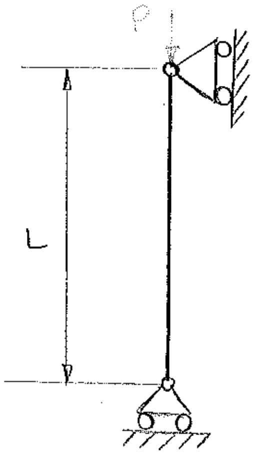
PANDEO

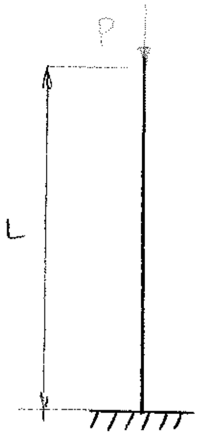
① COLUMNA DE EULER



$$\boxed{P_{cr} = \frac{\pi^2 E I}{L^2}}$$

$$\boxed{L_k = L}$$

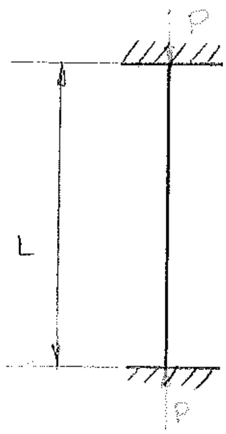
② COLUMNA EMPOTRADA Y LIBRE



$$\boxed{P_{cr} = \frac{\pi^2 E I}{4L^2}}$$

$$\boxed{L_k = 2L}$$

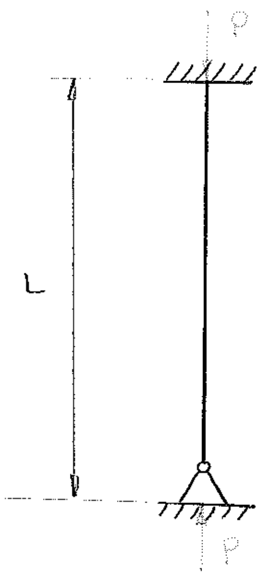
③ COLUMNA CON AMBOS EXTREMOS EMPOTRADOS



$$\boxed{P_{cr} = \frac{4\pi^2 E I}{L^2}}$$

$$\boxed{L_k = \frac{L}{2}}$$

④ COLUMNA EMPOTRADA EN UN EXTREMO Y ARTICULADA EN EL OTRO.

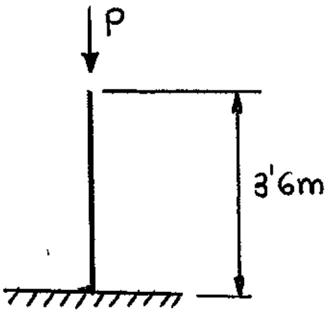


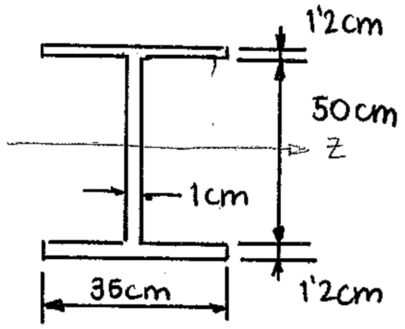
$$\boxed{P_{cr} = \frac{\pi^2 E I}{(0'7L)^2}}$$

$$\boxed{L_k \approx 0'7L}$$

4. Inestabilidad: Pandeo.

4.1





- Acero S-235
- E = 210GPa
- σ_f = 240MPa

TABLA 4.2

{ Perfil laminado en I
S-235
$\frac{h}{b} = \frac{52'4cm}{35cm} = 1'4971 > 1'2$

$t = 1'2cm < 40mm$

$I_z = \frac{1}{12} 35cm (52'4cm)^3 - 2 \frac{1}{12} \cdot 17cm (50cm)^3 = 6'547699 \, 10^{-4} m^4$

$I_y = \frac{1}{12} \cdot 50cm \cdot (1cm)^3 + 2 \frac{1}{12} \, 1'2cm (35cm)^3 = 8'579167 \cdot 10^{-5} m^4$

$I_y < I_z \Rightarrow$ Se producirá el fenómero de pandeo en el plano xz

- La curva de pandeo es la b. $\boxed{\alpha = 0'34}$

$L_K = 2L = 7'2m$; $A = 2 \cdot 1'2cm \, 35cm + 1cm \cdot 50cm = 0'0134m^2$

$P_{cr} = \frac{\pi^2 E I}{L_k^2} = 3430039'877N$

$\bar{\lambda} = \sqrt{\frac{A \, \sigma_f}{P_{cr}}} = 0'968297 \quad \lambda \to A4$

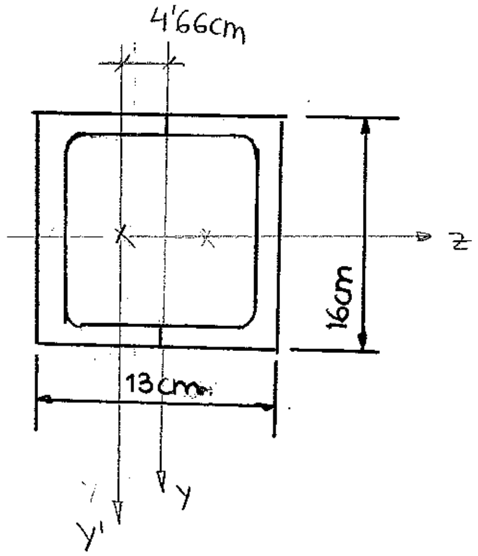
$$\Phi = 0'5\left[1 + \alpha \cdot (\bar{\lambda} - 0'2) + \bar{\lambda}^2\right] = 1'09941$$

$$\chi = \frac{1}{\Phi + \sqrt{\Phi^2 - \bar{\lambda}^2}} = 0'61725$$

$$P_b = \chi \cdot A \cdot \sigma_f = 1985080,652 N$$

$$\boxed{P_b = 1985,081 kN}$$

4.2



- Dos perfiles UPN-160
- Extremos articulados : $L = 8m$
- Acero S-235
- $E = 210 GPa$
- ~~2100~~ $\sigma_f = 240 MPa$

TABLA 4.2 → Agrupación de perfiles laminados soldados → C : $\boxed{\alpha = 0'49}$

$I_z = 2.925 cm^4 = 1'85 . 10^{-5} m^4$

$\left.\begin{array}{l} I_{y'_1} = 85'3 cm^4 \\ A_1 = 24 cm^2 \end{array}\right\} I_{y_1} = 85'3 cm^4 + 24 cm^2 . (4'66 cm)^2 = 606,4744 cm^4 = 6'064744 . 10^{-6} m^4$

$I_y = 2 I_{y_1} = 1'2129488 . 10^{-5} m^4$

- $I_y < I_z$: El pandeo se produce en la sección xz

$\left.\begin{array}{l} P_{cr} = \dfrac{\pi^2 E I_y}{L_k^2} \\ L_k = L = 8m \end{array}\right\} P_{cr} = 392809,0955 N$

$\left.\begin{array}{l} \bar{\lambda} = \sqrt{\dfrac{A \sigma_f}{P_{cr}}} \\ A = 2A_1 = 48 cm^2 \end{array}\right\} \bar{\lambda} = 1'7125191 \to A$

$\phi = 0'5 [1 + \alpha (\bar{\lambda} - 0'2) + \bar{\lambda}^2] = 2'3369284 \to B$

$\chi = \dfrac{1}{\phi + \sqrt{\phi^2 - \bar{\lambda}^2}} = 0'25464$

$P_b = \chi . A . \sigma_f = 293'349 kN$

$\boxed{P_b = 293'349 kN}$

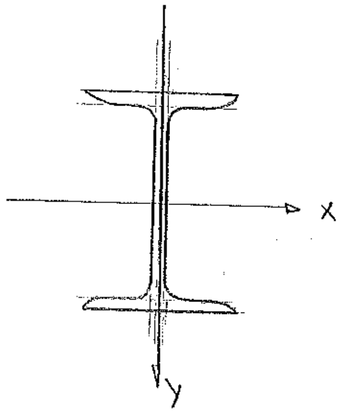
4.3

- Perfil IPN
- $L = 4m$
- $P = 300 kN$
- S-235 ; $E = 210 GPa$; $\sigma_f = 240 MPa$

1) EXTREMOS EMPOTRADOS.

$L_K = \frac{L}{2} = 2m$

1 Elegimos IPN-100



$\begin{cases} I_x = 171 cm^4 \\ I_y = 12'2 cm^4 \end{cases}$

$A = 10'6 cm^2$
$h = 100$
$b = 50$
$t < 40mm$

$I_y < I_x$ El pandeo se produce en el plano xz

$P_{cr} = \frac{\pi^2 E I_y}{L_K^2} = 63214'81619 \ N$

TABLA 4.2 $\begin{cases} \frac{h}{b} = 2 > 1'2 \\ t < 40mm \\ \text{eje de pandeo } y \end{cases}$ $\Rightarrow$ curva b $\boxed{\alpha = 0'34}$

$\bar{\lambda} = \sqrt{\frac{A \sigma_f}{P_{cr}}} = 2{,}006$ {→A}

$\phi = 0'5 [1 + \alpha(\bar{\lambda} - 0'2) + \bar{\lambda}^2] = 2'8192$ {→B}

$\chi = \frac{1}{\phi + \sqrt{\phi^2 - \bar{\lambda}^2}} = 0'2083$

$P_b = \chi \cdot A \cdot \sigma_f = 52999{,}59 N < 300 kN$ (No nos sirve)

2 Elegimos IPN-180

$I_x = 1450 cm^4$
$I_y = 81'3 cm^4$

$A = 27'9 cm^2$
$h = 180 cm$
$b = 82 cm$
$t < 40 mm$

$I_y < I_x$: El pandeo se produce en el plano xz

$$P_{cr} = \frac{\pi^2 E I}{L_k^2} = 421259{,}3898$$

TABLA 4.2 $\left\{ \begin{array}{l} \frac{h}{b} = 2'195 > 1'2 \\ t < 40\text{mm} \\ \text{eje de pandeo } y \end{array} \right.$ $\Rightarrow$ curva a, $\alpha = 0'34$

$$\bar{\lambda} = \sqrt{\frac{A\sigma_f}{P_{cr}}} = 1'26076 \ \{\rightarrow A\}$$

$$\phi = 0'5\,[1 + \alpha(\bar{\lambda} - 0'2) + \bar{\lambda}^2] = 1'47511 \ \{\rightarrow B\}$$

$$\chi = \frac{1}{\phi + \sqrt{\phi^2 - \lambda^2}} = 0'44626$$

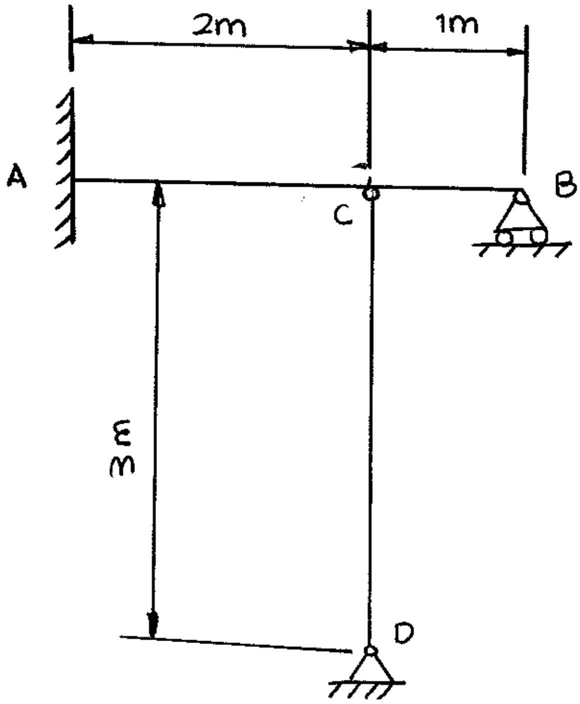
$P_b = \chi \cdot \sigma_f \cdot A = 298817{,}012 \text{ kN} > P$. No nos sirve; el inmediatamente superior sí. (habría que repetir el proceso y comprobar)

$\boxed{\text{Necesitamos un perfil IPN-200}}$

2) EXTREMOS ARTICULADOS

sería lo mismo pero con $L_k = L = 8\text{m}$.

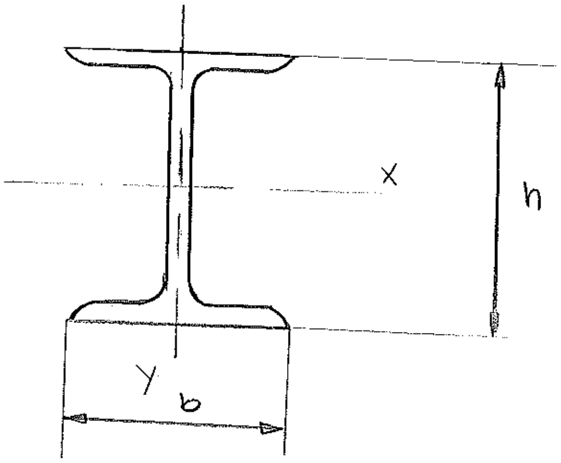
4.4



- VIGA AB: $I_z = 19610\ cm^4$
- SOPORTE CD: IPN-200
- Acero S-275 ; $G = 210GPa$; $\sigma_f = 260MPa$
  $\alpha = 10^{-5}\ °C^{-1}$

T(CD)↑

SOPORTE CD

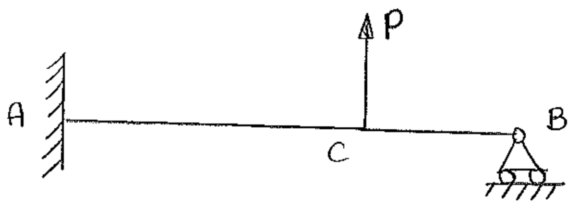


$$\begin{cases} h = 200cm \\ b = 90cm \\ t < 40mm \\ A = 33'5cm^2 \\ I_x = 2140\ cm^4 \\ I_y = 117\ cm^4 \end{cases}$$

TABLA 2.4

$$\begin{cases} \frac{h}{b} = 2'22 > 1'2 \\ t < 40mm \\ I_y < I_x : \text{el pandeo se produce en el plano xz : eje y} \end{cases} \Rightarrow \boxed{\begin{matrix} \text{curva b} \\ \alpha = 0'34 \end{matrix}}$$

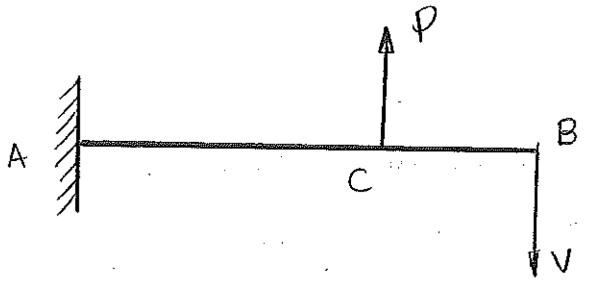
1



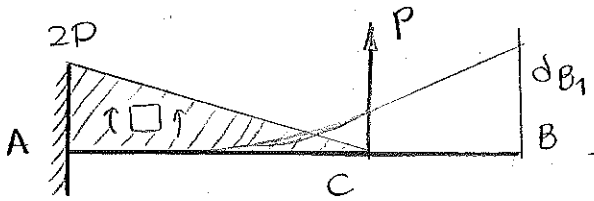
2

$$\boxed{\text{Ec compatibilidad: } \overset{\downarrow}{y}_{c_1} = \overset{\downarrow}{y}_{c_2}}$$

