Problemas Tema 2: Concepto de Tensión

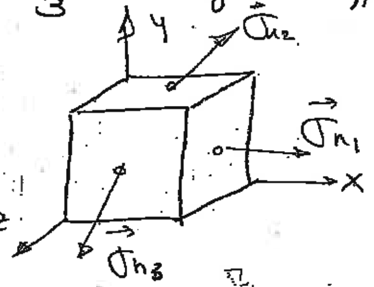
Problema 2.1.

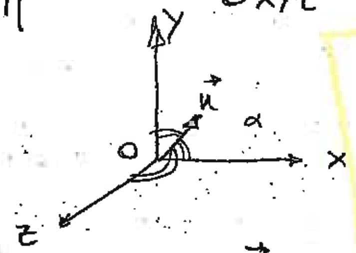
Tensiones principales en el punto P de un sólido elástico, referidas a un sistema cartesiano ortogonal OXYZ (MPa):

$$\vec{\sigma}_1 = \frac{50}{3}(2\vec{i} + 2\vec{j} + \vec{k})$$
$$\vec{\sigma}_2 = 10(2\vec{i} - \vec{j} - 2\vec{k})$$
$$\vec{\sigma}_3 = -\frac{20}{3}(\vec{i} - 2\vec{j} + 2\vec{k})$$

son vect. ortog. ortog. en sist. de ref. ppal

$\vec{\sigma}_n$: $\vec{n}$ (ext.) ángulos agudos $\ominus$ con los semiejes $\oplus$ del triedro OXYZ





$\alpha^2 + \alpha^2 + \alpha^2 = 1$

$3\alpha^2 = 1$

$\boxed{\alpha = 1/\sqrt{3}}$ cosenos directores

$\vec{n} = (1/\sqrt{3},\ 1/\sqrt{3},\ 1/\sqrt{3})$

$$\{\vec{\sigma}_n\} = \begin{bmatrix} \frac{100}{3} & \frac{100}{3} & \frac{50}{3} \\ 20 & -10 & -20 \\ -\frac{20}{3} & \frac{40}{3} & -\frac{40}{3} \end{bmatrix} \begin{Bmatrix} 1 \\ 1 \\ 1 \end{Bmatrix} 1/\sqrt{3} = \frac{1}{\sqrt{3}} \begin{Bmatrix} \frac{250}{3} \\ -10 \\ -\frac{20}{3} \end{Bmatrix}$$

DUDA ¿? ¿?

$$[T_{ij}] = \begin{bmatrix} \frac{100}{3} & 20 & -\frac{20}{3} \\ 100/3 & -10 & \frac{40}{3} \\ 50/3 & -20 & -\frac{40}{3} \end{bmatrix}$$

$$\vec{\sigma}_n = \frac{10}{\sqrt{3}}\left(25/3\,\vec{e}_1 - \vec{e}_2 - \frac{2}{3}\vec{e}_3\right)$$

$$\{\sigma_n\} = [T_{ij}]\{n\} = \begin{bmatrix} \frac{100}{3} & 20 & -20/3 \\ 100/3 & -10 & 40/3 \\ 50/3 & -20 & -40/3 \end{bmatrix} \begin{Bmatrix} 1 \\ 1 \\ 1 \end{Bmatrix} 1/\sqrt{3} = 1/\sqrt{3} \begin{Bmatrix} \frac{100}{3} + 20 - \frac{20}{3} \\ \frac{100}{3} - 10 + \frac{40}{3} \\ \frac{50}{3} - 20 - \frac{40}{3} \end{Bmatrix}$$

$$\{\sigma_n\} = 1/\sqrt{3} \begin{Bmatrix} \frac{100+60-20}{3} \\ \frac{100-30+40}{3} \\ \frac{50-60-40}{3} \end{Bmatrix} = 1/\sqrt{3} \begin{Bmatrix} 140/3 \\ 110/3 \\ -50/3 \end{Bmatrix} = \frac{10}{3\sqrt{3}} \begin{Bmatrix} 14 \\ 11 \\ -5 \end{Bmatrix} = \frac{10\sqrt{3}}{9} \begin{Bmatrix} 14 \\ 11 \\ -5 \end{Bmatrix}$$

3x3 3x1

Otra forma:

⇨ Cambio de base → tensiones principales:

Conozco $[T_{ij}']$, Busco $[T_{ij}]$

$$\begin{bmatrix} 50/3 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & -20/3 \end{bmatrix}$$

	X	Y	Z
x'	2	2	1
y'	2	-1	-2
z'	1	-2	2

$$[N] = \begin{bmatrix} 2 & 2 & 1 \\ 2 & -1 & -2 \\ 1 & -2 & 2 \end{bmatrix}$$

$$[T_{ij}] \cdot \{n'\} = \frac{1}{\sqrt{3}}(2\bar{e}_1 + 2\bar{e}_2 + \vec{e}_3) + \frac{1}{\sqrt{3}}(2\bar{e}_1 - \bar{e}_2 - 2\bar{e}_3) + \frac{1}{\sqrt{3}}(\bar{e}_1 - 2\bar{e}_2 + 2\bar{e}_3) =$$

$\{x\} = [N]^T \cdot \{x'\}$

$\{\sigma_n'\}^T = [N][T_{ij}][N^T]\{n'\}$

$$\{n'\} = \bar{e}_1\left(2/\sqrt{3} + 2/\sqrt{3} + 1/\sqrt{3}\right) + \bar{e}_2\left(2/\sqrt{3} - 1/\sqrt{3} - 2/\sqrt{3}\right) + \bar{e}_3\left(1/\sqrt{3} - 2/\sqrt{3} + 2/\sqrt{3}\right)$$

$$\{n'\} = 5/\sqrt{3}\,\bar{e}_1 - 1/\sqrt{3}\,\bar{e}_2 + 1/\sqrt{3}\,\bar{e}_3$$

$$\vec{\sigma}_n' = \begin{bmatrix} 50/3 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & -20/3 \end{bmatrix} \begin{Bmatrix} 5/\sqrt{3} \\ -1/\sqrt{3} \\ 1/\sqrt{3} \end{Bmatrix} = \begin{Bmatrix} \frac{250}{3\sqrt{3}} \\ -10/\sqrt{3} \\ -20/\sqrt{3} \end{Bmatrix} = \frac{10}{\sqrt{3}}\left(25/3, -1, -2\right)$$

$$\vec{\sigma}_n = \frac{10}{\sqrt{3}}\left[\frac{25}{3}(2\vec{i} + 2\vec{j} + \vec{k}) - (2\vec{i} - \vec{j} - 2\vec{k}) - 2(\vec{i} - 2\vec{j} + 2\vec{k})\right] =$$

$$\vec{\sigma}_n = \frac{10}{\sqrt{3}}\left[\left(\frac{50}{3} - 2 - 2\right)\vec{i} + \left(\frac{50}{3} + 1 + 4\right)\vec{j} + \left(25/3 + 2 - 4\right)\vec{k}\right] =$$

$$\vec{\sigma}_n = \frac{10}{\sqrt{3}}\left[\left(\frac{50 - 12}{3}\right)\vec{i} + \left(\frac{50 + 15}{3}\right)\vec{j} + \left(\frac{25 - 6}{3}\right)\vec{k}\right] =$$

$$\boxed{\vec{\sigma}_n = 10\sqrt{3}\,(48\,\vec{i} + 65\,\vec{j} + 19\,\vec{k})\ \text{MPa}}$$ Mal !!

Problema 22.

Problema de valores y vectores propios:

$$1\,\frac{N}{mm^2}\cdot\frac{10^6\,mm^2}{1\,m^2} = 10^6\,\frac{N}{m^2}$$

$$1\,\frac{N}{mm^2} = 10^6\,Pa$$

$$\boxed{1\,\frac{N}{mm^2} = 1\,MPa}$$

$$[T_{ij}] = \begin{bmatrix} 10 & 0 & 0 \\ 0 & 30 & 20 \\ 0 & 20 & 0 \end{bmatrix}$$

$$\left| [T_{ij}] - \sigma[I] \right| = 0$$

$$\begin{vmatrix} 10-\sigma & 0 & 0 \\ 0 & 30-\sigma & 20 \\ 0 & 20 & -\sigma \end{vmatrix} = 0$$

$$(10-\sigma)(30-\sigma)(-\sigma) - 400(10-\sigma) = 0$$

$$-\sigma(300 - 10\sigma - 30\sigma + \sigma^2) - 4000 + 400\sigma = 0$$

$$-\sigma^3 + 40\sigma^2 + 100\sigma - 4000 = 0 \qquad \text{Rufini}$$

$$\sigma^3 - 40\sigma^2 - 100\sigma + 4000 = 0$$

	1	-40	-100	4000
-10		-10	500	-4000
	1	-50	400	0

$$\boxed{\sigma_1 = -10}$$

$$(\sigma+10)\cdot(\sigma^2 - 50\sigma + 400) = 0$$

$$\sigma^2 - 50\sigma + 400 = 0 \qquad \sigma = \frac{50 \pm \sqrt{2500-1600}}{2} = \frac{50 \pm 30}{2} \begin{cases} \boxed{\sigma_2 = 40} \\ \boxed{\sigma_1 = 10} \end{cases}$$

$$\boxed{\sigma_1 = -10}$$

$$\{[T_{ij}] - \sigma_1[I]\}\{n_1\} = 0 \quad + \quad \{n_{1x}^2 + n_{1y}^2 + n_{1z}^2 = 0\}$$

$$\begin{bmatrix} 20 & 0 & 0 \\ 0 & 40 & 20 \\ 0 & 20 & 10 \end{bmatrix} \begin{Bmatrix} n_{1x} \\ n_{1y} \\ n_{1z} \end{Bmatrix} = 0$$

$$20\,n_{1x} = 0 \qquad \boxed{n_{1x} = 0}$$

$$40\,n_{1y} + 20\,n_{1z} = 0$$

$$20\,n_{1y} + 10\,n_{1z} = 0$$

$$2\,n_{1y} + n_{1z} = 0 \qquad \boxed{n_{1z} = -2\,n_{1y}}$$

$$n_{1y}^2 + (-2n_{1y})^2 = 1$$

$$n_{1y}^2 + 4n_{1y}^2 = 1$$

$$5n_{1y}^2 = 1 \qquad n_{1y}^2 = 1/5 \qquad n_{1y} = \pm\, 1/\sqrt{5} \qquad n_{1z} = -2/\sqrt{5}$$

Da igual

$$\boxed{\sigma_1 = -10} \quad \vec{n}_1 = \left(0, \tfrac{1}{\sqrt{5}}, -\tfrac{2}{\sqrt{5}}\right) \quad \boxed{MPa} \rightarrow \Pi_1 = \boxed{y - 2z = 0}$$

$$\boxed{\sigma_2 = 40\,MPa}$$

$$\begin{bmatrix} -30 & 0 & 0 \\ 0 & -10 & 20 \\ 0 & 20 & -40 \end{bmatrix} \begin{Bmatrix} n_{2x} \\ n_{2y} \\ n_{2z} \end{Bmatrix} = \{0\} \qquad \underline{n_{2x} = 0}$$

$$-10n_{2y} + 20n_{2z} = 0 \qquad (2n_{2z})^2 + (n_{2z})^2 = 1$$

$$-n_{2y} + 2n_{2z} = 0 \rightarrow \underline{n_{2y} = 2n_{2z}} \qquad 4n_{2z}^2 + n_{2z}^2 = 1$$

$$5n_{2z}^2 = 1$$

$$\Pi_2 = \boxed{2y + z = 0} \qquad \underline{n_{2z} = 1/\sqrt{5}}$$

$$\underline{n_{2y} = 2/\sqrt{5}}$$

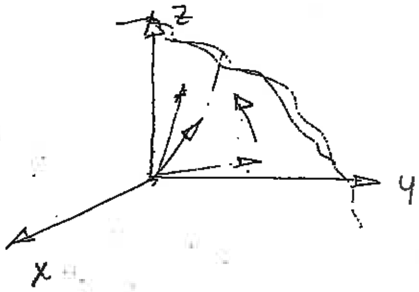
$$\boxed{\sigma_3 = 10\,MPa}$$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 20 & 20 \\ 0 & 20 & -10 \end{bmatrix} \begin{Bmatrix} n_{3x} \\ n_{3y} \\ n_{3z} \end{Bmatrix} = \{0\} \qquad 0n_{3x} + 0n_{3y} + 0n_{3z} = 0$$

no aporta nada
cualquier valor.

$$n_{3y} + n_{3z} = 0 \qquad n_{3y} + 2n_{3y} = 0 \qquad 3n_{3y} = 0$$

$$2n_{3y} - n_{3z} = 0 \qquad n_{3z} = 2n_{3y} \qquad \boxed{n_{3y} = n_{3z} = 0}$$



plano en el que los vectores $\in$ a él forman 0° con ejes y y z

Plano YOZ ó

Plano $\boxed{X = 0} = \Pi_3$

Problema 24.

$$[T] = \begin{bmatrix} 40x+30y & -60(x+y+z) & 10(y+z) \\ -60(x+y+z) & 100(y-z) & 30x \\ 10(y+z) & 30x & 50z \end{bmatrix} \quad MPa / N/mm^2$$

1°) Calcular las fuerzas de volumen * AL FINAL *

2°) Hallar las matrices esférica y desviadora en P (0, 1, -1)

$$[T] = \begin{bmatrix} 30 & 0 & 0 \\ 0 & 200 & 0 \\ 0 & 0 & -50 \end{bmatrix} \qquad \sigma_m = \frac{30+200-50}{3} = \frac{180}{3} = 60$$

Todo en $\boxed{MPa}$

$$[T_{ij}]_e = \begin{bmatrix} 60 & 0 & 0 \\ 0 & 60 & 0 \\ 0 & 0 & 60 \end{bmatrix} \qquad [T_{ij}]_d = \begin{bmatrix} -30 & 0 & 0 \\ 0 & 140 & 0 \\ 0 & 0 & -110 \end{bmatrix}$$

Problema 25.

$$[T_{ij}] = \begin{bmatrix} -10 & 0 & -8 \\ 0 & 2 & 0 \\ -8 & 0 & 2 \end{bmatrix} \quad (MPa)$$

1°) Tensiones pples:
$$\begin{vmatrix} -10-\sigma & 0 & -8 \\ 0 & 2-\sigma & 0 \\ -8 & 0 & 2-\sigma \end{vmatrix} = 0$$

$$(-10-\sigma)(4+\sigma^2-4\sigma) - 64(2-\sigma) = 0$$

$$-40 - \cancel{10\sigma^2} + \cancel{40}\sigma - \cancel{4\sigma} - \cancel{\sigma^3} + \cancel{4\sigma^2} - 128 + \underset{100}{\overset{36}{\cancel{64}}}\sigma = 0$$

$$-\sigma^3 - 6\sigma^2 + 100\sigma - 168 = 0$$

$$\sigma^3 + 6\sigma^2 - 100\sigma + 168 = 0$$

Ruffini :

	1	6	-100	168
2		2	16	-168
	1	8	-84	0

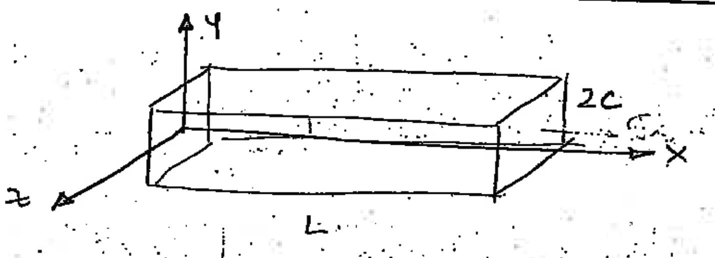
$\boxed{\sigma_1 = 2}$ MPa

$(\sigma-2)(\sigma^2+8-84)=0 \qquad \sigma = \dfrac{-8 \pm \sqrt{64+336}}{2}$

$\sigma = \dfrac{-8 \pm \sqrt{400}}{2} = \dfrac{-8 \pm 20}{2}$

$\boxed{\sigma_2 = 6}$ MPa $\boxed{\sigma_3 = -14}$ MPa

Problema 2.6.

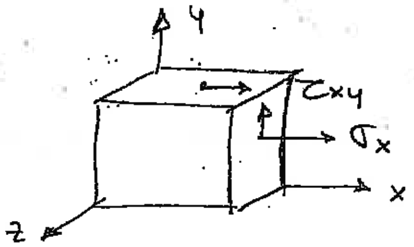


Flexión → $\sigma_x = C_1 y + C_2 xy$

$\tau_{xy} = C_3 (c^2 - y^2)$

$\sigma_y = \sigma_z = \tau_{xz} = \tau_{yz} = 0$

1) ¿Bajo qué condiciones hay equilibrio? Despreciando fuerzas de Volumen.



$$\left\{\begin{aligned} \frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{yz}}{\partial z} + \phi_x = 0 \\ \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \tau_{zy}}{\partial z} + \phi_y = 0 \\ \frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \sigma_{zz}}{\partial z} + \phi_z = 0 \end{aligned}\right\}$$

$C_2 \cancel{y} + C_3(-2\cancel{y}) = 0$

$C_2 - 2C_3 = 0 \qquad \boxed{C_2 = 2C_3}$

2) Superficie $x=0$:

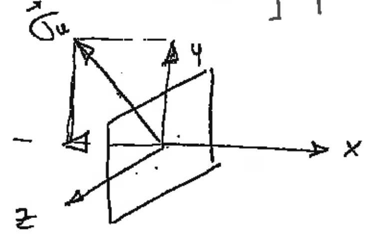
$$\sigma_x = C_1 \cdot y$$
$$\tau_{xy} = C_3\,(c^2-y^2)$$

$$[T_{ij}] = \begin{bmatrix} C_1 y & C_3(c^2-y^2) & 0 \\ C_3(c^2-y^2) & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\vec{n} = (-1,0,0)$$

$$\{\vec{\sigma}_u\} = \begin{bmatrix} C_1 y & C_3(c^2-y^2) & 0 \\ C_3(c^2-y^2) & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} -1 \\ 0 \\ 0 \end{Bmatrix} = \begin{Bmatrix} -C_1 y \\ C_3(c^2-y^2) \\ 0 \end{Bmatrix}$$

$$\}\, C_3 = C_2/2 \,\{$$



$$\sigma_{nx} = \{-C_1 y \quad C_2(c^2-y^2) \quad 0\} \begin{Bmatrix} -1 \\ 0 \\ 0 \end{Bmatrix}$$

$$\sigma_{nx} = \underline{\underline{C_1 y}}$$

$$\tau_{ny} = \{-C_1 y \quad C_2(c^2-y^2) \quad 0\} \begin{Bmatrix} 0 \\ 1 \\ 0 \end{Bmatrix} = \underline{\underline{C_2(c^2-y^2)}}$$

Superficie $x=L$: $\sigma_x = C_1 y + C_2 L y$

$\tau_{xy} = C_3(c^2-y^2)$ $\qquad \vec{n} = (1,0,0)$

$$\{\vec{\sigma}_u\} = \begin{bmatrix} C_1 y + C_2 L y & C_3(c^2-y^2) & 0 \\ C_3(c^2-y^2) & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} 1 \\ 0 \\ 0 \end{Bmatrix} = \begin{Bmatrix} C_1 y + C_2 L y \\ C_3(c^2-y^2) \\ 0 \end{Bmatrix}$$

$$\sigma_{ux} = \vec{\sigma}_u \circ \vec{n}_x = \{C_1 y + C_2 L y \quad C_3(c^2-y^2) \quad 0\} \begin{Bmatrix} 1 \\ 0 \\ 0 \end{Bmatrix} = \underline{\underline{(C_1 + 2C_2)\,y}}$$

$$\tau_{uy} = \vec{\sigma}_u \circ \vec{n}_y = \{C_1 y + C_2 L y \quad C_3(c^2-y^2) \quad 0\} \begin{Bmatrix} 0 \\ 1 \\ 0 \end{Bmatrix} = C_3\,(c^2-y^2) =$$

$$= \underline{\underline{\frac{C_2}{2}(c^2-y^2)}}$$

Superficie lateral superior: $Y=C$ $\quad \vec{u}=(0,1,0)$

$\sigma_x = C_1 c + C_2 c x$

$\tau_{xy} = C_3 (c^2 - c^2) = 0$

$$\{\vec{\sigma}_u\} = \begin{pmatrix} C_1c+C_2cx & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{Bmatrix} 0 \\ 1 \\ 0 \end{Bmatrix} = \{\vec{0}\}$$

$\sigma_{uy} = 0$

$\tau_{nx} = 0$

Superficie lateral inferior $Y=-C$ $\quad \vec{u}=(0,-1,0)$

$\sigma_x = -C_1 c - C_2 c x$

$\tau_{xy} = 0$

$$\{\vec{\sigma}_u\} = \begin{pmatrix} -C_1c-C_2cx & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{Bmatrix} 0 \\ -1 \\ 0 \end{Bmatrix} = \{0\}$$

$\boxed{\begin{matrix}\sigma_{ny}=0 \\ \tau_{ux}=0\end{matrix}}$

Problema 2.7.

$$[T_{ij}] = \begin{pmatrix} 1 & -3 & \sqrt{2} \\ -3 & 4 & -\sqrt{2} \\ \sqrt{2} & -\sqrt{2} & 4 \end{pmatrix} \text{MPa}$$

1) Tensiones pples y sus direcciones.

$$\begin{vmatrix} 1-\sigma & -3 & \sqrt{2} \\ -3 & 4-\sigma & -\sqrt{2} \\ \sqrt{2} & -\sqrt{2} & 4-\sigma \end{vmatrix} = 0 \qquad (1-\sigma)(16+\sigma^2-8\sigma)+6+6-2(4-\sigma)-9(4-\sigma)-2(1-\sigma)=0$$

$$\cancel{16}+\cancel{\sigma^2}-\cancel{8\sigma}-16\cancel{\sigma}-\cancel{\sigma^3}+8\cancel{\sigma^2}+1\cancel{2}-8+2\cancel{\sigma}-3\cancel{6}+9\cancel{\sigma}-\cancel{2}+2\cancel{\sigma}=0$$

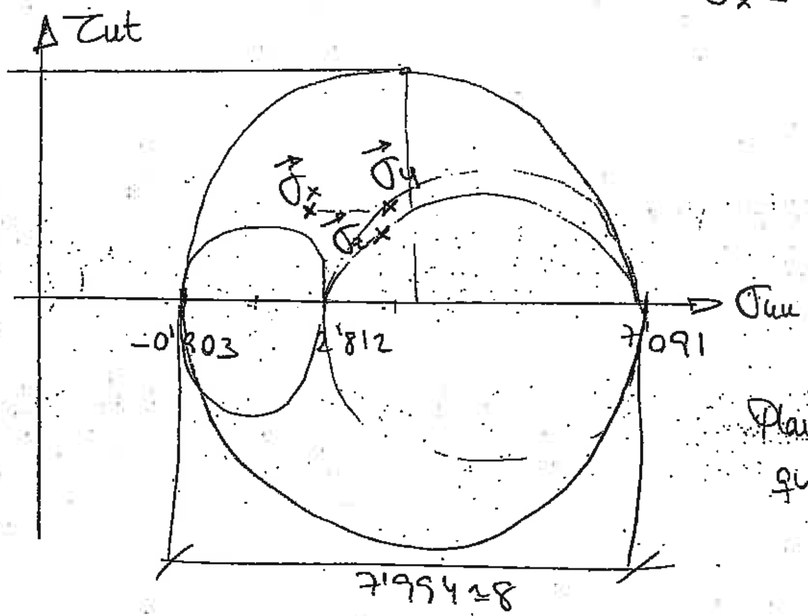
$$-\sigma^3+9\sigma^2-11\sigma-10=0 \longrightarrow \sigma^3-9\sigma^2+11\sigma+10=0$$

Raíces:
$$\begin{cases} \sigma_1 = 7'091 \\ \sigma_2 = 2'812 \\ \sigma_3 = -0'903 \end{cases}$$

σ_1:
$$\begin{pmatrix} 1-7'091 & -3 & \sqrt{2} \\ -3 & 4-7'091 & -\sqrt{2} \\ \sqrt{2} & -\sqrt{2} & 4-7'091 \end{pmatrix} \begin{Bmatrix} n_{1x} \\ n_{1y} \\ n_{1z} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \qquad n_{1x}^2 + n_{1y}^2 + n_{1z}^2 = 1$$

Resolver ... $\vec{u_1} = (0'469, -0'699, 0'535)$

2) $\vec{\sigma_x} = [T_{ij}]\{\vec{n}\}$



Planos con τ_{ut} máx → son los que tienen $\sigma_{uu} = 3'094$ MPa

$$\vec{\sigma_x} = \begin{bmatrix} 1 & -3 & \sqrt{2} \\ -3 & 4 & -\sqrt{2} \\ \sqrt{2} & -\sqrt{2} & 4 \end{bmatrix} \begin{Bmatrix} 1 \\ 0 \\ 0 \end{Bmatrix} = \begin{Bmatrix} 1 \\ -3 \\ \sqrt{2} \end{Bmatrix}$$

$\sigma_{xx} = \vec{\sigma_x} \cdot \vec{n_x} = 1$

$\left.\begin{matrix} \tau_{xy} = -3 \\ \tau_{xz} = \sqrt{2} \end{matrix}\right\} \quad \tau_{ut} = \sqrt{9+2} = \sqrt{11}$

⇩ Su punto corresp. $(1, \sqrt{11})$ $(1, 3'316)$

MPa

$\vec{\sigma_y} = \{-3, 4, -\sqrt{2}\} \longrightarrow \begin{cases} \sigma_{yy} = \sigma_{uu} = 4 \\ \tau_{yt} = \tau_{ut} = \sqrt{9+2} = \sqrt{11} \end{cases}$ $(4, \sqrt{11})$

$\vec{\sigma_z} = \{\sqrt{2}, -\sqrt{2}, 4\} \longrightarrow \begin{cases} \sigma_{zz} = \sigma_{uu} = 4 \\ \tau_{zt} = \tau_{ut} = \sqrt{2+2} = 2 \end{cases}$ $(4, 2)$

Problema 2.8.

P: $[T_{ij}] = \begin{bmatrix} 2 & 1 & 0 \\ 1 & -1 & 2 \\ 0 & 2 & 3 \end{bmatrix}$ (MPa)

1) $\vec{\sigma}_n$? $\vec{n}$ angulos iguales de 45º con X, Y. siendo ⊕.

$\vec{n} = (1/\sqrt{2}, 1/\sqrt{2}, 0)$

$$\vec{\sigma}_n = \begin{bmatrix} 2 & 1 & 0 \\ 1 & -1 & 2 \\ 0 & 2 & 3 \end{bmatrix} \begin{Bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \\ 0 \end{Bmatrix} = 1/\sqrt{2} \begin{Bmatrix} 3 \\ 0 \\ 2 \end{Bmatrix} \quad \underline{\underline{MPa}}$$

2) Tensiones y direcc. pples.

$$\begin{vmatrix} 2-\sigma & 1 & 0 \\ 1 & -1-\sigma & 2 \\ 0 & 2 & 3-\sigma \end{vmatrix} = 0$$

$(2-\sigma)(-1-\sigma)(3-\sigma) - (3-\sigma) - 4(2-\sigma) = 0$

$\underbrace{(-2-2\sigma+\sigma+\sigma^2)}_{(\sigma^2-\sigma-2)(3-\sigma)}(3-\sigma) - 3 + \sigma - 8 + 4\sigma = 0$

$3\sigma^2 - \sigma^3 - 3\sigma + \sigma^2 - 6 + 2\sigma - 3 + \sigma - 8 + 4\sigma = 0$

$-\sigma^3 + 4\sigma^2 + 4\sigma - 17 = 0$

$\sigma^3 - 4\sigma^2 - 4\sigma + 17 = 0$

	1	-4	-4	-17
7		7	21	
	1	3	7...	

Resultado: $\sigma_1 = 3'91$ $\quad \bar{n}_1 = (0'212, 0'405, 0'889)$

$\sigma_2 = 2'13$ $\quad \bar{n}_2 = (0'951, 0'123, -0'283)$ $\quad \underline{\underline{MPa}}$

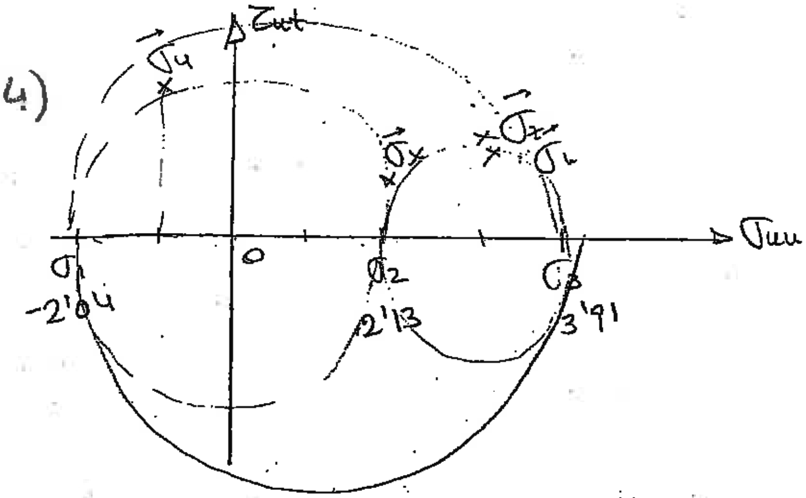
$\sigma_3 = -2'04$ $\quad \bar{n}_3 = (-0'224, 0'906, -0'359)$

3) $\sigma_{uu} = \vec{\sigma}_u \cdot \vec{n} = \{\sigma_u\}^T \cdot \{n\} = \frac{1}{\sqrt{2}}\{3 \quad 0 \quad 2\} \begin{Bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \\ 0 \end{Bmatrix} = \frac{1}{2}\cdot 3 = \frac{3}{2}$ M

$|\vec{\sigma}_u| = \sqrt{\frac{9}{2} + \frac{4}{2}} = \sqrt{\frac{13}{2}}$

$\tau_{ut} = \sqrt{\frac{13}{2} - \frac{9}{4}} = \sqrt{\frac{26-9}{4}} = \sqrt{\frac{17}{4}} = \frac{\sqrt{17}}{2}$ MPa

4)



a) Planos coordenados:

$\vec{n} = \vec{e}_1$:

$\vec{\sigma}_x = \{2, 1, 0\}$

$\sigma_{xx} = 2$

$\tau_{xt} = \sqrt{1^2 + 0^2} = 1$

$\vec{n} = \vec{e}_2 \quad \vec{\sigma}_y = \{1, -1, 2\} \quad \begin{cases} \sigma_{yy} = -1 \\ \tau_{xt} = \sqrt{1^2+2^2} = \sqrt{5} = 2'236 \end{cases}$

$\vec{n} = \vec{e}_3 \quad \vec{\sigma}_z = \{0, 2, 3\} \quad \begin{cases} \sigma_{zz} = 3 \\ \tau_{zt} = \sqrt{4} = 2 \end{cases}$

b) $\vec{\sigma}_u \quad \vec{u}\,(1/2,\ 1/\sqrt{2},\ 1/2)$

$$\{\vec{\sigma}_u\} = \begin{bmatrix} 2 & 1 & 0 \\ 1 & -1 & 2 \\ 0 & 2 & 3 \end{bmatrix} \begin{Bmatrix} 1/2 \\ 1/\sqrt{2} \\ 1/2 \end{Bmatrix} = \begin{Bmatrix} 1 + 1/\sqrt{2} \\ 1/2 - 1/\sqrt{2} + 1 \\ 2/\sqrt{2} + 3/2 \end{Bmatrix} = \begin{Bmatrix} \frac{\sqrt{2}+1}{\sqrt{2}} \\ \frac{3-\sqrt{2}}{2} \\ \frac{2\sqrt{2}+3}{2} \end{Bmatrix} = \begin{Bmatrix} 1'70 \\ 0'792 \\ 2'914 \end{Bmatrix}$$

$$\sigma_{uu} = \left\{ \frac{\sqrt{2}+1}{\sqrt{2}} \quad \frac{3-\sqrt{2}}{\sqrt{2}} \quad \frac{2\sqrt{2}+3}{2} \right\} \begin{Bmatrix} 1/2 \\ 1/\sqrt{2} \\ 1/2 \end{Bmatrix} = \frac{\sqrt{2}+1}{2\sqrt{2}} + \frac{3-\sqrt{2}}{2\sqrt{2}} + \frac{2\sqrt{2}+3}{4} =$$

$$= \frac{\cancel{2\sqrt{2}}+2+6-\cancel{2\sqrt{2}}+4+3\sqrt{2}}{4\sqrt{2}} = \frac{12+3\sqrt{2}}{4\sqrt{2}} = 2'8713 \text{ MPa}$$

$\tau_{ut} = 1'9474$ MPa

$|\vec{\sigma}_u| = \sqrt{\left(\frac{\sqrt{2}+1}{\sqrt{2}}\right)^2 + \left(\frac{3-\sqrt{2}}{2}\right)^2 + \left(\frac{2\sqrt{2}+3}{2}\right)^2} = 3'469202$ MPa

5) Tensión hidrostática o tensión media en P

$$\sigma_m = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3} = \frac{3'91 + 2'13 - 2'04}{3} = \underline{\underline{4/3 \text{ MPa}}}$$

Problema 2.9.

a) $\sigma_{xx} = ax + by \quad \sigma_{yy} = cx + dy \quad \tau_{xy} = fx + gy$

$$\left\{\begin{array}{l} \frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z} + \cancel{\phi_x}^{0} = 0 \\ \frac{\partial \tau_{yx}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \tau_{yz}}{\partial z} + \phi_y = 0 \\ \frac{\partial \tau_{zx}}{\partial x} + \frac{\partial \tau_{zy}}{\partial y} + \frac{\partial \sigma_{zz}}{\partial z} + \phi_z = 0 \end{array}\right\}$$

$$\left\{\begin{array}{l} a + g = 0 \\ f + d = 0 \end{array}\right\} \checkmark \text{ Sí es posible}$$

b) $\sigma_{xx} = ax^2y^2 + bx \quad \sigma_{yy} = cy^2 \quad \tau_{xy} = dxy$

$$\left\{\begin{array}{l} 2ax^2y^2 + b + dx = 0 \\ d\cancel{y} + 2c\cancel{y} = 0 \quad d + 2c = 0 \quad d = -2c \checkmark \end{array}\right.$$

← No es un campo válido para todo el sólido, pero sólo es válido para puntos que lo cumplan

c) $\sigma_{xx} = a[y^2 + b(x^2 - y^2)] \quad \sigma_{yy} = a[x^2 + b(y^2 - x^2)] \quad \sigma_{zz} = ab(x^2 + y^2) \quad \tau_{xy} = 2ab$

$$\left\{\begin{array}{l} 2abx + 2abx = 0 \\ 2aby + 2aby = 0 \quad 4aby = 0 \left\langle \begin{array}{l} a = 0 \rightarrow 0 = 0 \checkmark \\ \underline{b = 0} \rightarrow 0 = 0 \checkmark \end{array}\right. \\ 0 = 0 \text{ ?} \end{array}\right.$$

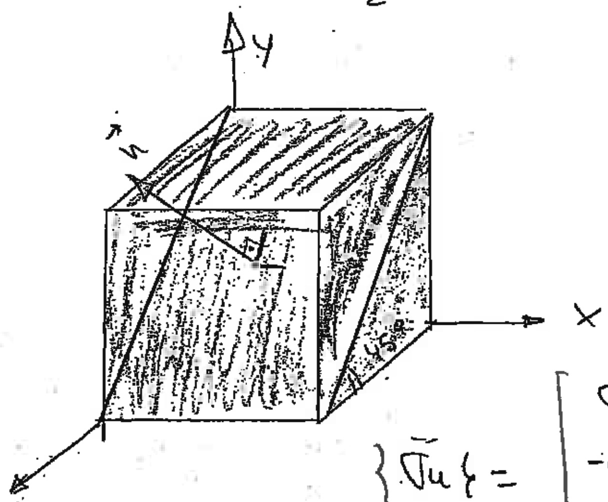
Problema 2.10

Conjunto: $[T_{ij}] = \begin{bmatrix} \sigma & -\sigma & \sigma/2 \\ -\sigma & 2\sigma & 3\sigma \\ \sigma/2 & 3\sigma & \sigma \end{bmatrix}$

Máximo valor de σ para que no se separen las piezas.

$\sigma_{nn\,máx} = 100.000\ N/m^2$

$\tau_{nt\,máx} = 75000\ N/m^2$



$\vec{n} = \{0,\ 1/\sqrt{2}\ \ 1/\sqrt{2}\}$

$$\{\sigma_n\} = \begin{bmatrix} \sigma & -\sigma & \sigma/2 \\ -\sigma & 2\sigma & 3\sigma \\ \sigma/2 & 3\sigma & \sigma \end{bmatrix} \begin{Bmatrix} 0 \\ 1 \\ 1 \end{Bmatrix} 1/\sqrt{2} =$$

$$= \frac{1}{\sqrt{2}} \begin{Bmatrix} -\sigma/2 \\ 5\sigma \\ 4\sigma \end{Bmatrix} \longrightarrow \sigma_{nn} = \{\sigma_n\}^T\{n\} = \frac{1}{\sqrt{2}}\{-\sigma/2\ \ 5\sigma\ \ 4\sigma\}\begin{Bmatrix} 0 \\ 1 \\ 1 \end{Bmatrix} 1/\sqrt{2} =$$

$$= \frac{1}{2}(9\sigma) = \frac{9\sigma}{2}$$

$$\sigma_{nn\,máx} = \frac{9\sigma_{máx}}{2} = 100.000\ N/m^2 \rightarrow \underline{\sigma = 22\,222'22\ N/m^2}$$

$$\rightarrow \tau_{nt}: \quad |\vec{\sigma}_n| = \sqrt{\frac{1}{2}\left[\frac{\sigma^2}{4} + 25\sigma^2 + 16\sigma^2\right]} = \sqrt{\frac{1}{2}\left(\frac{\sigma^2 + 100\sigma^2 + 64\sigma^2}{4}\right)} = \sqrt{\frac{165\sigma^2}{8}}$$

$$\tau_{nt} = \sqrt{|\vec{\sigma}_n|^2 - \sigma_{nn}^2} = \sqrt{\frac{165\sigma^2}{8} - \frac{81\sigma^2}{4}} = \sqrt{\frac{8.4\,\sigma^2}{8}} = \sigma \cdot 3'2403$$

$$\tau_{nt\,máx} = \sigma_{máx}\ 3'2403 = 75000\ N/m^2 \longrightarrow \sigma_{máx} = \underline{23\,145'5\ N/m^2}$$

$\sigma_{máx}$ es el más restrictivo $\quad \underline{\sigma_{máx} = 22\,222'22\ N/m^2}$

Problema 2.11.

