LAPLACE

- FUNTSEZKO FUNTZIOEN LAPLACE TRANSFORMATUA:

- $L[1] = \frac{1}{s}$
- $L[t^n] = \frac{n!}{s^{n+1}}$
- $L[e^{at}] = \frac{1}{s-a}$
- $L[\sin(at)] = \frac{a}{s^2+a^2}$
- $L[\cos(at)] = \frac{s}{s^2+a^2}$
- $L[\sinh(at)] = \frac{a}{s^2-a^2}$
- $L[\cosh(at)] = \frac{s}{s^2-a^2}$

- ESKALOIA:
- $L[H(t)] = \frac{1}{s}$ · $L[H(t-a)] = \frac{e^{-as}}{s}$
- $L[H(t-a) \cdot f(t-a)] = e^{-as} \cdot F(s)$

- INPULTSOA:
- $L[\delta(t)] = 1$ · $L[\delta(t-a)] = e^{-as}$

- PROPIETATEAK

1.- LINEALTASUNA:

$$T[c_1 f_1(t) + c_2 f_2(t)] = c_1 T[f_1(t)] + c_2 T[f_2(t)]$$

2.- TRANSLAZIOA S DOMEINUAN

$$L[e^{ct} f(t)] = F(s-c)$$

3.- TRANSFORMATUAREN DERIBATUA

$$\frac{d^n F(s)}{ds^n} = (-1)^n \cdot L[t^n f(t)]$$

4.- DERIBATUAREN TRANSFORMATUA

$$L[f'(t)] = sF(s) - f(0)$$

$$L[f''(t)] = s L[f'(t)] - f'(0)$$

5.- INTEGRALAREN TRANSFORMATUA: $L\left[\int_0^t f(x)\,dt\right] = \frac{F(s)}{s}$

6.- TRANSLAZIOA t DOMEINUAN:

$$L[H(t-a) \cdot f(t-a)] = e^{-as} \cdot F(s)$$

- ALDERANTZIZKO TRANSFORMATUA FAKTORIZATZEA

A KASUA: FAKTORE LINEAL BAKUNAK

$$\boxed{F(s) = \frac{1}{(s+5)(s-2)}} = \frac{A_1}{(s+5)} + \frac{A_2}{(s+2)} \Rightarrow$$

$$1 = A_1 \cdot (s-2) + A_2 (s+5) = (A_1 + A_2)s + (5A_2 - 2A_1)$$

$$\begin{cases} A_1 + A_2 = 0 \\ 5A_2 - 2A_1 = 1 \end{cases} \rightarrow A_1 = -1/7 \quad A_2 = 1/7$$

B KASUA: FAKTORE LINEAL ANIZKOITZAK

$$\boxed{F(s) = \frac{2s+1}{(s+1)^3 (s-1)(s-2)}} = \frac{A_{11}}{(s+1)} + \frac{A_{12}}{(s+1)^2} + \frac{A_{13}}{(s+1)^3} + \frac{A_2}{(s-1)} + \frac{A_3}{(s-2)}$$

$$2s+1 = A_{11}(s+1)^2(s-1)(s-2) + A_{12}(s+1)\cdot(s-1)(s-2) +$$

$$+ A_{13}(s-1)(s-2) + A_2(s+1)^3(s-2) + A_3(s+1)^3(s-1)$$

$$2s+1 = (A_{11} + A_2 + A_3)s^4 + (-A_{11} + A_{12} + A_2 + 2A_3)s^3 + \ldots$$

$$\begin{cases} A_{11} + A_2 + A_3 = 0 \\ -A_{11} + A_{12} + A_2 + 2A_3 = 0 \\ \ldots \end{cases} \quad A_{11} = \ldots, \ \ldots$$

