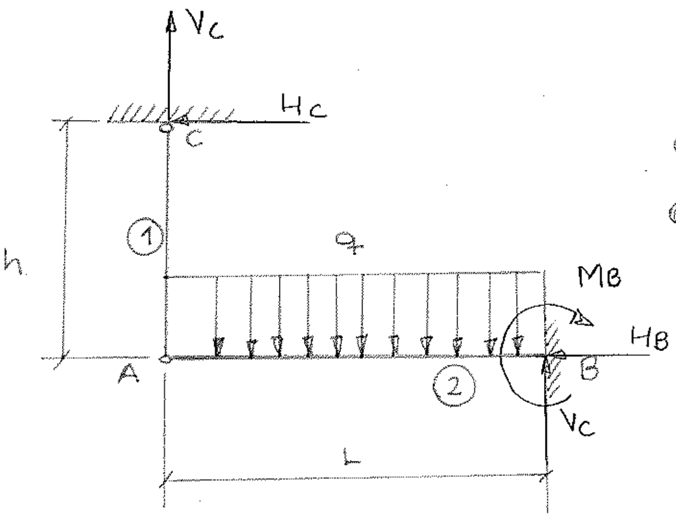
2.1



- Cable: A_1, E_1
- viga: EI. (rigidez a flexión)

- Incógnitas: 5
- Ecuaciones: 4

$\left.\right\} \boxed{h=1}$

1° ECUACIONES DE EQUILIBRIO.

$\Sigma F_x = 0: \; H_C + H_B = 0 \Rightarrow \boxed{H_B = 0}$

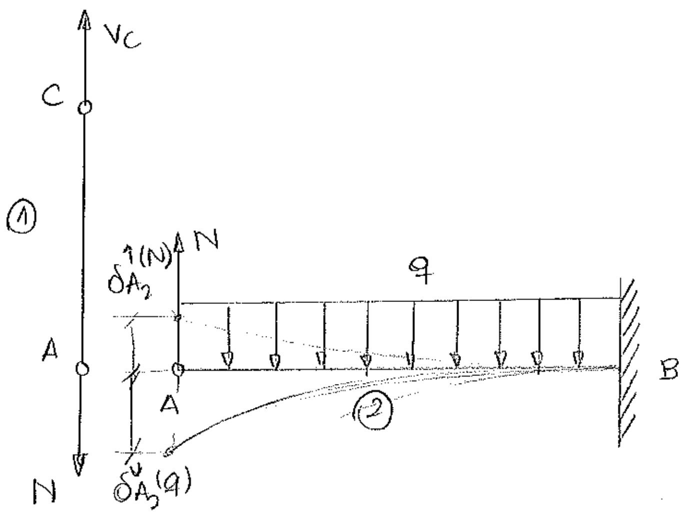
$\Sigma F_y = 0: \; V_B + V_C = q.L$

$M_A^{(AC)} = 0: \; H_C . h = 0 \Rightarrow \boxed{H_C = 0}$

$M_A^{(AB)} = 0: \; M_B + q.L.\frac{L}{2} - V_C.L = 0.$

2° ELEGIMOS UNA INCÓGNITA Y HALLAMOS LA ESTRUCTURA BÁSICA

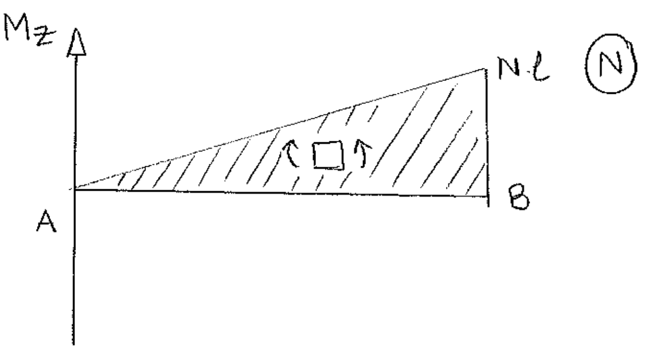
- Incógnita: N

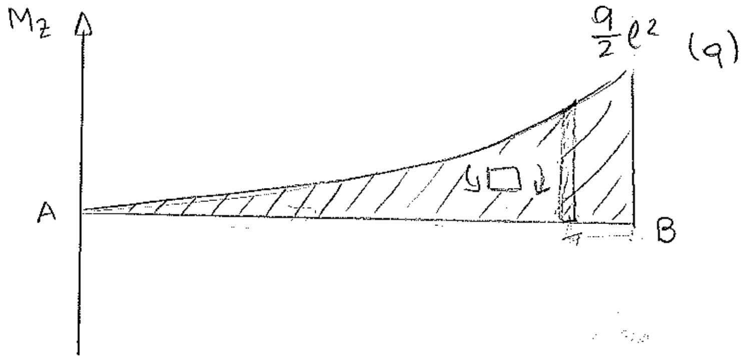


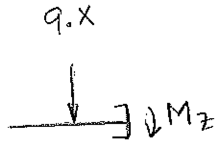
- Ecuación de compatibilidad

$\delta_{A_1}^{\downarrow} = \delta_{A_2}^{\downarrow} = -\delta_{A_2}^{\uparrow(N)} + \delta_{A_2}^{\downarrow(q)}$

Diagrama de esfuerzos de la viga.







$M_z = \frac{q}{2} x^2$

$$\delta_{A_1}^{\downarrow} = \frac{N.h}{E_1 . A_1}$$

$$\delta_{A_2}^{\uparrow (N)} = \frac{1}{EI}\left(\frac{1}{2} \cdot N\ell . \ell . \frac{2}{3}\ell\right) = \frac{N\ell^3}{3EI}$$

$$\delta_{A_2}^{\downarrow (q)} = \frac{q\ell^4}{8EI}$$

Mirar en Resis I cuál era la fórmula para la integral

$$\delta = \frac{1}{EI}\int_0^{\ell} \frac{qx^2}{2} dx \cdot X$$

$$\frac{1}{EI}\int_0^{\ell} \frac{qx^2}{2} dx \, (\ell - x)$$

$$\frac{N.h}{E_1 . A_1} = \frac{-N\ell^3}{3EI} + \frac{q\ell^4}{8EI}$$

$$N\left(\frac{h}{E_1 A_1} + \frac{\ell^3}{3EI}\right) = \frac{q.\ell^4}{8EI}$$

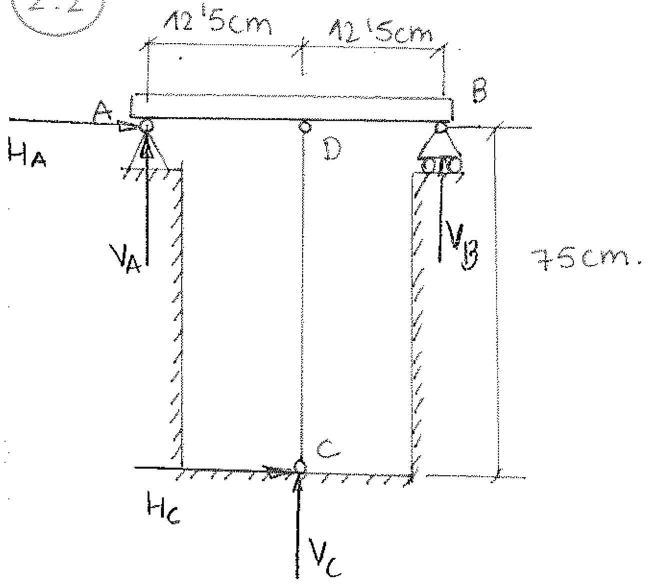
$$N = \frac{q\ell^4}{8EI\left(\frac{h}{E_1 A_1} + \frac{\ell^3}{3EI}\right)}$$

$$\overset{\downarrow}{\delta_1} = \frac{N.h}{E_1 A_1} = \frac{q.\ell^4}{8EI\left(\frac{h}{E_1 A_1} + \frac{\ell^3}{3EI}\right)} \cdot \frac{h}{E_1 . A_1} = \frac{q.\ell^4 . h}{8\cancel{EI}\left(\frac{3EIh + E_1 A_1 \ell^3}{3 \cancel{E_1 A_1} \cancel{EI}}\right)\cancel{E_1 A_1}} =$$

$$= \frac{3q\ell^4 h}{24EI.h + 8E_1 A_1 \ell^3}$$

$$\boxed{\text{El alargamiento del cable es } \delta_1 = \frac{3q.\ell^4 h}{24EIh + 8E_1 A_1 \ell^3}}$$

2.2



- Viga AB: $EI = 30\,kN.cm^2$

  $T_f = 10°C \Rightarrow \Delta T = -55°C$ (alambre)

- Datos: $A_c = 6.10^{-2} mm^2$

  $E = 200 GPa$

  $\alpha = 11{,}7.10^{-6}\,C^{-1}$

- Incógnitas: 5
- Ecuaciones: 4

$\left.\begin{matrix}\\ \\ \end{matrix}\right\} \boxed{h=1}$

1' ECUACIONES DE EQUILIBRIO.

$\Sigma F_x = 0: \quad H_A + H_C = 0 \Rightarrow \boxed{H_A = 0}$

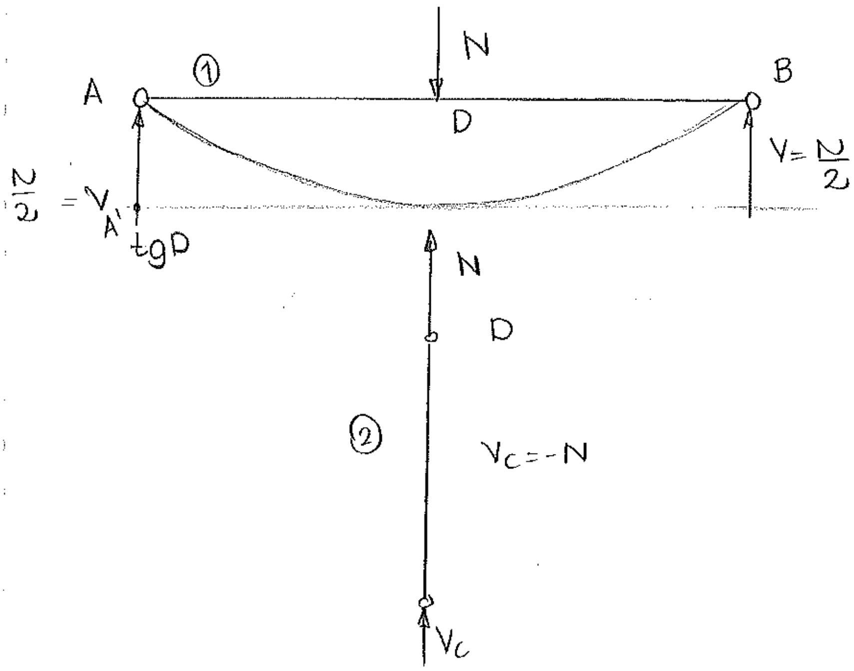
$\Sigma F_y = 0: \quad V_A + V_C + V_B = 0.$

$M_D^{(DC)} = 0: \; H_C . 75cm = 0. \Rightarrow \boxed{H_C = 0}$

$M_D^{(ADB)} = 0: \; V_A . 12'5cm - V_B . 12'5cm = 0 \Rightarrow V_A = V_B = V$

2' ELEGIMOS UNA INCÓGNITA Y HALLAMOS LA ESTRUCTURA BÁSICA.

- Incógnita N

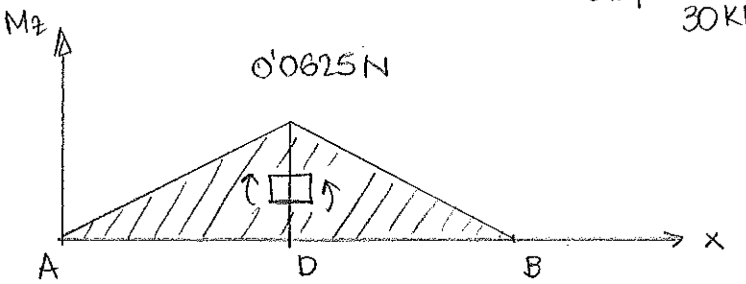


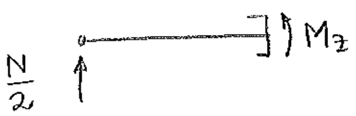
- Ecuación de compatibilidad

$\boxed{\delta_{D_1}^{\downarrow} = \delta_{D_2}^{\downarrow}}$

- Diagrama de esfuerzos de la viga

$$\overset{\downarrow}{\delta}_{D_1} = \frac{1}{30\,KN \cdot cm^2} \cdot \left(\frac{1}{2}\, 6'25cm \cdot N \cdot 12'5cm \cdot \frac{2}{3}\, 12'5cm \right)$$





: $M_z = \frac{N}{2}x$

$$\overset{\downarrow}{\delta}_{D_2} = -\frac{N \cdot 75cm}{200GPa \cdot 6 \cdot 10^{-2} mm^2} + 11'7 \cdot 10^{-6}\, C^{-1} \cdot 75cm \cdot 55°C$$

$$\frac{1}{30\,KN \cdot cm^2}\left(\frac{1}{2}\, 6'25cm\, N \cdot 12'5cm\, \frac{2}{3}\, 12'5\,cm \right) = -\frac{N \cdot 75cm}{200GPa \cdot 6 \cdot 10^{-4} cm^2} + 11'7 \cdot 10^{-6}\, C^{-1}\, 75cm\, 55C$$

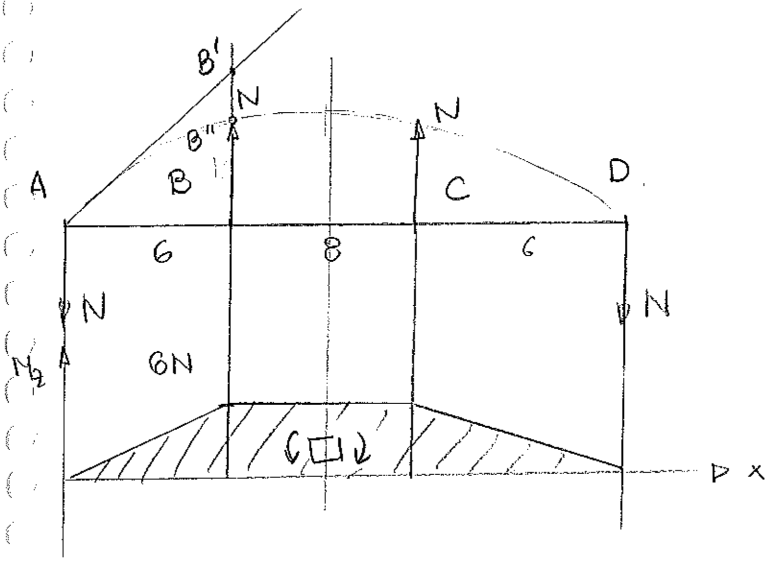
$200GPa = 200 \cdot 10^5 \frac{N}{cm^2}$

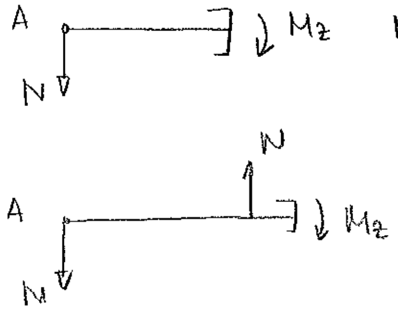
$N = 2{,}822N$

$\sigma_{xx} = \frac{N}{A} = 47'038\,MPa$

$\boxed{\text{La tensión en el alambre será de } \sigma_{xx} = 47'038MPa}$

Diagrama de esfuerzos debido a N





$M_z = Nx$

$M_z = Nx - N(x-6) =$

$= 6N$

$$\delta_{B_2}^{\uparrow(N)} = BB' - B'B''$$

$$B'B'' = \frac{1}{EI}\left(\frac{1}{2}\cdot 6 \cdot 6N \cdot \frac{1}{3}6\right) = \frac{36\,N}{EI}$$

$$BB' = \theta_A \cdot 6 = \frac{252\,N}{EI}$$

$$\theta_A = \frac{\delta_{DA}}{20} = \frac{1}{20}\,\frac{1}{EI}\left(\frac{1}{2}\cdot 6 \cdot 6N \cdot 2 + 8 \cdot 6N\right)\cdot 10 = \frac{42N}{EI}$$

$$\delta_{B_2}^{\uparrow(N)} = \frac{216\,N}{EI}$$

$$\delta_{B_1}^{\downarrow} = \frac{10\,N}{E \cdot 3\cdot 10^{-3}}$$

$$\frac{10\,N}{E\cdot 3\cdot 10^{-3}} = 0'47407\text{ m} - \frac{216\,N}{EI}$$

$N = 197325,9567\,N$

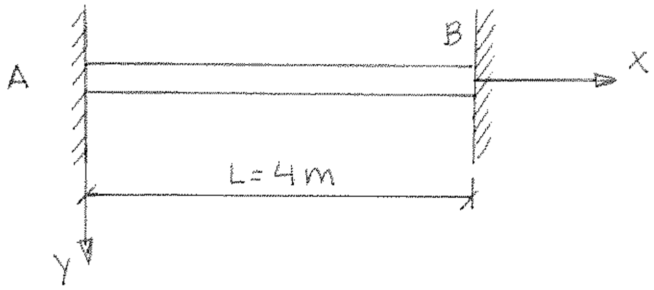
$$\boxed{\sigma_v = 77'86\,MPa}$$

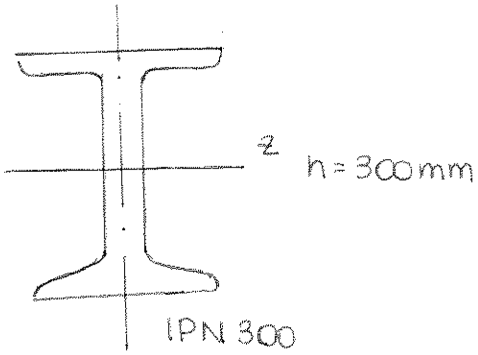
M,máx (viga) = 201567,1368 N.m (sección B)

$$\boxed{\sigma_{xx,max} = 111'982\,MPa}$$

Las tensiones en la varilla son de $\sigma_v = 77'86$ MPa y tensión máxima en la viga es de $\sigma_{xx} = 111'982$ MPa en la sección B.