$[T] = \begin{bmatrix} 2 & 2 \\ 2 & 5 \end{bmatrix}$ MPa (Estado de tensión plana)

~~1) $\vec{\sigma}_u\ (P, \vec{n})$ Plano Π a la bisectriz 1er cuadrante~~

$\vec{u} = \left(1/\sqrt{2}, 1/\sqrt{2}\right)$

$$\vec{\sigma}_u = \begin{bmatrix} 2 & 2 \\ 2 & 5 \end{bmatrix} \begin{Bmatrix} 1 \\ 1 \end{Bmatrix} 1/\sqrt{2} = \frac{1}{\sqrt{2}} \begin{Bmatrix} 4 \\ 7 \end{Bmatrix}$$

2) $\sigma_{uu} = \frac{1}{\sqrt{2}} \{4 \quad 7\} \begin{Bmatrix} 1 \\ 1 \end{Bmatrix} 1/\sqrt{2} = \frac{1}{2}(11) = 11/2$ MPa

τ_{ut}: $|\vec{\sigma}_u|^2 = \frac{1}{2}(16 + 49) = \frac{65}{2}$

$\tau_{ut} = \sqrt{|\vec{\sigma}_u|^2 - \sigma_{uu}^2} = \sqrt{\frac{65}{2} - \frac{121}{4}} = \sqrt{\frac{130 - 121}{4}} = \sqrt{\frac{9}{4}} = \frac{3}{2}$ MPa

3) Tensiones y direcciones ppls.

$$\begin{vmatrix} 2-\sigma & 2 \\ 2 & 5-\sigma \end{vmatrix} = 0 \qquad (2-\sigma)(5-\sigma) - 4 = 0$$

$\cancel{10} - 2\cancel{\sigma} - 5\cancel{\sigma} + \cancel{\sigma^2} - \cancel{4} = 0$

$\sigma^2 - 7\sigma + 6 = 0 \qquad \sigma = \frac{7 \pm \sqrt{49-24}}{2} = \frac{7 \pm \sqrt{25}}{2}$

$\boxed{\sigma_1 = 6}$

$\boxed{\sigma_2 = 1}$

$\boxed{\sigma_1 = 6}$; $\begin{bmatrix} -4 & 2 \\ 2 & -1 \end{bmatrix} \begin{Bmatrix} n_{1x} \\ n_{1y} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$

$-2n_{1x} - n_{1y} = 0 \qquad 2n_{1x} = n_{1y} \qquad \vec{u}\left(1/\sqrt{5}, 2/\sqrt{5}\right)$

$n_{1x}^2 + n_{1y}^2 = 1 \qquad 5n_{1x}^2 = 1 \qquad n_{1y} = 2/\sqrt{5}$

$n_{1x}^2 + 4n_{1x}^2 = 1 \qquad n_{1x} = \frac{1}{\sqrt{5}}$ (Tomamos el $\oplus$, p.e)

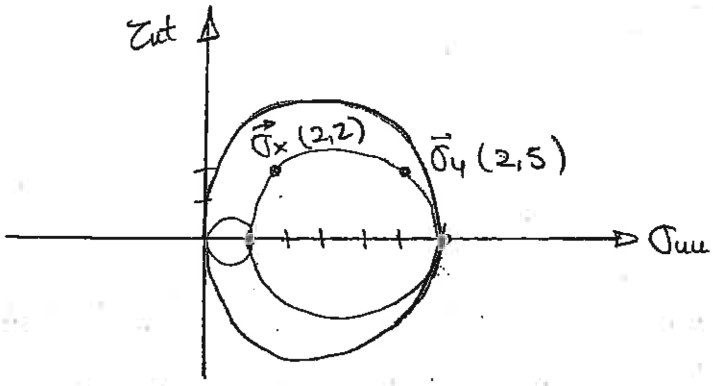
$\boxed{\sigma = 1}$ $\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} \begin{Bmatrix} n_{2x} \\ n_{2y} \end{Bmatrix} = \{0\}$ $n_{2x} + 2n_{2y} = 0$

$n_{2x} = -2n_{2y}$

$n_{2x}^2 + n_{2y}^2 = 1$ $\bar{u} = (-2/\sqrt{5}, 1/\sqrt{5})$

$4n_{2y}^2 + n_{2y}^2 = 1$ $n_{2y} = 1/\sqrt{5}$ $u_{2x} = -2/\sqrt{5}$

4) Diagrama de Mohr. y tensiones $\vec{\sigma}_x$, $\vec{\sigma}_y$



5) $I_1 = \sigma_1 + \sigma_2 = 7$
$I_2 = \sigma_1\sigma_2 = 6$
$I_3 = \sigma_1\sigma_2\sigma_3 = 0$ ✓

6) Tensor desviador. $\sigma_{m} = \dfrac{\sigma_1 + \sigma_2 + \sigma_3}{3} = \dfrac{7}{3}$ $[T_{ij}]_{m/h} = \begin{bmatrix} 7/3 & 0 & 0 \\ 0 & 7/3 & 0 \\ 0 & 0 & 7/3 \end{bmatrix}$

$[T_{ij}]_d = [T_{ij}] - [T_{ij}]_{m/h} = \begin{bmatrix} -1/3 & 2 & 0 \\ 2 & 8/3 & 0 \\ 0 & 0 & -7/3 \end{bmatrix} = \dfrac{1}{3} \begin{bmatrix} -1 & 2 & 0 \\ 2 & 8 & 0 \\ 0 & 0 & -7 \end{bmatrix}$

Tens. pples: $\dfrac{1}{3} \begin{vmatrix} -1-\sigma & \cancel{2}6 & 0 \\ \cancel{2}6 & 8-\sigma & 0 \\ 0 & 0 & -7-\sigma \end{vmatrix} = 0$

op. mal

$\dfrac{1}{3}\left((-1-\sigma)(8-\sigma)(-7-\sigma) - 3\cancel{6}(-7-\sigma) \right) = 0$ $\cancel{\dfrac{1}{3}}\left(\cancel{56} \cancel{+8\sigma} + 4\cancel{5\sigma} + \cancel{7\sigma^2} - \cancel{7\sigma^2} - \cancel{\sigma^3} + \cancel{28} + \cancel{4\sigma}\right) =$ $252 + 36\sigma$:

$\dfrac{1}{3}\left[(\overbrace{-8 + \sigma - 8\sigma}^{-7\sigma} + \sigma^2)(-7-\sigma) + \overbrace{\cancel{28} + \cancel{4\sigma}}^{252 + 36\sigma} \right] = 0$

$-\sigma^3 + 93\sigma + 308 = 0$ Mal ¡ DUDA

$\sigma^3 - 93\sigma - 308 = 0 \longrightarrow \sigma_1 = 11/3$

$\sigma_2 = -4/3$

$\sigma_3 = -7/3$

Problema 2.12.

$T = \begin{bmatrix} 2 & 2 \\ 2 & -1 \end{bmatrix}$ MPa

1) $y = 3x$ $\quad \vec{u} = \dfrac{(1,\ 3,\ 0)}{\sqrt{1^2+3^2}}$ $\qquad \vec{\sigma}_u = \begin{bmatrix} 2 & 2 \\ 2 & -1 \end{bmatrix} \begin{Bmatrix} 1 \\ 3 \end{Bmatrix} 1/\sqrt{10} = \begin{Bmatrix} 8 \\ -1 \end{Bmatrix} 1/\sqrt{10}$

$\vec{u} = \dfrac{(1,3,0)}{\sqrt{10}}$

2) $|\vec{\sigma}_u|^2 = \frac{1}{10}(64+1) = 65/10$ $\qquad \sigma_{uu} = \frac{1}{\sqrt{10}} \{8 \quad -1\} \begin{Bmatrix} 1 \\ 3 \end{Bmatrix} 1/\sqrt{10} = \frac{1}{10}(5) =$

$\sigma_{uu} = \underline{0'5 \text{ MPa}}$

$\tau_{ut} = \sqrt{\frac{65}{10} - \frac{1}{4}} = \underline{2'5 \text{ MPa}}$

3) $\begin{vmatrix} 2-\sigma & 2 \\ 2 & -1-\sigma \end{vmatrix} = 0$ $\qquad (2-\sigma)(-1-\sigma) - 4 = 0$

$-2 - \cancel{2\sigma} + \cancel{\sigma} + \sigma^2 - 4 = 0$

$\sigma^2 - \sigma - 6 = 0 \qquad \sigma = \dfrac{1 \pm \sqrt{1+24}}{2}$ $\qquad \boxed{\begin{matrix}\sigma_1 = 3 \\ \sigma_2 = -2\end{matrix}}$

$\boxed{\sigma_1 = 3}$ $\qquad \begin{pmatrix} -1 & 2 \\ 2 & -4 \end{pmatrix} \begin{Bmatrix} n_{1x} \\ n_{1y} \end{Bmatrix} = \{0\}$

$-n_{1x} + 2n_{1y} = 0 \qquad n_{1x}^2 + n_{1y}^2 = 1$

$n_{1x} = 2n_{1y} \qquad 4n_{1y}^2 + n_{1y}^2 = 1 \qquad n_{1y} = 1/\sqrt{5} \quad n_{1x} = 2/\sqrt{5}$

$\vec{n}_1 = \left(2/\sqrt{5},\ 1/\sqrt{5} \right)$

$\boxed{\sigma_2 = -2}$

$$\begin{bmatrix} 4 & 2 \\ 2 & 1 \end{bmatrix} \begin{Bmatrix} n_{2x} \\ n_{2y} \end{Bmatrix} = \{0\} \qquad n_{2x}^2 + n_{2y}^2 = 1$$

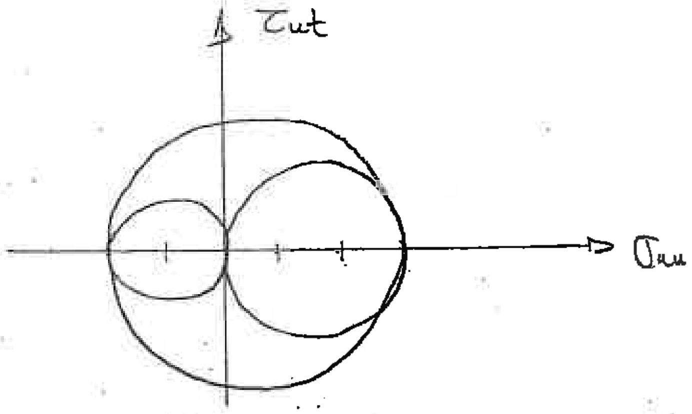
$$n_{2x}^2 + 4n_{2x}^2 = 1$$

$$2n_{2x} + n_{2y} = 0 \qquad n_{2x} = 1/\sqrt{5} \quad n_{2y} = -2/\sqrt{5}$$

$$n_{2y} = -2n_{2x}$$

$$\vec{n}\left(1/\sqrt{5},\ -2/\sqrt{5}\right)$$

4)



$\vec{\sigma}_x = (2, 2, 0)$

$X\ (2,2)$

$\vec{\sigma}_y = (2, -1, 0)$

$Y\ (-1, 2)$

5) $I_1 = \sigma_1 + \sigma_2 + \sigma_3 = 3 - 2 + 0 = 1$

$I_2 = \sigma_1\sigma_2 + \sigma_1\sigma_3 + \sigma_2\sigma_3 = -6$

$I_3 = \sigma_1\sigma_2\sigma_3 = 0$

6) $\sigma_m = \frac{1}{3}$ $\qquad [T_{ij}]_d = \begin{bmatrix} 5/3 & 2 & 0 \\ 2 & -4/3 & 0 \\ 0 & 0 & -1/3 \end{bmatrix}$

Tens. pples:

$$(5/3 - \sigma)(-4/3 - \sigma)(-1/3 - \sigma) - 4(-1/3 - \sigma) = 0$$

$$\begin{vmatrix} 5/3 - \sigma & 2 & 0 \\ 2 & -4/3 - \sigma & 0 \\ 0 & 0 & -1/3 - \sigma \end{vmatrix} = 0$$

$$\left(-\frac{20}{9} + \frac{5}{3}\sigma + \frac{4}{3}\sigma + \sigma^2\right)$$

$$\left(\sigma^2 - \frac{\sigma}{3} - \frac{20}{9}\right)(-1/3 - \sigma) + 4/3 + 4\sigma = 0$$

$$\cancel{-\frac{\sigma^2}{3}} - \cancel{\sigma^3} + \cancel{\frac{\sigma}{9}} + \cancel{\frac{\sigma^2}{3}} + \cancel{\frac{20}{27}} + \cancel{\frac{20}{9}\sigma} + \cancel{4/3} + \cancel{4\sigma} = 0$$

$$-\sigma^3 + \frac{19}{3}\sigma + \frac{56}{27} = 0 \qquad \sigma^3 - \frac{19}{3}\sigma - \frac{56}{27} = 0$$

$\sigma_1 = 8/3$ ✓

$\sigma_2 = -7/3$ ✓

$\sigma_3 = -1/3$ ✓

Problema 2.13.

$\vec{u}$:- trisectriz del primer octante $\quad n_1^2 + n_1^2 + n_1^2 = 1$

$\vec{n} = (1/\sqrt{3}, 1/\sqrt{3}, 1/\sqrt{3})$

$\vec{\sigma}_u (P, \vec{n}) = 150\ MN/m^2\ \vec{u} \quad \underbrace{\vec{u} / 60^\circ \text{ y } 45^\circ \text{ con ejes } X, Y.\ \oplus}$

1º) σ_{un}, τ_{ut}

$\vec{u} = \left(1/2, 1/\sqrt{2}, u_z \right)$

$\left(\frac{1}{2}\right)^2 + \left(1/\sqrt{2}\right)^2 + (u_z)^2 = 1$

$\frac{1}{4} + \frac{1}{2} + u_z^2 = 1$

$\frac{1+2}{4} + u_z^2 = 1$

$u_z^2 = 1 - 3/4 = 1/4$

$u_z = 1/2$

$\vec{\sigma}_u = 150 \left(1/2, 1/\sqrt{2}, 1/2 \right)$

$\sigma_{nu} = \{\vec{\sigma}_u\} \{\vec{n}\} =$

$= 150 \{ 1/2 \quad \sqrt{2}/2 \quad 1/2 \} \begin{Bmatrix} 1 \\ 1 \\ 1 \end{Bmatrix} 1/\sqrt{3} = \frac{150}{\sqrt{3}} \left(\frac{1}{2} + \frac{\sqrt{2}}{2} + \frac{1}{2} \right) = 147'839\ MN/m^2$

$\tau_{ut} = \sqrt{150^2 - \sigma_{nn}^2} = 25'36\ MN/m^2$

2) $\tau_{xy} = 40 \quad \tau_{yz} = -20 \quad \tau_{xz} = 15 \quad \sigma_{xx}\ \sigma_{yy}\ \sigma_{zz}$?

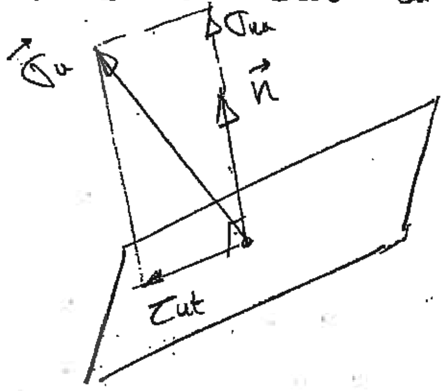
$$150 \begin{Bmatrix} 1/2 \\ \sqrt{2}/2 \\ 1/2 \end{Bmatrix} = \begin{bmatrix} \sigma_{xx} & 40 & 15 \\ 40 & \sigma_{yy} & -20 \\ 15 & -20 & \sigma_{zz} \end{bmatrix} \begin{Bmatrix} 1 \\ 1 \\ 1 \end{Bmatrix} 1/\sqrt{3}$$

$1/\sqrt{3} \left(\sigma_{xx} + 55 \right) = 1/2 \cdot 150 \qquad \sigma_{xx} = 74'903\ MN/m^2$

$\frac{1}{\sqrt{3}}\left(20+\sigma_{yy}\right)=\frac{150\sqrt{2}}{2}$ $\quad \underline{\sigma_{yy}=163'71 \ MN/m^2}$

$\frac{1}{\sqrt{3}}\left(-5+\sigma_{zz}\right)=150/2$ $\quad \underline{\sigma_{zz}=134'90 \ MN/m^2}$

Cosenos directores de τ_{ut} en el plano oblicuo:



¿Es de la parte que no hemos visto? DUDA

Problema 2.14.

$$[T_{ij}]=\begin{bmatrix}109 & -22 & 47\\ -22 & -54 & 63\\ 47 & 63 & 83\end{bmatrix} \ MPa$$

1) a) Plano igualmente inclinado respecto ejes coordenados

$\vec{u}\,(1/\sqrt{3},\,1/\sqrt{3},\,1/\sqrt{3})$ $\qquad \{\vec{\sigma}_u\}=[T_{ij}]\{\vec{n}\}$

b) Plano // eje Z igualmente inclinado respecto a X, Y

$\vec{u}=(1/\sqrt{2},\,1/\sqrt{2},\,0)$ / $\qquad 2u^2=1 \quad u=1/\sqrt{2}$

$\{\vec{\sigma}_u\}=[T_{ij}]\{\vec{n}\}$

2) Tensiones pples $\left|[T_{ij}]-\sigma[I]\right|=0 \rightarrow \sigma_1,\sigma_2\,\sigma_3$

σ_1: $\left[[T_{ij}]-\sigma_1[I]\right]\{n_1\}=\{0\}$, $n_{1x}^2+n_{1y}^2+n_{1z}^2=1$

σ_2, σ_3

3) a) $I_1 = \sigma_{xx} + \sigma_{yy} + \sigma_{zz}$

$$I_2 = \begin{vmatrix} \sigma_{yy} & \tau_{yz} \\ \tau_{yz} & \sigma_{zz} \end{vmatrix} + \begin{vmatrix} \sigma_{xx} & \tau_{xz} \\ \tau_{xz} & \sigma_{zz} \end{vmatrix} + \begin{vmatrix} \sigma_{xx} & \tau_{xy} \\ \tau_{xy} & \sigma_{yy} \end{vmatrix}$$

$$I_3 = \begin{vmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{xy} & \sigma_{yy} & \tau_{yz} \\ \tau_{xz} & \tau_{yz} & \sigma_{zz} \end{vmatrix}$$

b) $I_1 = \sigma_1 + \sigma_2 + \sigma_3$

$I_2 = \sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_1\sigma_3$

$I_3 = \sigma_1\sigma_2\sigma_3$

4) $\sigma_m = \dfrac{\sigma_1 + \sigma_2 + \sigma_3}{3} \rightarrow [T_{ij}]_d = [T_{ij}] - [T_{ij}]_e$

$$[T_{ij}]_e = \begin{bmatrix} \sigma_m & 0 & 0 \\ 0 & \sigma_m & 0 \\ 0 & 0 & \sigma_m \end{bmatrix}$$

Teniendo $[T_{ij}]_d$: $\sigma_{1d}, \sigma_{2d}; \sigma_{3d}$: $\left| [T_{ij}]_d - \sigma[I] \right| = 0$

$I_{1d} = \sigma_{1d} + \sigma_{2d} + \sigma_{3d}$

$I_{2d} = \sigma_{1d}\sigma_{2d} + \sigma_{1d}\sigma_{3d} + \sigma_{2d}\sigma_{3d}$

$I_{3d} = \sigma_{1d}\sigma_{2d}\sigma_{3d}$

Problema 15

$$[T_{ij}] = \begin{bmatrix} 127 & -17 & 0 \\ -17 & 29 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$ Estado de tensión plano.

1) σ_{nn}, τ_{nt} plano q forma 45° con X, Y, y es // al Z.

$$\{\vec{\sigma}_n\} = \begin{bmatrix} 127 & -17 \\ -17 & 29 \end{bmatrix} \begin{Bmatrix} 1 \\ 1 \end{Bmatrix} 1/\sqrt{2} = 1/\sqrt{2} \begin{Bmatrix} 110 \\ 12 \end{Bmatrix} \qquad |\vec{\sigma}_n|^2 = \frac{1}{2}\cdot(110^2 + 12^2)$$

$$= 6122$$

$$\sigma_{nn} = \frac{1}{2}\{110 \quad 12\}\begin{Bmatrix} 1 \\ 1 \end{Bmatrix} = \frac{1}{2}122 = \underline{\underline{61 \text{ MPa}}}$$

$$\tau_{nt} = \underline{\underline{49 \text{ MPa}}}$$

2) direcc. y tens. ppales

$$\begin{vmatrix} 127-\sigma & -17 \\ -17 & 29-\sigma \end{vmatrix} = 0$$

$$(127-\sigma)(29-\sigma) - 289 = 0$$

$$3683 - 127\sigma - 29\sigma + \sigma^2 - 289 = 0$$

$$\sigma^2 - 156\sigma + 3394 = 0$$

$$\sigma = \frac{156 \pm \sqrt{156^2 - 4\cdot 3394}}{2} = \frac{156 \pm 103'73}{2}$$

$$\boxed{\begin{matrix} \sigma_1 = 129'86 \\ \sigma_2 = 26'135 \end{matrix}}$$

$\boxed{\sigma_1 = 129'86}$ $$\begin{bmatrix} -2'86 & -17 \\ -17 & -100'86 \end{bmatrix} \begin{Bmatrix} n_{1x} \\ n_{1y} \end{Bmatrix} = \{0\}$$ $\boxed{\bar{n}_1 = (-0'9861, 0'1656)}$

$$-2'86\, n_{1x} - 17\, n_{1y} = 0 \qquad n_{1y}^2 + 35'33\, n_{1y}^2 = 1$$

$$n_{1x} = -\frac{17 n_{1y}}{2'86} = -5'944\, n_{1y} \qquad n_{1y} = 0'1656$$

$$n_{1x} = -0'9861$$

$$\boxed{\sigma_2 = 26'135} \qquad \begin{pmatrix} 100'87 & -17 \\ -17 & 2'865 \end{pmatrix} \begin{Bmatrix} n_{2x} \\ n_{2y} \end{Bmatrix} = \{0\}$$

$-17\, n_{2x} + 2'865\, n_{2y} = 0$

$17\, n_{2x} = 2'865\, n_{2y}$ $\qquad \begin{cases} n_{2y} = 0'9860 \\ n_{2x} = 0'16615 \end{cases}$

$n_{2x} = 0'1685\, n_{2y}$ $\qquad \vec{n}_2 = (0'16615,\ 0'9860)$

$\sigma_3 = 0$ en dirección de z $(0,0,1)$

3) $\begin{cases} I_1 = \sigma_1 + \sigma_2 + \sigma_3 = 156 \\ I_2 = \sigma_1 \sigma_2 = 3393'89 \\ I_3 = 0 \end{cases}$

Problema 2.16

$$[\sigma] = \begin{bmatrix} 0 & 0 & -C_y \\ 0 & 0 & C_x \\ -C_y & C_x & 0 \end{bmatrix} \qquad P\ (1,2,4) \qquad [\sigma] = \begin{bmatrix} 0 & 0 & -2c \\ 0 & 0 & c \\ -2c & c & 0 \end{bmatrix}$$

1) $\begin{vmatrix} -\sigma & 0 & -2c \\ 0 & -\sigma & c \\ -2c & c & -\sigma \end{vmatrix} = 0 \qquad -\sigma^3 + 4c^2\sigma + c^2\sigma = 0$

$\boxed{\sigma_1 = 0}$ $\qquad -\sigma^3 + 5c^2\sigma = 0$

$\boxed{\sigma_2 = \sqrt{5}\, c}$ $\qquad -\sigma^2 + 5c^2 = 0$

$\boxed{\sigma_3 = -\sqrt{5}\, c}$ $\qquad \sigma^2 = 5c^2$

$\sigma = \pm\sqrt{5c^2} = \pm\sqrt{5}\cdot c$

$\boxed{\sigma_1 = 0}$ $\qquad \begin{bmatrix} 0 & 0 & -2c \\ 0 & 0 & c \\ -2c & c & 0 \end{bmatrix} \begin{Bmatrix} n_{1x} \\ n_{1y} \\ n_{1z} \end{Bmatrix} = \{0\}$

$-2c \cdot n_{1z} = 0$ $\quad \boxed{n_{1z} = 0}$

$c\, n_{1z} = 0$

$-2\cancel{c}\, n_{1x} + \cancel{c}\, n_{1y} = 0$ $\quad \boxed{n_{1y} = 2n_{1x}}$

$n_{1x}^2 + 4n_{1x}^2 = 1$

$\boxed{n_{1x} = 1/\sqrt{5}}$

$\boxed{n_{1y} = 2/\sqrt{5}}$

$\vec{n}_1\ (\underbrace{1/\sqrt{5}}_{0'447},\ \underbrace{2/\sqrt{5}}_{0'89},$

$\boxed{\sigma_2 = \sqrt{5}c}$

$$\begin{pmatrix} -\sqrt{5}c & 0 & -2c \\ 0 & -\sqrt{5}c & c \\ -2c & c & -\sqrt{5}c \end{pmatrix} \begin{Bmatrix} n_{2x} \\ n_{2y} \\ n_{2z} \end{Bmatrix} = \{0\}$$

$$-\sqrt{5}c\, n_{2x} - 2c\, n_{2z} = 0 \qquad n_{2x} = -\frac{2n_{2z}}{\sqrt{5}}$$

$$-\sqrt{5}c\, n_{2y} + c\, n_{2z} = 0 \qquad n_{2y} = \frac{n_{2z}}{\sqrt{5}}$$

$$-2c\, n_{2x} + c\, n_{2y} - \sqrt{5}c\, n_{2z} = 0$$

$$\frac{4}{5} n_{2z}^2 + \frac{n_{2z}^2}{5} + n_{2z}^2 = 1$$

$$2n_{2z}^2 = 1 \qquad \boxed{n_{2z} = 1/\sqrt{2}} \qquad n_{2x} = \frac{-2}{\sqrt{10}} \qquad n_{2y} = \frac{1}{\sqrt{10}}$$

$$\boxed{\vec{n}_2 = \left(-2/\sqrt{10},\ 1/\sqrt{10},\ 1/\sqrt{2}\right)}\ ✓$$

$\boxed{\sigma_3 = -\sqrt{5}c}$

$$\begin{bmatrix} \sqrt{5}c & 0 & -2c \\ 0 & \sqrt{5}c & c \\ -2c & c & \sqrt{5}c \end{bmatrix} \begin{Bmatrix} n_{3x} \\ n_{3y} \\ n_{3z} \end{Bmatrix} = \{0\}$$

$$\sqrt{5}\cancel{c}\, n_{3x} - 2\cancel{c}\, n_{3z} = 0 \longrightarrow n_{3x} = \frac{2n_{3z}}{\sqrt{5}}$$

$$\sqrt{5}\cancel{c}\, n_{3y} + \cancel{c}\, n_{3z} = 0 \longrightarrow n_{3y} = -n_{3z}/\sqrt{5}$$

$$-2c\, n_{3x} + c\, n_{3y} + \sqrt{5}c\, n_{3z} = 0$$

$$\left(\frac{4}{5} + \frac{1}{5} + 1\right) n_{3z}^2 = 1$$

$$n_{3z} = 1/\sqrt{2}$$

$$n_{3x} = 2/\sqrt{10} \qquad n_{3y} = -1/\sqrt{10}$$

$$\boxed{\vec{n}_3 = \left(2/\sqrt{10},\ -1/\sqrt{10},\ 1/\sqrt{2}\right)}$$

El de signo contrario tb ✓

Problema 2.17.

$$[T_{ij}] = \begin{pmatrix} 400 & 100 & 0 \\ 100 & 200 & 0 \\ 0 & 0 & 0 \end{pmatrix} \text{ (MPa)} \quad \text{Plano} \longrightarrow [T_{ij}] = \begin{bmatrix} 4 & 1 \\ 1 & 2 \end{bmatrix} \times 100 \text{ MPa}$$

1) $\vec{n}_1 : (1/\sqrt{2},\ 1/\sqrt{2},\ 0)$ $\quad \vec{\sigma}_u = \begin{bmatrix} 4 & 1 \\ 1 & 2 \end{bmatrix} \begin{Bmatrix} 1 \\ 1 \end{Bmatrix} 1/\sqrt{2} = 1/\sqrt{2} \begin{Bmatrix} 5 \\ 3 \end{Bmatrix}$

$|\vec{\sigma}_u|^2 = \frac{1}{2}(25+9) = \frac{34}{2}$

$\sigma_{nn} = \frac{1}{\sqrt{2}} \{5 \quad 3\} \begin{Bmatrix} 1 \\ 1 \end{Bmatrix} 1/\sqrt{2} = \frac{1}{2}(8) = 4 \longrightarrow$ 400 MPa

$\tau_{nt} = \sqrt{\frac{34}{2} - 16} = \sqrt{\frac{34-32}{2}} = 1 \longrightarrow$ 100 MPa

$\vec{n}_2 = (-1/\sqrt{2},\ 1/\sqrt{2})$ $\quad \vec{\sigma}_u = \begin{bmatrix} 4 & 1 \\ 1 & 2 \end{bmatrix} \begin{Bmatrix} -1 \\ 1 \end{Bmatrix} 1/\sqrt{2} = \frac{1}{\sqrt{2}} \begin{Bmatrix} -3 \\ 1 \end{Bmatrix}$

$|\vec{\sigma}_u|^2 = \frac{1}{2}(9+1) = 5$

$\sigma_{nn} = \frac{1}{\sqrt{2}} \{-3 \quad 1\} \begin{Bmatrix} -1 \\ 1 \end{Bmatrix} 1/\sqrt{2} = \frac{1}{2}(3+1) = \frac{4}{2} = 2 \longrightarrow$ 200 MPa

$\tau_{nt} = \sqrt{5-4} = 1 \longrightarrow$ 100 MPa

$\vec{n}_3 = (0, 0, 1) \longrightarrow \vec{\sigma}_z = 0$

2) Magnitud y dirección de las tensiones principales.

$\begin{vmatrix} 4-\sigma & 1 \\ 1 & 2-\sigma \end{vmatrix} = 0$ $\quad (4-\sigma)(2-\sigma) - 1 = 0$ $\quad \to \sigma_1 = 441'42$ MPa

$8 - 4\sigma - 2\sigma + \sigma^2 - 1 = 0$ $\quad \to \sigma_2 = 158'57$ MPa

$\sigma^2 - 6\sigma + 7 = 0$

$\sigma_1 = 4'4142$ $\quad \sigma = \frac{6 \pm \sqrt{36-28}}{2}$

$\sigma_2 = 1'5857$

$$\boxed{\sigma_1 = 4'4142} \qquad \begin{vmatrix} -0'4142 & 1 \\ 1 & -2'4142 \end{vmatrix} \begin{Bmatrix} n_{1x} \\ n_{1y} \end{Bmatrix} = \{0\}$$

$$-0'4142\, n_{1x} + n_{1y} = 0 \qquad (0'4142 n_{1x})^2 + n_{1x}^2 = 1$$

$$n_{1y} = 0'4142\, n_{1x} \qquad n_{1x} = 0'9238 \qquad n_{1y} = 0'3826$$

$$\boxed{\bar{n}_1 = (0'9238,\ 0'3826)}$$

$$\sigma_2 = 1'5857 \qquad \begin{vmatrix} 2'4143 & 1 \\ 1 & 0'4143 \end{vmatrix} \begin{Bmatrix} n_{2x} \\ n_{2y} \end{Bmatrix} = \{0\}$$

$$n_{2x} + 0'4143\, n_{2y} = 0 \qquad n_{2y} = 0'923851$$

$$n_{2x} = -0'413\, n_{2y} \qquad n_{2x} = -0'3827$$

$$\boxed{\bar{n}_2 = (-0'3827,\ 0'923851)}$$

$\sigma_3 = 0$ en $(0, 0, 1)$ Dirección $z \rightarrow$ No hay tensión

Problema 2.4.

1)
$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z} + \phi_x = 0 \rightarrow 40 + 60 + 10 + \phi_x = 0 \rightarrow \phi_x = 10$$

$$\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \tau_{yz}}{\partial z} + \phi_y = 0 \rightarrow -60 + 100 + \phi_y = 0 \rightarrow \phi_y = -40$$

$$\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{zy}}{\partial y} + \frac{\partial \sigma_{zz}}{\partial z} + \phi_z = 0 \rightarrow 50 + \phi_z = 0 \qquad \phi_z = -50$$

$$\vec{\Phi} = (10, -40, -50) = 10\vec{i} - 40\vec{j} - 50\vec{k}$$

$$\boxed{\vec{\Phi} = (1, -4, -5) \times 10^7\ N/m^3}$$

PROBLEMAS TEMA 3

Problema 3.1.

a) $u = K(y-z)$, $v = K(x-y)$, $w = Kxz$ K ct $\ll 1$.

¿Condición para que sean posibles? → DUDA

↓

Condiciones de compatibilidad o continuidad

$\varepsilon_{xx} = \frac{\partial u}{\partial x} = 0$ $\varepsilon_{xy} = \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) = \frac{1}{2}(K+K) = K$

$\varepsilon_{yy} = \frac{\partial v}{\partial y} = -K$ $\varepsilon_{xz} = \frac{1}{2}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right) = \frac{1}{2}(-K+Kz) = \frac{K}{2}(z-1)$

$\varepsilon_{zz} = \frac{\partial w}{\partial z} = Kx$ $\varepsilon_{yz} = \frac{1}{2}(0+0) = 0$

6 ecuaciones de compatibilidad: ✓ $\boxed{\text{SÍ}}$

$$\frac{\partial^2 \varepsilon_{xx}}{\partial y^2} + \frac{\partial^2 \varepsilon_{yy}}{\partial x^2} = 2\frac{\partial^2 \varepsilon_{xy}}{\partial x \partial y} \qquad 0+0=0 \checkmark$$

$$\frac{\partial^2 \varepsilon_{xx}}{\partial z^2} + \frac{\partial^2 \varepsilon_{zz}}{\partial x^2} = 2\frac{\partial^2 \varepsilon_{xz}}{\partial x \partial z} \qquad 0+0=0 \checkmark$$

$$\frac{\partial^2 \varepsilon_{yy}}{\partial z^2} + \frac{\partial^2 \varepsilon_{zz}}{\partial y^2} = 2\frac{\partial^2 \varepsilon_{yz}}{\partial y \partial z} \qquad 0+0=0 \checkmark$$

$$\frac{\partial^2 \varepsilon_{xx}}{\partial y \partial z} = \frac{\partial}{\partial x}\left(-\frac{\partial \varepsilon_{yz}}{\partial x} + \frac{\partial \varepsilon_{xz}}{\partial y} + \frac{\partial \varepsilon_{xy}}{\partial z}\right) \qquad 0 = \frac{\partial}{\partial x}(-0+0+0) \checkmark$$

$$\frac{\partial^2 \varepsilon_{yy}}{\partial x \partial z} = \frac{\partial}{\partial y}\left(\frac{\partial \varepsilon_{yz}}{\partial x} - \frac{\partial \varepsilon_{xz}}{\partial y} + \frac{\partial \varepsilon_{xy}}{\partial z}\right) \qquad 0 = \frac{\partial}{\partial y}(0-0+0) \checkmark$$

$$\frac{\partial^2 \varepsilon_{zz}}{\partial x \partial y} = \frac{\partial}{\partial z}\left(\frac{\partial \varepsilon_{yz}}{\partial x} + \frac{\partial \varepsilon_{xz}}{\partial y} - \frac{\partial \varepsilon_{xy}}{\partial z}\right) \qquad 0 = \frac{\partial}{\partial z}(0+0-0) \checkmark$$

b) $u = ayz \quad v = bxz \quad w = cxy \qquad a, b, c \ll 1$

$\varepsilon_{xx} = 0 \qquad \varepsilon_{xy} = \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) = \frac{1}{2}(0 + bz) = \frac{b}{2}z$

$\varepsilon_{yy} = 0 \qquad \varepsilon_{xz} = \frac{1}{2}(ay + cy) = y/2\,(a+c)$

$\varepsilon_{zz} = 0 \qquad \varepsilon_{zy} = \frac{1}{2}(cx + bx) = x/2\,(c+b)$

Ecs. de compatibilidad. $\boxed{SI}$

$$\begin{cases} 0 + 0 = 2 \cdot 0 \ \checkmark \\ 0 + 0 = 2 \cdot 0 \ \checkmark \\ 0 + 0 = 2 \cdot 0 \ \checkmark \end{cases}$$

$$0 = \frac{\partial}{\partial x}\left(-\frac{(c+b)}{2} + \frac{(c+a)}{2} + b/2\right) = 0 \ \checkmark$$

$$0 = \frac{\partial}{\partial y}\Big(\underbrace{\frac{c+b}{2} - \frac{(c+a)}{2} + b/2}_{cte}\Big) = 0 \ \checkmark$$

$$0 = \frac{\partial}{\partial z}\left(\frac{c+b}{2} + \frac{(c+a)}{2} - b/2\right) = 0 \ \checkmark$$

c) $u = -k_1 xy \quad v = k_2(x^2 + \gamma y^2 - \gamma z^2) \quad w = k_3 \gamma yz \qquad k_1, k_2, k_3 \ll 1$ (c)

γ cte

$$\begin{cases} \varepsilon_{xx} = -k_1 y \\ \varepsilon_{yy} = 2k_2 y \\ \varepsilon_{zz} = k_3 \gamma y \end{cases}$$

$\varepsilon_{xy} = \frac{1}{2}(-k_1 x + 2k_2 x) = \frac{x}{2}(2k_2 - k_1)$

$\varepsilon_{xz} = \frac{1}{2}(0 + 0) = 0$

$\varepsilon_{yz} = \frac{1}{2}(-2k_2\gamma z + k_3\gamma z) = \frac{z}{2}(k_3\gamma - 2\gamma k_2)$

$$\begin{cases} 0 + 0 = 2 \cdot 0 \ \checkmark \\ 0 + 0 = 2 \cdot 0 \ \checkmark \\ 0 + 0 = 2 \cdot 0 \ \checkmark \end{cases}$$

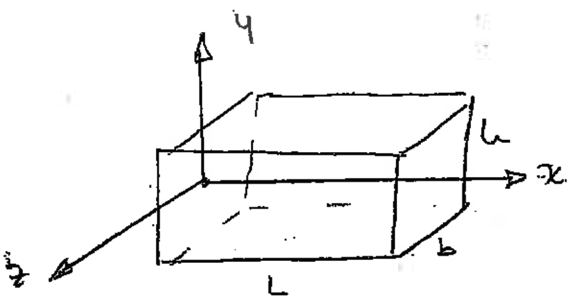
$$0 = \frac{\partial}{\partial x}\left(-0+0+0\right)=0$$ ✓

$$0 = \frac{\partial}{\partial y}\left(0-0+0\right)=0$$ ✓ $\rightarrow$ $\boxed{SI}$

$$0 = \frac{\partial}{\partial z}\left(0+0-0\right)$$ ✓

Problema 3.2

Paralelepipedo $\left|\begin{array}{l} 0 \le x \le L \\ -h \le y \le h \\ -b \le z \le b \end{array}\right.$



$$P(x,y,z) \xrightarrow[\text{Se desplaza}]{def} P^*(x^*, y^*, z^*)$$

$$\left|\begin{array}{l} x^* = (C-y)\cos\frac{x}{C} \\ y^* = (C-y)\operatorname{sen}\frac{x}{C} \\ z^* = z \end{array}\right.$$

Condiciones de compatibilidad

$$\left\{\begin{array}{l} u = x^* - x = (C-y)\cos\frac{x}{c} - x \\ v = y^* - y = (C-y)\operatorname{sen}\frac{x}{c} - y \\ z = z^* - z = 0 \end{array}\right.$$

$\Rightarrow$

$$\varepsilon_{xx} = -\left(\frac{C-y}{C}\right)\operatorname{sen}\frac{x}{c} - 1$$

$$\varepsilon_{yy} = -\operatorname{sen}\frac{x}{c} - 1$$

$$\varepsilon_{zz} = 0$$

$$\varepsilon_{xy} = \frac{1}{2}\left(-\cos\frac{x}{c} - \frac{(c-y)}{c}\cos\frac{x}{c}\right) =$$

$$\varepsilon_{xy} = -\frac{\cos x/c}{2}\left(1 + \frac{c-y}{c}\right)$$

$$\varepsilon_{yz} = \frac{1}{2}\left(0+0\right) = 0$$

$$\varepsilon_{xz} = 0$$ ✓

$$\frac{\partial^2 \varepsilon_{xx}}{\partial y^2} + \frac{\partial^2 \varepsilon_{yy}}{\partial x^2} = \frac{\partial^2 \varepsilon_{xy}}{\partial x \partial y}$$

$$-\frac{1}{c}\cos x/c \rightarrow \frac{1}{c^2}\operatorname{sen} x/c = \frac{1}{2c^2}\operatorname{sen} x/c \; !$$

$$\Rightarrow -\frac{1}{2c}\cos\frac{x}{c} \rightarrow \frac{1}{2c^2}\operatorname{sen} x/c$$

No es necesario

Problema 3.3

1) $\varepsilon_x = C(x^2+y^2)$

$\varepsilon_y = Cy^2$

$\gamma_{xy} = 2Cxy \rightarrow \varepsilon_{xy} = \frac{\gamma_{xy}}{2} \rightarrow \varepsilon_{xy} = Cxy$

$\gamma_{xz} = \gamma_{yz} = \varepsilon_z = 0$ $\quad \varepsilon_{xz} = \varepsilon_{yz} = 0$

Ecs. Comp:

$$\begin{cases} 2C + 0 = 2C \ \checkmark \\ 0 + 0 = 0 \ \checkmark \\ 0 + 0 = 0 \ \checkmark \end{cases}$$

$\boxed{\text{SÍ}}$

$$\begin{cases} 0 = \frac{\partial}{\partial x}(-0+0-0) \ \checkmark \\ 0 = \frac{\partial}{\partial y}(0-0+0) \ \checkmark \\ 0 = \frac{\partial}{\partial z}(0+0-0) \ \checkmark \end{cases}$$

2) $\varepsilon_x = Cz(x^2+y^2)$

$\varepsilon_y = Cy^2z$

$\varepsilon_{xy} = Cxyz$

$\varepsilon_z = \gamma_{xz} = \varepsilon_{xz} = \varepsilon_{yz} = 0$

$\boxed{\text{NO}}$

$$\begin{cases} 2Cz + 0 = 2Cz \ \checkmark \\ 0 + 0 = 0 \ \checkmark \\ 0 + 0 = 0 \ \checkmark \end{cases}$$

$2Cy = \frac{\partial}{\partial x}(-Cyz + 0 + 0)$

$\boxed{2Cy = 0}$ Sólo para punto $y=0$, no para el sólido entero

$0 = \frac{\partial}{\partial y}(Cyz - 0 + 0) \neq 0$ ✗

$0 = \frac{\partial}{\partial z}(Cyz + 0 - 0) \neq 0$ ✗

Problema 3.4.

