TEOREMAS ENERGÉTICOS

1 ENERGÍA ELÁSTICA DE DEFORMACIÓN EN PIEZAS PRISMÁTICAS.

A) Esfuerzo axial.

$$U_0 = \frac{N_x^2}{2EA^2} \Rightarrow U_{(N_x)} = \frac{N_x^2 L}{2EA}$$

B) Momento flector

$$U_0 = \frac{M_z^2 y^2}{2EJ_z^2} \Rightarrow U_{(M_z)} = \int_L \frac{M_z^2}{2EJ_z} dx$$

C) Esfuerzo cortante.

$$U_0 = \frac{V_y^2 \cdot Q_z^2}{2Gb^2 I_z^2} \Rightarrow U_{(V_y)} = \int_L \frac{f_z V_y^2}{2GA} dx$$

D) Momento torsor

$$U_0 = \frac{M_x^2 r^2}{2GJ} \Rightarrow U_{(M_x)} = \int_L \frac{M_x^2}{2GJ} dx$$

2 TEOREMA DE CLAPEYRON

$$\boxed{W = \frac{1}{2} \sum_{k=1}^{n} Q_k \Delta_k = U}$$ (comportamiento elástico lineal)

3 TEOREMAS DE RECIPROCIDAD

* Teorema de reciprocidad de Rayleigh y Betti.

$$\boxed{\sum_{i=1}^{n} Q_i \cdot q_i^{(2)} = \sum_{j=1}^{m} P_j \cdot p_j^{(1)}}$$ (comportamiento elástico lineal)

* Teorema de reciprocidad de Maxwell.

$$\boxed{q^{(2)} = p^{(1)}}$$ (comportamiento elástico lineal)

4 PRIMER TEOREMA DE CASTIGLIANO

$$\boxed{Q_i = \frac{\partial U}{\partial \Delta_i} \quad (i = 1, 2 \ldots n)}$$

5) PRIMER TEOREMA DE ENGESSER Y SEGUNDO TEOREMA DE CASTIGLIANO.

* Primer teorema de Engesser

$$\boxed{\Delta_i = \frac{\partial U^*}{\partial Q_i} \quad (i = 1, 2 \ldots n)}$$

* Segundo teorema de Castigliano

$$\boxed{\Delta_i = \frac{\partial U}{\partial Q_i} \quad (i = 1, 2 \ldots n)}$$ (comportamiento elástico lineal)

6) SEGUNDO TEOREMA DE ENGESSER: TEOREMA DEL TRABAJO MÍNIMO.

(Estructuras hiperestáticas)

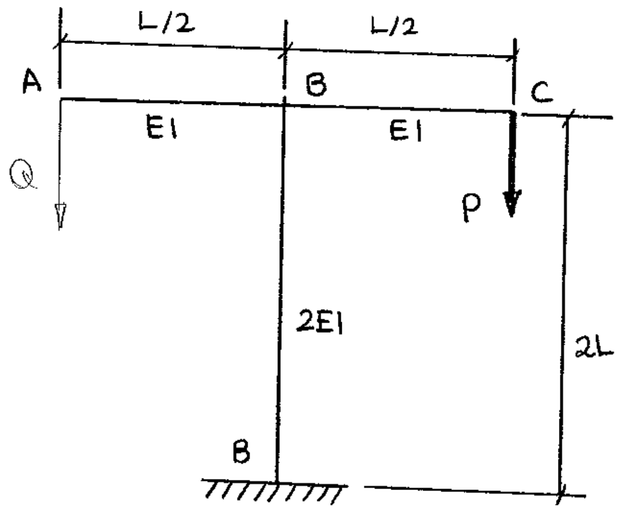
$$\boxed{\begin{array}{ll} \dfrac{\partial U^*}{\partial Q_i} = \Delta_i & (i = 1, 2 \ldots n) \\[2ex] \dfrac{\partial U^*}{\partial R_j} = 0 & (j = 1, 2 \ldots r) \end{array}}$$

* Teorema de Castigliano del trabajo mínimo

$$\boxed{\begin{array}{ll} \dfrac{\partial U}{\partial Q_i} = \Delta_i & (i = 1, 2 \ldots n) \\[2ex] \dfrac{\partial U}{\partial R_j} = 0 & (j = 1, 2 \ldots r) \end{array}}$$ (comportamiento elástico lineal)

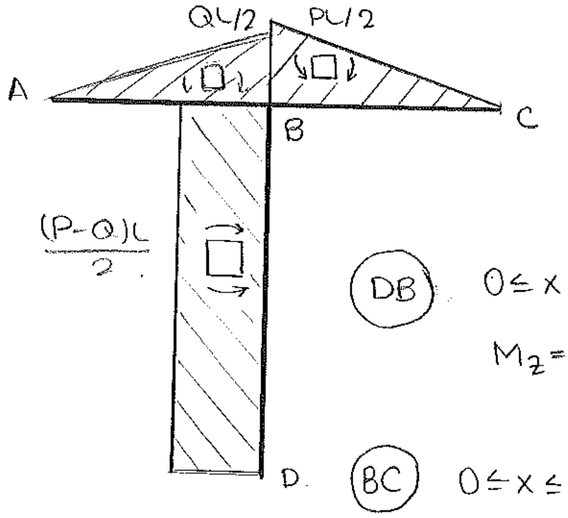
5. Teoremas energéticos

5.2



- se desprecian los efectos debidos a los esfuerzos axiales y cortantes

DIAGRAMAS DE ESFUERZOS



SEGUNDO TEOREMA DE CASTIGLIANO.

$$\partial_{Y_A} = \frac{\partial U}{\partial Q} = \frac{1}{EI}\int M_z \frac{\partial M_z}{\partial Q} dl \,;\; \partial_{Y_C} = \frac{\partial U}{\partial P}$$

(DB) $0 \le x \le 2L$

$$M_z = \frac{(P-Q)\cdot L}{2} \;;\; \frac{\partial U}{\partial Q} = -\frac{L}{2} \;;\; \frac{\partial U}{\partial P} = \frac{L}{2}$$

(BC) $0 \le x \le \frac{L}{2}$

$$M_z = P\,x \;;\; \frac{\partial U}{\partial Q} = 0 \;;\; \frac{\partial U}{\partial P} = x$$

(AB) $0 \le x \le \frac{L}{2}$

$$M_z = Q\cdot x \;;\; \frac{\partial U}{\partial Q} = x \;;\; \frac{\partial U}{\partial P} = 0$$

$$\delta Y_A = \frac{1}{2EI}\int_0^{2L} \frac{(P-Q)L}{2}\cdot\frac{(-L)}{2}\,dx + \frac{1}{EI}\int_0^{L/2} Qx^2\,dx =$$

$$= \frac{1}{2EI}\cdot\frac{(Q-P)L^2}{4}(2L) + \frac{1}{EI}\cdot\frac{Q\cdot(L/2)^3}{3}$$

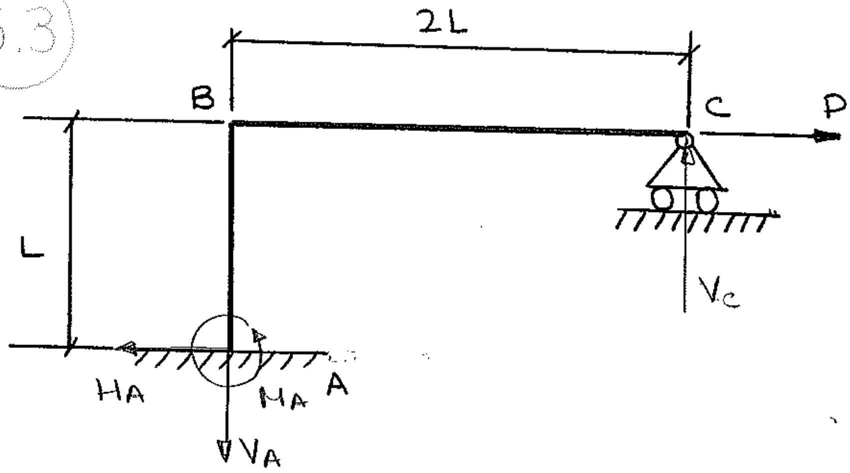
$$\delta Y_A = \{Q=0\} = -\frac{2PL^3}{8EI} = -\frac{PL^3}{4EI} \Longrightarrow \boxed{\overset{\uparrow}{Y_A} = \frac{PL^3}{4EI}}$$

$$\delta Y_C = \frac{1}{2EI}\int_0^{2L} \frac{(P-Q)L}{2}\,\frac{(+L)}{2}\,dx + \frac{1}{EI}\int_0^{L/2} Px^2\,dx =$$

$$= \frac{1}{2EI}\,\frac{(P-Q)L^2}{4}(2L) + \frac{1}{EI}\cdot\frac{P(L/2)^3}{3}$$

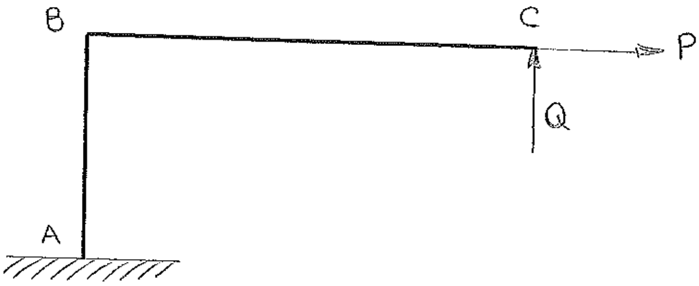
$$\delta Y_C = \{Q=0\} = \frac{7PL^3}{24EI} \Longrightarrow \boxed{\overset{\downarrow}{Y_C} = \frac{7PL^3}{24EI}}$$

5.3



- Rigidez a flexión: EI
- Se desprecian esfuerzos axiales y cortantes.