C KASUA: FAKTORE KUADRATIKO BAKUNAK

$$\boxed{F(s) = \frac{1}{(s^2+4)(s^2+1)}} = \frac{A_1 s + B_1}{(s^2+4)} + \frac{A_2 s + B_2}{(s^2+1)}$$

$$1 = (A_1 s + B_1)\cdot(s^2+1) + (A_2 s + B_2)\cdot(s^2+4) = \ldots \Rightarrow \begin{cases} A_1 + A_2 = 0 \\ B_1 + B_2 = 0 \\ \ldots \\ B_1 + 4B_2 = 1 \end{cases}$$

D KASUA: FAKTORE KUADRATIKO ANIZKOITZAK

$$\boxed{F(s) = \frac{1}{(s^2+1)^2 \cdot (s-1)}} = \frac{A_{11} s + B_{11}}{(s^2+1)} + \frac{A_{12} s + B_{12}}{(s^2+1)^2} + \frac{A_2 s + B_2}{(s-1)}$$

$$1 = (A_{11}s + B_{11})(s^2+1)(s-1) + \ldots$$

- KONBOLUZIOA

$$\left[f * g = \int_0^t f(u) \cdot g(t-u)\, du \right] \quad \left[L[f*g] = F(s) \cdot G(s) \neq L[f(t)] \cdot L[g(t)] \right]$$

ADB

$$\sin(at) * \sin(at) = \int_0^t \sin(au) \cdot \sin(a \cdot (t-u)) \cdot du$$

- EKUAZIO DIFERENTZIALAK

•

T EREMUA		S EREMUA
$y''(t) - y(t) = 1$ $y(0) = y'(0) = 1$ EKUAZIO DIFERENTZIALA	$\xrightarrow{L}$	$(s^2-1)\,Y(s) = s + 1 + 1/s^2$ EKUAZIO LAGUNTZAILEA
$y(t) = \dots$ PROBLEMAREN EMAITZA	$\xleftarrow{L^{-1}}$	$Y(s) = \dots$ EKUAZIO ALJEBRAIKOA

- TRANSFERENTZIA FUNTZIOA

- $r(t)$: SARRERA
- $y(t)$: IRTEERA
- $Q(s)$: TRANSFERENTZIA
- $y(0), y''(0)$: HASTAPEN BALDINTZA NULUAK

ADB $\quad y''(t) + a y'(t) + b y = r(t)$

$$Y(s) = \frac{R(s)}{s^2+as+b} \qquad Q(s) = \frac{1}{s^2+as+b}$$

- $Y(s) = Q(s) \cdot R(s) \qquad Q(s) = \dfrac{L[y(t)]}{L[r(t)]}$,

1) $r(t) = \delta(t)$ "DIRAC"

$\boxed{R(s) = 1} \longrightarrow Y(s) = Q(s) \longrightarrow \boxed{y(t) = g(t)}$

UNITATE BULKADAREKIKO ERANTZUNA

2) $r(t) = H(t)$ "ESKALOIA"

$\boxed{R(s) = \frac{1}{s}} \longrightarrow Y(s) = \dfrac{Q(s)}{s} \longrightarrow \boxed{y(t) = a(t)}$

SISTEMAREN ADMITANTZIA

ALDAGAI KONPLEXUKO FUNTZIOAK

$$w = f(z) = u(x,y) + i\,v(x,y)$$

- CAUCHY - RIEMANN

• $$\frac{\partial f(z)}{\partial x} = \frac{\partial f(z)}{\partial y} \qquad \boxed{\begin{array}{l} u_x = v_y \\ v_x = -u_y \end{array}}$$