$\varepsilon_x = a_0 + a_1 x + a_2 y + a_3 x^2 + a_4 xy + a_5 y^2$

$\varepsilon_y = b_0 + b_1 x + b_2 y + b_3 x^2 + b_4 xy + b_5 y^2$

$\varepsilon_{xy} = \frac{1}{2}\left(c_0 + c_1 x + c_2 y + c_3 x^2 + c_4 xy + c_5 y^2 \right)$

$\varepsilon_z = \varepsilon_{xz} = \varepsilon_{zy} = 0$

$a_2 + a_4 x + 2a_5 y \rightarrow \quad 2a_5$

$b_1 + 2b_3 x + b_4 y \rightarrow \quad 2b_3$

$c_1 + 2c_3 x + c_4 y \quad c_4$

$\boxed{2a_5 + 2b_3 = c_4}$

$\left.\begin{array}{l} 0+0=0 \\ 0+0=0 \end{array}\right\}$ ✓

$0 = \frac{\partial}{\partial x}\left(-0 + 0 + 0 \right)$ ✓

✓

Problema 3.5.

$u = u_0 + u_1 x^4 y^4 + u_2 \sin yz + u_3 e^z$

$v = v_0 + v_1 x^3 y^3 + v_2 \cos y + v_3 \operatorname{sh} z$

$w = w_0 + w_1 x^5 + w_2 \operatorname{ch} zx$

$\varepsilon_{xx} = 4u_1 y^4 x^3$ ...

$\varepsilon_{yy} = 3v_1 x^3 y^3 - v_2 \sin y$

ε_{zz}

6 ecs de compatibilidad con ε_{ij} y ε_{ii}

Todas se satisfacen

Problema 36

$u = K(3x^2 + y) \qquad v = K(2y^2 + z) \qquad w = K(4z^2 + x) \qquad K > 0$

Alargamiento de $ds \longrightarrow P(1,1,1)$ en dirección: $\underbrace{n_x = n_y = n_z = 1/\sqrt{3}}$

$\varepsilon_{xx} = 6Kx \qquad \varepsilon_{xy} = \frac{1}{2}(K + 0) = K/2$

$\varepsilon_{yy} = 4Ky \qquad \varepsilon_{xz} = \frac{1}{2}(0 + K) = K/2$

$\varepsilon_{zz} = 8Kz \qquad \varepsilon_{yz} = \frac{1}{2}(K + 0) = K/2$

$$[D_{ij}] = K \begin{bmatrix} 6x & 1/2 & 1/2 \\ 1/2 & 4y & 1/2 \\ 1/2 & 1/2 & 8z \end{bmatrix} \longrightarrow P(1,1,1) \Rightarrow [T_{ij}] = K \begin{bmatrix} 6 & 1/2 & 1/2 \\ 1/2 & 4 & 1/2 \\ 1/2 & 1/2 & 8 \end{bmatrix}$$

$$\{\vec{\varepsilon}_u\} = \frac{K}{\sqrt{3}} \begin{bmatrix} 6 & 1/2 & 1/2 \\ 1/2 & 4 & 1/2 \\ 1/2 & 1/2 & 8 \end{bmatrix} \begin{Bmatrix} 1 \\ 1 \\ 1 \end{Bmatrix} = K/\sqrt{3} \begin{Bmatrix} 7 \\ 5 \\ 9 \end{Bmatrix}$$

Alargamiento $\rightarrow$ $\varepsilon_{uu} = \{\varepsilon_u\}^T \cdot \{n\} = \frac{K}{3} \{7 \quad 5 \quad 9\} \begin{Bmatrix} 1 \\ 1 \\ 1 \end{Bmatrix} = \frac{K}{3}(21)$

$\boxed{\varepsilon_{uu} = 7K}$ Alargamiento

Problema 3.7.

$\varepsilon_{xx} = 2\cdot 10^{-3}$ (1) ε_{uu} $\vec{u}\left(2/\sqrt{5}, 0, 1/\sqrt{5}\right) \rightarrow 4\cdot 10^{-3}$

$\varepsilon_{yy} = 2\cdot 10^{-3}$ Además (2) ε_{uu} $\vec{u}\left(3/\sqrt{10}, -1/\sqrt{10}, 0\right) \rightarrow 3\cdot 10^{-3}$

$\varepsilon_{zz} = -2\cdot 10^{-3}$ (3) ε_{nn} $\vec{n}\left(1/\sqrt{3}, 1/\sqrt{3}, 1/\sqrt{3}\right) \rightarrow 1\cdot 10^{-3}$

(1):
$$\begin{bmatrix} 2 & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{xy} & 2 & \varepsilon_{yz} \\ \varepsilon_{xz} & \varepsilon_{yz} & -2 \end{bmatrix} \begin{Bmatrix} 2/\sqrt{5} \\ 0 \\ 1/\sqrt{5} \end{Bmatrix} = \begin{Bmatrix} 4/\sqrt{5} + \frac{\varepsilon_{xz}}{\sqrt{5}} \\ \frac{2\varepsilon_{xy}}{\sqrt{5}} + \varepsilon_{yz}/\sqrt{5} \\ \frac{2\varepsilon_{xz}}{\sqrt{5}} - 2/\sqrt{5} \end{Bmatrix} = \{\varepsilon_u\}$$

$$\{\varepsilon_u\}^T\{n\} = \frac{8}{5} + \frac{2}{5}\varepsilon_{xz} + \frac{2\varepsilon_{xz}}{5} - \frac{2}{5} = 4$$

$$6 + 4\varepsilon_{xz} = 20 \qquad \varepsilon_{xz} = 3'5 \longrightarrow \boxed{\varepsilon_{xz} = 3'5\,10^{-3}}$$

$$\boxed{\varepsilon_{xz} = 0'0035}$$

(2)
$$\begin{bmatrix} 2 & \varepsilon_{xy} & 3'5 \\ \varepsilon_{xy} & 2 & \varepsilon_{yz} \\ 3'5 & \varepsilon_{yz} & -2 \end{bmatrix} \begin{Bmatrix} 3/\sqrt{10} \\ -1/\sqrt{10} \\ 0 \end{Bmatrix} = \begin{Bmatrix} \frac{6}{\sqrt{10}} - \frac{\varepsilon_{xy}}{\sqrt{10}} \\ \frac{3\varepsilon_{xy}}{\sqrt{10}} - \frac{2}{\sqrt{10}} \\ \frac{10'5}{\sqrt{10}} - \frac{\varepsilon_{yz}}{\sqrt{10}} \end{Bmatrix}$$

$$\{\varepsilon_u\}^T\{n\} = \frac{18}{10} - \frac{3\varepsilon_{xy}}{10} - \frac{3\varepsilon_{xy}}{10} + \frac{2}{10} = 3$$

$$20 - 6\varepsilon_{xy} = 30 \qquad \varepsilon_{xy} = -1'67 \rightarrow \boxed{\varepsilon_{xy} = -0'00167}$$

(3)
$$\begin{bmatrix} 2 & -1'67 & 3'5 \\ -1'67 & 2 & \varepsilon_{yz} \\ 3'5 & \varepsilon_{yz} & -2 \end{bmatrix} \begin{Bmatrix} 1/\sqrt{3} \\ 1/\sqrt{3} \\ 1/\sqrt{3} \end{Bmatrix} = \begin{Bmatrix} 2'2115 \\ 0'1905 + \varepsilon_{yz}/\sqrt{3} \\ 0'86602 + \varepsilon_{yz}/\sqrt{3} \end{Bmatrix}$$

$$\boxed{\varepsilon_{yz} = -0'0013302}$$

$$1'2768 + \frac{2\varepsilon_{yz}}{3} + 0'11 + 0'5 = 1 \qquad \varepsilon_{yz} = -1'3302$$

Problema 3.8.

$u = 2x \quad v = 3y + z \quad w = z - y$

1º) Verificar que es un conjunto de desp. físicamente posible.

↳ Lo es. u, v, w son funciones lineales de x, y, z.
(Todas las derivadas de 2º orden son nulas) → Las ecuaciones de compatibilidad ambos miembros a cero. ✓

2º) $\vec{\varepsilon}_u$ $\vec{u}\,(1/\sqrt{3}, 1/\sqrt{3}, 1/\sqrt{3})$ Deformación.

Y def. tangencial entre esta dirección y $(-1/\sqrt{2}, 1/\sqrt{2}, 0)$.

$\varepsilon_{xx} = 2 \qquad \varepsilon_{xy} = \frac{1}{2}(0+0) = 0$

$\varepsilon_{yy} = 3 \qquad \varepsilon_{xz} = \frac{1}{2}(0+0) = 0 \qquad [D_{ij}] = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\varepsilon_{zz} = 1 \qquad \varepsilon_{yz} = \frac{1}{2}(1-1) = 0$

$$\vec{\varepsilon}_u = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} 1 \\ 1 \\ 1 \end{Bmatrix} 1/\sqrt{3} = 1/\sqrt{3} \begin{Bmatrix} 2 \\ 3 \\ 1 \end{Bmatrix} \checkmark$$

$$\varepsilon_{uu} = \frac{1}{3} \cdot 6 = 2$$

$$\varepsilon_{un} = \{\varepsilon_u\}^T \{n\} = 1/\sqrt{3}\,\{2 \quad 3 \quad 1\} \begin{Bmatrix} -1 \\ 1 \\ 0 \end{Bmatrix} 1/\sqrt{2}$$

$$\varepsilon_{nu} = 1/\sqrt{6}\,(-2+3+0) = 1/\sqrt{6} \longrightarrow \text{Deformación angular}$$

↓ el doble

$$\boxed{\gamma_{nu} = 2/\sqrt{6}} \checkmark$$

Problema 3.9.

$u = 4ax^2 \qquad v = 8az^2 \qquad w = -2ay^2$ coord. en metros

$a = 10^{-4}\ m^{-1}$

1) Matriz de deformación

$\varepsilon_{xx} = 8ax \qquad \varepsilon_{xy} = \frac{1}{2}(0+0) = 0$

$\varepsilon_{yy} = 0 \qquad \varepsilon_{xz} = \frac{1}{2}(0+0) = 0$

$\varepsilon_{zz} = 0 \qquad \varepsilon_{yz} = \frac{1}{2}(16az - 4ay) = 8az - 2ay = 2a(4z - y)$

2) Alargam. y def. pples en $P(1/2, 1, 1)$

$$[D_{ij}] = \begin{bmatrix} 4a & 0 & 0 \\ 0 & 0 & 6a \\ 0 & 6a & 0 \end{bmatrix} \qquad \begin{vmatrix} 4a-\varepsilon & 0 & 0 \\ 0 & -\varepsilon & 6a \\ 0 & 6a & -\varepsilon \end{vmatrix} = 0$$

$$\varepsilon^2(4a-\varepsilon) - 36a^2(4a-\varepsilon) = 0$$

$$4a\varepsilon^2 - \varepsilon^3 - 144a^3 + 36a^2\varepsilon = 0$$

$$\varepsilon^3 - 4a\varepsilon^2 - 36a^2\varepsilon + 144a^3 = 0$$

	1	$-4a$	$-36a^2$	$144a^3$
$6a$		$6a$	$12a^2$	$-144a^3$
	1	$2a$	$-24a^2$	0

$$\boxed{\varepsilon_1 = 6a}$$

$$\varepsilon^2 + 2a\varepsilon - 24a^2 = 0$$

$$\varepsilon = \frac{-2a \pm \sqrt{4a^2 + 96a^2}}{2}$$

$$\varepsilon = \frac{-2a \pm 10a}{2}$$

$$\boxed{\varepsilon_2 = 4a}$$

$$\boxed{\varepsilon_3 = -6a}$$

$\boxed{\varepsilon_1 = 6a}$:

$$\begin{pmatrix} -2a & 0 & 0 \\ 0 & -6a & 6a \\ 0 & 6a & -6a \end{pmatrix} \begin{Bmatrix} n_{1x} \\ n_{1y} \\ n_{1z} \end{Bmatrix} = \{0\}$$

$-2a\, n_{1x} = 0 \longrightarrow \boxed{n_{1x} = 0}$

$2 n_{1z}^2 = 1$

$-\cancel{6a}\, n_{1y} + \cancel{6a}\, n_{1z} = 0$

$\boxed{n_{1y} = n_{1z}}$

$\boxed{n_{1y} = n_{1z} = 1/\sqrt{2}}$

$$\vec{n}\ (0,\ 1/\sqrt{2},\ 1/\sqrt{2})$$

$\boxed{\varepsilon_2 = 4a}$

$$\begin{vmatrix} 0 & 0 & 0 \\ 0 & -4a & 6a \\ 0 & 6a & -4a \end{vmatrix} \begin{Bmatrix} n_{2x} \\ n_{2y} \\ n_{2z} \end{Bmatrix} = \{0\}$$

$-4\cancel{a}\, n_{2y} + 6\cancel{a}\, n_{2z} = 0 \qquad 6 n_{2z} = 4 n_{2y} \qquad n_{2z} = \frac{2}{3} n_{2y} = \frac{4}{6} n_{2y}$

$6\cancel{a}\, n_{2y} - 4\cancel{a}\, n_{2z} = 0$

$\frac{6 n_{2y}}{3} - \frac{8}{3} n_{2y} = 0 \qquad \frac{10}{3} n_{2y} = 0 \rightarrow \boxed{\begin{matrix} n_{2y} = 0 \\ n_{2z} = 0 \end{matrix}}$

$\boxed{n_{2x} = 1}$

$$\vec{n}\ (1,\ 0,\ 0)$$

$\boxed{\varepsilon_3 = -6a}$

$$\begin{vmatrix} 10a & 0 & 0 \\ 0 & 6a & 6a \\ 0 & 6a & 6a \end{vmatrix} \begin{Bmatrix} n_{3x} \\ n_{3y} \\ n_{3z} \end{Bmatrix} = \{0\}$$

$10a\, n_{3x} = 0 \rightarrow \boxed{n_{3x} = 0}$

$2 n_{3y}^2 = 1 \qquad n_{3y} = 1/\sqrt{2} \qquad n_{3z} = -1/\sqrt{2}$

$\cancel{6a}\, n_{3y} + \cancel{6a}\, n_{3z} = 0$

$n_{3y} = -n_{3z}$

$$\vec{n} = (0,\ 1/\sqrt{2},\ -1/\sqrt{2})$$

Problema 8.10.

Estado de ~~tensión~~ plana y uniforme:
deformació

1) $[D_{ij}]$

$$\left.\begin{aligned} \varepsilon_{xx} &= \frac{\Delta l}{l} = \frac{80}{1200} = \frac{8}{120} = 0'0\widehat{66} \\ \varepsilon_{yy} &= \frac{\Delta l}{l} = \frac{60}{900} = 0'0\widehat{66} \end{aligned}\right\}$$

$$\varepsilon_{xy} = \frac{1}{2}(\alpha+\beta) \qquad tg\,\alpha \approx \alpha = \frac{60}{1200} = 0'05$$

$$\varepsilon_{xy} = \frac{1}{2}(\alpha+\beta) = -0'0528 \qquad \beta = \frac{60}{900} = 0'0\widehat{55}$$

$$\varepsilon_{xy} = -\frac{1}{2}\left(\frac{180+200}{3600}\right) = \frac{-19\cancel{0}}{360\cancel{0}} = -19/360$$

el ángulo aumenta

$$[D_{ij}] = \begin{bmatrix} 8/120 & -19/360 \\ -19/360 & 6/90 \end{bmatrix}$$

2) Deformaciones y direcciones principales.

$$\begin{vmatrix} \frac{8}{120}-\varepsilon & -\frac{19}{360} \\ \frac{-19}{360} & \frac{6}{90}-\varepsilon \end{vmatrix} = 0 \qquad \left(\frac{8}{120}-\varepsilon\right)\left(\frac{6}{90}-\varepsilon\right) - \frac{361}{129600} = 0$$

$$4'44\cdot 10^{-3} - \frac{8}{120}\varepsilon - \frac{6}{90}\varepsilon + \varepsilon^2 - 2'785\cdot 10^{-3} =$$

$$\varepsilon^2 - 0'134\,\varepsilon + 1'655\cdot 10^{-3} = 0$$

$$\varepsilon = \frac{0'134 \pm \sqrt{0'134^2 - 4\cdot 1'655\cdot 10^{-3}}}{2} \quad \begin{aligned} &\varepsilon_1 = 0'1202 = 12\cdot 10^{-2} \\ &\varepsilon_2 = 0'01376 = 1'37\cdot 10^{-2} \end{aligned}$$

→ Resolución $\left\{\begin{matrix} n_1 \\ n_2 \end{matrix}\right\}$ $\quad \big[[D_{ij}] - n_1[I]\big]\{n_1\} = \{0\}$

$$n_{1x}^2 + n_{1y}^2 + n_{1z}^2 = 1$$

Problemas 3.11.

O(0,0,0)   A(100, 0, 0)   B(0, 100, 0)   C(0, 0, 100)   (cm)

en metros: O(0,0,0)   A(1,0,0)   B(0,1,0)   C(0,0,1)

$$[D_{ij}] = \begin{pmatrix} 2Kx & K & 0 \\ K & 2Ky & 0 \\ 0 & 0 & 2Kz \end{pmatrix} \leftarrow x, y, z \text{ en cm.} \quad K = 10^{-6}$$

1) Estado de def. físicamente posible: ε_{cc}, ε_{ij} son funciones lineales de x, y, z → En las ecuaciones de compatibil. → deriv 2as → todas nulas → se cumplen ✓

2) ε_{nn} → dirección AC → Me ~~dicen ε_{nn} en los puntos~~ de la arista. en A:

$$D_{ij} \begin{pmatrix} 200K & K & 0 \\ K & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\vec{n} = \frac{(-100, 100, 0)}{\sqrt{100^2 + 100^2}} =$$

$$\vec{n} = \frac{(-100, 100, 0)}{100\sqrt{2}} = (-1/\sqrt{2}, 1/\sqrt{2}, 0)$$

MAL → Tengo q. generaliza para los punt.

$$\vec{\varepsilon}_n = \begin{pmatrix} 200K & K \\ K & 0 \end{pmatrix} \begin{Bmatrix} -1 \\ 1 \end{Bmatrix} 1/\sqrt{2} = \frac{1}{\sqrt{2}} \begin{Bmatrix} -199K \\ -K \end{Bmatrix}$$

$$\varepsilon_{nn} = \frac{1}{2} \{-199K \quad -K\} \begin{Bmatrix} -1 \\ 1 \end{Bmatrix} = \frac{1}{2}(199K - K) = \frac{198K}{2} = 99K$$

$$\varepsilon_{nn} = 99 \cdot 10^{-6}$$

$$\vec{\varepsilon}_n = \begin{pmatrix} 2Kx & K & 0 \\ K & 2Ky & 0 \\ 0 & 0 & 2Kz \end{pmatrix} \begin{Bmatrix} -1 \\ 1 \\ 0 \end{Bmatrix} 1/\sqrt{2} = \frac{1}{\sqrt{2}} \begin{Bmatrix} -2Kx + K \\ -K + 2Ky \\ 0 \end{Bmatrix} = \frac{K}{\sqrt{2}} \begin{Bmatrix} 1-2x \\ 2y-1 \\ 0 \end{Bmatrix} \leftarrow \text{Mal?}$$

¿

3) Variación del ángulo formado por OB y OC

$\vec{OB} = (0, 100, 0)$
$\vec{OC} = (0, 0, 100)$ } Perpendiculares → Variación del ángulo

$\rightarrow \quad \gamma_{BC} = 2\varepsilon_{BC} = 2 \cdot \underline{\varepsilon_{yz} = 0}$ ✓

4) Deformación transversal máxima en el punto P(10,10,10) por Mohr

$$[D_{ij}] = \begin{bmatrix} 20K & K & 0 \\ K & 20K & 0 \\ 0 & 0 & 20K \end{bmatrix} \rightarrow \begin{vmatrix} 20K-\varepsilon & K & 0 \\ K & 20K-\varepsilon & 0 \\ 0 & 0 & 20K-\varepsilon \end{vmatrix} = 0$$

$$(400K^2 + \varepsilon^2 - 40K\varepsilon)(20K-\varepsilon) - K^2(20K-\varepsilon) = 0$$

$$\cancel{8000K^3} - \cancel{400K^2}\varepsilon + \cancel{20K\varepsilon^2} - \cancel{\varepsilon^3} - \cancel{800K^2}\varepsilon + 40K\cancel{\varepsilon^2} - \cancel{20K^3} + K^2\cancel{\varepsilon} = 0$$

$$-\varepsilon^3 + 60K\varepsilon^2 - 1199K^2\varepsilon^2 + 7980K^3 = 0 \qquad ?$$

$\varepsilon_1, \varepsilon_2, \varepsilon_3 \rightarrow$ Mohr → Máximo punto (más alto) → $\underline{\varepsilon_{ab}} = \left(\frac{D_{max}}{2}\right)$

$\gamma \rightarrow 2 \cdot \varepsilon_{ab}$

5)
$$\begin{bmatrix} 20K & K & 0 \\ K & 20K & 0 \\ 0 & 0 & 20K \end{bmatrix} \begin{Bmatrix} 1 \\ 1 \\ 0 \end{Bmatrix} 1/\sqrt{2} = 1/\sqrt{2} \begin{Bmatrix} 21K \\ 21K \\ 0 \end{Bmatrix}$$

$$\varepsilon_{uu} = \frac{1}{2}\{21K \quad 21K \quad 0\} \begin{Bmatrix} 1 \\ 1 \\ 0 \end{Bmatrix} = \frac{1}{2}(21K + 21K) = \underline{\underline{21K}}$$

Problema 3.12.

$u = Axyz \quad v = Bxyz \quad w = Cxyz$

1) $[D_{ij}]$ en $E \rightarrow E^*(1'504,\ 1'002,\ 1'996)$ $\quad E(1,5,\ 1,\ 2)$

$\varepsilon_{xx} = Ayz \qquad \varepsilon_{xy} = \frac{1}{2}(Axz + Byz)$

$\varepsilon_{yy} = Bxz \qquad \varepsilon_{xz} = \frac{1}{2}(Axy + Cyz)$ DUDA

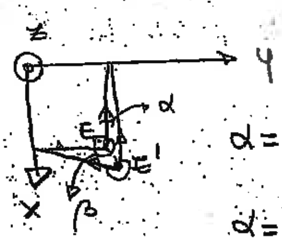
$\varepsilon_{zz} = Cxy \qquad \varepsilon_{yz} = \frac{1}{2}(Bxy + Cxz)$

2) ~~Def. unitaria EA~~

$\varepsilon_{xx} = \frac{1'504 - 1'5}{1'5} = 2'6\widehat{6} \cdot 10^{-3}$ ✓ $= 0'0026\widehat{6}$

$\varepsilon_{yy} = \frac{1'002 - 1}{1} = 0'002$ ✓

$\varepsilon_{zz} = \frac{1'996 - 2}{2} = -0'002$ ✓



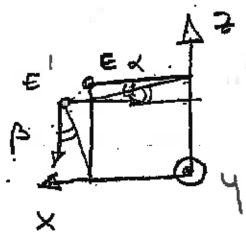
$\alpha = \frac{y_2 - y_1}{x_1}$

$\alpha = \frac{0'002}{1'5}$

$\alpha = 0'001\widehat{33}$

$\beta = \frac{0'004}{1} = 0'004$ ✓

$\varepsilon_{xy} = \frac{1}{2}(\alpha + \beta) = 2'6\widehat{6} \cdot 10^{-3} = 0'0026\widehat{6}$



$\alpha = \frac{-0'004}{1'5}$

$\alpha = -0'0026\widehat{6}$

$\varepsilon_{xz} = \frac{1}{2}(\beta + \alpha) = \frac{1}{2}(0'002 - 0'0026\widehat{6})$

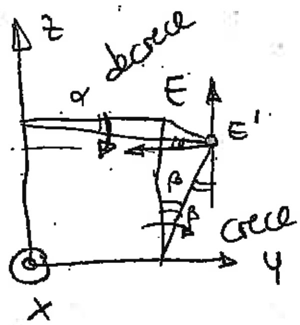
$\varepsilon_{xz} = -0'000\widehat{33}$ ✓

$\beta = \frac{0'004}{2} = 0'002$

$\varepsilon_{yz} = \frac{1}{2}(\beta + \alpha) =$

$\frac{1}{2}(0'001 - 0'004)$

$\varepsilon_{yz} = -0'0015$ ✓



$\alpha, \beta \rightarrow$ Sentidos de variación $(\neq) \rightarrow$ Signo!

$\alpha = \frac{1'996 - 2}{1} = -0'004$

$\beta = \frac{1'002 - 1}{2} = 0'001$

2) Def. long. unitaria en la dirección EA.

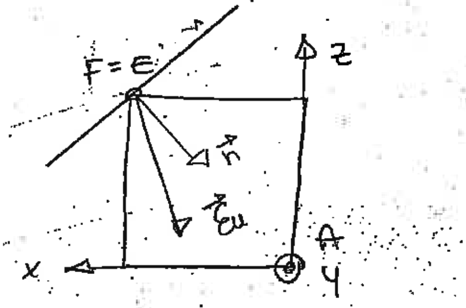
$\vec{EA} = (1'5;0;0) - (1'5;1,2) = (0,-1,-2)$ $|\vec{EA}| = \sqrt{1+4} = \sqrt{5}$

$\vec{n} = (0, -1/\sqrt{5}, -2/\sqrt{5})$

$$\vec{\varepsilon}_u = \begin{bmatrix} 0'00266 & 0'00266 & -0'00033 \\ 0'00266 & 0'002 & -0'0015 \\ -0'00033 & -0'0015 & -0'002 \end{bmatrix} \begin{Bmatrix} 0 \\ -1 \\ -2 \end{Bmatrix} 1/\sqrt{5} = 1/\sqrt{5} \begin{Bmatrix} -0'002 \\ 0'001 \\ 0'0055 \end{Bmatrix}$$

$$\varepsilon_{nn} = \frac{1}{\sqrt{5}} \{-0'002 \quad 0'001 \quad 0'0055\} \begin{Bmatrix} 0 \\ -1 \\ -2 \end{Bmatrix} 1/\sqrt{5} = \frac{1}{5}(-0'012) = \underline{-2'4\cdot 10^{-3}}$$

3) Def. transversal unitaria en E, para las líneas ortogonales EA y EF



Conozco $\vec{\varepsilon}_u$

$\vec{EF} = (0,1,2) - (1'5,1,2)$

$\vec{EF} = (-1'5,0,0)$

$\hookrightarrow \vec{\mu} = (-1,0,0)$

$$\underbrace{\vec{\varepsilon}_u \cdot \vec{\mu}}_{} = 1/\sqrt{5} \{-0'002 \quad 0'001 \quad 0'0055\} \begin{Bmatrix} -1 \\ 0 \\ 0 \end{Bmatrix} =$$

Siendo $\vec{\mu}$ y $\vec{n}$ $\perp$

$$\varepsilon_{EAEF} = \vec{\varepsilon}_u \cdot \vec{\mu} = 1/\sqrt{5}(0'002) = 8'94\cdot 10^{-4}$$

Problema 3.13

$$\begin{cases} u = ax + 4ay \\ v = 2az^2 - ay^2 \\ w = ay^2 - 2ax^2 - (a/3)z^2 \end{cases}$$

1) Matriz $[D_{ij}]$ en $P(2, 1, -3/2)$

$\varepsilon_{xx} = a$ $\qquad \varepsilon_{xy} = \frac{1}{2}(4a) = 2a$

$\varepsilon_{yy} = -2ay$ $\qquad \varepsilon_{xz} = \frac{1}{2}(-4ax) = -2ax$

$\varepsilon_{zz} = -\frac{2az}{3}$ $\qquad \varepsilon_{yz} = \frac{1}{2}(4az + 2ay)$

$2az + ay = -3a + a = -2a$

$$[D_{ij}] = \begin{bmatrix} a & 2a & -4a \\ 2a & -2a & -2a \\ -4a & -2a & a \end{bmatrix}$$

2) $\varepsilon_1, \varepsilon_2, \varepsilon_3$. Direcciones pples en P.

$$\begin{vmatrix} (a-\sigma) & 2a & -4a \\ 2a & (-2a-\sigma) & -2a \\ -4a & -2a & (a-\sigma) \end{vmatrix} = 0$$

$(a^2 + \sigma^2 - 2a\sigma)(-2a-\sigma) + 16a^3$

$+ 16a^3 - 16a^2(-2a-\sigma) - 8a^2(a-\sigma) = 0$

$\cancel{-4a^2(a-\sigma) = 0}$

$-\cancel{2a^3} - a^2\cancel{\sigma} - \cancel{2a\sigma^2} - \sigma^3 + 4a^2\cancel{\sigma} + \cancel{2a\sigma^2}$

$+ 3\cancel{2}a^3 + 32\cancel{a^3} + 16a^2\cancel{\sigma} - \cancel{8a^3} + 8a^2\cancel{\sigma} = 0$

¡Ojo! En vez de σ → ε

$-\sigma^3 + 27a^2\sigma + 54a^3 = 0$

$\sigma^3 - 27a^2\sigma - 54a^3 = 0$

	1	0	$-27a^2$	$-54a^3$
$-3a$		$-3a$	$9a^2$	$54a^3$
	1	$-3a$	$-18a^2$	0

$\boxed{\varepsilon_1 = -3a}$

$(\sigma + 3a)(\sigma^2 - 3a\sigma - 18a^2) = 0$

$18 \cdot 4 = 72$

$$\sigma = \frac{3a \pm \sqrt{9a^2 + 72a^2}}{2} = \frac{3a \pm 9a}{2} =$$

$\boxed{\varepsilon_2 = 6a}$

$\boxed{\varepsilon_3 = -3a}$

<u>Raíz doble</u> $\varepsilon = -3a \rightarrow$ <u>Plano</u>.

$\boxed{\varepsilon_2 = 6a} \rightarrow \vec{n}$ (al plano)

$$\begin{bmatrix} -5a & 2a & -4a \\ 2a & -8a & -2a \\ -4a & -2a & -5a \end{bmatrix} \begin{Bmatrix} n_x \\ n_y \\ n_z \end{Bmatrix} = \{0\}$$

$-5n_x + 2n_y - 4n_z = 0$

$\cancel{2}n_x - \cancel{8}^{4}n_y - \cancel{2}n_z = 0$

$-4n_x - 2n_y - 5n_z = 0$

$\boxed{n_z = n_x - 4n_y}$

$-5\cancel{n_x} + 2n_y - 4(\cancel{n_x} - 4n_y) = 0$

$-9n_x + 18n_y = 0$

$-n_x + 2n_y = 0$

$\boxed{n_x = 2n_y}$

$\rightsquigarrow n_z = 2n_y - 4n_y = \underline{\underline{-2}}$

$n_x^2 + n_y^2 + n_z^2 = 1$

$4n_y^2 + n_y^2 + 4n_y^2 = 1$

$9n_y^2 = 1 \quad n_y^2 = 1/9 \quad n_y = \pm\sqrt{1/9} = \pm\frac{1}{\sqrt{9}} = \pm 1/3 = 1/3$

$n_x = 2/3$

$n_z = -2/3$

$\vec{n} = (2/3, 1/3, -2/3)$

Las otras 2 direcciones cualesquiera contenidas en el plano

$\frac{2}{3}x + \frac{1}{3}y - \frac{2}{3}z = 0$

$\boxed{2x + y - 2z = 0}$

Problema 3.14

$$\begin{cases} u = 2x - y \\ v = y - 2x \\ w = z \end{cases}$$

1) ¿Cto de desplazamiento físicamente posible? Sí, u, v, w son funciones lineales. Las ecs. de compatibilidad son en derivadas 2ªs, todas ellas serían nulas → se cumplirían las ecs.

2) Def. ppales.

$\varepsilon_{xx} = 2$ $\qquad \varepsilon_{xy} = \frac{1}{2}(-1-2) = -3/2$

$\varepsilon_{yy} = 1$ $\qquad \varepsilon_{xz} = \frac{1}{2}(0+0) = 0$

$\varepsilon_{zz} = 1$ $\qquad \varepsilon_{yz} = \frac{1}{2}(0+0) = 0$

$$[D_{ij}] = \begin{bmatrix} 2 & -3/2 & 0 \\ -3/2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{vmatrix} 2-\varepsilon & -3/2 & 0 \\ -3/2 & 1-\varepsilon & 0 \\ 0 & 0 & 1-\varepsilon \end{vmatrix} = 0$$

$$(1+\varepsilon^2-2\varepsilon)(2-\varepsilon) - \frac{9}{4}(1-\varepsilon) = 0$$

$$2-\cancel{\varepsilon}+\cancel{2\varepsilon^2}-\cancel{\varepsilon^3}-4\cancel{\varepsilon}+2\cancel{\varepsilon^2}-9/\cancel{4}+9/\cancel{4}\varepsilon = 0$$

$$-\varepsilon^3+4\varepsilon^2-11/4\,\varepsilon-9/4 = 0$$

$$-5+\frac{9}{4} = \frac{-20+9}{4} = -11/4$$

$\varepsilon_1 = 3'0811$

$\varepsilon_2 = 1$

$\varepsilon_3 = -0'0811$

3) Dirección de la def. ppal máxima

$\varepsilon_1 = 3'0811 \longrightarrow \vec{n}_1$

$$\begin{cases} \left\{[D_{ij}] - \varepsilon_1 [I]\right\}\{\vec{n}_1\} = \{\vec{0}\} \\ n_{1x}^2 + n_{1y}^2 + n_{1z}^2 = 1 \end{cases}$$

$\vec{n}_1 = (0'81, -0'58, 0)$

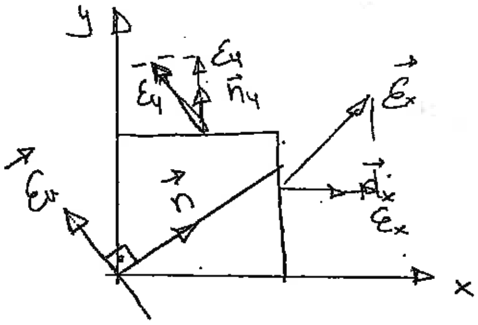
4)

Problema 3.15.

$\varepsilon_x = 0'01$

$\varepsilon_y = 0'02$

$\varepsilon_{30°} = 0 \rightarrow$ Def. de un elemento lineal a $30°$ medidos desde el eje X en sentido antihorario. Calcular $\varepsilon_{60°}$.



$\vec{n} = \left(\sqrt{3}/2,\ 1/2,\ 0\right)$

$\vec{\varepsilon_u} \cdot \vec{n} = 0 \qquad \vec{\varepsilon_u} = (\vec{\varepsilon}_{nx}, \vec{\varepsilon}_{ny})$

$\downarrow$

$\vec{\varepsilon_u} \perp \vec{n}$

$$\{\vec{\varepsilon}_n\} = \begin{bmatrix} 0'01 & \varepsilon_{xy} \\ \varepsilon_{xy} & 0'02 \end{bmatrix} \begin{Bmatrix} \sqrt{3}/2 \\ 1/2 \end{Bmatrix}$$

$\frac{\sqrt{3}}{2}\varepsilon_{nx} + \frac{1}{2}\varepsilon_{ny} = 0$

$\boxed{\sqrt{3}\,\varepsilon_{nx} + \varepsilon_{ny} = 0} \qquad \varepsilon_{ny} = -\sqrt{3}\,\varepsilon_{nx}$

$0'01 \cdot \frac{\sqrt{3}}{2} + \varepsilon_{xy} \cdot \frac{1}{2} = \varepsilon_{nx}$

$\varepsilon_{xy}\frac{\sqrt{3}}{2} + 0'01 = -\sqrt{3}\,\varepsilon_{nx}$

$0'866\ \varepsilon_{xy} + 0'01 = -\sqrt{3}\left(0'01\frac{\sqrt{3}}{2} + \varepsilon_{xy}\frac{1}{2}\right)$

$\cancel{2'0}\overset{\sqrt{3}}{}\ 866\ \varepsilon_{xy} = -0'01 - 0'015$

$\underline{\varepsilon_{xy} = -0'01443}$

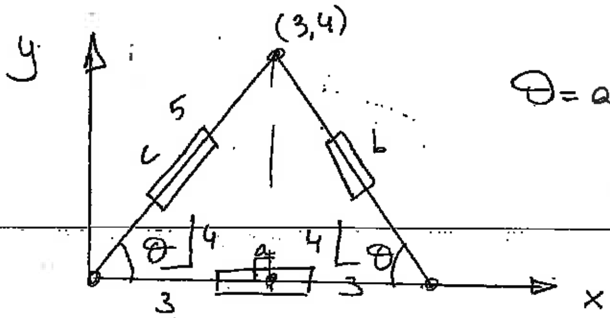
$\varepsilon_{60°}$:

$\vec{u} = \left(1/2,\ \sqrt{3}/2,\ 0\right)$

$$\vec{\varepsilon_u} = \begin{bmatrix} 0'01 & -0'01443 \\ -0'01443 & 0'02 \end{bmatrix} \begin{Bmatrix} 1/2 \\ \sqrt{3}/2 \end{Bmatrix} = \begin{Bmatrix} -7'49 \cdot 10^{-3} \\ 0'0101 \end{Bmatrix}$$

$\underline{\underline{\varepsilon_{60} = \vec{\varepsilon_u} \cdot \vec{u} = 0'005}}$

Problema 3.16.



$\theta = \mathrm{arctg}(4/3)$

Conozco: ε_a, ε_b, ε_c

ε_{xx}

ε_{xy} ?