2.4



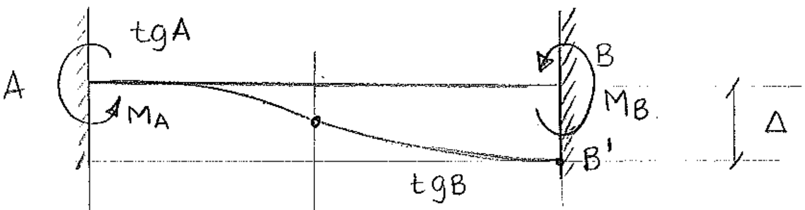


$I_z = 9800\,cm^4 = 9'8 \cdot 10^{-5}\,m^4$

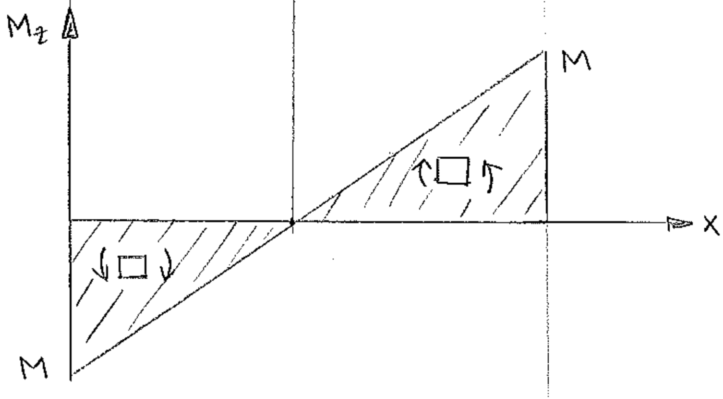
$\sigma_{adm} = 100\,MPa$

$E = 200\,GPa.$

¿Por qué no calcula las reacciones?
Habría que calcularlas para el diagrama de esfuerzo cortante...



- Diagrama de esfuelzos flectores



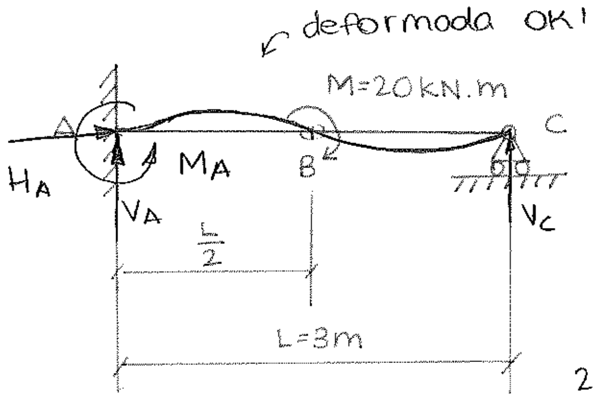
$$\Delta = \delta_{BA} = \frac{1}{EI}\left[\frac{1}{2}\cdot M\cdot\frac{L}{2}\cdot\left(L-\frac{L}{6}\right) - \frac{1}{2}\cdot M\cdot\frac{L}{2}\cdot\frac{L}{6}\right] = \frac{M\cdot L^2}{6EI}$$

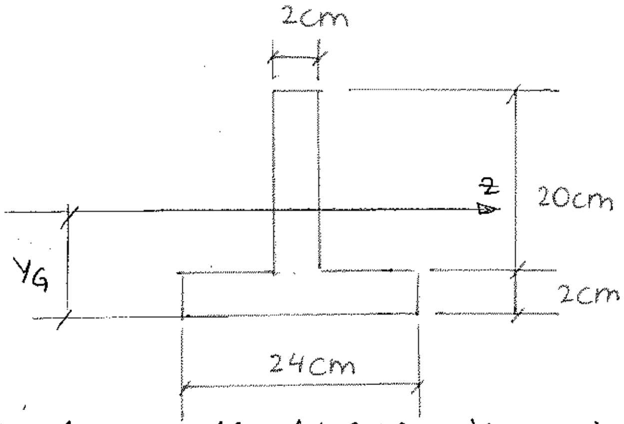
$$\sigma_{xx,máx} = \sigma_{adm} = \frac{M\cdot y}{I_z} = \frac{M\cdot 150\cdot 10^{-3}\,m}{9'8\cdot 10^{-5}\,m^4} = 100\,MPa \Rightarrow M = 65'33\,kN\cdot m$$

$\Delta = 8'889\,mm$

El descenso máximo que podrá soportar será de $\Delta = 8'889\,mm$

2.5





$24.2.1+20.2.11=(24.2+20.2)Y_G \Rightarrow Y_G=5'54cm$

E = 100 GPa. $I_z = 4000\ cm^4 = 4.10^{-5} m^4$

1) DIAGRAMA DE MOMENTOS FLECTORES Y ESFUERZOS CORTANTES.

- Incógnitas: 4
- Ecuaciones: 3

$\boxed{h=1}$

1° EC EQUILIBRIO

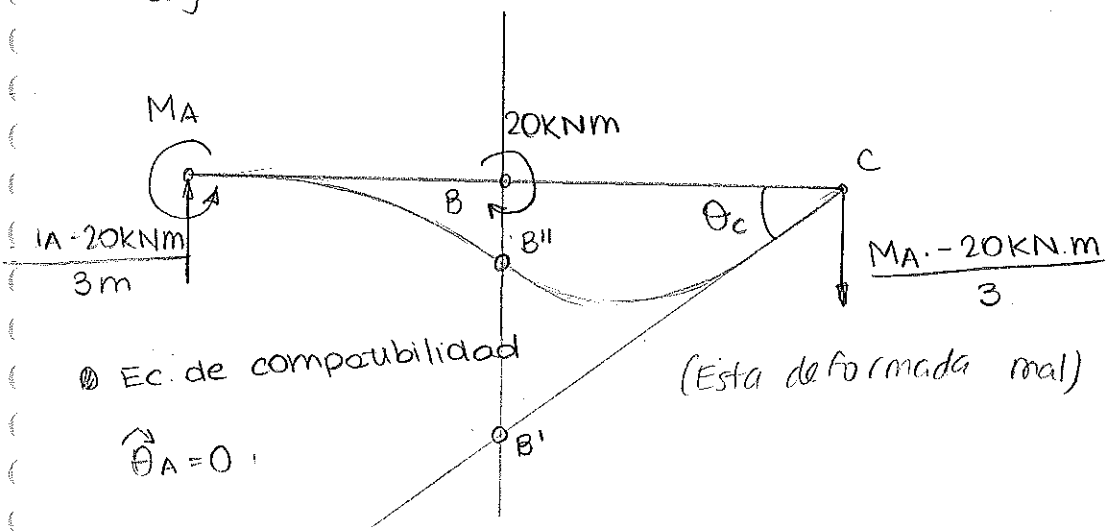
(1) $\Sigma F_x=0: \boxed{H_A=0}$

(2) $\Sigma F_y=0: V_A+V_C=0 \Rightarrow \boxed{V_A=-V_C=V}$

(3) $\Sigma M_A=0: M_A-20kN.m+V_C.3m=0 \Rightarrow V=\dfrac{M_A-20kNm}{3m}$

2° ELEGIMOS UNA INCÓGNITA Y HALLAMOS LA ESTRUCTURA AUXILIAR.

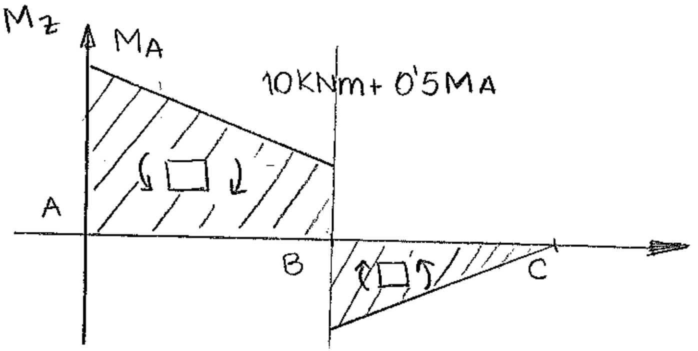
- Incógnita: M_A.

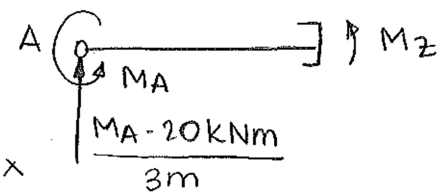


- Ec. de compatibilidad

$\hat{\theta}_A=0$

● Diagrama de momentos flectores



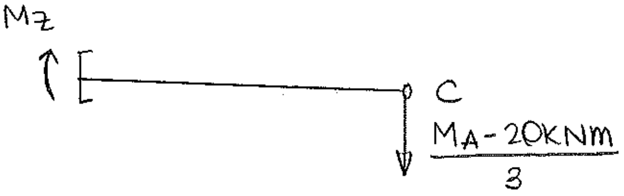


$$M_z = \frac{M_A - 20kNm}{3m} x - M_A$$

$x = 0 : M_z = -M_A$

$x = 1'5 : M_z = \frac{M_A}{2} - 10kNm - M_A =$

$= -10kNm - \frac{M_A}{2}$



$$M_z = -\frac{M_A - 20kNm}{3} x$$

$x = 0 : M_z = 0$

$x = 1'5 : M_z = -\frac{M_A}{2} + 10\,kNm$

$$\delta_{CA} = 0 : \quad \delta_{CA} = \frac{1}{EI}\left[\frac{1}{2}\cdot(10kNm - 0'5M_A)\cdot 1'5m \cdot \frac{2}{3} 1'5m - \right.$$

$$\left. -\left(\frac{1}{2}\cdot 1'5m\,(0'5M_A - 10kNm)\cdot\frac{5}{3} 1'5m + 1'5m(10kNm + 0'5M_A)\cdot 2'25m\right)\right] = 0$$

$$\boxed{\begin{array}{l} M_A = -2500\,N.m \\ V = -7500\,N.m \end{array}}$$

● Diagrama de momentos flectores

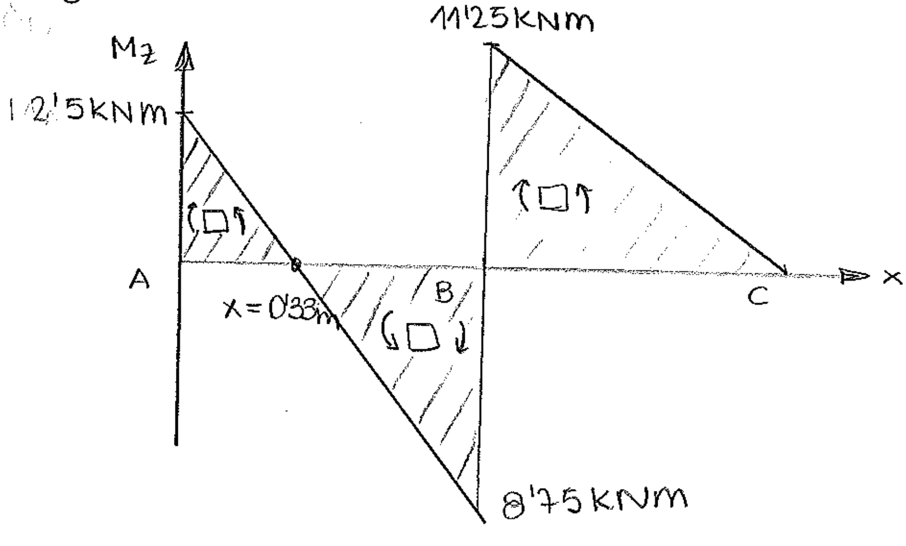
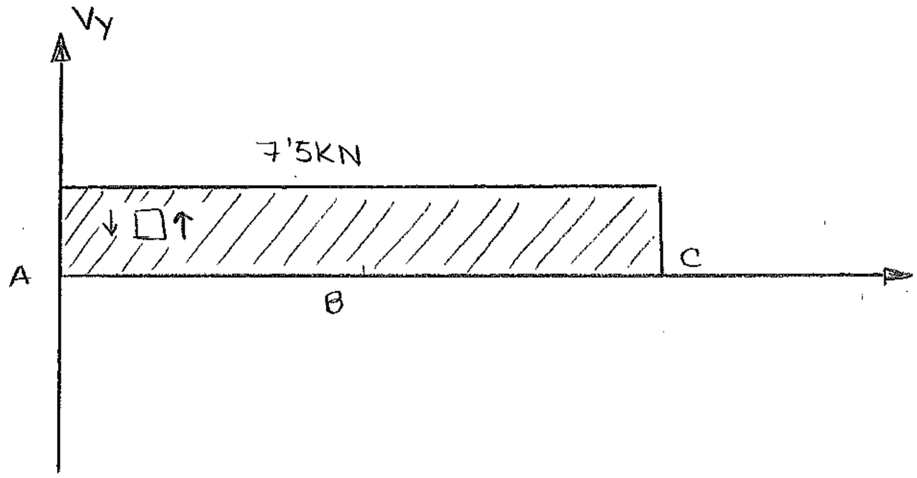
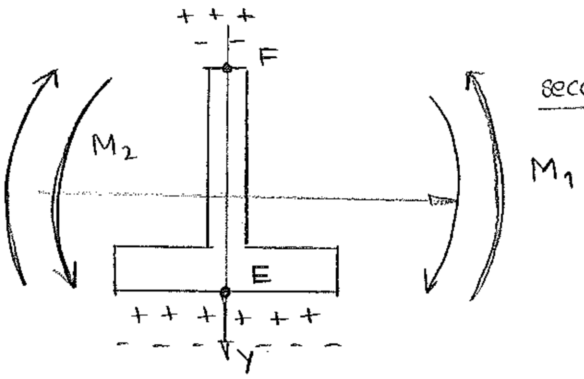


Diagrama de esfuerzos cortantes



2) TENSIONES NORMALES DE MÁXIMA TRACCIÓN Y MÁXIMA COMPRESIÓN.



sección B de BC

$M_{máx} = 11'25 KNm = M_1$

$$\boxed{\sigma_{xx,máx}^{(M_1)}} = \sigma_{xx}^{(M_1)}(E) = \frac{11'25 KN \cdot 5'54\,cm}{4 \cdot 10^{-5} m^4} =$$

$$= \boxed{15'597 MPa\ (+)}$$

$$\boxed{\sigma_{xx,máx}^{(M_2)}} = \sigma_{xx}^{(M_1)}(F) = \frac{11'25 KN\,(22cm - 5'54cm)}{4 \cdot 10^{-5} m^4} = \boxed{46'278\ MPa\ (-)}$$

$M_{máx} = 8'75 KN \cdot m = M_2$

Sección B de AB

$$\boxed{\sigma_{xx,máx}^{(M_2)}} = \sigma_{xx}^{(M_2)}(F) = \frac{8'75 KN \cdot m(22cm - 5'54cm)}{4 \cdot 10^{-5} m^4} = \boxed{35'994 MPa (+)}$$

$$\boxed{\sigma_{xx,máx}^{(M_2)}} = \sigma_{xx}^{(M_2)}(E) = \frac{8'75 KN \cdot m\ 5'54cm}{4 \cdot 10^{-5} m^4} = \boxed{12'131 MPa (-)}$$

La maxima tensión de tracción es $\sigma_{xx} = 35'994$ MPa y la máxima tensión de tracción será de $\sigma_{xx} = 46'278$ MPa. Ambas en la sección B.

