longrightarrow \boxed{\omega_n = \sqrt{8} = 2\sqrt{2} = 2{,}83} \\ 2\delta\omega_n = 1 \Rightarrow 2\delta \cdot 2\sqrt{3} = 1 \Rightarrow \boxed{\delta = \frac{1}{4\sqrt{3}} = 0{,}17} \end{cases} \qquad \boxed{0 < \delta < 1}$$

AZPIMOTELDUA

$$K \cdot \omega_n^2 = 6 \longrightarrow \boxed{K = \frac{6}{8} = \frac{3}{4} = 0{,}75}$$

$$y_{ss} = K \cdot u = 0{,}75 \cdot \underset{\substack{\uparrow \\ \text{espaloi} \\ \text{unitarioa}}}{1} = 0{,}75$$

$$\theta = \arccos\delta = \operatorname{arctg}\frac{\sqrt{1-\delta^2}}{\delta} = 1{,}4 \text{ rad}$$

$$\boxed{t_r = \frac{\pi-\theta}{\omega_d} = \frac{\pi-\theta}{\omega_n \cdot \sqrt{1-\delta^2}} = \frac{\pi-1{,}4}{\sqrt{8}\cdot\sqrt{1-0{,}17^2}} = 0{,}62s}$$

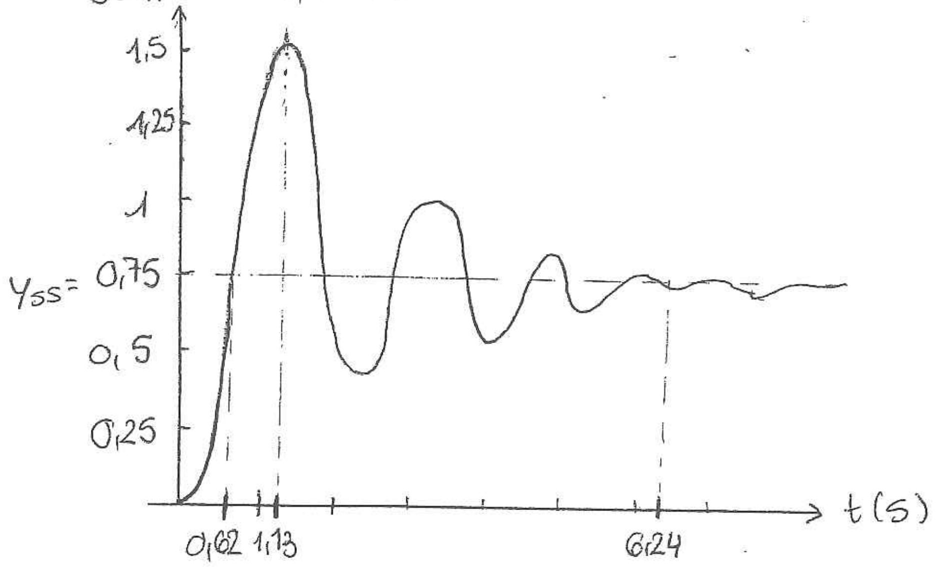
$$\boxed{t_p = \frac{\pi}{\omega_d} = \frac{\pi}{\sqrt{8}\cdot\sqrt{1-0{,}17^2}} = 1{,}13s}$$

$$\boxed{M_p = e^{-\frac{\pi\delta}{\sqrt{1-\delta^2}}} = 0{,}58 \Rightarrow M_p = \% 58}$$

$$M_p = \frac{y(t_p)-y_{ss}}{y_{ss}} \Rightarrow y(t_p) = \frac{0{,}58}{0{,}75} + 0{,}75$$

$$y(t_p) = 1{,}52$$

$$\boxed{t_s = \frac{3}{\delta\omega_n} = \frac{3}{0{,}17\cdot\sqrt{8}} = 6{,}24s \simeq 6s} \quad (\%5)$$

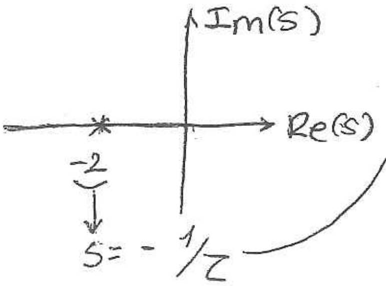


⑧

$$G_{inpultsu} = s \cdot G_{espaloi}$$

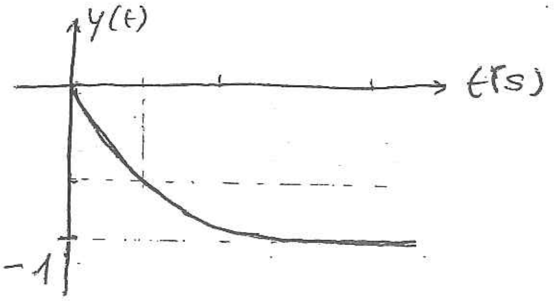
a) $G_1(s) = \dfrac{-s}{s+2} \Rightarrow$ espaloi $\longrightarrow$ inputsu aldaketa

Inpultsu erantzuna daukagu: Poloak $\rightarrow$ $s+2=0 \longrightarrow s = -2$



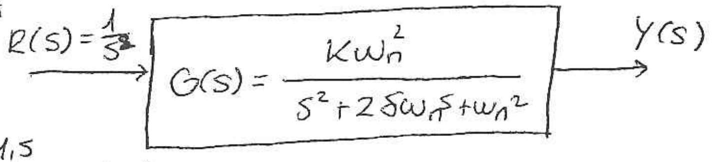
$K = -1$

$Z = 1/2$



㉗

Anapola unitarioa ezarhen da.