ε_{yy}

$\vec{n}_c = \dfrac{(3,4)}{\sqrt{9+16}} = \dfrac{(3,4)}{5} = \left(3/5, 4/5\right)$ $\qquad \varepsilon_c = \vec{\varepsilon}_{nc} \cdot \vec{u}_c$

$\vec{n}_a = \dfrac{(6,0)}{6} = (1,0)$ $\qquad \varepsilon_a = \vec{\varepsilon}_{na} \cdot \vec{n}_a$

$\vec{n}_b = \left(-3/5, 4/5\right)$ $\qquad \varepsilon_b = \vec{\varepsilon}_{nb} \cdot \vec{n}_b$

Incógnitas: $\begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} \\ \varepsilon_{xy} & \varepsilon_{yy} \end{bmatrix}$

ε_c:

$$\{\vec{\varepsilon}_{nc}\} = \begin{pmatrix} \varepsilon_{xx} & \varepsilon_{xy} \\ \varepsilon_{xy} & \varepsilon_{yy} \end{pmatrix} \begin{Bmatrix} 3/5 \\ 4/5 \end{Bmatrix} = \begin{Bmatrix} 3/5\,\varepsilon_{xx} + 4/5\,\varepsilon_{xy} \\ 3/5\,\varepsilon_{xy} + 4/5\,\varepsilon_{yy} \end{Bmatrix}$$

$$\varepsilon_c = \left\{ 3/5\,\varepsilon_{xx} + 4/5\,\varepsilon_{xy} \quad 3/5\,\varepsilon_{xy} + 4/5\,\varepsilon_{yy} \right\} \begin{Bmatrix} 3/5 \\ 4/5 \end{Bmatrix}$$

$$\varepsilon_c = \frac{9}{25}\varepsilon_{xx} + \frac{12}{25}\varepsilon_{xy} + \frac{12}{25}\varepsilon_{xy} + \frac{16}{25}\varepsilon_{yy}$$

$$\varepsilon_c = 9/25\,\varepsilon_{xx} + 24/25\,\varepsilon_{xy} + \frac{16}{25}\varepsilon_{yy}$$

$$\boxed{9\varepsilon_{xx} + 24\varepsilon_{xy} + 16\varepsilon_{yy} = 25\varepsilon_c}$$

ε_a: $\begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} \\ \varepsilon_{xy} & \varepsilon_{yy} \end{bmatrix} \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} = \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{xy} \end{Bmatrix}$

$$\varepsilon_a = \{\varepsilon_{xx} \quad \varepsilon_{xy}\} \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} = \varepsilon_{xx} \qquad \boxed{\varepsilon_{xx} = \varepsilon_a}$$ ✓

ε_b: $\begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} \\ \varepsilon_{xy} & \varepsilon_{yy} \end{bmatrix} \begin{Bmatrix} -3/5 \\ 4/5 \end{Bmatrix} = \begin{Bmatrix} -3/5\,\varepsilon_{xx} + 4/5\,\varepsilon_{xy} \\ -3/5\,\varepsilon_{xy} + 4/5\,\varepsilon_{yy} \end{Bmatrix}$

$$\varepsilon_b = \{-3/5\,\varepsilon_{xx} + 4/5\,\varepsilon_{xy} \quad -3/5\,\varepsilon_{xy} + 4/5\,\varepsilon_{yy}\} \begin{Bmatrix} -3/5 \\ 4/5 \end{Bmatrix} =$$

$$\varepsilon_b = \frac{9}{25}\varepsilon_{xx} - \frac{12}{25}\varepsilon_{xy} - \frac{12}{25}\varepsilon_{xy} + \frac{16}{25}\varepsilon_{yy}$$

$$\boxed{9\varepsilon_{xx} - 24\varepsilon_{xy} + 16\varepsilon_{yy} = 25\varepsilon_b}$$

$$\begin{cases} \cancel{24\varepsilon_{xy}} + 16\varepsilon_{yy} = 25\varepsilon_c - 9\varepsilon_a \\ \cancel{-24\varepsilon_{xy}} + 16\varepsilon_{yy} = 25\varepsilon_b - 9\varepsilon_a \end{cases} \qquad \varepsilon_{yy} = \frac{25}{32}(\varepsilon_c + \varepsilon_b) - \frac{18}{32}\varepsilon_a$$ ✓

$$32\varepsilon_{yy} = 25\varepsilon_c + 25\varepsilon_b - 18\varepsilon_a \qquad \boxed{\varepsilon_{yy} = \frac{25\varepsilon_c + 25\varepsilon_b - 18\varepsilon_a}{32}}$$

$$24\varepsilon_{xy} = -\frac{25(\varepsilon_c + \varepsilon_b) - 18\varepsilon_a}{2} + 25\varepsilon_c - 9\varepsilon_a \qquad \left\{\frac{18}{25} = 0'72\right\}$$ ✓

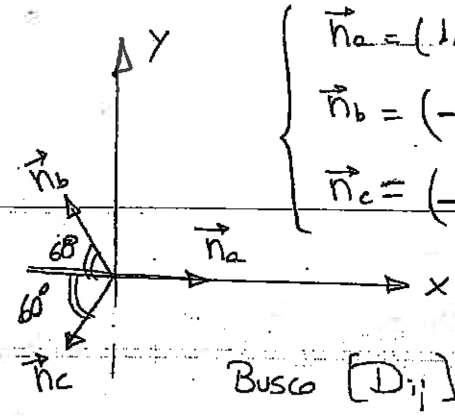
$$24\varepsilon_{xy} = \frac{25(\varepsilon_c - \varepsilon_b)}{2} \qquad \boxed{\varepsilon_{xy} = \frac{25}{48}(\varepsilon_c - \varepsilon_b)}$$ ✓

$$\varepsilon_{yy} = \frac{25\left(\varepsilon_c + \varepsilon_b - \frac{18}{25}\varepsilon_a\right)}{32} \qquad \boxed{\varepsilon_{xy} = \frac{(\varepsilon_c - \varepsilon_b)}{1'92}}$$ ✓

$$\boxed{\varepsilon_{yy} = (\varepsilon_c + \varepsilon_b - 0'72\varepsilon_a)}$$

Problema 3.17.

Dirección	Ángulo ϕ	ε
a	0°	0'002
b	120°	0'002
c	240°	-0'001



$$\begin{cases} \vec{n}_a = (1,0,0) \\ \vec{n}_b = \left(-\frac{1}{2}, \frac{\sqrt{3}}{2}, 0\right) \\ \vec{n}_c = \left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}, 0\right) \end{cases}$$

Busco $[D_{ij}]$

$$\vec{\varepsilon}_{na} = \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} \\ \varepsilon_{xy} & \varepsilon_{yy} \end{bmatrix} \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} = \{\varepsilon_{xx} \ \ 0\} \qquad \varepsilon_a = \{\varepsilon_{xx} \ \ 0\} \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} = \underline{\varepsilon_{xx} = 0'002}$$

$$\vec{\varepsilon}_{nb} = \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} \\ \varepsilon_{xy} & \varepsilon_{yy} \end{bmatrix} \begin{Bmatrix} -1/2 \\ \sqrt{3}/2 \end{Bmatrix} = \begin{Bmatrix} -\frac{\varepsilon_{xx}}{2} + \frac{\sqrt{3}}{2}\varepsilon_{xy} \\ -\frac{\varepsilon_{xy}}{2} + \frac{\sqrt{3}}{2}\varepsilon_{yy} \end{Bmatrix}$$

$$\left\{ -\frac{\varepsilon_{xx}}{2} + \frac{\sqrt{3}}{2}\varepsilon_{xy} \quad -\frac{\varepsilon_{xy}}{2} + \frac{\sqrt{3}}{2}\varepsilon_{yy} \right\} \begin{Bmatrix} -1/2 \\ \sqrt{3}/2 \end{Bmatrix} = 0'002$$

$$\frac{\varepsilon_{xx}}{4} - \frac{\sqrt{3}}{4}\varepsilon_{xy} - \frac{\sqrt{3}}{4}\varepsilon_{xy} + \frac{3}{4}\varepsilon_{yy} = 0'002$$

$$\boxed{-\frac{\sqrt{3}}{2}\varepsilon_{xy} + \frac{3}{4}\varepsilon_{yy} = 0'0015}$$

$$\vec{\varepsilon}_{nc} = \begin{Bmatrix} -\frac{\varepsilon_{xx}}{2} - \frac{\sqrt{3}}{2}\varepsilon_{xy} \\ -\frac{\varepsilon_{xy}}{2} - \frac{\sqrt{3}}{2}\varepsilon_{yy} \end{Bmatrix} \qquad \left\{ -\frac{\varepsilon_{xx}}{2} - \frac{\sqrt{3}}{2}\varepsilon_{xy} \quad -\frac{\varepsilon_{xy}}{2} - \frac{\sqrt{3}}{2}\varepsilon_{yy} \right\} \begin{Bmatrix} -1/2 \\ -\sqrt{3}/2 \end{Bmatrix} = -0'001$$

$$\frac{\varepsilon_{xx}}{4} + \frac{\sqrt{3}}{4}\varepsilon_{xy} + \frac{\sqrt{3}\varepsilon_{xy}}{4} + \frac{3}{4}\varepsilon_{yy} = -0'001$$

$$\boxed{\frac{\sqrt{3}\varepsilon_{xy}}{2} + \frac{3}{4}\varepsilon_{yy} = -0'0015}$$

$$\frac{3}{2}\varepsilon_{yy} = 0 \qquad \boxed{\varepsilon_{yy} = 0}$$

$$\boxed{\varepsilon_{xy} = -0'00173}$$

$$[D_{ij}] = \begin{bmatrix} 0'002 & -0'00173 \\ -0'00173 & 0 \end{bmatrix} \qquad \begin{vmatrix} 0'002-\varepsilon & -0'00173 \\ -0'00173 & -\varepsilon \end{vmatrix} = 0$$

$$-0'002\,\varepsilon + \varepsilon^2 - 3\cdot 10^{-6} = 0 \qquad \boxed{0'004 = 0'0039996}$$

$$\varepsilon = \frac{0'002 \pm \sqrt{0'002^2 + 4\cdot 3\cdot 10^{-6}}}{2} \quad \begin{cases} \boxed{\varepsilon_1 = 0'003} \\ \boxed{\varepsilon_2 = -0'001} \end{cases}$$

$\boxed{\varepsilon_1 = 0'003}$

$$\begin{bmatrix} -0'001 & -0'00173 \\ -0'00173 & -0'003 \end{bmatrix} \begin{Bmatrix} n_{1x} \\ n_{1y} \end{Bmatrix} = \{0\}$$

$$-0'001\,n_{1x} - 0'00173\,n_{1y} = 0 \qquad n_{1x} = -1'73\,n_{1y}$$

$$2'9929\,n_{1y}^2 + n_{1y}^2 = 1$$

$$3'9929\,n_{1y}^2 = 1 \qquad \boxed{n_{1y} = 0'5} \quad \boxed{n_{1x} = -0'8657}$$

$\bar{n}_1 = \boxed{\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)}$ ✓ (Me hablan de superficie)

Estado de deformación plana.

$\boxed{\varepsilon_2 = -0'001}$

$$\begin{bmatrix} 0'003 & -0'00173 \\ -0'00173 & 0'001 \end{bmatrix} \begin{Bmatrix} n_{2x} \\ n_{2y} \end{Bmatrix} = \{0\}$$

$$-0'00173\,n_{2x} + 0'001\,n_{2y} = 0$$

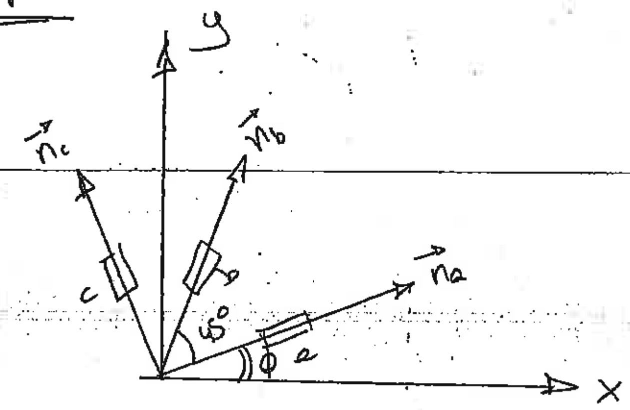
$$n_{2x} = 0'578\,n_{1y} \qquad n_{2x} = 0'5$$

$$0'33\,n_{2y}^2 + n_{2y}^2 = 1 \qquad n_{2y} = 0'8657 \qquad n_{2y} = \frac{\sqrt{3}}{2}$$

$$\boxed{\vec{n}_2 = \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)}$$

Problema 8.18.

Superficie



$\varepsilon_a = \vec{\varepsilon}_{na} \cdot \vec{n}_a$ $\quad \vec{n}_a = (\cos\phi, \sin\phi)$

$\varepsilon_b = \vec{\varepsilon}_{nb} \cdot \vec{n}_b$ $\quad \vec{n}_b = (\cos(45+\phi), \sin(45+\phi))$

$\varepsilon_c = \vec{\varepsilon}_{nc} \cdot \vec{n}_c$ $\quad \vec{n}_c = (\cos(\phi+90), \sin(\phi+90))$

$\varepsilon_a)$ $\begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} \\ \varepsilon_{xy} & \varepsilon_{yy} \end{bmatrix} \begin{Bmatrix} \cos\phi \\ \sin\phi \end{Bmatrix} \Rightarrow \{\vec{\varepsilon}_{na}\}^T\{n_a\} = \{\varepsilon_{xx}\cos\phi + \varepsilon_{xy}\sin\phi \quad \varepsilon_{xy}\cos\phi + \varepsilon_{yy}\sin\phi\} \begin{Bmatrix} \cos \\ \sin \end{Bmatrix}$

$$\varepsilon_{xx}\cos^2\phi + \varepsilon_{xy}\sin\phi\cos\phi + \varepsilon_{xy}\sin\phi\cos\phi + \varepsilon_{yy}\sin\phi^2 = 0'002$$

$$\varepsilon_{xx}\cos^2\phi + \varepsilon_{xy}\sin(2\phi) + \varepsilon_{yy}\sin\phi^2 = 0'002 \qquad (1)$$

$\varepsilon_b)$ $\{\varepsilon_{xx}\cos(45+\phi) + \varepsilon_{xy}\sin(45+\phi) \quad \varepsilon_{xy}\cos(\phi+45) + \varepsilon_{yy}\sin(\phi+45)\} \begin{Bmatrix} \cos(45+\phi) \\ \sin(45+\phi) \end{Bmatrix}$

$$\varepsilon_{xx}\cos^2(45+\phi) + \varepsilon_{xy}\sin(90+2\phi) + \varepsilon_{yy}\sin^2(\phi+45) = 0'00135 \qquad (2)$$

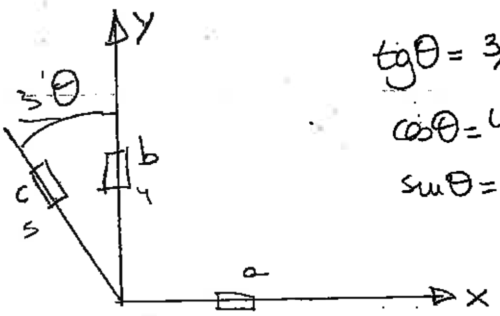
$\varepsilon_c)$ $\varepsilon_{xx}\cos^2(90+\phi) + \varepsilon_{xy}\,\mathrm{sen}(180+2\phi) + \varepsilon_{yy}\sin^2(\phi+90) = 0'00095 \qquad (3)$

Despejamos ε_{xx}, ε_{xy}, ε_{yy} en función de ϕ

↳ $[D_{ij}]$ en función de ϕ

↳ $\boxed{\varepsilon_1, \varepsilon_2, \vec{n}_1, \vec{n}_2 \text{ en función de } \phi}$ Problema → despejar

Problema 3.19.



$\left.\begin{array}{l} tg\theta = 3/4 \\ \cos\theta = 4/5 \\ \sin\theta = 3/5 \end{array}\right\}$ $\left|\begin{array}{l} \varepsilon_a = 3\cdot 10^{-3} = \varepsilon_{xx} \\ \varepsilon_b = 2\cdot 10^{-3} = \varepsilon_{yy} \\ \varepsilon_c = -4\cdot 10^{-3} \end{array}\right.$

1) Def. angular entre a, b $\rightarrow \gamma_{ab} = 2\varepsilon_{ab} = 2\varepsilon_{xy}$ $\qquad \varepsilon_{xy} = \varepsilon_{xy}\cdot 10^{-3}$

$$\begin{bmatrix} 3 & \varepsilon_{xy} \\ \varepsilon_{xy} & 2 \end{bmatrix} 10^{-3} \begin{Bmatrix} -3/5 \\ 4/5 \end{Bmatrix} = \frac{1}{5}\cdot 10^{-3} \begin{Bmatrix} -9+4\varepsilon_{xy} \\ -3\varepsilon_{xy}+8 \end{Bmatrix}$$

$$\{-9+4\varepsilon_{xy} \quad -3\varepsilon_{xy}+8\} \frac{10^{-3}}{25} \begin{Bmatrix} -3 \\ 4 \end{Bmatrix} = (27-12\varepsilon_{xy}-12\varepsilon_{xy}+32)\frac{10^{-3}}{25} = -4$$

$$59-24\varepsilon_{xy} = -100 \qquad \underline{\varepsilon_{xy} = 6'625\cdot 10^{-3}}$$

$$\gamma_{ab} = 2\varepsilon_{xy} = 13'25\cdot 10^{-3} = \underline{0'01325}$$

2) Def. pples.:

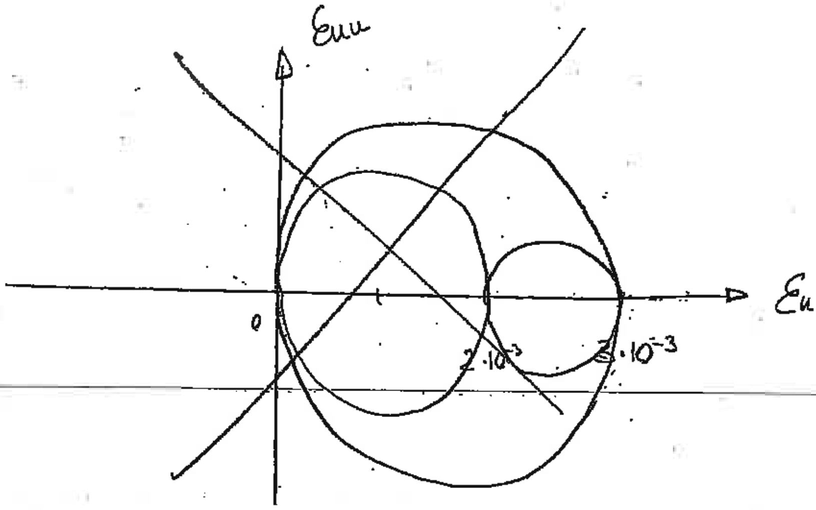
$$\begin{vmatrix} 3-\varepsilon & 6'625 \\ 6'625 & 2-\varepsilon \end{vmatrix} = 0 \qquad \begin{array}{l} (3-\varepsilon)(2-\varepsilon) - 43'89 = 0 \\ 6 - \cancel{3\varepsilon} - \cancel{2\varepsilon} + \cancel{\varepsilon^2} - 43'89 = 0 \\ \varepsilon^2 - 6\varepsilon - 37'89 = 0 \end{array}$$

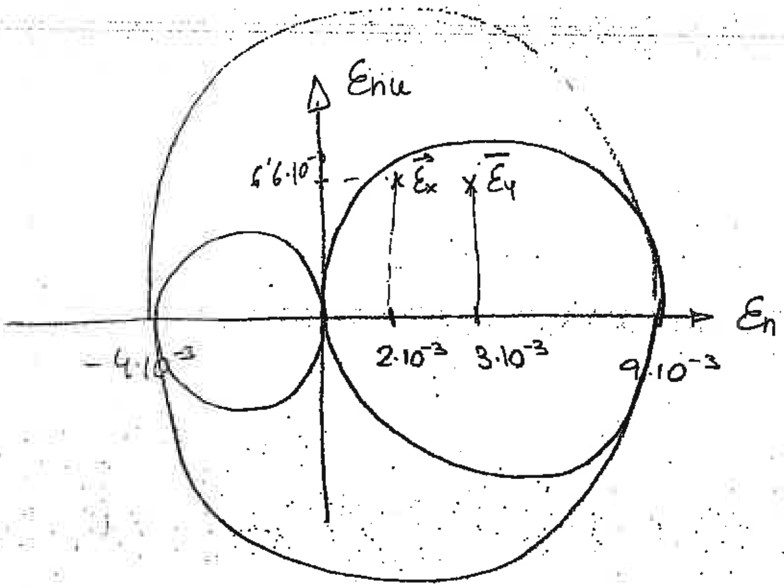
$$\varepsilon = \frac{6 \pm \sqrt{36+4\cdot 37'89}}{2} \left\langle \begin{array}{l} \varepsilon_1 = 9'847 \\ \varepsilon_2 = -3'847 \end{array} \right\} \rightarrow \left.\begin{array}{l} \varepsilon_1 = 9'847\cdot 10^{-3} \\ \varepsilon_2 = -3'847\cdot 10^{-3} \end{array}\right\} \rightarrow$$

$$\vec{n}_1: \left\{ [D_{ij}] - \varepsilon_1 [I] \right\} \{n_1\} = \{0\} \qquad n_{1x}^2 + n_{1y}^2 = 1 \rightarrow$$

$\varepsilon_1 \quad \vec{n}_1\ (0'733,\ 0'680) \qquad \varepsilon_2 \quad \vec{n}_2\ (-0'680,\ 0'733)$

3)





4) $\vec{n} = (1/\sqrt{2},\ 1/\sqrt{2},\ 0)$ $\quad \vec{\varepsilon}_{\vec{u}} = \begin{bmatrix} 3 & 6'625 \\ 6'625 & 2 \end{bmatrix} \begin{Bmatrix} 1 \\ 1 \end{Bmatrix} \times \frac{10^{-3}}{\sqrt{2}} = \begin{Bmatrix} 9'625 \\ 8'625 \end{Bmatrix} \frac{10^{-3}}{\sqrt{2}}$

$\vec{\varepsilon}_u \cdot \vec{u} = \frac{10^{-3}}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} \{ 9'625 \quad 8'625 \} \begin{Bmatrix} 1 \\ 1 \end{Bmatrix} = \underline{9'126 \cdot 10^{-3}}$ ✓

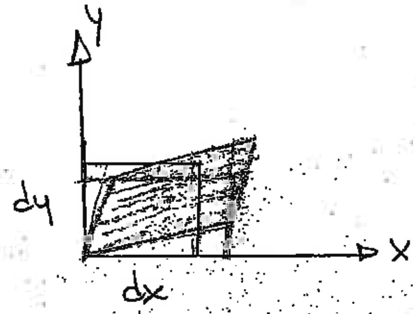
Problema 3.20.

$$u = C(2x+y^2)$$
$$v = C(x^2-3y^2)$$
$$w=0$$

Def. plana. $\quad C = 10^{-2}$

1) $M(2,1,0)$

$$\varepsilon_{xx} = 2C \qquad \varepsilon_{xy} = \frac{1}{\cancel{2}}(\cancel{2}Cy + \cancel{2}Cx) \qquad \varepsilon_{yy} = -6Cy \quad \Rightarrow \quad \varepsilon_{xx} = 2\cdot 10^{-2} \qquad \varepsilon_{xy} = 3\cdot 10^{-2} \qquad \varepsilon_{yy} = -6\cdot 10^{-2}$$



2) M después?

$$u = X^* - X \qquad v = Y^* - Y$$
$$X^* = 2 + 2\cdot 10^{-2} = 2'02$$
$$Y^* = 1 - 6\cdot 10^{-2} = 0'94$$

3) $\vec{n} = (1/\sqrt{3}, 1/\sqrt{3}, 1/\sqrt{3})$

$$\begin{bmatrix} 2\cdot 10^{-2} & 3\cdot 10^{-2} & 0 \\ 3\cdot 10^{-2} & -6\cdot 10^{-2} & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1/\sqrt{3} \\ 1/\sqrt{3} \end{bmatrix} = \begin{Bmatrix} 2'88\,10^{-2} \\ -1'732\cdot 10^{-2} \\ 0 \end{Bmatrix}$$

$$\underline{\varepsilon_u = 0'662}$$

Mal? Resultado?

$$10^{-2}\begin{pmatrix} 2 & 3 & 0 \\ 3 & -6 & 0 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}\frac{1}{\sqrt{3}} = \begin{pmatrix} 5 \\ -3 \\ 0 \end{pmatrix}\frac{1}{\sqrt{3}}\cdot 10^{-2}$$

$$\boxed{\{\varepsilon_n\} = \frac{1}{\sqrt{3}}(0'05, -0'03, 0)}$$

Problema 3.21.

$$\begin{pmatrix} 1'51 & -0'475 & 0'155 \\ -0'475 & 0'63 & 0'21 \\ 0'155 & 0'21 & -0'54 \end{pmatrix} 10^{-3} = [D_{ij}]$$

1) Def. long. unitarias.

a) $\vec{n}\ (1/\sqrt{3},\ 1/\sqrt{3},\ 1/\sqrt{3})$ ---

b) $\vec{n}\ (0'5,\ 0'2888,\ 0'8264)$ ---

2) Direccion pples. (deformacion) $|[D_{ij}] - \varepsilon[I]| = 0$

--- Ec. característica: $\varepsilon_1 = 1'719 \cdot 10^{-3}$

$\varepsilon_2 = 0'486 \cdot 10^{-3}$

$\varepsilon_3 = -0'604 \cdot 10^{-3}$

3) $\begin{cases} I_1 = 1'601 \cdot 10^{-3} \\ I_2 = -4'96 \cdot 10^{-7} \\ I_3 = -5'04 \cdot 10^{-10} \end{cases}$

4) Dilatación cúbica unitaria.

$$e = (1+\varepsilon_1)(1+\varepsilon_2)(1+\varepsilon_3) - 1 \Rightarrow e = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 = 1'601 \cdot 10^{-3}$$

Problema 3.22.

$$[D_{ij}] = \begin{bmatrix} 1'73\cdot10^{-3} & -0'175\cdot10^{-3} \\ -0'175\cdot10^{-3} & 0'59\cdot10^{-3} \end{bmatrix}$$

1) $\vec{n}\left(\frac{\sqrt{3}}{2}, \frac{1}{2}, 0\right)$

$$\{\varepsilon_u\} = \begin{Bmatrix} 1'41 \\ 0'143 \end{Bmatrix} 10^{-3} \qquad \begin{cases} \varepsilon_u = 1'29\cdot10^{-3} \\ \varepsilon_\tau = 0'586\cdot10^{-3} \xrightarrow[\gamma]{} \times 2 = \underline{1'17\cdot10^{-3}} \end{cases}$$

2) Def. pples

$$10^{-3}\begin{vmatrix} 1'73-\varepsilon & -0'175 \\ -0'175 & 0'59-\varepsilon \end{vmatrix} = 0$$

$$\hookrightarrow \begin{cases} \varepsilon_1 = 1'756\cdot10^{-3} \\ \varepsilon_2 = 0'564\cdot10^{-3} \end{cases}$$

3) Invariantes

$$\begin{cases} I_1 = 2'32\cdot10^{-3} \\ I_2 = 9'9\cdot10^{-7} \\ I_3 = 0 \end{cases}$$

PROBLEMAS TEMA 4: EL SÓLIDO ELÁSTICO

Problema 4.1.

¿Hasta qué deformación total el error de utilizar la deformación ingenieril frente a la deformación logarítmica es inferior al 10% en tracción uniaxial?

Deformación ingenieril: $\varepsilon_{xx} = \frac{\Delta l}{l_0}$

Deformación logarítmica o real: $\varepsilon_{xx} = \ln \frac{l}{l_0}$

Deformación total $(l - l_0) = \Delta l$

error: $\dfrac{\Delta l / l_0 - \ln(l/l_0)}{\ln(l/l_0)} < 0'1$

$$\frac{\Delta l}{l_0} - \ln\left(\frac{l}{l_0}\right) < 0'1 \cdot \ln(l/l_0)$$

$$\frac{\Delta l}{l_0} < 0'1 \cdot \ln(l/l_0) + \ln(l/l_0)$$

$$\boxed{\frac{l - l_0}{l_0}} < 0'1 \ln l - 0'1 \ln l_0 + \ln l - \ln l_0$$

$$l - l_0 < l_0(1'1 \cdot \ln l - 1'1 \cdot \ln l_0)$$

$$\boxed{l - l_0 < \ln l^{1'1 l_0} - \ln l_0^{1'1 l_0}}$$

si $l_0 = 1\,m$

$$l - 1 < \ln \cdot l^{1'1} - \cancel{\ln 1}^{0}$$

$$l < 1 + 1'1 \ln l$$

$$l - 1'1 \ln l < 1$$

$l = 1'5 \rightarrow 1'053 < 1$ NO

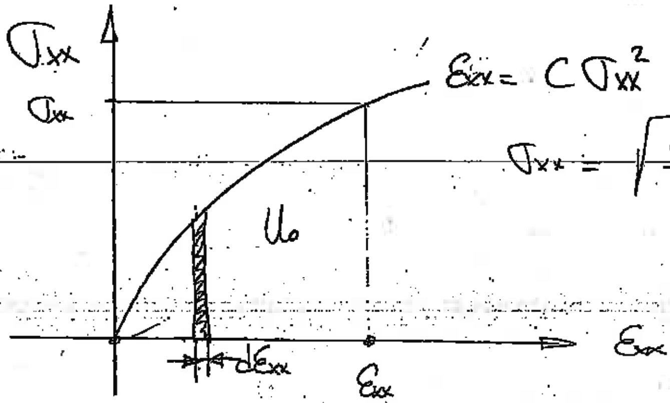
$l = 1'053 \rightarrow 0'996.14 < 1$ SÍ (Valor límite)

$\Delta l = l - l_0 = 1'053 - 1 = 0'053$ MAL

Problema 4.2.

$\varepsilon_{xx} = C\sigma_{xx}^2$

U_0, U_0^* ?



$\sigma_{xx} = \sqrt{\frac{\varepsilon_{xx}}{C}}$ $\quad f(\varepsilon_{xx}) = \sigma_{xx}$

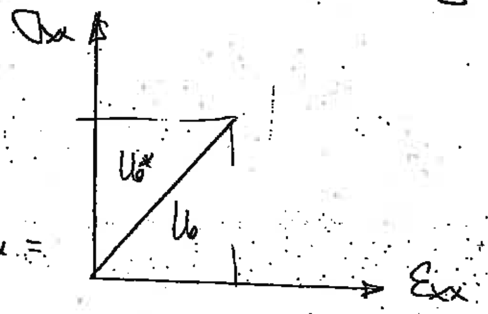
$$U_0 = \int_0^{\varepsilon_{xx}} \sqrt{\frac{\varepsilon_{xx}}{C}} \cdot d\varepsilon_{xx} = \int_0^{\varepsilon_{xx}} \left(\frac{\varepsilon_{xx}}{C}\right)^{1/2} d\varepsilon_{xx} = \frac{\varepsilon_{xx}^{3/2}}{C^{1/2}\,3/2}$$

$$\boxed{U_0^* = \frac{1}{3}\varepsilon_x\sigma_x}$$

$$U_0 = \frac{2}{3C^{1/2}}\,\varepsilon_{xx}^{3/2} = \frac{2}{3}\,\varepsilon_x \cdot \sqrt{\frac{\varepsilon_x}{C}} = \frac{2}{3}\,\varepsilon_x\sigma_x \qquad \boxed{U_0 = 2/3\,\varepsilon_x\sigma_x}$$

$U_0^* = \varepsilon_x\sigma_x - U_0$

Si fuera lineal:



$\varepsilon_{xx} = C\,\sigma_{xx}$

$\sigma_{xx} = \frac{\varepsilon_{xx}}{C}$

$$U_0^* = U_0 = \int_0^{\varepsilon_{xx}} \frac{\varepsilon_{xx}}{C} \cdot d\varepsilon_{xx} =$$

$$= \frac{1}{C} \cdot \frac{\varepsilon_{xx}^2}{2} = \frac{1}{2} \cdot \varepsilon_{xx} \cdot \frac{\varepsilon_{xx}}{C} = \frac{1}{2}\,\varepsilon_{xx}\,\sigma_{xx} \qquad \boxed{U_0 = U_0^* = \frac{1}{2}\,\varepsilon_x\sigma_x}$$

Problema 4.3.

$$U_0 = a\,\varepsilon_{xx} + b\,\varepsilon_{yy} + c\,\varepsilon_{zz} + d\,\varepsilon_{xx}^2 + e\,\varepsilon_{yy}^2 + f\,\varepsilon_{zz}^2$$

1) Lineal: U_0 es función cuadrática de ε_{ij} → $\boxed{a=b=c=0}$

2º) Lineal e isótropo: $U_0 = d\,\varepsilon_{xx}^2 + e\,\varepsilon_{yy}^2 + f\,\varepsilon_{zz}^2$

$$U_0 = \frac{\lambda}{2}(\varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz})^2 + G(\varepsilon_{xx}^2 + \varepsilon_{yy}^2 + \varepsilon_{zz}^2 + 2\varepsilon_{xy}^2 + 2\varepsilon_{xz}^2 + 2\varepsilon_{yz}^2)$$

Vemos que los coeficientes de ε_{xx}^2, ε_{yy}^2, ε_{zz} → iguales $= (\frac{\lambda}{2} + G$

Problema 4.4.

Estado de deformación plana: $\begin{cases} u = 3xy \\ v = x^2+y^2 \\ w = 0 \end{cases}$ $\left(\begin{array}{l} \varepsilon_x = \frac{\partial u}{\partial x} = 3y \\ \varepsilon_y = \frac{\partial v}{\partial y} = 2y \\ \varepsilon_{xy} = \frac{1}{2}(3x+2x) \end{array} \right)$ ✓

$$U_0 = \frac{1}{2}\left(b_{11}\varepsilon_x^2 + b_{22}\varepsilon_y^2 + b_{33}\gamma_{xy}^2 + 2b_{12}\varepsilon_x\varepsilon_{yy} + 2b_{13}\varepsilon_x\gamma_{xy} + 2b_{23}\varepsilon_{yy}\gamma_{xy} \right) \qquad \gamma_{xy} = 2\varepsilon_{xy}$$

$$U_0 = \frac{1}{2}\left[b_{11}\varepsilon_x^2 + b_{22}\varepsilon_y^2 + 4b_{33}\varepsilon_{xy}^2 + 2b_{12}\varepsilon_x\varepsilon_y + 4b_{13}\varepsilon_x\varepsilon_{xy} + 4b_{23}\varepsilon_y\varepsilon_{xy} \right]$$

Obtener las ecuaciones de equilibrio, si $\exists$ fuerzas de volumen.

$$\begin{cases} \frac{\partial\sigma_x}{\partial x} + \frac{\partial\tau_{xy}}{\partial y} + \frac{\partial\tau_{xz}}{\partial z} + \phi_x = 0 \\ \frac{\partial\tau_{xy}}{\partial x} + \frac{\partial\sigma_{yy}}{\partial y} + \frac{\partial\tau_{yz}}{\partial z} + \phi_y = 0 \\ \frac{\partial\tau_{xz}}{\partial x} + \frac{\partial\tau_{zy}}{\partial y} + \frac{\partial\sigma_{zz}}{\partial z} + \phi_z = 0 \end{cases}$$

$$\sigma_{xx} = \frac{\partial U_0}{\partial\varepsilon_x} = \frac{1}{2}(2b_{11}\varepsilon_x + 2b_{12}\varepsilon_y + 4b_{13}\varepsilon_{xy})$$

$$\sigma_{yy} = \frac{\partial U_0}{\partial\varepsilon_{yy}} = \frac{1}{2}(2b_{22}\varepsilon_y + 2b_{12}\varepsilon_x + 4b_{23}\varepsilon_{xy})$$

$$\tau_{xy} = \frac{1}{2}\frac{\partial U_0}{\partial\varepsilon_{xy}} = \frac{1}{2}\left[\frac{1}{2}(8\varepsilon_{xy}b_{33} + 4b_{13}\varepsilon_x + 4b_{23}\varepsilon_y)\right]$$

$$\sigma_{xx} = b_{11}\cdot 3y + b_{12}\cdot 2y + b_{13}\cdot 5x$$

$$\sigma_{yy} = b_{22}\cdot 2y + b_{12}\cdot 3y + b_{23}\cdot 5x$$

$$\tau_{xy} = b_{33}\cdot 5x + b_{13}3y + b_{23}2y$$

(2) $5b_{33} + 2b_{22} + 3b_{12} + \phi_y$

$$\boxed{5b_{33} + 2b_{22} + 3b_{12} + \phi_y = 0}$$

(1). $5b_{13} + 3b_{13} + 2b_{23} + \phi_x = 0$

$$\boxed{8b_{13} + 2b_{23} + \phi_x = 0}$$

Problema 4.5.

$\sigma_x = K \varepsilon_x^{1/3}$

ε_x independiente de las coordenadas x, z, respondiendo a una función lineal de y. Además, y=0 no se deforma

U? [Eexternas en $y=a$, $y=-a$, V, K]

???

Problema 4.6.

$a \times b = 100 \times 50 \, mm^2$

$L = 2m.$

Tracción $F = 500 \, kN$

$\Delta L = 1 mm$

$\Delta b = -0'007 \, mm$

1) Módulo de elasticidad de la barra

$$\sigma = \frac{500 \cdot 10^3 \, N}{(0'1 \cdot 0'05) m} = 100 \cdot MPa$$

$$E = \frac{\sigma}{\varepsilon_{xx}} \qquad \varepsilon_{xx} = \frac{\Delta l}{l} = \frac{0'001}{2} = 0'0005$$

$$E = \frac{100 \cdot 10^6}{0'0005} = 200.000 \cdot 10^6 = \underline{200 \, GPa}$$

2) Coeficiente de Poisson:

$$\nu = -\frac{\varepsilon_{yy}}{\varepsilon_{xx}} = -\frac{\varepsilon_{zz}}{\varepsilon_{xx}} = -\frac{\Delta b/b}{\Delta \ell/\ell} = -\frac{(-0'007/50)}{0'001/2} = +0'28$$

$$\boxed{\nu = 0'28}$$

3) Variación de a

$$\nu = -\frac{\varepsilon_{yy}}{\varepsilon_{xx}} = -\frac{\varepsilon_{zz}}{\varepsilon_{xx}} \rightarrow \varepsilon_{zz} = -\nu \cdot \varepsilon_{xx}$$

$$\varepsilon_{zz} = -0'28 \cdot 0'0005 = -0'00014 = \frac{\Delta a}{a} \qquad \underline{\Delta a = -0'014 \text{ mm}}$$

4) $F = 400$ KN → dimensiones a y b

$E = cte$: $\quad E = \frac{\sigma}{\varepsilon_{xx}}$ $\quad \sigma = \frac{400 \cdot 10^3}{0'1 \cdot 0'05} = 80 MPa$

$$\varepsilon_{xx} = \frac{\sigma}{E} = \frac{80 \text{ MPa}}{200 \cdot 10^3 \text{ MPa}} = 0'0004$$

$$\varepsilon_{yy} = \varepsilon_{zz} = -\nu\, \varepsilon_{xx} = -0'28 \cdot 0'0004 = -0'00012$$

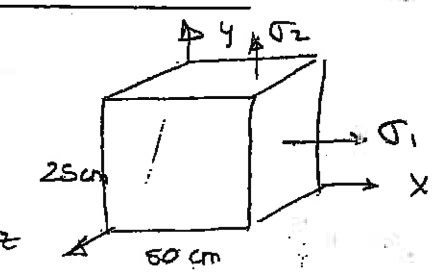
$$\varepsilon_{yy} = \frac{\Delta b}{b} \rightarrow \Delta b = -0'00012 \cdot 50 = \underline{-0'0056 \text{ mm}}$$

$$\varepsilon_{zz} = \frac{\Delta a}{a} \rightarrow \Delta a = -0'00012 \cdot 100 = \underline{-0'012 \text{ mm}}$$

Sección A: b: $50 - 0'0056 = 49'9944$ mm
a: $100 - 0'012 = 99'988$ m
} $4'99884$ mm²

Problema 4.7.

$$\begin{cases} \sigma_1 = 70 \text{ MPa} \\ \sigma_2 = 30 \text{ MPa} \\ \sigma_3 = 0 \text{ MPa} \end{cases}$$



$E = 200$ GPa
$\nu = 0'3$

1) Variación del área de la placa.

$$\varepsilon_{xx} = \frac{1}{E}\left[\sigma_{xx} - \nu\,(\sigma_{yy} + \sigma_{zz})\right] = \frac{1}{200 \cdot 10^3}\left[70 - 0'3 \cdot 30\right] = 0'000305$$

$$\varepsilon_{yy} = \frac{1}{200\cdot 10^3}\left[30 - 0'3\,(70)\right] = 0'000045$$

$$\varepsilon_{zz} = \frac{1}{200\cdot 10^3}\left[0 - 0'3\,(70+30)\right] = 0'00015$$

$\varepsilon_{xx} = \frac{\Delta a}{a} \rightarrow \quad \Delta a = 0'01525 \text{ cm} \rightarrow a = 49'9847 \text{ cm}$

$\varepsilon_{yy} = \frac{\Delta b}{b} \rightarrow \quad \Delta b = 0'00125 \text{ cm} \rightarrow b = 22'9988 \text{ cm}$

$1249'56 \text{ cm}^2 = A'$

Variación área $= 50\cdot 25 - 1249'56 =$
$= \underline{0'4387 \text{ cm}^2} = \underline{43'87 \text{ mm}^2}$

2) Deformación unitaria del espesor: $\varepsilon_{zz} = -0'00015$

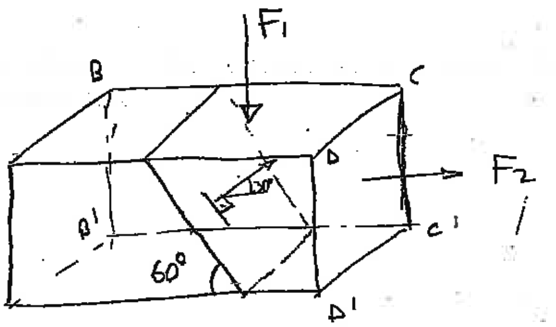
¡¡O!! Sería negativa, ya que cuando está sometido a estas cargas (en eje z no) → se produce una "estricción" llevada a ... un estado biaxial de carga.