BALDINTZA BEHARREZKO

$f(z)$ DERIBAGARRIA

• $\exists$: $u(x,y)$; $v(x,y)$; u_x, u_y, v_x, v_y } JARRAITUAK z_0

BALDINTZA NAHIKOA

$[\exists\, f'(z_0)]$

- FUNTSEZKO FUNTZIOAK

• FUNTZIO POLINOMIKOAK: $p(z) = a_0 + a_1 z + a_2 z^2 + \ldots + a_n z^n$

$a_0, a_1, a_2, \ldots$ KONPLEXUAK

• FUNTZIO ARRAZIONALAK: $\frac{p(z)}{q(z)}$

• FUNTZIO ESPONENTZIALA: $f(z) = e^z$

$$e^z = e^{x+iy} = e^x \cdot e^{iy} = e^x \cdot (\cos(y) + i\sin(y))$$

$$Re\{e^z\} = e^x \cdot \cos y \qquad |e^z| = e^x$$

$$Im\{e^z\} = e^x \cdot \sin y \qquad \arg(e^z) = y + 2\pi k \quad \forall k \in \mathbb{Z}$$

• FUNTZIO TRIGONOMETRIKOAK:

$$\cos(z) = \frac{e^{iz} + e^{-iz}}{2} \qquad ch(x) = \frac{e^x + e^{-x}}{2} \qquad ch(y) = \frac{e^{-y} + e^{y}}{2}$$

$$\sin(z) = \frac{e^{iz} - e^{-iz}}{2i} \qquad sh(x) = \frac{e^x - e^{-x}}{2} \qquad i\,sh(y) = \frac{e^{-y} - e^{y}}{2i}$$

$$\cos(z) = \cos(x) \cdot ch(y) - i\sin(x)\, sh(y)$$

$$\sin(z) = \sin(x)\, ch(y) + i\cos(x) \cdot sh(y)$$

ZEROAK:

$$\sin(z) \to \begin{cases} \sin(x)\cdot Ch(y) = 0 & // \; Ch \neq 0 \;;\; \sin(x) = 0 \quad x = k\cdot\pi \; // \\ \cos(x)\cdot Sh(y) = 0 & // \; \cos(k\pi)\, Sh(y) = 0 \quad y = 0 \; // \end{cases}$$

$$\boxed{z = k\cdot\pi \;,\; k \in \mathbb{Z}}$$

$$\cos(z) \to \begin{cases} \cos(x)\cdot ch(y) = 0 & // \; Ch \neq 0 \;;\; \cos(x) = 0 \quad x = \frac{\pi}{2} + k\pi \; // \\ \sin(x)\cdot sh(y) = 0 & // \; \sin\left(\frac{\pi}{2} + k\pi\right)\cdot sh(y) = 0 \quad y = 0 \; // \end{cases}$$

$$\boxed{z = \frac{\pi}{2} + k\pi \;;\; k \in \mathbb{Z}}$$

PROPIETATEAK: $\cos^2(x) + \sin^2(x) = 1$

$$Ch^2(y) - sh^2(y) = 1$$

• FUNTZIO HIPERBOLIKOAK:

$Ch(y) = \cos(yi)$ | $i\,Sh(y) = \sin(iy)$

$Ch(iy) = \cos(y)$ | $Sh(iy) = i\sin(y)$

$[y \cup x \cup z]$

$$Sh(z) = Sh(x)\cdot\cos(y) + i\, Ch(x)\cdot\sin(y)$$

$$Ch(z) = Ch(x)\cdot\cos(y) + i\, Sh(x)\sin(y)$$

ZEROAK:

$Sh(z) \to \boxed{z = k\pi i \;;\; k \in \mathbb{Z}}$ $\boxed{ch(z) \to z = \left(\frac{\pi}{2} + k\pi\right) i \;;\; k \in \mathbb{Z}}$

• LOGARITMO FUNTZIOA:

$$w = \log_e(z) \iff z = e^{w}$$

$$z = |z| \cdot e^{i\arg(z)} \longrightarrow \begin{cases} |z| = e^{u(x,y)} \longrightarrow L|z| = u(x,y) \\ \arg(z) + 2k\cdot\pi = v(x,y) \end{cases}$$

$$\boxed{\log z = L(|z|) + i\,(\arg(z) + 2k\pi)}$$

NAGUSIA: $\arg(z) \in (-\pi, \pi]$

• BERRETURA KONPLEXUAK:

$$z^{w} = e^{w\cdot\log(z)} \longrightarrow \text{BALIO NAGUSIA}: \; e^{w(L|z| + i\,\mathrm{Arg}(z))}$$