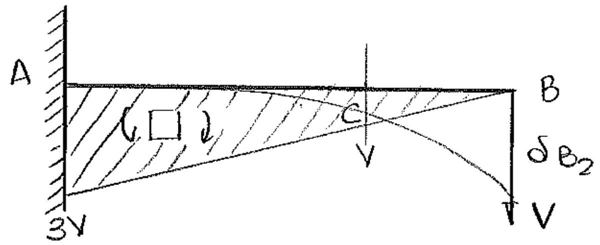
1



$\boxed{\text{Ec. compatibilidad: } \delta_B = 0}$



+



$$\delta_B = \delta_{B_1} - \delta_{B_2} = 0$$

$$\delta_{B_1} = \delta_{BA} = \frac{1}{EI}\left[\frac{1}{\cancel{2}}\cancel{2}\cdot 2P\cdot\left(1+\frac{2}{3}2\right)\right] = \frac{14P}{3EI}$$

$$\delta_{B_2} = \delta_{BA} = \frac{1}{EI}\left[\frac{1}{\cancel{2}}3\cdot 3V\,\frac{\cancel{2}}{\cancel{3}}\cancel{3}\right] = \frac{9V}{EI}$$

$$\frac{14P}{3EI} = \frac{9V}{EI}$$

$$\boxed{V = \frac{14}{27}P}$$

$$\overset{\downarrow}{y}_{C_1} = \delta_{C_2} - \delta_{C_1}$$

$$\delta_{C_1} = \frac{1}{EI}\cdot\left[\frac{1}{\cancel{2}}2\cdot 2P\cdot\frac{\cancel{2}}{3}2\right] = \frac{8P}{3EI}$$

$$\delta_{C_2} = \frac{1}{EI}\left[\frac{1}{\cancel{2}}\cancel{2}\cdot 2V\cdot\frac{2}{3}2 + 2\cdot V\cdot 1\right] = \frac{14V}{3EI} = \frac{14\cdot 14P}{3\cdot 27EI} = \frac{196P}{81EI}$$

$$\boxed{\overset{\downarrow}{y}_{C_1} = -\frac{20P}{81EI}}$$

2

$$L_K = L = 3m$$

$$P_{cr} = \frac{\pi^2 EI}{L_K^2} = 269440{,}2001\,N$$

$$\bar{\lambda} = \sqrt{\frac{A\sigma_f}{P_{cr}}} = 1'79795 \to A4$$

$$\phi = 0'5\left[1+\alpha(\bar{\lambda}-0'2)+\bar{\lambda}^2\right] = 2'38797 \to B4$$

$$\chi = \frac{1}{\phi+\sqrt{\phi^2-\bar{\lambda}^2}} = 0'252556$$

$$\boxed{P_b = \chi\cdot\sigma_f\cdot A = 219976{,}6906N}$$

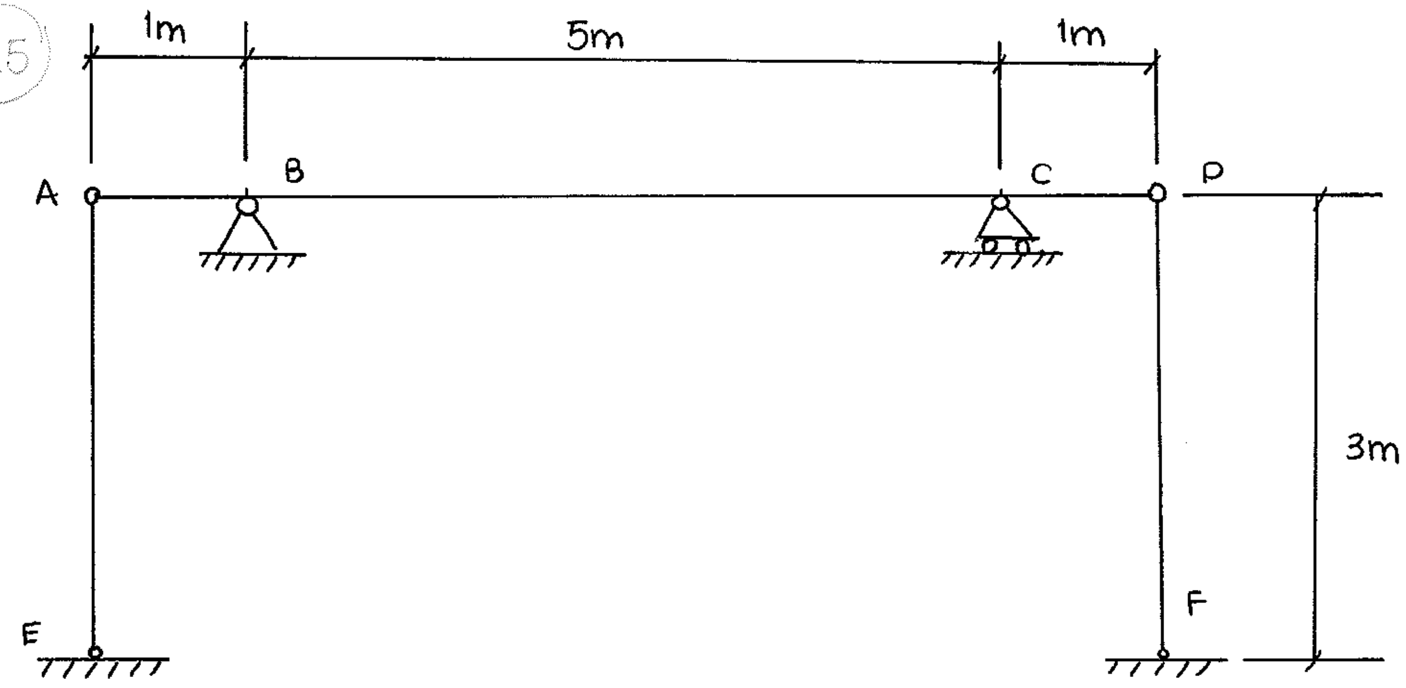
$$\boxed{\delta_{c_2} = \frac{P.L}{AE} - \alpha \Delta T . L = \overset{\downarrow}{y}_{c_2}}$$

$$-\frac{20P}{81EI} = \frac{PL}{AE} - \alpha \Delta T . L$$

$$-\frac{20 \cdot 219976{,}6906}{81 \cdot 210 \cdot 10^9 \; 19610 \; \frac{1}{100^4}} = \frac{219976{,}6906 \cdot \overset{3m}{}}{33'5 \cdot \frac{1}{100^2} \; 210 \cdot 10^9} - 10^{-5} \, \Delta T . 3$$

$$\boxed{\Delta T = 75'234 \,^{\circ}C}$$

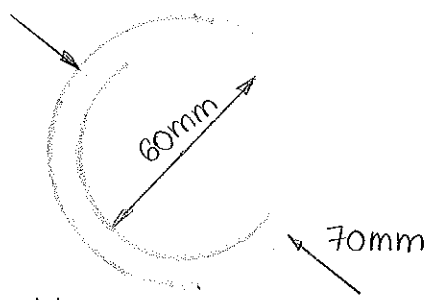
4.5



- $I(ABCD) = 19610\,cm^4$
- Barras AE y DF: sección tubular $\begin{cases} \phi_i = 60mm \\ \phi_e = 70mm \end{cases}$
- Acero S-355
- $E = 210GPa$, $\sigma_f = 360\,MPa$, $\alpha = 10^{-5}\,C^{-1}$

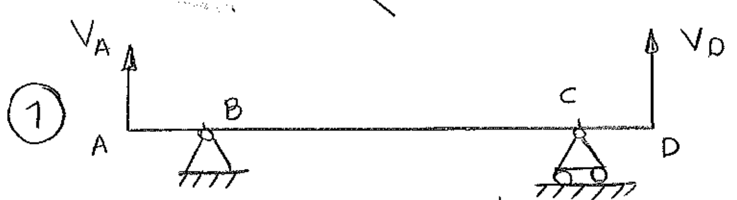
BARRAS AE y CF

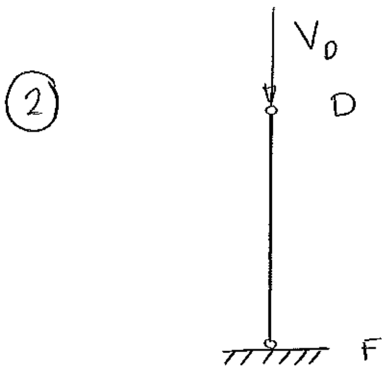
- suponemos conformado en frío $\longrightarrow$ $\boxed{\text{curva } c \quad \alpha = 0'49}$

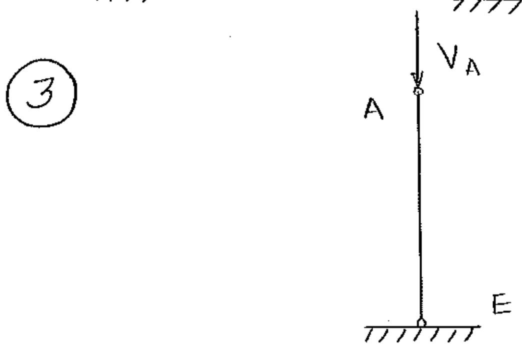


$$I = \frac{\pi(35mm)^4 - \pi(30mm)^4}{4} = 5'424156 \cdot 10^{-7} m^4$$

$$A = \pi(35mm)^2 - \pi(30mm)^2 = 1'0210176121 \cdot 10^{-3} m^2$$



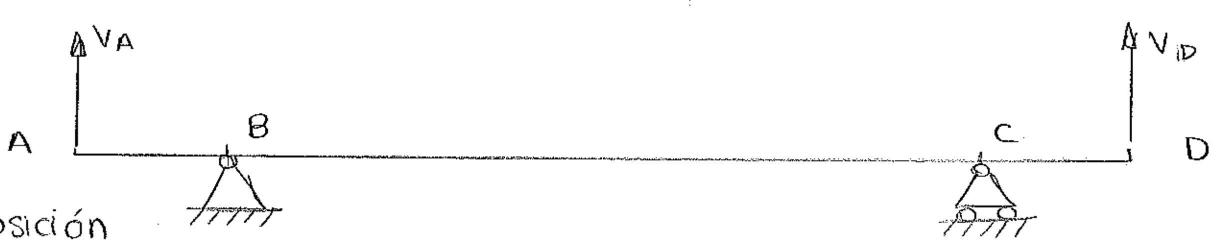




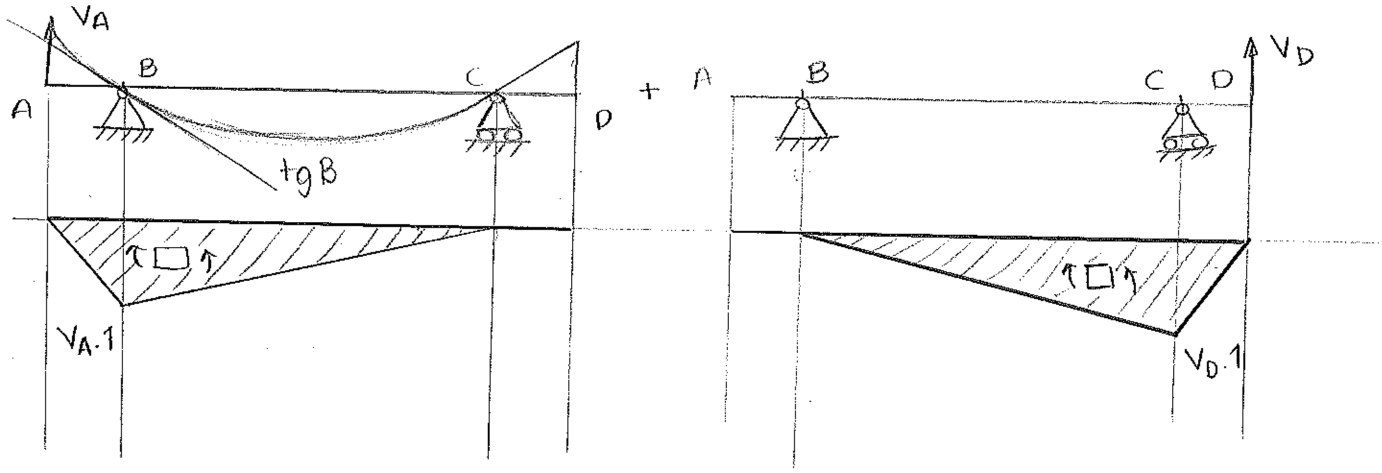
EC compatibilidad: (1) $\overset{\uparrow}{Y}_{A_1} = \overset{\uparrow}{Y}_{A_3}$

(2) $\overset{\uparrow}{Y}_{D_1} = \overset{\uparrow}{Y}_{D_2}$

(1)



⊘ superposición



⊘ EQUILIBRIO: $V_A = R_{BA} + R_{BC} \Rightarrow R_{BA} = \frac{4}{5} V_A$

$V_A \, 1m = R_{BC} \cdot 5m \Rightarrow R_{BC} = \frac{1}{5} V_A$

⊘ EQUILIBRIO: $V_D = R_{BD} + R_{CD} \Rightarrow R_{CD} = \frac{4}{5}$

$V_D \cdot 1m = R_{BD} \cdot 5m \Rightarrow R_{BD} = \frac{V}{5}$

$\overset{\uparrow}{Y}_{A1} = \delta_{AB} + \theta_B \cdot 1m$

$$\left.\begin{array}{l} \theta_B = \dfrac{\delta_{CB}}{5m} \\ \delta_{CB} = \dfrac{1}{EI}\left[\dfrac{1}{2} V_A \, 5m \cdot \dfrac{2}{3} 5m\right] = \dfrac{25 V_A}{3EI} \end{array}\right\} \theta_B = \frac{5V_A}{3EI}$$

$$\delta_{AB} = \frac{1}{EI}\left[\frac{1}{2} V_A \cdot 1m \, \frac{2}{3} \, 1m\right] = \frac{V_A}{3EI}$$

$$\boxed{\overset{\uparrow}{Y}_{A1} = \frac{2V_A}{EI}}$$

$\overset{\uparrow}{Y}_{D1} = \theta_C \cdot 1m$

$$\left.\begin{array}{l} \theta_C = \dfrac{\delta_{BC}}{5m} \\ \delta_{BC} = \dfrac{1}{EI}\left[\dfrac{1}{2} V_A 5m \, \dfrac{1}{3} 5m\right] = \dfrac{25 V_A}{6EI} \end{array}\right\} \theta_C = \frac{5V_A}{6EI}$$

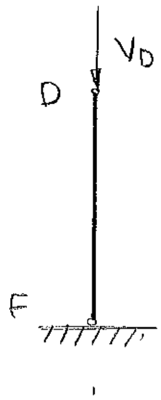
$$\boxed{\overset{\uparrow}{Y}_{D1} = \frac{5V_A}{6EI}}$$

Por simetría:

$$\boxed{\overset{\uparrow}{y}_{D_{21}} = \frac{2V_D}{EI} \qquad \overset{\uparrow}{y}_{A_{21}} = \frac{5V_D}{6EI}}$$

$$\boxed{\overset{\uparrow}{y}_{D_1} = \frac{5V_A}{6EI} + \frac{2V_D}{EI} = \frac{1}{EI}\left(\frac{5}{6}V_A + 2V_D\right) \qquad \overset{\uparrow}{y}_{A_1} = \frac{2V_A}{EI} + \frac{5V_D}{6EI} = \frac{1}{EI}\left(2V_A + \frac{5}{6}V_D\right)}$$

2



$L_K = L = 3m$

$$P_{cr} = \frac{\pi^2 E \cdot I_{OF}}{L_K^2} = \frac{\pi^2 \cdot 210\,GPa \cdot 5'424156 \cdot 10^{-7} m^4}{(3m)^2} = 124,9133058\,kN$$

$$\bar{\lambda} = \sqrt{\frac{A \cdot \sigma_f}{P_{cr}}} = 1'715392537 \rightarrow A4$$

$$\phi = 0'5\left[1 + \alpha(\bar{\lambda} - 0'2) + \bar{\lambda}^2\right] = 2'342556949 \rightarrow B4$$

$$\chi = \frac{1}{\phi + \sqrt{\phi^2 - \bar{\lambda}^2}} = 0'2539448405$$

$$\boxed{P_b = \chi \cdot \sigma_f \cdot A = 933,4157567\,kN}$$ (Para ambas barras)

$$\overset{\uparrow}{y}_{D_2} = -\frac{V_D \cdot 3m}{AE} + \alpha \cdot \Delta T \cdot 3m$$

3 $$\overset{\uparrow}{y}_{A_2} = -\frac{V_A \cdot 3m}{AE} + \alpha \Delta T \cdot 3m$$