3) ESFUERZO TOTAL DE DESGARRAMIENTO ENTRE ALA Y ALMA.

what!

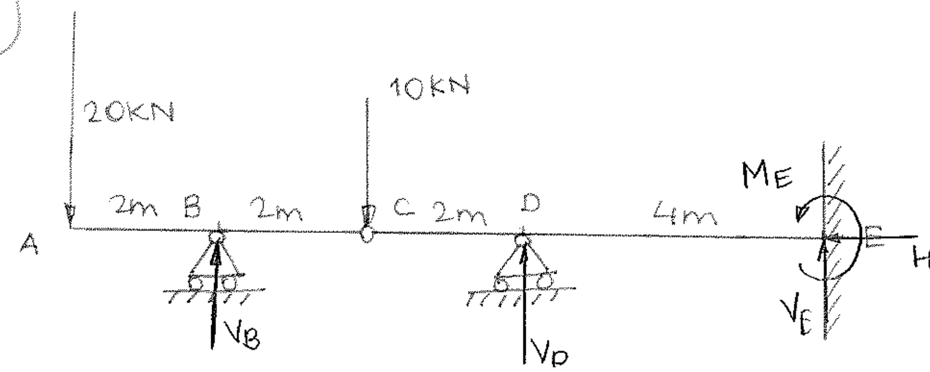
4) FLECHA EN B Y DIBUJO APROXIMADO DE LA DEFORMADA

$$\delta_{BA} = \frac{1}{EI}\left[-\frac{1}{2}\cdot 0'33m \cdot 2'5KNm \cdot 1'388m + \frac{1}{2}\cdot 1'667m \cdot 8'75KNm \cdot 0'388m\right] =$$

$$= 3'5156 \cdot 10^{-4} m = 0'035156 cm$$

$\boxed{\text{La flecha en B es } \delta_{BA} = 0'035156 cm}$

2.7



$I = 150.000 \, cm^4$

$E = 200 \, GPa$

- Incógnitas: 5
- Ecuaciones: 4

$\boxed{h=1}$

1) ECUACIONES DE EQUILIBRIO

(1) $\Sigma F_x = 0$: $\boxed{H_E = 0}$

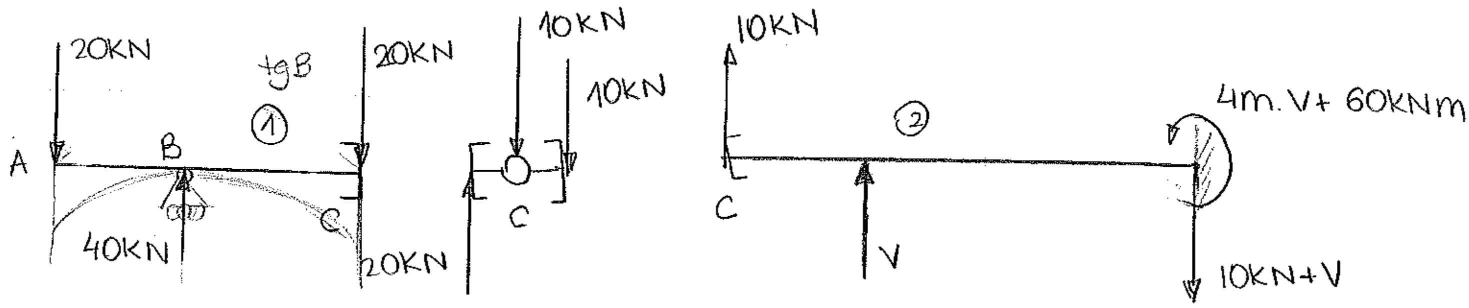
(2) $\Sigma F_y = 0$: $V_B + V_D + V_E = 30 \, KN \Rightarrow V_D + V_E = -10 \, KN \Rightarrow V_E = -10KN - V$

(3) $M_C^{(ABC)} = 0$: $20KN \cdot 4m - V_B \cdot 2m = 0 \Rightarrow \boxed{V_B = 40KN}$

(4) $M_C^{(CDE)} = 0$: $M_E + V_E \cdot 6m + V_D \cdot 2m = 0 \Rightarrow M_E = -2m \cdot V - 6m(-10KN - V) =$
$= 4m \cdot V + 60 \, KN \cdot m$

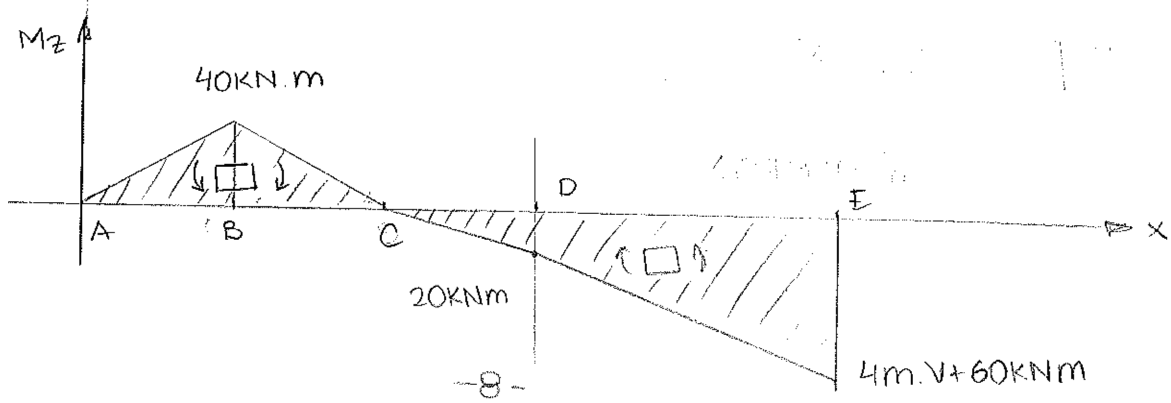
2) ELEGIMOS UNA INCÓGNITA Y OBTENEMOS LA ESTR. BÁSICA

- Incógnita: $V_D = V$



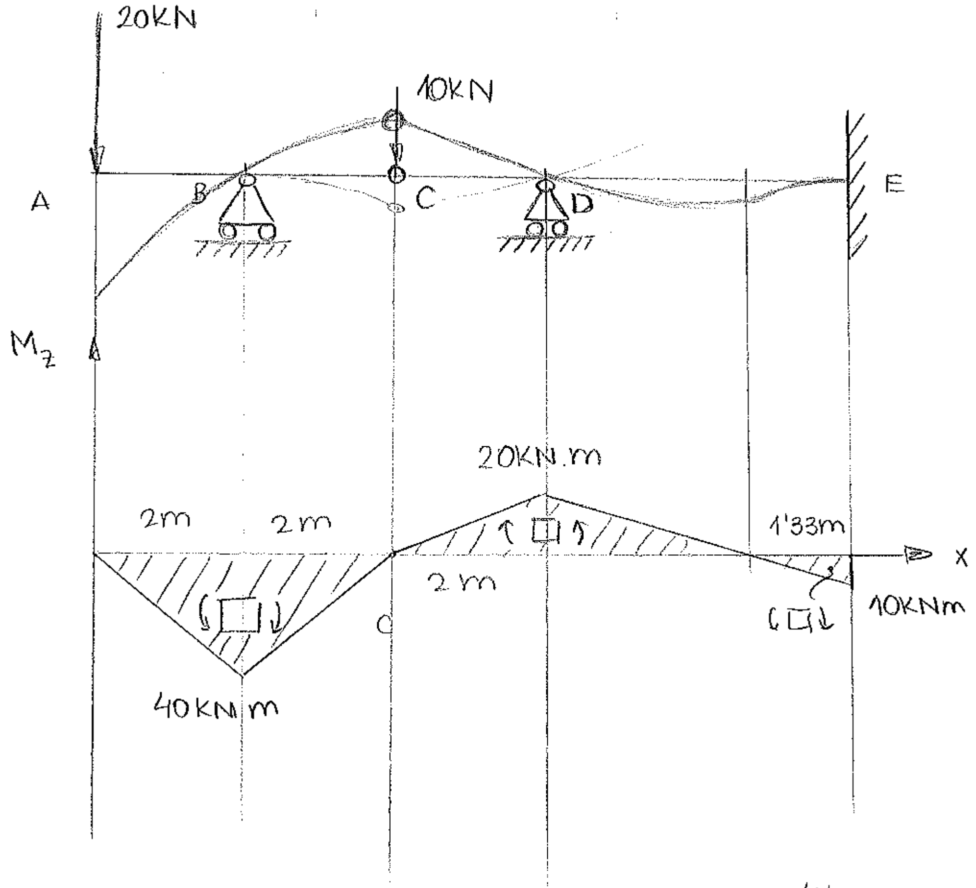
- Ecuación de compatibilidad: $\delta_D = 0$
- Diagramas de momentos $f(V)$.

1



$$\delta_D = 0 = \delta_{DE} = \frac{1}{EI}\left[4m \cdot 20KN\,m \cdot 2m + \frac{1}{2}4m(4m \cdot V + 40KNm)\frac{2}{3}4m\right] = 0$$

$\boxed{V = -17500\,N}$



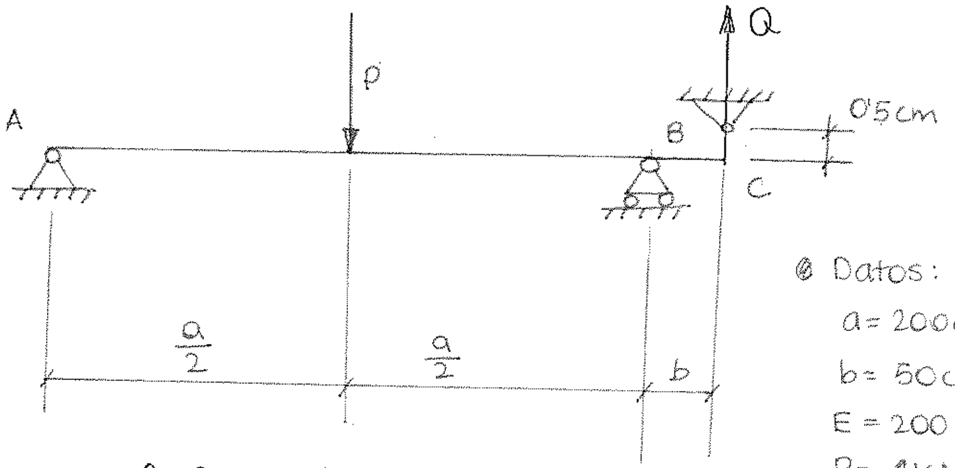
$$\delta_C = \delta_{CE} = \frac{1}{EI}\cdot\left[-\frac{1}{2}\cdot 1'33m \cdot 10KNm \cdot (6m - \frac{1}{3}1'33m) + \frac{1}{2}(4m - 1'33m)\cdot 20KNm \cdot (2m + \frac{1}{3}\cdot\right.$$

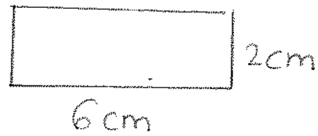
$$\left.(2'67m) + \frac{1}{2}2m\,20KNm\cdot\frac{2}{3}\cdot 2m\right] = \frac{1}{EI}\left[-\frac{10^6}{27} + \frac{208 \cdot 10^4}{27} + \frac{8 \cdot 10^4}{3}\right] = 2,22 \cdot 10^{-4}\,m$$

$\delta_C = 0'022cm$

La flecha en el punto C es de 0'22cm

2.10





Datos:

$a = 200\text{cm} = 2\text{m}$

$b = 50\text{cm} = 0'5\text{m}$

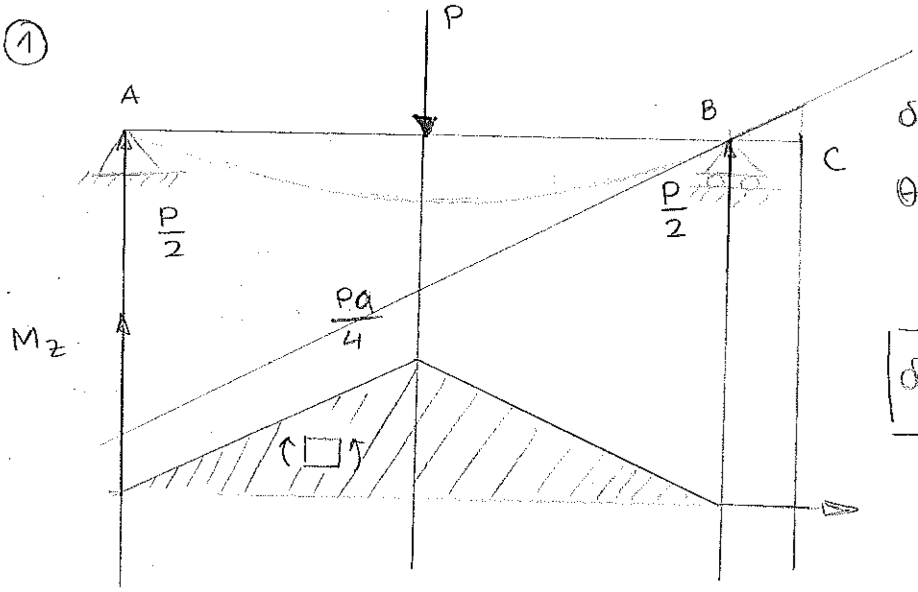
$E = 200\ \text{GPa}$

$P = 1\text{KN}.$

$I_z = 4 \cdot 10^{-8}\text{m}^4$

superposición: ① P + ② Q

①



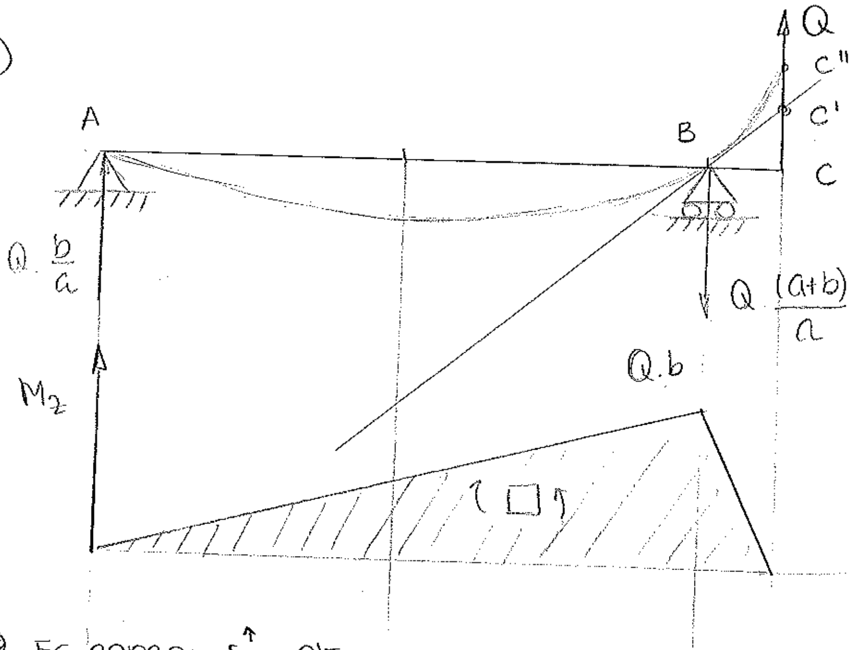
$\delta_{c_1}^{\uparrow} = \theta_{c_1} \cdot b.$

