

$$V = V_g + V_e$$

$$\sqrt{g} = 2 + g_2 L \cos \theta + g_3 L \cos^2 \theta$$

$$V_g = 7 + g \cos \theta$$

$$U_e = \frac{1}{2} k L^2 x u^2 \theta$$

$$V = 7\pi g L \cos \theta + \frac{1}{2} k L^2 \sin^2 \theta$$

$E_H = T + V = \text{cte}$? Holónomo? \rightarrow Sí

Sn

Ecklonianos? - Si

Perkelt? → Si

Lesson Nine

Acaciaes África → Peso, elástica (vezatius?)

$$E = \frac{1}{2} k L^2 \theta^2 \left[\frac{5}{6} + 8 \zeta u^2 \theta \right] + 7 \pi g L \cos \theta + \frac{1}{2} k L^2 \dot{\theta}^2$$

$$\theta = 0 \quad \dot{\theta} = 0 \quad \rightarrow E = 7 \text{ kJ/L}$$

$$7\pi GL = \pi L^2 \left[\frac{5}{6} + \gamma \sin^2 \phi \right] + 7\pi GL \cos \phi + \frac{1}{2} kL^2 \sin^2 \phi$$

$$\dot{\theta} = 0 \quad , \quad \theta + \alpha x = 60^\circ$$

$$\pi g L = \pi g \left(\frac{1}{2} + \frac{1}{2} k L^2 \right) \Rightarrow \frac{\pi g}{k} = \frac{1}{2} k L^2 \cdot \frac{3}{4}$$

$$k = \frac{2\pi}{3} \frac{\pi g}{L}$$

$\overset{oo}{\theta}, \quad \dot{\theta} = 0, \quad \ddot{\theta} = 0$

$$7\pi g L = \pi L^2 \left[\frac{5}{6} + 8 \sin^2 \theta \right] + 7\pi g L \cos \theta + \frac{1}{2} k L^2 \sin^2 \theta$$

$$0 = \cancel{\pi L^2} \left[2\overset{oo}{\theta} \left(\frac{5}{6} + 8 \sin^2 \theta \right) + 16 \overset{oo}{\theta}^2 \sin \theta \cos \theta \right] - \cancel{7\pi g L} \overset{oo}{\theta} \sin \theta + \cancel{k L^2} \overset{oo}{\theta} \sin \theta \cos \theta$$

$$0 = \overset{oo}{\theta} \left\{ \left[2\overset{oo}{\theta} \left(\frac{5}{6} + 8 \sin^2 \theta \right) + 16 \overset{oo}{\theta}^2 \sin \theta \cos \theta \right] - \frac{7g}{L} \sin \theta + \frac{k}{\pi} \sin \theta \cos \theta \right\}$$

Ecuaçā depende de L^0 orden $\Rightarrow x$ tiene que cumplirse y como no cumple $\overset{oo}{\theta} = 0$

$$\left[2\overset{oo}{\theta} \left(\frac{5}{6} + 8 \sin^2 \theta \right) + 16 \overset{oo}{\theta}^2 \sin \theta \cos \theta \right] - \frac{7g}{L} \sin \theta + \frac{k}{\pi} \sin \theta \cos \theta = 0$$

$$\overset{oo}{\theta} = 0, \quad \overset{oo}{\theta}$$

$$2\overset{oo}{\theta} \cdot \frac{5}{6} = 0 \Rightarrow \overset{oo}{\theta} = 0$$

Ejemplo 6.2

Un semidisco de masa m y radio R rueda sobre un plano inclinado, como se muestra en la figura 6.4. Se desea determinar la aceleración angular del semidisco en la posición representada, sabiendo que en ese instante su velocidad angular es ω .

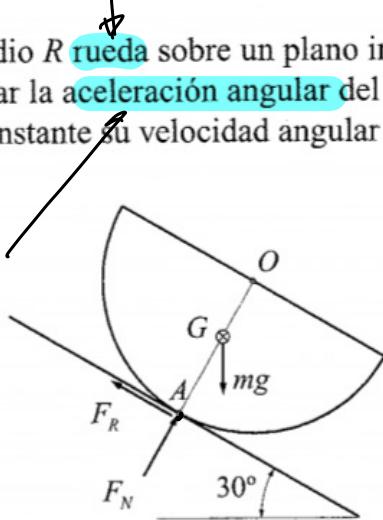


FIGURA 6.4

(Ahora quiere la α no vale con ecuaciones energéticas)

$$\delta W_r = \vec{F}_R \cdot \vec{\delta r}^0 ; \quad \vec{F}_N = 0 \Rightarrow \vec{\delta r}_I = \frac{\vec{\delta r}^0}{dt} \Rightarrow \delta W_r = 0$$

\Rightarrow fórmula de movimiento de energía $\Rightarrow E = T + U = \text{cte}$

$$\text{fijo punto } i \Rightarrow I \rightarrow A \Rightarrow \underbrace{\vec{v}_I = \vec{v}_A = 0}_{\text{y}} \quad \underbrace{\vec{a}_I \neq 0}_{\text{y}}$$

$$\vec{F}_f = I_6 \vec{\alpha}$$

ω

$$-F_r |\vec{A}_6| = I_6 \alpha$$

+ 2º Thuc.