Ec. compatibilidad

$$\frac{1}{EI}\left[2V_A + \frac{5}{6}V_D\right] = -\frac{V_A \cdot 3r}{AE} + 3\alpha\Delta T$$

por simetría $V_A = V_D$

Pandeo cuando $V_A = P_b$

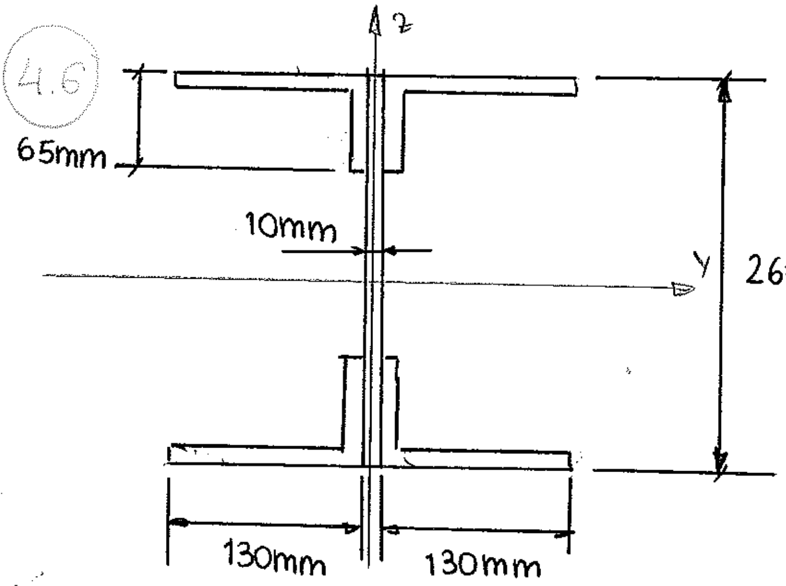
$$\boxed{\Delta T = 257'608\,C}$$

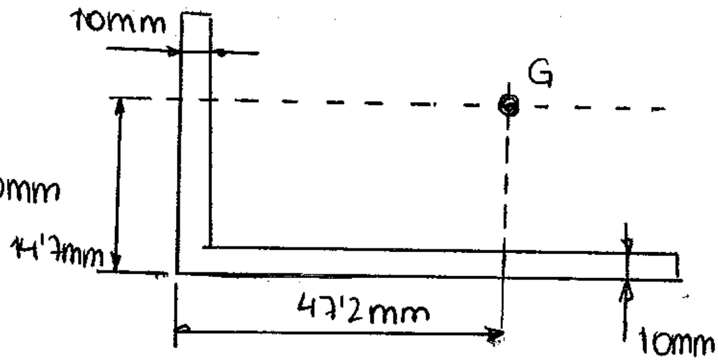
Si lo haces suponiendo que esta laminado en caliente:

curva a $\Rightarrow \alpha = 0'21$

$$\boxed{\Delta T = 298'904\,C}$$

4.6





- Acero S-235
- $E = 210\,GPa$, $\sigma_f = 240MPa$.
- $L = 3'90m$ (viga articulada en los extremos)

$$L_k = L = 3'90m$$

$$P_{cr} = \frac{\pi^2 \cdot E \cdot I}{L_k^2} = 4505432{,}231\,N.$$

$$I_y = \frac{1}{12}\left[(130mm \cdot 2 + 10mm)(260mm)^3\right] - 2 \cdot \frac{1}{12}\, 120mm(260mm - 2\,10mm)^3 -$$

$$- 2 \cdot \frac{1}{12}\left[10mm \cdot (260mm - 65mm \cdot 2)^3\right] = 1'153183\hat{3}\; 10^{-4} m^4$$

$$I_z = 2 \cdot \frac{1}{12}(10mm) \cdot (130mm\; 2 + 10mm)^3 + 2\frac{1}{12}(65mm - 10mm) \cdot (3 \cdot 10mm)^3 +$$

$$+ \frac{1}{12}(260mm - 2 \cdot 65mm)(10mm)^3 = 3'3063\hat{3} \cdot 10^{-5} m^4$$

$I_z < I_y$: El pandeo se produce en el plano xy: eje z
Agrupación de perfiles laminados soldados. } curva c $\Rightarrow$ $\boxed{\alpha = 0'49}$

$$\bar{\lambda} = \sqrt{\frac{A\sigma_f}{P_{cr}}} = 0'7298563486 \leftrightarrow A$$

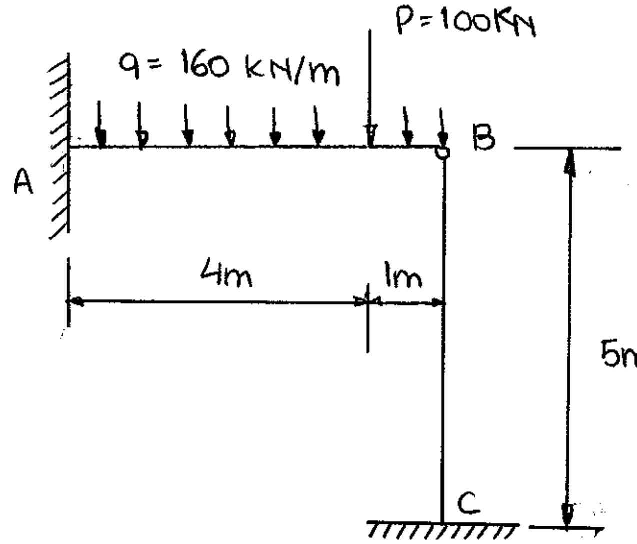
$$A = 4\left[5mm \cdot 130mm + 10mm \cdot 65mm + 120mm \cdot 10mm\right] - 0'01m^2$$

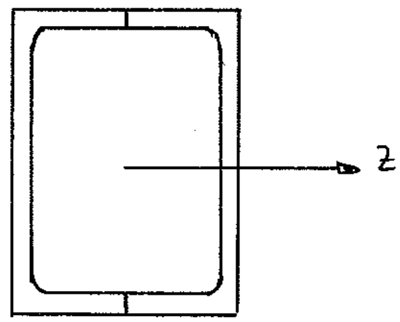
$$\phi = 0'5\left[1 + \alpha(\bar{\lambda} - 0'2) + \bar{\lambda}^2\right] = 0'8564207241 \leftrightarrow B$$

$$\chi = \frac{1}{\phi + \sqrt{\phi^2 - \bar{\lambda}^2}} = 0'7665830494$$

$$\boxed{P_b = \chi \cdot \sigma_f \cdot A = 1694{,}709\,kN}$$

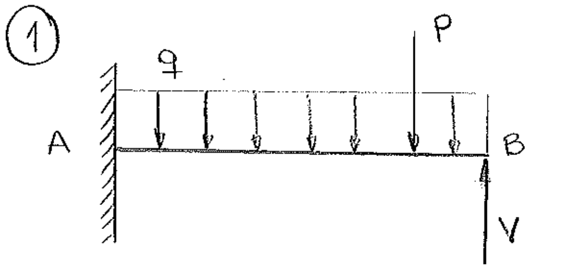
4.7

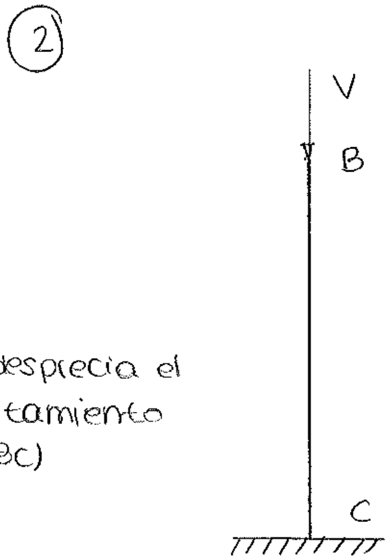




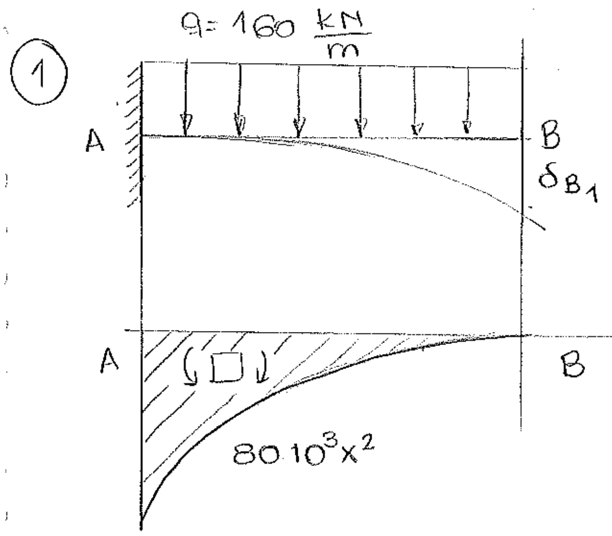
sección soporte (BC)
(perfiles UPN soldados)

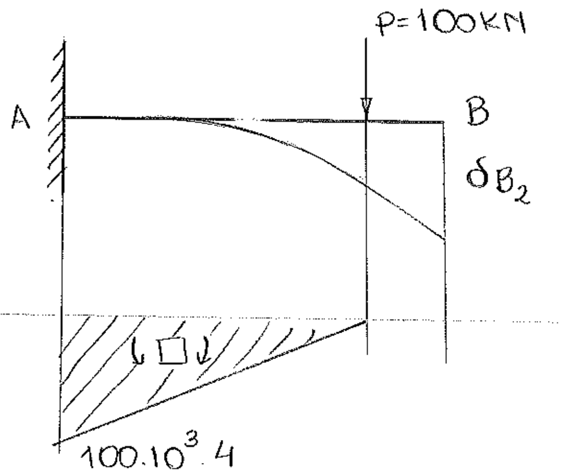
- Acero S-235
- $E = 210\,GPa$ $\quad \sigma_f = 240\,MPa$.

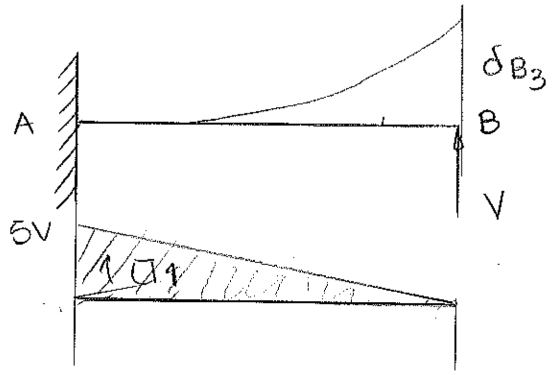




- Ec. compatibilidad $\quad \overset{\downarrow}{y}_{B_1} = \overset{\downarrow}{y}_{B_2} = 0$ (se desprecia el acortamiento en BC)







$$\overset{\downarrow}{Y_{B_1}} = \delta_{B_1} + \delta_{B_2} - \delta_{B_3}$$

$$\delta_{B_1} = \frac{1}{EJ}\int_0^5 80\cdot10^3 x^3\,dx = \frac{1}{EI}\cdot 125\cdot10^5$$

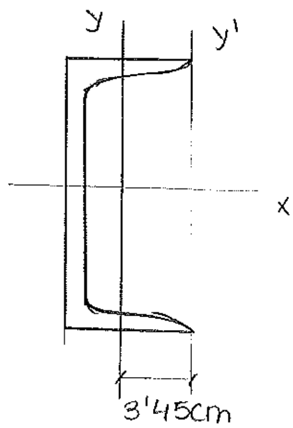
$$\delta_{B_2} = \frac{1}{EI}\left[\frac{1}{\cancel{2}}\, 400\,10^3\cdot \cancel{4}^{2}\,(1+\tfrac{2}{3}4)\right] = \frac{88\cdot10^5}{3EI}$$

$$\delta_{B_3} = \frac{1}{EI}\left[\frac{1}{\cancel{2}}\, V\cdot5\cdot5\cdot\frac{\cancel{2}}{3}\,5\right] = \frac{125V}{3EI}$$

$$\overset{\downarrow}{Y_{B_1}} = \frac{463\,10^5 - 125V}{3EI} = 0 \Rightarrow \boxed{V = 26997{,}84017\,N}$$

(2) $L_K = L = 5m$

- Suponemos UPN-100.



$$\begin{cases} I_x = 206\,cm^4 \\ I_y = 29'3\,cm^4 \\ A = 13'5\,cm^2 \longrightarrow A_1 = 27\,cm^2 \end{cases}$$

$I_{y'} = I_y + A(3'45cm)^2 = 189'98375\,cm^4$

$$\left.\begin{array}{l} I_{x,TOTAL} = 412\,cm^4 \\ I_{y',TOTAL} = 379'9675\,cm^4 \end{array}\right\} I_x > I_{y'} \Rightarrow$$ El pandeo se produce en el plano $YZ \rightarrow$ eje y'

$\Downarrow$

curva c $\Rightarrow \boxed{\alpha = 0'49}$

$$P_{cr} = \frac{\pi^2 E I_{y'}}{L_K^2} = 315010{,}8285\,N$$

$$\bar{\lambda} = \sqrt{\frac{A\cdot\sigma_f}{P_{cr}}} = 1'434249679 \leftarrow A4$$

$$\phi = 0'5\left[1 + \alpha(\bar{\lambda} - 0'2) + \bar{\lambda}^2\right] = 1'830927243 \longrightarrow B4$$

$$\chi = \frac{1}{\phi + \sqrt{\phi^2 - \bar{\lambda}^2}} = 0'3368131028$$

$P_b = \chi \cdot A \cdot \sigma_f = 218254{,}8906\ N < V \Rightarrow$ No nos sirve

- Suponemos UPN-120.

$I_x = 364 cm^4$

$I_y = 43'2 cm^4$

$A = 17 cm^2 \longrightarrow A_T = 34 cm^2$

$I_{y'} = I_y + A \cdot (5'5cm - 1'6cm)^2 = 301{,}77\ cm^4$

$\left. \begin{array}{l} I_{x,TOTAL} = 728 cm^4 \\ I_{y',TOTAL} = 603'54 cm^4 \end{array} \right\}$ $I_x > I_{y'} \Rightarrow$ El pandeo se produce en el plano $Y'Z \Rightarrow$ eje Z

curva c $\Rightarrow \boxed{\alpha = 0'49}$

$$P_{cr} = \frac{\pi^2 E I_{y'}}{L_k^2} = 500362{,}8874 N$$

$$\bar{\lambda} = \sqrt{\frac{A \sigma_f}{P_{cr}}} = 1'277034218 \longleftrightarrow A4$$

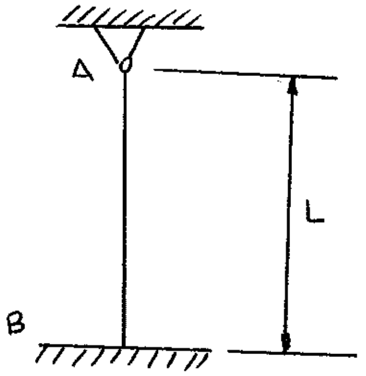
$$\phi = 0'5 [1 + \alpha(\bar{\lambda} - 0'2) + \bar{\lambda}^2] = 1'579281581 \longleftrightarrow B4$$

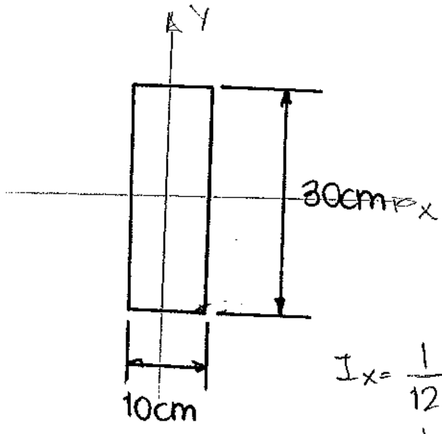
$$\chi = \frac{1}{\phi + \sqrt{\phi^2 - \bar{\lambda}^2}} = 0'3986559782$$

$P_b = \chi \cdot A \sigma_f = 325303{,}2782 N < V \Rightarrow$ Nos sirve

El perfil más adecuado es el UPN-120

4.8





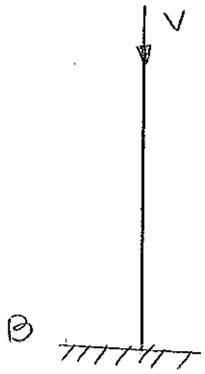
- Acero S-355
- E = 210 GPa
- $\sigma_f = 300$ MPa

$\alpha = 10^{-5}\ C^{-1}$

$I_x = \frac{1}{12} 10cm(30cm)^3 = 2'25 \cdot 10^{-4} m^4$

$I_y = \frac{1}{12} 30cm(10cm)^3 = 2'5 \cdot 10^{-5} m^4$

1) PLASTIFICACIÓN



- Ec. compatibilidad $\delta_{AB} = 0$.

$$\delta = -\frac{V \cdot L}{EA} + \Delta T \cdot \alpha \cdot L = 0.$$

$\sigma_f = 300\,MPa \Rightarrow V_{máx} = \sigma_f \cdot A = 9MN$

$$-\frac{\sigma_f \cdot L}{E} + \Delta T \alpha L = 0 \Rightarrow \boxed{\Delta T = 142,857\,°C}$$

2) PANDEO.