Problema 4.8

$AB = 4$ cm

$AD = 10/3$ cm

$AA' = 2$ cm

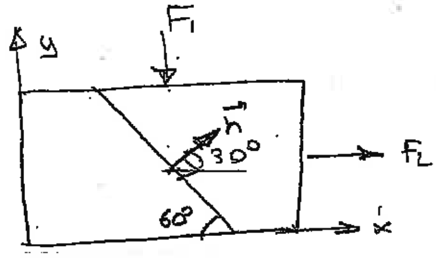


$F_1 = 100$ kN
$F_2 = 20$ kN } Uniformemente repartidas en cada cara.

$E = 210$ GPa
$\nu = 0'25$

1) Componentes intrínsecas del vector tensión correspondiente al plano.

$\vec{n} = \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$



$$\vec{\sigma_u} = \begin{bmatrix} 25 & 0 & 0 \\ 0 & -75 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} \sqrt{3}/2 \\ 1/2 \\ 0 \end{Bmatrix} = \begin{Bmatrix} 21'65 \\ -37'5 \\ 0 \end{Bmatrix} \quad (MPa)$$

$$\sigma_2 = \frac{F_2}{A} = \frac{20 \cdot 10^3}{0'04 \cdot 0'02} = 25 MPa$$

$$\sigma_1 = \frac{F_1}{A} = \frac{-100 \cdot 10^3}{0'04 \cdot \frac{10}{3} \cdot 10^{-2}} = -75 MPa$$

$$\sigma_{nn} = \left\{ \frac{25\sqrt{3}}{2} \quad -\frac{75}{2} \right\} \begin{Bmatrix} \sqrt{3}/2 \\ 1/2 \end{Bmatrix} = \frac{25 \cdot 3}{4} - \frac{75}{4} = \underline{\underline{0}}$$

$$\tau_{ut} = \sqrt{\left(\frac{25\sqrt{3}}{2}\right)^2 + \left(\frac{75}{2}\right)^2} = \underline{\underline{43'30 \ MPa}}$$

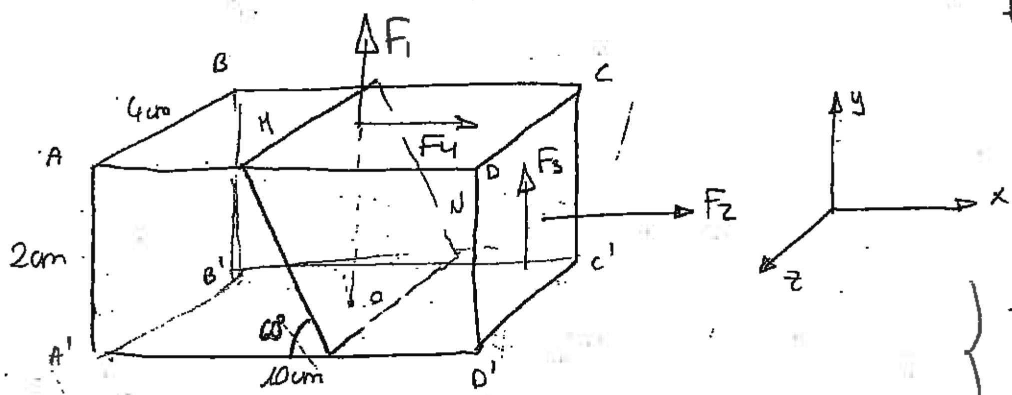
2°) Alargamientos principales unitarios.

$$\varepsilon_{xx} = \frac{1}{E}\left[\sigma_{xx} - \nu(\sigma_{yy} + \sigma_{zz})\right] = \frac{1}{210 \cdot 10^3}\left[25 - 0'25 \cdot (-75)\right] = 2'083 \cdot 10^{-4}$$

$$\varepsilon_{yy} = \frac{1}{210 \cdot 10^3}\left[-75 - 0'25 \cdot 25\right] = -3'86 \cdot 10^{-4}$$

$$\varepsilon_{zz} = \frac{1}{210 \cdot 10^3}\left[-0'25\,(25 - 75)\right] = 5'95 \cdot 10^{-5}$$

Problema 4.9.



$F_1 = 100$ KN $\Rightarrow$ 25 MPa

$F_2 = 20$ kN $\Rightarrow$ 25 MPa

$F_3 = 60$ KN $\Rightarrow$ 75 MPa

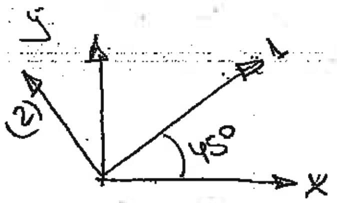
$F_4 = 300$ KN $\Rightarrow$ 75 MPa

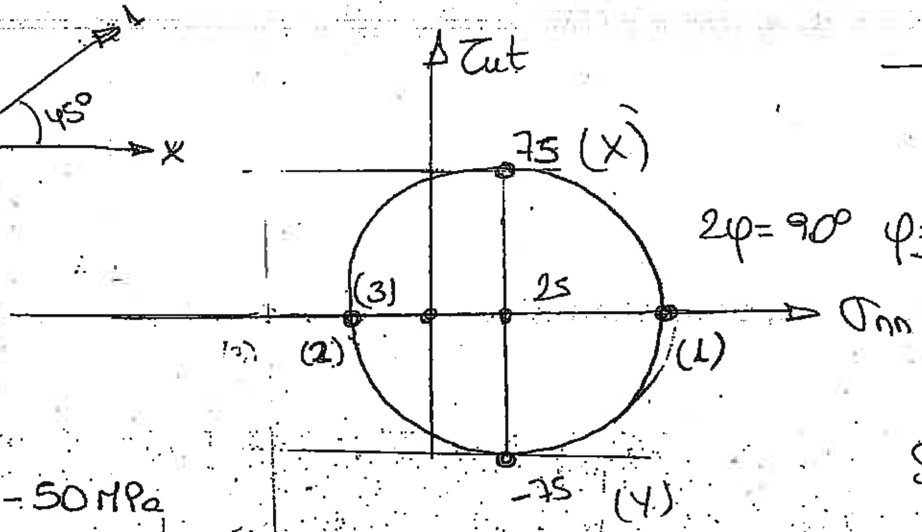
$E = 200$ GPa

$\nu = 0'25$

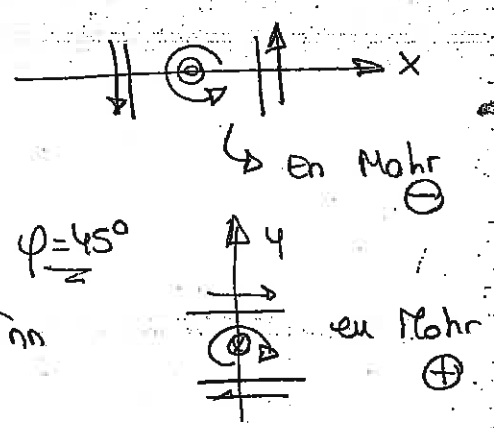
1) Apoyándose en Mohr, tensiones pples y dirección correspondiente a la máxima tensión principal de tracción

$$[T_{ij}] = \begin{bmatrix} 25 & 75 & 0 \\ 75 & 25 & 0 \\ 0 & 0 & 0 \end{bmatrix} \text{ (MPa)} \Rightarrow \begin{matrix} X(25,75) \\ Y(25,75)? \end{matrix}$$





$2\varphi = 90^\circ \quad \varphi = 45^\circ$



Sé que deben ser diametralmente opuestos

$\sigma_2 = -50$ MPa

$\sigma_1 = 100$ MPa → dirección principal: a 45°

$\sigma_3 = 0$

$n_1 = (1/\sqrt{2}, 1/\sqrt{2}, 0)$

2) Deformaciones principales.

$$\varepsilon_{xx} = \frac{1}{200 \cdot 10^3}\left[25 - 0'25 \cdot 25\right] = 9'375 \cdot 10^{-5} = 0'9375 \cdot 10^{-4}$$

$$\varepsilon_{yy} = \frac{1}{200 \cdot 10^3}\left[25 - 0'25\, 25\right] = 9'375 \cdot 10^{-5} = 0'9375 \cdot 10^{-4}$$

$$\varepsilon_{xy} = \frac{1+\nu}{E} \cdot \tau_{xy} = \frac{1+0'25}{200 \cdot 10^{-3}} \cdot 75 = 4'6875 \cdot 10^{-4}$$

$$\varepsilon_{zz} = \frac{1}{200 \cdot 10^3}\left[0 - 0'25 \cdot (25+25)\right]$$

$$\varepsilon_{zz} = -0'625 \cdot 10^{-4}$$

$$\cancel{\begin{vmatrix} 9'375 \cdot 10^{-5} - \varepsilon & 4'6875 \cdot 10^{-4} \\ 4'6875 \cdot 10^{-4} & 9'375 \cdot 10^{-5} - \varepsilon \end{vmatrix}} = 0$$

$$\varepsilon^2 + 8'789\cdot10^{-9} + 1'875\cdot10^{-4}\varepsilon - \underbrace{2'1972\cdot10^{-7}}=0$$

$$\varepsilon = \frac{-1'875\cdot10^{-4} \pm \sqrt{(1'875\cdot10^{-4})^2 + 4\cdot 2'197\cdot10^{-7}}}{2}$$

$\varepsilon_1 = 3'8428\cdot10^{-4}$

$\varepsilon_2 = -5'71\cdot10^{-4}$

$\varepsilon_{xz}=0$
$\varepsilon_{yz}=0$

$$10^{-4}\begin{vmatrix} 0'9375-\varepsilon & 4'6875 & 0 \\ 4'6875 & 0'9375-\varepsilon & 0 \\ 0 & 0 & -0'625-\varepsilon \end{vmatrix}=0$$

$$(-0'625-\varepsilon)\left[\underbrace{(0'9375-\varepsilon)^2 - 4'6875^2}\right]=0$$

$\underline{\varepsilon_1 = -0'625\cdot10^{-4}}$ ✓

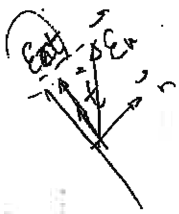
$[(0'9375-\varepsilon)-4'6875][(0'9375-\varepsilon)+4'6875]$

↳ $\varepsilon_2 \Rightarrow (0'9375-\varepsilon)-4'6875=0$

$\underline{\varepsilon_2 = -3'75\cdot10^{-4}}$

↳ $\varepsilon_3 \Rightarrow \varepsilon_3 = \underline{5'625\cdot10^{-4}}$

3) Def. angular en OM, ON



$\vec{n} = \left(\frac{\sqrt{3}}{2}, \frac{1}{2}, 0\right)$

$$\vec{\varepsilon}_n = 10^{-4}\begin{pmatrix} 0'9375 & 4'6875 & 0 \\ 4'6875 & 0'9375 & 0 \\ 0 & 0 & -0'625 \end{pmatrix}\begin{Bmatrix} \frac{\sqrt{3}}{2} \\ \frac{1}{2} \\ 0 \end{Bmatrix} = \begin{Bmatrix} 3'1556 \\ 4'5282 \\ 0 \end{Bmatrix}10^{-4}$$

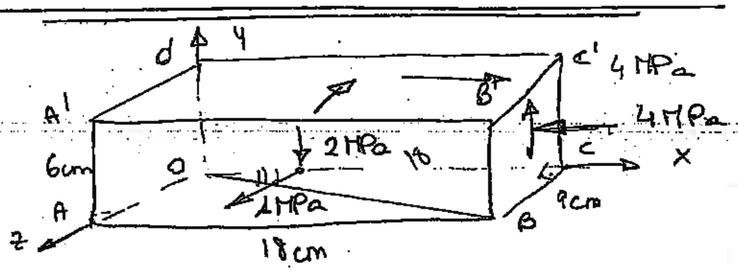
$\varepsilon_{nn} = 4'99695\cdot10^{-4} \rightarrow |\vec{\varepsilon}_n| = 5\cdot10^{-4} \rightarrow$ $\boxed{\text{Def. angular nula}}$

$|\varepsilon_{nn}| \approx |\vec{\varepsilon}_n|$

4) Def. angular lados AA' AD

$\gamma_{xy} = 2\varepsilon_{xy} = 2 \cdot 4'6875 \cdot 10^{-4} = \underline{9'375 \cdot 10^{-4}}$ ¿Seguro?

Problema 4.10.



$$[T_{ij}] = \begin{pmatrix} -4 & 4 & 0 \\ 4 & 0 & -2 \\ 0 & -2 & 1 \end{pmatrix}$$

$E = 200$ GPa $\nu = 0'2$

1) Tensiones pples y tens. correspondiente a máxima ppal de tracción

$$\begin{vmatrix} -4-\sigma & 4 & 0 \\ 4 & -\sigma & -2 \\ 0 & -2 & 1-\sigma \end{vmatrix} = 0$$

$(-4-\sigma)(-\sigma)(1-\sigma) - 16(1-\sigma) - 4(-4-\sigma)$

$(4\sigma+\sigma^2)(1-\sigma) - 16 + 16\sigma + 16 + 4\sigma = 0$

$\cancel{4\sigma} - 4\sigma^2 + \cancel{\sigma^2} - \sigma^3 - \cancel{16} + 16\sigma + \cancel{16} + 4\sigma = 0$

$-\sigma^3 - 3\sigma^2 + 24\sigma = 0$ $\boxed{\sigma_1 = 0}$

$-\sigma^2 - 3\sigma + 24 = 0$

$\sigma^2 + 3\sigma - 24 = 0$ $\sigma = \dfrac{-3 \pm \sqrt{9+4\cdot 24}}{2}$ $\boxed{\sigma_2 = 3'6234}$ $\boxed{\sigma_3 = -6'6234}$

$\sigma_2 = 3'6234 \rightarrow$

$$\begin{pmatrix} 7'62 & 4 & 0 \\ 4 & -3'62 & -2 \\ 0 & -2 & -2'62 \end{pmatrix} \begin{Bmatrix} n_{1x} \\ n_{1y} \\ n_z \end{Bmatrix} = \{0\}$$

$n_x^2 + n_y^2 + n_z^2 = 1$

$-2n_y - 2'62 n_z = 0$ $n_y = -1'31 n_z$

$0' \ldots 3'18 n_z^2 = 1$

$4n_x - 3'62 n_y - 2n_z = 0$

$\boxed{n_z = 0'56}$

$4n_x - 3'62(-1'31)n_z - 2n_z = 0$

$\boxed{n_y = -0'7339}$

$4n_x + 2'7422 n_z = 0$

$n_x = -0'68555 n_z$

$\boxed{n_x = -0'3839}$

2) Tensor de deformaciones

Hooke:

$$\left\{\begin{array}{l}
\varepsilon_{xx} = \dfrac{1}{200\cdot 10^3}\left[-4 - 0'2\,(0+1)\right] = -2'1\cdot 10^{-5} = -21\cdot 10^{-6} \\
\varepsilon_{yy} = 3\cdot 10^{-6} \\
\varepsilon_{zz} = 9\cdot 10^{-6} \\
\varepsilon_{xy} = \dfrac{\tau_{xy}}{2G} = \dfrac{(1+\nu)}{E}\tau_{xy} = \dfrac{1+0'2}{200\cdot 10^3}\cdot 4 = 24\cdot 10^{-6} \\
\varepsilon_{xz} = 0 \\
\varepsilon_{yz} = -12\cdot 10^{-6}
\end{array}\right.$$

✓

3) ε_{nn} en dirección OB' $\vec{n} = \dfrac{(18,-6,9)}{\sqrt{18^2+6^2+9^2}} = \left(\dfrac{18}{21}, \dfrac{-6}{21}, \dfrac{9}{21}\right)$

$\vec{n} = \left(\dfrac{6}{7}, \dfrac{2}{7}, \dfrac{3}{7}\right)$

$$\{\varepsilon_n\} = \frac{10^{-6}}{7}\begin{bmatrix} -21 & 24 & 0 \\ 24 & 3 & -12 \\ 0 & -12 & 9 \end{bmatrix}\begin{Bmatrix} 6 \\ 2 \\ 3 \end{Bmatrix} = \frac{10^{-6}}{7}\begin{Bmatrix} -78 \\ 114 \\ 3 \end{Bmatrix}$$

$$\varepsilon_{nn} = \frac{10^{-6}}{7}\{-78 \quad 114 \quad 3\}\frac{1}{7}\begin{Bmatrix} 6 \\ 2 \\ 3 \end{Bmatrix} = -\frac{10^{-6}}{49}\cdot 231 = -4'71\cdot 10^{-6}$$

$\dfrac{\Delta \ell}{\ell} = \varepsilon_{nn}$ $\Delta \ell = \varepsilon_{nn}\cdot \ell = -4'71\cdot 10^{-6}\cdot 21\text{ cm} = \underline{-99\cdot 10^{-6}\text{ cm}}$

Problema 4.11.

$\nu = 0'3$

$\sigma_z = -100$ MPa

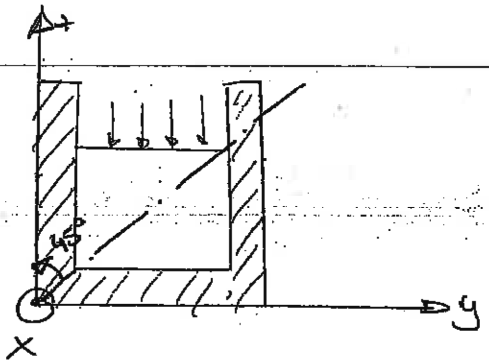
σ_y ?

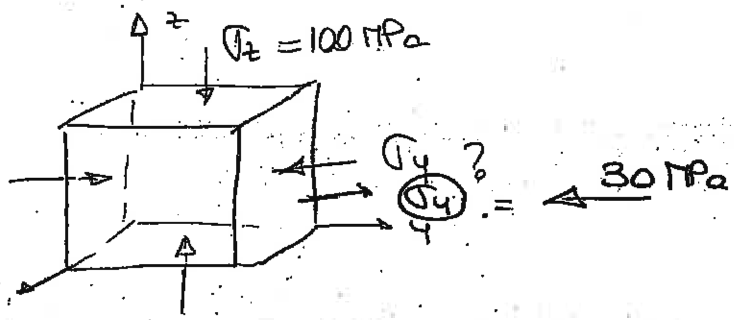
σ_x (libre) $= 0$

$\boxed{\text{Contacto sin rozamiento}}$

$\tau_{nt\,máx}$. → plano ppal. τ_{nt} en este punto = 0

Estado de tensión plana en XZ:





$\varepsilon_{zz} = \frac{1}{E}\left[\sigma_{zz} - \nu(\sigma_{xx}+\sigma_{yy})\right]$

$\varepsilon_{yy} = \frac{1}{E}\left[\sigma_{yy} - \nu(\sigma_{xx}+\sigma_{zz})\right] = 0$

$\sigma_{yy} - 0'3 \cdot [0 + (-100)] = 0$

$\sigma_{yy} = 0'3 \cdot (-100) = -30$ MPa

~~$\frac{1}{E}(-100 - 0'3\,\sigma_{yy}) = 0$~~

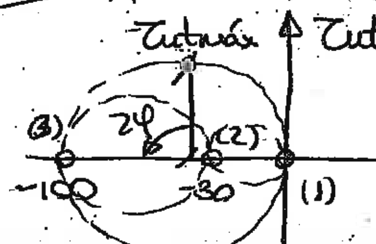
~~$E \neq 0$~~

~~$-100 - 0'3\,\sigma_{yy} = 0 \quad \sigma_{yy} = -333'33$~~

~~$\tau_{nt\,máx} = \frac{-333'33 - (-100)}{2} = 166'67$~~ MAL

$\tau_{nt\,máx} = \frac{100}{2} = 50$ MPa

~~DUDA~~



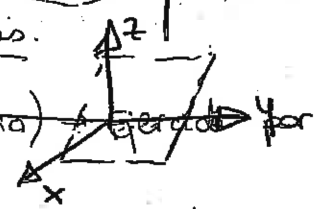
$2\varphi = 90°$ $\quad \varphi = 45°$ ~~PLANO,~~

? → ~~Como es un cubo → igual área todas sus caras.~~

en el círculo de planos // al eje Y

~~Cara inferior → tensión~~ (por equilibrio) ~~generada por~~

100 MPa

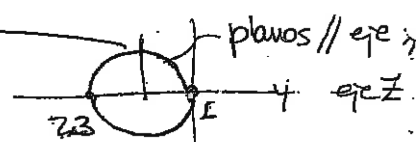


~~el material indeformable~~ Caras laterales: el mismo material

indeformable $\vec{n}_1 = \left(-\frac{\sqrt{2}}{2}, 0, \frac{\sqrt{2}}{2}\right)$ ~~misma área~~ igual presión. $\sigma_2 = \sigma_3 = -100$

Además planos de ~~máx~~ tensión ~~tangencial~~ $\vec{n}_2 = \left(\frac{\sqrt{2}}{2}, 0, \frac{\sqrt{2}}{2}\right)$ ~~tangencial~~

~~diametralmente~~ opuesto

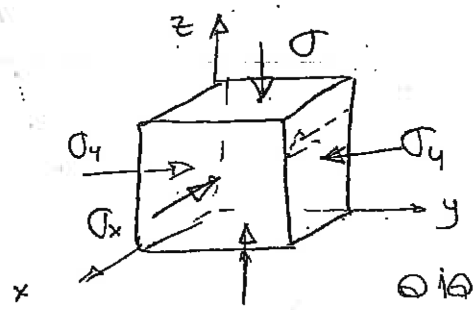


τ_{nt} máx opuesto ~~$\frac{100}{2} = 50$ MPa → plano $2\varphi = 90°$ → $\varphi = 45°$~~ ? $\left(\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right)$

Problema 4.12.

Cubo : a

E, ν



Sin rozamiento

$\sigma_x = \sigma_y$

⊖ ¡Ojo! Sé que es a compresión → o pongo con signo ⊖ y me da el resultado

$$\varepsilon_{zz} = \frac{1}{E}\left[-\sigma - \nu(2\sigma_x)\right]$$

$$\varepsilon_{xx} = \frac{1}{E}\left[+\sigma_x - \nu(-\sigma + \sigma_x)\right] = 0$$

$$+\sigma_x = \nu(-\sigma + \sigma_x)$$

$$+\sigma_x(1-\nu) = -\nu\sigma$$

$$\boxed{\sigma_{xx} = \sigma_{yy} = -\frac{\nu\sigma}{1-\nu}}$$

¡A compresión! numérico, al que tendré que añadir un signo ⊖, o lo dejo como incógnita y me saldrá el signo.

$$\varepsilon_{zz} = \frac{1}{E}\left[-\sigma + \nu\,\frac{2\nu\sigma}{1-\nu}\right] =$$

$$\varepsilon_{zz} = \frac{-\sigma(1-\nu) + 2\sigma\nu^2}{E(1-\nu)}$$

$$\boxed{\varepsilon_{zz} = \frac{2\sigma\nu^2 - \sigma + \nu\sigma}{E(1-\nu)}}$$

Problema 4.13.

Arista : $\ell = 1$

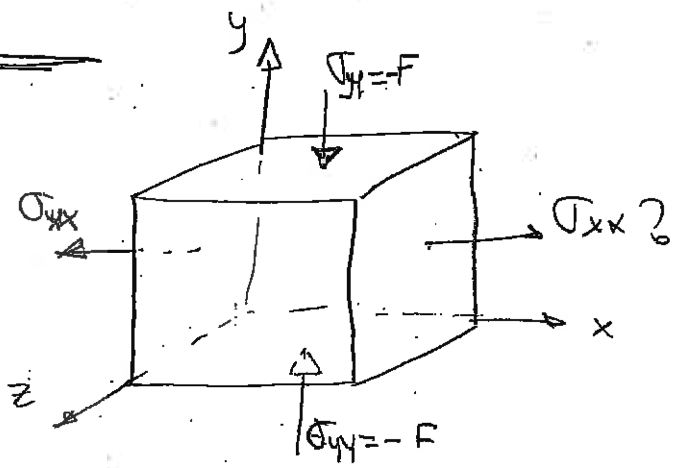
E, ν

Sin rozamiento

Deformaciones unitarias y tensiones

$\boxed{\sigma_{zz} = 0}$

$\boxed{\sigma_{yy} = -F}$



$$\varepsilon_{xx} = \frac{1}{E}\left[\sigma_{xx} - \nu(\sigma_{yy} + \sigma_{zz})\right] = 0$$

$$\sigma_{xx} = \nu(-F)$$

$$\boxed{\sigma_{xx} = -\nu F}$$

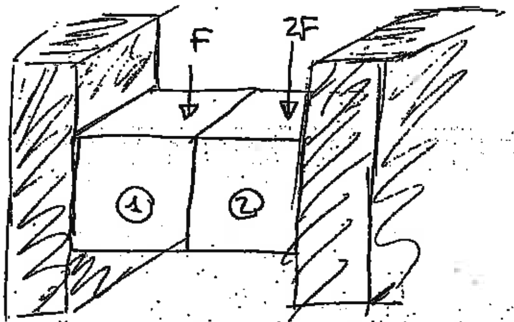
$$\begin{cases} \varepsilon_{xx} = 0 \\ \varepsilon_{yy} = \dfrac{1}{E}\left[-F - \nu(0 - \nu F)\right] = \dfrac{-F + \nu^2 F}{E} \\ \cancel{\varepsilon_{zz} = \dfrac{1}{E}\left[0 - \nu(-F - \nu F)\right] = \dfrac{\nu F + \nu^2 F}{E}} \end{cases}$$

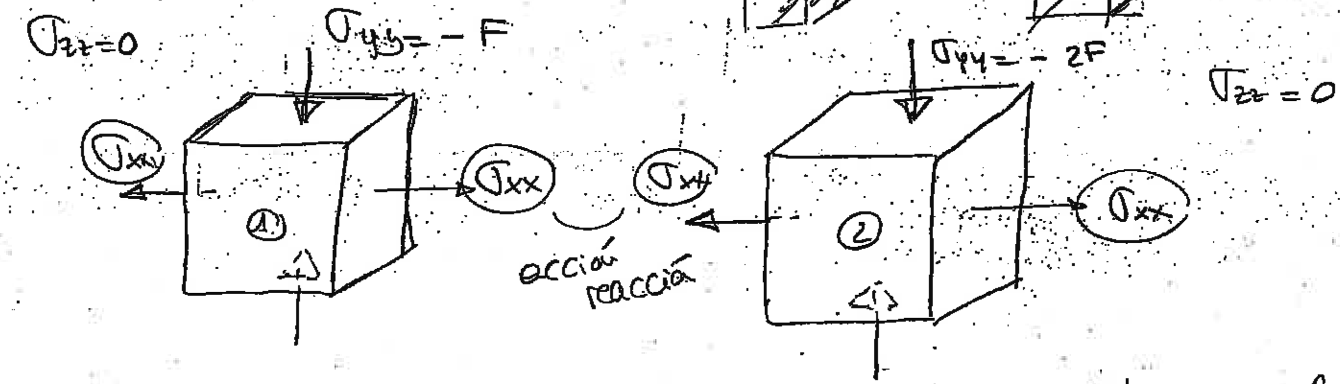
Problema 4.14.

2 cubos $\ell = 1$

E, ν

Contacto sin rozamiento





$\varepsilon_{1,2} \Rightarrow$ ¡¡Q!! Se suman siempre cantidades dimensionales

$$\varepsilon_{xx}^1 \cdot \ell^{1} + \varepsilon_{xx}^2 \cdot \ell^{1} = 0$$

$$\frac{1}{E}\left[\sigma_{xx} + \nu(\sigma_{yy} + \sigma_{zz})\right]_1 + \frac{1}{E}\left[\sigma_{xx} - \nu(\sigma_{yy} + \sigma_{zz})\right]_2 = 0$$

$$\frac{1}{E}\left[\sigma_{xx} + \nu(-F + 0)\right] + \frac{1}{E}\left[\sigma_{xx} - \nu(-2F + 0)\right] = 0$$

$$\frac{1}{E}\left[2\sigma_{xx} + \nu F + 2\nu F\right] = 0 \qquad 2\sigma_{xx} + 3\nu F = 0$$

$$\boxed{\sigma_{xx} = -\frac{3\nu F}{2}}$$

① $\varepsilon_{xx} = \frac{1}{E}\left[-\frac{3\nu F}{2} - \nu(-F)\right] = \frac{\nu F}{E}\left(-\frac{3}{2}+1\right)$ ✓

$\varepsilon_{yy} = \frac{1}{E}\left[-F - \nu\left(-\frac{3\nu F}{2}\right)\right] = \frac{F}{E}\left[-1 + \frac{3\nu^2}{2}\right]$ ✓

$\varepsilon_{zz} = \frac{1}{E}\left[0 - \nu\left(-\frac{3\nu F}{2} - F\right)\right] = \frac{\nu F}{E}\left[\frac{3\nu}{2}+1\right]$ ✓

② $\varepsilon_{xx} = \frac{1}{E}\left[-\frac{3\nu F}{2} - \nu(-2F)\right] = \frac{\nu F}{E}\left(-\frac{3}{2}+2\right) = \frac{\nu F}{2E}$ ✓

$\varepsilon_{yy} = \frac{1}{E}\left[-2F - \nu\left(-\frac{3\nu F}{2}\right)\right] = \frac{F}{E}\left[-2 + \frac{3\nu^2}{2}\right]$ ✓

$\varepsilon_{zz} = \frac{1}{E}\left[0 - \nu\left(-\frac{3\nu F}{2} - 2F\right)\right] = \frac{\nu F}{E}\left(\frac{3\nu}{2}+2\right)$ ✓

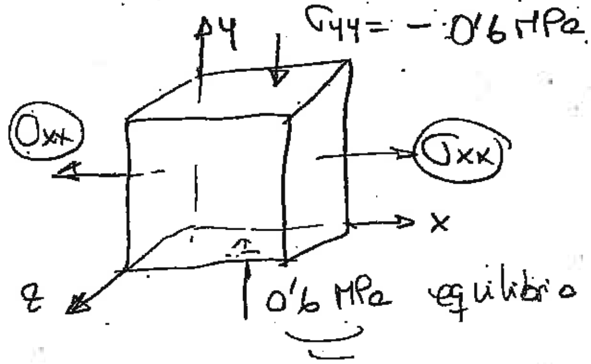
Problema 4.15

$\ell = 1\,cm$

1º) Descenso cara superior del cubo por aplicación de $P = 6000\,N$

$|\sigma_{yy}| = \frac{6000}{0'01^2} = 600000$ [illegible]

$|\sigma_{yy}| = \underline{0'6\,MPa}$



$\sigma_{zz} = 0$

Se desprecian los rozamientos

$E = 70\,GPa$

$\nu = 1/3$

$\varepsilon_{xx} = \frac{1}{E}\left[\sigma_{xx} - \nu(\sigma_{yy} + \sigma_{zz})\right]$

$\varepsilon_{xx} = \frac{1}{E}\left[\sigma_{xx} - \frac{1}{3}(-0'6 + 0)\right] = 0$

$\sigma_{xx} = \frac{1}{3}(-0'6) = \underline{-0'2\,MPa}$

$\varepsilon_{yy} = \frac{1}{7\cdot 10^{3}}\left[-0'6 - \frac{1}{3}(-0'2)\right] =$

$\varepsilon_{yy} = -7'61 \cdot 10^{-5}$

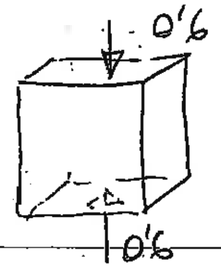
↳ $\frac{\Delta\ell}{\ell} = \varepsilon_{yy}$

$\Delta\ell = \varepsilon_{yy} \cdot 1\,cm = -71'61 \cdot \underline{10^{-5}\,cm}$

$\Delta\ell = -0'7161 \cdot 10^{-3}\,cm$

| desciende $0'71 \cdot 10^{-3}\,cm$ |

2) Ranura 1'1 cm de ancho.



$\sigma_{yy} = E \cdot \varepsilon_{yy}$

$\dfrac{-0'6}{70 \cdot 10^{+3}} = \varepsilon_{yy} = -8'57 \cdot 10^{-6}$

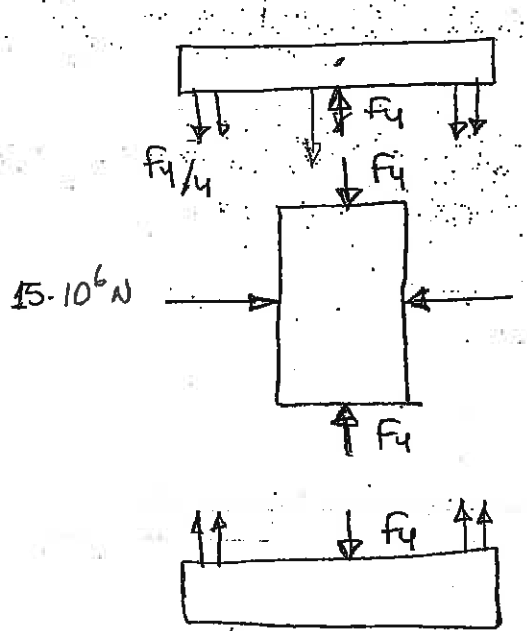
$\nu = -\dfrac{\varepsilon_{xx}}{\varepsilon_{yy}}$ $\quad \varepsilon_{xx} = -\varepsilon_{yy} \cdot \nu = +2'85 \cdot 10^{-6}$

Δl (ancho) $= l \cdot \varepsilon_{xx} = 2'85 \cdot 10^{-6}$ cm < 1'1

$\Delta l = l \cdot \varepsilon_{yy} = 8'57 \cdot 10^{-6}$ cm. (desciende)

Problema 4.16.

$E = 28$ GPa
$\nu = 0'1$
$a = 20$ cm
$A_1 = 1$ cm^2
$E = 200$ GPa
$L = 1$ m
$p = 75$ MPa



$F = 75 \cdot 10^6$ Pa $\cdot 0'2$m $=$
$F = 15 \cdot 10^6$ N

Prisma: $\varepsilon_{yy} = 0$

$\varepsilon_{yy} = \dfrac{1}{E}\left[\sigma_{yy} - \nu(\sigma_{xx} + \sigma_{zz})\right] = \dfrac{1}{28 \cdot 10^3}\left[\sigma_{yy} - 0'1(-75 + 0)\right] = 0$

$\sigma_{yy} = 0'1(-75) = -7'5$ MPa

DUDA

$F_y = 7'5 \cdot 10^6 \dfrac{N}{m^2} \cdot 0'2^2 \cancel{m^2} = 0'3 \cdot 10^6$ N $\rightarrow$ Cada cable: 75000 N

Problema 4.17.

$$\sigma_x = C\left[y^2 + \mu(x^2 - y^2)\right] \qquad \tau_{xy} = -2C\mu xy$$

$$\sigma_y = C\left[x^2 + \mu(y^2 - x^2)\right] \qquad \tau_{xz} = \tau_{yz} = 0$$

$$\sigma_z = C\mu\left[x^2 + y^2\right]$$

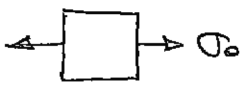
¿Tiene solución el problema de elasticidad?

TEMA 5 DE PROBLEMAS:

EL PROBLEMA ELÁSTICO

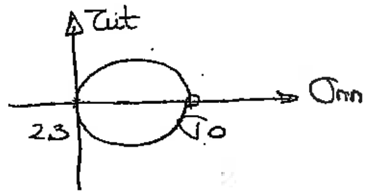
Problema 5.1.

A)

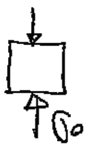


$\sigma_1 = \sigma_0$

$\sigma_2 = \sigma_3 = 0$



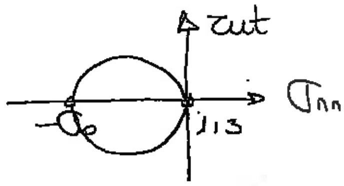
B)



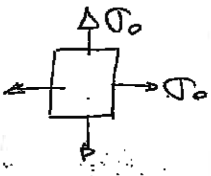
$\sigma_1 = 0$

$\sigma_2 = -\sigma_0$

$\sigma_3 = 0$



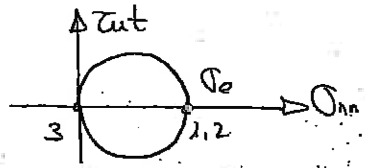
C)



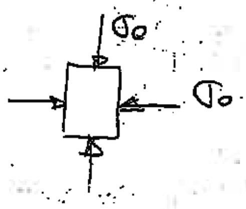
$\sigma_1 = \sigma_0$

$\sigma_2 = \sigma_0$

$\sigma_3 = 0$



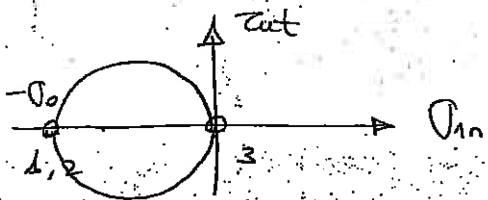
D)



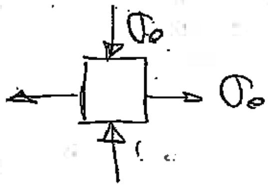
$\sigma_1 = -\sigma_0$

$\sigma_2 = -\sigma_0$

$\sigma_3 = 0$



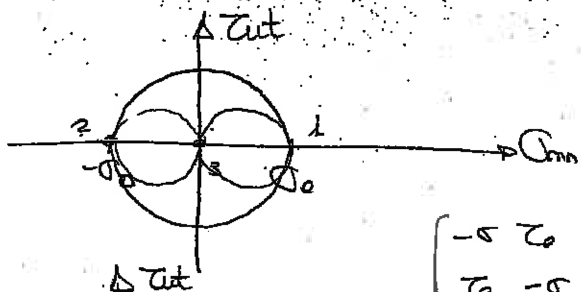
E)



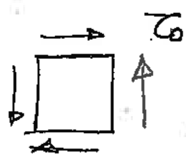
$\sigma_1 = \sigma_0$

$\sigma_2 = -\sigma_0$

$\sigma_3 = 0$



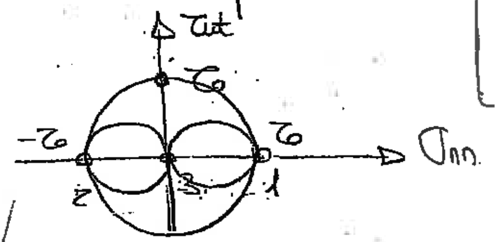
F)



$\sigma_1 = \tau_0$

$\sigma_2 = -\tau_0$

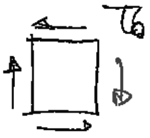
$\sigma_3 = 0$



$$\begin{vmatrix} -\sigma & \tau_0 & 0 \\ \tau_0 & -\sigma & 0 \\ 0 & 0 & -\sigma \end{vmatrix} = 0$$

$-\sigma^3 + \tau_0 \sigma = 0$

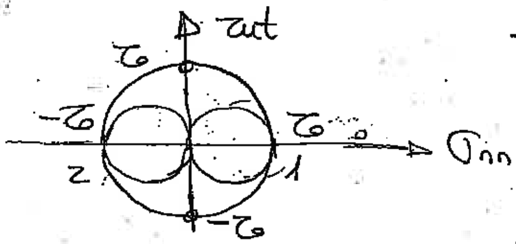
$\sigma_1 = 0$



$\sigma_1 = \tau_0$

$\sigma_2 = -\tau_0$

$\sigma_3 = 0$



$-\sigma^2 + \tau_0 = 0$

$\boxed{\sigma = \pm\tau_0}$

$$\begin{vmatrix} -\sigma & -\tau_0 & 0 \\ -\tau_0 & -\sigma & 0 \\ 0 & 0 & -\sigma \end{vmatrix} = 0$$

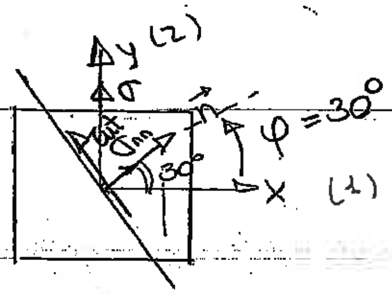
$-\sigma^3 + \tau_0 \sigma = 0$

$\sigma = 0$

$\sigma = \pm \tau_0$

Problema 5.2.