$$-F_r \left(R - \frac{4R}{3\pi} \right) = \left[I_2 \eta \alpha^2 + \eta \left(\frac{4R}{3\pi} \right)^2 \right] \alpha$$

$$I_0 = \frac{1}{2} \pi R^2 ; \quad I_0 = I_6 + \eta \left(\frac{4R}{3\pi} \right)^2$$

$$I_6 = \underbrace{\frac{1}{2} \pi R^2}_{\text{ }} - \eta \left(\frac{4R}{3\pi} \right)^2$$

$$F_{\parallel} = \eta a_{6\parallel}$$

$$Mg \& n \theta - F_R = \eta a_{6\parallel}$$

$$A \equiv I \equiv \text{cir}$$

$$\vec{a}_f = \underbrace{\vec{a}_A}_{\perp} + \alpha \hat{k} \wedge \vec{a}_6 - \underbrace{\omega^2 \vec{a}_6}_{\parallel}$$

; $\vec{a}_A \rightarrow$ perpendicular a la tangente curva a la base y recta en el CIR.

y tendra su eje paralelo a la recta de la base.

$$a_{6\parallel} = \alpha \left(R - \frac{4R}{3\pi} \right)$$

per Thuc

$$Mg \& n \theta - F_R = \eta \alpha R \left(1 - \frac{4R}{3\pi} \right)$$



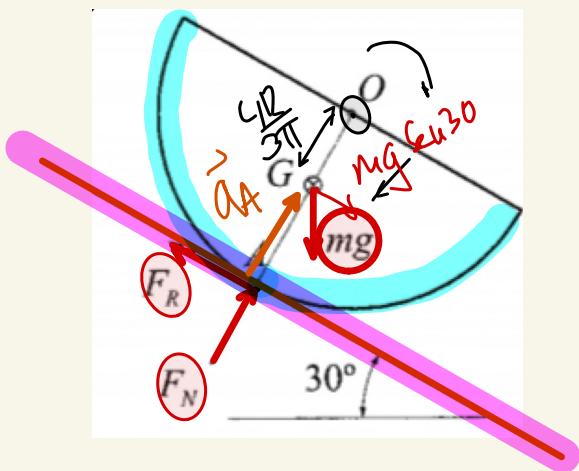
TURINIENTO

F_p PLANO

$$\vec{F}_p = I_p \vec{\alpha} + \eta \vec{P}_6 \wedge \vec{a}_p$$

$$p=6 \Rightarrow \vec{F}_f > I_6 \vec{\alpha} + \eta \vec{a}_6 \wedge \vec{a}_6$$

$$\vec{F}_f = I_6 \vec{\alpha}$$



$$0 \equiv j_0^{\wedge} \vec{v}_0 \Rightarrow \vec{v}_0 = 0 \quad \vec{a}_0 = 0 \quad ; \quad \vec{J}_0 \neq 0 \quad a_0 = 0$$

$$\vec{F}_0 = I_0 \vec{\alpha} + \tau A_0 \vec{\omega} \wedge \vec{a}_0 \quad \Rightarrow \quad \vec{F}_0 = I_0 \vec{\alpha}$$

$$\vec{F}_A = I_A \vec{\alpha} + \tau \underbrace{A_0 \vec{\omega} \wedge \vec{a}_A}_{\downarrow} \quad \vec{a}_A \parallel \vec{A}_0 \Rightarrow \vec{F}_A = I_A \vec{\alpha}$$

$$Mg \times n30 \left(R - \frac{4R}{3\pi} \right) = \left(\frac{1}{2} \pi R^2 - \tau \left(\frac{4R}{3\pi} \right)^2 + \tau \left(R - \frac{4R}{3\pi} \right)^2 \right) \vec{\alpha}$$

$$I_A = I_0 + \tau \left(R - \frac{4R}{3\pi} \right)^2 \Rightarrow I_A = \underbrace{\left(\frac{1}{2} \pi R^2 - \tau \left(\frac{4R}{3\pi} \right)^2 + \tau \left(R - \frac{4R}{3\pi} \right)^2 \right)}_{\downarrow}$$

$$\alpha = \frac{g}{R} \frac{(3\pi - 4)}{8\pi - 16}$$

α ? a) Rueda con ω en sentido antihorario

b) Se abandona a la gravedad en reposo

a) Rueda con $\omega \uparrow$

$$\vec{F}_A = I_A \vec{\alpha} + \tau \vec{A}_0 \wedge \vec{a}_A$$

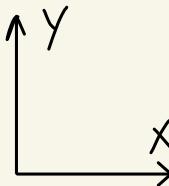
aceleración del C.R.