1) Hiperestático de grado 1. (h=1)



Ec. compatibilidad:
$\delta y_C = 0$

SEGUNDO TEOREMA DE CASTIGLIANO.

$$\delta y_C = \frac{\partial U}{\partial Q} = \frac{1}{EI}\int M_z \frac{\partial M_z}{\partial Q}\, dl$$

CB $(0 \le x \le 2L)$ $\quad M_z = Q \cdot x \,; \quad \frac{\partial M_z}{\partial Q} = x$

BA $(0 \le x \le L)$ $\quad M_z = P x - Q\, 2L \,; \quad \frac{\partial M_z}{\partial Q} = -2L$

$$\delta y_C = \frac{\partial U}{\partial Q} = \frac{1}{EI}\left[\int_0^{2L} Q x^2 dx + \int_0^{L} (Px - 2QL)(-2L)\, dx\right] = \frac{1}{EI}\left[\frac{Q(2L)^3}{3} - PL^3 + 4QL^3\right]$$

- Ec. Compatibilidad: $\delta y_C = \frac{1}{EI}\left[\frac{8QL^3}{3} - PL^3 + 4QL^3\right] = 0$

$$\frac{20}{3} Q = P \Rightarrow Q = \frac{3P}{20} = V_C$$

EQUILIBRIO.

$V_A = V_C = \frac{3P}{20}$

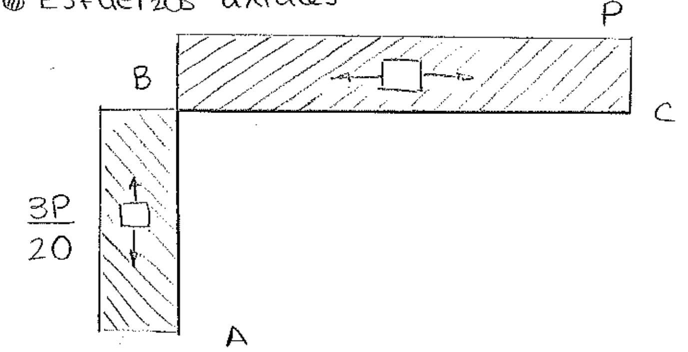
$H_A = P$

$M_A + V_C \cdot 2L = P \cdot L \Rightarrow M_A = P \cdot L - \frac{6PL}{20} = \frac{7}{10} PL$

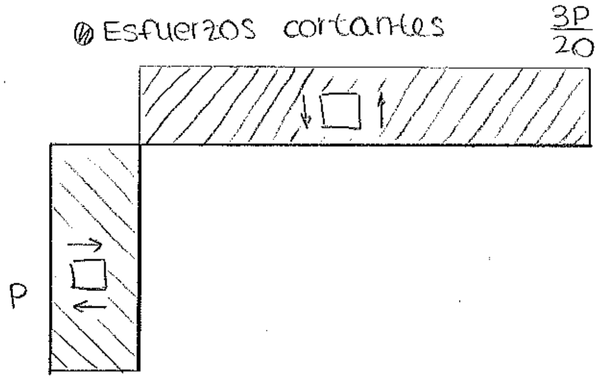
- Reacciones:
  $V_A = V_C = \frac{3P}{20}$
  $H_A = P$
  $M_A = \frac{7}{10} PL$

DIAGRAMAS DE ESFUERZOS

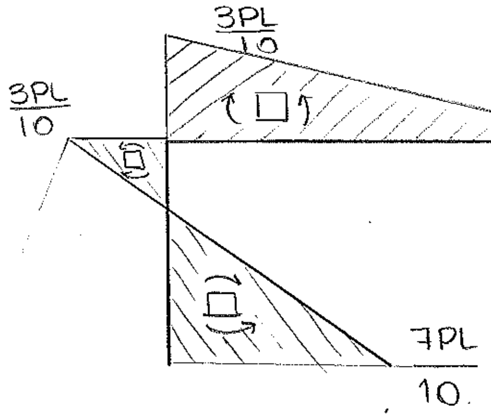
- Esfuerzos axiales



- Esfuerzos cortantes



- Esfuerzos flectores.



2) SEGUNDO TEOREMA DE CASTIGLIANO

$$\vec{\delta}_{x_C} = \frac{\partial U}{\partial P}$$

$$\frac{\partial U}{\partial P} = \frac{1}{EI}\int M_z \frac{\partial M_z}{\partial P}\,\partial l$$

(CB) $\quad 0 \le x \le 2L \qquad M_z = Q \cdot x = \frac{3P}{20}x\,, \quad \frac{\partial M_z}{\partial P} = \frac{3x}{20}.$

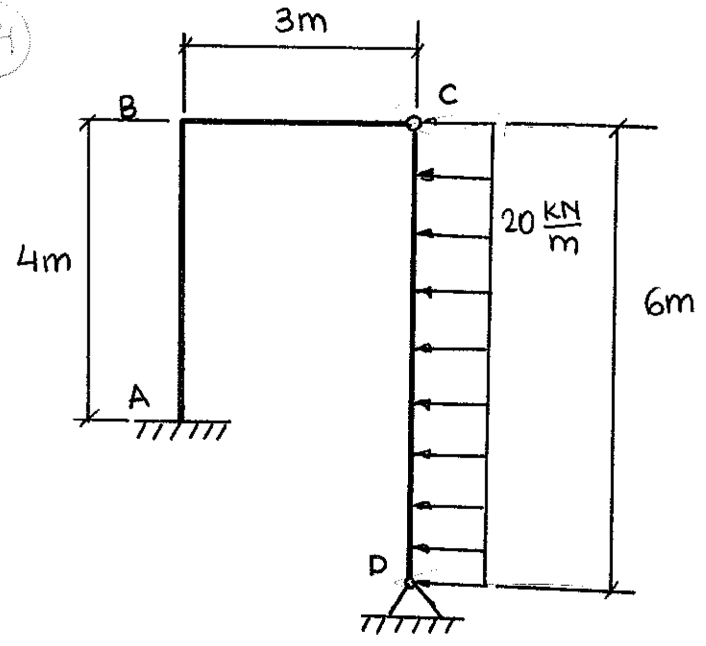
(BA) $\quad 0 \le x \le L \qquad M_z = Px - Q \cdot 2L = P\,x - \frac{3P}{10}L\,; \quad \frac{\partial M_z}{\partial P} = x - \frac{3L}{10}$

$$\frac{\partial U}{\partial P} = \frac{1}{EI}\left[\int_0^{2L} \frac{3P}{20}x\left(\frac{3x}{20}\right)dx + \int_0^{L}\left(Px - \frac{3P}{10}L\right)\left(x - \frac{3L}{10}\right)dx\right] =$$

$$= \frac{1}{EI}\left[\frac{9P}{400}\cdot\frac{8L^3}{3} + \frac{37PL^3}{300}\right] = \frac{11PL^3}{60.}$$

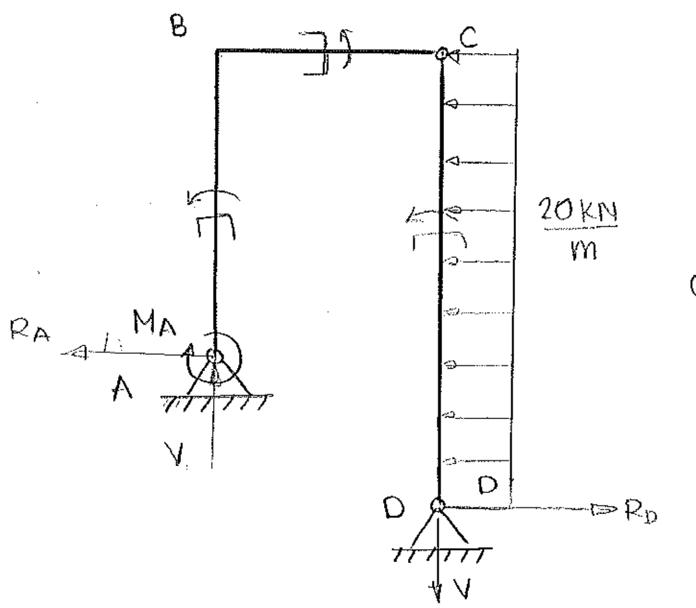
$$\boxed{\vec{\delta}_{x_C} = \frac{11PL^3}{60}}$$

5.4



- Rigidez a flexión : EI
- Se desprecian los esfuerzos axiales y cortantes.