$I_y < I_x \Rightarrow$ el pandeo se produce en el eje y.

$L_K = 0'7L.$

$$P_{cr} = \frac{\pi^2 \cdot EI}{(0'7L)^2} = \frac{105745761'4}{L^2}$$

- Perfil simple $\Rightarrow$ curva c: $\alpha = 0'49$

$$\bar{\lambda} = \sqrt{\frac{A\sigma_f}{P_{cr}}} = 0'2917358296 \cdot L = A \cdot L$$

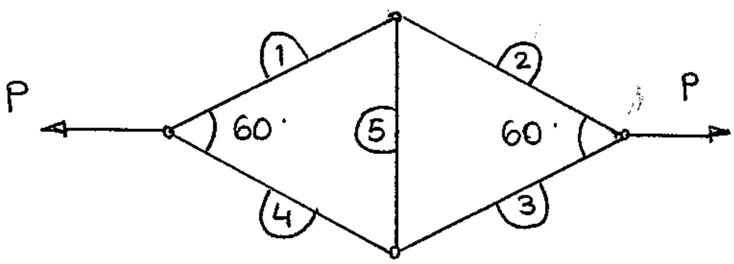
$$-\frac{VL}{EA} + \Delta T \cdot \alpha \cdot L = 0 \Rightarrow \boxed{V = \Delta T \cdot \alpha \cdot E} = V = \chi \cdot A \cdot \sigma_f \ ?$$

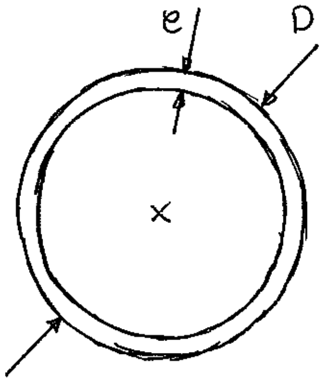
$$\boxed{\Delta T = \frac{\chi A \sigma_f}{\alpha \cdot E}}$$

$$\phi = 0'5(1 + \alpha(\bar{\lambda} - 0'2) + \bar{\lambda}^2) = 0'5(1 + 0'49 \cdot (A \cdot L - 0'2) + A^2 L^2)$$

$$\chi = \frac{1}{\phi + \sqrt{\phi^2 - \bar{\lambda}^2}}$$

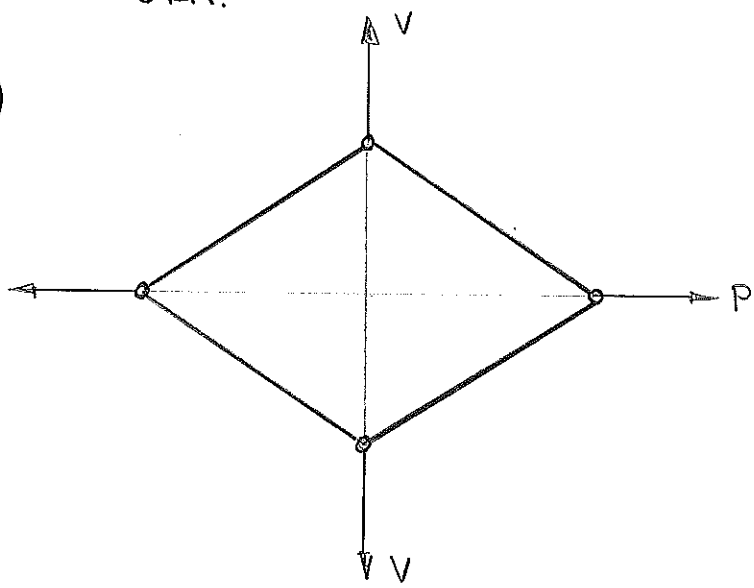
4.9





- $L_1 = L_2 = L_3 = L_4 = L_5 = L = 2m$
- Acero S-275
- $E = 210GPa$
- $\sigma_f = 260MPa$
- $e \leq 5mm$
- $P = 260\sqrt{3}\ kN.$

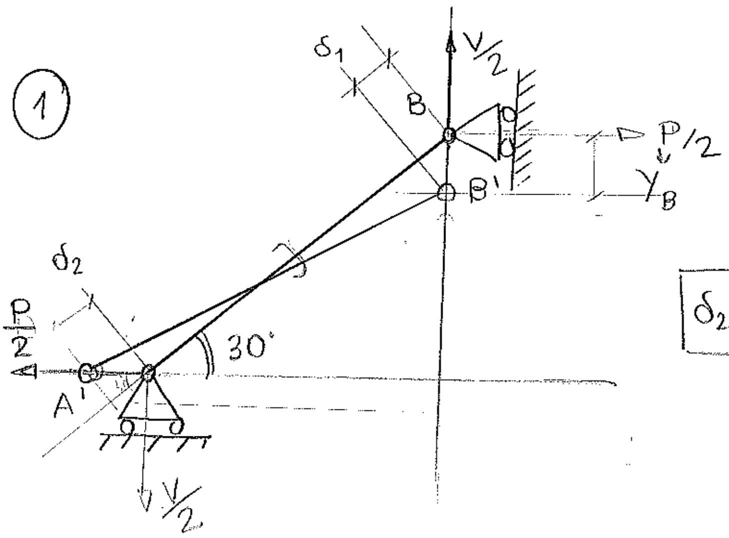
1



2

- Ec. compatibilidad $\boxed{L'_{5_1} = L'_{5_2}}$ $(\Delta L'_{51} = \Delta L'_{52})$

1



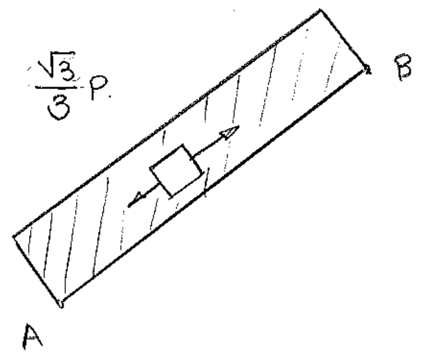
$\boxed{\delta_2 - \delta_1 = \Delta L_1}$

EQUILIBRIO: $\frac{P}{2}\frac{L}{2} = \frac{V}{2}.L.\cos 30$

$\frac{P}{2}\frac{1}{2} = \frac{V}{2}\frac{\sqrt{3}}{2} \Rightarrow \boxed{V = \frac{P}{\sqrt{3}}}$

Diagrama de esfuerzos

$\boxed{\text{Esfuerzos normales}}$



$$N = \frac{P}{2}\cos 30 + \frac{V}{2}\cdot\cos 60 = \frac{\sqrt{3}\,P}{4} + \frac{P}{2\sqrt{3}}\cdot\frac{1}{2} = \frac{\sqrt{3}}{3}P$$

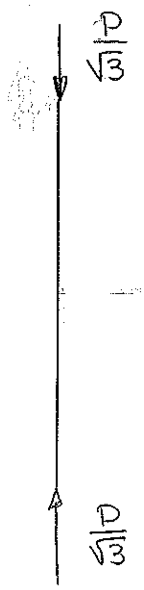
$$\frac{\sqrt{3}}{3}P = \sigma_f \cdot A_1$$

$\boxed{A_1 = 10^{-3} m^2}$ $\Rightarrow$ TABLAS: $\boxed{\begin{array}{l} D = 90mm \\ e = 4mm \end{array}}$

- La barra no trabaja a flexión

$$\Delta L_1 = \frac{N \cdot L_{AB}}{E \cdot A} = \delta_2 - \frac{Y_{B1}}{\cos 30} \rightarrow ; Y_{B1} = \frac{\delta_2 - \Delta L_1}{\cos 30} = \frac{\delta_2 - \frac{N \cdot L}{EA}}{\cos 30} \Rightarrow \boxed{\Delta L_{S1} = 2 \cdot \frac{\delta_2 - \frac{N \cdot L}{EA_1}}{\cos 30}}$$

2



$$\boxed{\Delta L_{S2} = \frac{\frac{2P}{\sqrt{3}}\cdot L}{EA_2} = \frac{2PL}{\sqrt{3}\,EA_2}}$$

- suponemos laminado en caliente: curva a: $\boxed{\alpha = 0'21}$

$L_K = 2m$.

- suponemos $D = 100\,mm$, $e = 4mm$, $A = 12{,}06 cm^2$

$I = 139\, cm^4$

$$P_{cr} = \frac{\pi^2 \cdot E \cdot I}{L_K^2} = 720234{,}3812\, N.$$

$$\bar{\lambda} = \sqrt{\frac{A \cdot \sigma_f}{P_{cr}}} = 0'6598168519 \rightarrow A$$

$$\phi = 0'5\,[1 + \alpha(\bar{\lambda} - 0'2) + \bar{\lambda}^2] = 0'7659599085 \rightarrow B$$

$$\chi = \frac{1}{\phi + \sqrt{\phi^2 - \bar{\lambda}^2}} = 0'8658161746$$

$P_b = \chi \cdot \sigma_f \cdot A = 271485{,}3197\, N$ } Nos sirve, probamos con otra de menor resistencia

$V = 260000\, N$

• suponemos $D = 94\,mm$, $e = 4\,mm$, $A = 11'30\,cm^2$

$$I = 114\,cm^4$$

$$P_{cr} = \frac{\pi^2 E I}{L_k^2} = 590695,8234\,N$$

$$\bar{\lambda} = \sqrt{\frac{A \sigma_f}{P_{cr}}} = 0'7052513817 \quad \{\to A\}$$

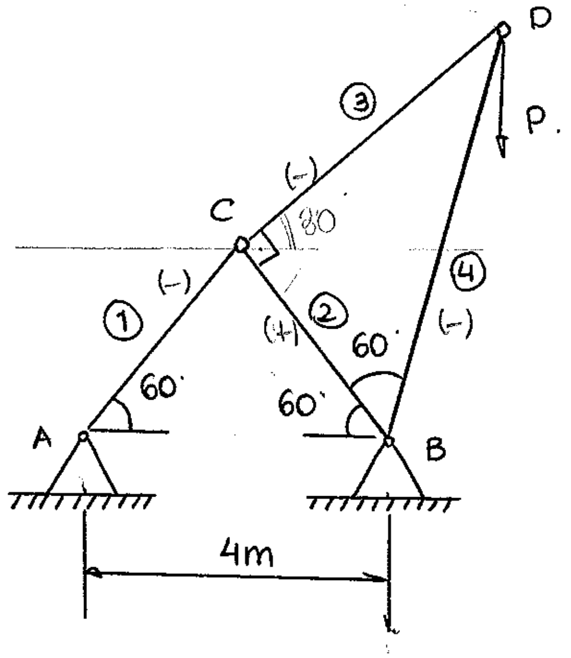
$$\phi = 0'5\left[1 + \alpha(\bar{\lambda} - 0'2) + \bar{\lambda}^2\right] = 0'8017411507 \quad \{\to B\}$$

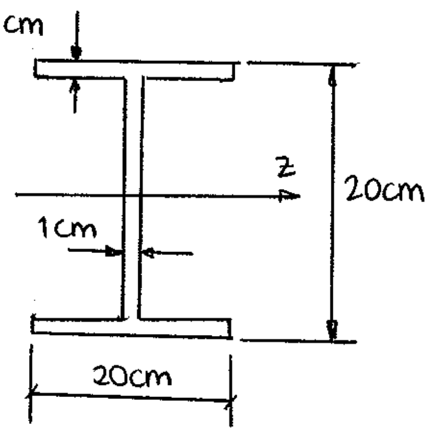
$$\chi = \frac{1}{\phi + \sqrt{\phi^2 - \bar{\lambda}^2}} = 0'8452607049$$

$P_b = \chi \cdot \sigma_f \cdot A = 248337,5951\,N < V$: No nos sirve.

$$\boxed{\text{Las barras 1, 2, 3 y 4: } D = 90\,mm,\ e = 4\,mm \quad \text{y la barra 5: } D = 100\,mm,\ e = 4\,mm}$$

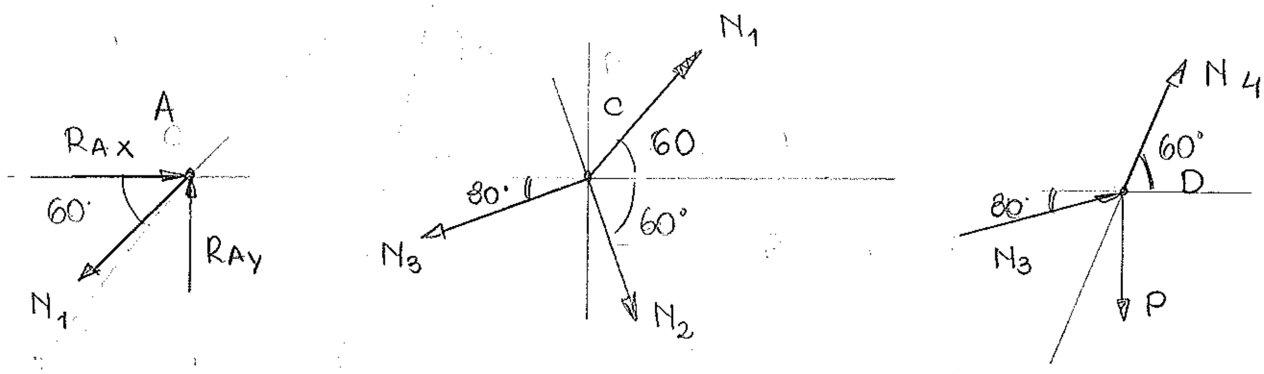
4.10

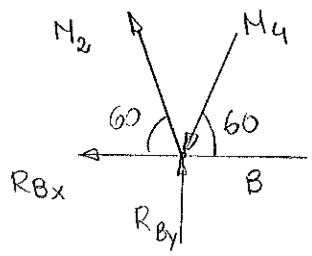




- Acero S-235
- $E = 210 GPa$
- $\sigma_f = 240 MPa$.

1





(A) $N_1 \cdot \cos 60 = R_{Ax}$

$N_1 \cdot \operatorname{sen} 60 = R_{Ay}$

(B) $N_2 \cdot \cos 60 + N_4 \cos 60 = -R_{Bx}$

$N_2 \cdot \operatorname{sen} 60 - N_4 \cdot \operatorname{sen} 60 + R_{By} = 0$

$\boxed{N_1 = -\dfrac{2\sqrt{3}P}{3}}$ $\boxed{N_2 = -\dfrac{P}{\sqrt{3}}}$

(C) $N_1 \cos 60 + N_2 \cos 60 = N_3 \cdot \cos 30 \Rightarrow N_1 \cdot \cos 60 + N_2 \cdot \cos 60 = -P \cdot \cos 30$

$N_1 \cdot \operatorname{sen} 60 - N_2 \cdot \operatorname{sen} 60 + N_3 \cdot \operatorname{sen} 30 \Rightarrow N_1 \cdot \operatorname{sen} 60 - N_2 \cdot \operatorname{sen} 60 = -P \cdot \operatorname{sen} 30$

(D) $N_4 \cos 60 + N_3 \cdot \cos 30 = 0 \Rightarrow N_4 = -N_3 \dfrac{\cos 30}{\cos 60} = -\sqrt{3} \cdot N_3 \Rightarrow \boxed{N_4 = \sqrt{3}P}$

$N_4 \cdot \operatorname{sen} 60 + N_3 \cdot \operatorname{sen} 30 = P$.

