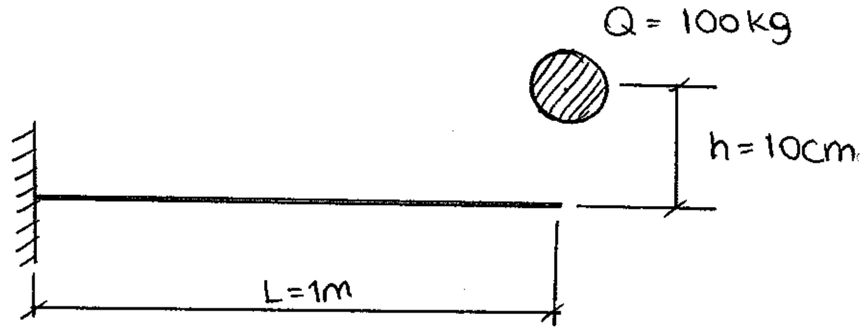
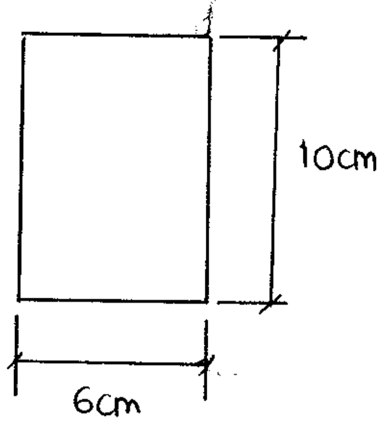
6. Teoría elemental del choque.

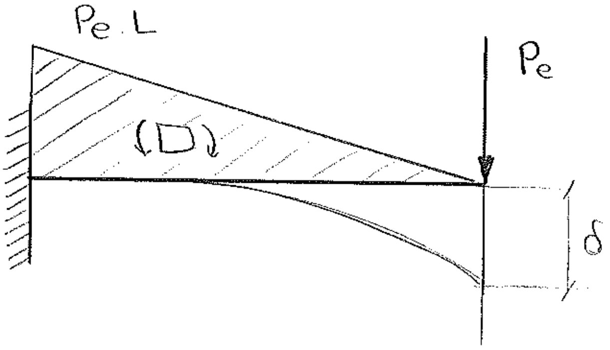
6.1





$$I = \frac{1}{12}\, 6cm\, (10cm)^3 = 5.10^{-6} m^4$$

E = 200GPa.



$$\boxed{\delta = \frac{1}{EI}\left[\frac{1}{2} P_e\, L.L\, \frac{2}{3}L\right) = \frac{P_e L^3}{3EI}}$$

$$P_e = \frac{3\delta EI}{L^3}$$

$U_s = W_e$.

$$\left.\begin{array}{l} U_s = Q(\delta + h) \\ W_e = \frac{1}{2} P_e.\delta = \frac{3EI\delta^2}{2L^3} \end{array}\right\} \quad Q(\delta+h) = \frac{3EI\delta^2}{2L^3}$$

$$\boxed{\delta_{ST} = \frac{QL^3}{3EI}}$$

$$\frac{Q.L^3}{3EI}(\delta+h) = \frac{\delta^2}{2}$$

$$2\delta_{ST}(\delta+h) = \delta^2$$

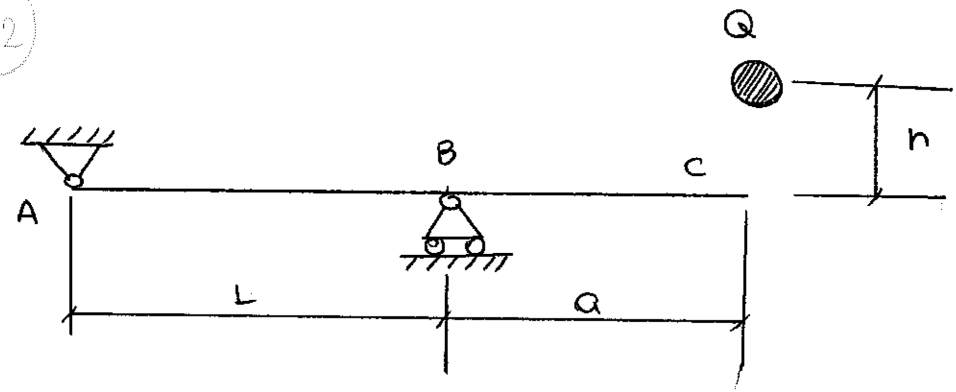
$$\delta \lll h:\ \delta = \sqrt{2\delta_{ST}.h}$$

$$\delta = \sqrt{\frac{2QL^3.h}{3EI}} \Rightarrow P_e = \frac{3EI}{L^3}\sqrt{\frac{2QL^3h}{3EI}} = 24494{,}897\,N$$

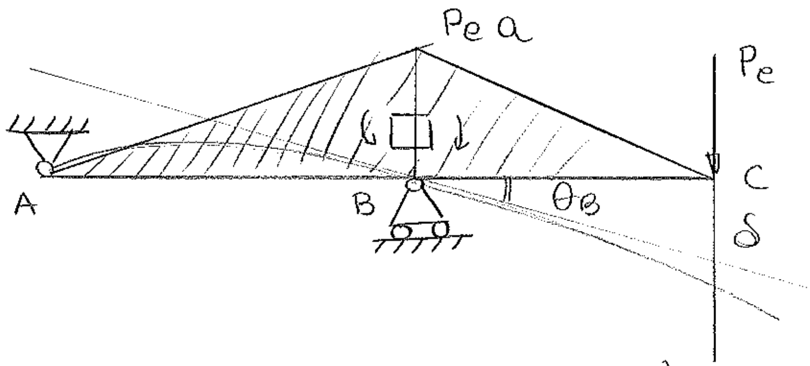
MOMENTO MÁXIMO: Empotramiento $(M_z = 24494{,}897\,Nm)$

$$\sigma_{xx} = \frac{M_z\ y}{I_z} = 244'95MPa \qquad \boxed{\sigma_{máx} = 244'95MPa}$$

6.2



- $EI = 500\ kN.m^2$
- $Q = 200kg$
- $h = 10cm$
- $a = 80cm$
- $L = 1m$



$$\delta = \theta_B\, a + \delta_{cB}$$

$$\theta_B = \delta_{AB}\ \frac{1}{L}$$

$$\delta_{AB} = \frac{1}{EI}\left[\frac{1}{2}\, Pe.a.L.\frac{2}{3}L\right] = \frac{Pe\,a\,L^2}{3EI}$$

$$\theta_B = \frac{Pe.aL}{3EI}$$

$$\delta_{CB} = \frac{1}{EI}\left[\frac{1}{2}\,Pe\,a.a.\frac{2}{3}a\right] = \frac{Pe\,a^3}{3EI}$$

$$\boxed{\delta = \frac{Pe\,a^2L}{3EI} + \frac{Pe\,a^3}{3EI}}$$

$$\delta = \frac{Pe\,a^2}{3EI}\left(L + a\right)$$

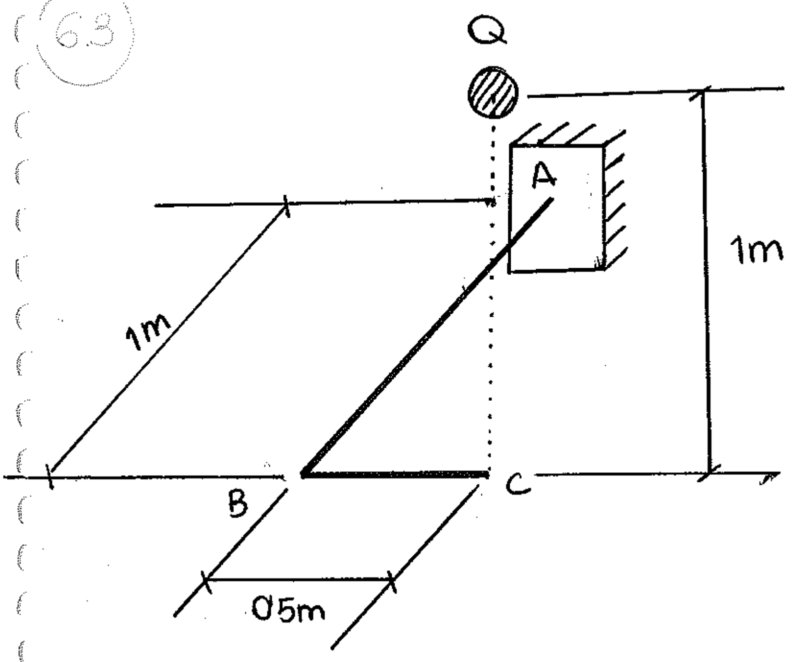
$$U_s = W_e$$
$$U_s = Q(h+\delta)$$
$$W_e = \frac{1}{2}\,P_e\,.\delta$$

$$\boxed{P_e = \frac{3\,\delta\, EI}{a^2(L+a)}}$$

$$W_e = \frac{3EI\delta^2}{2a^2(L+a)} = Q(h+\delta) \Rightarrow \boxed{\delta = 0'0191m}$$

$$\boxed{M_{max} = Pe.a = 19927{,}393\,Nm}$$

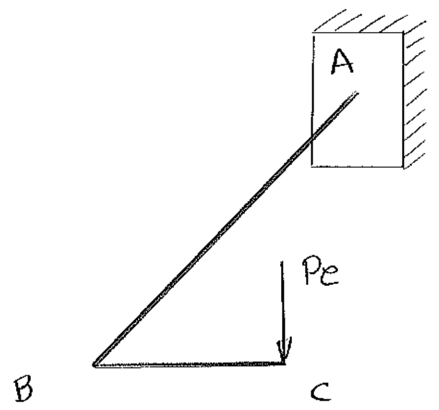
6.3



- $d = 10cm$
- $Q = 10kg$
- $E = 200GPa$
  $G = 80GPa$

$$\left.\begin{array}{l} U_s = W_e \\ U_s = Q(\delta + h) \\ W_e = \frac{1}{2} P_e \cdot \delta \end{array}\right\} \quad Q(\delta + h) = \frac{1}{2} P_e \cdot \delta$$

$\delta = \delta(P_e)$



$\delta = \delta_C = \delta(\text{flexión } BC) + \delta(\text{flexión } AB) +$
$+ \delta(\text{torsión } AB)$.

DIAGRAMA DE ESFUERZOS FLECTORES

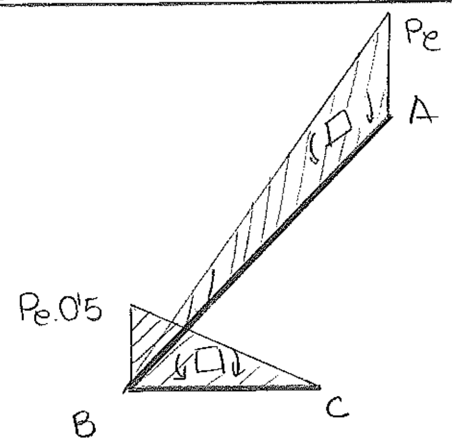
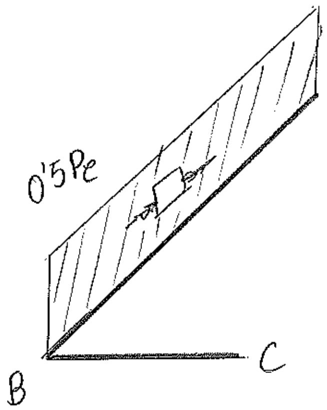


DIAGRAMA DE ESFUERZOS TORSORES



$$\delta_{Fl,BC} = \frac{1}{EI}\left[\frac{1}{2}\,0'5Pe.\,0'5\,.\,\frac{2}{3}\,0'5\right] = \frac{Pe}{24EI}$$

$$\delta_{Fl,AB} = \frac{1}{EI}\left[\frac{1}{2}\cdot Pe.\,1.\,\frac{2}{3}\,1\right] = \frac{Pe}{3EI}$$

$$\left.\begin{aligned} &\delta_{T,AB} = \varphi_{BA}\,.\,0'5 \\ &\varphi_{BA} = \frac{M_x\,.\,\Delta x_{BC}}{GI_p} = \frac{0'5Pe.\,1}{2GI} = \frac{Pe}{4GI} \end{aligned}\right\}\ \delta_{T,AB} = \frac{Pe}{8GI}$$

$$I = \frac{\pi R^4}{4} = 4{,}90873521{,}10^{-6}\,m^4$$

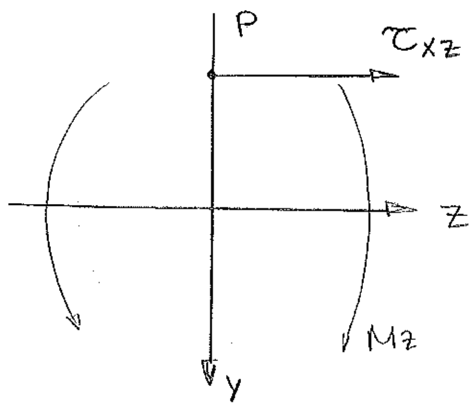
$$\delta = \frac{Pe}{24EI} + \frac{Pe}{EI.3} + \frac{Pe}{8GI} = 7{,}00281749610^{-7}\,Pe$$

$$\boxed{Pe = Pe(\delta) = 1427996{,}661.\,\delta}$$

$$Q(h+\delta) = \frac{1}{2}\,Pe\,.\,\delta = 713998{,}3304\,\delta^2 \Rightarrow \delta = 0'0119m.$$

$$P_e = 16999{,}98N \approx 17kN.$$

<u>SECCIÓN MÁS DESFAVORABLE</u>: A(empotramiento)