• FUNTZIO TRIGONOMETRIKO ETA HIPERBOLIKOEN ALDERANTZIZKOA

$$\arcsin(z) = -i\log\left(iz \pm \sqrt{1-z^2}\right)$$

$$\arccos(z) = -i\log\left(z \pm i\sqrt{1-z^2}\right)$$

$$\arctan(z) = i\,\frac{1}{2}\log\left(\frac{i+z}{i-z}\right)$$

$$\mathrm{argSh}(z) = \log\left(z \pm \sqrt{1+z^2}\right)$$

$$\mathrm{argCh}(z) = \log\left(z \pm \sqrt{z^2-1}\right)$$

$$\mathrm{argth}(z) = \frac{1}{2}\log\left(\frac{1+z}{1-z}\right)$$

FOURIERREN SERIEAK

- SARRERA: • PERIODOA $T = T + \tau$

MAIZTASUNA $\omega = \dfrac{2\pi}{T}$

• POLINOMIO / SERIE TRIGONOMETRIKO: $a_0 + \sum_{n=1}^{k/\infty} (a_n \cos(n\omega\tau) + b_n \sin(n\omega\tau))$

• FUNTZIO BIKOITIA: $f(-\tau) = f(\tau)$

$\left[\int_{-a}^{a} f(\tau)\,d\tau = 2\int_{0}^{a} f(\tau)\,d\tau\right]$

FUNTZIO BAKOITIA: $f(-\tau) = -f(\tau)$

$\left[\int_{-a}^{a} f(\tau)\,d\tau = 0\right]$

• bik · bik = bik

bak · bak = bik

bik · bak = bak

• cos → bik.

sin → bak.

• Bik ± bik = bik

bak ± bak = bak

bik ± bak = PARITATEA GALDU

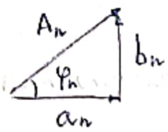
- ERA TRIGONOMETRIKOA:

1) $$\boxed{S_f(\tau) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(\omega_n \cdot \tau) + b_n \sin(\omega_n \cdot \tau)}$$

$$\left[a_0 = \frac{2}{T}\int_{-T/2}^{T/2} f(\tau)\,d\tau \qquad b_n = \frac{2}{T}\int_{-T/2}^{T/2} f(\tau)\sin(\omega_n\tau)\,d\tau \qquad a_n = \frac{2}{T}\int_{-T/2}^{T/2} f(\tau)\cos(\omega_n\tau)\,d\tau \right]$$

2) $$\boxed{S_f(\tau) = \frac{a_0}{2} + \sum_{n=1}^{\infty} A_n \cos(n\omega\tau + \varphi_n)}$$

$$\left[A_n = \sqrt{a_n^2 + b_n^2} \qquad \varphi_n = \arctan\left(-\frac{b_n}{a_n}\right) \right]$$



- ERA KONPLEXUA

$$\boxed{S_f(\tau) = \sum_{n=-\infty}^{\infty} \frac{a_n - ib_n}{2} e^{in\omega\tau}}$$

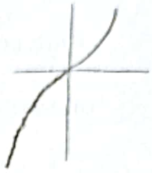
$$\left[c_n = \frac{1}{T}\int_{-T/2}^{T/2} f(\tau)\, e^{-in\omega\tau}\, d\tau \quad \text{(EULER)} \right]$$

$|C_n|$ = amplitudeen espektro konplexua.

$$\left[C_n = \frac{a_n - ib_n}{2} \qquad C_0 = \frac{a_0}{2} \right]$$

- LUZAPENAK

BIKOITIA: 

BAKOITIA: 

PERIODIKOA: 