$$\theta_{c_1} = \frac{\delta_{AB}}{a} = \frac{1}{a} \cdot \frac{1}{EI} \cdot \frac{1}{2} \cdot \frac{Pa}{4} \cdot a \cdot \frac{a}{2} = $$

$$= \frac{Pa^2}{16EI}$$

$$\boxed{\delta_{c_1}^{\uparrow} = \frac{Pa^2 b}{16EI}}$$

②



$M_A = 0.\ Q(a+b) - V_B \cdot a = 0$

$$V_B = \frac{Q(a+b)}{a}$$

$M_B = 0:\ Q\,b = V_A \cdot a$

$$V_A = Q\,\frac{b}{a}$$

$\delta_{c_2} = CC' + C'C''$

$$\begin{cases} CC' = \theta_{c_2} \cdot b \\ C'C'' = \delta_{ce} \end{cases}$$

$$\theta_{c_2} = \frac{\delta_{AB}}{a} = \frac{1}{\cancel{a}} \cdot \frac{1}{EI}\left[\frac{1}{2} \cdot \cancel{a} \cdot Qb \cdot \frac{2}{3}a\right] =$$

$$= \frac{Qab}{3EI}$$

$$CC' = \frac{Qab^2}{3EI}$$

Ec. comp: $\delta_c^{\uparrow} = 0'5\text{cm}.$

$\delta_{c_1}^{\uparrow} + \delta_{c_2}^{\uparrow} = 0'5\text{cm}$

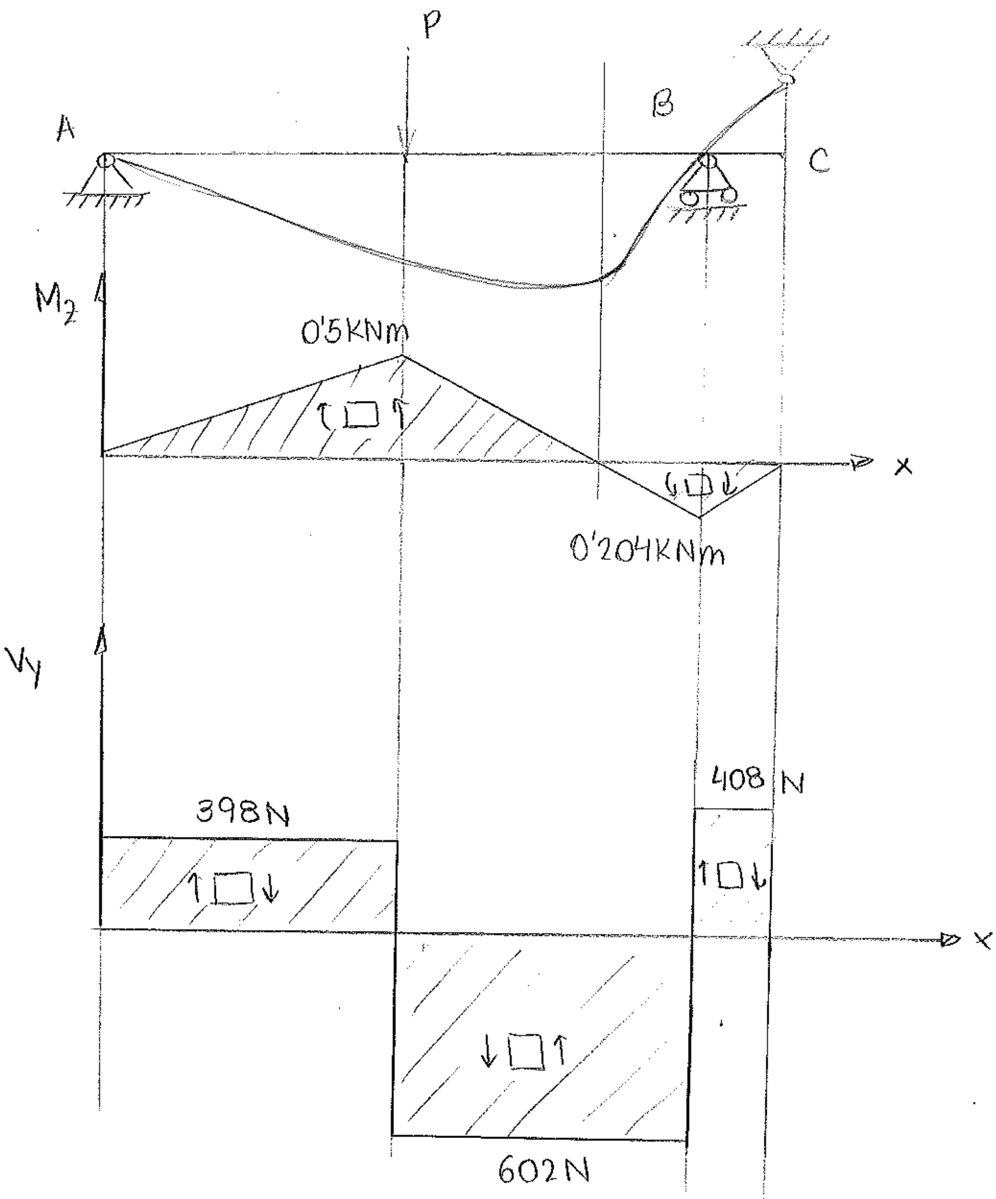
$$\delta_{BC} = \frac{1}{EI} \cdot \frac{1}{2} b \cdot Qb \cdot \frac{2}{3} b = \frac{Qb^3}{3EI}$$

$$\boxed{\delta_{C_2}^{\uparrow} = \frac{Q\,ab^2}{3EI} + \frac{Qb^3}{3EI}}$$

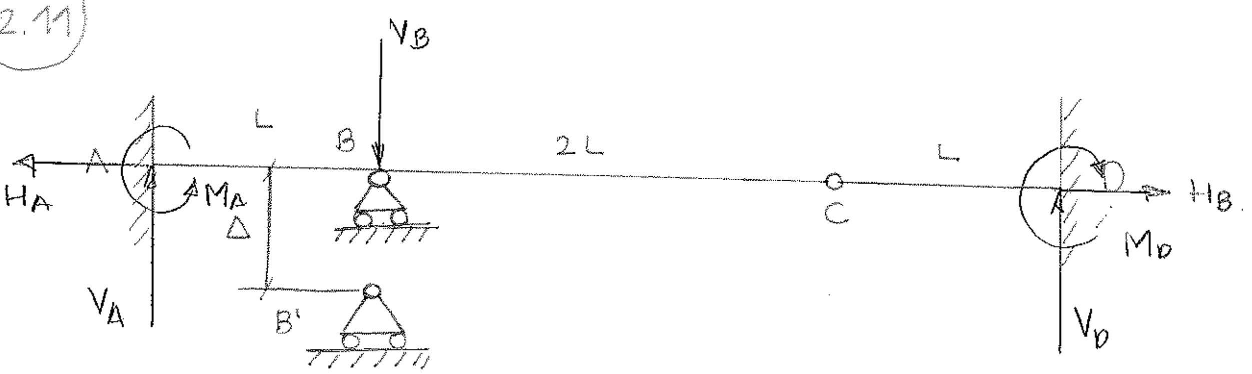
$$\frac{Pa^2b}{16EI} + \frac{Qab^2}{3EI} + \frac{Qb^3}{3EI} = 0'5\,cm$$

$$\boxed{Q = -408N}$$

En C aparecerá una reacción Q = 408N



2.11



$\Delta = 10 mm$

$E = 200 GPa$

$L = 2m$

$S = 6cm \times 10cm$

- Incógnitas: 7
- Ecuaciones: 4

$\} \; h = 3$

1) EC. EQUILIBRIO

(1) $\Sigma F_x = 0$: $H_A = H_D$

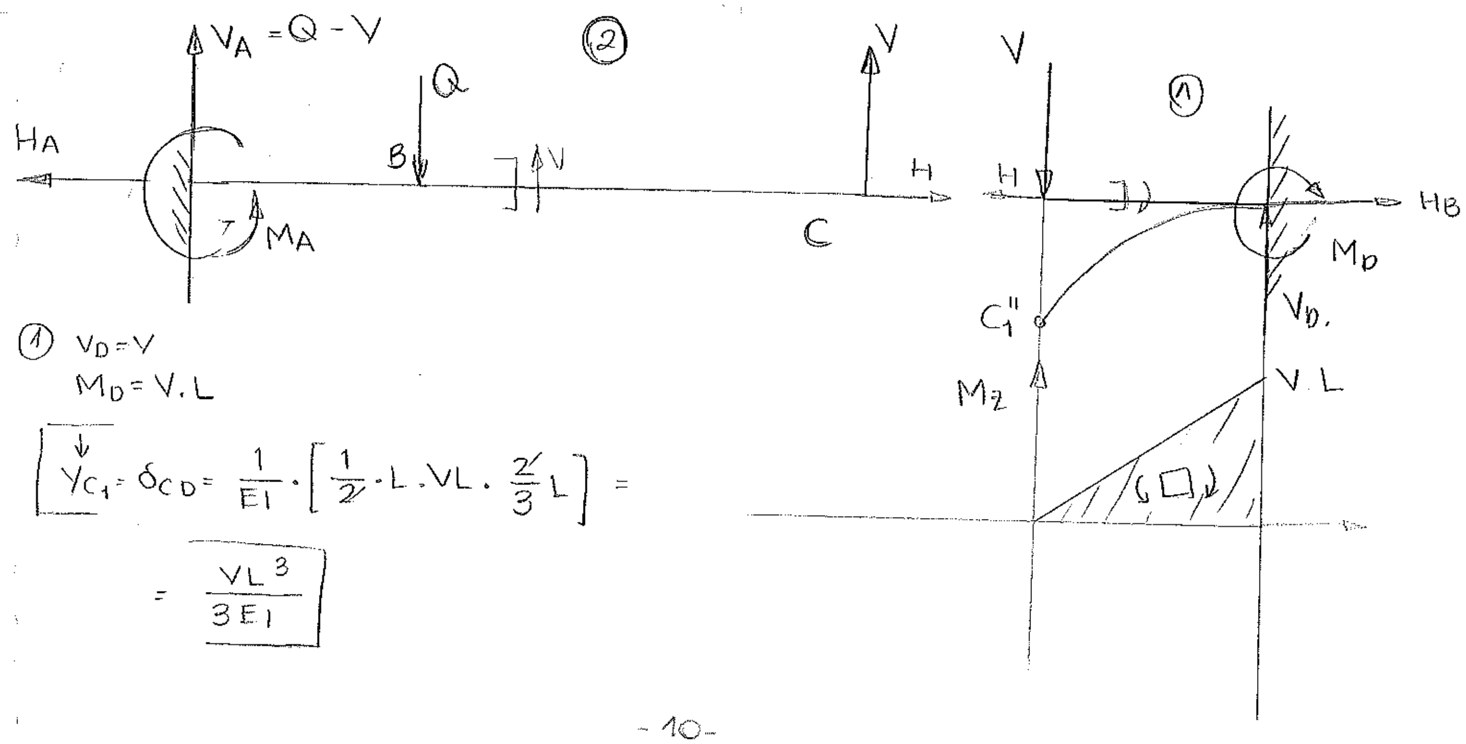
(2) $\Sigma F_y = 0$: $V_A + V_D = V_B$

(3) $M_C^{(ABC)} = 0$: $M_A + 2L V_B - 3L V_A = 0$.

(4) $M_C^{(CD)} = 0$: $M_D + V_D L = 0$

2) ELIJO TRES INCÓGNITAS Y HALLO LA ESTR BÁSICA

- Incógnitas: V, Q, H
- Ec comp: (1) $Y_B^{\downarrow} = 10 mm$ (2) $Y_{C_1}^{\downarrow} = Y_{C_2}^{\downarrow}$



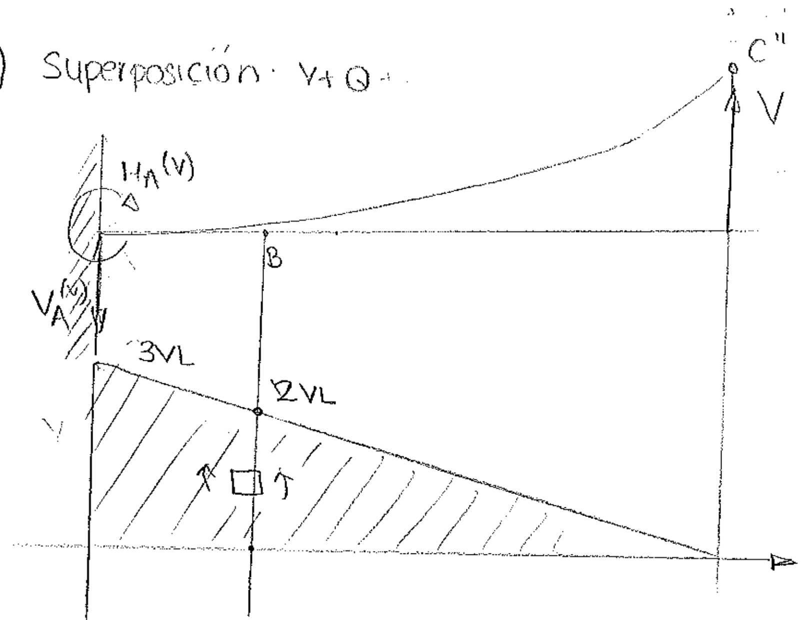
1 $V_D = V$

$M_D = V.L$

$\boxed{Y_{C_1}^{\downarrow}} = \delta_{CD} = \frac{1}{EI} \cdot \left[\frac{1}{2} \cdot L . VL . \frac{2}{3} L \right] =$

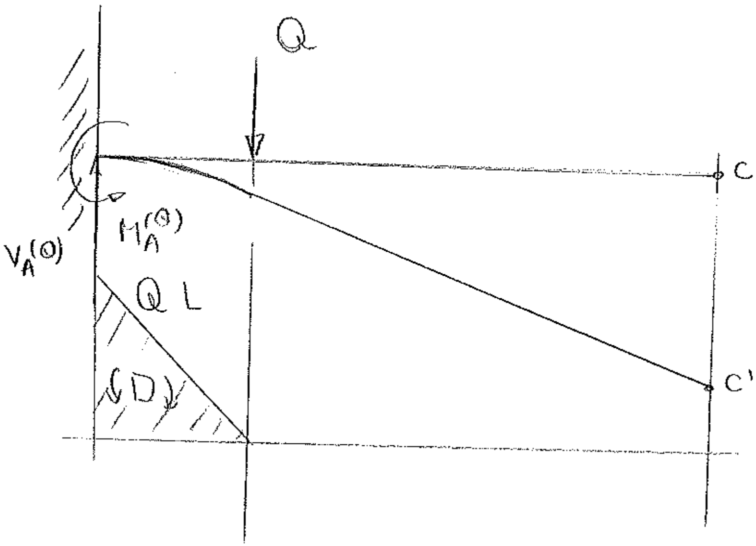
$= \boxed{\frac{VL^3}{3EI}}$

2) Superposición: V + Q



$$\boxed{y_C^{\uparrow(V)}} = \delta_{CA} = \frac{1}{EI}\left[\frac{1}{2}3L\cdot\frac{2}{3}3L\cdot 3VL\right] = \boxed{\frac{9VL^3}{EI}}$$

$$\boxed{y_B^{\uparrow(V)} = \delta_{BA} = \frac{1}{EI}\left[\frac{1}{2}L\cdot VL\cdot\frac{2}{3}L + L\cdot 2VL\cdot\frac{L}{2}\right] = \frac{4VL^3}{3EI}}$$



$$\boxed{y_C^{\downarrow(Q)} = \delta_{CA} = \frac{1}{EI}\left[\frac{1}{2}QL\cdot L\cdot\frac{8}{3}L\right] = \frac{4QL^3}{3EI}}$$

$$\boxed{y_B^{\downarrow(Q)} = \delta_{BA} = \frac{1}{EI}\left[\frac{1}{2}QL\cdot L\cdot\frac{2}{3}L\right] = \frac{QL^3}{3EI}}$$