G(s)?

$$M_p=\frac{\overbrace{y(t_p)-y_{ss}}^{\text{grafikotik}}}{y_{63}}=\frac{1{,}65-1{,}5}{1{,}5}=0{,}1$$

$$\delta=\sqrt{\frac{\ln^2 M_p}{\pi^2+\ln^2 M_p}}=0{,}6$$

$$G(s)=\frac{K\cdot 2{,}1^2}{s^2+2\cdot 0{,}6\cdot 2{,}1s+2{,}1^2}$$

$$t_p=\frac{\pi}{\omega_n\sqrt{1-\delta^2}}=1{,}5\longrightarrow \omega_n=2{,}1\ \text{rad}/s$$

---

⑥

Sarrean: 2 anpiltudeko maila $\Rightarrow \Delta u=2$
lehen ordenako sistema.

Grafikotik: $\Delta y=0-3=-3$

$$K=\frac{\Delta y}{\Delta u}=-\frac{3}{2}\qquad y_{63}=3+0{,}63\cdot(-3)=1{,}11\simeq 1\rightarrow t_{63}=\tau=5\,s$$

$$G(s)=\frac{K}{\tau s+1}=\frac{-3/2}{5s+1}$$

Esaten digute $\overset{\text{escaloi}}{\text{maila}}$ sarrera dela, baina grafika inpultsu erantzunarena da. $\Rightarrow$

$\rightarrow$ eskaloi sarrera: $u_1(s)=\frac{u}{s}$
inpultsu sarrera: $u_2(s)=u$

|| $u_1(s)\cdot s=u_2(s)$ (eskaloi sarrera inpultsu sarrera bihurtzeko)

$$y_{ss}=3\ (\text{grafikotik})\Rightarrow y_{ss}=\frac{Ku}{\tau}=3=\frac{\Delta y'}{\tau}$$

$$\Delta y'=3\cdot 5\Rightarrow K'=\frac{3\cdot 5}{2}=7{,}5$$

$$\boxed{G'(s)=\frac{K'}{\tau s+1}\cdot s=\frac{7{,}5s}{5s+1}}$$

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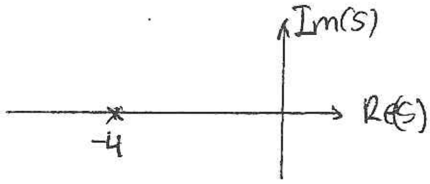
⑧

b) $G_2(s)=\frac{s-2}{s+4}$ $\qquad s+4=0\rightarrow s=-4$

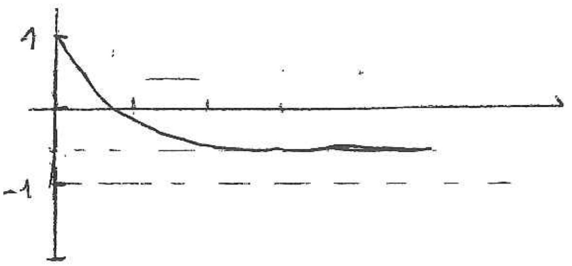


$$s=-\frac{1}{\tau}\Rightarrow +4\tau=+1\Rightarrow \tau=1/4$$

$$G_2(s)=\frac{s+4}{s+4}-\frac{\quad}{s+4}=1-\frac{6}{s+4}\Rightarrow 1-\frac{\cancel{6}3}{(\frac{1}{4}s+\ )\cancel{4}2}=1-\frac{1{,}5}{\frac{1}{4}s+1}$$

$$K=-1{,}5=\frac{\Delta y}{\Delta u}=\frac{\Delta y}{1}\Rightarrow \Delta y=-1{,}5$$

$$y_{63}=1-0{,}63\cdot 1{,}5=0{,}055$$



c) $G_3(s) = \frac{1,25}{s^2+s+2,5} \Rightarrow \begin{cases} K \cdot \omega_n^2 = 1,25 \rightarrow K = 0,5 \\ 2\delta\omega_n = 1 \rightarrow \delta = 1/\sqrt{10} = 0,316 \\ \omega_n^2 = 2,5 \longrightarrow \omega_n = 1,58 \text{ rad/s} \end{cases}$

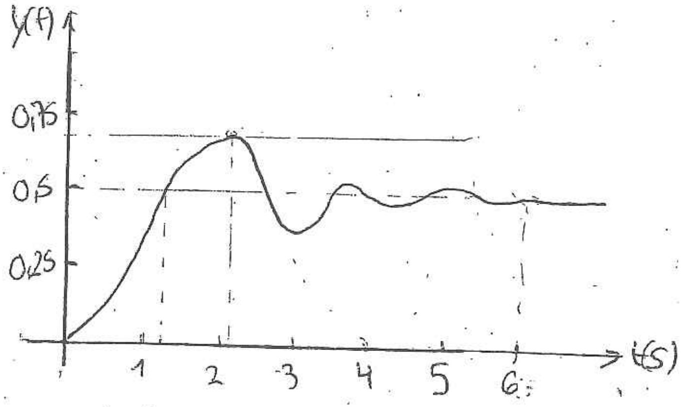
$\theta = \arccos \delta = 1,249$ rad

$y_{ss} = k \cdot u = 0,5$

$t_p = \frac{\pi}{\omega_d} = \frac{\pi}{\sqrt{2,5} \cdot \sqrt{1-(\frac{1}{\sqrt{10}})^2}} = \frac{2}{3}\pi = 2,09s$

$M_p = e^{-\frac{\pi\delta}{\sqrt{1-\delta^2}}} = 0,3509 = \% 35,09$

$M_p = \frac{y(t_p) - y_{ss}}{y_{ss}} \Rightarrow 0,3509 \cdot 0,5 + 0,5 = y(t_p) = 0,675$