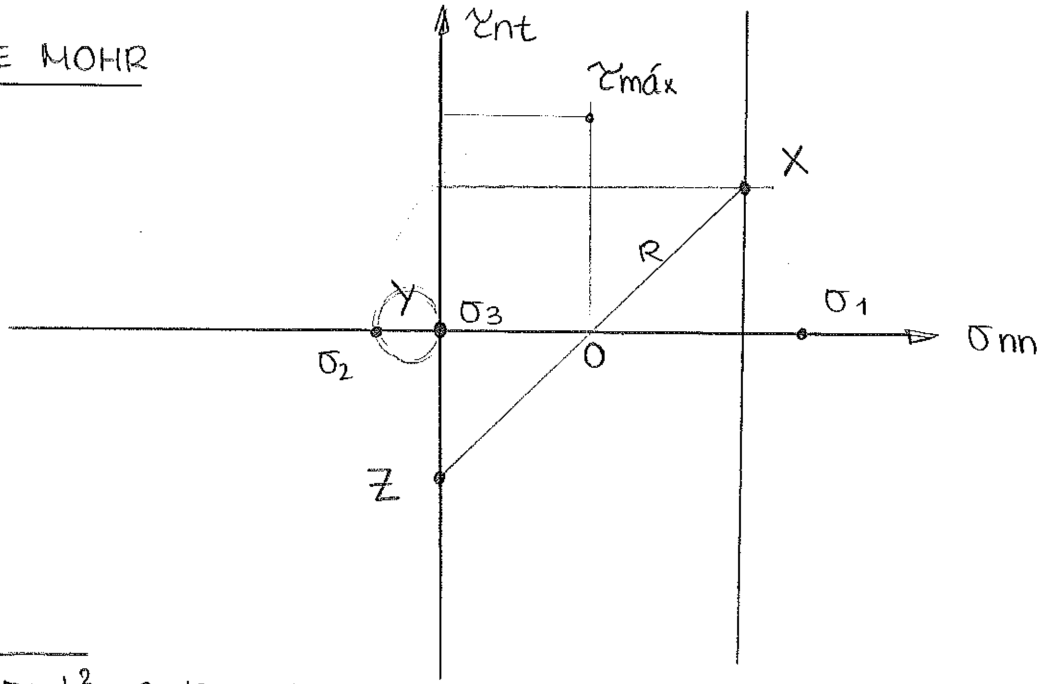


- Punto más desfavorable: P.

$$\sigma_{xx} = \frac{M_z\,.\,y}{I_z} = 173{,}1607MPa\ (+).$$

$$\tau_{xz} = \frac{M_x\,.\,R}{2I_z} = 43{,}290MPa$$

CÍRCULO DE MOHR



$$R = \sqrt{\tau_{xz}^2 + \left(\frac{\sigma_{xx}}{2}\right)^2} = 96'800\,MPa$$

$$\sigma_1 = \frac{\sigma_{xx}}{2} + R = 183'38\,MPa.$$

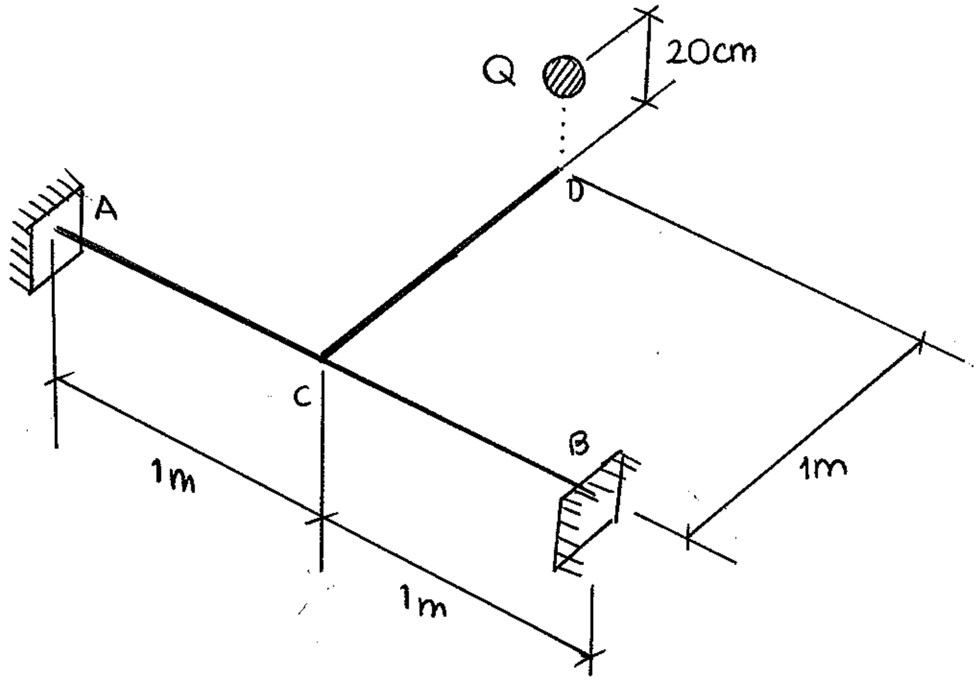
$$\tau_{máx} = 96'8\,MPa$$

$$\sigma_2 = \frac{\sigma_{xx}}{2} - R = -10'22\,MPa$$

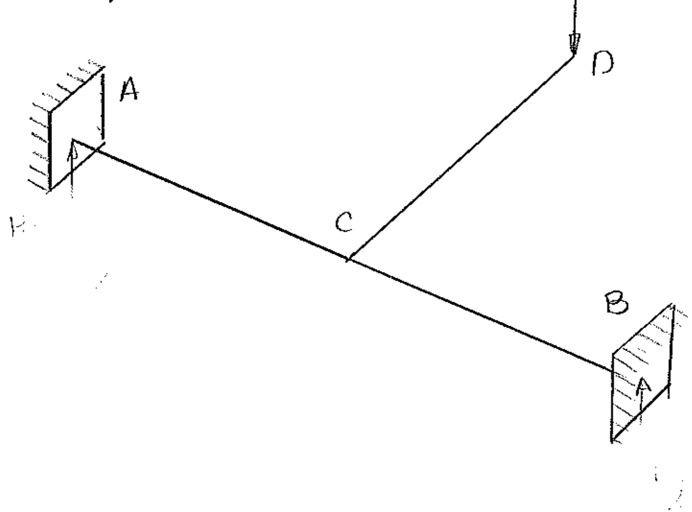
$$\sigma_3 = 0.$$

Tensiones principales:	tensión tangencial máxima
$\sigma_1 = 183'38\,MPa$	$\tau_{máx} = 96'8\,MPa$
$\sigma_2 = -10'22\,MPa$	
$\sigma_3 = 0.$	

6.4



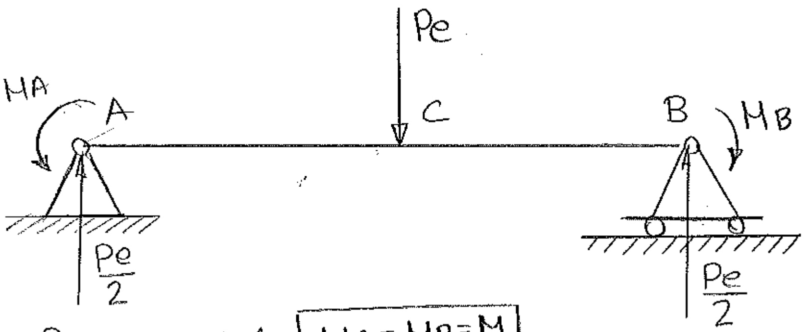
- $d = 10$ cm
- CD: indeformable.
- $n = 1'6$ (Tresca).
- $E = 200$ GPa, $G = 80$ GPa, $\sigma_f = 256$ MPa



$$\left.\begin{array}{l} U_s = W_e \\ U_s = Q(h+\delta) \\ W_e = \frac{1}{2} P_e \delta \end{array}\right\} \boxed{Q(h+\delta) = \frac{1}{2} P_e \delta}$$

$\delta_D = \delta$(flexión AB) + δ(torsión AB)

HIPERESTATICIDAD DE GRADO (h=2)



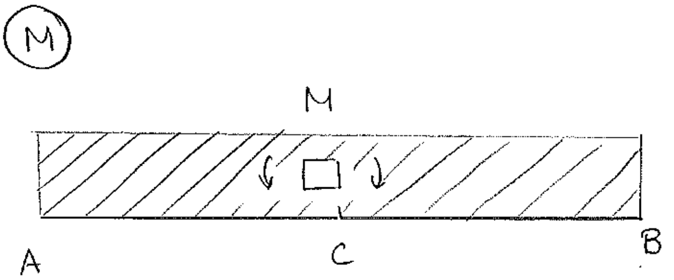
Ec. compatibilidad:

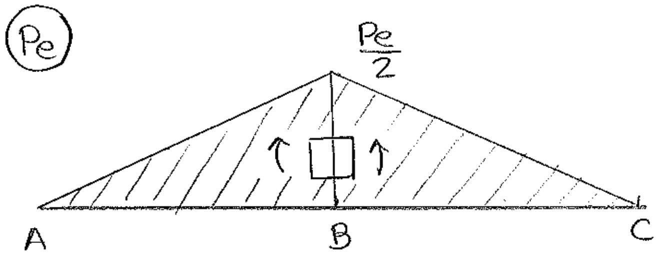
$\theta_A = \theta_B = 0$

$\hookrightarrow \boxed{\delta_{BA} = 0}$

Por simetría $\boxed{M_A = M_B = M}$

Diagrama de momentos flectores.



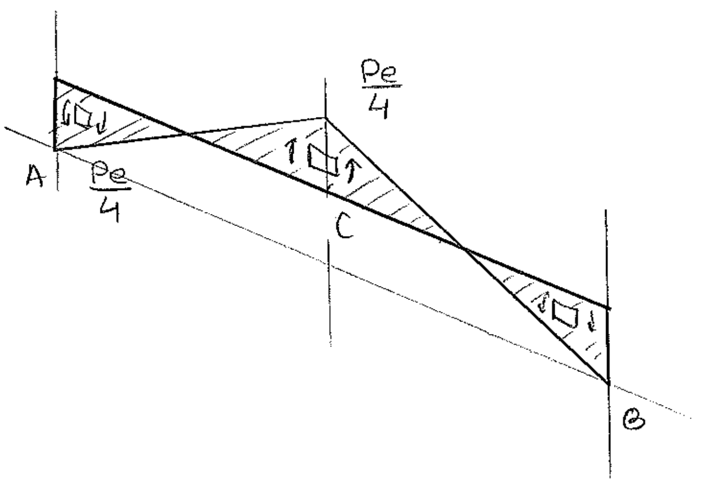


$$\overset{\downarrow}{\delta}_{BA_1} = \frac{1}{EI}\left[M \cdot 2 \cdot 1\right] = \frac{2M}{EI}$$

$$\overset{\uparrow}{\delta}_{BA_2} = \frac{1}{EI}\left[\frac{1}{2}\,\frac{Pe}{2} \cdot 2m \; 1m\right] = \frac{Pe}{2EI}$$

$$\overset{\downarrow}{\delta}_{B} = \frac{2M}{EI} - \frac{Pe}{2EI} = 0 \Rightarrow \boxed{M_A = M_B = \frac{Pe}{4}}$$

DIAGRAMA DE ESFUERZOS FLECTORES (AB)



$$\delta_C = \overset{\downarrow}{\delta}_{C,M} + \overset{\downarrow}{\delta}_{C,Pe}$$