- Estructura hiperestática de grado 1 h=1 (M_A)



Ec compatibilidad

$\theta_A = 0$

EQUILIBRIO

(1) $M_A + V \cdot 3m + R_A\, 4m = 0 \Rightarrow \boxed{V = \frac{240\,kNm - M_A}{3m}}$

(2) $20 \cdot 6\,kN \cdot 3m - R_D\, 6m = 0$

$\boxed{R_D = 60\,kN}$

(3) $R_D = R_A + 6 \cdot 20\,kN \Rightarrow \boxed{R_A = -60\,kN}$

SEGUNDO TEOREMA DE CASTIGLIANO.

$$\theta_A = \frac{\partial U}{\partial M_A} = \frac{1}{EI}\int M_z \frac{\partial M_z}{\partial M_A}\, dl$$

AB $\quad 0 \leq x \leq 4m$

$M_z = M_A - 60\,kN\,x \;;\quad \frac{\partial M_z}{\partial M_A} = 1$

$0 \leq x \leq 3m$

BC $\quad M_z = M_A - 60\,kN \cdot 4m + \frac{240\,kN\,m - M_A}{3}x \;;\quad \frac{\partial M_z}{\partial M_A} = 1 - \frac{x}{3}$

CD $\quad 0 \leq x \leq 6m$

$M_z = -20\,kN\, x \cdot \frac{x}{2} + 60\,kN\,x \,,\quad \frac{\partial M_z}{\partial M_A} = 0$

$$\theta_A = \frac{\partial U}{\partial M_A} = \frac{1}{EI}\int_0^4 (M_A - 60x)\,dx + \frac{1}{EI}\int_0^3 \left[(M_A - 240) + \frac{240 - M_A}{3}x\right]\left(1 - \frac{x}{3}\right)dx = 0$$

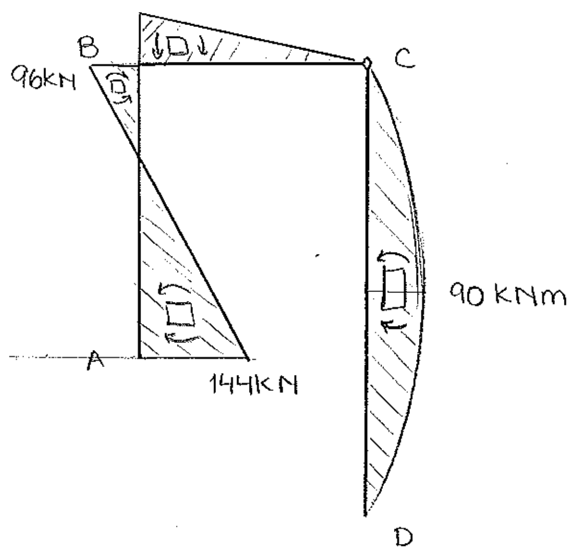
$$4M_A - \frac{4^2}{2}\,60 + [M_A - 240]\int_0^3 \left(1 - \frac{x}{3}\right)^2 dx = 0$$

$$4M_A - 480 + (M_A - 240)\int_0^3 \left(1 - \frac{2x}{3} + \frac{x^2}{9}\right)dx = 0$$

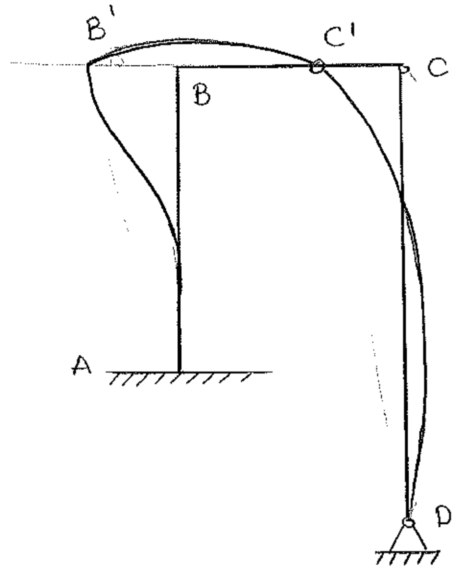
$$4M_A - 480 + (M_A - 240)\cdot\left(3 - \frac{2 \cdot 3^2}{3 \; 2} + \frac{3^3}{3 \; 9}\right) = 0$$

$$\boxed{M_A = 144\,\text{KN}}$$

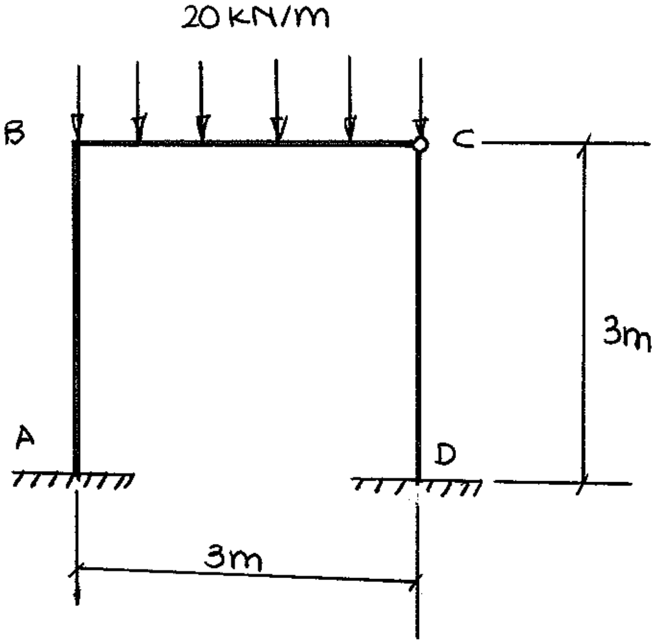
DIAGRAMA DE ESFUERZOS



DEFORMADA

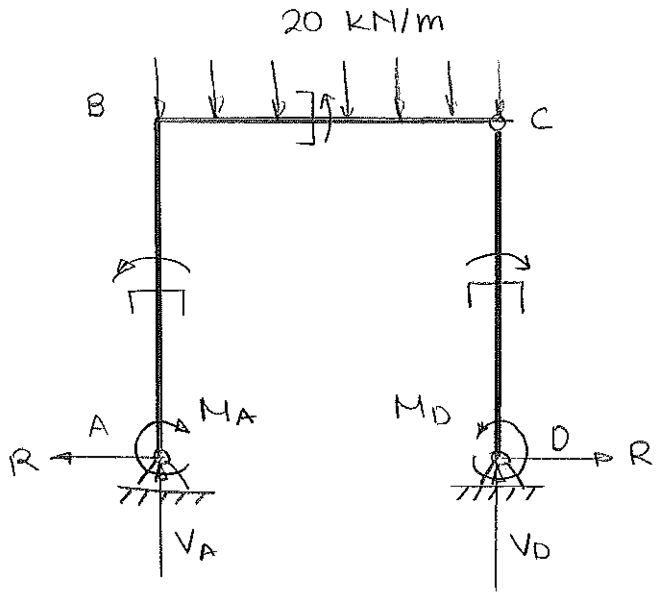


5.5



- Rigidez EI
- Se despreciará el efelcto de esfuerzos cortantes y axiales.