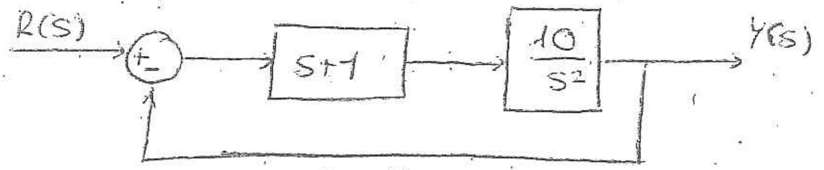


$t_s = \frac{3}{\delta\omega_n} = \frac{3}{\frac{1}{\sqrt{10}} \cdot \sqrt{2,5}} = 6s$

$t_r = \frac{\pi - \theta}{\omega_d} = 1,26s$

$s^2+s+2,5 = 0 \left\langle \begin{matrix} -0,5+3j \\ -0,5-3j \end{matrix} \right.$

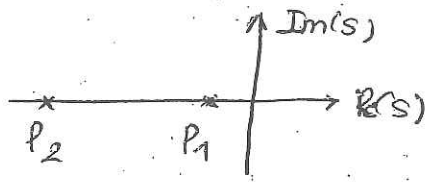
⑩



M_p?

t_p?

t_s?



$$G(s) = \frac{\frac{10(s+1)}{s^2}}{1+\frac{10(s+1)}{s^2}} = \frac{10(s+1)}{s^2+10(s+1)} = \frac{(s+1)10}{s^2+10s+10}$$

$$s^2+10s+10=0 \Rightarrow s = \frac{-10 \pm \sqrt{100-40}}{2} = \frac{-\overset{5}{\cancel{10}} \pm \cancel{2}\sqrt{15}}{\cancel{2}} \left\langle \begin{matrix} -1,127 = p_1 \\ -8,873 = p_2 \end{matrix} \right.$$

$\begin{cases} K\omega_n^2 = 10 \longrightarrow K=1 \\ 2\delta\omega_n = 10 \longrightarrow 2\delta \cdot \sqrt{10} = 10 \Rightarrow \delta = 1,58 > 1 \rightarrow \text{POLO ERREALAK} \Rightarrow \\ \omega_n^2 = 10 \longrightarrow \omega_n = \sqrt{10} \qquad \Rightarrow \text{GAINMOTELDUA} \end{cases}$

$M_p = e^{-\frac{\pi\delta}{\sqrt{1-\delta^2}}} = e^{-\frac{\pi \cdot 1,58}{\sqrt{1-1,58^2}}} = \nexists \Rightarrow$ ezin ditugo formulak erabili balio esanguratsuak ateratzeko.

$$G'(s) = \frac{10(s+1)}{(s+5-\sqrt{15})(s+5+\sqrt{15})} \cdot \frac{1}{s} = \frac{K\omega_n^2}{(s+\delta\omega_n - \omega_n\sqrt{\delta^2-1})(s+\delta\omega_n+\omega_n\sqrt{\delta^2-1})}$$

$$G'(s) = \frac{10(s+1)}{s(s+1,13)(s+8,87)} = \frac{A}{s} + \frac{B}{s+1,93} + \frac{C}{s+8,87} \longrightarrow$$

$$A = \left.\frac{10(s+1)}{(s+1,13)(s+8,87)}\right|_{s=0} = 1$$

$$C = \left.\frac{10(s+1)}{s(s+1,13)}\right|_{s=8,87} = -1,15$$

$$B = \left.\frac{10(s+1)}{s(s+8,87)}\right|_{s=1,13} = 0,15$$

$$\Longrightarrow G'(s) = \frac{1}{s} + \frac{0,15}{s+1,13} - \frac{1,15}{s+8,87}$$

$\mathcal{L}^{-1}$

$$g'(t) = 1 + 0,15e^{-1,13t} - 1,15e^{-8,87t} = c(t)$$

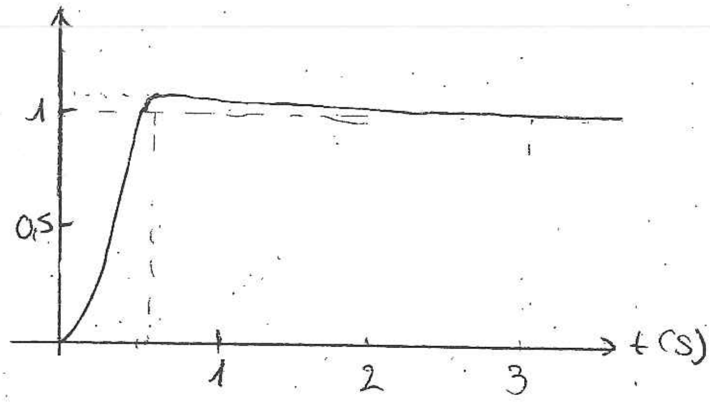
t_p kalkulatzeko $\Longrightarrow c'(t) = \frac{dc(t)}{dt} = 0$

$$-0,17e^{-1,13t} + 10,2e^{-8,87t} = 0 \Longrightarrow \underbrace{t_p = 0,53}$$

$$y_{ss} = K \cdot u = 1$$

$$c(t_p) = y(t_p) = 1 + 0,15e^{-1,13 \cdot 0,53} - 1,15e^{-8,87 \cdot 0,53} = 1,07$$