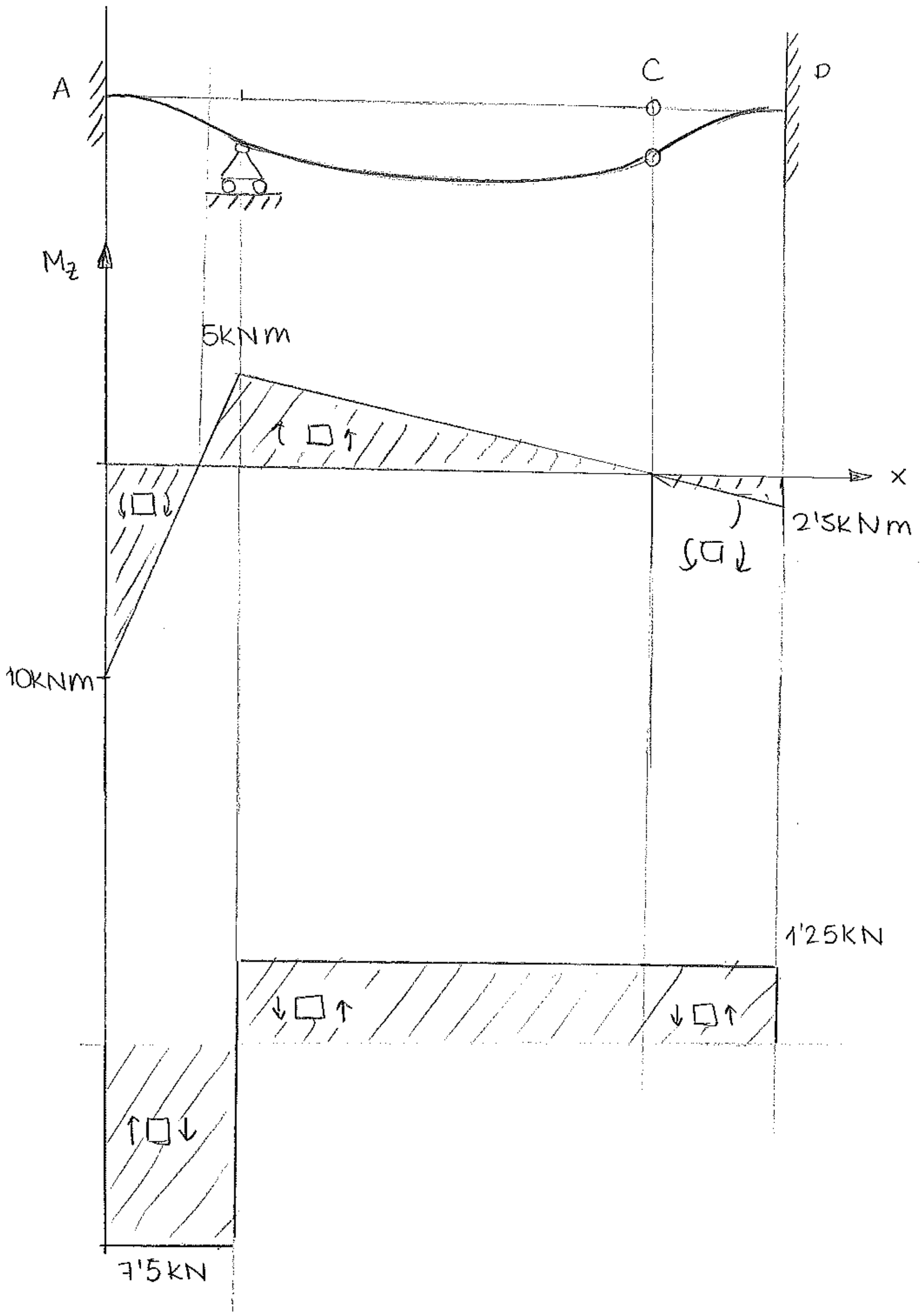
$$y_{C_1}^{\downarrow} = y_{C_2}^{\downarrow}: \quad \frac{VL^3}{3EI} = -\frac{9VL^3}{EI} + \frac{4QL^3}{3EI} \Rightarrow V = -27V + 4Q \Rightarrow 28V - 4Q = 0 \quad (1)$$

$$y_B^{\downarrow} = 10\,mm: \quad -\frac{4VL^3}{3EI} + \frac{QL^3}{3EI} = 10\cdot10^{-3} \Rightarrow -4VL^3 + QL^3 = 3EI(10\cdot10^{-3}) \quad (2)$$

$$I = \frac{1}{12}\cdot 6cm(10cm)^3 = 5\cdot10^{-6}\,m^4$$

$$\boxed{\begin{array}{l} V = 1250\,N \\ Q = 8750\,N \end{array}}$$

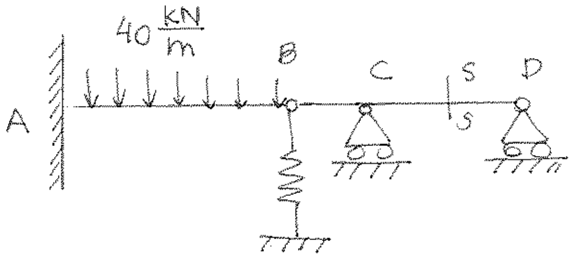
Diagramas.

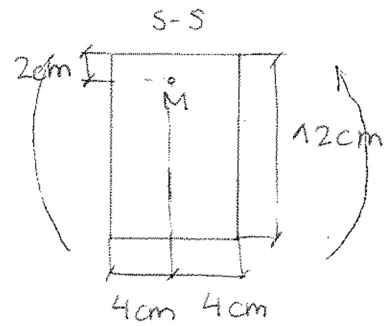


$$\sigma_{xx,máx} = \sigma_{xx,máx}(A) = \frac{10kN\,m \cdot 5cm}{5 \cdot 10^{-6} m^4} = 100MPa.$$

La tensión máxima es de 100MPa en la sección A

2.12





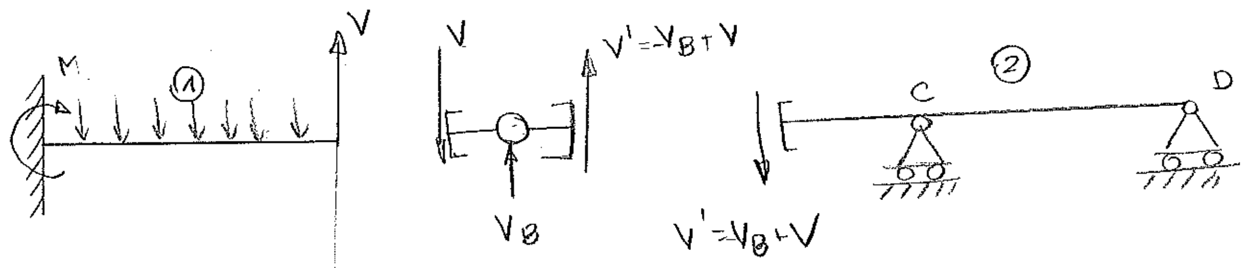
$K_{muelle} = 1 \frac{MN}{m}$

$E = 200 GPa$.

1) DESCENSO DE B.

- incógnitas: 6
- ecuaciones: 4

$\Big\}\ h = 2\ (V_B, V)$

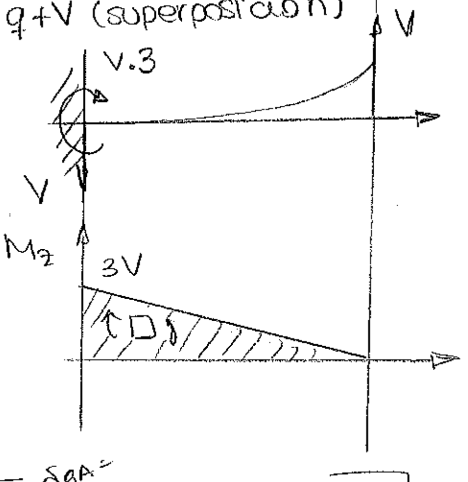


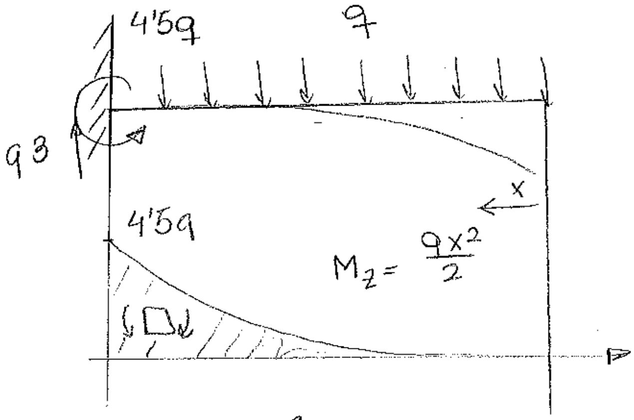
Ec. compatibilidad:

(1) $\overset{\downarrow}{\delta B_1} = \overset{\downarrow}{\delta B_2}$

(2) $V_B = K . \delta B$

(1) q+V (superposición)





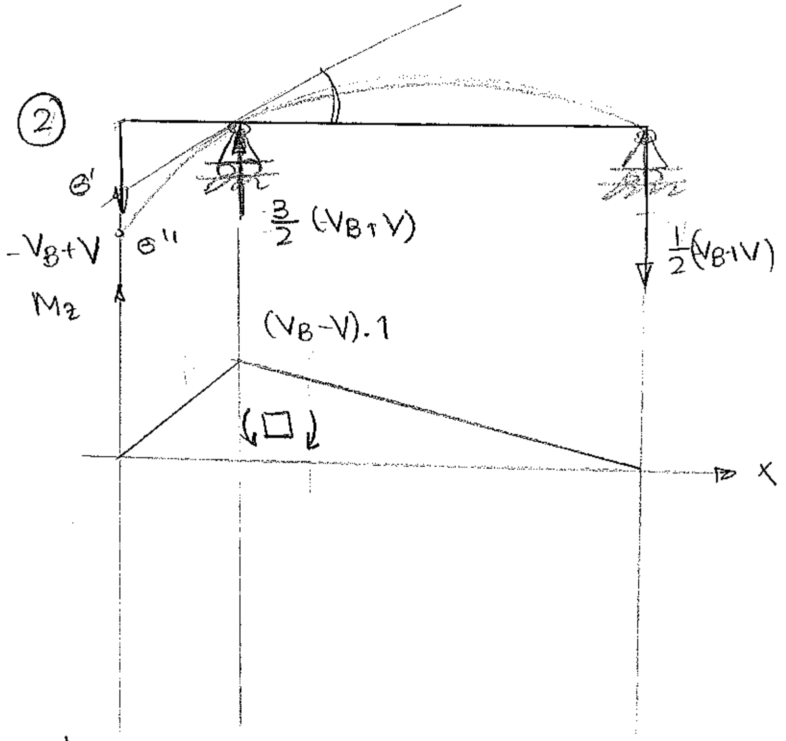
$\delta_{BA} =$

$$\boxed{y_{B_1}^{\uparrow (V)}} = \frac{1}{EI} \cdot \frac{1}{2} \cdot 3V \cdot 3 \cdot \frac{2}{3} \cdot 3 = \boxed{\frac{9V}{EI}}$$

$$\boxed{y_{B_1}^{\downarrow (q)}} = \frac{1}{EI} \int_0^3 \frac{qx^2}{2} dx . (x) = \frac{1}{EI} \left[\frac{qx^4}{8} \right]_0^3 =$$

$$= \boxed{\frac{81q}{8EI}}$$

(2)



$$\begin{cases} V_B - V = V_C + V_D = -2V_D \\ V_C \cdot 1 + V_D \cdot 3 = 0 \Rightarrow V_C = -3V_D \end{cases}$$

$V_B - V = -2V_D \Rightarrow V_D = -\frac{1}{2}(V_B - V)$

$V_C = \frac{3}{2}(V_B - V)$

$\overset{\downarrow}{y}_{B_2} = BB' + B'B''$

$B'B'' = \delta_{BC}$

$BB' = \theta_C \cdot 1m = \frac{2(-V_B+V)}{3EI}$

$\theta_C = \frac{\delta_{DC}}{2m} = \frac{1}{\cancel{2}} \cdot \frac{1}{EI} \cdot \frac{1}{\cancel{2}}(-V_B+V) \cdot 2 \cdot \frac{\cancel{2}}{3} \cdot \cancel{2} = \frac{2(-V_B+V)}{3EI}$

$\delta_{BC} = \frac{1}{EI} \cdot \frac{1}{\cancel{2}}(-V_B+V) \cdot 1 \cdot \frac{\cancel{2}}{3} \cdot 1 = \frac{-V_B+V}{3EI}$

$$\boxed{\overset{\downarrow}{y}_{B_2} = \frac{2(-V_B+V)}{3EI} + \frac{-V_B+V}{3EI} = \frac{-V_B+V}{EI}}$$

$$\boxed{\overset{\downarrow}{y}_{B_1} = \frac{81q}{8EI} - \frac{9V}{EI}}$$

- Ec compatibilidad

(1) $\frac{-V_B+V}{EI} = \frac{81q}{8EI} - \frac{9V}{EI} \Rightarrow 8(-V_B+V) = 81q - 72V \Rightarrow -8V_B + 80V = 81q$

(2) $\left(\frac{81q}{8EI} - \frac{9V}{EI}\right)K = V_B \Rightarrow \frac{8EI}{K}V_B = 81q - 72V \Rightarrow \frac{8EI}{K}V_B + 72V = 81q$

$V_B = 12640,45 N$

$V = 11764,04 N$

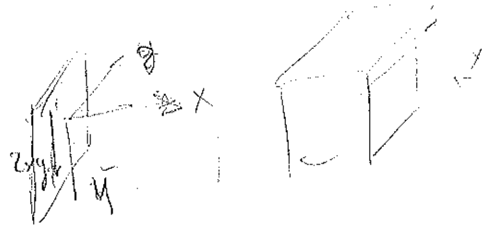
$\delta_B = 0'01264 m = 1'264 cm$

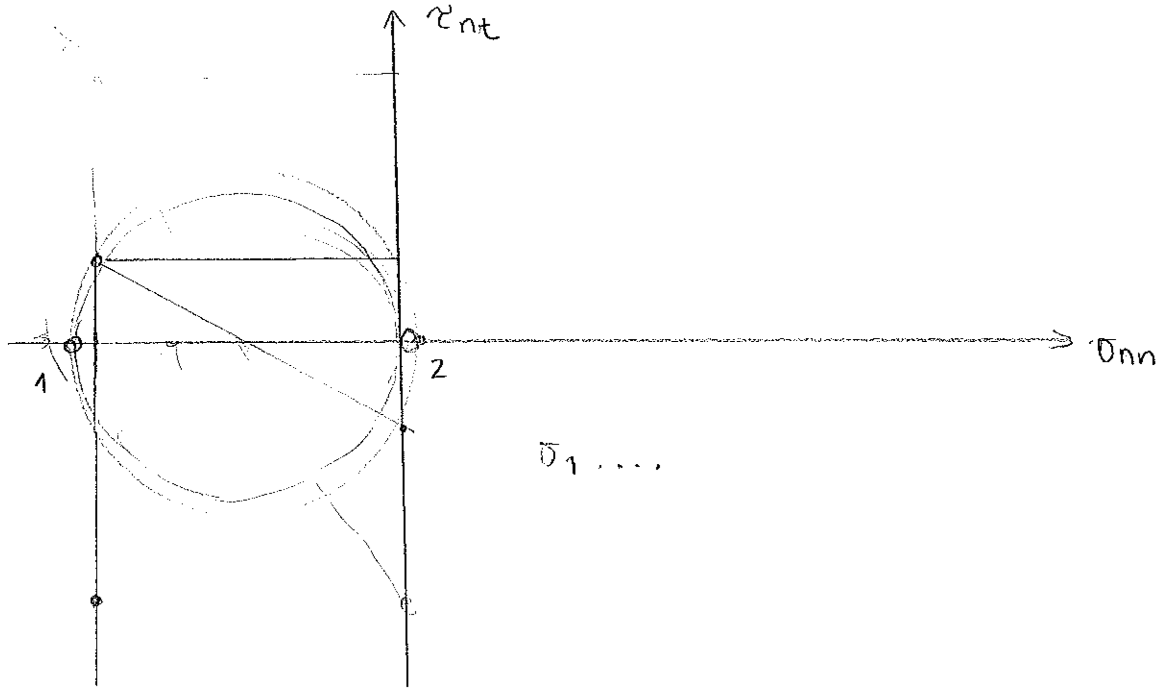
El descenso de la articulación C es de 1'264cm

