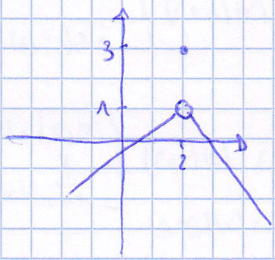
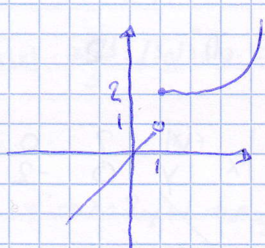
Limiteak:

→ $\lim_{x\to a} f(x)$ $\begin{cases} x<a: & \lim_{x\to a^-} f(x) \\ x>a: & \lim_{x\to a^+} f(x) \end{cases}$ } Alboko limiteak



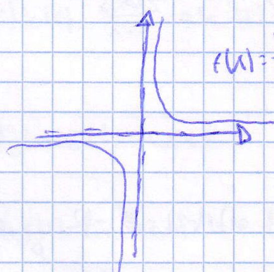
$f(2)=3$

$\lim_{x\to 2} f(x) = \begin{cases} \lim_{x\to 2^-} f(x)=1 \\ \lim_{x\to 2^+} f(x)=1 \end{cases}$ → $\lim_{x\to 2} f(x)=1$

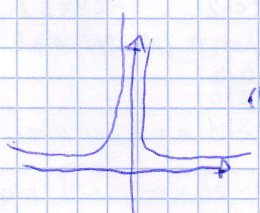


$f(1)=2$

$\lim_{x\to 1} f(x) = \begin{cases} \lim_{x\to 1^-} f(x)=1 \\ \lim_{x\to 1^+} f(x)=3 \end{cases}$ $\neq$ → $\nexists \lim_{x\to 1} f(x)$



$f(x)=\frac{1}{x}$ $\quad \lim_{x\to 0} f(x) \begin{cases} \lim_{x\to 0^+} f(x)=\infty \\ \lim_{x\to 0^-} f(x)=-\infty \end{cases}$ $\neq$ $\quad \nexists \lim_{x\to 0} f(x)$



$f(x)=\frac{1}{x^2}$ $\quad \lim_{x\to 0} f(x) \begin{cases} \lim_{x\to 0^+} f(x)=\infty \\ \lim_{x\to 0^-} f(x)=\infty \end{cases}$ } $\lim_{x\to 0} f(x)=\infty$

→ $\lim_{x\to \infty} f(x)$

→ $\lim_{x\to -\infty} f(x)$

Limiteen kalkulua:

$\lim_{x \to a} f(x) \rightarrow \lim_{x \to 1} (x^2 - 3x - 1) = 1^2 - 3 - 1 = -3$

$\lim_{x \to \infty} f(x) \rightarrow \lim_{x \to \infty} (x-1) = \infty$

$\lim_{x \to -\infty} f(x) \rightarrow \lim_{x \to -\infty} (x-1) = -\infty$

$\lim_{x \to \infty} (x^2 - 3x - 1) \rightarrow \infty^2 - \infty - 1 = \boxed{\infty - \infty}$

Indeterminazioak:

$\infty - \infty$:
- $\infty - K = \infty$
- $K - \infty = -\infty$
- $K - K = 0$

$\frac{\infty}{\infty}$:
- $\frac{\infty}{K} = \infty$
- $\frac{K}{\infty} = 0$
- $\frac{K}{K} = 1$

$\frac{0}{0}$:
- $\frac{0}{K} = 0$
- $\frac{K}{0} = ?$
- $\frac{K}{K}$

$0 \cdot \infty$

1^{∞}

0^0

∞^0

$\lim_{x \to \infty} (x^2 - 3x - 1) = \lim_{x \to \infty} x^3 = \infty$

$\boxed{\frac{K}{0}}$ $\lim_{x \to 0} \frac{1}{x} = \frac{1}{0}$:
- $x<0$: $\lim_{x \to 0^-} \frac{1}{x} = -\infty$ ($x = -0'0000001$)
- $x>0$: $\lim_{x \to 0^+} \frac{1}{x} = \infty$ ($\rightarrow x = 0'00000001$)