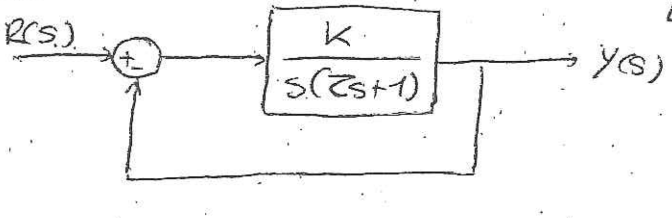
$$1 = y(t_s) = c(t_s) = 1 + 0,15e^{-1,93t_s} - 1,15e^{-8,87 \cdot t_s}$$



$$M_p = \frac{y(t_p) - y_{ss}}{y_{ss}} = \frac{1,07-1}{1} = 0,0$$

$$\underbrace{M_p = \%7}$$

⑪



$\Delta u = 2$

$$G(s) = \frac{\frac{K}{s(Zs+1)}}{1 + \frac{K}{s(Zs+1)}} =$$

$$= \frac{K}{s(Zs+1)+K} = \frac{K}{Zs^2+s+K} =$$

$$= \frac{K_p/Z}{s^2 + \frac{1}{Z}s + \frac{K_p}{Z}}$$

Irudtik:

$B = y(t_p) - y_{ss} = 0,25$ || $y_{t_p} = 1,25$

$A = y_{ss} = 1$

$t_p = 3s$

$$\boxed{M_p = \frac{0,25}{1} = 0,25 = \%25}$$

[illegible] $M_p = e^{-\frac{\pi \delta}{\sqrt{1-\delta^2}}} = 0{,}25$

$$\frac{\pi\delta}{\sqrt{1-\delta^2}} = \ln(0{,}25) \rightarrow \frac{(\pi\delta)^2}{1-\delta^2} = (\ln 0{,}25)^2$$

$$\pi^2\delta^2 = (\ln 0{,}25)^2(1-\delta^2) \Rightarrow \delta^2 = \frac{(\ln(0{,}25))^2}{(\ln 0{,}25)^2+\pi^2} = 0{,}159$$

$$\Rightarrow \delta = 0{,}40$$

$$K = \frac{\Delta y}{\Delta u} = \frac{1}{2} = 0{,}5$$

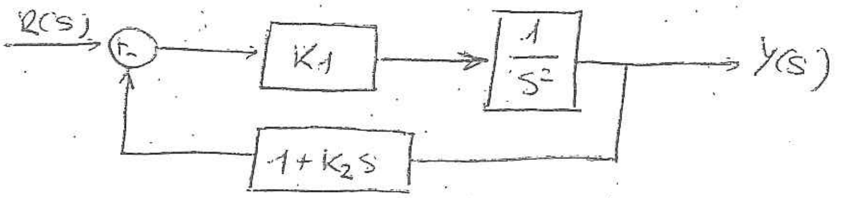
$$t_p = 3 = \frac{\pi}{\omega_n\sqrt{1-\underset{0{,}4^2}{\underline{\delta^2}}}} \Rightarrow \omega_n = \frac{\pi}{3\sqrt{1-0{,}4^2}} = 1{,}14 \text{ rad/s}$$

$$\frac{1}{\tau} = 2\delta\omega_n \rightarrow \tau = \frac{1}{2\cdot 0{,}4\cdot 1{,}14} = 1{,}09\text{s} \rightarrow \boxed{\tau = 1{,}09\text{s}}$$

$$K\omega_n^2 = \frac{K_p}{\tau} \rightarrow K_p = \tau\cdot K\cdot\omega_n^2 = 1{,}09\cdot 0{,}5\cdot 1{,}14$$

$$\boxed{K_p = 1{,}41}$$

⑫



$$G(s) = \frac{\frac{K_1}{s^2}}{1+(1+K_2 s)\frac{K_1}{s^2}} =$$

$M_p = 0{,}25$

$\Delta u = 1$

$t_p = 2s$

$$= \frac{K_1}{s^2 + K_1 + K_1K_2 s} \quad \begin{cases} K_1 = K\omega_n^2 \Rightarrow K = 1 \\ K_1 = \omega_n^2 \\ 2\delta\omega_n = K_1K_2 \end{cases}$$

$$K = \frac{\Delta y}{\Delta u} = 1 \Rightarrow \Delta y = 1$$

$$y_{ss} = 1 \Longrightarrow 0{,}25 = \frac{y(t_p) - y_{ss}}{y_{ss}} \Rightarrow y(t_p) = 1{,}25$$

$$0{,}25 = e^{-\frac{\pi\delta}{\sqrt{1-\delta^2}}} \Rightarrow (\ln 0{,}25)^2(1-\delta^2) = \pi^2\cdot\delta^2$$

$$\delta^2 = \frac{(\ln 0{,}25)^2}{(\ln(0{,}25))^2+\pi^2} = 0{,}159 \rightarrow \delta = 0{,}4$$

$$t_p = 2 = \frac{\pi}{\omega_n\sqrt{1-0{,}4^2}} \Rightarrow \omega_n = 1{,}714 \text{ rad/s} \Rightarrow \boxed{K_1 = 2{,}94}$$

$$K_2 = \frac{2\delta\omega_n}{K_1} = \frac{2\cdot 0{,}4\cdot 1{,}714}{2{,}94} \Rightarrow \boxed{K_2 = 0{,}47}$$

⑬ $$\frac{d^2y(t)}{dt^2} + K\frac{dy(t)}{dt} + 4y(t) = x(t)$$

a) Laplace:

$$s^2 Y(s) + k\,Y(s)\cdot s + 4Y(s) = X(s)$$

$$G(s) = \frac{Y(s)}{X(s)} , \quad Y(s)(s^2+ks+4) = X(s)$$

$$G(s) = \frac{1}{s^2+ks+4} \quad \Big\} \; \underbrace{w_n = 2} \quad K_p = \frac{1}{4} \qquad 2\delta \overset{2}{w_n} = k \;\; \hookrightarrow \underbrace{\delta = \frac{k}{4}}$$

b)

- $-10 \leq K < 0 \Rightarrow \delta < 0 \Rightarrow$ sistema ezegonkorra.
- $0 < K < 4 \Rightarrow 0 < \delta < 1 \Rightarrow$ sistema azpimoteldua
- $K = 0 \rightarrow \delta = 0 \rightarrow$ sistema kritikoki egonkorra
- $k = 4 \rightarrow \delta = 1 \Rightarrow$ sistema kritikoki moteldua
- $4 < K \leq 10 \Rightarrow \delta > 1 \Rightarrow$ sistema gainmoteldua

⑭ $$G(s) = \frac{Y(s)}{X(s)} = \frac{18}{s^2+3s+9}$$

a) maila aldaketa $x(t)$-n : 3-ko anplitudea.