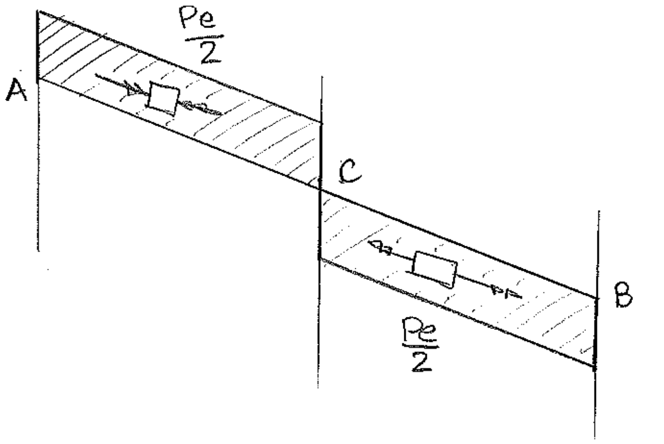
$$\delta_{C,M} = \frac{1}{EI}\left[M \cdot 1 \cdot 0'5\right] = \frac{M}{2EI} = \frac{Pe}{8EI}$$

$$\overset{\downarrow}{\delta}_{C,Pe} = -\frac{1}{EI}\left[\frac{1}{2}\,\frac{Pe}{2} \cdot 1\,\frac{1}{3}\right] = \frac{-Pe}{12EI}$$

$$\overset{\downarrow}{\delta}_C = \frac{Pe}{24EI} \text{ (flexión AB)}$$

$$\boxed{\delta_D(\text{flexión AB}) = \frac{Pe}{24EI}}$$

DIAGRAMA DE ESFUERZOS TORSORES (AB)



$$\varphi_{CA} = \frac{\frac{Pe}{2} \cdot 1m}{2GI} = \frac{Pe}{4GI}$$

$$\delta_D = \varphi_{CA} \cdot 1m = \frac{Pe}{4GI}$$

$$\boxed{\delta_D(\text{torsión AB}) = \frac{Pe}{4GI}}$$

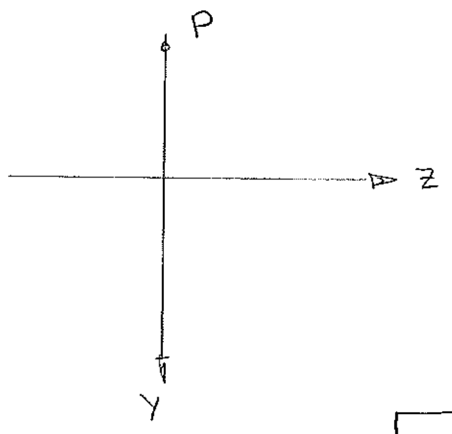
$$I = \frac{\pi R^4}{4} = 4{,}908738521 \cdot 10^{-6}\, m^4$$

$$\delta = \delta(Pe) = \frac{Pe}{24EI} + \frac{Pe}{4GJ} = 6{,}790610907 \cdot 10^{-7}\, Pe$$

$$\boxed{Pe = Pe(\delta) = 1472621{,}556\,\delta}$$

SECCIÓN MÁS DESFAVORECIDA DE LA BARRA: EMPOTRAMIENTO (A)

Sección A



Punto más desfavorecido: P.

$$\sigma_{xx} = \frac{Pe/4 \cdot 5cm}{I_z} = 2546{,}47909\, Pe$$

$$\tau_{xz} = \frac{Pe/2 \cdot 5cm}{2I_z} = 2546{,}47909\, Pe$$

Tresca: $\sigma_{adm} = \sqrt{\sigma_{xx}^2 + 4\tau_{xz}^2} = 5694{,}100348\, Pe \leq \frac{\sigma_f}{n}$

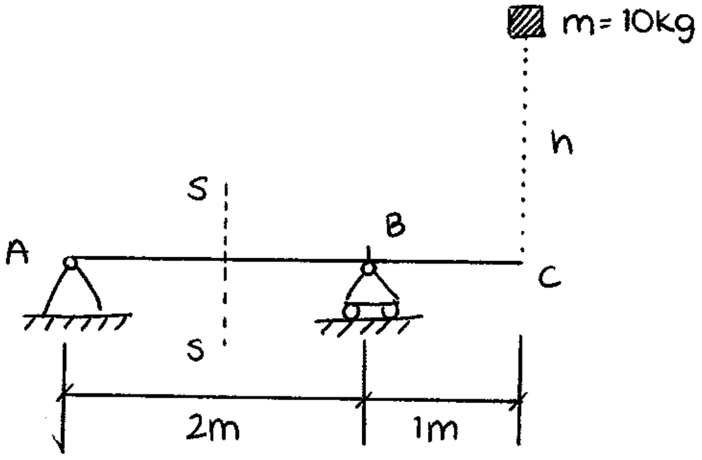
$$\boxed{Pe \leq 28099{,}25892\, N}$$

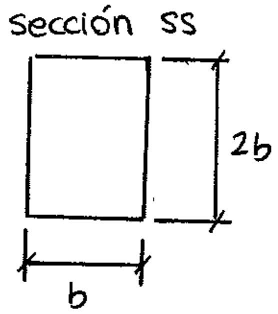
$$\boxed{\delta = 0'01908111341\, m}$$

$Q(h+\delta) = \frac{1}{2} Pe \cdot \delta \Rightarrow Q = 1223{,}668\, N = 122{,}367\, kg$

$$\boxed{Q = 122{,}367\, kg}$$

6.5





- $\sigma_r = 100 MPa$
- $E = 100 GPa$
- $b = 6cm$

$$I = \frac{1}{12} b(2b)^3 = \frac{8}{12} b^4 = 8'64 \cdot 10^{-6} m^4$$

$$\left.\begin{array}{l} U_s = Q(h+\delta) \\ W_e = \frac{1}{2} P_e \cdot \delta \\ U_s = W_e \end{array}\right\} \quad Q(h+\delta) = \frac{1}{2} P_e \cdot \delta$$

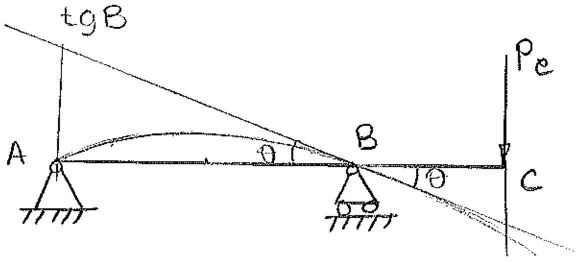
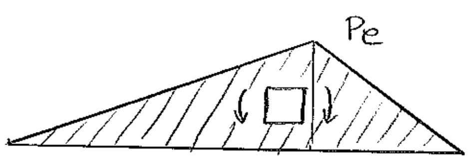


Diagrama de esfuerzos flectores.



$$\delta_C = \delta_{CB} + \theta \cdot 1m$$

$$\theta = \frac{\delta_{AB}}{2m}$$

$$\delta_{AB} = \frac{1}{EI}\left[\frac{1}{2} P_e \cdot 2 \cdot \frac{2}{3} 2\right] = \frac{4P_e}{3EI}$$

$$\theta = \frac{2P_e}{3EI}$$

$$\delta_{CB} = \frac{1}{EI}\left[\frac{1}{2} P_e \cdot 1 \cdot \frac{2}{3} \cdot 1\right] = \frac{P_e}{3EI}$$

$$\delta_C = \frac{P_e}{3EI} + \frac{2P_e}{3EI} = \frac{P_e}{EI}$$

SECCIÓN MÁS DESFAVORECIDA: sección B.

$$\sigma_{xx} = \frac{Pe \cdot 6cm}{I} < \sigma_r \Rightarrow Pe = 14400\,N$$

$$\delta = \frac{Pe}{EI} = 0'0167m$$

$$Q(h+\cancel{\delta}^{\,0}) = \frac{1}{2}Pe\cdot\delta \Rightarrow h = 1'2m \Rightarrow \boxed{h_{máx} = 120cm}$$

se desprecia

h = 100cm

$\sigma_{xx} < 60MPa$

$$\sigma_{xx} = \frac{Pe \cdot b}{I} = \frac{Pe \cdot b}{\frac{8}{12}b^4} = \frac{3Pe}{2b^3} = 60MPa$$

$$\boxed{Pe = 40 \cdot 10^6 \cdot b^3}$$

$$\delta = \frac{Pe}{EI} = \frac{40 \cdot 10^6 \cdot b^3}{100 \cdot 10^9 \frac{8}{12} b^4} = 6 \cdot 10^{-4} \frac{1}{b}$$

$$Q \cdot h = \frac{1}{2} Pe \cdot \delta = \frac{1}{2} \cdot 40\,10^6 b^3 \cdot 6\,10^{-4} \frac{1}{b} = 12000\,b^2$$

$b = 0{,}09120\ m = 9'129cm$ $\boxed{b = 9'129cm}$

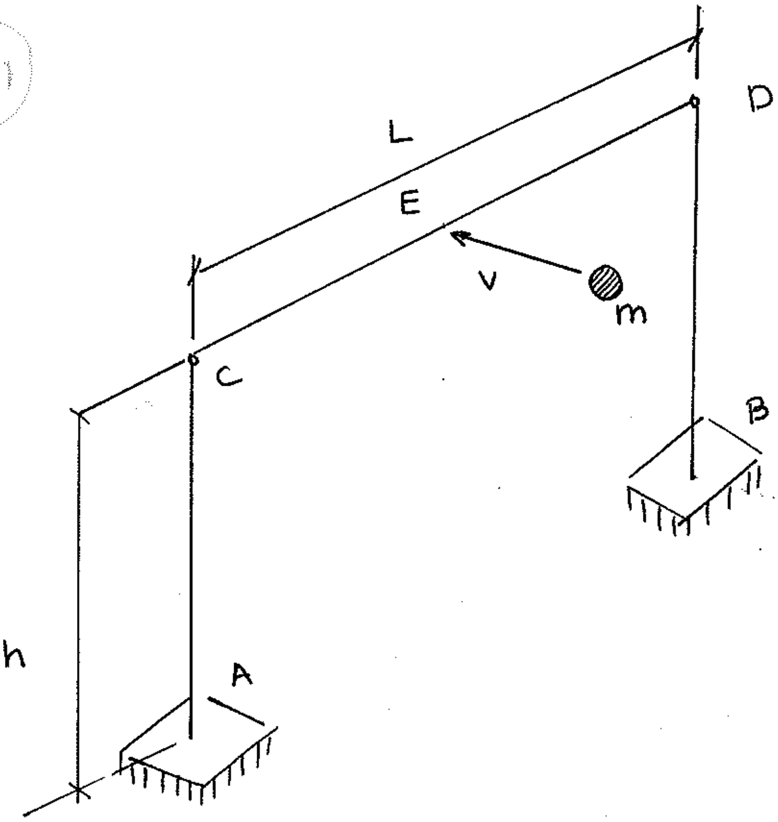
h = 50cm

b = 6cm

$$Qh = \frac{1}{2}Pe\cdot\delta = \frac{1}{2}EI\cdot\delta^2 \Rightarrow \delta = 0'010758m = 1'0758cm$$

$$\boxed{\delta = 1'0758cm}$$

C-6.1

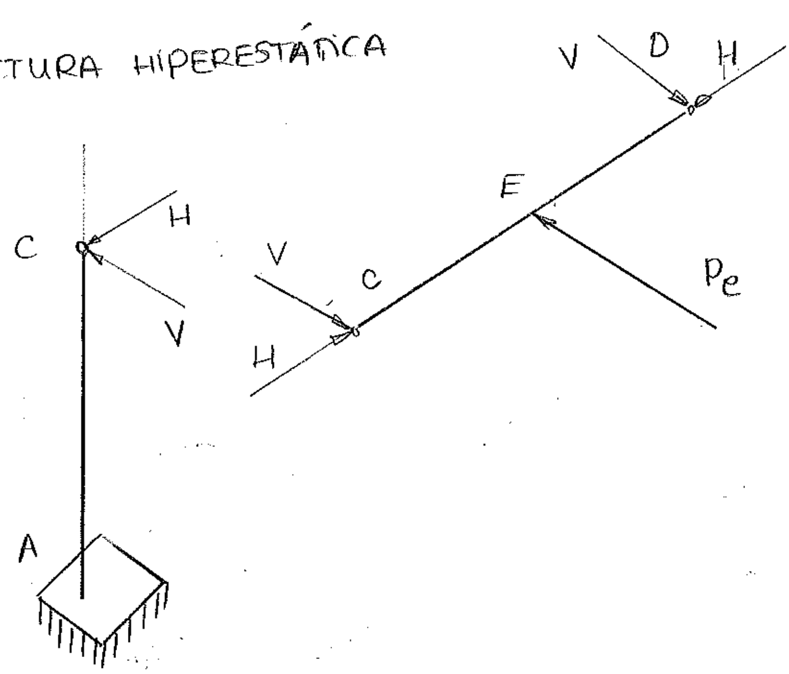


- sección barras: 4cm x 4cm
- $L = 30cm$, $h = 20cm$
- $m = 1kg$, $v = 10\ \frac{m}{s}$
- $g = 10\ \frac{m}{s^2}$
- $E = 100GPa$ $\sigma_r = 200MPa$

$U_s = W_e$

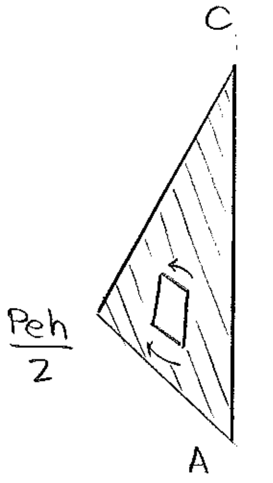
$$\left.\begin{array}{l} U_s = \frac{1}{2} m \cdot v^2 \\ W_e = \frac{1}{2} P_e \cdot \delta \end{array}\right\} \quad m \cdot v^2 = P_e \cdot \delta$$