$-\sqrt{3} \cdot N_3 \operatorname{sen} 60 + N_3 \cdot \operatorname{sen} 30 = P$

$-N_3 = P \Rightarrow \boxed{N_3 = -P}$

• Barras

(1) $N_1 = \sigma_f \cdot A$

$\frac{2\sqrt{3}}{3} P = \sigma_f \cdot A \Rightarrow \boxed{P = 1205{,}507 \text{ kN}}$

(2) $N_2 = \sigma_f A$

$\frac{P}{\sqrt{3}} = \sigma_f A \Rightarrow \boxed{P = 2411{,}015 \text{ kN}}$

(3) $N_3 = \sigma_f A$

$P = \sigma_f A \Rightarrow \boxed{P = 1392 \text{ kN}}$

(4) $N_4 = \sigma_f A$

$\sqrt{3} P = \sigma_f A \Rightarrow \boxed{P = 803'672 \text{ kN}}$

$A = 2 \cdot 1\text{cm} \cdot 20\text{cm} + 1\text{cm} \cdot 18\text{cm} =$
$= 5'8 \cdot 10^{-3} m^2$

$I_z = 4{,}099\hat{3} \cdot 10^{-5} m^4$

$I_y = 1'33483\hat{3} \cdot 10^{-5} m^4$

$I_y < I_z \Rightarrow$ Eje z

• Barras a compresión: barra 2, barra 4

$\boxed{\text{BARRA 2}}$

$L_K = 4m$

$$\left.\begin{array}{l} \frac{h}{b} = 1 \leq 1'2 \\ t \leq 100mm \\ S\text{-}235 \\ \text{eje } z \end{array}\right\} \text{curva } b \Rightarrow \boxed{\alpha = 0'34}$$

$$P_{cr} = \frac{\pi^2 \cdot E \cdot I_y}{L_K^2} = 1729123{,}849 \text{ N}$$

$$\bar{\lambda} = \sqrt{\frac{A \sigma_f}{P_{cr}}} = 0'8972357453 \; \{\rightarrow A\}$$

$$\phi = 0'5[1 + \alpha(\bar{\lambda} - 0'2) + \bar{\lambda}^2] = 1'021046068 \; \{\rightarrow B\}$$

$$\chi = \frac{1}{\phi + \sqrt{\phi^2 - \bar{\lambda}^2}} = 0'6629589776$$

$$P_b = \chi \cdot \sigma_f A = 922{,}839 \text{kN} = N_2 = \frac{P}{\sqrt{3}} \Rightarrow \boxed{P = 1598{,}404 \text{ kN}}$$

BARRA 4

$$\left.\begin{array}{l} L_K = 8m \\ \frac{h}{b} = 1 \leq 1'2 \\ t \leq 40mm \\ S\text{-}235 \\ \text{eje } z \end{array}\right\} \text{curva } a \Rightarrow \boxed{\alpha = 0'21}$$

$$P_{cr} = \frac{\pi^2 . E . I_y}{L_k^2} = 432280{,}9621 N$$

$$\bar{\lambda} = \sqrt{\frac{A . \sigma_f}{P_{cr}}} = 1'794471491 \} \rightarrow A4$$

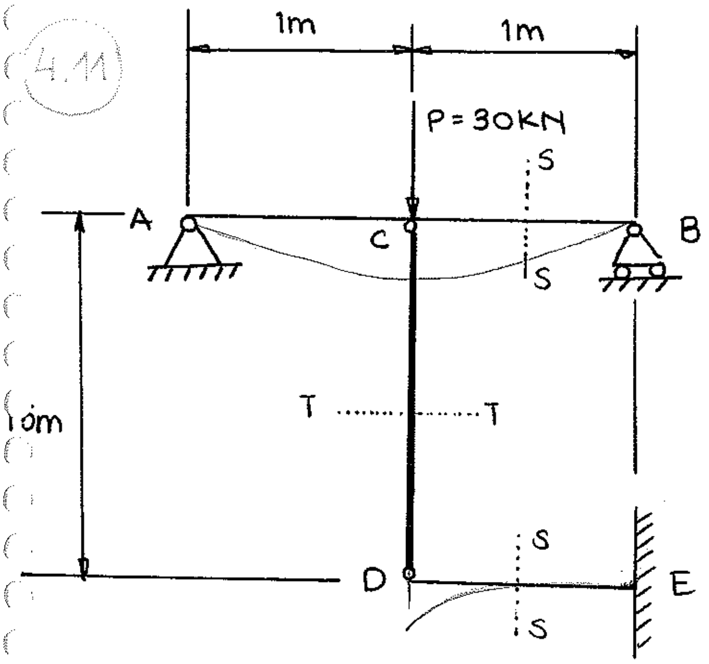
$$\phi = 0'5 [1 + \alpha (\bar{\lambda} - 0'2) + \lambda^2] = 2'381124119$$

$$\chi = \frac{1}{\phi + \sqrt{\phi^2 - \bar{\lambda}^2}} = 0'2534050265$$

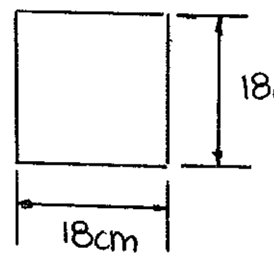
$$P_b = \chi . \sigma_f . A = 352{,}740 kN = \sqrt{3}\, P$$

$$P = 203{,}654\ kN$$

4.11

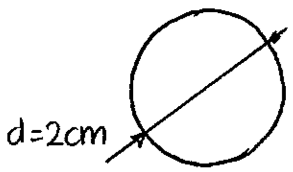


S-S



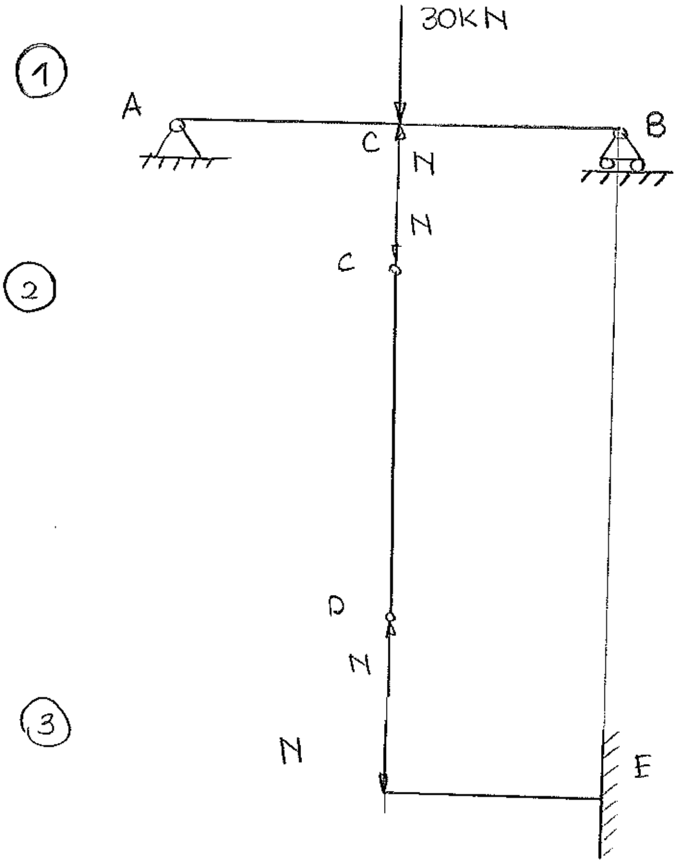
$I_S = \frac{1}{12}(18cm)^4 = 8'748 \cdot 10^{-5} m^4$

T-T



$I_T = \frac{\pi (1cm)^4}{4} = 7'853981634 \cdot 10^{-9} m^4$

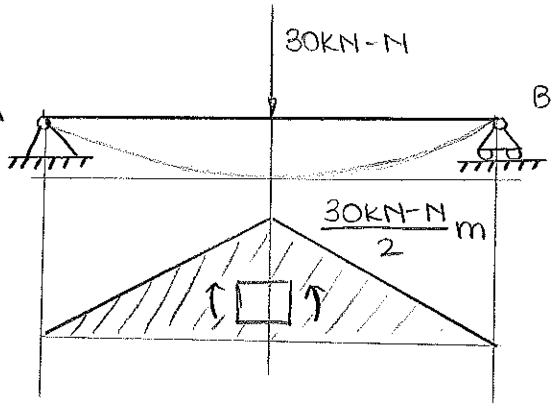
- Acero S-275
- $E = 210 GPa$
- $\sigma_f = 260 MPa$



- Ec. compatibilidad

$$\boxed{\delta_C - \delta_D = \Delta L_{CD}}$$

1

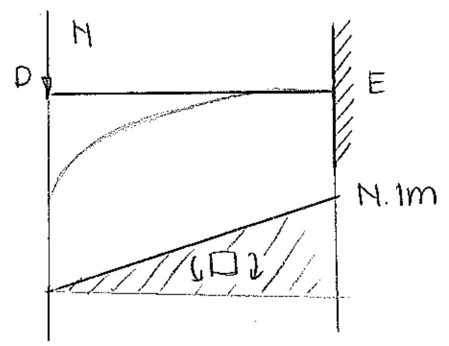


$$\delta_B = \delta_{AB} = \frac{1}{EI_S}\left[\frac{1}{2}\,\frac{30KN-N}{2}m \cdot 1m \cdot \frac{2}{3}m\right] =$$

$$= \frac{1}{6EI_S}(30KN-N)m^3$$

(2) $\Delta L_{CD} = \dfrac{N \cdot 1'5m}{E \cdot A_T} = \dfrac{N\, 1'5\, m}{E \cdot \pi (1cm)^2} = 2'273642044\, 10^{-8}_{N}\, \dfrac{m}{N}$

(3)



$\delta_D = \dfrac{1}{E\, I_s}\left[\dfrac{1}{2} N \cdot 1m \cdot 1m \cdot \dfrac{2}{3} 1m\right] = \dfrac{N}{3 E I_s}\, m^3$

Ec. compatibilidad

$\dfrac{1}{6EI_s}(30KN - N)m^3 - \dfrac{N}{3EI_s} m^3 = 2'273642044 \cdot 10^{-8} N \dfrac{m}{N}$

$\boxed{N = 5448{,}485356 N}$

Pandeo en CD

$L_K = 1'5m$

redondo macizo: curva $c \Rightarrow \boxed{\alpha = 0'49}$

$P_{cr} = \dfrac{\pi^2 \cdot E\, I_T}{L_K^2} = 7234{,}797892\ N.$

$\bar{\lambda} = \sqrt{\dfrac{A \sigma_f}{P_{cr}}} = 3'3600070646 \to A4$

$\phi = 0'5\,[1 + \alpha(\bar{\lambda} - 0'2) + \bar{\lambda}^2] = 6'9192546834 \to B4$

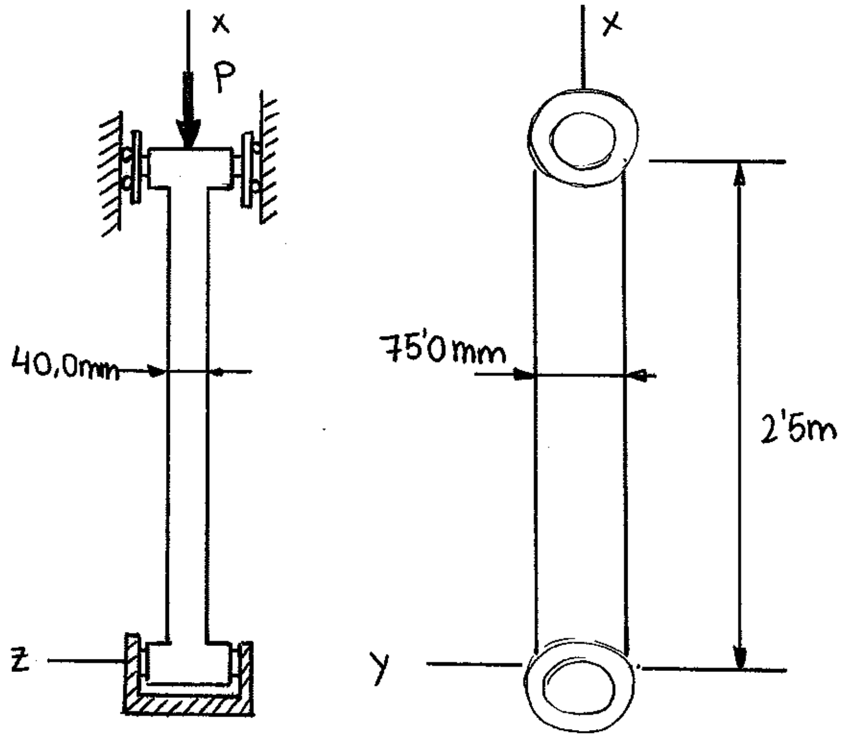
$\chi = \dfrac{1}{\phi + \sqrt{\phi^2 - \bar{\lambda}^2}} = 0'07711353686$

$P_b = \chi \cdot \sigma_f \cdot A = 6298{,}742$

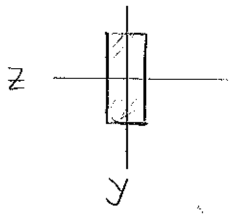
$n = \dfrac{P_b}{N} = 1'156$

$\boxed{n = 1'156}$

C-4.2



- $E = 70GPa$
- $n = 2'5$



$$I_z = \frac{1}{12}(40mm)(75mm)^3 = 1'40625\ 10^{-6}\ mm^4$$

$$I_y = \frac{1}{12}(75mm)(40mm)^3 = 4 \cdot 10^{-7} mm^4$$

PLANO XY

$L_K = 2'5m$

$$P_{cr} = \frac{\pi^2 \cdot E \cdot I_z}{L_K^2} = 155,446\ KN$$

PLANO XZ

$L_K = 1'25m$

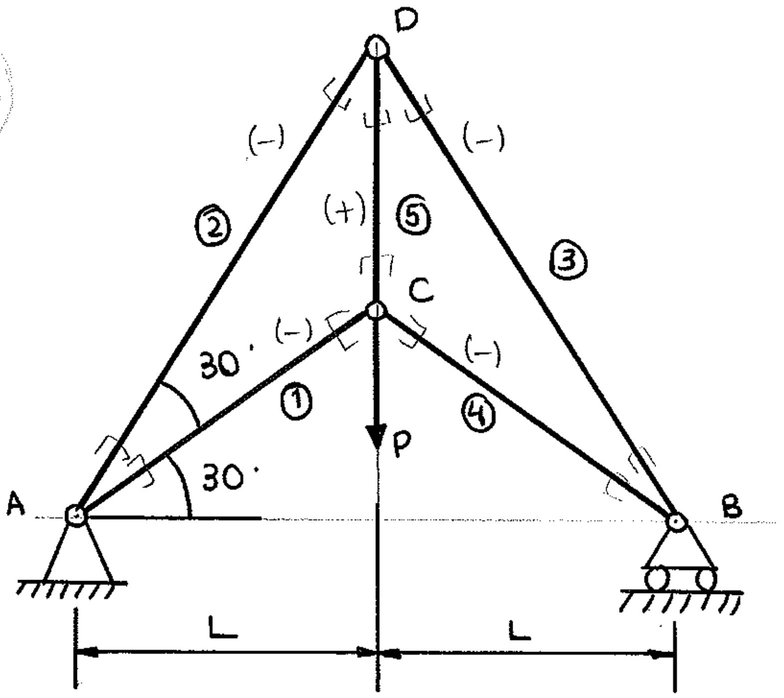
$$P_{cr} = \frac{\pi^2 \cdot E \cdot I_y}{L_K^2} = 176,863\ KN$$

El pandeo ocurre en el plano xy (He copiado los ejes al revés :))

$$P = \frac{P_{cr}}{n} = 62,178 KN$$

$$\boxed{P = 62,178 KN}$$

C-4.3

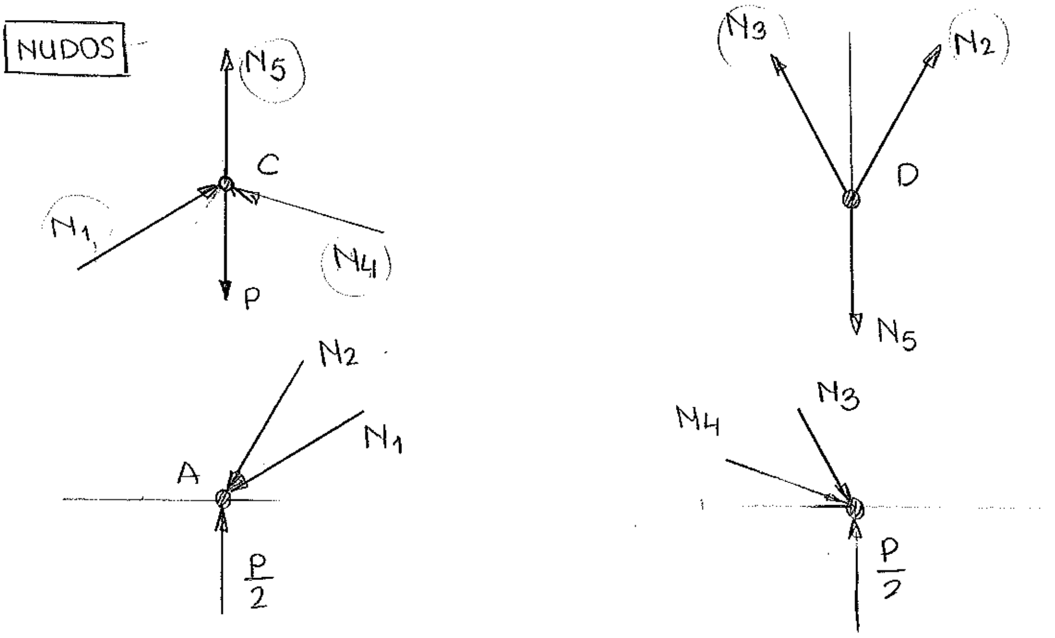


- $L = 2m$
- $P = 250 kN$
- Acero S=235
- $E = 210 GPa$, $\sigma_f = 240 MPa$

(UPN)