- KONBERGENTZIA

DIRICHLETEN BALDINTZAK

JARRAITUA: $S_f(t_0) = f(t_0)$

JAUZI FINITUA: $S_f(t_0) = \dfrac{f(t_1^-) + f(t_1^+)}{2}$

- KASU PARTIKULARRAK

• $f(t)$ BIKOITIA: $S_f(t) = \dfrac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(n w t)$

$C_n = \dfrac{a_n}{2} \longrightarrow$ ERREALAK

[KOSINUEN SERIEA]

• $f(t)$ BAKOITIA: $S_f(t) = \sum_{n=1}^{\infty} b_n \sin(n w t)$

$C_n = -\dfrac{b_n i}{2} \longrightarrow$ IRUDIKARI PURUAK

[SINUEN SERIEA]

- ARRAZOI TRIGONOMETRIKOAK

• $\sin(\alpha \pm \beta) = \sin\alpha \cos\beta \pm \cos\alpha \sin\beta$

• $\cos(\alpha \pm \beta) = \cos\alpha \cos\beta \mp \sin\alpha \sin\beta$

• $\sin\alpha \sin\beta = \frac{1}{2}[\cos(\alpha-\beta) - \cos(\alpha+\beta)]$

• $\sin\alpha \cos\beta = \frac{1}{2}[\sin(\alpha+\beta) + \sin(\alpha-\beta)]$

• $\cos\alpha \cos\beta = \frac{1}{2}[\cos(\alpha+\beta) + \cos(\alpha-\beta)]$

• $\sin 2\alpha = 2\sin\alpha \cos\alpha$

• $\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$

FOURIER TRANSFORMATUA

- DEF:

$$F[f(t)] = F(\omega) = \int_{-\infty}^{\infty} f(t)\, e^{-i\omega t}\, dt$$

$$F^{-1}[F(\omega)] = f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)\, e^{i\omega t}\, d\omega$$

- KASU PARTIKULARRAK:

· ERREALA ETA BIKOITIA:

$$F(\omega) = 2\int_0^{\infty} f(t)\cos(\omega t)\, dt$$

[KOSINU TRANSFORMATUA]

· ERREALA ETA BAKOITIA

$$F(\omega) = -2i\int_0^{\infty} f(t)\sin(\omega t)\, dt$$

[SINU TRANSFORMATUA]

- PROPIETATEAK:

1) LINEALTASUNA: $F[\alpha f(t) + \beta g(t)] = \alpha F[f(t)] + \beta F[g(t)]$

2) DUALTASUNA: $F[F(t)] = 2\pi f(-\omega)$

3) ESKALA ALDAKETA: $F[f(at)] = \frac{1}{|a|} F\left(\frac{\omega}{a}\right)$

4) t EREMUKO TRANSLAZIOA: $F[f(t-t_0)] = e^{-i\omega t_0} F(\omega)$

5) MAIZTASUN EREMUKO TRANSLAZIOA: $F(\omega-\omega_0) = F\left[e^{i\omega_0 t} f(t)\right]$

6) DERIBATUAREN TRANSFORMATUA: $F[f'(t)] = i\omega F(\omega)$

7) TRANSFORMATUAREN DERIBATUA: $F'(\omega) = F[(-it) f(t)]$

- SARRITAN ERABILITAKO FUNTZIOAK:

1] $F[A] = 2\pi A\,\delta(\omega)$

2] $F[\delta(t)] = 1$

3] $F[\delta(t-t_0)] = e^{-i\omega t_0}$

4] $F[e^{i\omega_0 t}] = 2\pi\,\delta(\omega-\omega_0)$

5] $F[\Pi_T(t)] = 2\,\frac{\sin(\omega T/2)}{\omega}$

6] $F[\mathrm{sinc}(at)]\ \frac{\pi}{a}\Pi_{2a}(\omega)$

7] $F[\sin(\omega_0 t)] = \pi i[\delta(\omega+\omega_0) - \delta(\omega-\omega_0)]$ $\left\{ \mathrm{sinc} = \frac{\sin(at)}{at} \right\}$