$w_n^2 = 9 \Rightarrow w_n = 3$

$2\delta w_n = 3 \Rightarrow \delta = \frac{3}{2w_n} = \frac{1}{2} = 0{,}5 \qquad \underbrace{0 < \delta < 1}$

$K\cdot w_n^2 = 18 \rightarrow K = 2$

$\boxed{y_{ss} = K\cdot u = 2\cdot 3 = 6}$

b) $y_p(t) \leq 10$

$$M_p = e^{\frac{-\pi\delta}{\sqrt{1-\delta^2}}} = e^{-\frac{\pi\cdot 0{,}5}{\sqrt{1-0{,}5^2}}} = 0{,}163 = \%16{,}3$$

$$0{,}163 = \frac{y_p - y_{ss}}{y_{ss}} \rightarrow 0{,}163\cdot y_{ss} + y_{ss} = 10$$

$$y_{ss}(0{,}163+1) = 10 \rightarrow y_{ss} = 8{,}6$$

$y_{ss} = K\cdot u \Rightarrow u = 4{,}3$

$\boxed{U(s) = \frac{4{,}3}{s}}$ (Maila sarrera: $U(s) = \frac{u}{s}$)

⑰ (2012-2013 Ekaina)

a) $G(s) = \frac{4,5}{s^2+2,4s+9}$ b) $G(s) = \frac{2}{s^2+2,4s+}$

$R(s) \longrightarrow \boxed{G(s)} \longrightarrow Y(s)$ $G(s)?$

c) $G(s) = \frac{0,5}{2,75s+e} e^{-4s}$ d) $G(s) = \frac{4,5 e^{-4}}{s^2+2,4s+9}$

Grafikoa

- → $r(t)$ sarrera : 4 anplitudeko espaloi-sarrera ⇒ $\Delta u = 4$
- → $y_{ss} = 4-2=2$
- → $y_{tp} = 4,5-2 = 2,5$

$$K = \frac{\Delta y}{\Delta u} = \frac{2}{4} = \frac{1}{2}$$

c) Aukera ezin da izan, 2. ordenekoa izan behar delako gure transferentzi funtzioa. Beraz, gainerakoetan daukagu $\omega_n^2 = 9$ dela.

$\omega_n = 3$ → $k\omega_n^2 = \frac{1}{2} \cdot 9 = 4,5$ → $2\delta \cdot \omega_n = 2,4$ (funtzioetan ikusten dugu)

$$\delta = \frac{2,4}{2 \cdot 3} = 0,4 \qquad \left[G(s) = \frac{4,5}{s^2+2,4s+9} \quad a) \right]$$

⑯

sarrera: −2 anplitudeko espaloi-sarrera. $\Delta u = 2$

$G(s)?$

Grafikoa $\longrightarrow \Delta y = 26-30 = -4$; $K = \frac{\Delta y}{\Delta u} = \frac{-4}{-2} = 2$

$$M_p = \frac{\boxed{26-25,2} \rightarrow B}{\boxed{30-26} \rightarrow A} = 0,2 \Rightarrow 0,2 = e^{\frac{-\pi\delta}{\sqrt{1-\delta^2}}} \Rightarrow \delta^2 = \frac{(\ln 0,2)^2}{(\ln 0,2)^2 + \pi^2} \longrightarrow \delta = 0,456$$

$$t_p = 1,7s = \frac{\pi}{\omega_n \sqrt{1-\delta^2}} \Rightarrow 1,7\omega_n\sqrt{1-\delta^2} = \pi$$

$$\omega_n = \frac{\pi}{1,7\sqrt{1-0,46^2}} = 2,076$$

$$G(s) = \frac{k \cdot \omega_n^2}{s^2 + 2\delta\omega_n s + \omega_n^2}$$

$$\boxed{G(s) = \frac{8,62}{s^2 + 1,89s + 4,3}}$$

(18)

sarrera: espaloi-sarrera -0,5 anplitudekoa.

$y_{ss} = 45 - 35 = 10$

$y_{ss} = K \cdot u \Rightarrow K = \frac{10}{-0,5} = -20$

G(s).

maila unitario sarrera

b) eta d) aukerak ezin dira izan lehenengo ordenakoak direlako.

$y(t_{63}) = 35 + 0,632(-20)(-0,5) = 41,32 \Rightarrow t_{63} \simeq 80s$

$y(t_{28}) = 35 + 0,28 \cdot 10 = 37,83 \Rightarrow t_{28} \simeq 65s$

$80 = tm + \tau$ ‖ $\tau = 80 - tm$

$65 \cdot 3 = tm \cdot 3 + \tau$ ‖ $195 = 3tm + 80 - tm \rightarrow tm = 57,5 \simeq 50$

$\boxed{\text{a) aukera da zuzena}}$

* * *

POLOEN KOKAPENA:

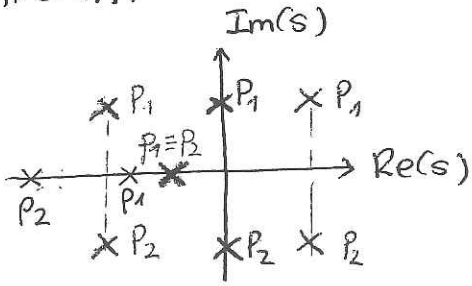
$\delta < 0$

$\delta = 0$

$0 < \delta < 1$

$\delta = 1$

$\delta > 1$



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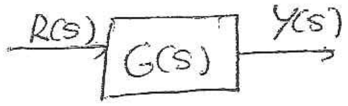
(17)



Maila sarrera, 4 anplitudekoa: $\Delta u = 4$

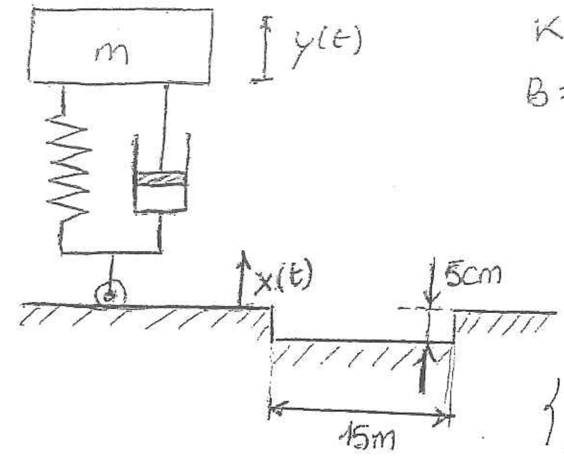
$\Delta y = 4 - 2 = 2 \rightarrow K = \frac{\Delta y}{\Delta u} = \frac{2}{4} = 0,5$

$Mp = \frac{B}{A} = \frac{4,5-4}{4-2} = 0,25 \rightarrow 0,25 = e^{\frac{-\pi\delta}{\sqrt{1-\delta^2}}} \Rightarrow \delta^2 = \frac{(\ln 0,25)^2}{(\ln 0,25)^2 + \pi^2} = 0,163$

$\delta = 0,4037$

$\boxed{\text{Aurretik eginda}}$

⑲



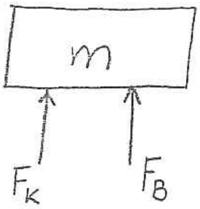
$M = 100\ kg$
$K = 50000\ N/m$
$B = 1000\ \frac{Ns}{m}$

x(t): lurrarekiko ukipen-puntua
y(t): masaren desplazamendu bertikala.

a)

$\sum \vec{F} = m\vec{a}$

$$\begin{cases} F_B = B\ \dfrac{d(x(t)-y(t))}{dt} \\ F_K = K(x(t)-y(t)) \end{cases}$$



$$K(x(t)-y(t)) + B\frac{d(x(t))}{dt} - B\cdot\frac{dy(t)}{dt} = M\frac{d^2y(t)}{dt^2}$$

$$\Rightarrow M\frac{d^2y(t)}{dt^2} + Ky(t) + B\frac{dy(t)}{dt} = Kx(t) + B\frac{dx(t)}{dt}$$

b) Laplace; $Ms^2Y(s) + KY(s) + BsY(s) = KX(s) + BsX(s)$

$$Y(s)\left(Ms^2+Bs+K\right) = X(s)\left(Bs+K\right)$$

$$G(s) = \frac{Y(s)}{X(s)} = \frac{Bs+K}{Ms^2+Bs+K} = \frac{10\cancel{0}\cancel{0}s + 500\cancel{0}\cancel{0}}{10\cancel{0}s^2 + 10\cancel{0}\cancel{0}s + 500\cancel{0}\cancel{0}}$$

$$G(s) = \frac{10s+500}{s^2+10s+500}$$

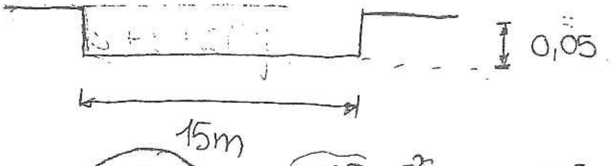
c)

- Poloak: $s^2+10s+500=0 \rightarrow s = \dfrac{-10 \pm \sqrt{10^2-4\cdot 500}}{2} = \dfrac{-10\pm 10\sqrt{19}\,j}{2} = -5 \pm 21{,}8j$

$p_1 = -5+21{,}8j$
$p_2 = -5-21{,}8j$ ‖ Polo konplexu konjokatuak $\Rightarrow 0 < \xi < 1 \rightarrow$ Azpimoteldua

- Zeroak. $10s+500=0 \longrightarrow 10s=-500$
$s = -50$

d)



$15\,m \longrightarrow v = 5\ m/s$ - tan $\Rightarrow 3s$

$$X(s) = -\frac{0{,}05}{s} + \frac{0{,}05}{s}e^{-3s} = \frac{0{,}05}{s}\left(e^{-3s}-1\right)$$

↳ espaloi-sarrera

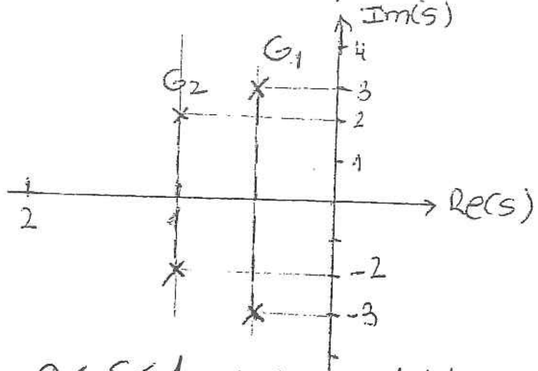
erantzunean: $u(s) = \dfrac{0{,}05}{s} - \dfrac{0{,}05}{s}e^{-3s}$

$\downarrow \mathcal{L}$

$x(t) = -0{,}05 + 0{,}05(t-3)$

⑳

a) Zein da 2 anplitudeko maila sarreran erantzuna?