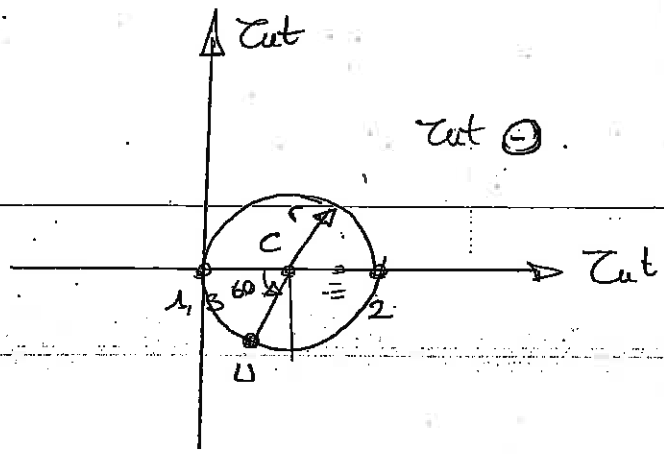
σ_{nn} ; τ_{uu} en U:



$\sigma_1 = 0$

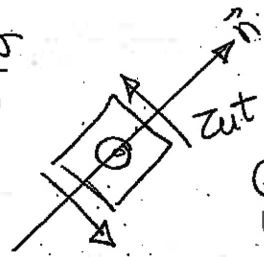
$\sigma_2 = \sigma$

$\sigma_3 = 0$



$$\begin{cases} C = \left(\frac{\sigma}{2}, 0\right) \\ r = \left|\frac{\sigma_1 - \sigma_2}{2}\right| = \frac{\sigma}{2} \end{cases}$$

$$\begin{cases} \sigma_{nn} = \frac{\sigma}{2} - \frac{\sigma}{2} \cdot \frac{1}{2} = \frac{\sigma}{2} - \frac{\sigma}{4} = \frac{\sigma}{4} \\ \tau_{ut} = \frac{\sigma}{2} \cdot \frac{\sqrt{3}}{2} = \frac{\sqrt{3}\sigma}{4} \end{cases}$$



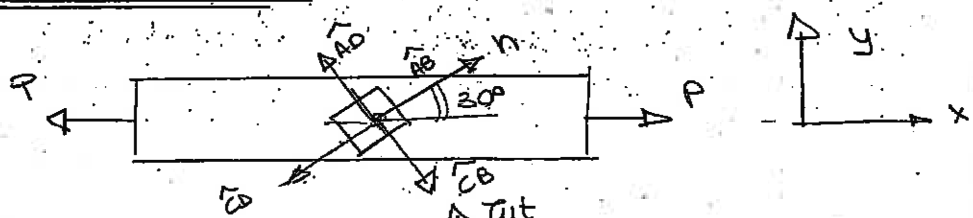
⊖ en Mohr

Problema 5.8.

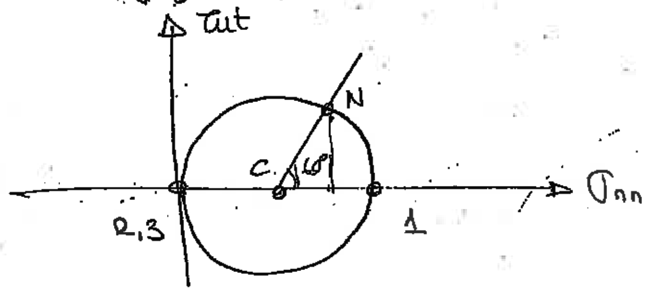
Sección $3 \times 3 \text{ cm}^2$

Tracción de 117 kN

Mohr → σ_{nn}, τ_{ut}

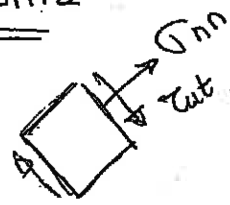


$$\sigma_{xx} = \frac{117 \cdot 10^3}{0'03^2} = 130 \text{ MPa} = \sigma_1$$

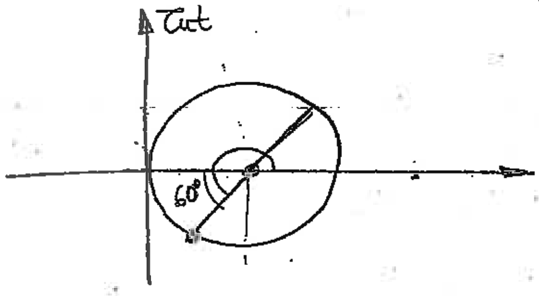


$\hat{AB} \to N$

$$\begin{cases} \sigma_{nn} = \frac{130}{2} + \frac{130}{2} \cdot \cos 60° = \frac{130}{2} + \frac{130}{4} = \underline{\underline{97'5 \text{ MPa}}} \\ \tau_{ut} = \frac{130}{2} \cdot \operatorname{sen} 60° = \frac{130\sqrt{3}}{4} = \underline{\underline{56'29 \text{ MPa}}} \end{cases}$$



$\vec{AD}$: $\varphi = (30+90) = 120^\circ$ $\quad 2\varphi = 240^\circ$

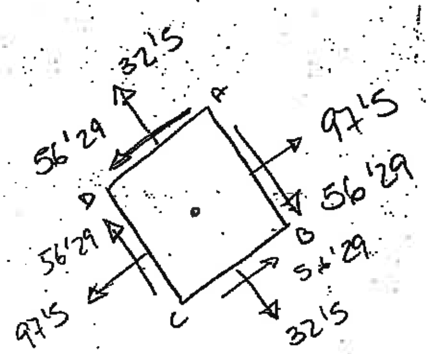


$$\sigma_{nn} = \frac{130}{2} - \frac{130}{2} \cdot \cos 60^\circ = 32'5 \text{ M}$$

$$\tau_{nt} = -\frac{130}{2} \operatorname{sen} 60^\circ = 56'29 \text{ MP}$$

CD : $\varphi = 210$ $\quad 2\varphi = 420^\circ = 360^\circ + 60^\circ \rightarrow$ igual que AB

CB : $\varphi = 300$ $\quad 2\varphi = 600 = 360^\circ + 240^\circ \rightarrow$ igual que AD

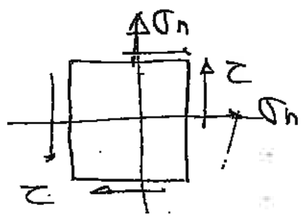


Problema 5.4.

$\sigma_n = 50$ MPa

τ ?

Tensión normal máxima y plano:



$$\begin{bmatrix} \sigma_n & \tau & 0 \\ \tau & \sigma_n & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{vmatrix} 50-\sigma & \tau & 0 \\ \tau & 50-\sigma & 0 \\ 0 & 0 & -\sigma \end{vmatrix} = 0$$

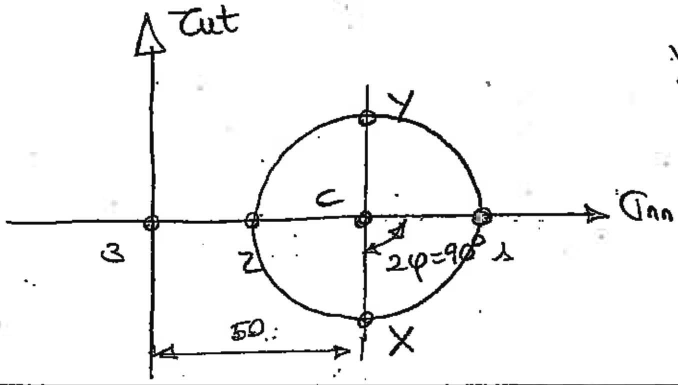
$$-\sigma(50^2 + \sigma^2 - 100\sigma) + \sigma\tau^2 = 0$$

$\sigma = 0$

$$\sigma^2 - 100\sigma + 50^2 - \tau^2 = 0$$

$$\sigma = \frac{100 \pm \sqrt{100^2 - 4(50^2 - \tau^2)}}{2}$$

Mohr :



X (50, −τ)

Y (50, τ)

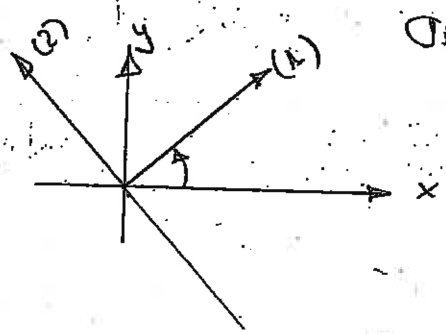
$$\sigma_c = \frac{\sigma_1+\sigma_2}{2} = 50$$

$$r = \left|\frac{\sigma_1-\sigma_2}{2}\right| = \tau$$

Además sé que la barra está sometida a una tracción simple uniaxial → $\boxed{\sigma_2 = 0}$ → En sist ppal:

$$\frac{\sigma_1}{2} = 50 \qquad \sigma_1 = 100 \text{ MPa}$$

$$\frac{\sigma_1}{2} = \tau \qquad \tau = \frac{100}{2} = 50 \text{ MPa}$$

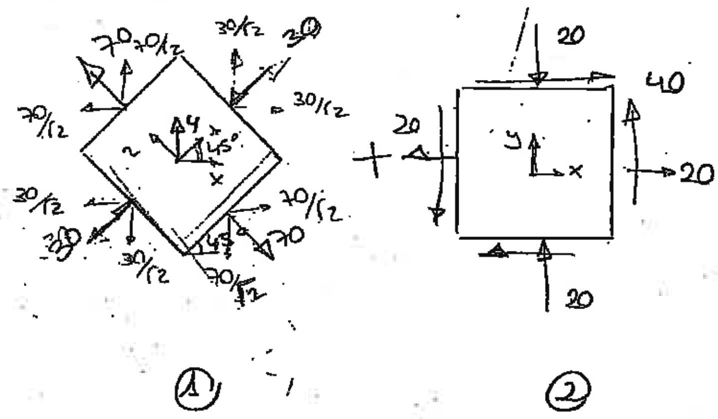


σ_1 en $\vec{n}\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, 0\right)$

$\left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}, 0\right)$??

Problema 5.5.

E = 200 GPa , ν = 0'25



① Cambio de base

x' $(+1/\sqrt{2}, -1/\sqrt{2})$

y $(1/\sqrt{2}, 1/\sqrt{2})$

Cambio de base: a sist. de ② x, y)

$$[T_{ij}]' = [N]\cdot[T_{ij}]\cdot[N]^T$$

$$[N] = \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}$$

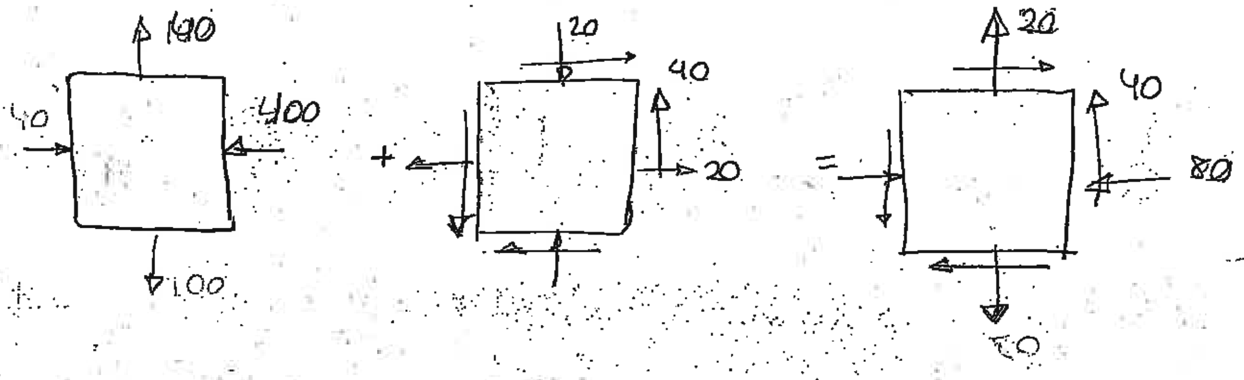
$$[T_{ij}]' = \begin{bmatrix} 1/\sqrt2 & -1/\sqrt2 \\ 1/\sqrt2 & 1/\sqrt2 \end{bmatrix} \begin{pmatrix} -30 & 70 \\ 70 & -30 \end{pmatrix} \begin{bmatrix} 1/\sqrt2 & 1/\sqrt2 \\ -1/\sqrt2 & 1/\sqrt2 \end{bmatrix}$$

$$[T_{ij}] = 1/\sqrt2 \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} -30 & 70 \\ 70 & -30 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} 1/\sqrt2$$

$$[T_{ij}] = \frac{1}{2} \begin{bmatrix} -100 & 100 \\ 40 & 40 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} -200 & 0 \\ 0 & 80 \end{bmatrix}$$

$$[T_{ij}] = \begin{bmatrix} -100 & 0 \\ 0 & 40 \end{bmatrix} \quad (MPa)$$

$$\begin{bmatrix} -80 & 40 \\ 40 & 20 \end{bmatrix}$$



$$\begin{vmatrix} -80-\sigma & 40 \\ 40 & 20-\sigma \end{vmatrix} = 0$$

$$(-80-\sigma)(+20-\sigma) - 1600 = 0$$

$$-1600 - 80\cancel{\sigma} - 20\cancel{\sigma} + \cancel{\sigma^2} - 1600 = 0$$

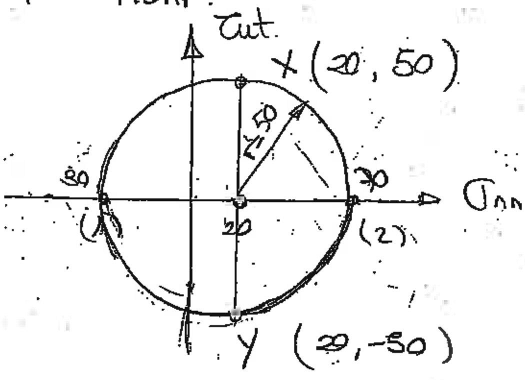
$$\sigma^2 - 100\sigma - 3200 = 0$$

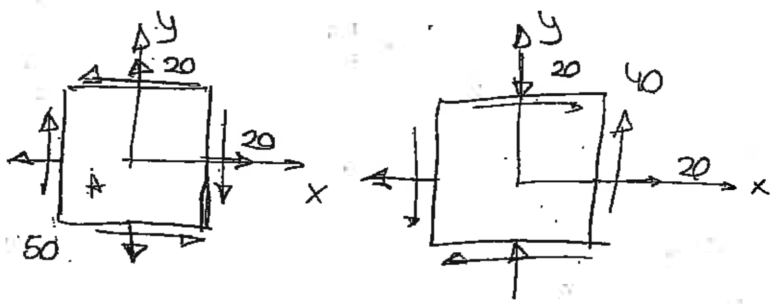
NO !

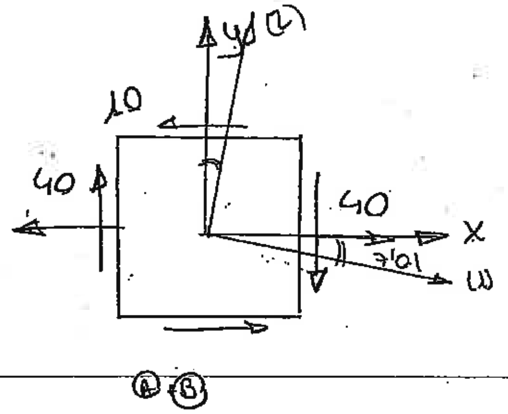
DUDA !

$$\sigma = \frac{100 \pm \sqrt{100^2 + 4\cdot 3200}}{2} \quad \begin{cases} \sigma_1 = 125'49\,MPa \\ \sigma_2 = -25'49\,MPa \end{cases}$$

Por Mohr:







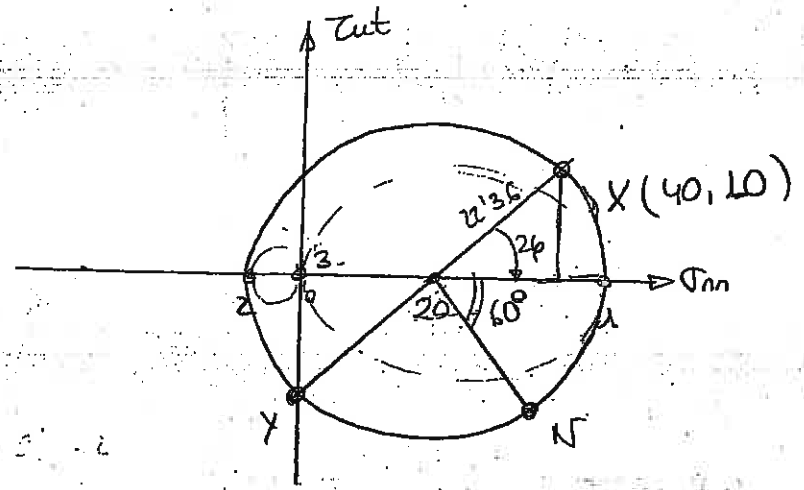
$\sigma_3 = 0$

$$\begin{vmatrix} 40-\sigma & -10 \\ -10 & -\sigma \end{vmatrix} = 0$$

$\cancel{-\sigma(40-\sigma) - 100 = 0}$

$-40\sigma + \sigma^2 - 100 = 0$

$\sigma = \dfrac{40 \pm \sqrt{40^2 + 400}}{2}$ $\sigma_1 =$ $\sigma_2 =$



$\sigma_1 = 42'36$ MPa

$\sigma_2 = -2'36$ MPa

$$\begin{cases} OC = \dfrac{42'36 + 2'36}{2} = 20' \\ r = \dfrac{42'36 - (-2'36)}{2} = 22'3 \end{cases}$$

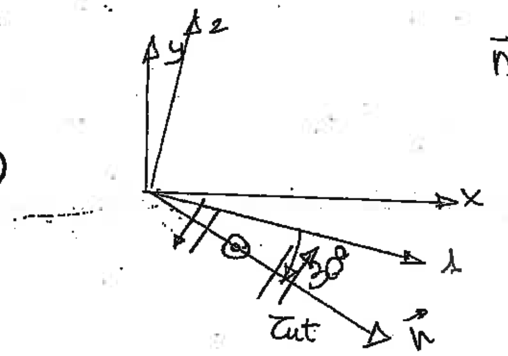
$tg\,2\varphi = \dfrac{10}{40} = 14'03$ $\quad \varphi = 7'01°$

$\vec{n}_1 = (0'9925\ ;\ -0'122)$

$\vec{n}_2 = (0'122\ ;\ 0'9925)$

$\vec{n}_3 = (0, 0, 1)$

2)



$\sigma_{nn} = 20 + 22'36 \cdot \cos 60° = 31'18$ MPa

$\tau_{nt} = 22'36 \cdot \sin 60° = 19'36$ MPa

3)

$$\begin{cases} \varepsilon_{xx} = \dfrac{1}{E}\left[\sigma_{xx} - \nu\,\sigma_{yy}\right] = \dfrac{1}{200\cdot 10^3}\left[40 - 0'25 \cdot 0\right] = 0'0002 \\ \varepsilon_{yy} = \dfrac{1}{E}\left[0 - \nu \cdot \sigma_{xx}\right] = \dfrac{1}{200\cdot 10^3}(-0'25 \cdot 40) = -0'00005 \\ \varepsilon_{zz} = \dfrac{1}{200\cdot 10^3}(-0'25 \cdot 40) = -0'00005 \end{cases}$$

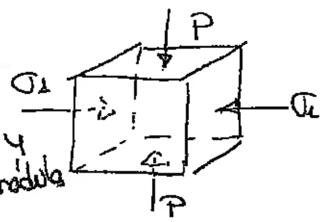
Problemas Tema 6: Teorías de Fallo

Problema 6.L.

Arista = 1

material equirresistente (isótropo) σ_f, $\nu = 0'3$ → sin rozamiento

P? / Fallo: Rankine, Saint-Venant, Tresca, Von Mises.



pongo signo y letras son módulo

$$\varepsilon_1 = \frac{1}{E}(-\sigma_1 + \nu P) = 0$$

$$\varepsilon_2 = \frac{1}{E}(-P + \nu\sigma_1)$$

Comparo $\varepsilon_1 = \varepsilon_2$ con ε_f

Rankine: $\sigma_{eq} = \sigma_{máx}$

fallo: $P = \sigma_f$

S-V: $\varepsilon_{máx} \rightarrow \sigma_{eq} = E \cdot \varepsilon_{máx}$

$$\sigma_{eq} = \cancel{E} \cdot \frac{1}{\cancel{E}}(-P + \nu\sigma_1) = \sigma_f$$

$$\sigma_1 = \nu P$$

$$\varepsilon_f = \frac{\sigma_f}{\cancel{E}} = \varepsilon_2 = \frac{1}{\cancel{E}}(\nu\sigma_1 - P)$$

$$\sigma_f = \nu^2 P - P \qquad \sigma_f = (\nu^2 - 1)P$$

$$P/\sigma_f = \frac{1}{\nu^2 - 1} = -1'0989$$

Signo → debido a que P y σ_f tienen signo contrario. Es compresión. He supuesto tracción

Tresca

$$\sigma_{eq} = \sigma_{máx} - \sigma_{mín}$$

$$\cancel{\sigma_{eq} = -P - (-\sigma_1) = \sigma_f}$$

$$\cancel{\varepsilon_1 = \frac{1}{E}(-\sigma_1 + \nu P) = 0} \qquad \sigma_1 = \nu P$$

$$\cancel{\sigma_f = -P + \nu P}$$

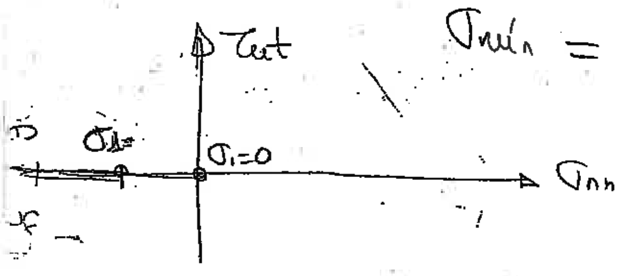
$$\cancel{\sigma_f = P(\nu - 1)}$$

$$\cancel{P/\sigma_f = \frac{1}{\nu - 1}} = -1'4285$$

tb

¡Ojo! $\sigma_{mín} \neq \sigma_1$

$\sigma_{mín} =$



$$\sigma_{eq} = 0 - (-P) = +\sigma_f$$

$$\boxed{P = \sigma_f} \qquad P/\sigma_f = 1$$

Von-Mises $\quad \sigma_{eq} = \sqrt{\frac{1}{2}\left[(\sigma_1-\sigma_2)^2+(\sigma_1-\sigma_3)^2+(\sigma_2-\sigma_3)^2\right]}$

$$\begin{cases} \sigma_1 = 0 \\ \sigma_2 = -\sigma_1 \\ \sigma_3 = -P \end{cases} \qquad \sigma_{eq} = \sqrt{\frac{1}{2}\left[\sigma_1^2+P^2+(-\sigma_1+P)^2\right]} = \sigma_f$$

$$\sigma_1 = \nu P \qquad \sqrt{\frac{1}{2}\left[\nu^2P^2+P^2+(-\nu P+P)^2\right]} = \sigma_f$$

$$2\sigma_f^2 = P^2(1+\nu^2)+P^2+\nu^2P^2-2\nu P^2$$

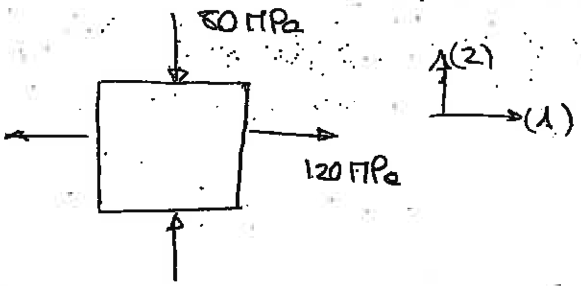
$$2\sigma_f^2 = P^2(1+\nu^2+1+\nu^2-2\nu)$$

$$\frac{P^2}{\sigma_f^2} = \frac{2}{2\nu^2-2\nu+2}$$

$$P/\sigma_f = \sqrt{\frac{2}{2\nu^2-2\nu+2}} = 1'125$$

Problema 6.2

$\nu = 0'3, \quad \sigma_f = 240$ MPa



$$\begin{cases} \sigma_1 = 120 \\ \sigma_2 = -80 \\ \sigma_3 = 0 \end{cases} \qquad \sigma_{eq} = \frac{\sigma_f}{n}, \quad n \geq 1$$

Rankine $\rightarrow \quad \sigma_1 = 120 = \sigma_{eq} = \frac{\sigma_f}{n} = \frac{240}{n} \qquad n = \frac{24\cancel{0}}{12\cancel{0}} = \underline{2}$

Saint-Venant $\rightarrow \quad \sigma_{eq} = E \cdot \varepsilon_{máx} = \frac{\sigma_f}{n} = \frac{E \cdot \varepsilon_f}{n}$

$$\varepsilon_1 = \frac{1}{E}\left[120-\nu(-80)\right] = \frac{144}{E} \rightarrow \text{máx} \rightarrow \qquad \cancel{E}\,\frac{144}{\cancel{E}} = \frac{\sigma_f}{n} = \frac{240}{n} \rightarrow \underline{n = 1'67}$$

$$\varepsilon_2 = \frac{1}{E}\left[-80-\nu(120)\right] = -116/E$$

$$\varepsilon_3 = \frac{1}{E}\left[0-\nu(120-80)\right] = -12/E$$

Tresca: $\sigma_{eq} = \sigma_{máx} - \sigma_{mín} = 120 - (-80) = 200 = \frac{\sigma_f}{n} = \frac{240}{n}$ $\quad \underline{n = 1'2}$

V-M: $\sigma_{eq} = \sqrt{\frac{1}{2}\left[(\sigma_1-\sigma_2)^2 + (\sigma_1-\sigma_3)^2 + (\sigma_2-\sigma_3)^2\right]} =$

$$= \sqrt{\frac{1}{2}\left[200^2 + 120^2 + 80^2\right]} = \frac{\sigma_f}{n} \rightarrow \underline{n = 1'376}$$

Problema 6.3.

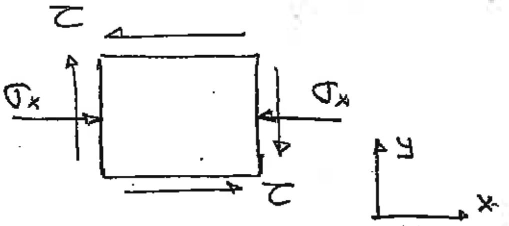
$\sigma_t = 3\ MPa$

$\sigma_c = -12\ MPa$

$\nu = 0'1$

n? / R, S-V, T

Estado más desfavorable



$$\left\{\begin{array}{l} \sigma_x = 1'8\ MPa \\ \sigma_y = 0 \\ \sigma_z = 0 \\ \tau = 1'2\ MPa \end{array}\right.$$

$$\begin{bmatrix} -1'8 & -1'2 & 0 \\ -1'2 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = [T_\sigma]$$

$$\begin{vmatrix} (-1'8-\sigma) & -1'2 & 0 \\ -1'2 & -\sigma & 0 \\ 0 & 0 & -\sigma \end{vmatrix} = 0$$

$+\sigma^2(-1'8-\sigma) + 1'2^2\sigma = 0$

$-\sigma_1 = 0$

$-1'8\sigma - \sigma^2 + 1'44 = 0$

$\sigma^2 + 1'8\sigma - 1'44 = 0$

$$\sigma = \frac{-1'8 \pm \sqrt{1'8^2 + 4 \cdot 1'44}}{2} \left\langle \begin{array}{l} \underline{\sigma_2 = 0'6\ MPa} \\ \underline{\sigma_3 = -2'4\ MPa} \end{array}\right.$$

$$\left\{\begin{array}{l} \sigma_1 = 0'6 \\ \sigma_2 = 0 \\ \sigma_3 = -2'4 \end{array}\right.$$

Rankine: $\sigma_{eq} = \sigma_1 = 0'6 = \frac{\sigma_t}{n} = \frac{3}{n} \rightarrow$ $\underline{n = 5 \text{ . A Tracción}}$

a compresión: $\sigma_{eq} = -2'4 = \frac{\sigma_c}{n} = \frac{-12}{n} \rightarrow$ $\underline{n = 5 \text{ A Compresión}}$

Saint-Venant:
$$\left\{\begin{array}{l} \varepsilon_1 = \frac{1}{E}(0'6 + 0'1 \cdot 2'4) = 0'84/E \\ \varepsilon_2 = \frac{1}{E}\left[-0'1(0'6 - 2'4)\right] = 0'18/E \\ \varepsilon_3 = \frac{1}{E}\left[-2'4 - 0'1(0'6)\right] = -2'46/E \end{array}\right.$$

A tracción: $\sigma_{eq} = E \cdot \varepsilon_{máx} = 0'84 = \frac{\sigma_t}{n} = \frac{3}{n} \rightarrow \underline{\underline{n = 3'57}}$ ✓

A compresión: $\sigma_{eq} = E \cdot \varepsilon_{máx} = -2'46 = \frac{\sigma_c}{n} = -\frac{12}{n} \rightarrow \underline{\underline{n = 4'87}}$

<u>Tresca</u> $\sigma_{eq} = \sigma_{máx} - \sigma_{mín} = 0'6 - (-2'4) = 3$

A Tracción $\sigma_{eq} = \frac{\sigma_t}{n} \rightarrow 3 = \frac{3}{n} \rightarrow \underline{\underline{n = 1}}$

A Compresión $\sigma_{eq} = \frac{\sigma_c}{n} \rightarrow 3 = -\frac{12}{n} \rightarrow \underline{\underline{n = -4}}$

Problema 6.4.

$E = 200$ GPa
$\sigma_f = 200$ MPa
$\nu = 0'3$

$P = 500$ kN $\qquad \sigma_z = \frac{-500 \cdot 10^3}{0'1^2} = \underline{-50 \text{ MPa}} = \sigma_3$

$\varepsilon_x = \varepsilon_1 = \frac{1}{E}[\sigma_1 - \nu(\sigma_2 + \sigma_3)] = 0$

$\sigma_1 - 0'3(\sigma_2 + 50) = 0 \qquad \sigma_1 = 0'3(\sigma_2 + 50)$

$\varepsilon_y = \varepsilon_2 = \frac{1}{E}[\sigma_2 - \nu(\sigma_1 + \sigma_3)] = 0$

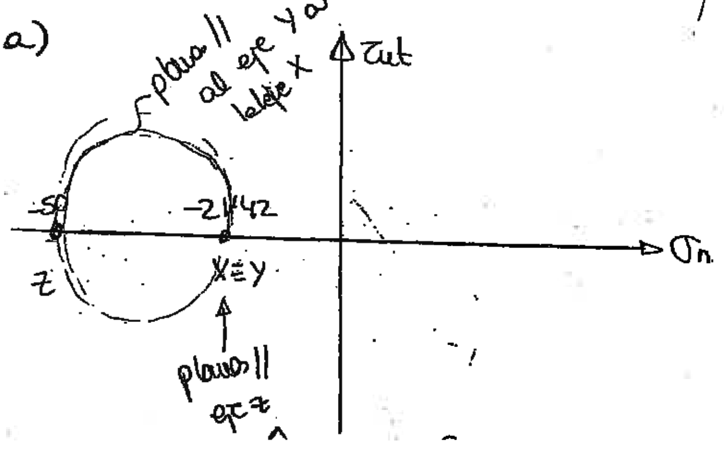
$\sigma_2 = 0'3[0'3(\sigma_2 - 50) - 50] \qquad \sigma_2 - 0'3(\sigma_1 - 50) = 0 \qquad \sigma_2 = 0'3(\sigma_1 - 50)$

$\sigma_2 - 0'09\sigma_2 = -0'09 \cdot 50 - 0'3 \cdot 50$

$\sigma_2(1 - 0'09) = -50(0'3 + 0'09) \rightarrow \sigma_2 = \underline{\underline{-21'42 \text{ MPa}}} = \sigma_y$

$\sigma_1 = 0'3[-21'42 - 50] \longrightarrow \sigma_1 = \underline{\underline{-21'42 \text{ MPa}}} = \sigma_x$

a)



b) n? R / S-V / T:

Rankine: $\sigma_{eq} = \sigma_{máx} = -50 = \frac{200}{n} \rightarrow n = 4$

Trabaja a compresión. La mayor tensión a compresión $\rightarrow$ 50 MPa

Saint-Venant:

$$\varepsilon_3 = \frac{1}{E}\left[\sigma_3 - \nu(\sigma_1+\sigma_2)\right] = \frac{1}{E}\left[-50 - 0'3(-21'43-21'4\right.$$

$$\varepsilon_3 = \frac{-37'142}{E}$$

$$\sigma_{eq} = E \cdot \varepsilon_3 = \frac{\sigma_f}{n} \rightarrow n = 5'38$$

Tresca:

$$\sigma_{eq} = -50 - (-21'43) = \frac{\sigma_f}{n} \rightarrow n = 7$$

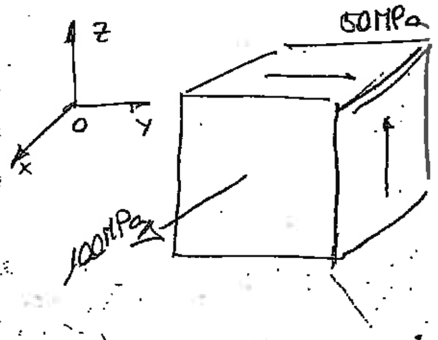
V-M:

$$\sigma_{eq} = \sqrt{\frac{1}{2}\left[(\sigma_1-\sigma_2)^2 + (\sigma_1-\sigma_3)^2 + (\sigma_2-\sigma_3)^2\right]} = \frac{\sigma_f}{n}$$

$$\sqrt{\frac{1}{2}\left[(-50+21'43)^2 + (-50+21'43)^2 + \cancel{(0)}\right]} = \frac{\sigma_f}{n} \rightarrow n = 7$$

Problema 6.5.

Arista = 1



$\sigma_x = 100$ MPa

$\tau_{yz} = \tau_{zy} = 50$ MPa

1) Tens. y direccion ppl.

$$\begin{vmatrix} 100-\sigma & 0 & 0 \\ 0 & -\sigma & 50 \\ 0 & 50 & -\sigma \end{vmatrix} = 0$$

$$\sigma^2(100-\sigma) - 2500(100-\sigma) = 0$$

$$\sigma^2 100 - \sigma^3 - 250000 + 2500\sigma = 0$$

$\sigma^3 - 100\sigma^2 - 2500\sigma + 250000 = 0 \Rightarrow (\sigma-50)[\sigma^2 - 50\sigma - 5000] = 0$

	1	-100	-2500	250000
50		50	-2500	-250000
	1	-50	-5000	0

$\sigma_1 = 50$ MPa

$\sigma = \dfrac{50 \pm \sqrt{50^2 + 20000}}{2}$

$\sigma = \dfrac{50 \pm 150}{2}$

$\sigma_2 = 100$ MPa

$\sigma_3 = -50$ MPa

$\left\{ \begin{array}{l} \sigma_1 = 100 \text{ MPa} \rightarrow \\ \sigma_2 = 50 \text{ MPa} \\ \sigma_3 = -50 \text{ MPa} \end{array} \right.$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & -100 & 50 \\ 0 & 50 & -100 \end{bmatrix} \begin{Bmatrix} n_{1x} \\ n_{1y} \\ n_{1z} \end{Bmatrix} = \{0\}$$

$50 n_{1y} - 100 n_{1z} = 0$

$\boxed{n_{1y} = 2n_{1z}}$

$50 \cdot n_{1y} - 100 n_{1z} = 0$

$\boxed{n_{1y} = 2n_{1z}}$

$x \rightarrow$ cualquier valor de x ✓

$\cancel{n_{1x}^2} + 2n_{1z}^2 + n_{1y}^2 = 0$

0

$3n_{1z}^2 = 0$

ó

$\left\{ \begin{array}{l} n_{1z} = 1/\sqrt{3} \\ n_{1y} = 1/\sqrt{3} \\ n_{1x} = 0 \end{array} \right.$

$\left\{ \begin{array}{l} n_{1x} = 1 \\ n_{1y} = 0 \\ n_{1z} = 0 \end{array} \right\}$

$\sigma_1 = 100 \rightarrow (1, 0, 0)$ Plano x plano ppal ¡¡¡

$\boxed{\sigma_2 = 50 \text{ MPa}}$

$$\begin{vmatrix} 50 & 0 & 0 \\ 0 & -50 & 50 \\ 0 & 50 & -50 \end{vmatrix} \begin{Bmatrix} n_{2x} \\ n_{2y} \\ n_{2z} \end{Bmatrix} = \{0\}$$

$n_{2x} = 0$

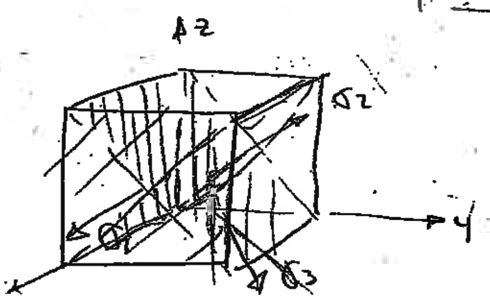
$-\cancel{50} n_{2y} + \cancel{50} n_{2z} = 0$

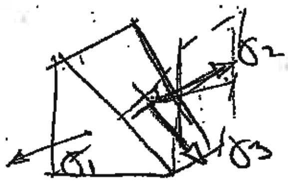
$\boxed{n_{2y} = n_{2z}}$

$2n_{2y}^2 = 0$

$\boxed{n_{2y} = 1/\sqrt{2} = n_{2z}}$

$(0, 1/\sqrt{2}, 1/\sqrt{2})$



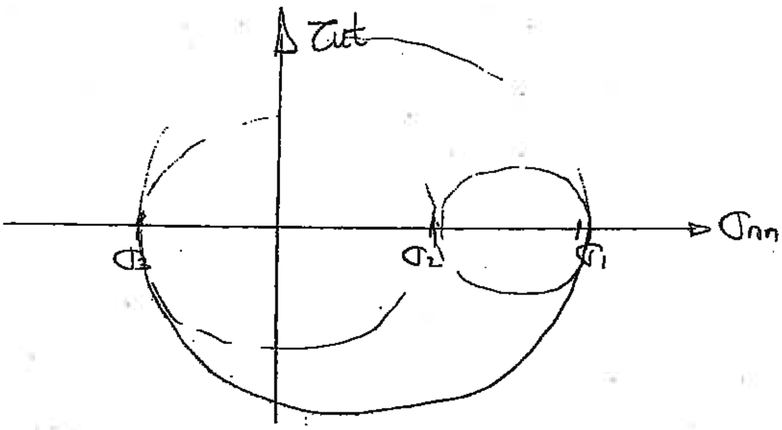


$\boxed{\sigma_3 = -50\,MPa}$

$$\begin{pmatrix} -150 & 0 & 0 \\ 0 & +50 & 50 \\ 0 & 50 & 50 \end{pmatrix} \begin{Bmatrix} n_{3x} \\ n_{3y} \\ n_{3z} \end{Bmatrix} = \{0\} \qquad \boxed{n_{3x} = 0} \quad n_{3y} = -n_{3z}$$

$$\left(0, \tfrac{1}{\sqrt{2}}, -\tfrac{1}{\sqrt{2}}\right)$$

2) Mohr $\quad \tau_{nt\,máx} = \dfrac{\sigma_{máx} - \sigma_{mín}}{2} = \dfrac{100 - (-50)}{2} = \dfrac{150}{2} = 75\ MPa$



3) n? / R, S-V, T, V-M: $\quad \begin{cases} E = 200\ GPa \\ \sigma_f = 200\ MPa \\ \nu = 0'3 \end{cases}$

R) $\quad \sigma_1 = \sigma_{eq} = 100 = \dfrac{200}{n} \longrightarrow \boxed{n = 2}$

S-V) $\quad \sigma_{eq} = E \cdot \varepsilon_{máx}$

$$\begin{cases} \varepsilon_1 = \frac{1}{E}\left[100 - 0'3\,(50 - 50)\right] = 100/E \\ \varepsilon_2 = \frac{1}{E}\left[50 - 0'3\,(100 - 50)\right] = 35/E \\ \varepsilon_3 = \frac{1}{E}\left[-50 - 0'3\,(100 + 50)\right] = -95/E \end{cases}$$

$\sigma_{eq} = 100 = \dfrac{200}{n} \longrightarrow \boxed{n = 2}$

T) $\quad \sigma_{eq} = \sigma_{máx} - \sigma_{mín} = 150 = \dfrac{200}{n} \longrightarrow \boxed{n = 1'34}$

V-M) $\quad \sigma_{eq} = \sqrt{\dfrac{1}{2}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_1 - \sigma_3)^2 + (\sigma_2 - \sigma_3)^2\right]} = \sqrt{\dfrac{1}{2}\left[(100-50)^2 + (100+50)^2 + (50+50)^2\right]} =$

$\boxed{n = 1'51}$

Problema 6.6

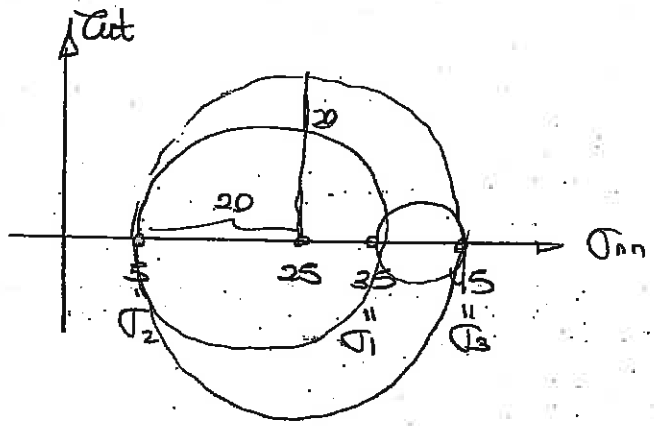
$$\begin{cases} \sigma_1 = 35 \text{ MPa} \\ \sigma_2 = 5 \text{ MPa} \\ \sigma_3 \geq 0 \end{cases}$$

$\tau_{nt\,máx} = 20$ MPa

1) Mohr   Si $\boxed{\sigma_1 \text{ es } \sigma_{máx}}$

- σ_2 es $\sigma_{mín}$: NO
  $\tau_{nt\,máx} = \dfrac{35-6}{2} = 15 \neq 20$ X
- σ_3 es $\sigma_{mín}$: $0 < \sigma_3 < 5$ NO
  $\tau_{nt\,máx} = \dfrac{35-\sigma_3}{2} = $ entre $35/2 = 17'5$ y $30/2 = 15$ → entre 17'5, 15 → 20 X

Por tanto, $\boxed{\sigma_3 \text{ es } \sigma_{máx}}$ y $\boxed{\sigma_2 \text{ es } \sigma_{mínima}}$: $\tau_{nt} = \dfrac{\sigma_3 - 5}{2} = 20$ $\boxed{\sigma_3 = 45 \text{ MPa}}$



2) σ_{nn} de $\tau_{nt\,máx}$: <u>25 MPa</u>

3) n? / R, S-V, T, V-M
$$\begin{cases} E = 210 \text{ GPa} \\ \sigma_f = 80 \text{ MPa} \\ \nu = 0'3 \end{cases}$$

R : $45 = \dfrac{80}{n} \rightarrow \boxed{n = 1'7}$

S-V : $\sigma_{eq} = E\,\varepsilon_{máx}$

$\varepsilon_3 = \frac{1}{E}\left[45 - 0'3\,(5+35)\right] = 33/E$ → $33 = 80/n \rightarrow \boxed{n = 2'44}$

$\varepsilon_1 = \frac{1}{E}\left[35 - 0'35\,(5+45)\right] = 20/E$

$\varepsilon_2 = \frac{1}{E}\left[5 - 0'3\,(45+35)\right] = -19/E$

T : $\sigma_{eq} = \sigma_{máx} - \sigma_{mín} = \dfrac{\sigma_f}{n} \rightarrow \boxed{n = 2}$

V-M : $\sigma_{eq} = \sqrt{\frac{1}{2}\left[(10^2 + 40^2 + 30^2\right]} = \dfrac{\sigma_f}{n} = \dfrac{80}{n} \longrightarrow \boxed{n = 2'21}$

Problema 6.7.