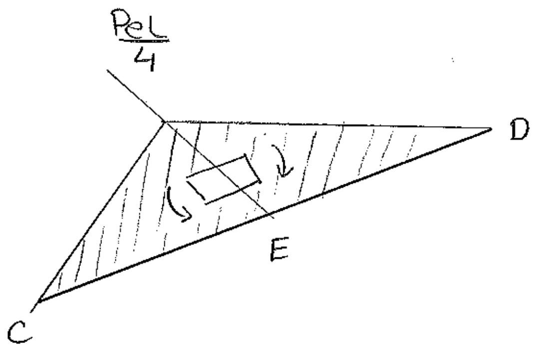
ESTRUCTURA HIPERESTÁTICA



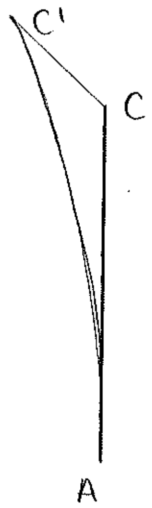
<u>EQUILIBRIO</u>: $P_e = 2V$; $V = \frac{P_e}{2}$

Diagrama de esfuerzos flectores



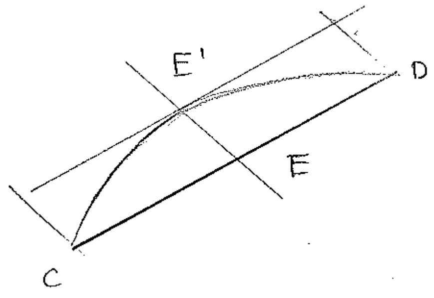


$\delta_E = \delta_E$ (flexión AC) + δ_E (flexión CED)



$$\delta_E = \delta_{CA} = \frac{1}{EI}\left[\frac{1}{2} \cdot \frac{P_e h}{2} \cdot h \cdot \frac{2}{3} h\right] = \frac{P_e h^3}{6EI}$$

$$\delta_E = \delta_{CE} = \frac{1}{EI}\left[\frac{1}{2} \; \frac{P_e L}{4} \cdot \frac{L}{2} \cdot \frac{2}{3} \; \frac{L}{2}\right] = \frac{P_e L^3}{48EI}$$



$$\boxed{\delta = P_e\left(\frac{h^3}{6EI} + \frac{L^3}{48EI}\right)}$$

Las secciones más frágiles: A y B (empotramientos)

$$\sigma_{xx,máx} = \frac{P_e \cdot \frac{h}{2} \cdot 2cm}{I} = 200MPa \Rightarrow P_e = 21333{,}33\,N$$

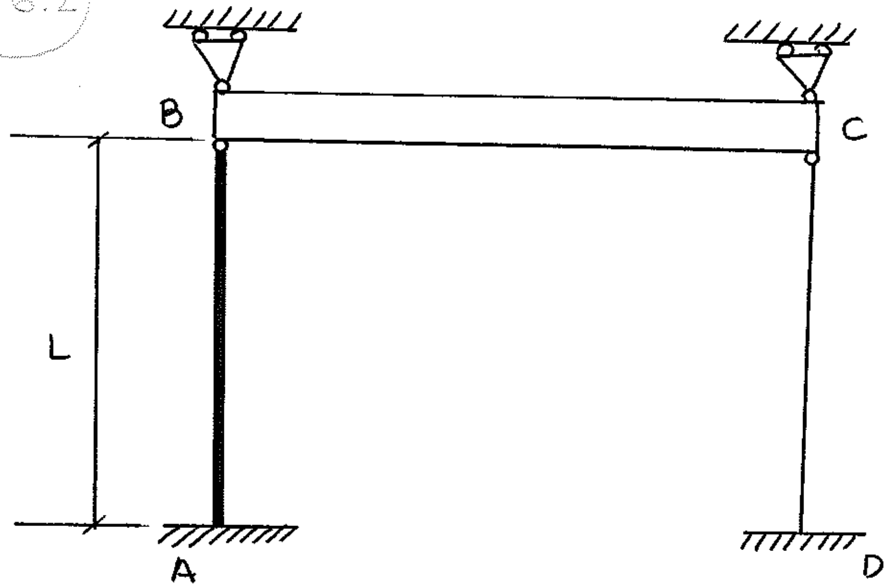
$$\delta = 1'895833 \cdot 10^{-3}$$

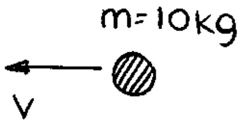
$$I = \frac{1}{12}(4cm)^4 = 2{,}133 \cdot 10^{-7} m^4$$

$$\left.\begin{array}{l} W_e = \frac{1}{2} P_e \cdot \delta = 20{,}222\ N \cdot m \\ U_s = 50Nm \end{array}\right\} U_s > W_e \ \boxed{\text{La estructura rompe en A y B}}$$

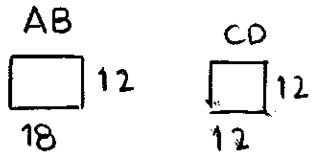
$$U_r = U_s - W_e = 29{,}778\ Nm = \frac{1}{2} m v_r^2 \Rightarrow \boxed{v_r = 7{,}717 \frac{m}{s}}$$

C-6.2





secciones (cm)

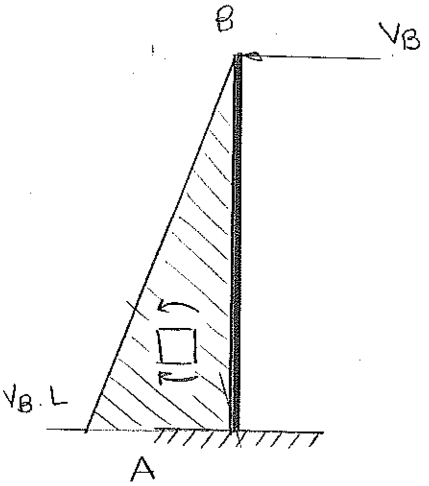


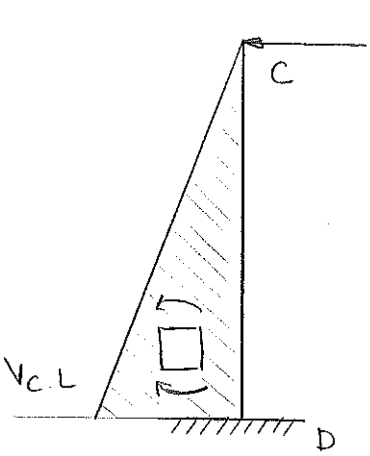
$I_{AB} = \frac{1}{12} 12cm (18cm)^3 = 5,832 \cdot 10^{-5} m^4$

$I_{CD} = \frac{1}{12} (12cm)^4 = 1728 \cdot 10^{-5} m^4$

- $L = 2m$
- $\sigma_f = 200 MPa$
- $E = 200 GPa$
- $g = 10 \frac{m}{s^2}$

$$\left. \begin{array}{l} U_s = \frac{1}{2} m v^2 \\ W_e \end{array} \right\} \boxed{U_s = W_e}$$





$$\boxed{V_B + V_C = P_e}$$

$$3,375 V_C + V_C = P_e$$

$$\boxed{\begin{array}{l} V_C = 0'22857 P_e \\ V_B = 0'77143 P_e \end{array}}$$

$$\boxed{\text{Ec. compatibilidad: } \delta_B = \delta_C}$$

$$\left. \begin{array}{l} \delta_B = \delta_{BA} = \frac{1}{E I_{AB}} \left[\frac{1}{2} V_B L \cdot L \cdot \frac{2}{3} L \right] = \frac{V_B L^3}{3 E I_{AB}} \\ \delta_C = \delta_{CD} = \frac{1}{E I_{CD}} \left[\frac{1}{2} V_C L L \cdot \frac{2}{3} L \right] = \frac{V_C L^3}{3 E I_{CD}} \end{array} \right\} V_B = \frac{I_{AB}}{I_{CD}} \cdot V_C = 3,375 V_C$$

● SECCIÓN A

$$\sigma_{xx} = \frac{0'77143 P_e L \cdot 9cm}{I_{AB}} = 200\,MPa : \quad P_e = 83999{,}84N\ldots$$

● SECCIÓN D

$$\sigma_{xx} = \frac{0'22857 P_e L \cdot 6cm}{I_{CD}} = 200\,MPa : \quad P_e = 126000{,}7875N$$

● Rompe primero la barra AB con $P_e = 83999{,}84N$

$$W_e = \frac{1}{2} V_B \cdot \delta_B + \frac{1}{2} V_C\, \delta_C = 622{,}22\,Nm$$

$$\delta = \frac{0'77143 P_e \cdot L^3}{3 E I_{AB}} = 0'0148148m$$

$\boxed{U_S = W_e}$ $\frac{1}{2} m v^2 = W_e \Rightarrow \boxed{V = 11{,}155\ \frac{m}{s}}$

$$W_e(\text{fractura total}) = \frac{1}{2}(V_B \cdot \delta_B) + \frac{1}{2} P_c \delta_c = 799{,}999\,Nm$$

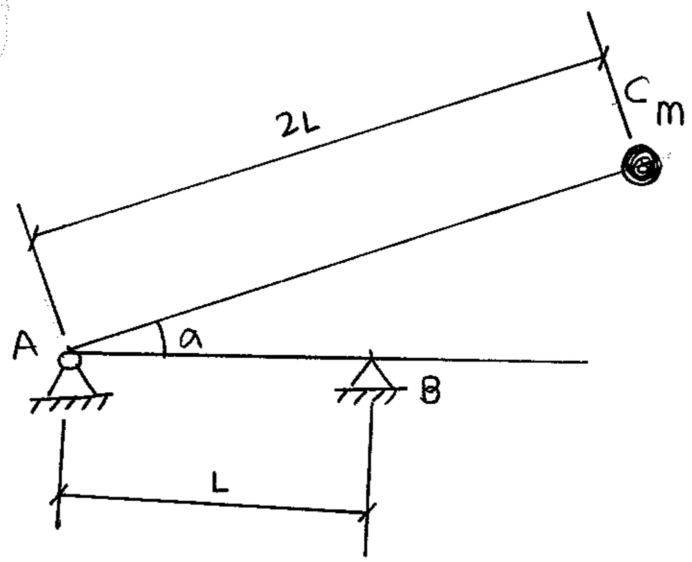
$$\frac{P_c \cdot L \cdot 6cm}{I_{CD}} = 200\,MPa \Rightarrow P_c = 28800\,N$$

$$\delta_c = 0'0222m$$

$\boxed{U_S = W_e}$ $\frac{1}{2} m u^2 = W_e \Rightarrow \boxed{V = 12'649\ \frac{m}{s}}$

$\boxed{\Delta = 2{,}22cm}$

C-6.3



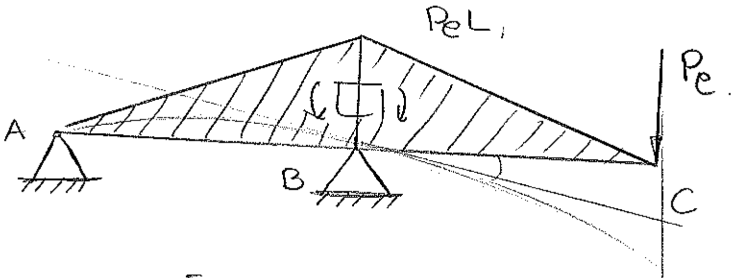
- $m = 10 kg$
- $\sigma_r = 200 MPa$
- $W_z = 36 cm^3$
- $EI = 100 m^2 kN$

(1) $U_s = W_e$

$U_s = m.(2L\,sen\alpha + \delta)$

$W_e = \frac{1}{2} P_e . \delta$

$$\boxed{m(2L.sen\alpha + \delta) = \frac{1}{2} P_e . \delta}$$



$\overset{\downarrow}{Y_c} = \theta . L + d_{CB}$

$\theta = \frac{\delta_{AB}}{L}$

$\delta_{AB} = \frac{1}{EI}\left[\frac{1}{2} P_e L . L . \frac{2}{3} L\right] = \frac{P_e L^3}{3EI}$

$\theta = \frac{P_e L^2}{3EI}$

$d_{CB} = \frac{1}{EI}\left[\frac{1}{2} P_e . L . L . \frac{2}{3} L\right] = \frac{P_e L^3}{3EI}$

$\overset{\downarrow}{Y_c} = \frac{2 P_e L^3}{3EI} = \delta$

SECCIÓN MÁS DESFAVORECIDA: sección B

$M_z = P_e . L \Rightarrow \sigma_{xx} = \frac{M_z}{W_z} = \frac{P_e . L}{W_z} = \sigma_r \Rightarrow$ $\boxed{P_e = 7200 N \quad \delta = 0'048 m}$