- ESTRUCTURA HIPERESTÁTICA: $n=2\,(M_A, M_B)$



Ec. compatibilidad
$\theta_A = 0$
$\theta_D = 0$

EQUILIBRIO

(1) $V_A + V_D = 60\,kN$

(2) $M_D + 3R = 0 \rightarrow \boxed{R = -\dfrac{M_D}{3m}}$

(3) $M_A + 3R + 3V_A - 60\,kN \cdot 1'5\,m = 0$

$M_A - M_D + 3V_A - 90\,kNm = 0 \Rightarrow \boxed{V_A = 30\,kN - \dfrac{M_A - M_D}{3m}}$

$$\boxed{V_D = 30\,kN + \frac{M_A - M_D}{3m}}$$

(AB) $0 \leq x \leq 3m$

$$M_z = M_A + Rx = M_A - \frac{M_D}{3}x$$

$$\frac{\partial M_z}{\partial M_A} = 1; \quad \frac{\partial M_z}{\partial M_D} = -\frac{x}{3}$$

(BC) $0 \leq x \leq 3$

$$M_z = M_A + 3R + V_A \cdot x - 20x \cdot \frac{x}{2} = M_A - M_D + 30x - \frac{M_A - M_D}{3}x - 10x^2$$

$$\frac{\partial M_z}{\partial M_A} = 1 - \frac{x}{3}; \quad \frac{\partial M_z}{\partial M_D} = -1 + \frac{x}{3}$$

(CD) $0 \leq x \leq 3$

$$M_z = M_D + R \cdot x = M_D - \frac{M_D}{3}x$$

$$\frac{\partial M_z}{\partial M_A} = 0; \quad \frac{\partial M_z}{\partial M_D} = 1 - \frac{x}{3}$$

$$\theta_A = \frac{\partial U}{\partial M_A} = \frac{1}{EI}\int M_z \frac{\partial M_z}{\partial M_A} dl = 0$$

$$\int_0^3 \left(M_A - \frac{M_D}{3}x\right)dx + \int_0^3 \left(M_A - M_D + 30x - \frac{M_A - M_D}{3}x - 10x^2\right)\left(1 - \frac{x}{3}\right)dx = 0.$$

$$3M_A - \frac{M_D}{3}\cdot\frac{3^2}{2} + \int_0^3 \left(M_A - M_D + 30x - 2\frac{M_A - M_D}{3}x - 20x^2 + \frac{10}{3}x^3 + \frac{M_A - M_D}{9}x^2\right)dx$$

$$3M_A - \frac{3}{2}M_D + 3M_A - 3M_D + 30\frac{3^2}{2} - 2\frac{M_A - M_D}{3}\cdot\frac{3^2}{2} - 20\frac{3^3}{3} + \frac{10}{3}\cdot\frac{3^4}{4} + \frac{M_A}{9}\cdot\frac{3^3}{3} - \frac{M_D}{9}3^3,$$

$$\boxed{4M_A - \frac{5}{2}M_D = -45/2}$$

$$\theta_B = \frac{\partial U}{\partial M_B} = \frac{1}{EI}\int M_z \frac{\partial M_z}{\partial M_B} dl = 0$$

$$\int_0^3 \left(M_A - \frac{M_D}{3}x\right)\left(-\frac{x}{3}\right)dx + \int_0^3 \left(M_A - M_D + 30x - \frac{M_A - M_D}{3}x - 10x^2\right)\left(-1 + \frac{x}{3}\right)dx + \int_0^3 M_D\left(1 - \frac{x}{3}\right)^2 dx = 0.$$

$$\int_0^3 \left(-M_A\frac{x}{3} + M_D\frac{x^2}{9}\right)dx - 3M_A + 3M_D - 30\cdot\frac{3^2}{2} + 2\frac{M_A - M_D}{3}\cdot\frac{3^2}{2} + 20\frac{3^3}{3} - \frac{10}{3}\cdot\frac{3^4}{4} - M_A + M_D$$

$$+ M_D\int_0^3 \left(1 - \frac{2x}{3} + \frac{x^2}{9}\right)dx = 0$$

$$-M_A \cdot \frac{3^2}{2 \cdot 3} + \frac{M_D}{9} \cdot \frac{3^3}{3} - 3M_A + 3M_D - 30 \cdot \frac{3^2}{2} + 2\frac{M_A - M_D}{3} \cdot \frac{3^2}{2} + 20\frac{3^3}{3} - \frac{10}{3} \cdot \frac{3^4}{4} - M_A + M_D$$

$$+ M_D \cdot 3 - M_D \cdot \frac{2}{3} \cdot \frac{3^2}{2} + M_D \cdot \frac{3^3}{9 \cdot 3} = 0$$

$$\boxed{-\frac{5}{2}M_A + 3M_D = +45/2}$$

$M_A = -1'9565 \text{ kNm}$

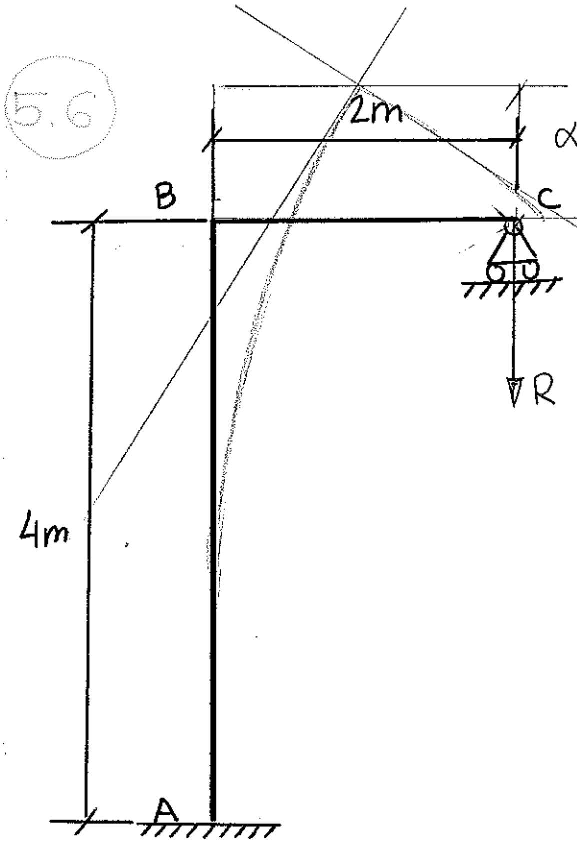
$M_B = +5'8696 \text{ kNm}$

$R = R_A = R_B = -1'9565 \text{ kN}$

$V_A = 32,6087 \text{ kN}$

$V_D = 27,3913 \text{ kN}$

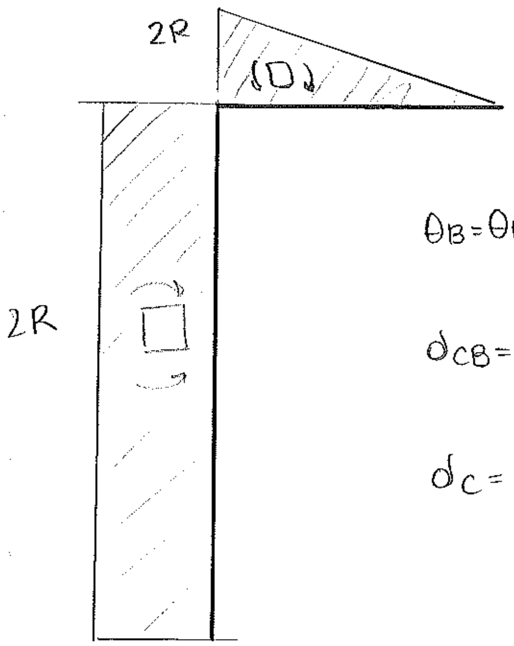
5.6



$\alpha . \Delta T . 4m$

- Sección : $6 \times 12 \, cm^2$
- $E = 200 GPa$
- $\alpha = 10^{-5} \, C^{-1}$; $\Delta T = 200 ^\circ C$
- Ec. compatibilidad $\boxed{\delta_C = \alpha \Delta T . 4m}$

DIAGRAMA DE ESFUERZOS FLECTORES



$$\delta_C = \theta_B . 2m + \delta_{CB}$$

$$\theta_B = \theta_{BA} = \frac{1}{EI}\left[2R . 4\right] = \frac{8R}{EI}$$

$$\delta_{CB} = \frac{1}{EI}\left[\frac{1}{\cancel{2}} \cancel{2}R . 2 . \frac{2}{3} 2\right] = \frac{8R}{3EI}$$

$$\delta_C = \frac{16R}{EI} + \frac{8R}{3EI} = 4 \alpha \Delta T$$

$$\boxed{R = 740{,}571 N}$$

$$\boxed{\theta_B = 3{,}429 . 10^{-3} rad}$$

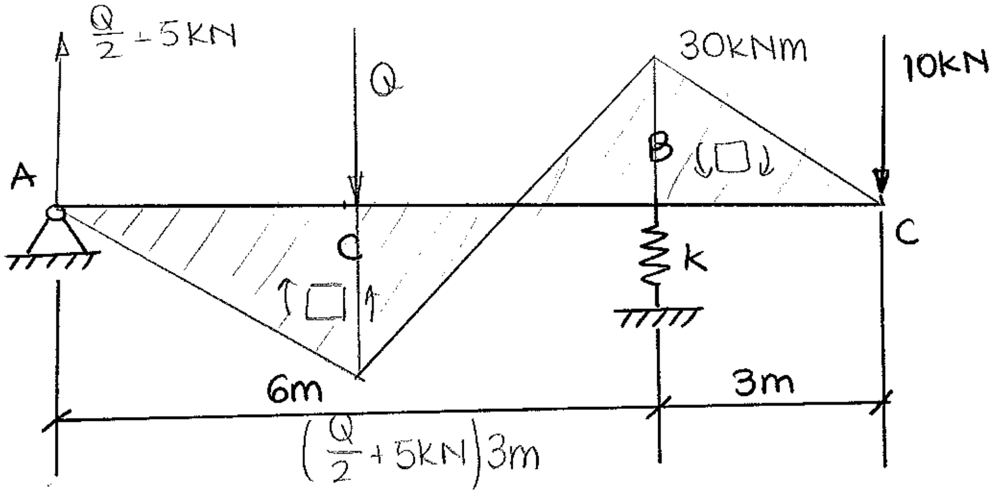
- Por energías...