2) TENSIONES PRINCIPALES.

$$\sigma_{xx,M} = \frac{M_A \cdot 4cm}{I_z} = -493,329 \text{ MPa}$$

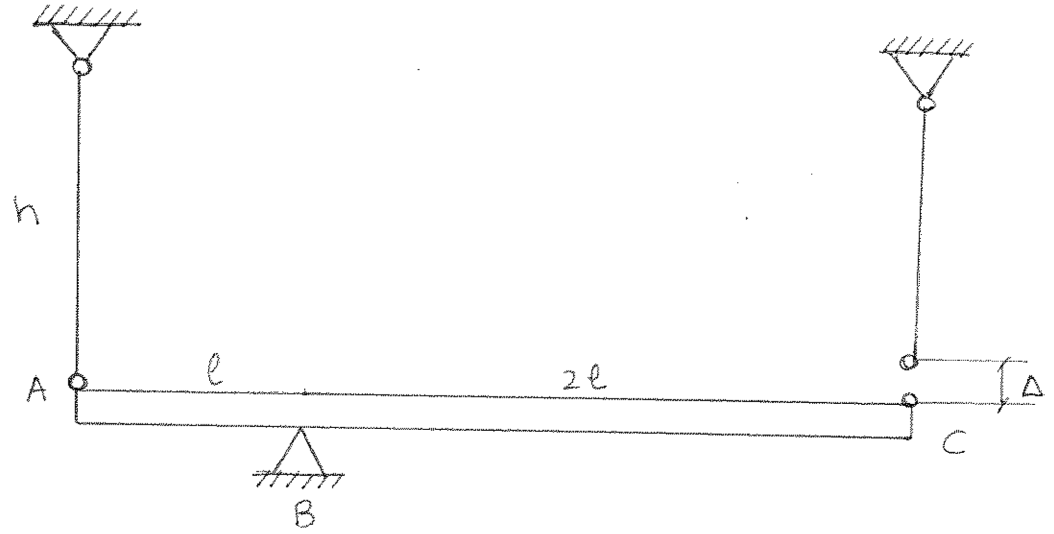
$$\tau_{xy} = \frac{V_y \cdot Q}{b \cdot I_z} = \frac{107359,55 \text{ N} \cdot 8 \cdot 10^{-5} m^3}{8 cm \quad 1'152 \cdot 10^{-5} \, m^4} = 9'54307 \text{ MPa}$$





$\sigma_1 \ldots$

C-2.1



$$8.10^{-5}m^4$$

$I = 8000 cm^4.$ $\quad E = 200\ GPa$

$A_t = 2\ cm^2\ 2.10^{-4} m^2$ $\quad l = 1m$

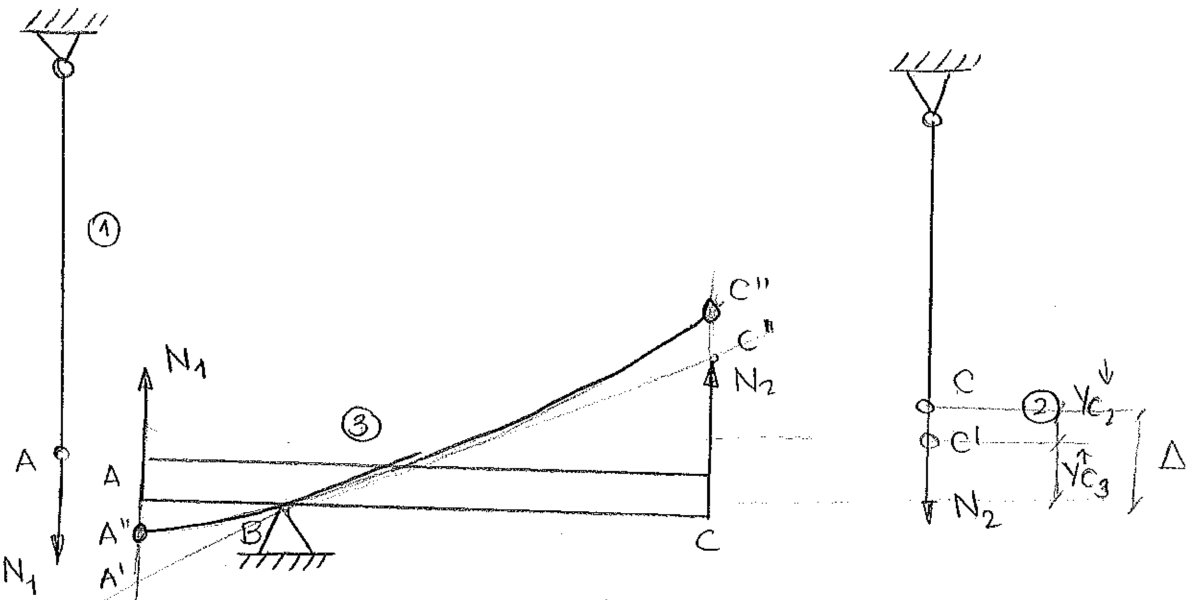
$h = 1m$

$\Delta = 5mm$

- Se desprecia el esfuerzo cortante.

- Incógnitas: 6
- Ecuaciones: 5

$\left.\begin{matrix} \\ \\ \end{matrix}\right\} \boxed{h=1}\ (N_1)$



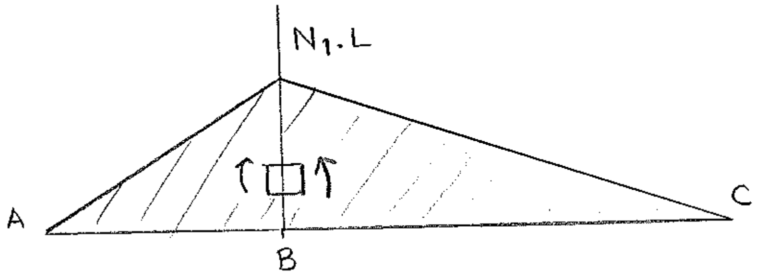
$\left\{N_1 = \frac{N_2}{2}\right\}$

- Ec. compatibilidad.

$$\delta_{A_1}^{\downarrow} = \delta_{A_3}^{\downarrow}$$

$$y_{C_2}^{\downarrow} + y_{C_3}^{\uparrow} = \Delta$$

③



$$\overset{\downarrow}{y}_{A_3} = AA' - A'A''$$

$$A'A'' = \delta_{AB} = \frac{1}{EI} \cdot \frac{1}{\cancel{2}} L \cdot N_1 \cdot L \cdot \frac{\cancel{2}}{3} L = \frac{N_1 L^3}{3EI}$$

$$\overset{\downarrow}{y}_{A_1} = \frac{N_1 \, h}{E A_t}$$

$$\overset{\downarrow}{y}_{C_2} = \frac{N_1 (h-\Delta)}{2 E A_t}$$

$$\overset{\uparrow}{y}_{C_3} = CC' + C'C'' = 2AA' + C'C''$$

$$C'C'' = \delta_{CB} = \frac{1}{EI}\left[\frac{1}{\cancel{2}} \cdot \cancel{2}L \cdot N_1 \cdot L \cdot \frac{2}{3} 2L\right] = \frac{4 N_1 L^3}{3EI}$$

Ec. compatibilidad

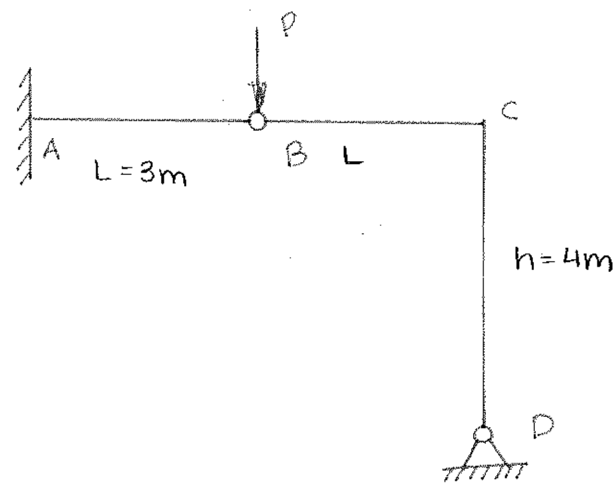
$$\left.\begin{aligned} (1) \quad & \frac{N_1 \, h}{E A_t} = AA' - \frac{N_1 L^3}{3EI} \\ (2) \quad & \frac{N_1 (h-\Delta)}{2 E A_t} + 2AA' + \frac{4 N_1 L^3}{3EI} = 5\,mm \end{aligned}\right\} \begin{aligned} N_1 &= 26675{,}559\,N \\ AA' &= 1'22 \cdot 10^{-3}\, m \end{aligned}$$

$$\sigma_1 = \frac{N_1}{A_t} = 133{,}378\,MPa \ (+) \qquad \theta_B = \frac{AA'}{L} = 1'22 \cdot 10^{-3}\, rad$$

$$\sigma_2 = \frac{N_2}{A_t} = 66{,}689\,MPa \ (+)$$

Las tensiones en los tirantes son de 133,378 MPa para el tirante ① y de 66,689 MPa para el ②. Ambas tracciones. El giro en B es de $\theta_B = 1'22 \cdot 10^{-3}$ rad

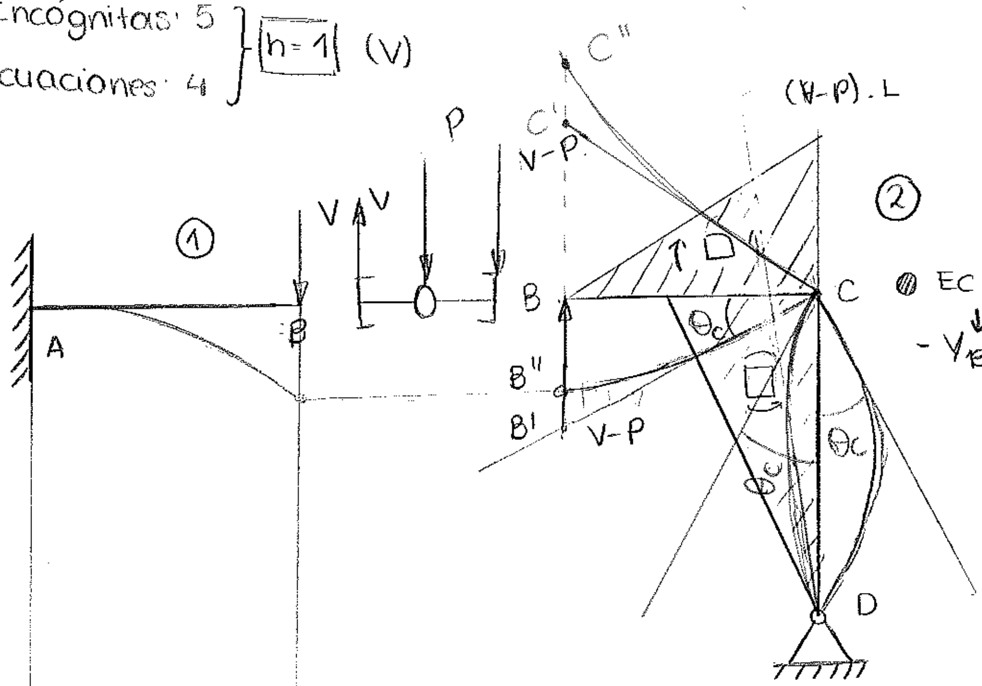
C-2.4



P = 50 kN

EI = 60 MN.m^2.

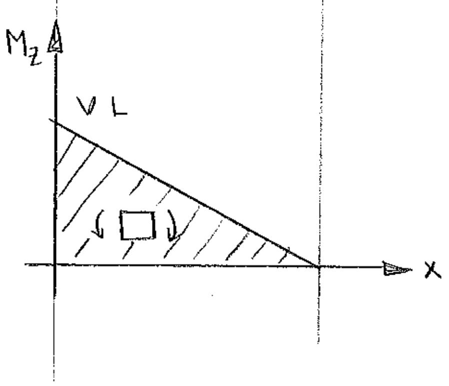
1) Incógnitas: 5 } $\boxed{h = 1}$ (V)
Ecuaciones: 4 }



Ec compatibilidad

$$-y_{B_1}^{\downarrow} = y_{B_2}^{\uparrow}$$

1



$$\boxed{y_{B_1}^{\downarrow} = \delta_{BA} = \frac{1}{EI}\cdot\frac{1}{2}VL.L.\frac{2}{3}L = \frac{VL^3}{3EI}}$$

2

$$\theta_C = \frac{\delta_{DC}}{h} = \frac{1}{\cancel{h}}\cdot\frac{1}{EI}\cdot(V-P).L.\frac{1}{\cancel{2}}\cdot\cancel{h}\cdot\frac{\cancel{2}}{3}h = \frac{(V-P)L.h}{3EI}$$

$$y_{B_2}^{\downarrow} = BB' + B''B' = \theta_C\cdot L + \delta_{BC}$$

$$\delta_{BC} = \frac{1}{EI}\cdot\frac{1}{2}(V-P)L\ L.\frac{2}{3}L = \frac{(V-P)L^3}{3EI}$$

$$\boxed{y_{B_2}^{\downarrow} = \frac{(V-P)L^2h}{3EI} + \frac{(V-P)L^3}{3EI}}$$

$$-\frac{V.L^3}{3EI} = \frac{(V-P)L^2h}{3EI} + \frac{(V-P)L^3}{3EI} \Rightarrow \boxed{V = 35000N}$$

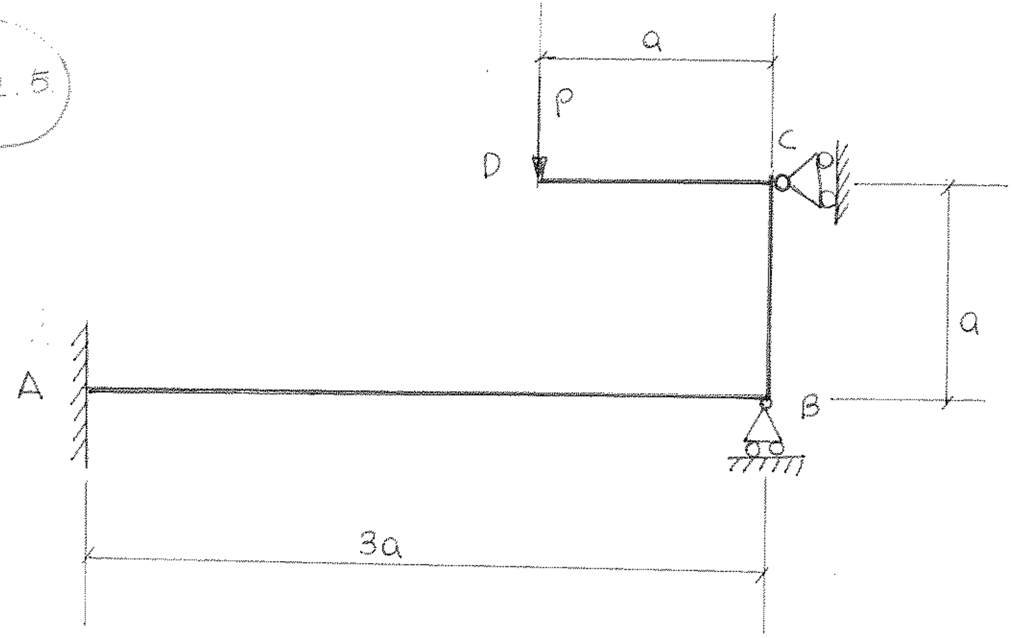
$$V_D = -(V+P) = -(35\,kN - 50\,kN) = 15\,kN$$

$$H_D = \frac{(V-P)\cdot L}{h} = 11'25\,kN$$

$$Y_B = 5,25 \cdot 10^{-3}\,m = 5,25\,mm$$

Las reacciones en B son $V_D = 15\,kN$ y $H_D = 11'25\,kN$. Y el desplazamiento vertical en B es de $\delta = 5,25\,mm$