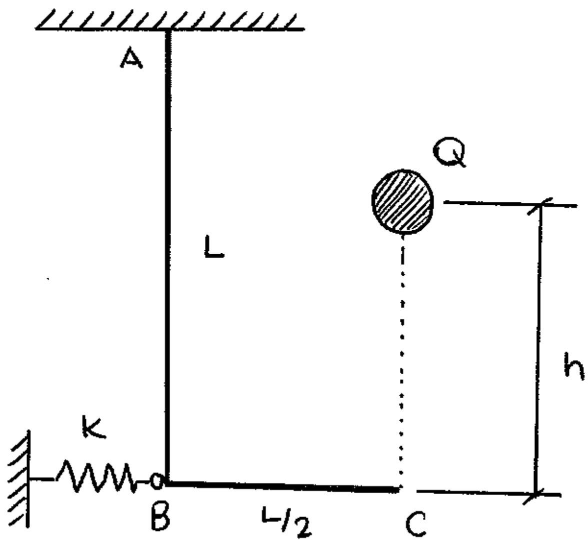
$m(2L.sen\alpha + \delta) = \frac{1}{2} P_e . \delta$

$$\boxed{\alpha = 57'140}$$

2) $\left.\begin{array}{l} U_s = m2L\,sen\,\alpha \\ W_e = \frac{1}{2} P_e \cdot \delta \end{array}\right\} \boxed{m2L\,sen\,\alpha = \frac{1}{2} P_e \cdot \delta}$

$$\boxed{\alpha = 59'769}$$

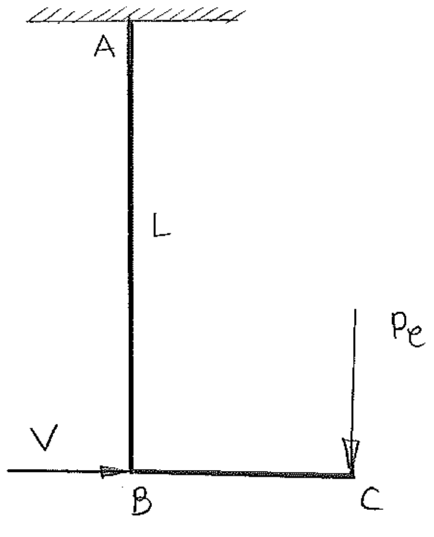
C-6.4



- $k(\text{muelle}) = 200\ \frac{kN}{m}$
- $Q = 20kg$
- $h = 80cm$
- $d = 8cm$
- $E = 200GPa$, $\sigma_f = 280MPa$
- $L = 1m$

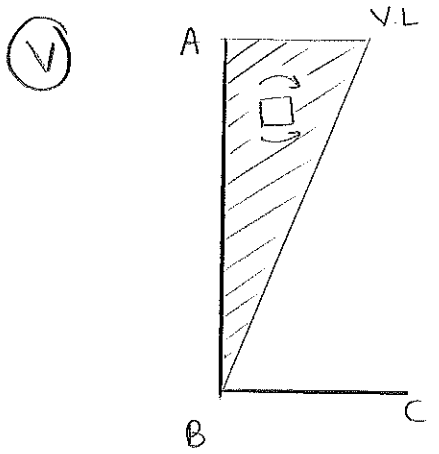
$$I = \frac{\pi (4cm)^4}{4} = 2{,}01061929810^{-6}\ m^4$$

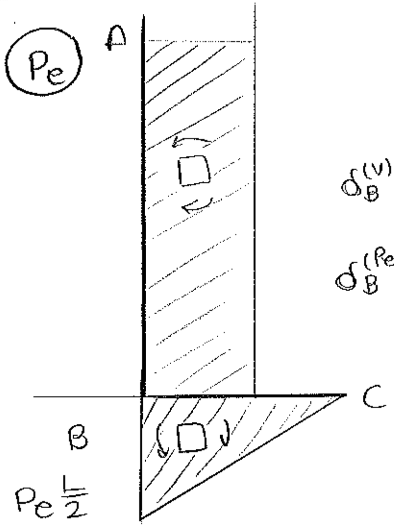
$$\left.\begin{array}{l} U_s = W_e \\ U_s = Q(h+\delta) \\ W_e = \frac{1}{2} P_e \delta + U_m \end{array}\right\} \quad Q(h+\delta) = \frac{1}{2} P_e \cdot \delta + \cancel{U_m}$$



$$\boxed{\text{Ec. compatibilidad: } \delta_B = \delta_m = \frac{V}{K}}$$

DIAGRAMAS DE ESFUERZOS FLECTORES





$$\delta_B = \delta_B(\text{flexión } V) + \delta_B(\text{flexión } Pe)$$

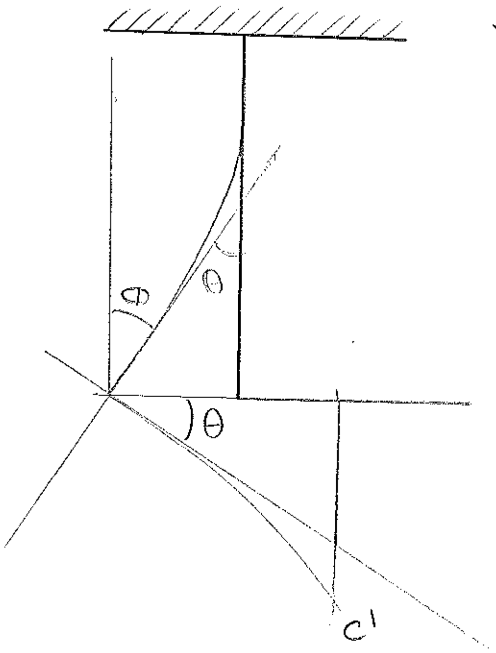
$$\delta_B^{(V)} = \frac{1}{EI}\left[\frac{1}{2} VL \cdot L \cdot \frac{2}{3} L\right] = \frac{VL^3}{3EI}$$

$$\delta_B^{(Pe)} = \frac{1}{EI}\left[Pe \frac{L}{2} L \cdot \frac{L}{2}\right] = \frac{PeL^3}{4EI}$$

$$\delta_B = -\frac{VL^3}{3EI} + \frac{P_e L^4}{4EI} = \frac{V}{K}$$

$$V\left(\frac{L^3}{3EI} + \frac{1}{K}\right) = + \frac{P_e L^4}{4EI}$$

$$\boxed{V = \frac{P_e L^4}{4EI} \cdot \frac{1}{+\frac{1}{K} + \frac{L^3}{3EI}}} = 0{,}1066574455\, P_e$$



$$Y_c = \theta_{BA} \cdot \frac{L}{2} + \delta_{CB}$$

$$\theta = \theta_{BA} = \frac{1}{EI}\left[P_e \frac{L}{2} \cdot L - \frac{1}{2} VL \cdot L\right] = \frac{0'4466712773\, P_e}{EI}$$

$$\delta_{CB} = \frac{1}{EI}\left[\frac{1}{2} P_e \cdot \frac{L}{2} \cdot \frac{L}{2} \cdot \frac{2}{3} \frac{L}{2}\right] = \frac{P_e L^3}{24EI}$$

$$Y_c = \boxed{\frac{P_e}{EI}\, 0'2650023053 = \delta}$$

$$\boxed{P_e = P_e(\delta) = 1517435{,}326\, \delta}$$

$$Q(h+\delta) = \frac{1}{2} P_e \cdot \delta = 758717{,}6631\, \delta^2$$

$$\delta = 0{,}01465 m$$

$$\boxed{\delta_c = 1'465 cm}$$

$$P_e = 22236{,}77164 N$$

$$\delta_m = \frac{V}{K} = 0{,}01186 m$$

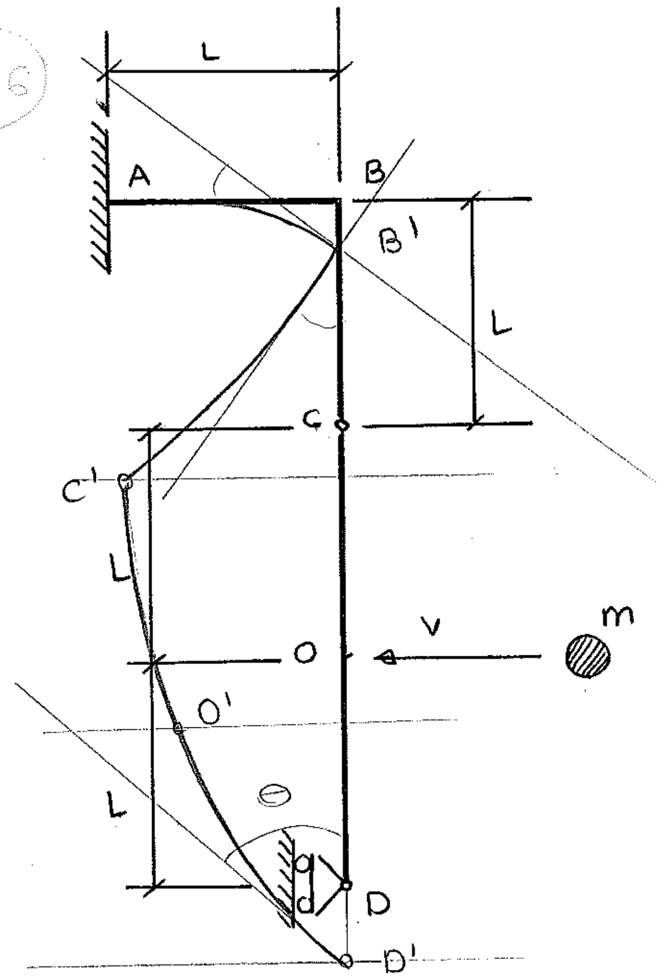
$$\boxed{\delta_m = 1'186 cm}$$

$$\sigma_{xx,max} = \frac{|M_{max}| R}{I} = 221{,}193 MPa$$

$$\boxed{n = 1'266}$$

$$n = \frac{\sigma_f}{\sigma_{xx\,max}} = 1'266$$

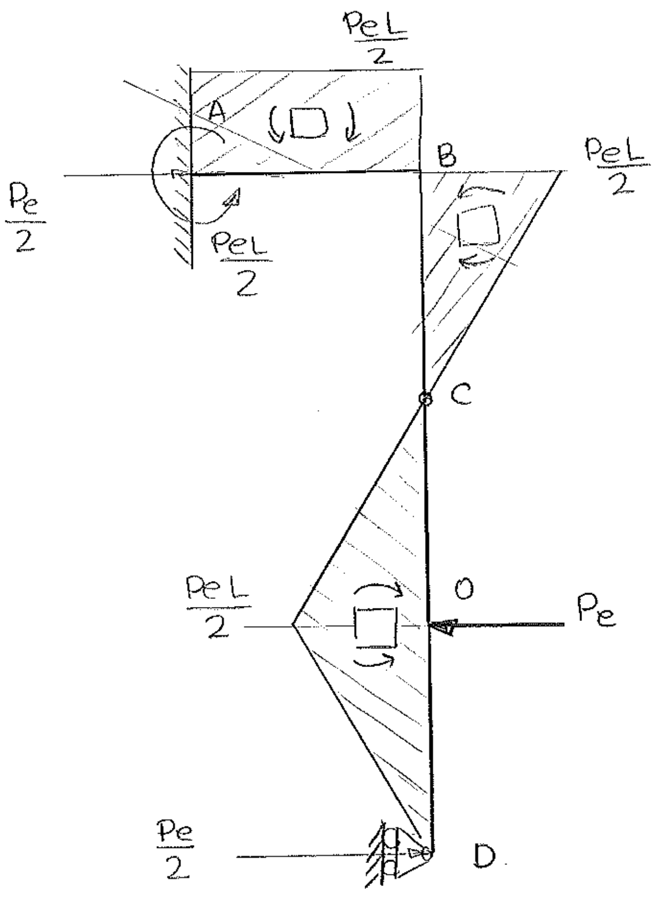
C-6.6



- $L = 100cm$
- sección: $6 \times 6 cm^2$
- $\sigma_r = 200MPa$
- $m = 1kg$
- $g = 10 \frac{m}{s^2}$

$$I = \frac{1}{12}(6cm)^4 = 1{,}08 \cdot 10^{-6} m^4$$

$$\left.\begin{array}{l} U_s = W_e \\ U_s = \frac{1}{2} m \cdot v^2 \\ W_e = \frac{1}{2} P_e \cdot \delta_o \end{array}\right\} \frac{1}{2} m \cdot v^2 = \frac{1}{2} P_e \cdot \delta_o : \quad \boxed{m v^2 = P_e \cdot \delta_o}$$



$$M_A = P_e \cdot 2L - \frac{P_e}{2} 3L = \frac{P_e L}{2}$$

$$\left\{\begin{array}{l} \delta x_O = \theta \cdot L - \delta_{OD} \\ \theta = \dfrac{\delta x_C + \delta_{CD}}{2L} \\ \delta x_C = \theta_{BA} \cdot L + \delta_{CB} \end{array}\right.$$

$$\theta_{BA} = \frac{1}{EI}\left[\frac{P_e L}{2} \cdot L\right] = \frac{P_e L^2}{2EI}$$

$$\delta_{CB} = \frac{1}{EI}\left[\frac{1}{2} \cdot \frac{P_e L}{2} \cdot L \cdot \frac{2}{3} L\right] = \frac{P_e L^3}{6EI}$$

$$\delta x_C = \frac{2 P_e L^3}{3EI}$$

$$\delta_{CD} = \frac{1}{EI}\left[\frac{1}{2} \frac{P_e L}{2} \cdot 2L \cdot L\right] = \frac{P_e L^3}{2EI}$$

$$\theta = \frac{7\, P_e L^2}{12\, EI}$$

$$\delta_{OD} = \frac{1}{EI}\left[\frac{1}{2}\,\frac{P_e L}{2}\cdot L\cdot\frac{1}{3}L\right] = \frac{P_e L^3}{12EI}$$