$$\delta_C = \frac{\partial U}{\partial R}$$

$$U = \int_0^4 \frac{(2R)^2}{2EI} dx + \int_0^2 \frac{(Rx)^2}{2EI} dx = \frac{7R^2}{1296000}$$

$$\delta_C = \frac{7R}{648000} = \alpha \Delta T . 4m \Rightarrow \boxed{R = 740{,}571 N}$$

5.7



- $E = 200\,GPa$
- $I = 1450\,cm^4$

$\boxed{AC}$ $0 \leq x \leq 3m$

$$M_z = \left(\frac{Q}{2}+5KN\right)x \qquad \frac{\partial M_z}{\partial Q} = \frac{x}{2}$$

$\boxed{CB}$ $3m \leq x \leq 6m$

$$M_z = \left(\frac{Q}{2}+5KN\right)x - Q(x-3m) = -\frac{Q}{2}x + 3m\,Q + 5KNx.$$

$$\frac{\partial M_z}{\partial Q} = -\frac{x}{2}+3$$

$\boxed{BC}$ $0 \leq x \leq 3$

$$M_z = 10K \cdot x \qquad \frac{\partial M_z}{\partial Q} = 0$$

$$\Delta_c = \frac{\partial U}{\partial Q} = \frac{1}{EI}\int_0^3 \left(\frac{\cancel{Q}}{2}+5KN\right)\frac{x^2}{2}\,dx + \frac{1}{EI}\int_3^6 \left(-\frac{\cancel{Q}}{2}x + 3\cancel{Q} + 5x\right)\left(3-\frac{x}{2}\right)dx + \frac{15}{2K} = 0$$

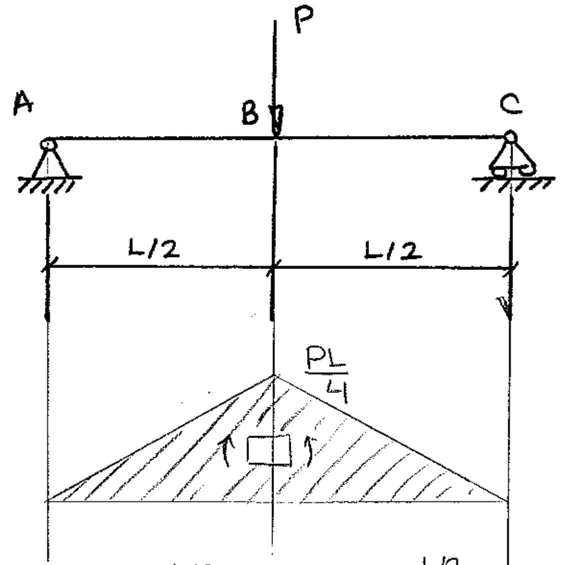
$$Um = \frac{1}{2}\frac{R^2}{K} = \frac{1}{2}\frac{\left(\frac{Q}{2}+15KN\right)^2}{K} = \frac{1}{2K}\left(\frac{Q^2}{4}+15Q+15^2\right)$$

$$\frac{\partial Um}{\partial Q} = \frac{+15}{2K} + \cancel{\frac{Q}{4K}}^{\,0}$$

$$\boxed{K = 322{,}22\,KN/m}$$

(...) Ponemos como incógnita 10KN y lo mismo.

C-5.1



- Rigidez: EI
- se desprecia el efecto del esfuerzo cortante.

A)

$$M_{z_1} = \frac{P}{2}x$$

$$M_{z_2} = -\frac{P}{2}x$$

$$U = \int_0^{L/2} \frac{M_{z_1}^2}{2EI_z}\,dz + \int_0^{L/2} \frac{M_{z_2}^2}{2EI_z}\,dz = \int_0^{L/2} \frac{(P^2/4)\,x^2}{2EI_z}\,dz + \int_0^{L/2} \frac{(P^2/4)x^2}{2EI_z}\,dz =$$

$$= 2 \cdot \int_0^{L/2} \frac{P^2}{4EI \cdot 2}\,x^2 dz = \frac{P^2}{4EI}\,\frac{(L/2)^3}{3} = \frac{P^2L^3}{96EI}$$

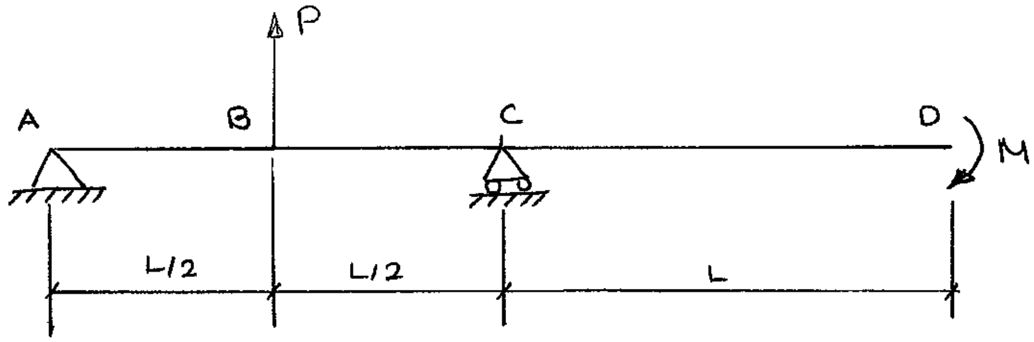
$$\boxed{U = \frac{P^2L^3}{96EI}}$$

B) $W = U = \frac{1}{2}P\,\delta_C$

$$\delta_C = \delta_{CB} = \frac{1}{EI}\left[\frac{1}{2}\,\frac{PL}{4}\cdot\frac{L}{\cancel{2}}\cdot\frac{\cancel{2}}{3}\,\frac{L}{2}\right] = \frac{PL^3}{48EI}$$

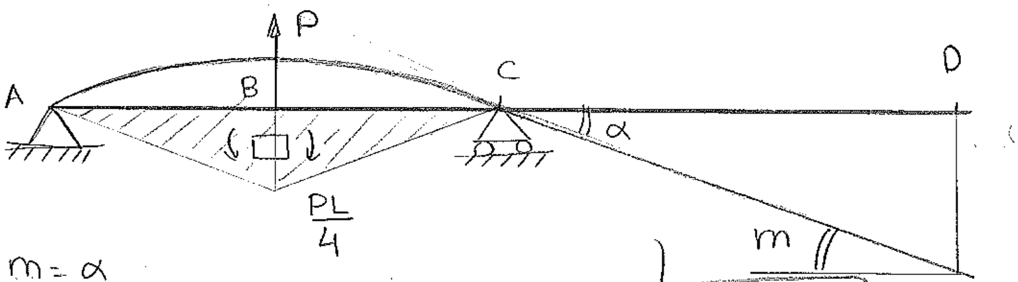
$$\boxed{U = \frac{P^2L^3}{96EI}}$$

C - 5.2



TEOREMA DE RECIPROCIDAD DE RAYLEIGH Y BETTI

$$\Sigma Q_i \cdot q_i^{(2)} = \Sigma P_j \cdot p_j^{(1)}$$



$$m = \alpha$$

$$\alpha = \frac{\delta_{AC}}{L}$$

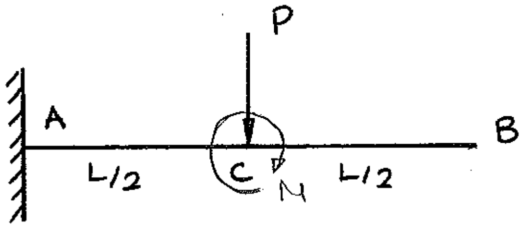
$$\delta_{AC} = \frac{1}{EI}\left(\frac{1}{2} \cdot \frac{PL}{4} \cdot \frac{L}{2} \cdot L\right) = \frac{PL^3}{16EI}$$