$\Rightarrow \nexists \lim_{x \to 0} \frac{1}{x}$

$\lim_{x \to 0} \frac{1}{x^2}$:
- $x<0$: $\lim_{x \to 0^-} \frac{1}{x^2} = \infty$
- $x>0$: $\lim_{x \to 0^+} \frac{1}{x^2} = \infty$

$\Rightarrow \lim_{x \to 0} \frac{1}{x^2} = \infty$

$\frac{0}{0}$ → a) funzione arrotondata

$$\lim_{x \to 3} \frac{x^2-9}{2x^2-4x-6} = \frac{3^2-9}{2\cdot 3^2-4\cdot 3-6} \left(\frac{0}{0} \text{IND}\right) = \lim_{x \to 3} \frac{(x-3)(x+3)}{(x-3)(x+1)2} = \frac{(x+3)}{2(x+1)} = \frac{3+3}{2\cdot 3+2} = \frac{6}{8} = \frac{3}{4}$$

$x^2-9 \to (x-3)\cdot(x+3)$

$2x^2-4x-6 = x^2-2x-3 \to \frac{2 \pm \sqrt{4+12}}{2} = \frac{2 \pm 4}{2}$ $\begin{cases} 3 \to x-3 \\ -1 \to x+1 \end{cases}$ $\Big\}$ $2(x-3)(x-1)$

$\frac{\infty}{\infty}$ → $\lim_{x\to\infty} \frac{x^2+3x+1}{1-2x^2} = \frac{\infty^2+\infty+1}{1-2\infty^2} = \frac{\infty}{-\infty}$ IND

$\lim_{x\to\infty} \frac{x^2}{-2x^2} = -\frac{1}{2}$

$\lim_{x\to\infty} \frac{x^2-3x+1}{1-2x^3} = \lim_{x\to\infty} \frac{x^2}{-2x^3} = \frac{1}{-2\infty} = \frac{1}{-\infty} = 0$

$\lim_{x\to\infty} \frac{1-x^4}{2x^3+x} = \frac{-x^4}{2x^3} = \frac{-\infty}{2} = -\infty$

$\lim_{x\to\infty} \frac{5x^2-2x+1}{\sqrt{4x^4-6}} = \lim_{x\to\infty} \frac{5x^2}{\sqrt{4x^4}} = \lim_{x\to\infty} \frac{5x^2}{2x^2} = \frac{5}{2}$

$\lim_{x\to\infty} \frac{5x^2-3}{x^3+\sqrt{4x^4+6}} = \frac{5x^2}{x^3} = \frac{5}{\infty} = 0$

$\lim_{x\to\infty} \frac{P(x)}{Q(x)} = \lim_{x\to\infty} \frac{ax^n \dots a_0}{bx^m \dots b_0}$

$= \lim_{x\to\infty} \frac{ax^n}{bx^m}$:
- $\frac{a}{b}$ $n=m$
- $\lim_{x\to\infty} \frac{a}{bx^{m-n}} = 0$ $n<m$
- $\lim_{x\to\infty} \frac{ax^{n-m}}{b} = \infty$ edo $-\infty$ $n>m$

$\infty-\infty$ → $\lim_{x\to1}\left(\frac{2x}{x^2-1} - \frac{1}{x-1}\right) = \frac{2}{1-1} - \frac{1}{1-1} = \left(\frac{2}{0} - \frac{1}{0} \text{ IND}\right) = \lim_{x\to1} \frac{2x-(x+1)}{(x-1)\cdot(x+1)} = \lim \frac{x-1}{(x-1)\cdot(x+1)} = \lim \frac{1}{(x+1)} = \frac{1}{2}$

$\frac{2x}{x^2-1} - \frac{x}{x^2-1} = \frac{2x-(x+1)}{(x-1)\cdot(x+1)} = \frac{-2x}{(x-1)}$