C-2.5

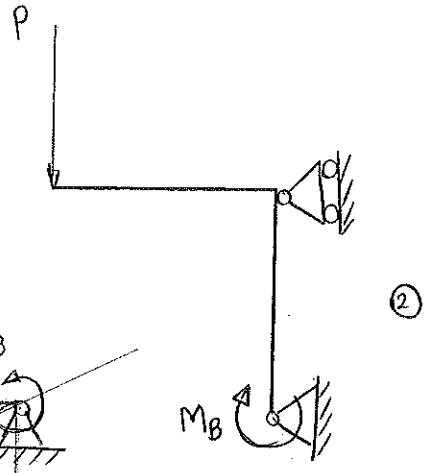


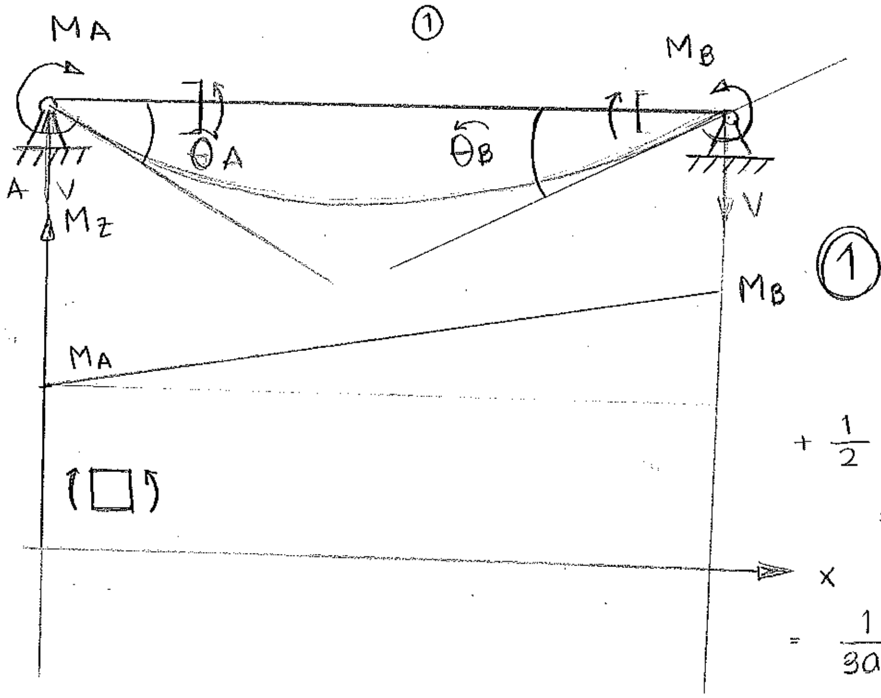
- Incógnitas: 5
- Ecuaciones: 3

$\Big\}\ \boxed{h=2}$

- Ec. compatibilidad: $\theta_A = 0$

$\theta_{B_1} = \theta_{B_2}$





1

$$\theta_A = \frac{\delta_{BA}}{3a} = \frac{1}{3a}\cdot\frac{1}{EI}\cdot\left[3a\,M_A\cdot\frac{3}{2}a + \frac{1}{2}(M_B - M_A)\cdot 3a\cdot\frac{1}{\cancel{3}}\,\cancel{3}a\right] =$$

$$= \frac{1}{3a}\cdot\frac{1}{EI}\left[\frac{9M_A}{2}a^2 + \frac{3M_B a^2}{2} - \frac{3M_A a^2}{2}\right] =$$

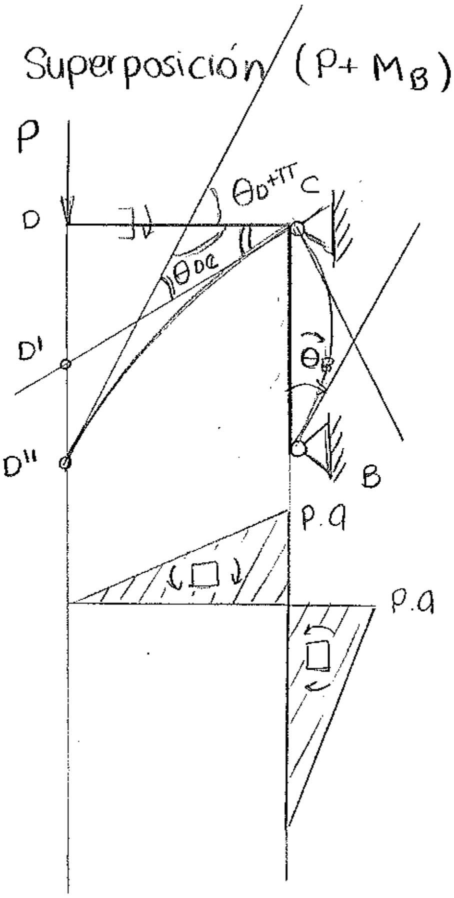
$$= \frac{1}{3a}\cdot\frac{1}{EI}\left[\frac{6M_A}{2}a^2 + \frac{3M_B a^2}{2}\right] \Rightarrow$$

$$\boxed{\overset{\frown}{\theta}_A = \frac{2M_A + M_B}{2EI}\,a^2}$$

$$\boxed{\overset{\frown}{\theta}_{B_1} =} \frac{\delta_{AB}}{3a} = \frac{1}{3a}\,\frac{1}{EI}\left[3a\,M_A\,\frac{3}{2}a + \frac{1}{\cancel{2}}(M_B - M_A)\,3a\,\frac{\cancel{2}}{\cancel{3}}\,\cancel{3}a\right] = \frac{1}{3a\,EI}\left[\frac{9M_A}{2}a^2 + 3M_B a^2 - 3M_A a^2\right] =$$

$$= \frac{1}{3a\,EI}\left[\frac{3}{2}M_A a^2 + 3M_B a^2\right] = \boxed{\frac{3M_A + 6M_B}{6EI}\,a}$$

② Superposición $(P + M_B)$



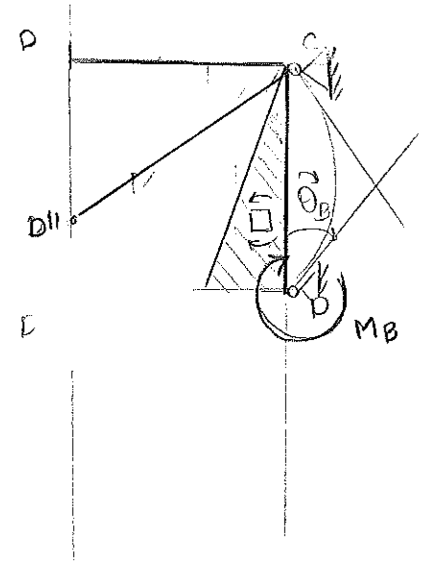
$$\boxed{\overset{\curvearrowright(P)}{\theta_{B_2}} = \frac{\delta_{CB}}{a} = \frac{1}{a}\cdot\frac{1}{EI}\left[\frac{1}{2}Pa\,a\cdot\frac{1}{3}a\right] = \frac{Pa^2}{6EI}}$$

$$\overset{\downarrow}{y}{}_D^{(P)} = DD' + D'D'' = \theta_C \cdot a + \delta_{DC}$$

$$\delta_{DC} = \frac{1}{EI}\cdot\frac{1}{\cancel{2}}\cdot Pa\,a\cdot\frac{\cancel{2}}{3}a = \frac{Pa^3}{3EI}$$

$$\theta_C = \frac{\delta_{BC}}{a} = \frac{1}{a}\left[\frac{1}{2}Pa\,a\,\frac{2}{3}a\right] = \frac{Pa^2}{3EI}$$

$$\boxed{\overset{\downarrow}{y}{}_D^{(P)} = \frac{Pa^3}{3EI} + \frac{Pa^3}{3EI} = \frac{2Pa^3}{3EI}}$$



$$\boxed{\overset{\curvearrowright(M_B)}{\theta_{B_2}} = \frac{\delta_{CB}}{a} = \frac{1}{a}\cdot\frac{1}{EI}\left[\frac{1}{\cancel{2}}M_B\,a\cdot\frac{\cancel{2}}{3}a\right] = \frac{M_B a}{3EI}}$$

$$\boxed{\overset{\downarrow}{y}{}_D^{(M_B)} = \theta_C \cdot a = \frac{M_B a^2}{6EI}}$$

$$\theta_C = \frac{\delta_{BC}}{a} = \frac{1}{a}\cdot\frac{1}{EI}\cdot\left[\frac{1}{2}M_B\,a\,\frac{1}{3}a\right] = \frac{M_B a}{6EI}$$

$\theta_A = 0$: $\frac{2M_A + M_B}{2EI}a^2 = 0 \rightarrow 2M_A + M_B = 0 \Rightarrow M_B = -2M_A$

$$\frac{3M_A + 6M_B}{6EI}a = -\frac{Pa^2}{6EI} - \frac{2M_B a}{6EI} \rightarrow 3M_A + 8M_B = -P.a$$

$$3M_A - 16M_A = -Pa$$

$$M_A = \frac{Pa}{13}$$

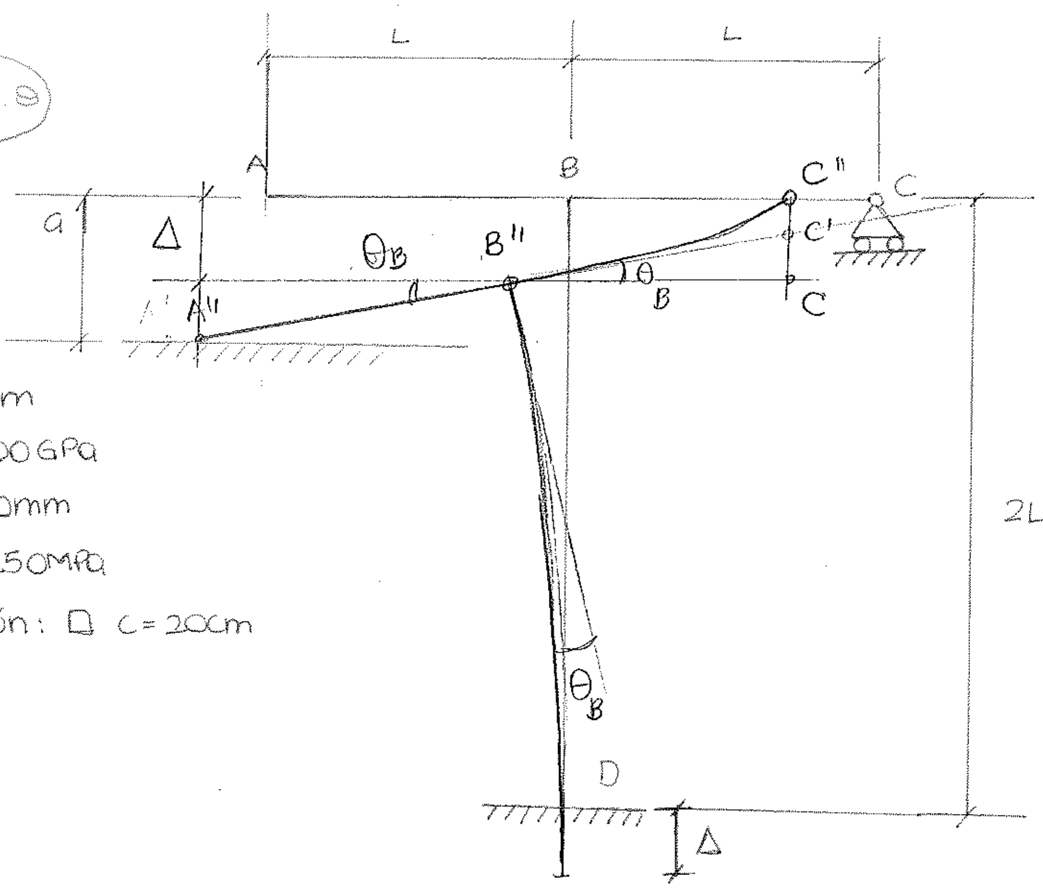
$$M_B = \frac{-2Pa}{13}$$

$$\boxed{Y_D^{\psi} = \frac{2Pa^3}{3EI} + \frac{(-2)Pa^3}{78EI} = \frac{25Pa^3}{39EI}}$$

$\boxed{\text{El despazamiento } Y_D^{\psi} = \frac{25Pa^3}{39EI}}$ (El giro más liadita pero se saca con ángulos)

(apunteetan)

C-2.9



$L = 2m$

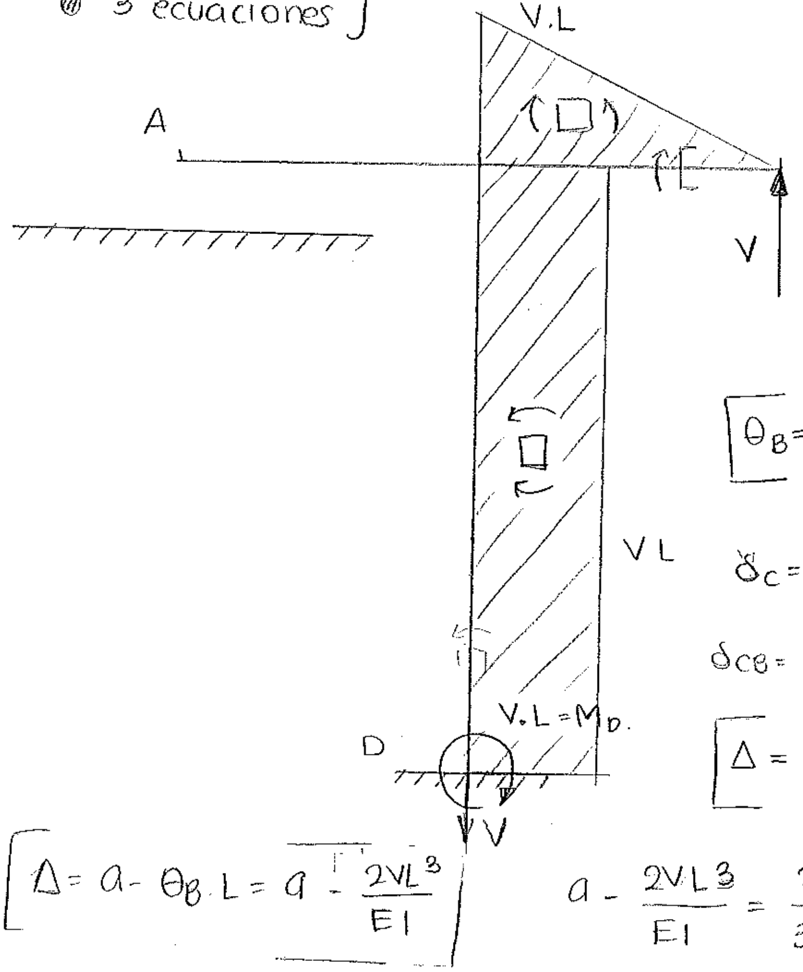
$E = 200\,GPa$

$a = 10mm$

$\sigma_f = 250MPa$

Sección: □ $c = 20cm$

1) 4 incógnitas, 3 ecuaciones $\Big\}$ $\boxed{h=1}$ (V_C)