$$\boxed{\delta_{XO} = \frac{P_e L^2}{2EI}}$$

② Esfuerzo flector máximo:

$$\sigma_{xx} = \frac{M_{max}\cdot 3cm}{I} = \frac{P_e \frac{L}{2}\cdot 3cm}{1'08\cdot 10^{-6} m^4} = \sigma_r \Rightarrow \boxed{P_e = 14400 N}$$

$$\delta_{XO} = 0'033 m.$$

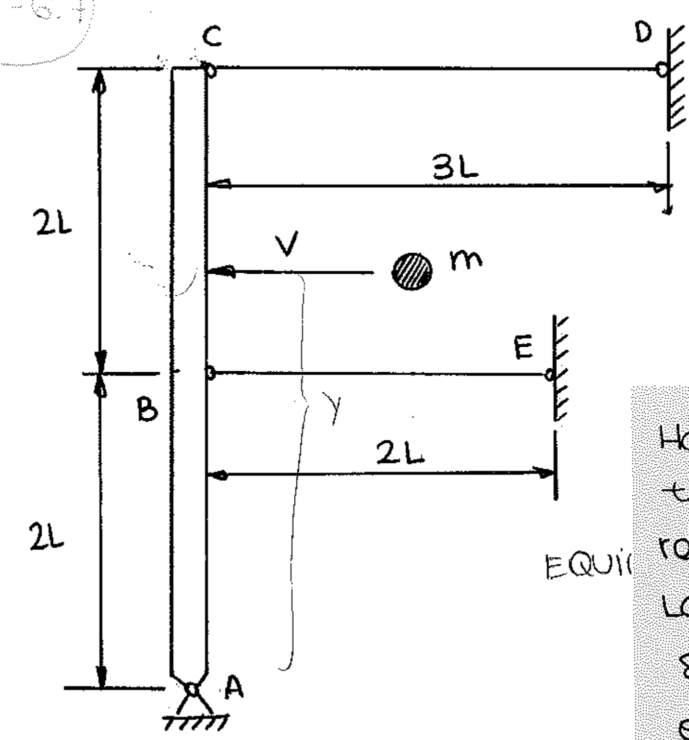
$$m\cdot v^2 = P_e\cdot\delta \Rightarrow \boxed{v = 21'909 \frac{m}{s}}$$

$$V = 25\,\frac{m}{s}$$

$$\left.\begin{array}{l} U_s = \frac{1}{2} m v^2 = 312{,}5\ Nm. \\ W_r = \frac{1}{2} P_e \cdot \delta_{X_O} = 240 Nm \end{array}\right\} U_r = U_s - W_r = 72'5\ Nm = \frac{1}{2} m v_r^2$$

$$\boxed{V_r = 12'042\,\frac{m}{s}}$$

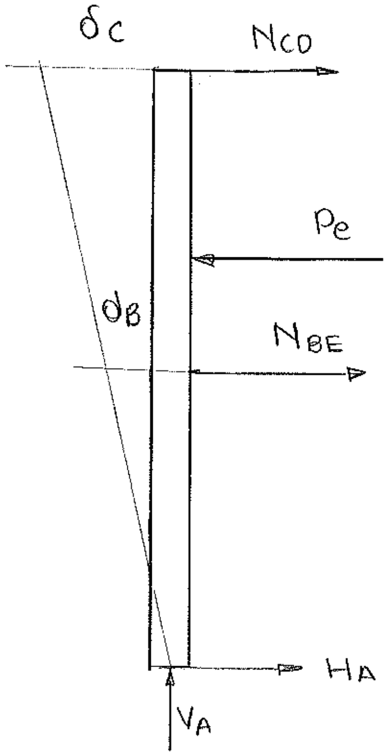
C-6.7



- $L = 1m$
- $\sigma_r = 200MPa$
- $E = 200GPa$
- $A_T = 2cm^2$
- $m = 10kg$

Hay que calcular el trabajo que supone romper cada tirante. La energía de la bola se va a traducir en eso.

EQUI... $\boxed{...L = P_e \cdot y}$



EC. COMPATIBILIDAD: $\boxed{\delta_C = 2 \cdot \delta_B}$

$$\left.\begin{aligned} \delta_C &= \frac{N_{CD} L_{CD}}{E \cdot A} \\ \delta_B &= \frac{N_{BE} L_{BE}}{E \cdot A} \end{aligned}\right\} \quad N_{CD} L_{CD} = 2 N_{BE} \cdot L_{BE} \qquad \boxed{N_{BE} = \frac{3}{4} N_{CD}}$$

- Para que rompa tirante CD (Rompe primero CD)

$$\sigma_{CD} = \frac{N_{CD}}{A} = 200MPa \Rightarrow \boxed{\begin{aligned} N_{CD} &= 40000N \\ N_{BE} &= 30000N \end{aligned}}$$

- Para que rompa tirante BE

$$\sigma_{xx} = \frac{N_{BE}}{A} = 200MPa \Rightarrow \boxed{\begin{aligned} N_{BE} &= 40000N \\ N_{CD} &= 53333{,}33N \end{aligned}} \; (*)$$

$\boxed{\text{Ambos tirantes rompen para } N_{BE} = 40000N \text{ y } N_{CD} = 53333{,}33N}$

$\delta_C(\text{rotura}) = 3.10^{-3}\, m$

$\delta_B(\text{rotura}) = 2.10^{-3}\, m$

$W_e = W_C(\text{rotura}) + W_B(\text{rotura}) = \frac{1}{2}\, 40.000 \cdot 3.10^{-3} + \frac{1}{2}\, 40000 \cdot 2.10^{-3} = 100\ Nm$

$\left.\begin{array}{l} U_s = W_e \\ U_s = \frac{1}{2} m v^2 \end{array}\right\}\ \frac{1}{2} m v^2 = 100$

$\boxed{V = 4{,}472\ \frac{m}{s}}$

$V = 6\ \frac{m}{s}$

$U_s = \frac{1}{2} m v^2 = 180\ Nm.$

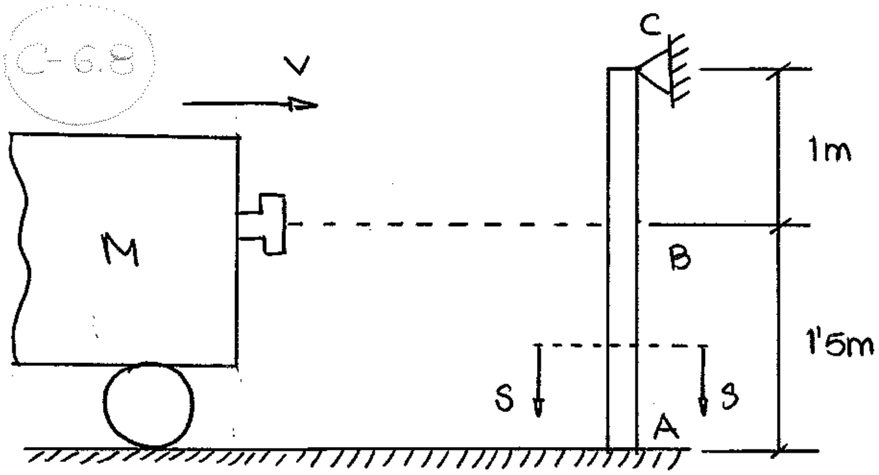
$W_e = W_C(\text{rotura}) + W_B(\text{deformación}) = \frac{1}{2}\, 40000 \cdot 3.10^{-3} + \frac{1}{2}\, 30.000 \cdot 1'5 \cdot 10^{-3} =$

$= 82{,}5\ Nm$

$U_r = U_s - W_e = 97'5\ Nm = \frac{1}{2} m v^2$

$\boxed{V = 4{,}416\ \frac{m}{s}}$

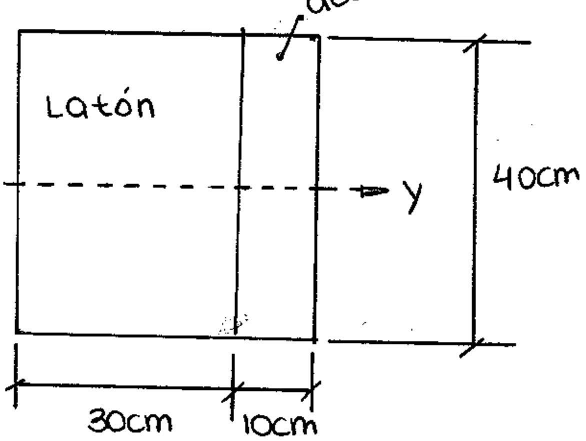
C-6.8



$M = 50000 kg$; $V = 0'3 \frac{m}{s}$

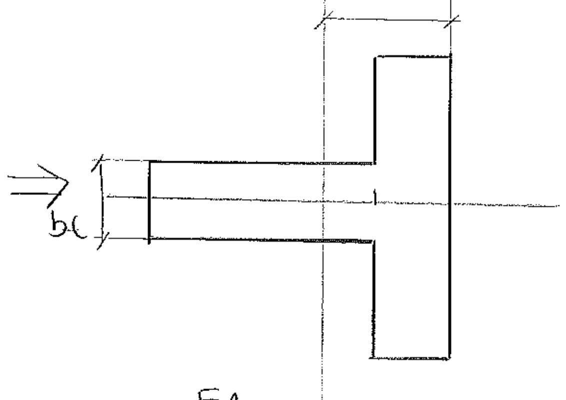
$E_a = 200 GPa$; $E_l = 100 GPa$

sección s-s



Z   Z'
Y_G

sección de acero equivalente

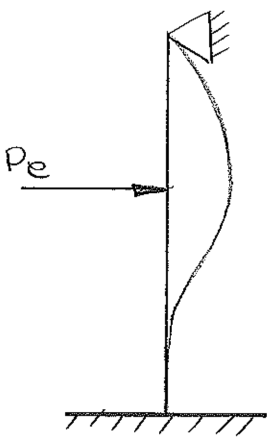


$b_t = b \cdot \frac{E_l}{E_a} = 20cm$.

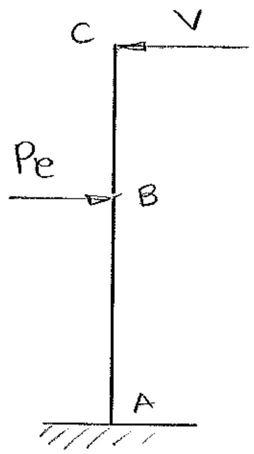
$$\left.\begin{array}{l} U_s = \frac{1}{2} M \cdot V^2 \\ W_e = \frac{1}{2} P_e \cdot \delta_e \end{array}\right\} \boxed{U_s = W_e}$$

$$Y_G = \frac{30cm \cdot 20cm \cdot 25cm + 10cm \cdot 40cm \cdot 5cm}{(30cm \cdot 20cm + 10cm \cdot 40cm)} = 17cm$$

$I_z' = 4{,}333 \cdot 10^{-3} m^4 \Rightarrow I_z = 1'44333 \cdot 10^{-3} m^4$



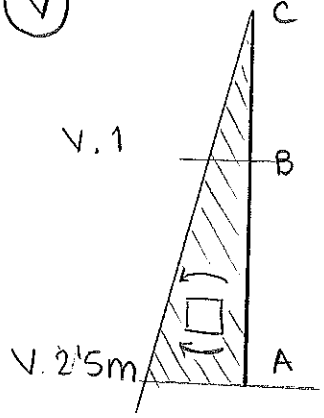
ESTRUCTURA HIPERESTÁTICA: (h=1) {H}



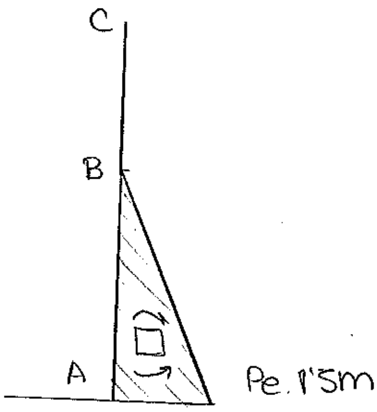
$\boxed{\text{Ec compatibilidad} : \delta_C = 0.}$