8] $F[\cos(\omega_0 t)] = \pi[\delta(\omega-\omega_0) + \delta(\omega+\omega_0)]$ $\left\{ S_i(t) = \int_0^t \frac{\sin(x)}{x}\, dx \right\}$

9] $F[e^{-at}H(t)] = \frac{1}{a+i\omega}$ $a>0$

10] $F[t\, e^{-at}H(t)] = \frac{1}{(a+i\omega)^2}$ $a>0$

11] $F[e^{-a|t|}] = \frac{2a}{a^2+\omega^2}$ $a>0$

12] $F[S_i(t)] = \frac{-i\pi}{\omega}\Pi_2(\omega)$

- KONBOLUZIOA: $(f * g)(t) = \int_{-\infty}^{\infty} f(u)\, g(t-u)\, du$

- [DLF] $F[f(t) * g(t)] = F(w) \cdot G(w)$

$$F[f(t) \cdot g(t)] = \frac{1}{2\pi} F(w) * G(w)$$

- PARSEVALEN TEOREMA:

- $\int_{-\infty}^{\infty} |f(t)|^2 dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} |F(w)|^2 dw$

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} A^2(w)\, dw \quad \{\text{A ENERGIA ESPEKTROA}\}$$

- $\int_{-\infty}^{\infty} f(t) \cdot \overline{g(t)}\, dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(w) \cdot \overline{G(w)}\, dw$

INTEGRALES INMEDIATAS

1	$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C \quad , n \neq -1$	$\int u^n \cdot u' \cdot dx = \frac{u^{n+1}}{n+1} + C \quad , n \neq -1$
2	$\int \frac{dx}{x} = Ln\,\lvert x \rvert + C$	$\int \frac{u'}{u} \cdot dx = Ln\,\lvert u \rvert + C$
3	$\int e^x\,dx = e^x + C$	$\int e^u \cdot u' \cdot dx = e^u + C$
4	$\int a^x\,dx = \frac{a^x}{Ln\,a} + C$	$\int a^u \cdot u' \cdot dx = \frac{a^u}{Ln\,a} + C$
5	$\int sen\,x \cdot dx = -\cos x + C$	$\int sen\,u \cdot u' \cdot dx = -\cos u + C$
6	$\int \cos x \cdot dx = sen\,x + C$	$\int \cos u \cdot u' \cdot du = sen\,u + C$
7	$\int \frac{1}{\cos^2 x} = tg\,x + C$	$\int \frac{u'}{\cos^2 u} \cdot dx = tg\,u + C$
8	$\int \frac{1}{sen^2 x} = -cotg\,x + C$	$\int \frac{u'}{sen^2 u} \cdot dx = -cotg\,u + C$
9	$\int Sh\,x \cdot dx = Ch\,x + C$	$\int Sh\,u \cdot u' \cdot dx = Ch\,u + C$
10	$\int Ch\,x \cdot dx = Sh\,x + C$	$\int Ch\,u \cdot u' \cdot dx = Sh\,u + C$
11	$\int \frac{1}{Ch^2 x} \cdot dx = Th\,x + C$	$\int \frac{u'}{Ch^2 u} \cdot dx = Th\,u + C$
12	$\int \frac{1}{Sh^2 x} \cdot dx = -Cth\,x + C$	$\int \frac{u'}{Sh^2 u} \cdot dx = -CTh\,x + C$
13	$\int \frac{dx}{a^2+x^2} = \frac{1}{a} \cdot arctg\,\frac{x}{a} + C$	$\int \frac{u'}{a^2+u^2} \cdot dx = \frac{1}{a} \cdot arctg\,\frac{u}{a} + C$
14	$\int \frac{dx}{\sqrt{a^2-x^2}} = arcsen\,\frac{x}{a} + C$	$\int \frac{u'}{\sqrt{a^2-u^2}} \cdot dx = arcsen\,\frac{u}{a} + C$
15	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a} \cdot Ln\,\left\lvert \frac{a+x}{a-x} \right\rvert + C = \frac{1}{a} \cdot Argth\,\frac{x}{a} + C$	$\int \frac{u'}{a^2-u^2}\,dx = \frac{1}{2a} \cdot Ln\,\left\lvert \frac{a+u}{a-u} \right\rvert + C = \frac{1}{a} \cdot Argth\,\frac{u}{a} + C$
16	$\int \frac{dx}{\sqrt{x^2+a^2}} = Ln\,\left\lvert x + \sqrt{x^2+a^2} \right\rvert + C = Argsh\,\frac{x}{a} + C$	$\int \frac{u'}{\sqrt{u^2+a^2}}\,dx = Ln\,\left\lvert u + \sqrt{u^2+a^2} \right\rvert + C = Argsh\,\frac{u}{a} + C$
17	$\int \frac{dx}{\sqrt{x^2-a^2}} = Ln\,\left\lvert x + \sqrt{x^2-a^2} \right\rvert + C = Argch\,\frac{x}{a} + C$	$\int \frac{u'}{\sqrt{u^2-a^2}}\,dx = Ln\,\left\lvert u + \sqrt{u^2-a^2} \right\rvert + C = Argch\,\frac{u}{a} + C$