$$\boxed{m = \frac{PL^2}{16EI}}$$

$$M \cdot m = P \cdot p \Rightarrow p = \frac{M \cdot \frac{PL^2}{16EI}}{P} = \frac{ML^2}{16EI}$$

$$\boxed{y_B = p = \frac{ML^2}{16EI}}$$

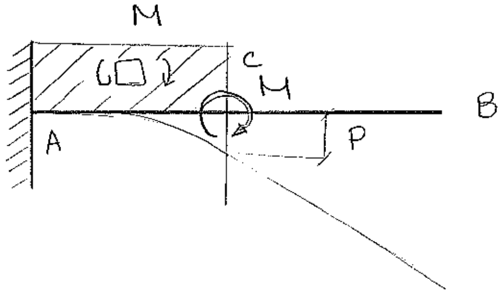
C-5.4



● Rigidez a flexión = EI

TEOREMA DE RECIPROCIDAD DE RAYLEIGH Y BETTI

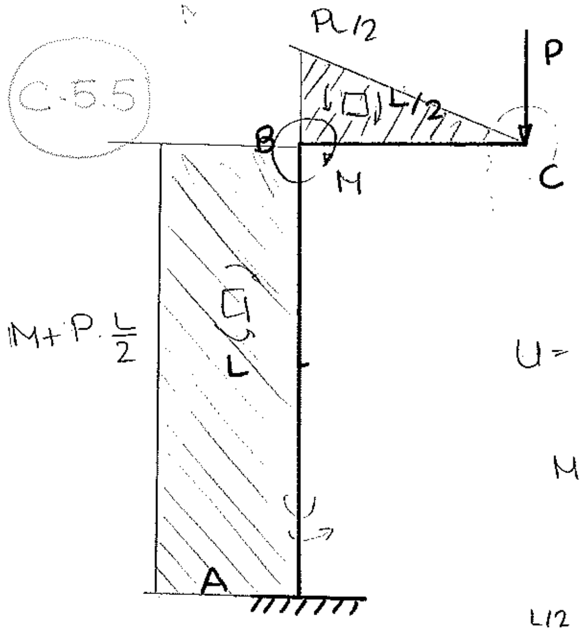
$M.m = P.p$



$$p = \delta_{CA} = \frac{1}{EI}\left[M \cdot \frac{L}{2} \cdot \frac{L}{4}\right] = \frac{ML^2}{8EI}$$

$$M.m = P.p. \Rightarrow m = \frac{P.p}{M} = \frac{PL^2}{8EI}$$

$$\boxed{\theta_C = m = \frac{PL^2}{8EI}}$$



SEGUNDO TEOREMA DE CASTIGLIANO.

$$\Delta i = \frac{\partial U}{\partial Q_i}$$

$$U = \int_0^L \frac{M_{z_1}^2}{2EI}\,dx + \int_0^{L/2} \frac{M_{z_2}^2}{2EI}\,dx$$

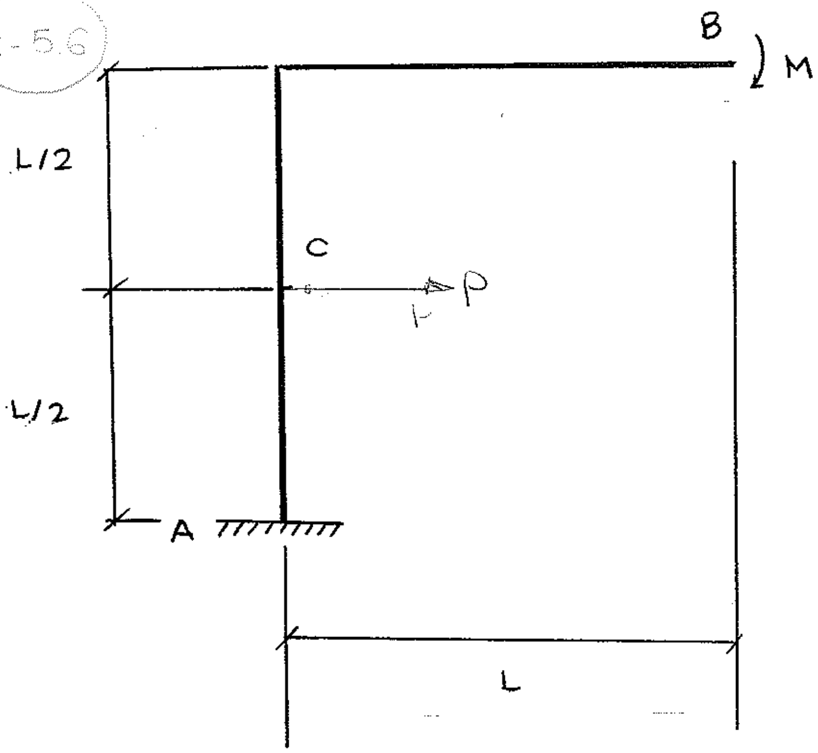
$$M_{z_1} = M + \frac{PL}{2} \;;\; M_{z_2} = P.x.$$

$$U = \int_0^L \frac{(M+PL/2)^2}{2EI}\,dx + \int_0^{L/2} \frac{(Px)^2}{2EI}\,dx = \frac{M^2 + MPL + (PL/2)^2}{2EI}\cdot L + \frac{P^2\,(L/2)^3}{6EI}$$

$$\theta_B = \frac{\partial U}{\partial M} = \frac{(2M+PL)L}{2EI}$$

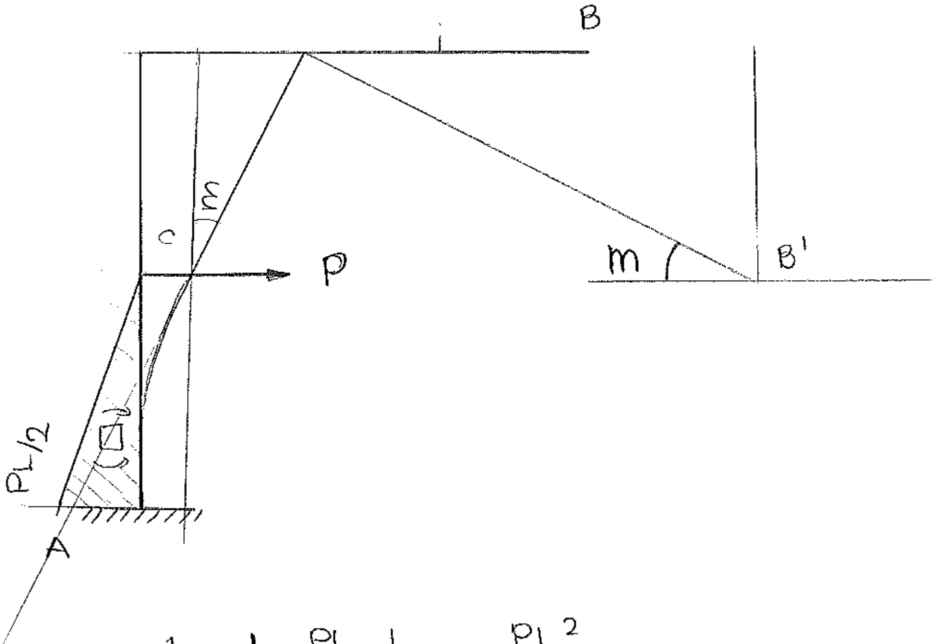
$$M=0 \Longrightarrow \boxed{\theta_B = \frac{PL^2}{2EI}}$$