$\omega_d^1 > \omega_d^2$

$\delta\omega_n^2 > \delta\omega_n^1$

$t_p = \dfrac{\pi}{\omega_d} \rightarrow \omega_d \uparrow \rightarrow t_p \downarrow \Rightarrow t_p^1 < t_p^2$

$\delta^1 < \delta^2$

$M_p^1 > M_p^2$

- $0 < \delta < 1 \rightarrow$ Azpimoteldua $\rightarrow$ C) eta D) aukerak ezin dira izan.
- B) Aukera ezin da izan $M_p^2 > M_p^1$ delako

$\boxed{\text{A) da aukera zuzena}}$

b) Zein da irabazpena? (A grafikotik) $\Delta u = 2$

$K = \dfrac{\Delta y}{\Delta u} = \dfrac{1}{2} = 0{,}5$ $\boxed{\text{A) aukera da zuzena}}$

c) Zein da egonkortze denbora (%2 irizpidea)?

$t_s = \dfrac{4}{\underbrace{\delta\omega_n}_{\text{zati erreala}}}$

$\left\{ \begin{array}{l} t_s^1 = \dfrac{4}{0{,}5} = 8s \\ t_s^2 = \dfrac{4}{1} = 4s \end{array} \right.$ $\Rightarrow$ $\boxed{\text{B) aukera da zuzena}}$

(21)

Anplitude 2 duen maila sarrera daukagu.

G(s)?

$\Delta u = 2$

$y_{ss} = 2{,}05 - 0{,}75 + 0{,}2 - 0{,}03 = 1{,}47$

$t_p \simeq 4{,}6s = \dfrac{\pi}{\omega_n\sqrt{1-\delta^2}}$

$y(t_p) = a_1 = 2{,}05$

$K = \dfrac{y_{ss}}{\Delta u} = \dfrac{1{,}47}{2} = 0{,}735$

$M_p = \dfrac{2{,}05 - 1{,}47}{1{,}47} = 0{,}395 = e^{\frac{-\pi\delta}{\sqrt{1-\delta^2}}} \rightarrow \delta = \sqrt{\dfrac{(\ln 0{,}395)^2}{(\ln 0{,}395)^2 + \pi^2}} = 0{,}284$

$G(s) = \dfrac{K \cdot \omega_n^2}{s^2 + 2\delta\omega_n s + \omega_n^2}$ non $\omega_n = \dfrac{\pi}{{,}6\sqrt{1-0{,}284^2}} = 0{,}712$

$$\boxed{G(s) = \dfrac{0{,}373}{s^2 + 0{,}41 s + 0{,}51}}$$

㉒

→ $G_1(s)$ eta $G_7(s)$ bigarren ordenako sistema azpimotelduen transferentzi funtzioak dira, beraz B eta C grafikoekin erlazionatuko ditugu.

C grafikoa ⟹ inpultsu erantzuna → $\boxed{C \longrightarrow G_1(s)}$

$\boxed{B \longrightarrow G_7(s)}$ B grafikoa → espaloi erantzuna.

→ $G_3(s)$-k $\delta > 1$ dauka, beraz sistema gainmoteldua daukagu eta espaloi erantzuna, beraz: $\boxed{I \longrightarrow G_3(s)}$

→ A grafikoan lehenengo ordenako sistema daukagu non $K=1$ den.

$$\Delta y = 1 \rightarrow y_{63} = 0 + 0{,}63 \cdot 1 = 0{,}63 \Rightarrow \tau \simeq 1$$

$$G(s) = \frac{1}{1+s} = G_2(s) \Rightarrow \boxed{A \longrightarrow G_2(s)}$$

→ $G_8(s)$ lehenengo ordenako sistema bati dagokio:

$$G_8(s) = \frac{-s+1}{s+1} = \frac{-(s+1)+1+1}{s+1} = -\frac{s+1}{s+1} + \frac{2}{s+1} = -1 + \frac{2}{s+1}$$

$\underline{K=2}$ eta $y=-1$-en hasten den grafikoa D da. $\boxed{D \longrightarrow G_8(s)}$
$\Delta y = 2$

→ $G_4(s)$ eta $G_5(s)$ funtzioak lehenengo ordeneko sistemetakoak dira, beraz F eta G grafikoenak izango dira.

$$G_5(s) = \frac{\cancel{10}}{\left(\frac{1}{10}s+1\right)\cancel{10}} = \frac{1}{0{,}1s+1} \Rightarrow \tau = 0{,}1 \quad , \quad K = \Delta y = 1 \quad \boxed{F \longrightarrow G_5(s)}$$

$$G_4(s) = \frac{1}{s+10} = \frac{1}{10(0{,}1s+1)} = \frac{0{,}1}{0{,}1s+1} \Rightarrow \tau = 0{,}1 \quad , \quad K = \Delta y = 0{,}1$$

$$\boxed{G \longrightarrow G_4(s)}$$

→ E grafikoak zero positiboa duen sistema batena da, beraz $G_6(s)$-rekin elkartuko dugu: $s = 1$