Escaneado con CamScanner

INTEGRALES POR PARTES

$$\int u\,dv = u.v - \int v\,du$$

u ↑ log , arcos
polinomios
exp / sen, cos (ciclicas)

INTEGRALES RACIONALES

$\boxed{\int \frac{P(x)}{Q(x)}\,dx}$; $P(x), Q(x)$: polinomios

Si GR. $P(x) \geq$ GR $Q(x)$: $\begin{array}{l|l} P(x) & Q(x) \\ \hline R(x) & C(x) \end{array}$ $\boxed{\frac{P(x)}{Q(x)} = C(x) + \frac{R(x)}{Q(x)}}$

RAICES REALES SIMPLES : $\frac{R(x)}{(x+2).(x+4).(x+5)} = \frac{A}{x+2} + \frac{B}{x+4} + \frac{C}{x+5}$

RAICES REALES MULTIPLES : $\frac{R(x)}{(x+2)^3} = \frac{A}{(x+2)^3} + \frac{B}{(x+2)^2} + \frac{C}{x+2}$

RAICES COMPLEJAS SIMPLES : $\frac{R(x)}{(x^2+4).(x^4+1)} = \frac{Ax+B}{x^2+4} + \frac{Cx^3+Dx^2+Ex+F}{x^4+1}$

INTEGRALES IRRACIONALES

$\int R\left(\sqrt{a^2-b^2x^2}\right).dx$ $\boxed{x = \frac{a}{b}.\operatorname{sen}t}$

$\int R\left(x, x^{m/n}, x^{p/q}, x^{r/s}, ---\right).dx$ $\boxed{x = t^M}$ $M = m.c.m\,(n, q, s, ---)$

INTEGRALES TRIGONOMETRICAS

IMPAR EN senx $\boxed{t = \cos x}$

IMPAR EN cosx $\boxed{t = \operatorname{sen} x}$

PAR $\boxed{t = \operatorname{tg} x}$ $\operatorname{sen}x = \frac{t}{\sqrt{1+t^2}}$ $\cos x = \frac{1}{\sqrt{1+t^2}}$ $dx = \frac{dt}{1+t^2}$

NINGUNO $\boxed{t = \operatorname{tg}\frac{x}{2}}$ $\operatorname{sen}x = \frac{2t}{1+t^2}$ $\cos x = \frac{1-t^2}{1+t^2}$ $dx = \frac{2dt}{1+t^2}$

$\operatorname{sen}^2 x = \frac{1-\cos 2x}{2}$ $\cos^2 x = \frac{1+\cos 2x}{2}$

Escaneado con CamScanner