C-5.6



Rigidez a flexión: EI

TEOREMA DE RECIPROCIDAD DE RAYLEIGH Y BETTI



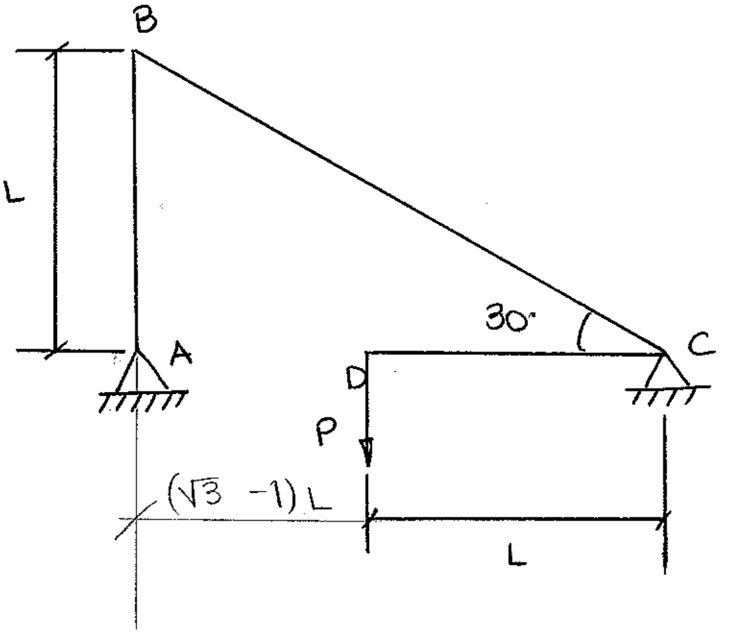
$$m = \theta_{CA} = \frac{1}{EI} \cdot \frac{1}{2} \cdot \frac{PL}{2} \cdot \frac{L}{2} = \frac{PL^2}{8EI}$$

$$P \cdot p = M \cdot m$$

$$p = \frac{M \; PL^2/(8EI)}{P} = \frac{ML^2}{8EI}$$

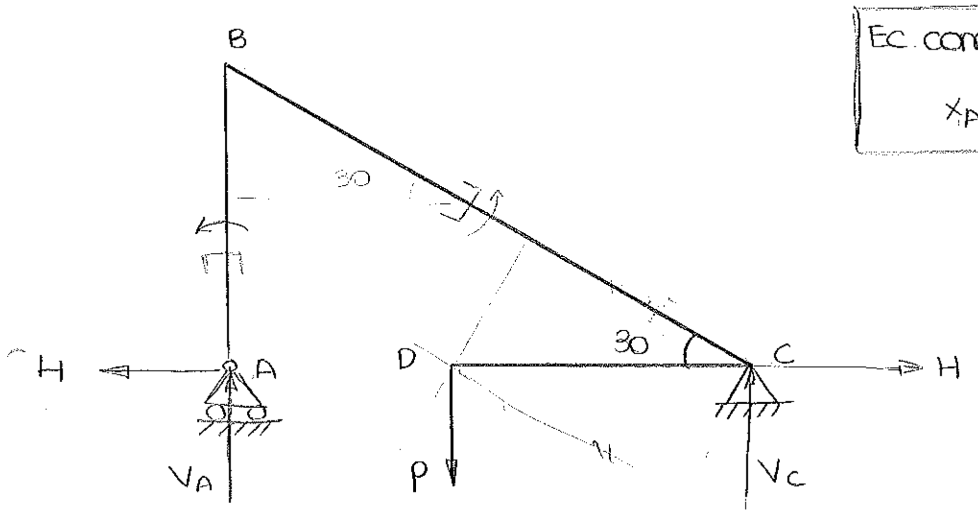
$$\boxed{\delta_C = p = \frac{ML^2}{8EI}}$$

C-5.7



- Se despreciará el efecto de los esfuerzos axial y cortante en el cálculo de las tensiones y deformaciones.
- $P = 32{,}4\,kN$
- $S = 10 \times 12\,cm^2$
- $L = 100\,cm$
- $E = 200\,GPa$, $\sigma_f = 270\,MPa$.

ESTRUCTURA HIPERESTÁTICA : h=1 (H)



Ec. compatibilidad:

$x_A = 0$.

EQUILIBRIO

$$M_A = 0:\ P(\sqrt{3}-1)L - V_C\sqrt{3}L = 0 \Rightarrow \boxed{V_C = \frac{P(\sqrt{3}-1)}{\sqrt{3}}}$$

$$\Sigma F_y = 0:\ V_A + V_C = P \Rightarrow V_A = P + \frac{1}{\sqrt{3}}P - P = \frac{P}{\sqrt{3}} \Rightarrow \boxed{V_A = \frac{P}{\sqrt{3}}}$$

(AB) $0 < x < L$

$$M_z = Hx, \quad \frac{\partial M_z}{\partial H} = x.$$

(BC) $0 < x < 2L$

$$M_z = V_A \cdot x \cdot \frac{\sqrt{3}}{2} + H \cdot \left(L - \frac{x}{2}\right)$$

$$\frac{\partial M_z}{\partial H} = \left(L - \frac{x}{2}\right)$$

(CD) $0 < x < L$

$$M_z = P \cdot x; \quad \frac{\partial M_z}{\partial H} = 0.$$

$$X_A = \frac{\partial U}{\partial H} = \int M_z \frac{\partial M_z}{\partial H} \partial l = 0$$

$$\frac{1}{EI}\int_0^L Hx^2 dx + \frac{1}{EI}\int_0^{2L}\left[\frac{Px}{2} + H\left(L - \frac{x}{2}\right)\right]\left(L - \frac{x}{2}\right)dx = 0.$$

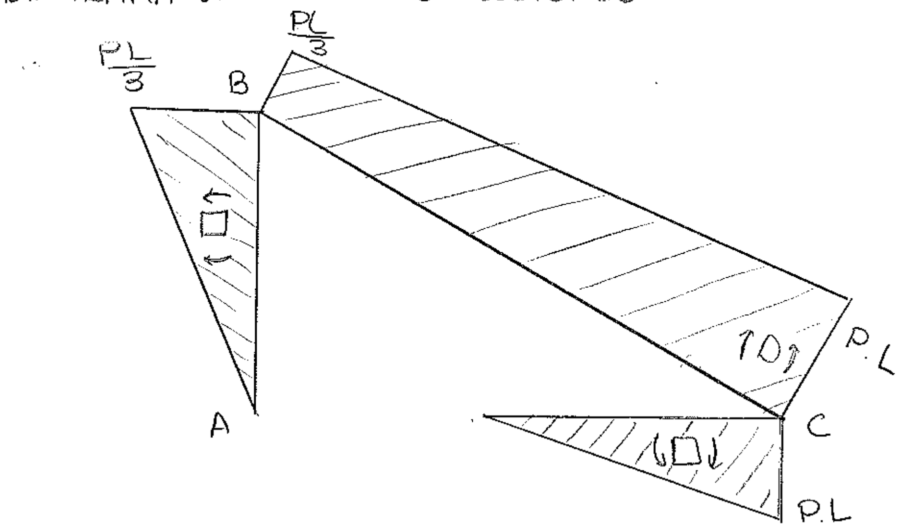
$$H \cdot \frac{L^3}{3} + \int_0^{2L} \frac{P}{2}\left(xL - \frac{x^2}{2}\right)dx + \int_0^{2L} H\left(L - \frac{x}{2}\right)^2 dx =$$

$$= H \cdot \frac{L^3}{3} + \frac{P}{2}\left[L \cdot \frac{(2L)^2}{2} - \frac{(2L)^3}{2 \cdot 3}\right] + HL^2(2L) - HL\frac{(2L)^2}{2} + \frac{H}{4} \cdot \frac{(2L)^3}{3} = 0.$$

$$H\left[\frac{L^3}{3} + 2L^3 - \frac{4L^3}{2} + \frac{8L^3}{12}\right] + \frac{P}{2}\left[\frac{4L^2}{2} - \frac{8L^3}{6}\right] = 0.$$

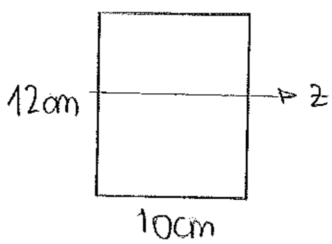
$$\boxed{H = \frac{-1}{3}P}$$

DIAGRAMA DE MOMENTOS FLECTORES.



Esfuerzo máximo: C.