Diagramas de esfuerzos flectores

V



Pe



$$\delta_C = \delta_C^{(V)} - \delta_C^{(Pe)} = 0.$$

$$\delta_C^{(V)} = \frac{1}{EI}\left[\frac{1}{2} \cdot V.2'5m.2'5m.\frac{2}{3}2'5m\right] = \frac{125V}{24EI}$$

$$\delta_C^{(Pe)} = \frac{1}{EI}\left[\frac{1}{2} Pe.1'5m.1'5m.\left(\frac{2}{3}1'5m+1\right)\right] = \frac{9Pe}{4EI}$$

$$\delta_C^{(V)} = \delta_C^{(Pe)} \Rightarrow \frac{125V}{24} = \frac{9Pe}{4} \Rightarrow \boxed{V = 0'432.Pe}$$

$$\delta_B = \delta_B^{(Pe)} - \delta_B^{(V)}$$

$$\delta_B^{(Pe)} = \frac{1}{EI}\left[\frac{1}{2} Pe.1'5m\ 1'5m.\frac{2}{3}1'5m\right) = \frac{9Pe}{8EI}$$

$$\delta_B^{(V)} = \frac{1}{EI}\left[V.1'5m.\frac{1'5m}{2} + \frac{1}{2}V.1'5m.1'5m.\frac{2}{3}1'5m\right] = \frac{9V}{4EI} = \frac{0'972Pe}{EI}$$

$$\boxed{\delta_B = 0'153\frac{Pe}{EI}}$$

$$U_s = W_e$$

$$M.V^2 = Pe.\delta = Pe.0'153.\frac{Pe}{EI}$$

$$Pe = 2913794,104N$$

Esfuerzo flector máximo ⇒ A, B

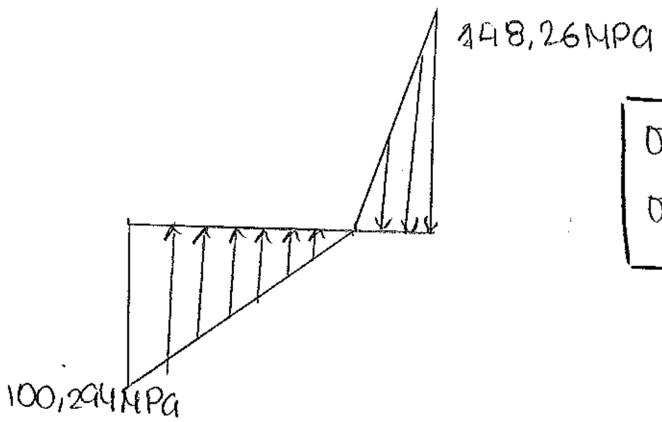
$$\left.\begin{array}{l} M_z(A) = 1223793{,}524\,Nm \\ M_z(B) = 1258759{,}053\,Nm \end{array}\right\} \text{(B)}$$

<u>Sección B</u>

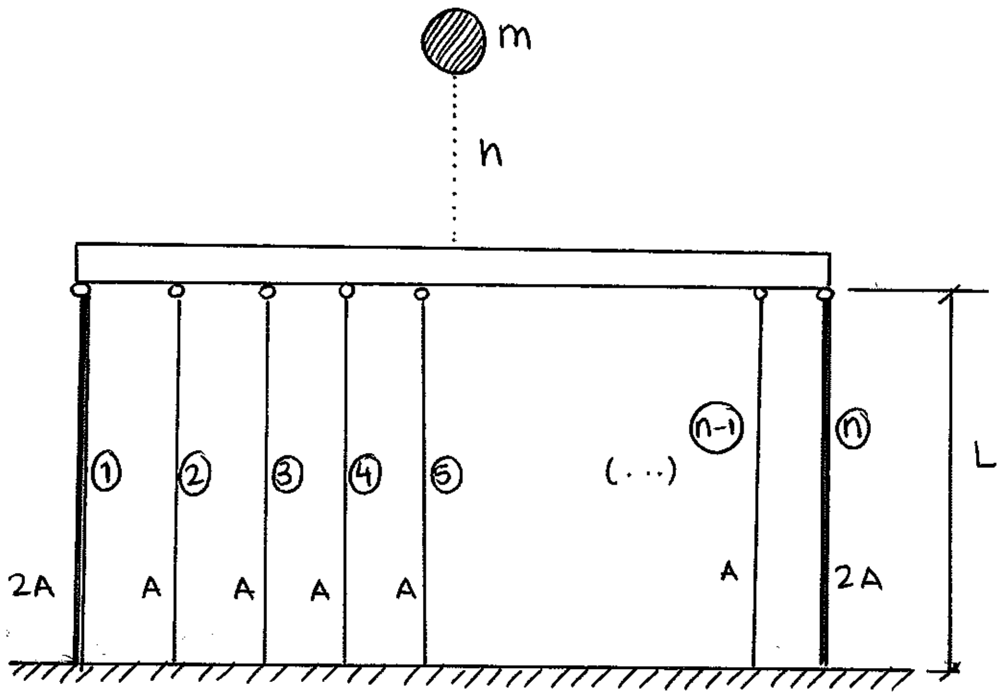
$$\sigma_{xx,\text{máx}}(\text{acero}) = \frac{M_z(B)\cdot 17cm}{I_z} = 148{,}26MPa$$

$$\sigma^{a}_{xx,\text{máx}}(\text{latón}) = \frac{M_z(B)\cdot 23cm}{I_z} = 200{,}588MPa$$

$$\sigma^{\ell}_{xx,\text{máx}}(\text{latón}) = \frac{E_L}{E_A}\cdot\sigma^{a}_{xx,\text{máx}}(\text{latón}) = 100{,}294MPa.$$



$$\boxed{\begin{array}{l} \sigma_{\text{máx}}(\text{acero}) = 148{,}26MPa \\ \sigma_{\text{máx}}(\text{latón}) = 100{,}294MPa \end{array}}$$



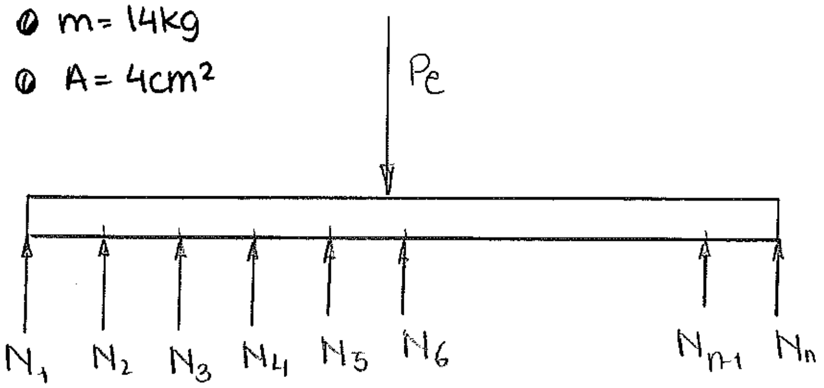
- $L = 1m$, $h = 1'5m$
- $g = 10 \ \frac{m}{s^2}$
- $E = 100 GPa$
- $m = 14kg$
- $A = 4cm^2$

$$\boxed{U_s = W_e}$$

$$I_e = \frac{1}{12}(2\sqrt{2}\ cm)^4 = 5{,}333 \cdot 10^{-8} m^4$$

$$I_i = \frac{1}{12}(2cm)^4 = 1'333 \cdot 10^{-8} m^4$$

$$L_K = 0'7L = 0'7m$$



$$\boxed{\Delta_1 = \Delta_2 = \ldots = \Delta n = \Delta}$$

$$\frac{N_1 \cdot L}{E \cdot A_1} = \frac{N_2 \cdot L}{E \cdot A_2} = (\ldots) = \frac{N_n \cdot L}{E A_n}$$

$$\begin{cases} N_1 = N_n = N_e \\ N_2 = N_3 = (\ldots) = N_{n-1} = \frac{1}{2} N_1 = N_i = \frac{1}{2} N_e \end{cases}$$

COLUMNAS EXTERIORES

$$P_{cr_e} = \frac{\pi^2 \cdot E \cdot I_e}{L_K^2} = 107424{,}2656\ N = N_e \Rightarrow N_i = 53712{,}1328 N > P_{cr_i}$$

COLUMNAS INTERIORES

$$P_{cr_i} = \frac{\pi^2 E \cdot I_i}{L_K^2} = 26856{,}0664 N =$$

$\boxed{\text{Las columnas interiores fallan primero por pandeo.}}$

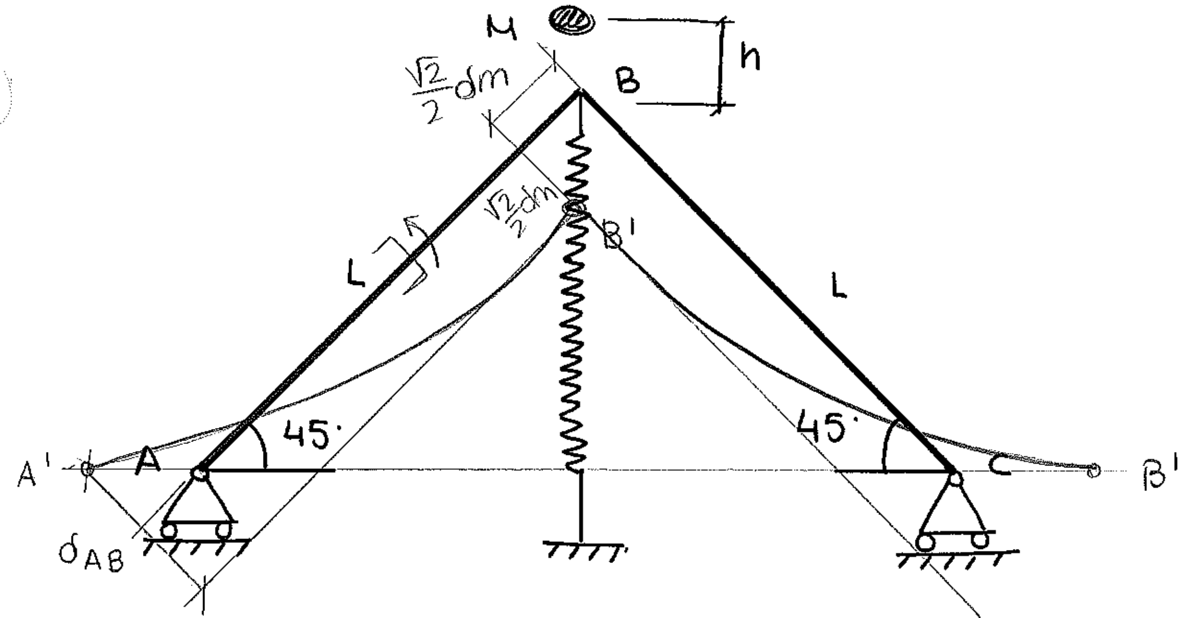
- Para que la estructura se derrumbe: las columnas exteriores han de fallar.

$N_e = 107424{,}2656\,N$.

$$W_e = 2 \cdot \frac{1}{2}(107424{,}2656\,N)^2 \frac{L}{E \cdot A_e} + (n-2)\frac{1}{2}(26856{,}0664\,N)^2 \frac{L}{EA_i} = U_s$$

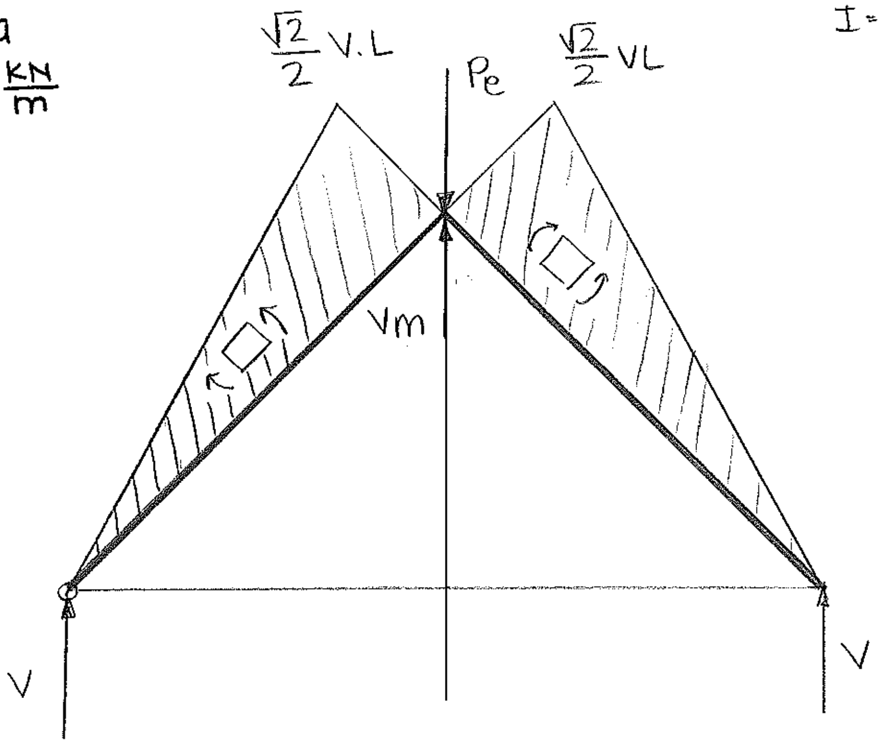
$$n = 9'293 \Rightarrow \boxed{n=10}$$

0610



- Sección cuadrada: $S = 10 \times 10 cm^2$
- $L = 1m$
- $E = 200 GPa$
- $k = 5000 \frac{KN}{m}$
- $h = 2m$
- $M = 10kg$