$E = 200\ GPa$

$\sigma_f = 240\ MPa$

$\nu = 0'3$

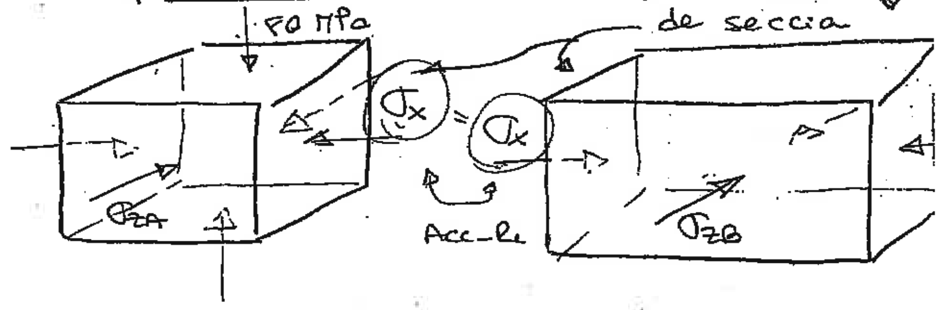
$\alpha = 12 \cdot 10^{-6}\ ºC^{-1}$

$\Delta T = 20ºC$

$8\ KN = 8 \cdot 10^3\ N \Rightarrow \sigma_{yy} = -\dfrac{8 \cdot 10^3}{0'010^2} = -80\ MPa$

↳ se dilata lateralmente

a) Equilibrio → de fuerzas → igual sección → de sección



Hipótesis { compresión }

b) Compatibilidad de deformaciones

A: { Dirección y → no hay restricción → $\boxed{\delta_{yA}}$ → $\boxed{\varepsilon_{yA}}$
Dirección x → δ_{xA}
Dirección z → 0 → $\boxed{\varepsilon_{zA} = 0}$

Supongo las z se dilat

$\delta_{xA} + \delta_{xB} = 0$

B: { Dirección y → no hay restricción → $\boxed{\delta_{yB}}$ → $\boxed{\varepsilon_{yB}}$
Dirección x → δ_{xB}
Dirección z → 0 → $\boxed{\varepsilon_{zB} = 0}$

$\varepsilon_{xA} \cdot 10 + \varepsilon_{xB} \cdot 20 = 0$

$\varepsilon_{xA} + 2\varepsilon_{xB} = 0$

c) Ley de comportamiento:

(Angulares nulas)

A
$$\varepsilon_{xA} = \frac{1}{E}\left[\sigma_x - \nu(\sigma_y + \sigma_z)\right] + \alpha \Delta T = \frac{1}{E}\left[-\sigma_x - 0'3(-80 - \sigma_{zA})\right] + 12 \cdot 10^{-6} \cdot 20 =$$
$$\varepsilon_{yA} = \frac{1}{E}\left[\sigma_y - \nu(\sigma_z + \sigma_x)\right] + \alpha \Delta T = \frac{1}{E}\left[-80 - 0'3(-\sigma_x - \sigma_{zA})\right] + 12 \cdot 10^{-6} \cdot 20 = \varepsilon_y$$
$$\varepsilon_{zA} = \frac{1}{E}\left[\sigma_z - \nu(\sigma_x + \sigma_y)\right] + \alpha \Delta T = \frac{1}{E}\left[-\sigma_{zA} - 0'3(-\sigma_x - 80)\right] + 12 \cdot 10^{-6} \cdot 20 = 0$$

B
$$\varepsilon_{xB} = \frac{1}{E}\left[-\sigma_x - 0'3(-\sigma_{zB})\right] + 12 \cdot 10^{-6} \cdot 20 = \varepsilon_{xB}$$
$$\varepsilon_{yB} = \frac{1}{E}\left[-0'3(-\sigma_{zB} - \sigma_x)\right] + 12 \cdot 10^{-6} \cdot 20 = \varepsilon_{yB}$$

$\varepsilon_{xA} + 2\varepsilon_{xB} = 0 \qquad \varepsilon_{xA} = -2\varepsilon_{xB}$

7 ecs 7 incóg:

$$\left\{\begin{array}{ll}
\frac{1}{200\cdot 10^{3}}\left[-\sigma_x+24+0'3\,\sigma_{zA}\right]+2'4\cdot 10^{-4}=-2\,\varepsilon_{xB} & (1) \\
\frac{1}{200\cdot 10^{-3}}\left[-80+0'3\,\sigma_x+\cancel{0'3\,\sigma_{zA}}\right]+2'4\cdot 10^{-4}=\varepsilon_{yA} & (2) \\
\varepsilon_{yA}=2'4\cdot 10^{-4} \text{ (libremente)} & \\
\frac{1}{200\cdot 10^{-3}}\left[-\sigma_{zA}+0'3\,\sigma_x+24\right]+2'4\cdot 10^{-4}=0 & (3) \\
\frac{1}{200\cdot 10^{-3}}\left[-\sigma_x+0'3\,\sigma_{zB}\right]+2'4\cdot 10^{-4}=\varepsilon_{xB} & (4) \\
\frac{1}{200\cdot 10^{-3}}\left[0'3\,\sigma_{zB}+0'3\,\sigma_x\right]+2'4\cdot 10^{-4}=\varepsilon_{yB} & (5) \\
\varepsilon_{yB}=2'4\cdot 10^{-4} & \\
\frac{1}{200\cdot 10^{-3}}\left[-\sigma_{zB}+0'3\,\sigma_x\right]+2'4\cdot 10^{-4}=0 & (6)
\end{array}\right.$$

(1), (4) $\frac{1}{\cancel{200\cdot 10^{-3}}}\left[-\sigma_x+24+0'3\,\sigma_{zA}\right]+\cancel{2'4\cdot 10^{-4}}=\frac{-2}{\cancel{200\cdot 10^{-3}}}\left[-\sigma_x+0'3\,\sigma_{zB}\right]\cancel{+2'4}$

$$-\sigma_x+24+0'3\,\sigma_{zA}=2\,\sigma_x-0'6\,\sigma_{zB}$$

$$24+0'3\,\sigma_{zA}=3\,\sigma_x-0'6\,\sigma_{zB}$$

(6) $-\sigma_{zB}+0'3\,\sigma_x=-48$ $\qquad \sigma_{zB}=0'3\,\sigma_x+48$

(3) $-\sigma_{zA}+0'3\,\sigma_x+24=-48$ $\qquad \sigma_{zA}=0'3\,\sigma_x+72$

$$24+0'3\left[0'3\,\sigma_x+72\right]=3\,\sigma_x-0'6\left[0'3\,\sigma_x+48\right]$$

$$24+21'6+28'8=(3-0'18-0'09)\,\sigma_x$$

a compresión

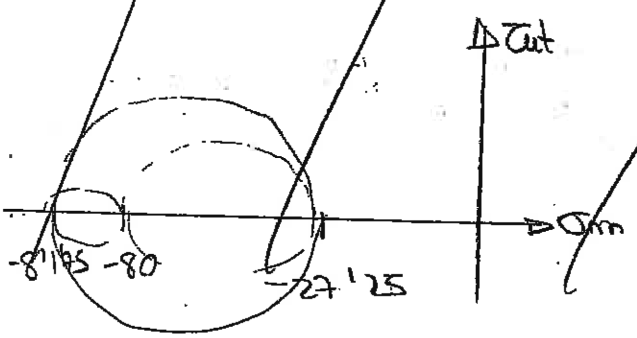
$\sigma_x=27'25$ MPa

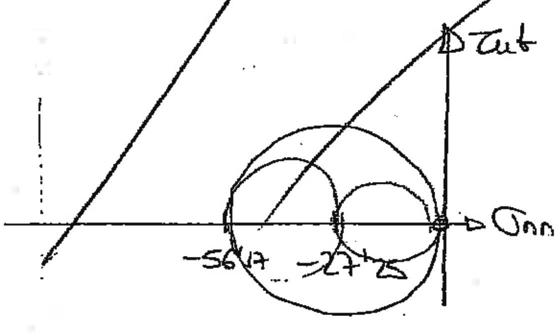
$\sigma_{zA}=80'175$ MPa ✓ A compresión

$\sigma_{zB}=56'1768$ MPa ✓ A compresión

Bloque A $\begin{cases} \sigma_1 = -80'175 = \sigma_{zA} \\ \sigma_2 = -80 = \sigma_{yA} \\ \sigma_3 = -27'25 = \sigma_{xA} \end{cases}$

Bloque B $\begin{cases} \sigma_1 = 0 = \sigma_{yB} \\ \sigma_2 = -56'17 = \sigma_{zB} \\ \sigma_3 = -27'25 = \sigma_{xB} \end{cases}$





$$\frac{1}{E}\left[-\sigma_x + 24 + 0'3\sigma_{zA}\right] + 2'4\cdot 10^{-4} = \frac{+2}{E}\left[-\sigma_x + 0'3\sigma_{zB}\right] - 4'8\cdot 10^{-4}$$

$$\frac{1}{E}\left[\cancel{-\sigma_x} + \cancel{24} + 0'3\sigma_{zA} - 2\cancel{\sigma_x} + 0'6\sigma_{zB}\right] = -7'2\cdot 10^{-4}$$

A $\begin{cases} \sigma_x = -80 \\ \sigma_y = -80 \\ \sigma_z = -96 \end{cases}$ MPa

$$-3\sigma_x + 24 + 0'3\sigma_{zA} + 0'6\sigma_{zB} = -144$$

$$-3\sigma_x + 0'3\sigma_{zA} + 0'6\sigma_{zB} = -168$$

B $\begin{cases} \sigma_x = -80 \\ \sigma_y = 0 \\ \sigma_z = -72 \end{cases}$ MPa

(6) $-\sigma_{zB} + 0'3\sigma_x = -48$ $\quad \sigma_{zB} = 0'3\sigma_x + 48$

(3) $-\sigma_{zA} + 0'3\sigma_x = -48 - 24$ $\quad \sigma_{zA} = 0'3\sigma_x + 72$

$\sigma_{zA} = 96$ MPa a c
$\sigma_{zB} = 72$ MPa a comp.

$$-3\cancel{\sigma_x} + 0'3\left[0'3\cancel{\sigma_x} + \cancel{72}\right] + 0'6\left[0'3\sigma_x + 48\right] = -168$$

~~$-2'01\sigma_x = -218'4$~~ $\quad$ ~~$\sigma_x = 108'65$ Resultado mal?~~

$-2'73\sigma_x = -218'4$!

$\sigma_x = 80$ MPa A compresión

2) Fácil → igual que todos xo con $\sigma_1, \sigma_2, \sigma_3$ de A y B.

A : $\sigma_{eq} = -80 - (-96) = \frac{\sigma_f}{n} = \frac{240}{n}$ $\quad n = 15$

B : $\sigma_{eq} = 0 - (-80) = 240/n$ $\quad n = 3$

Problema 6.8.

Datos:

$a = 80 \, mm$ $\quad E = 200 \, GPa$ $\quad \sigma_f = 240 \, MPa$

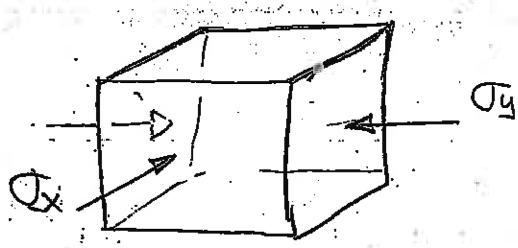
$b = 100 \, mm$ $\quad \alpha = 12 \cdot 10^{-6} \, °C^{-1}$ $\quad \Delta T = 80°C$

$c = 50 \, mm$ $\quad \nu = 0'3$ $\quad K = 10^5 \, N/mm$

Inicialmente no existen tensiones (muelle → x=0)

1) $\Delta T = 80°C$ → se dilata →

dirección X: restringida la dilatación lib.
dirección Y: se dilata xo el mu. se contrae y le empuja
dirección Z: se dilata libremente



a) Equilibrio. no dice nada

b) Compatibilidad de def.

$\varepsilon_z \checkmark = \alpha \Delta T = 9'6 \cdot 10^{-4}$

$\varepsilon_x = 0$

$\varepsilon_y \checkmark \; \underset{\downarrow}{y_{muelle}} = \varepsilon_y \cdot b$

$$\boxed{a \cdot c \sigma_y = K \cdot \varepsilon_y \cdot b}$$

c) Ley de comportamiento

$$\varepsilon_x = \frac{1}{E}\left[-\sigma_x - \nu(-\sigma_y)\right] + \alpha \Delta T = 0$$

$$\varepsilon_y = \alpha \Delta T = 9'6 \cdot 10^{-4} \longrightarrow$$

$$\sigma_y = \frac{K \cdot \varepsilon_y \, b}{a c} = \frac{10^5 N/\cancel{mm} \cdot 9'6 \cdot 10^{-4} \cdot 100}{80\cancel{mm} \cdot 50mm}$$

$$\sigma_y = 3'84 \frac{N}{mm^2} \cdot \frac{10^6 mm^2}{1 m^2} = 3'84 \, MPa$$

A Compresión

$$-\sigma_x - 0'3(-3'84) + 1'2 \cdot 10^{-6} \cdot 80 \cdot 200 \cdot 10^3 = 0$$

$\sigma_x = 193'15 \, MPa$ A Compresión

$\sigma_z = 0$

2) Componentes intrínsecas del vector tensión asociado al plano que forma ángulos iguales con los ejes XYZ.

$\vec{n}\,(1/\sqrt{3}, 1/\sqrt{3}, 1/\sqrt{3})$; $\{\vec{\sigma}_n\} = [T_{ij}]\{\vec{n}\}$

$$\begin{bmatrix} -193'15 & 0 & 0 \\ 0 & -3'84 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} 1/\sqrt{3} \\ 1/\sqrt{3} \\ 1/\sqrt{3} \end{Bmatrix} = \begin{Bmatrix} -111'51 \\ -2'21 \\ 0 \end{Bmatrix}$$

$\sigma_{nn} = \frac{1}{3}\{-193'15 \;\; -3'84 \;\; 0\}\begin{Bmatrix}1\\1\\1\end{Bmatrix}$

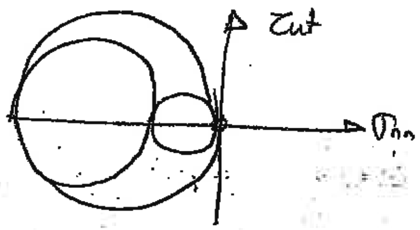
$\sigma_{nn} = -65'66$ MPa

$\tau_{nt} = \sqrt{|\sigma_n|^2 - |\sigma_{nn}|^2}$

$|\vec{\sigma}_n|^2 = 12439'36$

$\tau_{nt} = 90'1561$ MPa

3) Mohr: $\begin{cases} \sigma_1 = 0 \\ \sigma_2 = -3'84 \\ \sigma_3 = -193'15 \end{cases}$



$\tau_{nt\,max} = \frac{193'15}{2}$

$\sigma_{eq} = 193'15 = \frac{\sigma_f}{n} = \frac{240}{n} \rightarrow \boxed{n = 1'24}$

Problema 2000.17.

arista = 1cm

cavidad indeformable.

$E = 200\ GPa$

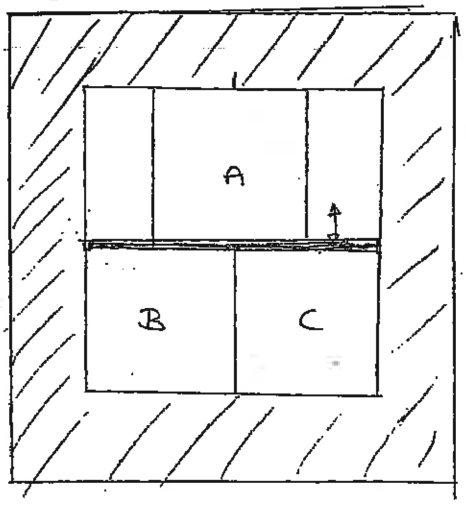
$\nu = 1/3$

$\sigma_f = 360\ MPa$

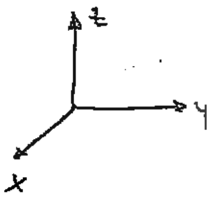
$\alpha = 10^{-5}\ ºC^{-1}$

∄ roz → $\varepsilon_{xy} = 0$ cortantes nulas

No se tiene en cuenta el peso



Sup = 1 cm²



$\boxed{A} \rightarrow \Delta T$

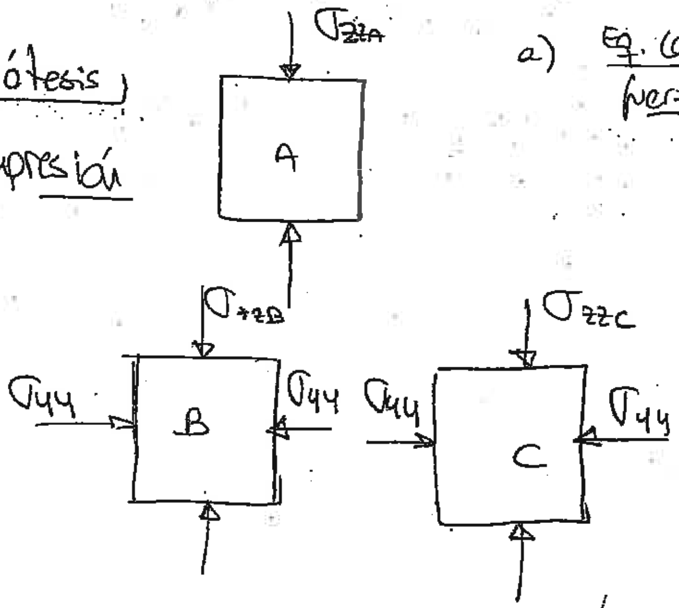
en dirección X → libre

problema plano. (∄ cortantes)

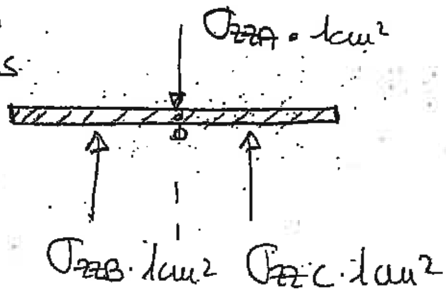
1º) Matrices tensiones 3 cubos si $\Delta T = 130ºC$

2º) ΔT máx / no se produzca el fallo según Tresca

Hipótesis, Compresión



a) Eq. con fuerzas



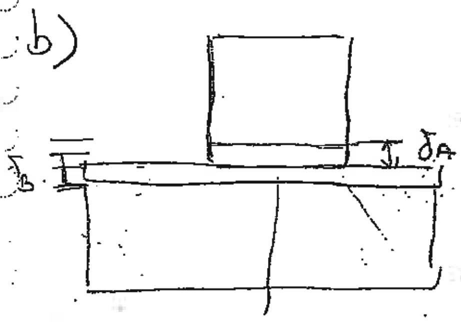
$\sigma_{zzA} = 2\sigma_f$

$1 \cdot \sigma_{zzA} = \sigma_{zzB} \cdot 1 + \sigma_{zzC} \cdot 1$

$\Sigma M_O = 0 \quad \sigma_{zzC} \cdot 1cm^2 \cdot \frac{1}{2} = \sigma_{zzB} \cdot 1cm^2$

$\boxed{\sigma_{zzC} = \sigma_{zzB}} = \sigma_f$

b)



A → alarga δ_A

B, C → acortan $\delta_B = \delta_C$

Como $\delta_{TOTAL} = 0$

$\delta_A = -(\delta_B)$

$\delta_A = \delta_B = \delta_C = \delta$

pero ¡ojo! con signo!!

Además, B, C → Se comportan igual

↳ tenderán a un ensancha-

miento lateral rest → deus -1- de comprensión ...

c) $\hat{A} \rightarrow \varepsilon_{zz} = \frac{1}{E}\left[\sigma_{zz_A} - \nu\,(\cancel{\sigma_{yy_A}} + \cancel{\sigma_{xx_A}})\right] + \alpha\,\Delta T$

$\varepsilon_{zz} = \frac{1}{E}\cdot\sigma_{zz_A} + \alpha\,\Delta T \quad (+)$

$\varepsilon_{zz} = -\frac{2\sigma_{zz}}{E} + (\alpha\,\Delta T) \quad (+)$

B, C (=) $\rightarrow \varepsilon_{zz} = \frac{1}{E}\left[\sigma_{zz_B} - \nu\,(\sigma_{yy_B} + \sigma_{xx_B})\right]$

$\varepsilon_{zz} = \frac{1}{E}\left[-\sigma_{zz} - \nu(-\sigma_{yy})\right] \quad (-)$

(=) en módulo

$-\frac{2\sigma_{zz}}{200\cdot 10^9} + 10^{-5}\,{}^\circ C^{-1}\cdot 130^\circ C = \frac{1}{200\cdot 10^9}\left[-\sigma_{zz} + \frac{1}{3}\sigma_{yy}\right]$ se compr.

Necesito otra ecuación:

B, C $\quad \varepsilon_{yy} = \frac{1}{E}\left[\sigma_{yy} - \nu\,(\sigma_{xx} + \sigma_{zz})\right] =$

$\varepsilon_{yy} = \frac{1}{E}\left(-\sigma_{yy} + \nu\,\sigma_{zz}\right) = 0$

$\nu\,\sigma_{zz} = \sigma_{yy}$

$-\frac{2\sigma_{zz}}{200\cdot 10^9} + 10^{-5}\cdot 130 = \frac{1}{200\cdot 10^9}\left(-\sigma_{zz} + \frac{1}{3}\cdot\frac{1}{3}\sigma_{zz}\right)$

$\delta_A = \varepsilon_{zz_A}\cdot \cancel{\ell_A} = -\varepsilon_{zz}\cdot \cancel{\ell_B}$ iguales = 1cm.

$\varepsilon_{zz_A} = -\varepsilon_{zz}$

$-\frac{2\sigma_{zz}}{200\cdot 10^9} + 10^{-5}\cdot 130 = \frac{-1}{200\cdot 10^9}\left[-\sigma_{zz} + \frac{1}{3}\cdot\frac{1}{3}\sigma_{zz}\right]$

$-2\sigma_{zz} + 260\cdot 10^6 = \sigma_{zz} + 0'12\,\sigma_{zz} \qquad \sigma_{zz} = \underline{80'2\ MPa}$ a compresi.

$$[A] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -180 \end{bmatrix} \qquad [B]=[C]= \begin{bmatrix} 0 & 0 & 0 \\ 0 & -30 & 0 \\ 0 & 0 & -90 \end{bmatrix}$$

en MPa

2º) ΔT ? Tresca : $\sigma_{eq} = \sigma_{max} - \sigma_{min} = \sigma_f = 360$ MPa

val. sin signo

$\boxed{A}$ $\sigma_{eq} = 0 - (-\sigma_{zz_A}) = \sigma_{zz_A} = 2\sigma_{zz} \leq 360$ (MPa)

$\boxed{B} = \boxed{C}$ $\sigma_{eq} = 0 - (-\sigma_{zz_B}) = \sigma_{zz_B} = \sigma_{zz} \leq 360$ (MPa)

si $360 = \sigma_{zz}$
$\sigma_{yy} = 0$
No puede ser

Lo más restrictivo $\sigma_{zz} = \frac{360}{2} = 180$

$\boxed{A}$

$$\frac{-2\sigma_{zz}}{200\cdot 10^9} + \Delta T \cdot \alpha = \frac{-1}{200\cdot 10^9}\left[-\sigma_{zz} + \frac{1}{9}\cdot \sigma_{zz}\right]$$

$$\frac{-2\cdot 180\cdot 10^6}{200\cdot 10^9} + \Delta T \cdot 10^{-5} = \frac{1}{200\cdot 10^9}\left(180\cdot 10^6 - \frac{1}{9}\cdot 180\cdot 10^6\right)$$

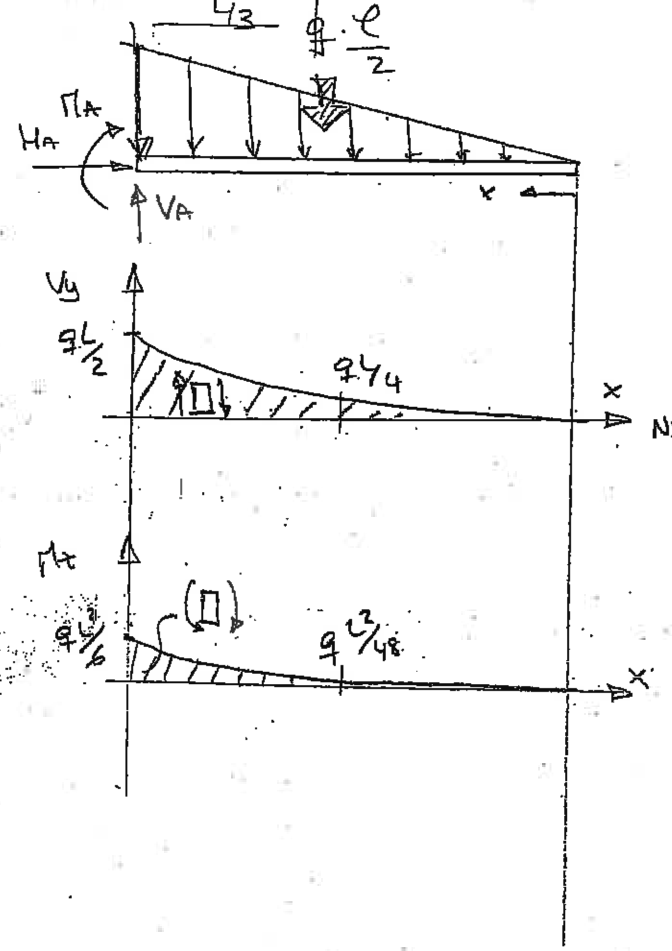
$$-2\cdot 180\cdot \cancel{10^6} + \Delta T \cdot \underset{2}{\cancel{200}}\cdot \cancel{10^4} = 180\cdot \cancel{10^6} - \frac{1}{9}\cdot 180\cdot \cancel{10^6}$$

$\Delta T = 260°C$

Problema Tema 7: Introducción a la Resistencia de Materiales

Problema 7.1.

Reacciones y diagramas de momento flector y cortante



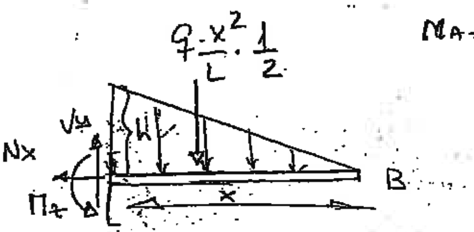
$H_A = 0$

$V_A = q \cdot \frac{L}{2}$

$M_A + q\frac{L}{2} \cdot \frac{L}{3} = 0 \qquad M_A = -\frac{qL^2}{6}$

$\downarrow$

$M_A = \frac{qL^2}{6}$ en $\oplus$



$h: \quad \frac{q}{L} = \frac{h}{x} \rightarrow h = \frac{q \cdot x}{L}$

$N_x = 0$

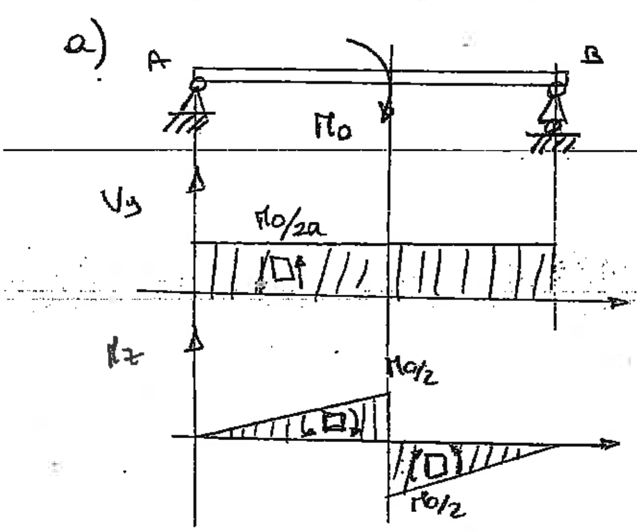
$$\boxed{V_y = \frac{q x^2}{2L}} \left\{ \begin{array}{l} x = L \rightarrow V_y = \frac{qL}{2} \\ x = L/2 \rightarrow V_y = \frac{q}{2} \rightarrow \frac{qL}{8} \\ x = 0 \rightarrow V_y = 0 \end{array} \right.$$

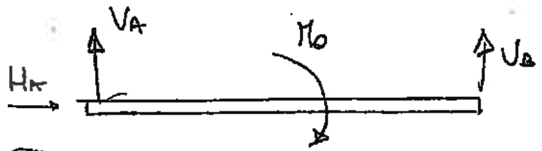
$M_z - \frac{q x^2}{2L} \cdot \frac{x}{3} = 0$

$$\boxed{M_z = \frac{q x^3}{6L}} \left\{ \begin{array}{l} x = L \rightarrow \frac{qL^2}{6} \\ x = L/2 \rightarrow \frac{qL^2}{48} \\ x = 0 \rightarrow 0 \end{array} \right.$$

Problema 7.2

a)





$\boxed{H_A = 0}$

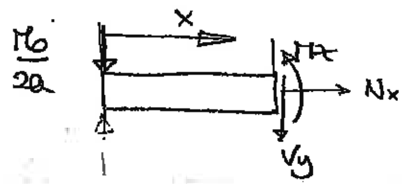
$\boxed{V_A = -\frac{M_0}{2a}}$

$V_A + V_B = 0$

$\Sigma M_A = 0 \rightarrow -M_0 + V_B \cdot 2a \qquad M_0 = V_B 2a$

$\boxed{V_B = \frac{M_0}{2a}}$

$\boxed{0 \le x \le a}$



$N_x = 0$

$V_y = -\frac{M_0}{2a} \rightarrow V_y = \frac{M_0}{2a}$

$M_z = -\frac{M_0}{2a} x$

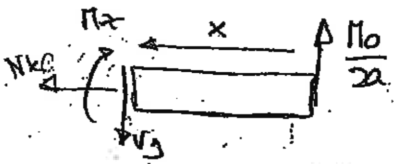
$M_z = \frac{M_0 x}{2a}$

$x = 0 \rightarrow M_z = 0$

$x = \frac{a}{2} \rightarrow M_0/4$

$x = a \rightarrow M_0/2$

$\boxed{0 \le x \le a}$



$N_x = 0$

$V_y = \frac{M_0}{2a}$

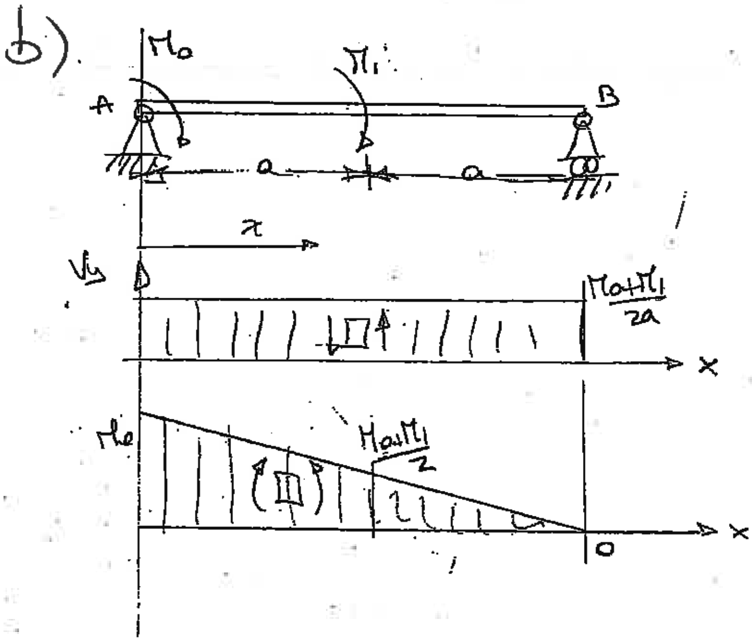
$M_z = \frac{M_0}{2a} x$

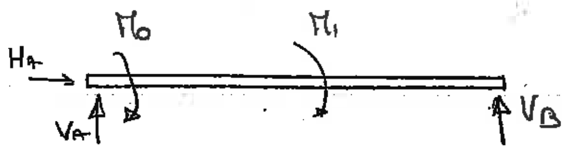
$x = a \quad M_z = \frac{M_0}{2}$

$x = a/2 \quad M_z = M_0/4$

$x = 0 \quad M_z = 0$

b)





$\boxed{H_A = 0}$

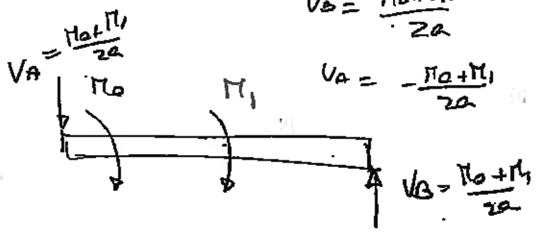
$\boxed{V_A = -V_B}$

$\Sigma M_A = 0$

$M_0 + M_1 = V_B \cdot 2a$

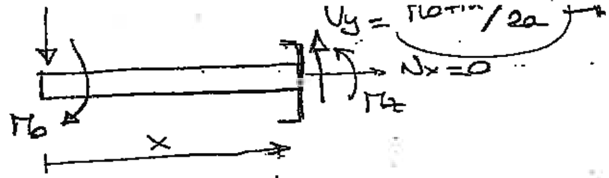
$V_B = \frac{M_0 + M_1}{2a}$

$V_A = -\frac{M_0 + M_1}{2a}$



$M_1 < M_0$

$\boxed{0 \le x \le a}$



$\frac{M_0+M_1}{2a}$

$$-M_0 + M_z + \frac{M_0+M_1}{2a} \cdot x = 0$$

$$M_z = M_0 - \frac{M_0+M_1}{2a} x$$

$$M_z = \frac{M_0(2a-x) + M_1 x}{2a}$$

$2a \ge x$ ✓

$x = 0 \rightarrow A$

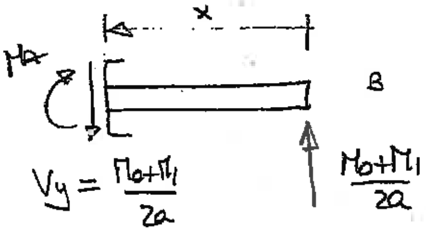
$M_{zA} = M_0$

$x = a/2 \rightarrow \frac{3\cancel{a}}{2} M_0 + \frac{\cancel{a}}{2} M_1 / \cancel{2a}$

$\frac{3M_0 + M_1}{8}$

$x = a \quad \frac{M_0+M_1}{2}$

$\boxed{0 \le x \le a}$



$\frac{M_0+M_1}{2a}$

$$V_y = \frac{M_0+M_1}{2a}$$

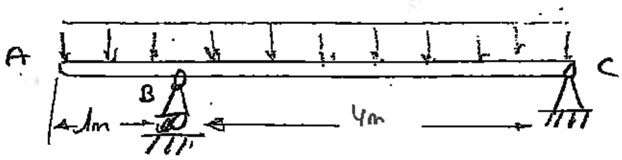
$$M_z = \frac{M_0+M_1}{2a} x$$

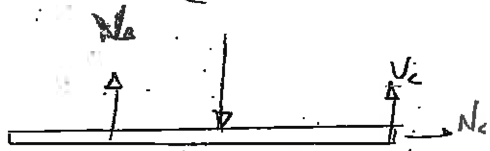
$x = 0 \rightarrow B \quad M_z = 0$

$x = a/2 \rightarrow M_z = \frac{M_0+M_1}{4}$

$x = a \rightarrow M_z = \frac{M_0+M_1}{2}$

Problema 7.3.