- HIPÓTESIS: Barras 1, 2, 3, 4 : Compresión
  Barra 5 : Tracción

NUDOS



- Por simetría: $N_2 = N_3$
  $N_1 = N_4$

NUDO A: $N_2 \cdot sen60 + N_1 \cdot sen30 = \frac{P}{2}$

$N_1 \cdot cos30 + N_2 \cdot sen30 = 0$

$\boxed{\begin{array}{l} N_1 = -125\ kN = N_4 \\ N_2 = 216{,}506\ kN = N_3 \end{array}}$

NUDO D: $N_5 = 2N_2 \cdot sen60 = 374{,}999\ kN \approx 375 kN.$

- RESISTENCIA A TRACCIÓN / COMPRESIÓN

$$\frac{N_5}{A}=\sigma_f \Rightarrow A=1{,}5625\cdot10^{-3}m^2=15'625\,cm^2 \Rightarrow \boxed{UPN\text{-}120}$$

PANDEO (Perfil UPN → curva c ⇒ $\alpha=0'49$)

- Trabajan a compresión las barras 2 y 3

$L_K=4m$; $N_2=216{,}506\,kN$

- Suponemos $\boxed{UPN\text{-}120}$

$$\left.\begin{array}{l} I_x=364\,cm^4 \\ I_y=43{,}2\,cm^4 \\ A=17\,cm^2 \end{array}\right\} I_y<I_x$$

$$P_{cr}=\frac{\pi^2\cdot E\cdot I_y}{L_K^2}=55960{,}65695\,N$$

$$\bar\lambda=\sqrt{\frac{A\sigma_f}{P_{cr}}}=2'700154902\ \{\to A\} \qquad \phi=0'5[1+\alpha(\bar\lambda-0'2)+\bar\lambda^2]=4{,}757956199\ \{\to B\}$$

$$\chi=\frac{1}{\phi+\sqrt{\phi^2-\bar\lambda^2}}=0'1152668581$$

$$P_b=\chi\cdot\sigma_f\cdot A=47028{,}87811 \ll N_2 \quad \boxed{\text{No nos sirve}}$$

- Suponemos $\boxed{UPN\text{-}200}$

$$\left.\begin{array}{l} I_x=1910\,cm^4 \\ I_y=148\,cm^4 \\ A=32{,}2\,cm^2 \end{array}\right\} I_y<I_x$$

$$P_{cr}=\frac{\pi^2\cdot E\cdot I_y}{L_K^2}=191717{,}0655\,N$$

$$\bar\lambda=\sqrt{\frac{A\sigma_f}{P_{cr}}}=2{,}007720116\ \{\to A\} \qquad \phi=0'5[1+\alpha(\bar\lambda-0'2)+\bar\lambda^2]=2'95836146\ \{\to B\}$$

$$\chi=\frac{1}{\phi+\sqrt{\phi^2-\bar\lambda^2}}=0'1948885014$$

$$P_b=\chi\cdot A\cdot\sigma_f=150609{,}8338\,N<N_2 \quad \boxed{\text{No nos sirve}}$$

Suponemos $\boxed{UPN\text{-}220}$

$$\left.\begin{aligned} I_x &= 2690\ cm^4 \\ I_y &= 197\ cm^4 \\ A &= 37'4\ cm^2 \end{aligned}\right\} I_y < I_x$$

$$P_{cr} = \frac{\pi^2 E\ I_y}{L_k^2} = 255190{,}9588\ N$$

$$\bar{\lambda} = \sqrt{\frac{A\,\sigma_f}{P_{cr}}} = 1'875464207 \quad \{\rightarrow A\} \qquad \phi = 0'5[1+\alpha(\bar{\lambda}-0'2)+\bar{\lambda}^2] = 2'669171727 \quad \{\rightarrow B\}$$

$$\chi = \frac{1}{\phi+\sqrt{\phi^2-\bar{\lambda}^2}} = 0'2188944669$$

$$P_b = \chi\,\sigma_f\,A = 196479{,}6735\ N < N_2$$: No nos sirve

Suponemos $\boxed{UPN\text{-}240}$

$$\left.\begin{aligned} I_x &= 3600\ cm^4 \\ I_y &= 248\ cm^4 \\ A &= 42'3\ cm^2 \end{aligned}\right\} I_y < I_x$$

$$P_{cr} = \frac{\pi^2 E\ I_y}{L_k^2} = 321255{,}6233\ N$$

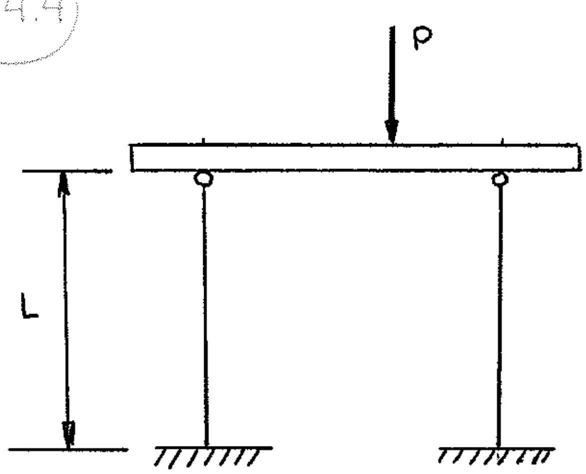
$$\bar{\lambda} = \sqrt{\frac{A\,\sigma_f}{P_{cr}}} = 1'777667102 \quad \{\rightarrow A\} \qquad \phi = 0'5[1+\alpha(\bar{\lambda}-0'21+\bar{\lambda}^2] = 2'466578603 \quad \{\rightarrow B\}$$

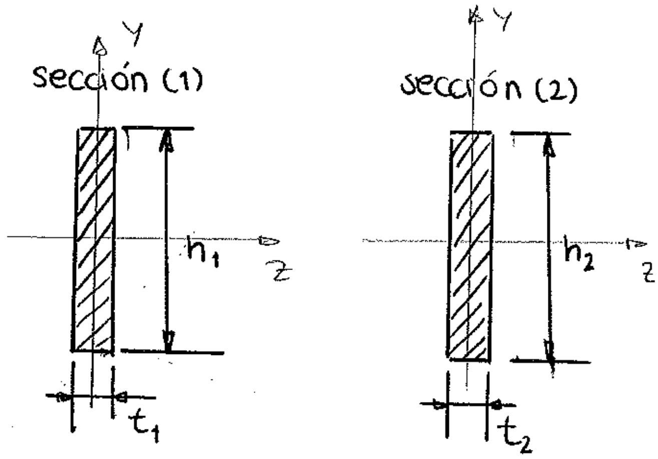
$$\chi = \frac{1}{\phi+\sqrt{\phi^2-\bar{\lambda}^2}} = 0'2394336177$$

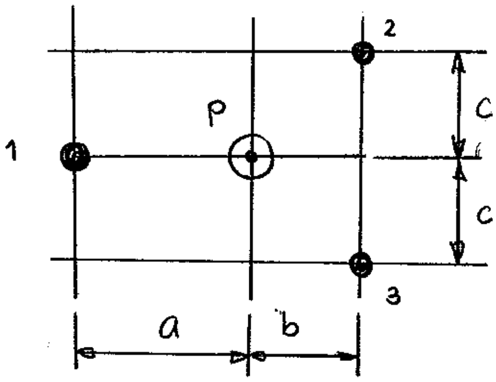
$$P_b = \chi\,\sigma_f \cdot A = 243073{,}0087\ N > N_2$$ $\boxed{\text{Nos sirve}}$

$\boxed{\text{El perfil más adecuado es el UPN-240.}}$

C-4.4

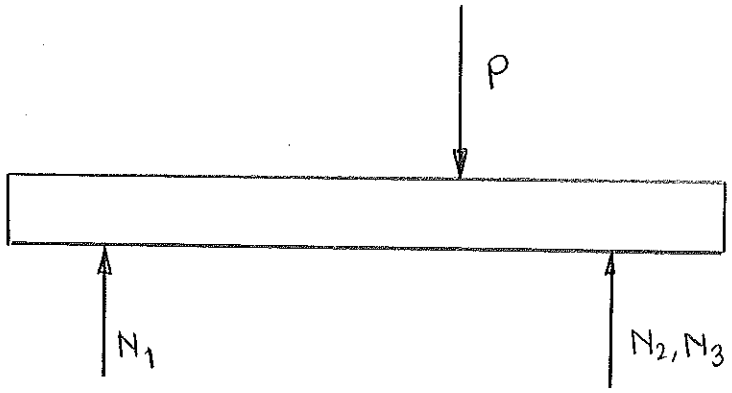






- $L = 20\,cm$
- $h_1 = 25mm$; $t_1 = 5mm$; $h_2 = 25mm$, $t_2 = 3mm$
- $a = 10cm$, $b = 4'5cm$, $c = 5cm$
- $E = 110\,GPa$, $\sigma_f = 495MPa$

A)



- Por simetría $N_2 = N_3$

$\Sigma F_y : N_1 + 2N_2 = P \Rightarrow N_2 = 0'344828\,P$

$\Sigma M = 0 : N_1\,(a+b) = P \cdot b \Rightarrow N_1 = \frac{b}{a+b} P = 0'310345P$

RESISTENCIA COMPRESIÓN

$\frac{N_2}{A_2} = \sigma_f$: $0'344828P = 3mm \cdot 25mm \; 495MPa \Rightarrow \boxed{107,662kN = P}$

$\frac{N_1}{A_1} = \sigma_f$: $0'310345P = 5mm \cdot 25mm \; 495MPa \Rightarrow \boxed{P = 199,375kN}$

PANDEO.

- Pilar (1)

$$P_{cr} = \frac{\pi^2 \cdot E\, I_{y_1}}{L_k^2} = 14424{,}645\,N.$$

$I_{z_1} = \frac{1}{12}\, 5mm (25mm)^3 = 6{,}510416667 \cdot 10^{-9} m^4$

$I_{y_1} = \frac{1}{12}\, 25mm (5mm)^3 = 2{,}604166667 \cdot 10^{-10} m^4$

$A = 25mm\, 5mm = 1'25 \cdot 10^{-4} m^2$

$I_{y_1} < I_{z_1}$

$L_k = 0'7L = 0'14m$

$N_1 = P_{cr}$

$0'310345P = 14424{,}645N$

$$\boxed{P = 46{,}479kN}$$

- Pilar (2)

$$P_{cr} = \frac{\pi^2\, E\, I_{y_2}}{L_k^2} = 3115{,}723328N$$

$I_{z_2} = \frac{1}{12}\, 3mm (25mm)^3 = 3'90625 \cdot 10^{-9} m^4$

$I_{y_2} = \frac{1}{12}\, 25mm (3mm)^3 = 5{,}625 \cdot 10^{-11} m^4$

$A = 25mm\, 3mm = 7'5 \cdot 10^{-5} m^2$

$L_k = 0'7L = 0'14m$

$I_{y_2} < I_{z_2}$

$N_2 = P_{cr}$

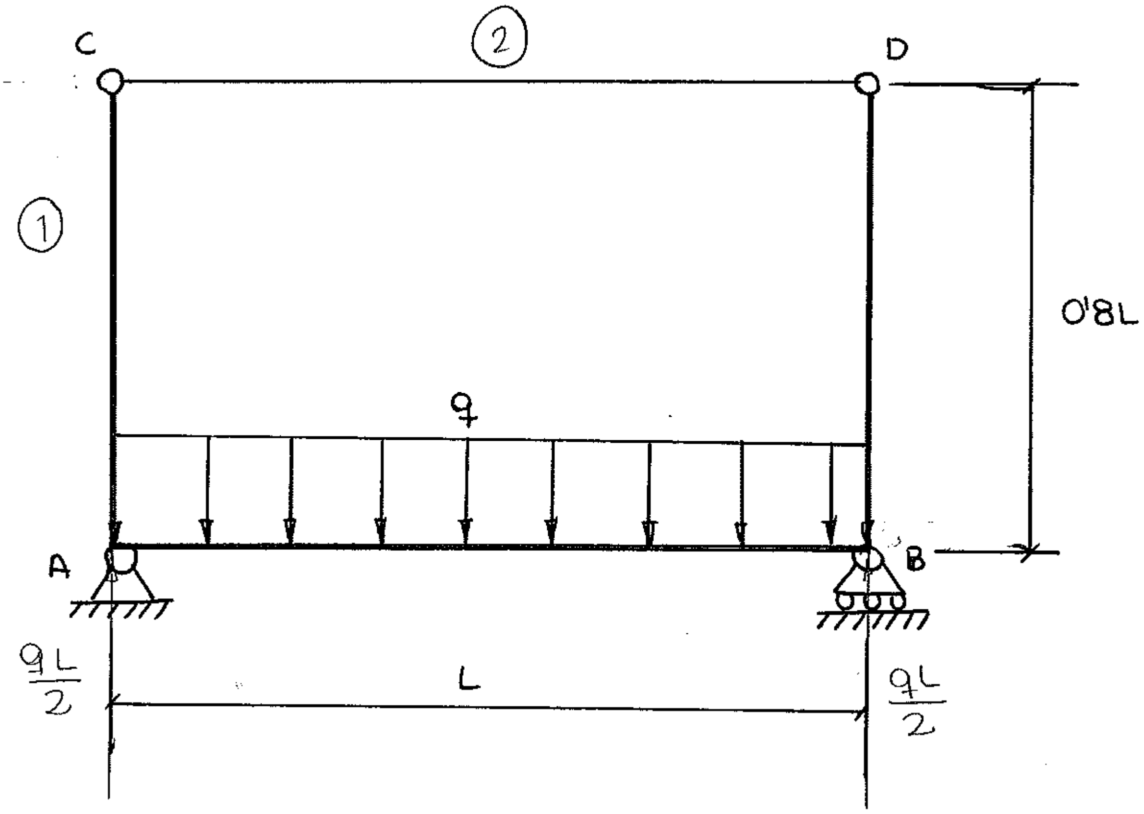
$0'344828P = 3115{,}723328N$

$$\boxed{P = 9035{,}587N}$$

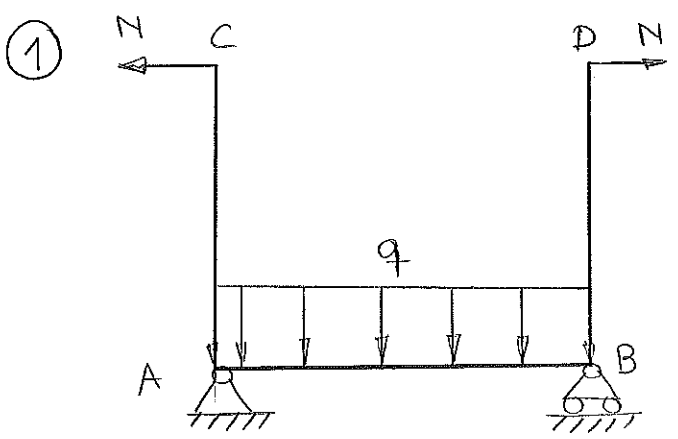
$\boxed{\text{La máxima carga P que puede resistir el soporte es de } P = 9035{,}587N}$

B)

C-4.5



- Barras AB, AC y CD: $S = 6 \times 8\,cm^2$
- Barra CD: sección circular: $d = 2\,cm$
- $L = 1\,m$
- Acero S-235 (curva c: $\alpha = 0'49$)
- $E = 210\,GPa$, $\sigma_f = 240\,MPa$.