$\boxed{E \longrightarrow G_6(s)}$

$\boxed{H \longrightarrow G_9(s)}$

㉓

$t_s < 2s$

$M_p < \%10$

$$G(s) = \frac{\omega_n^2}{s^2 + 2\delta\omega_n s + \omega_n^2}$$

$$M_p = e^{-\frac{\pi\delta}{\sqrt{1-\delta^2}}} = 0{,}1$$

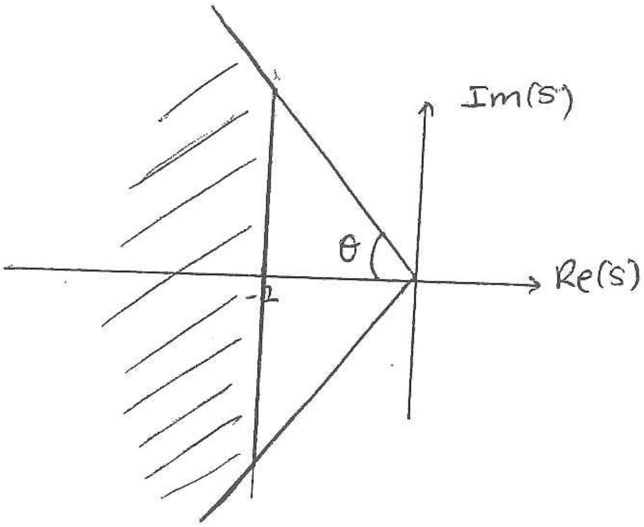
$$\left[\delta = \sqrt{\frac{(\ln 0{,}1)^2}{(\ln 0{,}1)^2 + \pi^2}} \geq \boxed{0{,}591}\right.$$

$$t_s = \frac{4}{\delta\omega_n} \quad (\%2)$$

$$\frac{4}{\delta\omega_n} \leq 2 \longrightarrow \left[\omega_n \geq \frac{4}{2\delta} = \boxed{3{,}38}\right.$$

↳ $2 \leq \delta\omega_n$ (zati erreala)

$\theta = \arccos\delta = 53{,}76$



㉔

a) Inpultsu-erantzuna eta lehenengo ordeneko sistema daukagu.

$$K = \frac{\Delta y}{\Delta u} = \frac{-1+0{,}5}{1} = -0{,}5$$

$\tau = 1{,}5$

$$\rightarrow \left[G(s) = \frac{-0{,}5}{1{,}5s+1} + 1 = \right.$$

$$= \frac{1{,}5s+1-0{,}5}{1{,}5s+1} \cdot \frac{2}{2} = \boxed{\frac{3s+1}{3s+2}}$$

b) Maila-erantzuna, $\delta = 0$ denean → bigarren ordenako sistema kritikoki egonkorra.

$$G(s) = \frac{K\omega_n^2}{s^2 + \omega_n^2}$$

$K = 0{,}25$

$\theta = \arccos\delta = \arccos 0 = \frac{\pi}{2}$

$$t_r = \frac{\pi - \theta}{\omega_d} = \frac{\pi - \theta}{\omega_n}$$

$t_p = \frac{\pi}{2}s$ (grafikotik) $\longrightarrow \frac{\pi}{4} = t_r \rightarrow \omega_n \frac{\pi}{4} = \pi - \frac{\pi}{2} \rightarrow \omega_n = 2$

$$\left[G(s) = \frac{0{,}25 \cdot 4}{s^2+4} = \boxed{\frac{1}{s^2+4}}\right.$$

c) $t_p = 1{,}51 = \frac{\pi}{\omega_d} \Rightarrow 1{,}51 = \frac{\pi}{\omega_n\sqrt{1-\delta^2}} \Rightarrow \omega_n = \frac{\pi}{1{,}51\sqrt{1-0{,}501^2}} = 2{,}4$

$y_{ss} = 0{,}5 \rightarrow K = 0{,}5$

$$M_p = \frac{0{,}581 - 0{,}5}{0{,}5} = 0{,}162 = e^{-\frac{\pi\delta}{\sqrt{1-\delta^2}}} \rightarrow \delta = 0{,}501$$

$$\left[G(s) = \frac{2{,}88}{s^2 + 2{,}41s + 5{,}7}\right.$$

d)

(25)

- Gaindikeb handienetik txikienera.

- Zeroa dominatzailea den sisteman egongo da gaindiketanik handiena. M_{p3}
- Bi polo dauden sisteman sistema atzeratuta egongo da eta gaindiketa txikiena rango da! $M_{p4} = M_{p2}$ $(\delta_4 = \delta_2)$
- Azkarragoa den sisteman $t_s \downarrow \rightarrow t_p \uparrow \rightarrow M_p \downarrow \Rightarrow \underbrace{M_{p2}}_{azkarragoa} < M_{p1}$

$M_{p3} > M_{p1} > M_{p2} = M_{p4}$

- Puntako denbora handienetik txikienera.

$t_{p2} > t_{p1}$

- Zeroek erantzuna aurreratzen dute, beraz t_{p3} txikiena rango da.
- M_p zenbat eta txikiagoa izan, t_p handiagoa.

$\underbrace{t_{p4} > t_{p2}} > t_{p1} > t_{p3}$

zenbat eta polo gehiago izan motelagoa rango d erantzu..

㉖

$u=2$

maila - sarrera.

grafikoa inpultsu-erantzun bati dagokio:

$$Y_{inpultsu}(s) = s \cdot Y_{maila}(s)$$

Maila - erantzunean: $\begin{cases} y(t_p) = a_1 = 2,3 \\ y_{ss} = 2,3-0,3 = 2 \end{cases}$ $M_p = \frac{2,3-2}{2} = 0,15$

$$K = \frac{y_{ss}}{u} = 1$$

$$\delta = \sqrt{\frac{(\ln 0,15)^2}{(\ln 0,15)^2+\pi^2}} = 0,517$$

$$\left[G(s) = \frac{1,039}{s^2+1,054s+1,039} \right]$$

$$t_p = 3,6 = \frac{\pi}{\omega_d} = \frac{\pi}{\omega_n\sqrt{1-\delta^2}} \Rightarrow \omega_n = \frac{\pi}{3,6\sqrt{1-0,517^2}} = 1,019 \text{ rad/s}$$