Ec compatibilidad

(1) $y_C = 0$

(2) $a = \Delta + \theta_B . L$

$$\boxed{\theta_B = \theta_{BD} = \frac{1}{EI} \cdot VL . 2L = \frac{2VL^2}{EI}}$$

$$\delta_C = \Delta = CC' + C'C'' = \theta_B . L + \delta_{CB}$$

$$\delta_{CB} = \frac{1}{EI} \frac{1}{2} . VL\,L . \frac{2}{3} L = \frac{VL^3}{3EI}$$

$$\boxed{\Delta = \frac{2VL^3}{EI} + \frac{VL^3}{3EI} = \frac{7VL^3}{3EI}}$$

$$\boxed{\Delta = a - \theta_B . L = a - \frac{2VL^3}{EI}}$$

$$a - \frac{2VL^3}{EI} = \frac{7VL^3}{3EI} \Rightarrow V = 7692{,}3076\,N$$

$$\Delta = 5'3846 . 10^{-3} m = 5'3846mm$$

$$\boxed{\text{El asiento del tencho es de } \Delta = 5'3846\,mm}$$

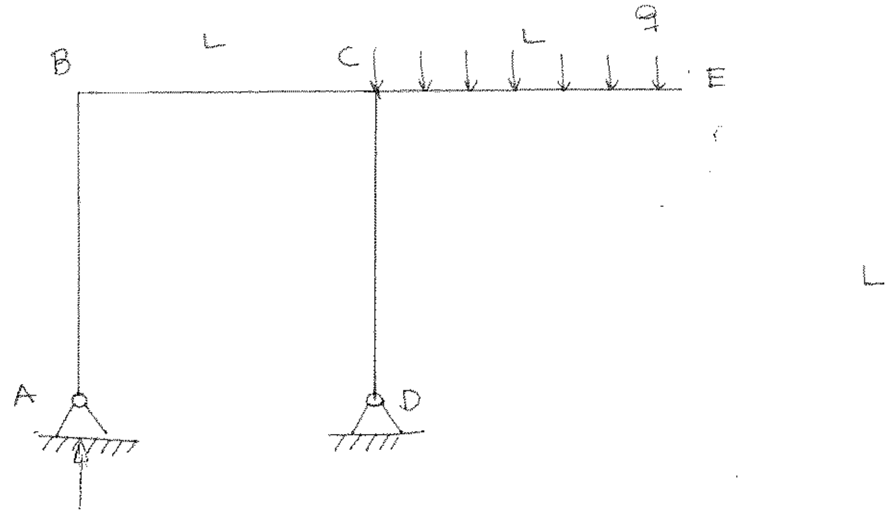
$$M_{máx} = V\,L = 15384,61538\ Nm$$

$$\sigma_{xx,máx} = \frac{M_{máx} \cdot 10cm}{I_z} = 11'538\ MPa$$

$$n = \frac{\sigma_f}{\sigma_{xx,máx}} = 21'66$$

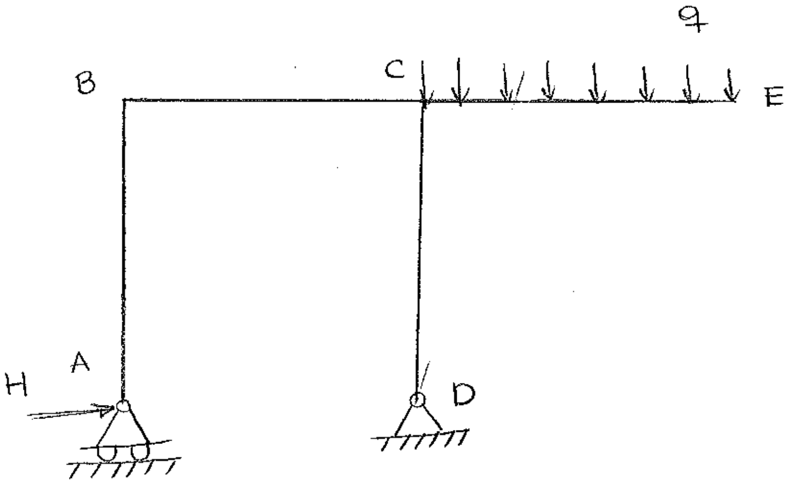
La tensión normal máxima es de 11'538MPa y la estructura trabaja con un coeficiente de seguridad de n= 21'67

C-2.11

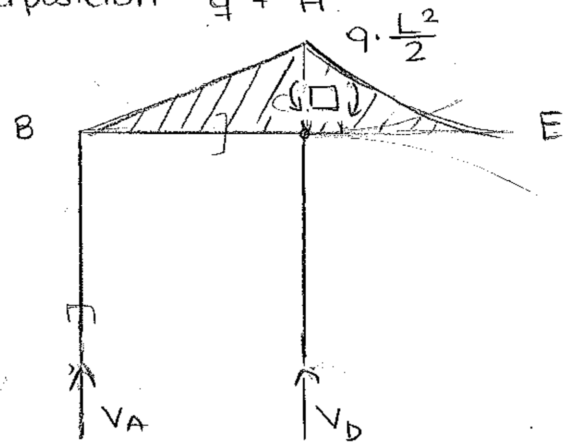


1) 4 incógnitas
3 reacciones } $\boxed{h=1}$ (H_A)

Ec. comp: $y_A = 0.$



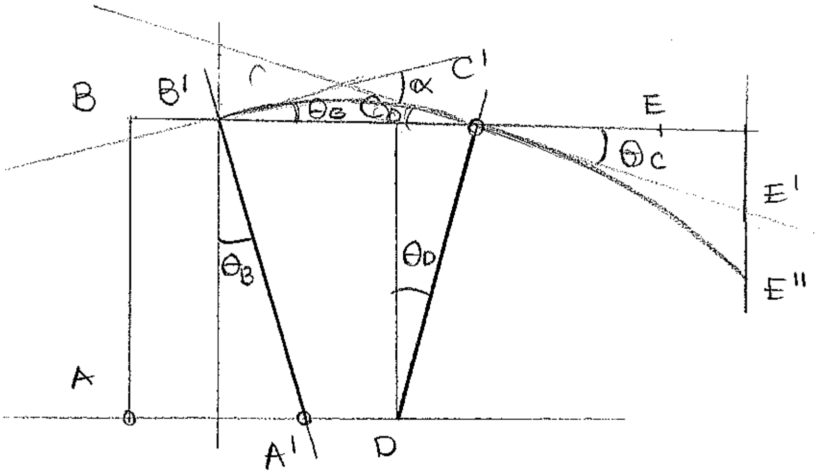
superposición $q + H$



$M_D=0:\ q\cdot L\cdot\frac{\cancel{L}}{2}+V_A\cdot\cancel{L}=0.$

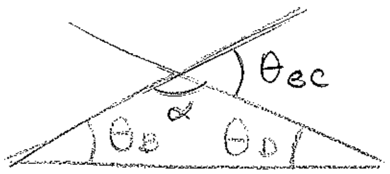
$$\boxed{V_A=-\frac{qL}{2}}$$

$$\boxed{V_D=\frac{3qL}{2}}$$



$$Y_A^{(q)} = AA' = BB' + \theta_B \cdot L = CC' + \theta_B \cdot L = \theta_C \cdot L + \theta_B \cdot L$$

$$\boxed{\theta_B = \frac{\delta_{CB}}{L} = \frac{1}{\cancel{L}} \cdot \frac{1}{EI}\left[\frac{1}{2}\,\frac{qL^2}{2}\cdot\cancel{L}\cdot\frac{1}{3}L\right] = \frac{qL^3}{12EI}}$$



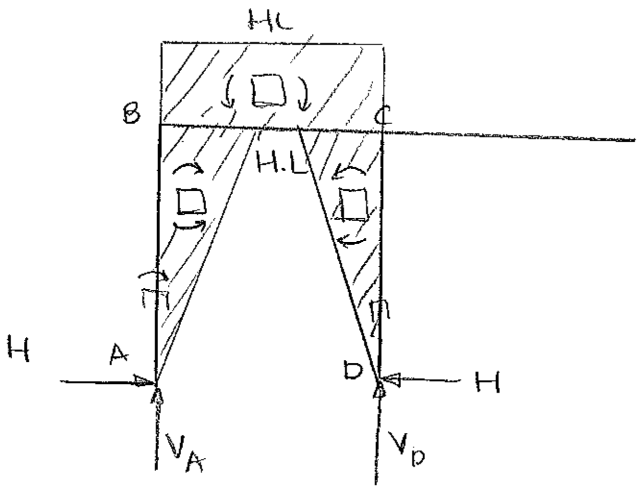
$$\theta_B + \theta_D = \theta_{BC}$$

$$\theta_D = \theta_C \Rightarrow \theta_B + \theta_C = \theta_{BC}$$

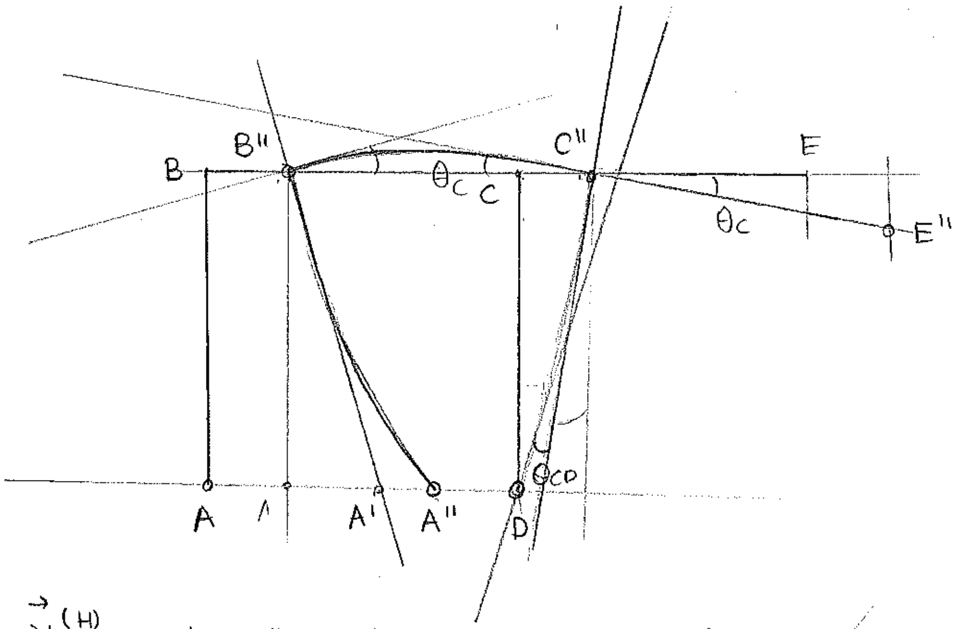
$$\boxed{\theta_{BC} = \frac{1}{2}\cdot L\cdot\frac{qL^2}{2}\cdot\frac{1}{EI} = \frac{qL^3}{4EI}}$$

$$\theta_C = \theta_{BC} - \theta_B = \frac{qL^3}{4EI} - \frac{qL^3}{12EI} = \frac{qL^3}{6EI}$$

$$\boxed{\vec{Y}_A^{(q)} = \frac{qL^4}{6EI} + \frac{qL^4}{12EI} = \frac{qL^4}{4EI}}$$



$$M_A = 0 \cdot V_D = 0 = V_A$$

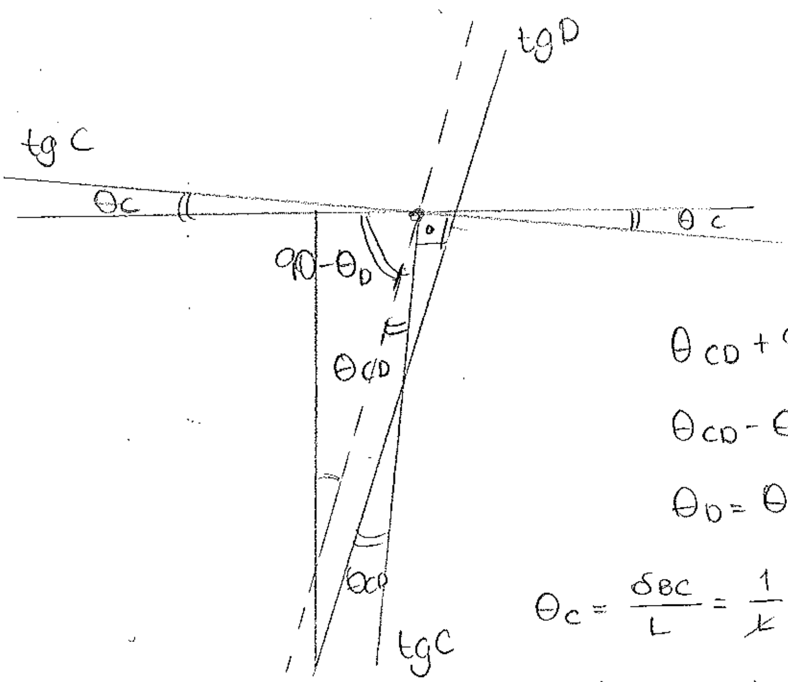


$$\vec{Y}_A^{(H)} = AA' + AA'' = BB'' + \theta_B \cdot L + \delta_{AB} = (\theta_D \cdot L - \delta_{CD}) + \theta_B \cdot L + \delta_{AB}$$

$$\boxed{\theta_B = \frac{\delta_{CB}}{L} = \frac{1}{k} \cdot \frac{1}{EI} \cdot HL \cdot k \cdot \frac{L}{2} = \frac{HL^2}{2EI}}$$

$$\boxed{\delta_{CD} = \frac{1}{EI} \cdot \frac{1}{2} HL \cdot L \cdot \frac{1}{3} L = \frac{HL^3}{6EI}}$$

$$\boxed{\delta_{AB} = \frac{1}{EI} \cdot \frac{1}{2} HL \cdot L \cdot \frac{2}{3} L = \frac{HL^3}{3EI}}$$



$$\theta_{CD} + 90 - \theta_D + \theta_C = 90$$

$$\theta_{CD} - \theta_D + \theta_C = 0$$

$$\theta_D = \theta_{CD} + \theta_C$$

$$\theta_C = \frac{\delta_{BC}}{L} = \frac{1}{k} \cdot \frac{1}{EI} \cdot HL \cdot \frac{k}{2} \cdot L = \frac{HL^2}{2EI}$$

$$\theta_{CD} = \frac{1}{2} HL \cdot L \cdot \frac{1}{EI} = \frac{HL^2}{2EI}$$

$$\boxed{\theta_D = \frac{HL^2}{EI}}$$

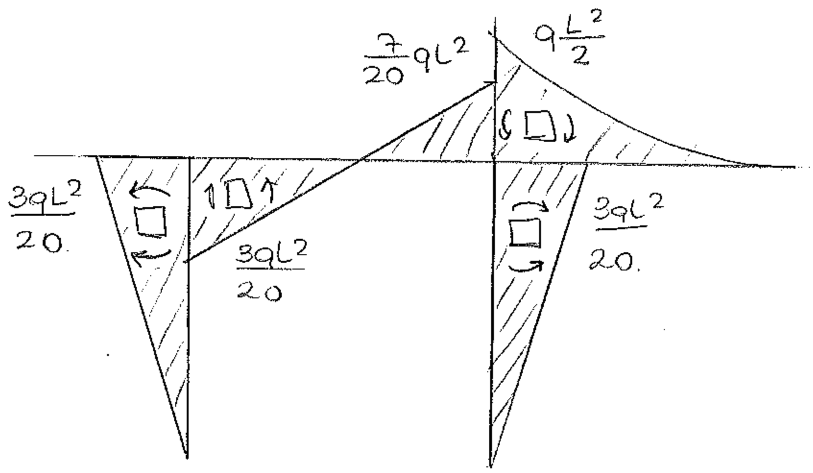
$$\boxed{\vec{Y}_A^{(H)} = \frac{HL^2}{EI}\cdot L - \frac{HL^3}{6EI} + \frac{HL^2}{2EI}L + \frac{HL^3}{3EI} = \frac{5HL^3}{3EI}}$$

$$Y_A^i = 0 \Rightarrow \frac{qL^4}{4EI} + \frac{5HL^3}{3EI} = 0$$

$$H = -\frac{q}{4}L\cdot\frac{3}{5} = -\frac{3qL}{20}$$

Las reacciones en los apoyos son $V_A = -\frac{qL}{2}$, $V_D = \frac{3qL}{2}$; $H_A = -\frac{3qL}{20}$ y $H_D = \frac{3qL}{20}$

2) Diagrama de momentos flectores.



3) Desplazamientos y giros de B.

$$BB'' = (\theta_D^{(H)}\cdot L - \delta_{CD}^{(H)}) + \theta_C^{(q)}\cdot L = \frac{HL^3}{EI} - \frac{HL^3}{6EI} + \frac{qL^4}{6EI} =$$

$$= -\frac{3qL^4}{20EI} + \frac{3qL^4}{120EI} + \frac{qL^4}{6EI} = \frac{qL^4}{24}$$

$$\theta_B = \frac{qL^3}{12EI} + \frac{HL^2}{2EI} = \frac{qL^3}{12EI} - \frac{3qL^3}{40EI} = \frac{qL^3}{120EI}$$

El desplazamiento del nudo B es $BB'' = \frac{qL^4}{24}$ y el giro $\theta_B = \frac{qL^3}{120EI}$