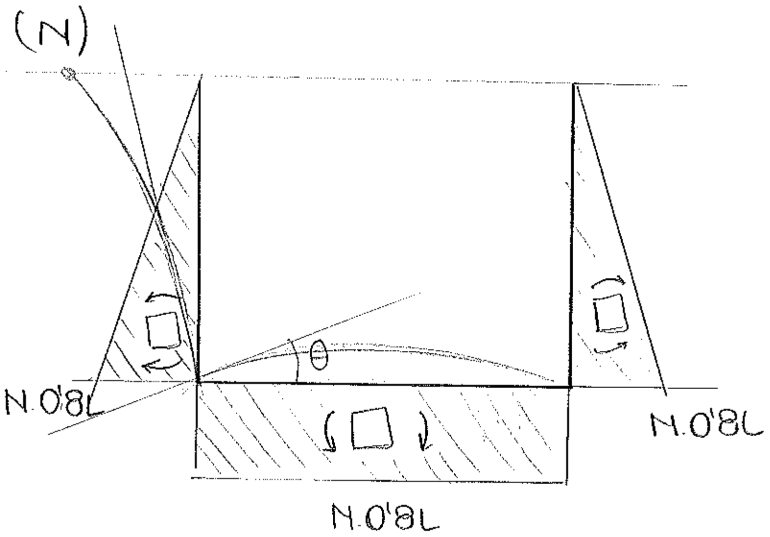


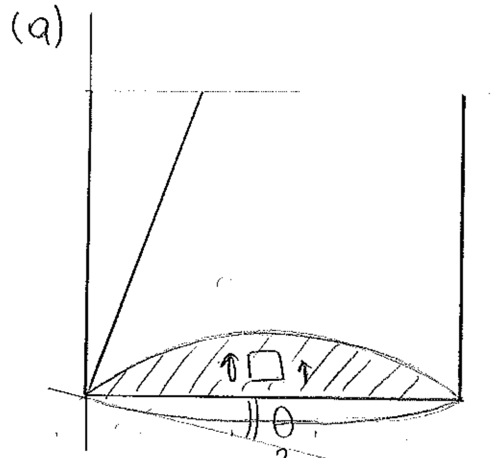
$$\delta_{c_2} = \frac{\Delta L}{2} = \frac{N \cdot L}{2EA}$$

$$I = 2{,}56 \cdot 10^{-6}\,m^4$$

$$\boxed{\text{Ec. compatibilidad: } \delta_{c_1} = \delta_{c_2}}$$

① Diagramas de esfuerzos flectores





$$\delta_C = \delta_C^{(N)} - \delta_C^{(q)}$$

$$\delta_C^{(N)} = \theta_A \cdot 0'8L + \delta_{CA}$$

$$\left.\begin{aligned} \theta_A &= \frac{\delta_{BA}}{L} \\ \delta_{BA} &= \frac{1}{EI}\left[N\,0'8L \cdot L \cdot \frac{L}{2}\right] = \frac{2NL^3}{5EI} \end{aligned}\right\} \theta_A = \frac{2NL^2}{5EI}$$

$$\delta_{CA} = \frac{1}{EI}\left[\frac{1}{2} N\,0'8L \cdot \frac{2}{3} 0'8L \cdot 0'8L\right] = \frac{64NL^3}{375EI}$$

$$\delta_C^{(N)} = \frac{184NL^3}{375EI}$$

$$\delta_C^{(q)} = \theta_A \cdot 0'8L$$

$$\theta_A = \frac{\delta_{BA}}{L}$$

$$\delta_{BA} = \frac{1}{EI}\int_0^L \left(\frac{qLx}{2} - \frac{qx^2}{2}\right) x\,dx = \frac{1}{EI}\left[\frac{qL}{2}\frac{L^3}{3} - \frac{qL^4}{8}\right] = \frac{qL^4}{24EI}$$

$$\delta_C^{(q)} = \frac{qL^4}{30EI}$$

$$\boxed{\delta_C^{(1)} = -\frac{184NL^3}{375EI} + \frac{qL^3}{30EI}}$$

$$\boxed{\frac{-184NL^3}{375EI} + \frac{qL^3}{30EI} = \frac{NL}{2EA}}$$

$$q = \frac{30I}{L^3}\left(\frac{184NL^3}{375I} + \frac{NL}{2A}\right) = 14{,}8422N \quad (B)$$

Esfuerzo flector máximo: (C)

$$\sigma_{xx} = \frac{0'8\,N.4cm}{I} = 240\,MPa \quad \boxed{\begin{array}{l} N_{máx} = 19200\,N \\ q_{máx} = 284970{,}8351\,N/m \end{array}}$$

Pandeo: tirante

$L_k = L$ $\qquad$ $I = 7'853981634\,10^{-9}$ (A)

$$P_{cr} = \frac{\pi^2 EI}{L_k^2} = 16278{,}29526N.$$

$$\bar{\lambda} = \sqrt{\frac{\sigma_f . A}{P_{cr}}} = 2{,}152167637 \quad (C)$$

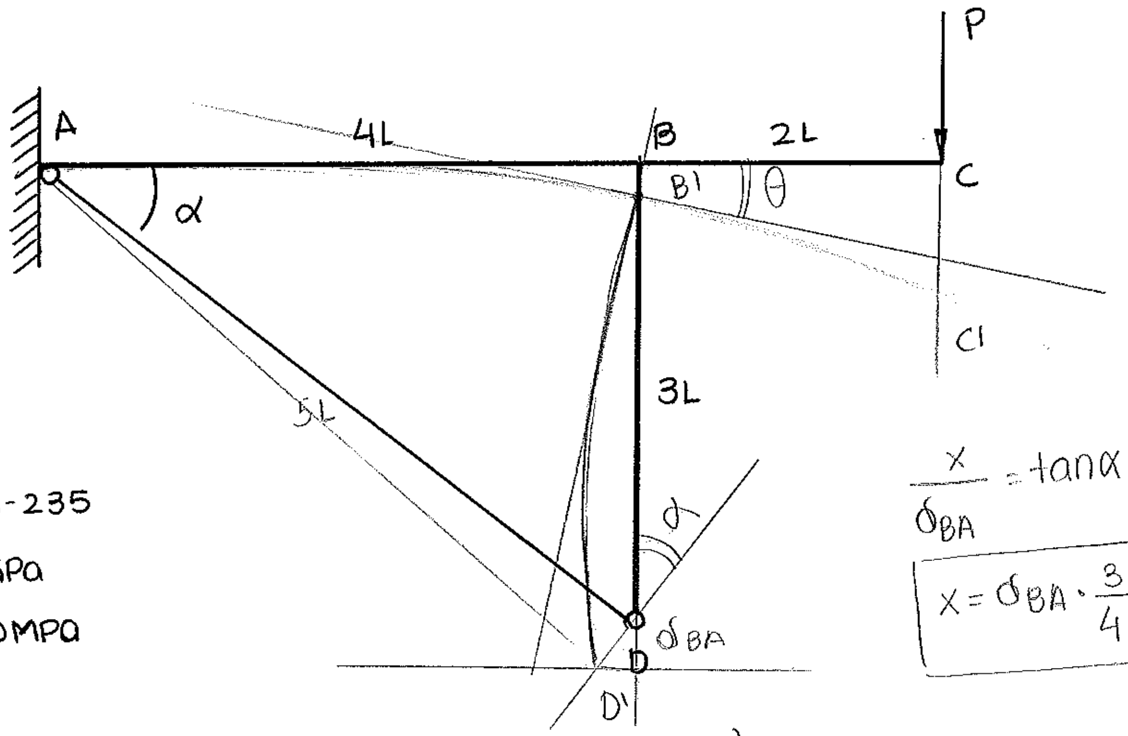
$$\phi = 0'5[1+\alpha(\bar{\lambda}-0'2)+\bar{\lambda}^2] = 3{,}29419384 \quad (D)$$

$$\chi = \frac{1}{\phi+\sqrt{\phi^2-\bar{\lambda}^2}} = 0{,}1727663535$$

$$P_b = \chi . A . \sigma_f = 13026{,}27616N = N \Rightarrow q = 193{,}33\,kN/m$$

$$\boxed{q_{máx} = 193{,}33\,\frac{kN}{m}}$$

C-4.6



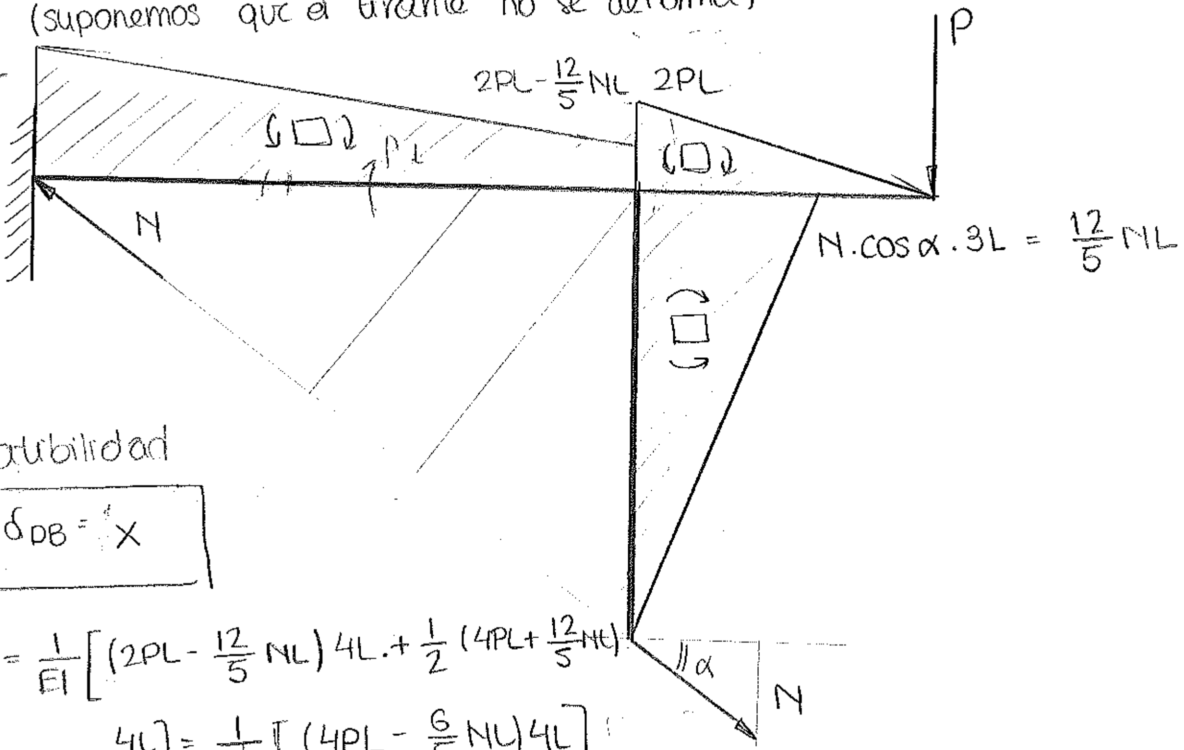
- Acero S-235
- $E = 210GPa$
- $\sigma_f = 240MPa$

$$\frac{x}{\delta_{BA}} = \tan\alpha$$

$$\boxed{x = \delta_{BA} \cdot \frac{3}{4}}$$

$N = f(P)$ (suponemos que el tirante no se deforma)

1



$$N \cdot \cos\alpha \cdot 3L = \frac{12}{5}NL$$

Ec. compatibilidad

$$\boxed{\theta \cdot 3L - \delta_{DB} = x}$$

$$\theta = \theta_{BA} = \frac{1}{EI}\left[\left(2PL - \frac{12}{5}NL\right)4L + \frac{1}{2}\left(4PL + \frac{12}{5}NL\right)4L\right] = \frac{1}{EI}\left[\left(4PL - \frac{6}{5}NL\right)4L\right]$$

$$\delta_{DB} = \frac{1}{EI}\left[\frac{1}{2}\,\frac{12}{5}NL \cdot 3L \cdot \frac{2}{3}3L\right] = \frac{36NL^3}{5EI}$$

$$\delta_{BA} = \frac{1}{EI}\left[\left(2PL - \frac{12}{5}NL\right)4L \cdot 2L + \frac{1}{2}\left(4PL + \frac{12}{5}NL\right) \cdot 4L \cdot \frac{2}{3}4L\right] =$$

$$= \frac{1}{EI}\left[\frac{112}{3}PL^3 - \frac{32}{5}NL^3\right]$$

$$\frac{12L^3}{EI}\left(4P-\frac{6}{5}N\right)-\frac{36NL^3}{5EI}=\frac{3}{4EI}\left[\frac{112PL^3}{3}-\frac{32}{5}NL^3\right]$$

$$20P=\frac{84}{5}N$$

$$\boxed{N=1'19P}$$

P= 10KN

L = 1'5m

$\boxed{\alpha = 0'49}$

sección 1: b= 50mm

$N=1'19P=11904{,}7619N$

$$P_{cr}=\frac{E\,\pi^2\,I}{L_K^{\,2}}=479772{,}4362N$$

$$\bar{\lambda}=\sqrt{\frac{\sigma_f A}{P_{cr}}}=1{,}118299108\ (A)$$

$$\phi=0'5\left[1+\alpha(\bar{\lambda}-0'2)+\bar{\lambda}^2\right]=1'350279729$$

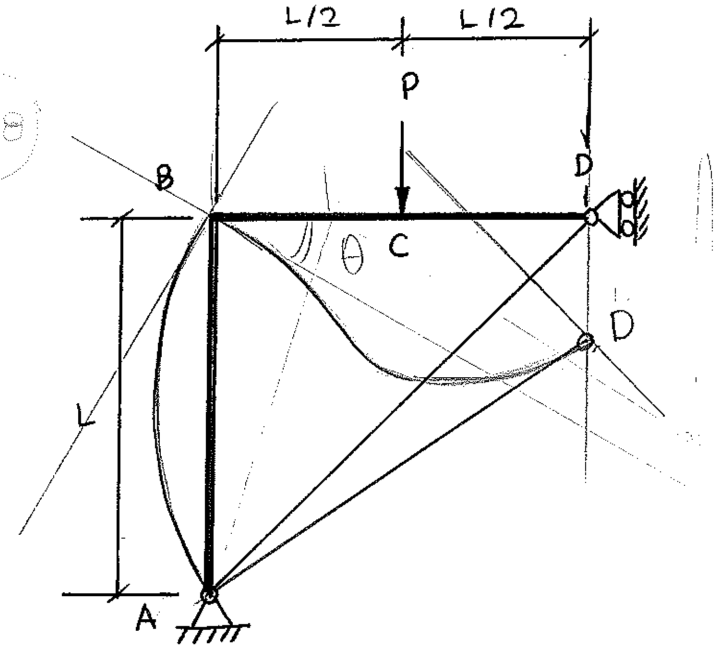
$$\chi=\frac{1}{\phi+\sqrt{\phi^2+\bar{\lambda}^2}}=0'37029$$

$$P_b=\chi\cdot\sigma_f\cdot A=222176{,}1858N$$

(...)