$$\vec{a}_A = \vec{v}_S \wedge \vec{\omega} \Rightarrow \vec{v}_S = \frac{\hat{\omega} \vec{r}_A}{\frac{1}{R_B} - \frac{1}{R_r}}$$

$\vec{a}_A \neq 0$ Descarto esta expresión $\Rightarrow b$

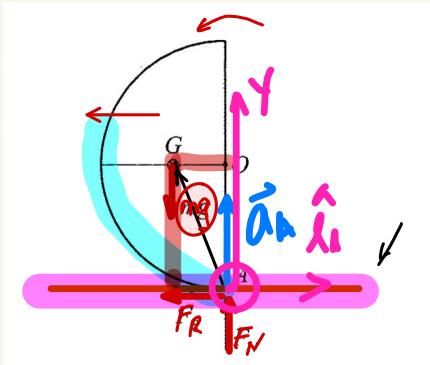
$$\vec{F}_0 = I_0 \vec{\alpha} \Rightarrow \boxed{I_0 = I_0 \alpha} \quad \downarrow$$

$$N \cdot \frac{4R}{3\pi} - F_R R = \left[\frac{1}{2} \pi R^2 - \tau \left(\frac{4R}{3\pi} \right)^2 \right] \alpha \quad (1)$$



$$-F_R = M \alpha_{bx} \quad (2)$$

$$N - Mg = M \alpha_{by} \quad (3)$$



$$\vec{a}_b = \vec{a}_0 + \vec{\alpha} \wedge \vec{v}_0 - \omega^2 \vec{v}_0$$

$$\vec{a}_b = \vec{a}_0 + \vec{\alpha} \hat{k} \wedge \left(-\frac{4R}{3\pi} \hat{i} + \omega \frac{4R}{3\pi} \hat{j} \right)$$

$$\vec{a}_0 = \vec{a}_A + \vec{\alpha} \hat{k} \wedge R \hat{j} - \omega^2 R \hat{j}$$

$$\vec{a}_A = \vec{v}_s \wedge \vec{\omega} \quad \vec{J}_S = \frac{\omega \hat{l}_1}{\frac{1}{\infty} - \frac{1}{R}} = -\omega R \hat{l}_1$$

$$\vec{a}_r = -\omega R \hat{i} \wedge \omega \hat{k} = \omega^2 R \hat{j} \quad \vec{J}_S = -\omega R \hat{i}$$

$$\vec{a}_0 = \omega^2 R \hat{j} - \vec{\alpha} R \hat{i} - \omega^2 R \hat{k}$$

$$\vec{a}_b = -\vec{\alpha} R \hat{i} - \vec{\alpha} \frac{4R}{3\pi} \hat{j} + \omega^2 \frac{4R}{3\pi} \hat{i}$$

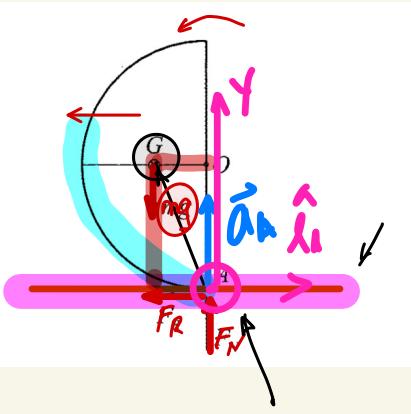
$$\alpha_{bx} = \omega^2 \frac{4R}{3\pi} - \vec{\alpha} R \quad (4)$$

$$\alpha_{by} = -\vec{\alpha} \frac{4R}{3\pi} \quad (5)$$

$$\vec{\alpha} = \frac{I}{9\pi} \left(\frac{g}{R} + \omega^2 \right)$$

b) Si medimos sin ω desde el eje central $\vec{a}_A = \omega^2 R \hat{j}$
 Si $\omega = 0 \Rightarrow \vec{a}_A = 0$

$$\vec{M}_A = I_A \vec{\alpha} + M_A \vec{a}_b \wedge \vec{a}_A$$



$$\vec{F}_A = I_A \vec{\alpha} + n \vec{I}_6 \wedge \vec{a}_A$$

$$I_A = I_A \alpha$$

$$n g \frac{4R}{3\pi} = I_A \alpha$$

$$I_6 = I_6 + n \left(\frac{4R}{3\pi} \right)^2$$

$$I_A = I_6 + n \left[\left(\frac{4R}{3\pi} \right)^2 + R^2 \right]$$

$$I_6 = I_6 - n \left(\frac{4R}{3\pi} \right)^2$$

$$I_6 = I_6 - n \left(\frac{4R}{3\pi} \right)^2 = \underbrace{\frac{1}{2} n R^2}_{\text{constant}} - n \left(\frac{4R}{3\pi} \right)^2$$

$$n g \frac{4R}{3\pi} = \left\{ \frac{1}{2} n R^2 - n \left(\frac{4R}{3\pi} \right)^2 + n \left[\left(\frac{4R}{3\pi} \right)^2 + R^2 \right] \right\} \alpha$$

$$\alpha = \frac{8}{9\pi} \frac{g}{R}$$