$I = \frac{1}{12}(10cm)^4 = 8{,}333 \cdot 10^{-6} m^4$



$$2V + V_m = P_e \Rightarrow \boxed{V_m = P_e - 2V}$$

Ec. compatibilidad: $\boxed{\delta m = \delta_B}$: $\frac{\sqrt{2}}{2}\delta m \cdot 2 = \delta_{AB}$: $\boxed{\delta_{AB} = \sqrt{2}\,\delta m}$

$$\delta_{AB} = \frac{1}{EI}\left[\frac{\sqrt{2}}{4} VL.L.\frac{2}{3}L\right] = \frac{\sqrt{2}VL^3}{6EI}$$

$$\delta m = \frac{V_m}{K} = \frac{P_e - 2V}{K}$$

$$\frac{P_e - 2V}{K}\cancel{\sqrt{2}} = \frac{\cancel{\sqrt{2}}VL^3}{6EI}$$

$$\boxed{P_e = \frac{VL^3}{6EI}.k + 2V}$$

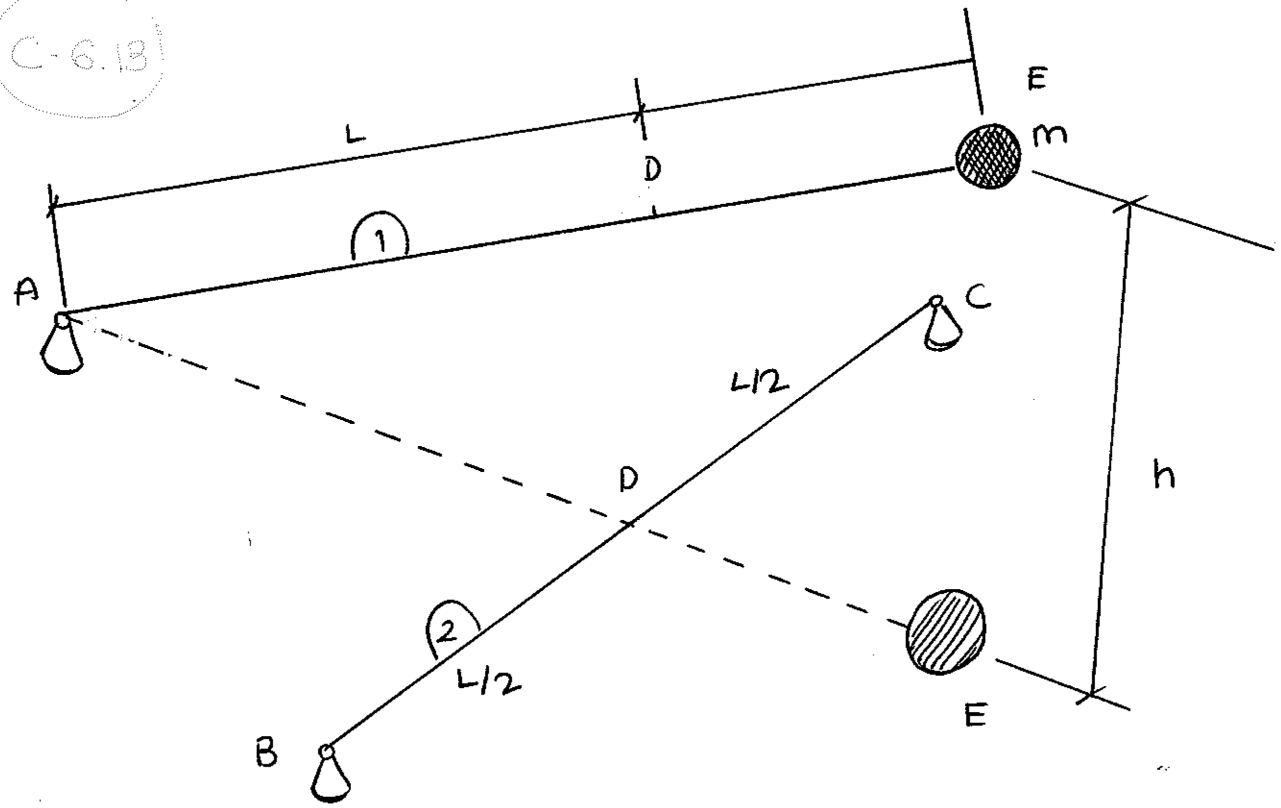
$$\left.\begin{array}{l} U_s = W_e \\ U_s = mgh \\ W_e = \frac{1}{2} P_e \cdot \delta m = \frac{1}{2}\left(\frac{VL^3}{6EI} k + 2V\right) \cdot \frac{VL^3}{6EI} \end{array}\right\}$$

$$\frac{VL^3}{12EI}\left(\frac{VL^3}{6EI} k + 2V\right) = 10.10.2m \Rightarrow V = 40.000N$$

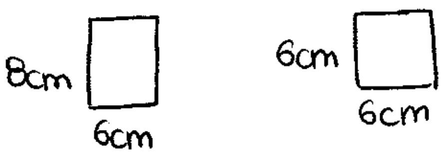
$$\delta m = 4.10^{-3} m$$

$$\boxed{\delta m = 4mm}$$

C-6.18



sección ① sección ②



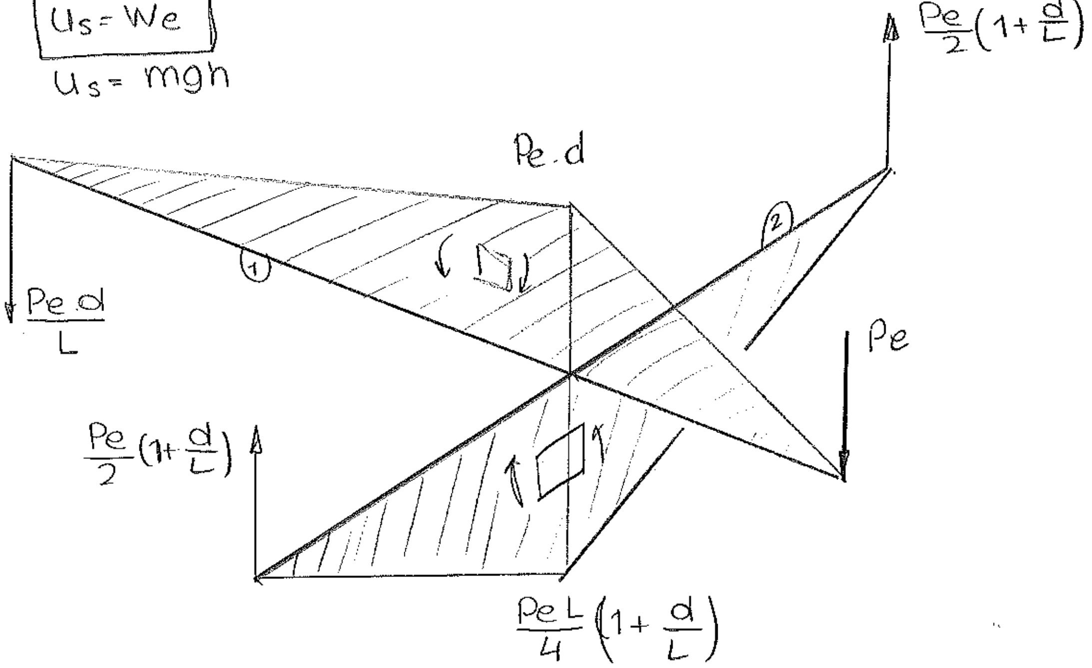
- $L = 1m$
- $E = 200GPa$
- $\sigma_r = 200MPa$
- $m = 10kg$, $h = 0'5m$

$I_1 = \frac{1}{12}\, 6cm\,(8cm)^3 = 2'56 \cdot 10^{-6} m^4$

$I_2 = \frac{1}{12}\,(6cm)^4 = 1'08 \cdot 10^{-6} m^4$

$\boxed{U_s = W_e}$

$U_s = mgh$



- Para que rompan simultaneamente $\sigma_{xx\,max}^{(adc)} = \sigma_{xx\,max}^{(bdc)}$.

$$\left.\begin{aligned} \sigma_{xx\,max}^{(adc)} &= \frac{Pe.d \cdot 4cm}{I_1} = Pe.d.15625 \\ \sigma_{xx,max}^{(bdc)} &= \frac{\left(\frac{PeL}{4}\right)\left(1+\frac{d}{L}\right)3cm}{J_2} = 6944,4\,PeL\left(1+\frac{d}{L}\right) \end{aligned}\right\}$$

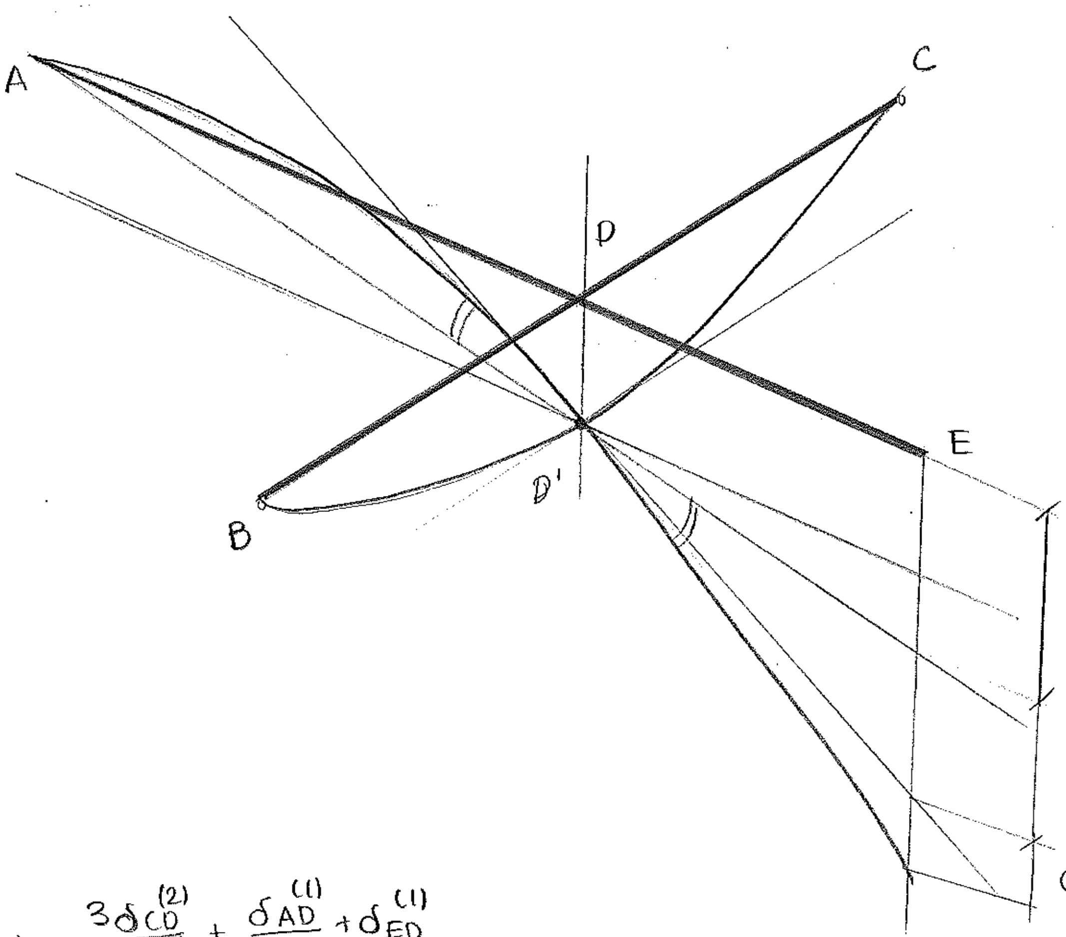
$15625d = 6944,444\,(1+\frac{d}{L}) \cdot L$

$15625d = 6944{,}444 \cdot L + 6944{,}444\, d$

$$\boxed{d = 0'8L}$$

$d = 0'5L$

$$\left.\begin{array}{l} W_e = \frac{1}{2} P_e \cdot \delta \\ U_s = mgh \end{array}\right\} \boxed{U_s = W_e}$$



$\frac{\delta_{CD}^{(2)}}{L} \cdot 1'5L$

$\frac{\delta_{AD}^{(1)}}{L} \cdot 0'5L$

$\delta_{ED}^{(1)}$

$$Y_E = \frac{3\delta_{CD}^{(2)}}{2} + \frac{\delta_{AD}^{(1)}}{2} + \delta_{ED}^{(1)}$$

$$\delta_{CD}^{(2)} = \frac{1}{EI}\left[\frac{1}{2}\ \frac{Pe.L.1'5}{4} \cdot \frac{L}{2} \cdot \frac{2}{3}\ \frac{L}{2}\right] = \frac{PeL^3}{32EI_2}$$

$$\delta_{AD}^{(1)} = \frac{1}{EI}\left[\frac{1}{2} Pe.0'5L \cdot L \cdot \frac{2}{3} L\right] = \frac{PeL^3}{6EI_1}$$

$$\delta_{ED}^{(1)} = \frac{1}{EI}\left[\frac{1}{2} Pe.0'5L.0'5L.\frac{2}{3} 0'5L\right] = \frac{PeL^3}{24EI_1}$$

$$\delta = \frac{Pe}{E}\left(\frac{L^3}{8I_1} + \frac{3L^3}{64I_2}\right)$$

$$mgh = \frac{1}{2E} P_e^2 \left(\frac{L^3}{8I_1} + \frac{3L^3}{64I_2} \right)$$

$$P_e = 14725{,}72779\,N$$

$$\sigma_{xx,máx}^{(odo)} = 115{,}045\,MPa$$

$$\sigma_{xx,máx}^{(bdc)} = 153{,}393\,MPa \Longrightarrow n = \frac{\sigma_f}{\sigma_{máx}} = 1'304$$

$$\boxed{n = 1'304}$$