$\boxed{b=45mm}$

C-4.8

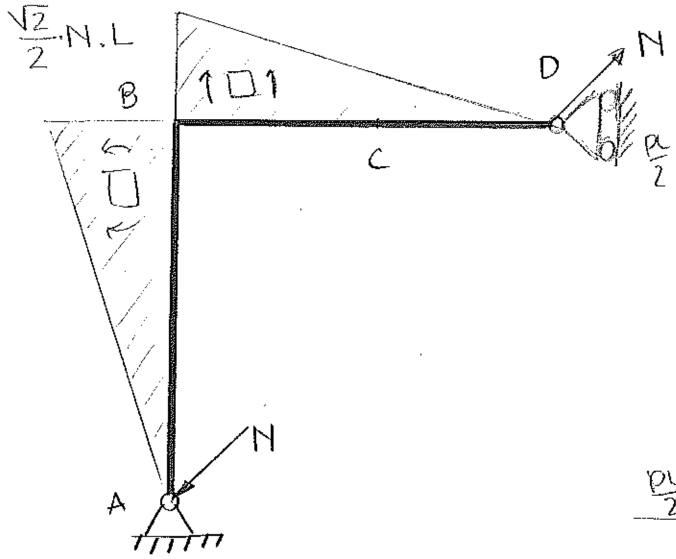


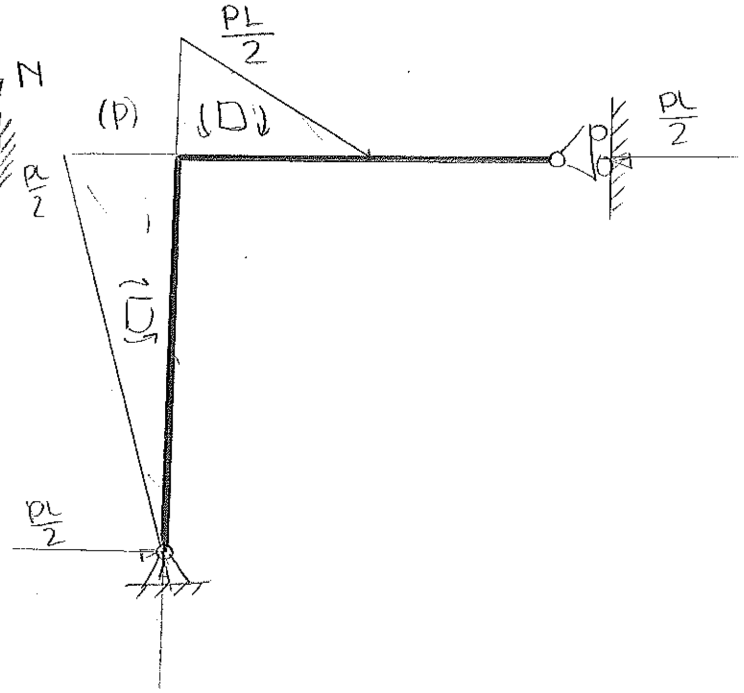
- $L = 1m$
- Sección: $S = 6 \times 8 cm^2$
- Tirante: $S = 3 \times 3 cm^2$
- Acero S-275
- $E = 210GPa$
- $\sigma_f = 260MPa$

$I = 2{,}56 \cdot 10^{-6} m^4$ (A)

1

(N)





Ec. compatibilidad

$$\boxed{\overset{\downarrow}{y}_D = \Delta_t \cdot \sqrt{2}}$$

$$\Delta_t = \frac{N \cdot \sqrt{2} L}{2EA}$$

$$\overset{\downarrow}{y}_D = \theta \cdot L - \delta_{DB}$$

$$\theta = \frac{\delta_{AB}}{L}$$

$$\delta_{AB} = \delta_{AB}^{(P)} - \delta_{AB}^{(N)}$$

$$\delta_{AB}^{(P)} = \frac{1}{EI}\left[\frac{1}{2}\frac{PL}{2} \cdot L \cdot \frac{2}{3}L\right] = \frac{PL^3}{6EI}$$

$$\delta_{AB}^{(N)} = \frac{1}{EI}\left[\frac{1}{2}\frac{\sqrt{2}}{2} N.L \cdot L \cdot \frac{2}{3}L\right] = \frac{\sqrt{2}NL^3}{6EI}$$

$$\delta_{DB}^{(P)} = \frac{1}{EI}\left[\frac{1}{2}\frac{PL}{2} \cdot \frac{L}{2} \cdot \left(\frac{2}{3}\frac{L}{2} + \frac{L}{2}\right)\right] = \frac{5PL^3}{48EI}$$

$$\delta_{DB}^{(N)} = \frac{\sqrt{2}NL^3}{6EI}$$

$$\frac{PL^3}{6EI} - \frac{\sqrt{2}NL^3}{6EI} - \frac{\sqrt{2}NL^3}{6EI} + \frac{5PL^3}{48EI} = \frac{NL}{EA}$$

$$5'037822421 \cdot 10^{-7} P = 8,821595335 \cdot 10^{-7} N$$

$$\boxed{N = 0'5710783854P}$$

Esfuerzo flector máximo

(B) $M_z = 0,09618660108 P.L \Rightarrow \sigma_{máx} = \frac{M_z \cdot 4cm}{I} = \sigma_f \Rightarrow \boxed{P = 186,304\,KN}$

(C) $M_z = \frac{\sqrt{2}}{4} NL \Rightarrow \sigma_{máx} = \frac{M_z \cdot 4cm}{I} = \sigma_f \rightarrow \boxed{P = 82'414\,KN}$

Pandeo $L_K = \sqrt{2}\,L$

$$P_{cr} = \frac{\pi^2 E I}{L_K^2} = 69950,82119N$$

$$\bar{\lambda} = \sqrt{\frac{\sigma_f A}{P_{cr}}} = 1'898033449 \ (B)$$

$$\phi = 0'5(1 + 0'49(\bar{\lambda} - 0'2) + \bar{\lambda}^2) = 2,717283682$$

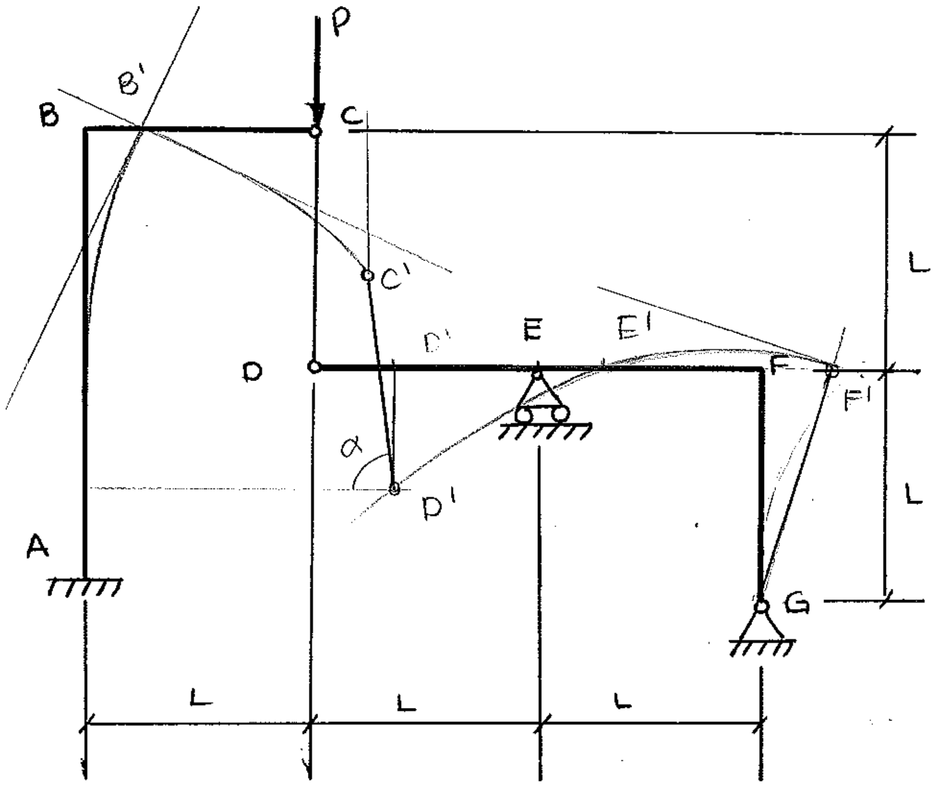
$$\chi = 0'2145099385$$

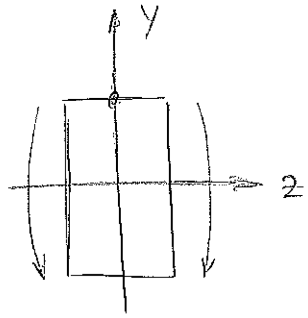
$$P_b = 54056,50449N = N \Rightarrow P = 94'657\,KN$$

Está hecho con $\sigma_f = 280MPa$

$$\boxed{\text{Falla por flexión en C con } P = 82'414KN}$$

C-4.9

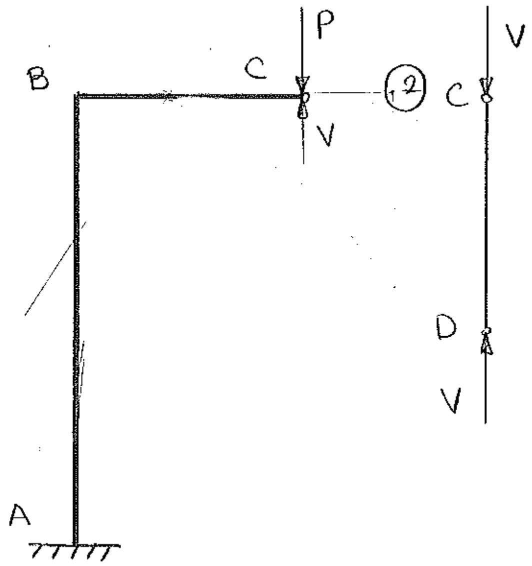




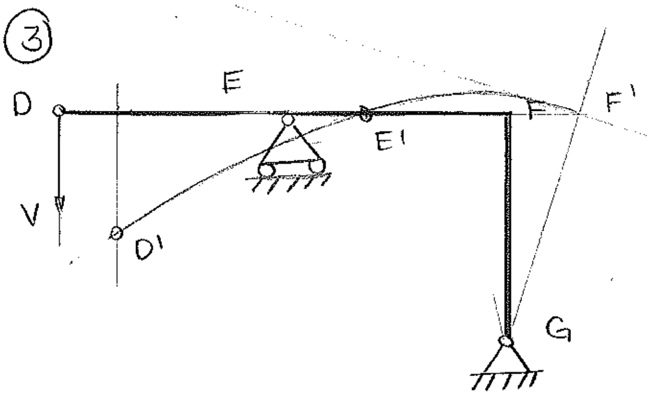
- Barra CD: $A_{CD} = 2 \times 2\ cm^2$
- Resto Barras: $A = 6 \times 8\ cm^2$; $I = \frac{1}{12} 6cm(8cm)^3 = 2{,}56 \cdot 10^{-6}\ m^4$
- $L = 1m$
- Acero S-275 ; $E = 200\ GPa$; $\sigma_f = 280\ MPa$.

(Se desprecian los efectos de los esfuerzos axial y cortante en los elementos que trabajan a flexión)

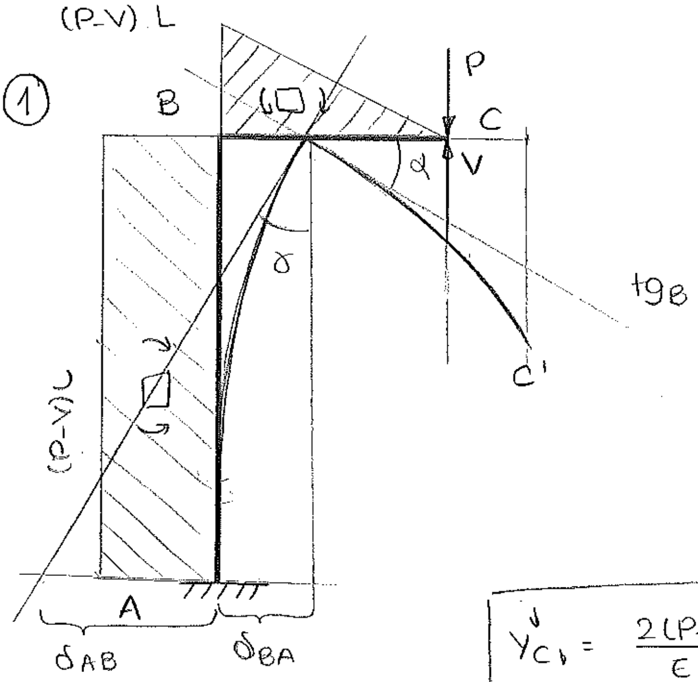
1



3



①



$$\overset{\downarrow}{Y}_{C1} = \alpha . L + \delta_{CB}$$

$$\alpha = \frac{\delta_{AB} + \delta_{BA}}{2L} = \frac{2(P-V)L^2}{EI}$$

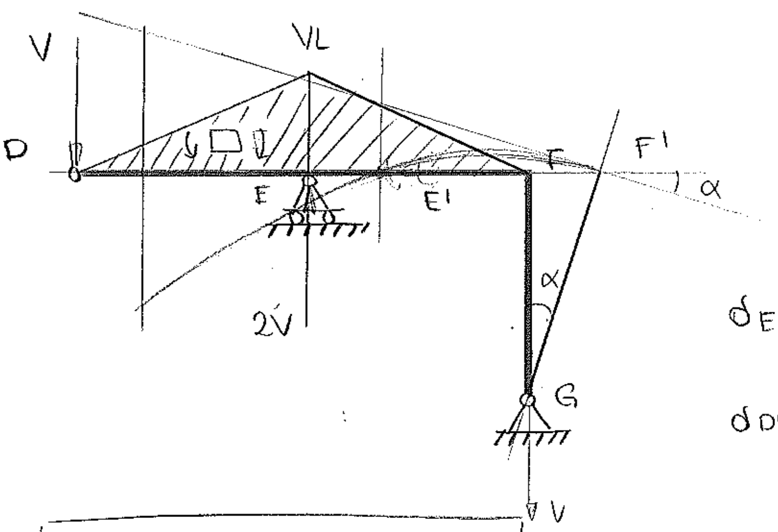
$$\delta_{AB} = \frac{1}{EI}\left[(P-V)L . 2L . L\right] = \frac{2(P-V)L^3}{EI} = \delta_{BA}$$

$$\delta_{CB} = \frac{1}{EI}\left[\frac{1}{2}(P-V)L . L \frac{\cancel{2}}{3}L\right] = \frac{(P-V)L^3}{6EI}$$

$$\overset{\downarrow}{Y}_{C1} = \frac{2(P-V)L^3}{EI} + \frac{(P-V).L^3}{6EI} = \frac{13(P-V)L^3}{6EI}$$

$$BB' = \delta_{BA} = \frac{2(P-V)L^3}{EI}$$

③



$$\overset{\downarrow}{Y}_{D3} = \delta_{DF} - \alpha . 2L$$

$$\alpha = \frac{\delta_{EF}}{L} = \frac{VL^2}{6EI}$$

$$\delta_{EF} = \frac{1}{EI}\left[\frac{1}{2}VL . L . \frac{1}{3}L\right] = \frac{VL^3}{6EI}$$

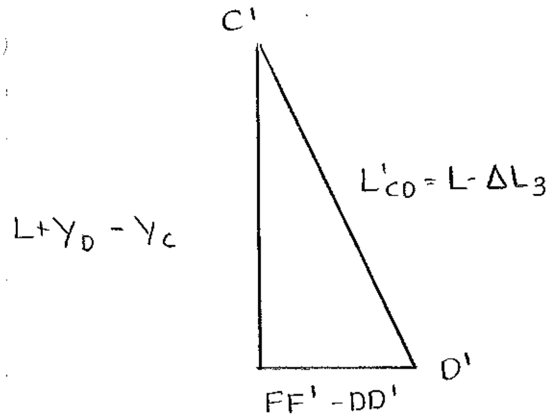
$$\delta_{DF} = \frac{1}{EI}\left[\frac{1}{2}VL . 2L . L\right] = \frac{VL^3}{EI}$$

$$\overset{\downarrow}{Y}_{D3} = \frac{VL^3}{EI} - \frac{VL^3}{3EI} = \frac{2VL^3}{3EI}$$

$$FF' = \alpha . L = \frac{VL^3}{6EI}$$

②

$$\Delta L_3 = \frac{P . L}{A_{co} . E}$$



● Ec. compatibilidad:

$$\underbrace{\left(L + \frac{2VL^3}{3EI} - \frac{13(P-V)L^3}{6EI}\right)^2 + \left(\frac{VL^3}{6EI}\right)^2 = \left(L - \frac{PL}{A_{CD}E}\right)^2}_{V = f(P)}$$

● Flexión máxima:

(E) $\sigma_{xx} = \dfrac{VL \cdot 4cm}{2'56 \cdot 10^{-6} m^4} = 15625 V = 280 MPa$

$V = 17920 N$ (A) $\Rightarrow \boxed{P = 23507,30 N}$

Para $P = 23507,30 N$

(A) $\sigma_{xx} = \dfrac{(P-V) \cdot L \cdot 4cm}{2,56 \cdot 10^{-6} m^4} = 87,302 MPa < \sigma_f$: OK!

PANDEO si $V = 17920 N$ ¿falla? $L_K = 1m$ curva c $\boxed{\alpha = 0'49}$

$$P_{cr} = \frac{\pi^2 E I}{L_K^2} = 26318,94507 N$$

$$\bar{\lambda} = \sqrt{\frac{\sigma_f A}{P_{cr}}} = 2,062883884 \quad \lambda \to A_S$$

$$\phi = 0'5[1 + \alpha(\bar{\lambda} - 0'2) + \bar{\lambda}^2] = 3'084151396 \,(-B)$$

$$\chi = \frac{1}{\phi + \sqrt{\phi^2 - \bar{\lambda}^2}} = 0'1859822484$$

$P_b = \chi \cdot \sigma_f \cdot A = 20830,01182 N > V \Rightarrow \boxed{\text{No falla}}$

> La estructura falla para $P = 23507,30 N$ y el fallo se produce en E por flexión