$M_z = P.L$



$$I_z = \frac{1}{12} 10cm\,(12cm)^3 = 1'44\,10^{-5} m$$

$$\sigma_{xx}(máx) = \frac{P.L.\,6cm}{I_z} = 135MPa$$

$$n = \frac{\sigma f}{\sigma_{xx} max} = 2$$

$$\boxed{n=2}$$

SEGUNDO TEOREMA DE CASTIGLIANO

$$\delta_D = \frac{\partial U}{\partial P} = \frac{1}{EI}\int M_z \frac{\partial M_z}{\partial P}\partial l$$

(AB) $0 < x < L$

$$M_z = H.x = -\frac{P}{3}x \; ; \; \frac{\partial M_z}{\partial P} = -\frac{x}{3}$$

(BC) $0 < x < 2L$

$$M_z = \frac{Px}{2} + \frac{P}{3}\left(L - \frac{x}{2}\right) = -\frac{PL}{3} + 2\frac{Px}{3}$$

$$\frac{\partial M_z}{\partial P} = \frac{2x + L}{3}$$

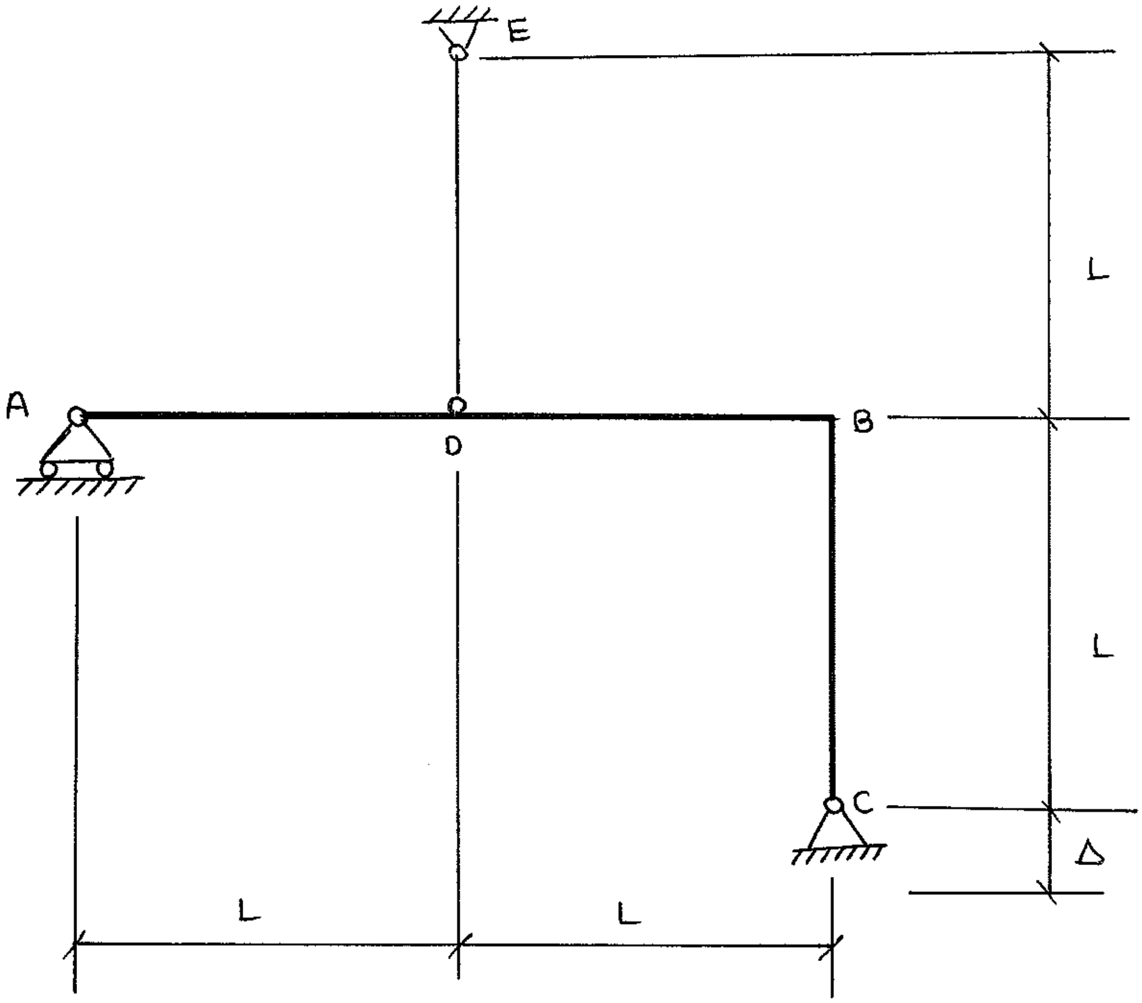
(CD) $0 < x < L$

$$M_z = P.x \; ; \; \frac{\partial M_z}{\partial P} = x$$

$$\delta_D = \frac{1}{EI}\left[\int_0^L \frac{P}{3}x \cdot \frac{x}{3}dx + \int_0^{2L}\left(-\frac{PL}{3} + 2\frac{Px}{3}\right)\frac{2x+L}{3} + \int_0^L Px^2 dx\right] = 0'01m$$

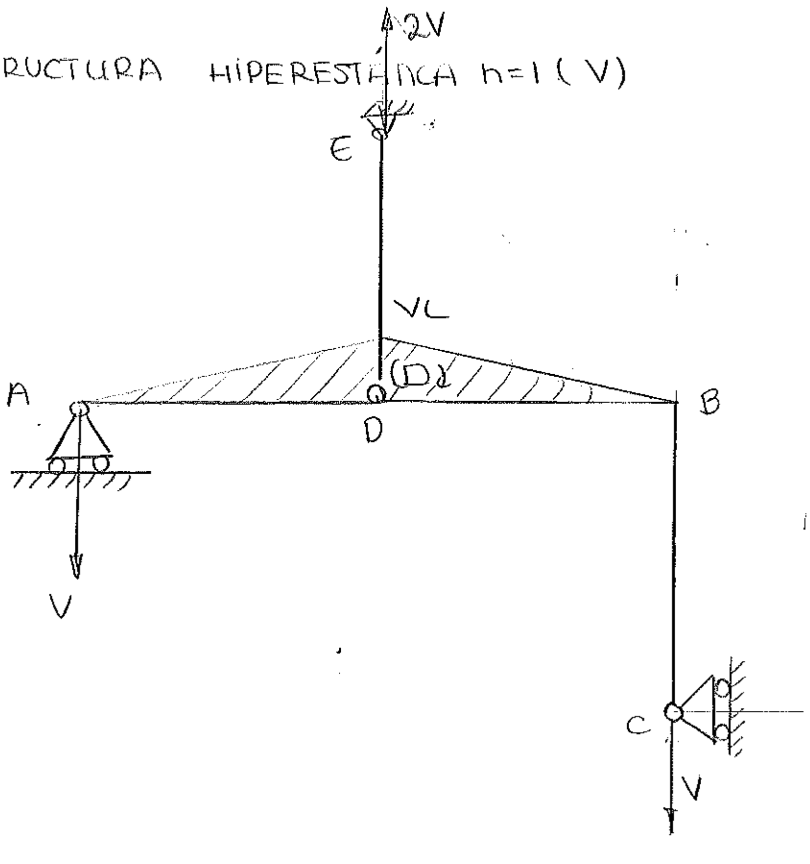
$$\boxed{\delta_D = 1 \text{ cm}}$$

C-58



- Barras: $S = 8 \times 12 cm^2$
- Tirante: $S = 2cm \times 2cm$
- $L = 1m$
- $E = 200 GPa$; $\sigma_f = 280 MPa$.

ESTRUCTURA HIPERESTÁTICA h=1 (V)



Ec. compatibilidad:

$$\delta_C = \Delta$$

$$U = 2\int_0^L \frac{M_z^2}{2EI}\,dx + \frac{N_x^2 L}{2EA_T} = 2\int_0^L \frac{V^2x^2}{2EI}\,dx + \frac{4V^2L}{2EA_T} =$$

$$= \frac{V^2}{EI}\cdot\frac{L^3}{3} + \frac{2V^2L}{EA_T}$$

$$\delta_c = \frac{\partial U}{\partial V} = \frac{2VL^3}{3EI} + \frac{4VL}{EA_T} = \Delta$$

$$V\left(\frac{2L^3}{3EI} + \frac{4L}{EA_T}\right) = \Delta \Rightarrow V = \frac{\Delta}{\left(\frac{2L^3}{3EI} + \frac{4L}{EA_T}\right)} = 2946793{,}997\,\Delta\ (N)$$

$$A_T = (2cm)^2 = 4\cdot10^{-4}m^2$$

$$I = \frac{1}{12}8cm(12cm)^3 = 1'152\cdot10^{-5}$$

$$N_x = 2V = 5893587{,}995\,\Delta(N) \Rightarrow \boxed{N = 5893'588\,\Delta\ (kN)}$$

$$\sigma_{xx} = \frac{N_x}{A_T} = 147{,}3397\,\Delta\ (MPa)$$

ESFUERZO AXIAL

$$\sigma_{xx} = 147{,}3397\,\Delta \qquad \frac{\sigma_f}{\sigma_{xx}} = n \Rightarrow \frac{\sigma_f}{147{,}3397\Delta} = 1'3 \Rightarrow \Delta_{máx} = 1'4618m$$

ESFUERZO FLECTOR

$$M_z = V\cdot L = 2946793{,}997\,\Delta\ (Nm)$$

$$\sigma_{xx} = \frac{M_z\cdot y}{I_z} = 1'53478854\cdot10^{10}\,\Delta\ (Pa) = 15347{,}8854\,\Delta\ (MPa)$$

$$\frac{\sigma_f}{\sigma_{xx}} = 1'3 \Rightarrow \Delta_{máx} = 0'014m = 14mm$$

$\boxed{\Delta_{máx} = 14mm \text{ Fallo en D por flexión}}$