$\lim_{x\to\infty}(\sqrt{x+4}-\sqrt{x-4}) = (\infty-\infty \text{ IND}) = \lim_{x\to\infty} \frac{(\sqrt{x+4}-\sqrt{x-4})\cdot(\sqrt{x+4}+\sqrt{x-4})}{\sqrt{x+4}+\sqrt{x-4}} = \lim_{x\to\infty} \frac{(\sqrt{x+4})^2-(\sqrt{x-4})^2}{\sqrt{x+4}+\sqrt{x-4}}$

$\lim_{x\to\infty} \frac{x+4-x+4}{\sqrt{x+4}+\sqrt{x-4}} = \lim_{x\to\infty} \frac{8}{\sqrt{x+4}+\sqrt{x-4}} = 0$

1^∞ → $\lim_{x\to\infty}\left(\frac{6+3x}{3x-8}\right)^{2x} = 1^\infty$ IND

$e = \lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x \to$

$\lim_{x\to\infty}\left(\frac{6+3x}{3x-8}\right)^{2x}$ ($2x = g(x)$, $\frac{6+3x}{3x-8} = f(x)$) $= \lim_{x\to\infty}\left(1+\frac{6+3x}{3x-8}-1\right)^{2x} = \lim_{x\to\infty}\left(1+\frac{6+3x-3x+8}{3x-8}\right)^{2x} = \lim_{x\to\infty}\left(1+\frac{14}{3x-8}\right)^{2x} =$

$\lim_{x\to\infty}\left(1+\frac{1}{\frac{3x-8}{14}}\right)^{2x} = \lim\left(1+\frac{1}{\frac{3x-8}{14}}\right)^{\frac{3x-8}{14}\cdot\frac{14}{3x-8}2x} = \lim_{x\to\infty}\Bigg[\underbrace{\left(1+\frac{1}{\frac{3x-8}{14}}\right)^{\frac{3x-8}{14}}}_{e}\Bigg]^{\frac{14}{3x-8}\cdot 2x} =$

$e^{\lim_{x\to\infty}\frac{14\cdot 2x}{3x-8}} = e^{\lim_{x\to\infty}\frac{28x}{3x}} = e^{28/3}$

edo $e^{\lim_{x\to\infty}(f(x)-1)\cdot g(x)}$

$\lim_{x\to0}(1-2x)^{\frac{1}{x}}$ ($\frac{1}{x}\to\infty$, $1-2x \to 1$) $= 1^\infty$ IND $= e^{\lim_{x\to0}\frac{1}{x}\cdot(1-2x-1)} = e^{\lim_{x\to0}-\frac{2x}{x}} = e^{-2}$

$\boxed{0 \cdot \infty}$ → $\lim_{x \to \infty} \frac{1}{x} \cdot x^2 = \lim_{x \to \infty} \frac{x^2}{x} = \infty$

($\frac{1}{x} \to 0$, $x^2 \to \infty$)

$\lim_{x \to \infty} \left(\frac{5x+1}{x^2-3} \cdot \frac{x^2-4}{4x} \right) = \lim_{x \to \infty} \frac{(5x+1) \cdot (x^2-4)}{(x^2-3) 4x} = \lim_{x \to \infty} \frac{5x^3 - 20x + x^2 - 4}{4x^3 - 12x} = \lim_{x \to \infty} \frac{5x^3}{4x^3} = \frac{5}{4}$

($\frac{5x+1}{x^2-3} \to 0$, $\frac{x^2-4}{4x} \to \infty$)

$\lim_{x \to \infty} \frac{3^x}{x^4 - 2} = \lim_{x \to \infty} \frac{3^x}{x^4}$

$a^x \gg x^n \gg \log(x^m)$

$\lim_{x \to \infty} \frac{\log(x^3 - 2x + 1)}{x^2 + 2} = \lim_{x \to \infty}$