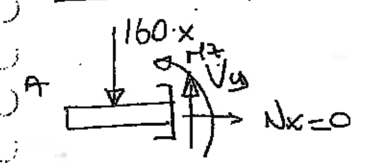
$N_c = 0 \qquad V_c = 300$ KN

$V_B + V_c - 800 = 0$

$\Sigma M_c = 0 \quad 800 \cdot 2'5 - V_B \cdot 4 = 0$

$V_B = 500$ KN

$0 \le x \le 1m$



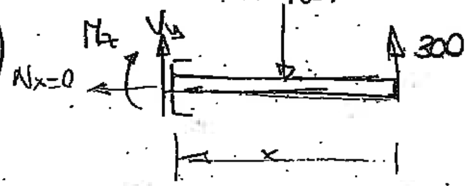
$V_y = 160x \rightarrow$

$\Sigma M = 0 \quad 160 \cdot x \cdot \frac{x}{2} + M_z = 0$

$M_z = -80x^2$

$M_z = 80x^2$

$0 \le x \le 4m$



$V_y = 300 - 160x$

$x = 0 \rightarrow 300$ (C)

$x = 2 \rightarrow -20$

$x = 4 \rightarrow (B) -340$

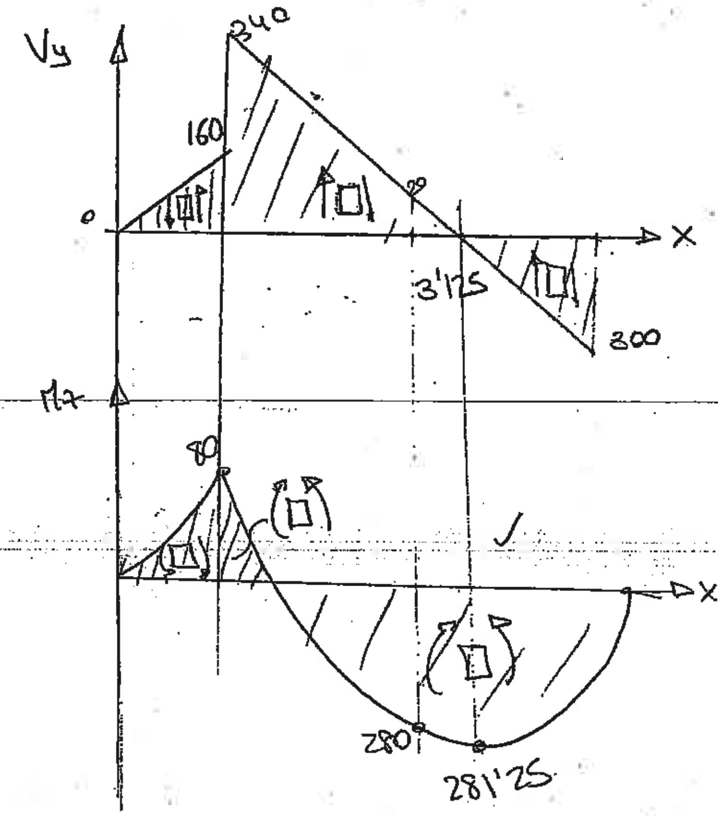
$\Sigma M_z = 0 \quad 300x - 160\frac{x^2}{2} - M_z = 0$

$M_z = 300x - 80x^2$

$x = 0 \rightarrow 0$

$x = 2 \rightarrow 280$

$x = 4 \rightarrow -80$



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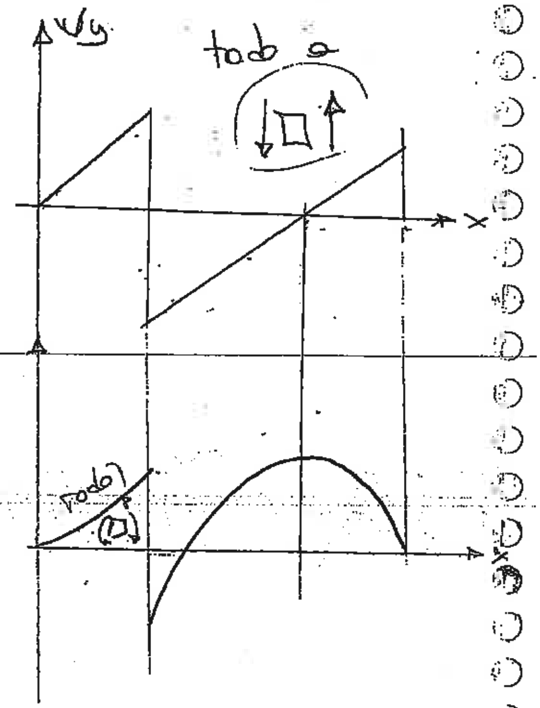
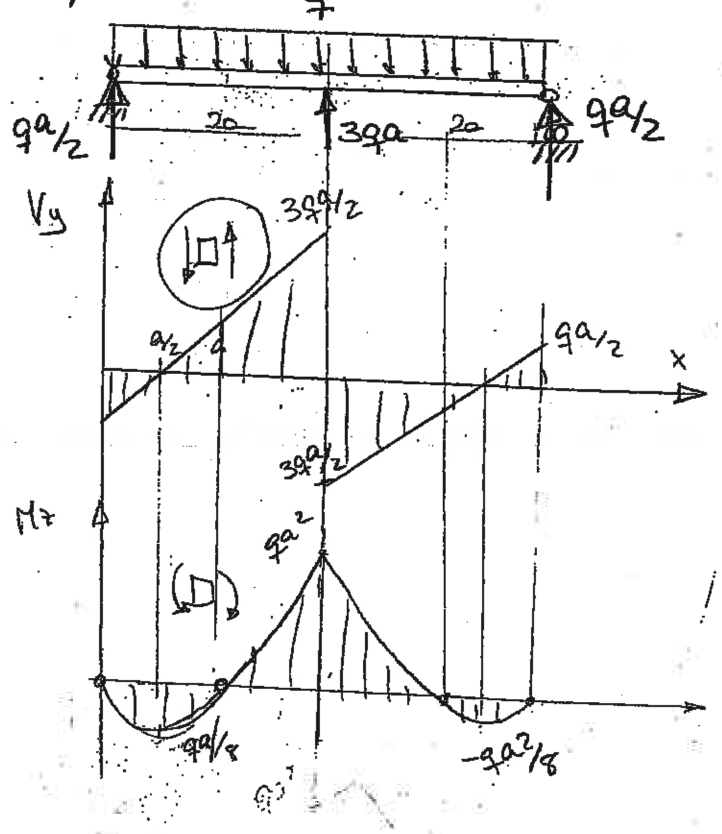
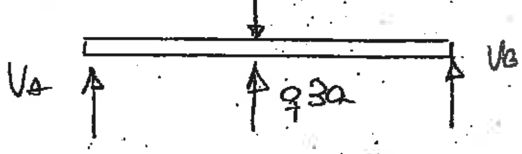


Problema 4.4.

a)



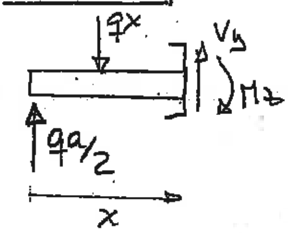


$V_A + V_B = qa$

$V_B \cdot 4\cancel{a} = qa \cdot 2\cancel{a}$ $V_B = \frac{qa}{2}$

$V_A = \frac{qa}{2}$

$\boxed{0 \le x \le 2a}$



$\frac{qa}{2} + V_y = qx$

$V_y = qx - \frac{qa}{2}$

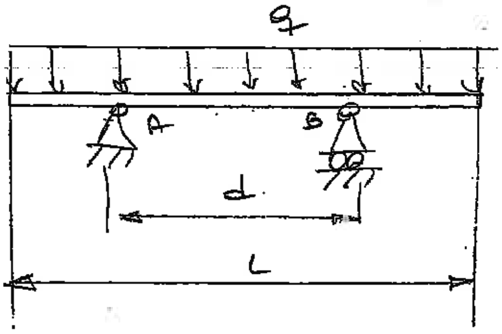
$\left\{\begin{array}{ll} x=0 & V_y = -\frac{qa}{2} \\ x=2a & V_y = \frac{3qa}{2} \end{array}\right.$

$\frac{qx^2}{2} - \frac{qa}{2}x = M_z$

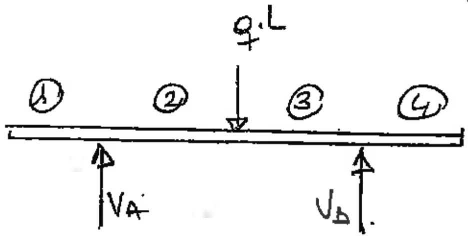
$M_z = \frac{q}{2}(x^2 - ax)$

$\left| \begin{array}{ll} x=0 & M_z = 0 \\ x=a & M_z = 0 \\ x=\frac{a}{2} & M_z = \frac{q}{2}\left(\frac{a^2}{4} - \right. \\ & M_z = -\frac{qa^2}{8} \\ x=2a & M_z = \frac{q}{2}(4a^2 - 2a \end{array}\right.$

Problema 7.5.



d / : $M_z(L/2) = M_A = M_B$

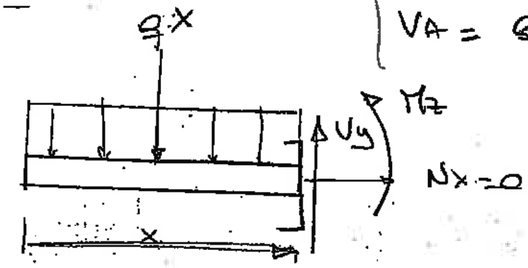


$V_A + V_B = q \cdot L$

$\Sigma M_A = 0 \qquad V_B \cdot d - q \cdot L \cdot d/2 = 0$

$$\begin{cases} V_B = qL/2 \\ V_A = qL/2 \end{cases}$$

① $\boxed{0 \leq x \leq \frac{L-d}{2}}$



$\boxed{V_y = qx}$

$V_y \quad x=0 \longrightarrow V_y = 0$

$x = \left(\frac{L-d}{2}\right) \longrightarrow V_y = q \cdot \left(\frac{L-d}{2}\right)$

$M_z + q \cdot x^2/2 = 0 \qquad M_z = -qx^2/2 \qquad \boxed{M_z = qx^2/2}$

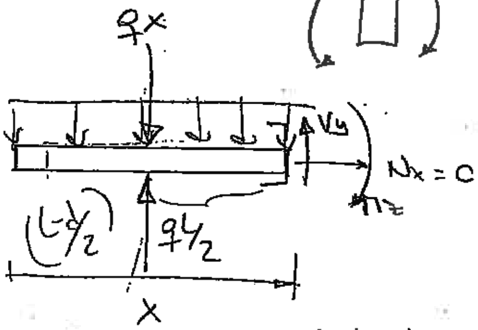
$x = 0 \longrightarrow 0$

$x = \frac{(L-d)}{4} \longrightarrow +q\frac{(L-d)}{32}$

$x = \left(\frac{L-d}{2}\right) \longrightarrow +q\frac{(L-d)}{8}$

" M_A "

② $\boxed{\left(\frac{L-d}{2}\right) \leq x \leq L/2}$



$V_y + qL/2 - qx = 0$

$\boxed{V_y = q(x - L/2)}$

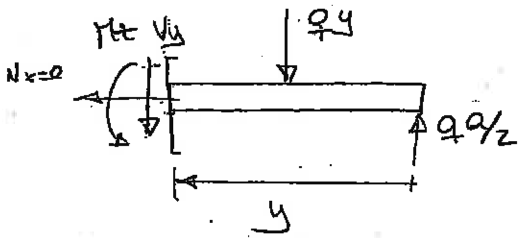
$x = \left(\frac{L-d}{2}\right) \longrightarrow q\left(\frac{L}{2} - \frac{d}{2} - \frac{L}{2}\right) \longrightarrow -q\frac{d}{2}$

$x = L/2 \longrightarrow q(L/2 - L/2) \longrightarrow 0$

$M_z - qL/2 \cdot \left[x - \left(\frac{L-d}{2}\right)\right] + q\,x^2/2 = 0$

$\boxed{2a \le x \le 4a}$ Tomo y!

$\boxed{2a \ge y \ge 0}$



$V_y + q\cdot y - q\,a/2 = 0$

$V_y = q\,a/2 - q\cdot y \qquad V_y = q(a/2 - y)$

$y=0$ (B) $\to q\,a/2$

$y=a/2 \to 0$

$y=a \to -qa/2$

$y=2a \to q = -3qa/2$

$+M_z - q\cdot y^2/2 + q\,a/2\cdot y = 0$

$M_z = -q\,a/2\cdot y + q\,y^2/2$

$M_z = q\,y/2\,(y-a)$

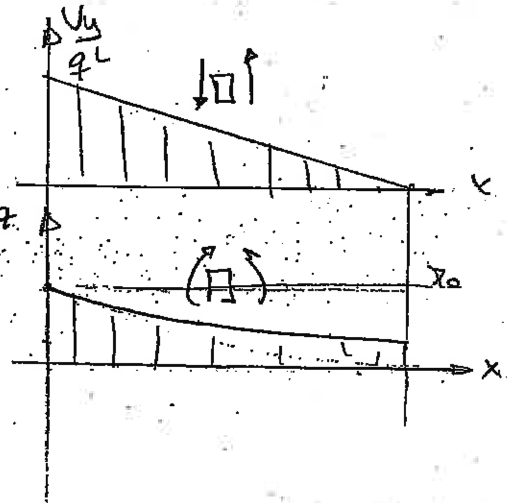
$y=0$ (B) $M_z = 0$

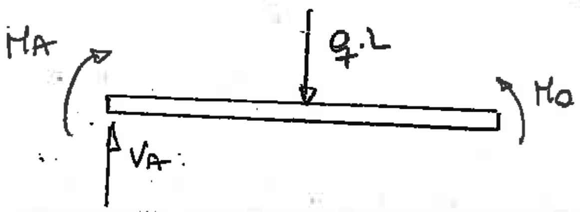
$y=a/2 \to q\,a/4\,(a/2 - a) \to -qa^2/8$

$y=a \to 0$

$y=2a \to q a^2$

b)



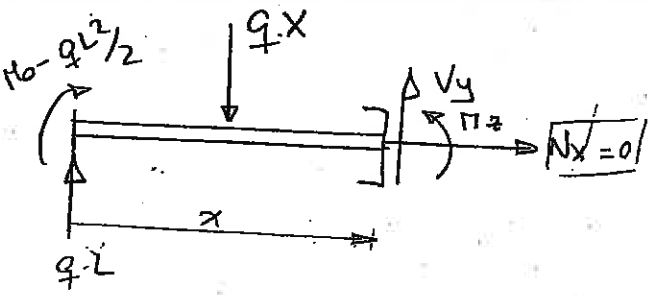


$\boxed{M_A=0}$

$\boxed{V_A = q\cdot L}$

$\Sigma M_A = 0 \quad -M_A + M_o - q\cdot L^2/2 = 0$

$\boxed{M_A = M_o - qL^2/2}$



$V_y - qx + qL = 0$

$\boxed{V_y = q(L-x)}$

$x=0 \to qL$

$x=L/2 \to qL/2$

$x=L \to 0$

$M_z - M_o + qL^2/2 + qx^2/2 - q\,Lx = 0$

$\boxed{M_z = M_o - qL^2/2 + qx\,(x/2 - L)}$

$x=0 \to M_o - qL^2/2$

$M_o - qL^2/2 - \frac{3qL^2}{8}$

$M_o - 7qL^2 \ldots$

$$M_z = q\frac{x^2}{2} - q\frac{Lx}{2} + q\frac{L}{2}\left(\frac{L-d}{2}\right)$$

$$\boxed{M_z = \frac{q}{2}\left[x(x-L) + \frac{L^2}{2} - \frac{Ld}{2}\right]} \quad (\text{II})$$

$x = \left(\frac{L-d}{2}\right)$ (A) $\rightarrow M_z(A) = \frac{q}{2}\left[\left(\frac{L-d}{2}\right)\left(\frac{L-d}{2}-L\right) + \frac{L^2}{2} - \frac{Ld}{2}\right] =$

$$= \frac{q}{2}\left[\left(\frac{L-d}{2}\right)^2 - \cancel{\frac{L^2}{2}} + \cancel{\frac{Ld}{2}} + \cancel{\frac{L^2}{2}} - \cancel{\frac{Ld}{2}}\right] = \underline{\frac{q}{8}(L-d)^2}$$

$x = L/2$ (Centro) $\rightarrow M_z(\text{centro}) = \frac{q}{2}\left[\frac{L}{2}\left(\frac{L}{2}-L\right) + \frac{L^2}{2} - \frac{Ld}{2}\right] =$

$$= \frac{q}{2}\left[\frac{L^2}{4} - \cancel{\frac{L^2}{2}} + \cancel{\frac{L^2}{2}} - \frac{Ld}{2}\right] = \underline{\frac{qL}{4}\left(\frac{L}{2}-d\right)}$$

$M_A = M_{centro}$: $\frac{\cancel{q}}{\cancel{4}\,2}(L^2+d^2-2Ld) = \frac{\cancel{q}L}{\cancel{4}}\left(\frac{L}{2}-d\right)$

$$\frac{1}{2}(\cancel{L^2}+d^2-2Ld) = \cancel{\frac{L^2}{2}} - dL$$

$$\frac{d^2}{2} - \cancel{Ld} = -\cancel{Ld} \qquad \boxed{d=0}$$

(3) Tomo y en vez de x: Problema simétrico:

$\frac{L}{2} \leq x \leq \left(\frac{L}{2}+\frac{d}{2}\right)\left(\frac{L+d}{2}\right)$

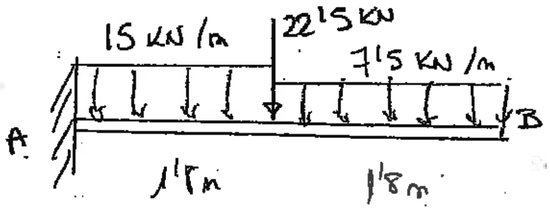
$\frac{L}{2} \geq y \geq \left(\frac{L-d}{2}\right)$

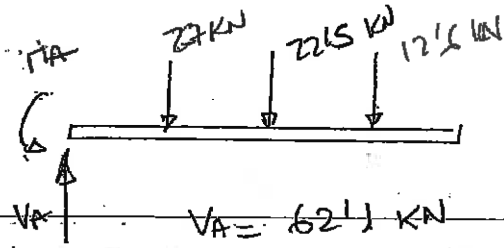
$$M_z = \frac{q}{2}\left[y(y-L) + \frac{L^2}{2} - \frac{Ld}{2}\right]$$

M_B

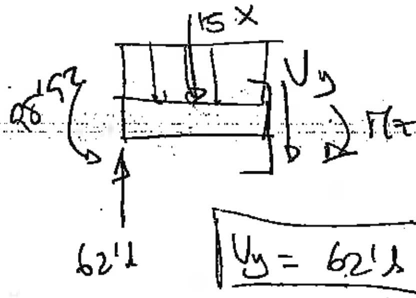
$$\frac{q}{8}(L-d)^2$$

Problema 7.6.





$V_A = 62'1$ KN



$$\boxed{V_y = 62'1 - 15x}$$

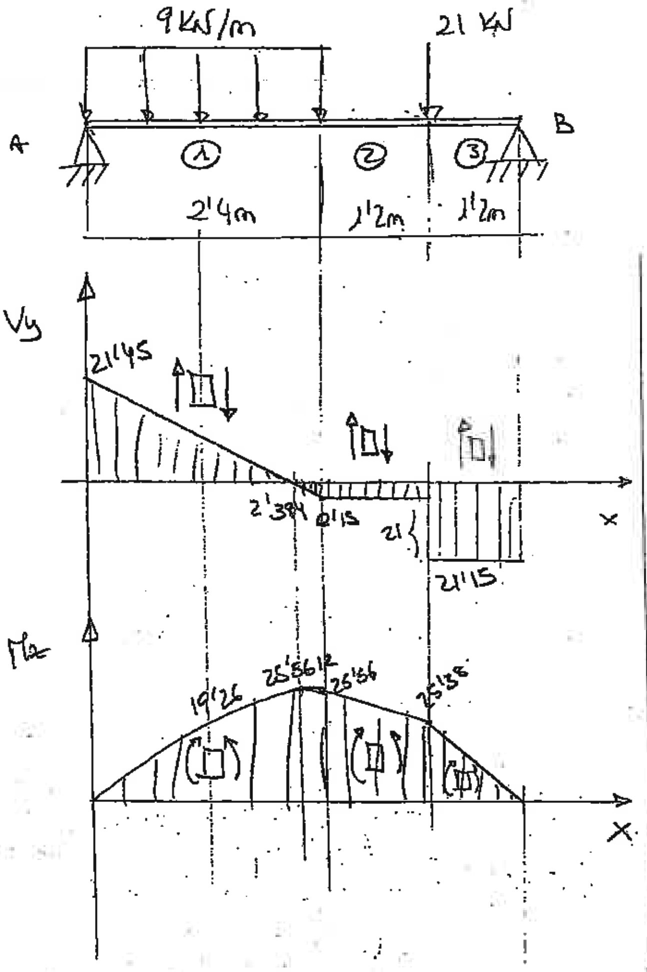
$$M_A = 27 \cdot 0'9 + 22'5 \cdot 1'8 + 12'6 \cdot 2'7$$

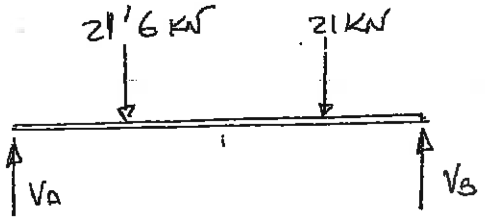
$$M_A = 98'82 \text{ KN} \cdot \text{m}$$

$$\boxed{M_z = 98'92 + \frac{15x^2}{2} - 62'1 \cdot x}$$

... igual q to?

Problema 7.7





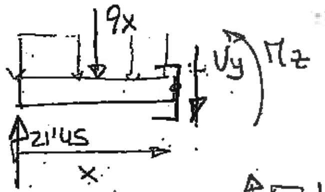
$V_A + V_B = 42'6\ kN$

$\Sigma M_A = 0. \quad V_B \cdot 4'8 = 21'6 \cdot 2'4 + 21 \cdot 3'6$

$V_B = 21'15\ kN$

$V_A = 21'45\ kN$

(1) $0 \le x \le 2'4\,m$



$V_y + 9x = 21'45$

$\boxed{V_y = 21'45 - 9x}$

$x = 0 \quad V_y = 21'45$

$x = 2'4 \quad V_y = -0'15$

$M_z + 9\frac{x^2}{2} - 21'45x = 0$

$M_z = 21'45x - 9x^2/2$

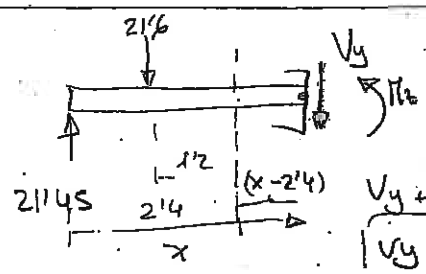
$x = 0 \quad M_z = 0$

$x = \frac{2'4}{2} = 1'2 \quad M_z = 19'26$

$x = 2'4 \quad M_z = 25'56$

$x = 2'384 \quad M_z = 25'56125$

(2)



$V_y + 21'6 = 21'45$

$\boxed{V_y = -0'15}$

$M_z + 21'6 \cdot [(x-2'4) + 1'2] - 21'45 \cdot x = 0$

$M_z = 21'45x - 21'6\,(x - 1'2) \quad 2'4 \le x \le 3'6$

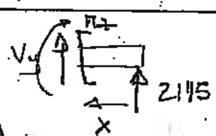
$M_z = -0'15x + 25'92$

$\boxed{M_z = 25'92 - 0'15x}$

$x = 2'4 \Rightarrow 25'56$

$x = 3'6 \Rightarrow 25'38$

(3)



$\boxed{V_y = -21'15}$

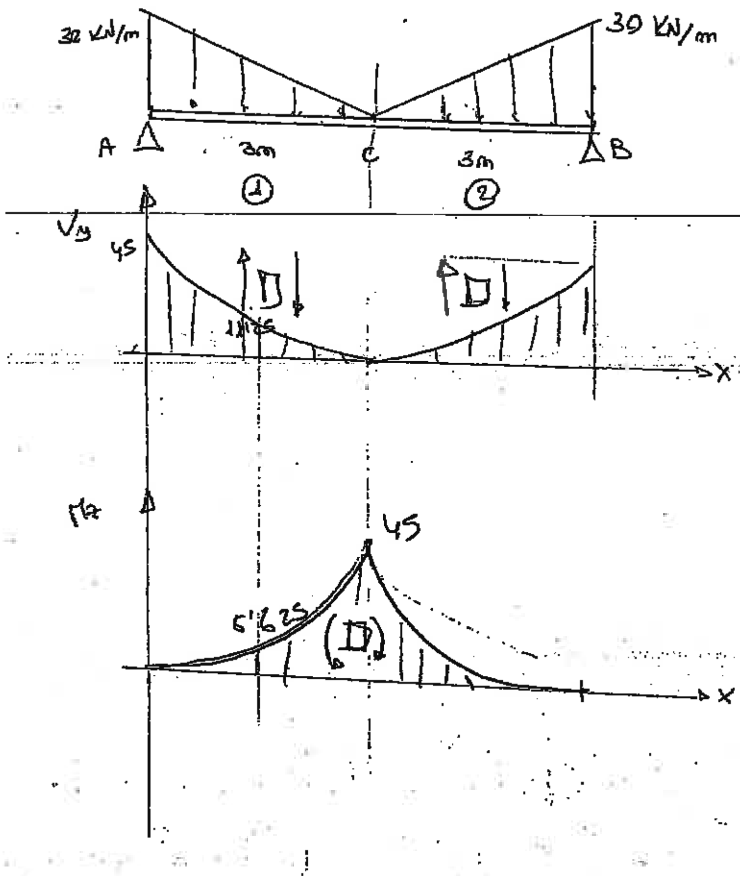
$-M_z + 21'15x = 0$

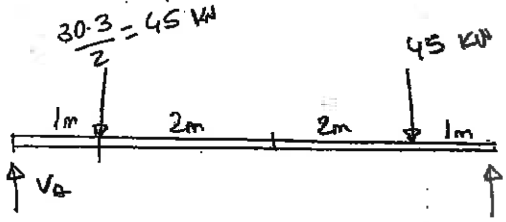
$\boxed{M_z = 21'15x}$

$x = 0 \quad (B)\ 0$

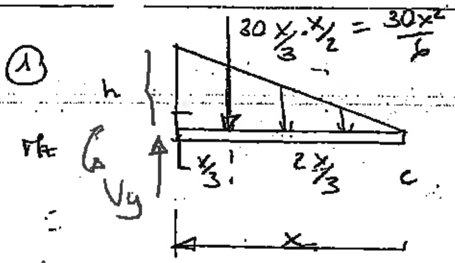
$x = 1'2 \quad \boxed{25'38}$

Problema 7.8.





$V_A = V_B = 45$ KN



$0 \leq x \leq 3$

$\frac{30}{3} = \frac{h}{x}$

$\boxed{h = \frac{30x}{3}}$

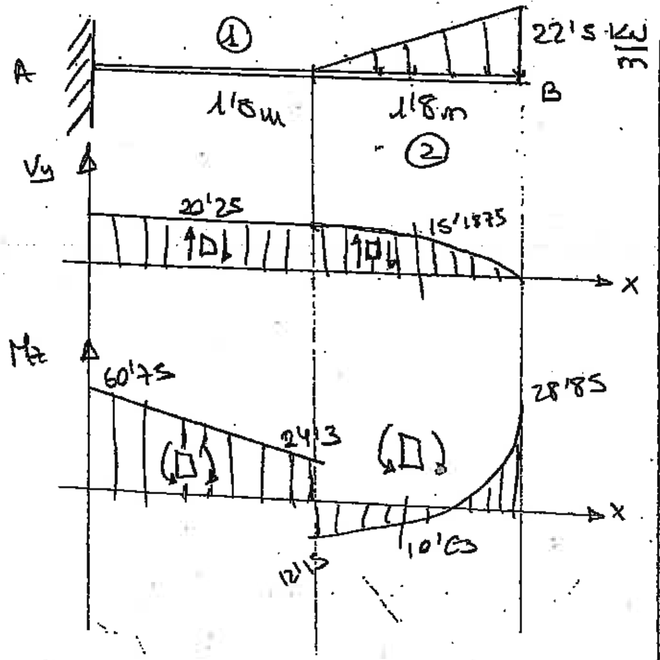
$V_y = \frac{30x^2}{6}$ $\left\{ \begin{array}{l} x=0 \;(C)\; 0 \\ x = 1'5 \rightarrow 11'25 \\ x = 3 \rightarrow 45 \end{array} \right.$

$+M_z - \frac{30x^2}{6}\cdot\frac{x}{3} = 0$

$M_z = \frac{30x^3}{18} = \frac{10x^3}{6}$

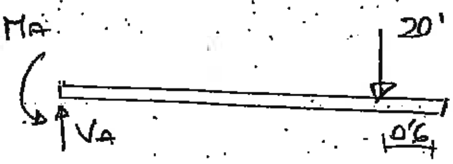
$\left| \begin{array}{l} x=0 \rightarrow 0 \\ x=1'5 \rightarrow 5'625 \\ x=3 \rightarrow 45 \end{array} \right.$

Problema 7.9.



$60'75 - M_z - 20'25\cdot 3'6 + \frac{22'5}{3'6}\cdot\frac{(x-1'8)^3}{3}$

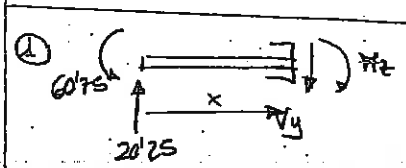
M \



$V_A = 20'25$ KN

$M_A = 20'25\cdot 3$

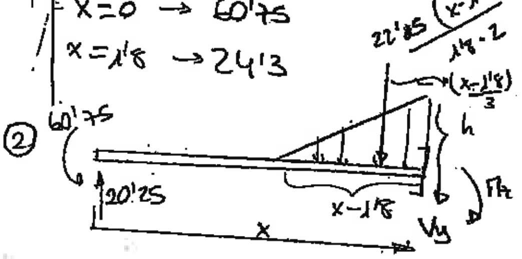
$M_A = 60'75$ KN



$\boxed{V_y = 20'25}$

$60'75 - 20'25x = M_z$

$\left| \begin{array}{l} x=0 \rightarrow 60'75 \\ x=1'8 \rightarrow 24'3 \end{array} \right.$



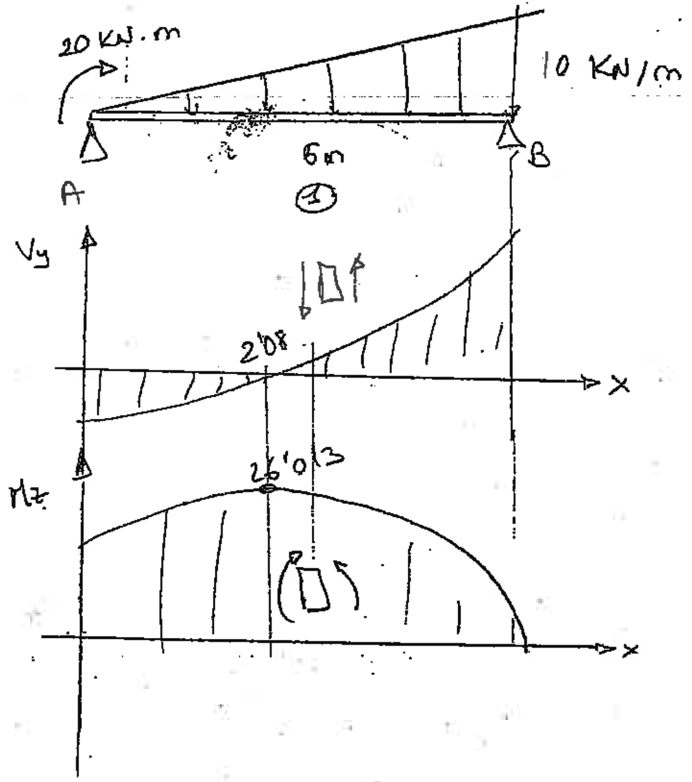
$\frac{22'5}{1'8} = \frac{h}{x-1'8}$

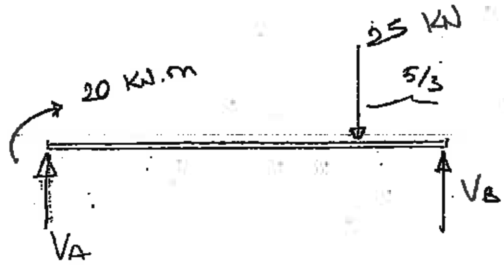
$h = \frac{22'5\,(x-1'8)}{1'8}$

$20'25 - \frac{22'5\,(x-1'8)^2}{1'8\cdot 2} = V_y$ $\left\{ \begin{array}{l} x=1'8 \rightarrow 20'25 \checkmark \\ x = 2'7 \rightarrow 15'1875 \\ x = 3'6 \rightarrow 0 \end{array} \right.$

$\left\} \begin{array}{l} x=1'8 \rightarrow -12'15 \\ x=2'7 \rightarrow -10'6312 \end{array} \right.$

Problema 7.11.



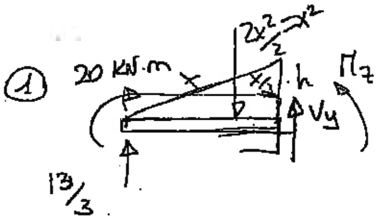


$V_A + V_B = 25$

$\Sigma M_A = 0 \quad V_B \cdot 5 - 20 - 25 \cdot 3'34 =$

$V_B = 20'67$ KN $= 62/3$

$V_A = 4'34$ KN $= 13/3$



$h: \quad \frac{10}{5} = \frac{h}{x}$

$h = \frac{10x}{5}$

$\boxed{h = 2x}$

$\boxed{V_y = x^2 - 13/3}$

$x = 0 \quad -13/3$

$x = 2'5 \quad 1'9167$

$x = 5 \quad 20'67$

$V_y = 0 \rightarrow 2'08$

$\frac{x^3}{3} - \frac{13}{3} \cdot x - 20 + M_z = 0$

$\boxed{M_z = 20 + \frac{13}{3}x - \frac{x^3}{3}}$

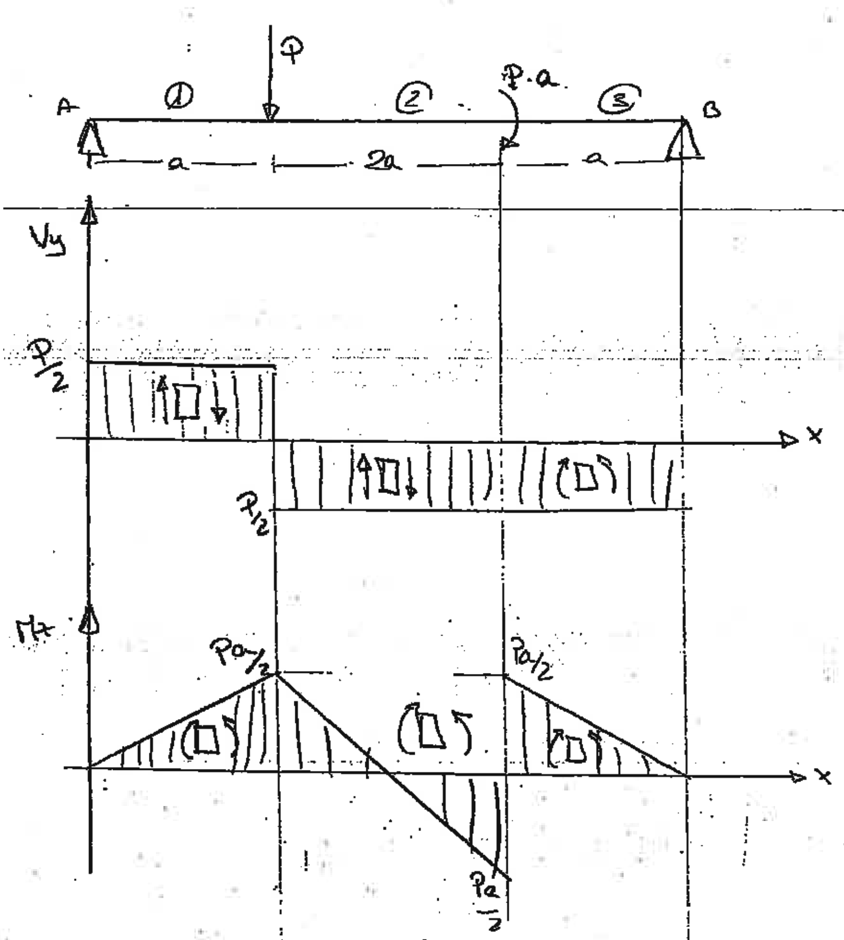
$x = 0 \rightarrow 20$

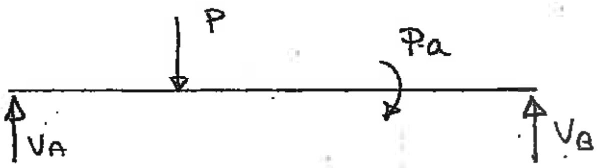
$x = 2'08 \rightarrow 26'013$

$x = 2'5 \rightarrow 25'62$

$x = 5 \rightarrow 0$

Problema 7.12.





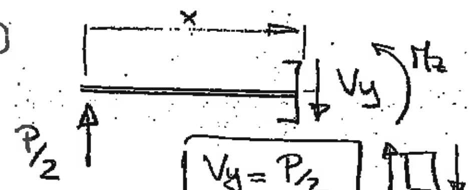
$V_A + V_B = P$

$V_B \cdot 4a - Pa - Pa = 0$

$V_B = \frac{\cancel{2}P\cancel{a}}{\cancel{2}4\cancel{a}} = P/2$ $\boxed{V_B = P/2}$

$\boxed{V_A = P/2}$

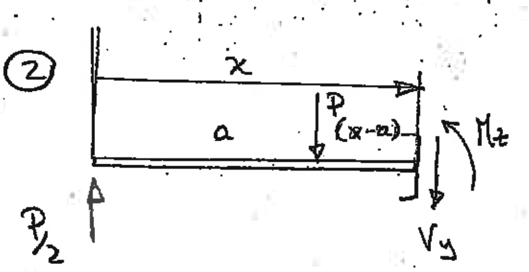
①



$\boxed{V_y = P/2}$

$-\frac{P}{2}x + M_z = 0$ $\boxed{M_z = \frac{Px}{2}}$

②



$P + V_y = P/2$

$\boxed{V_y = -P/2}$

$-P/2\,x + P(x-a) + M_z = 0$

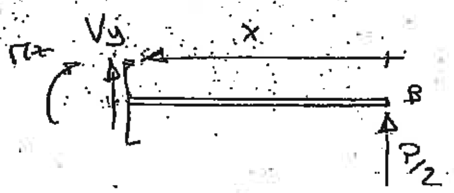
$\boxed{M_z = \frac{Px}{2} - P(x-a)}$

$x = a \quad Pa/2$

$x = 3a \quad \frac{3Pa}{2} - P(3a-a)$

$\frac{3Pa}{2} - 2Pa = -\frac{Pa}{2}$

③



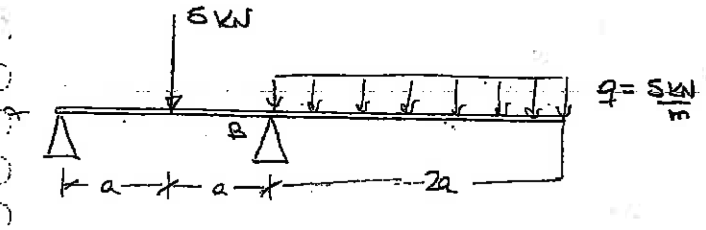
$V_y = -P/2$

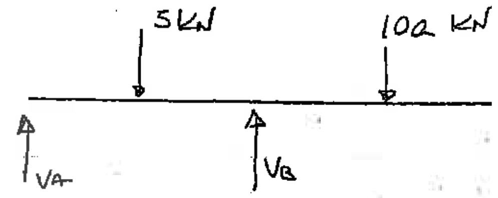
$-M_z + P/2\,x = 0 \qquad M_z = \frac{Px}{2}$

$x = 0 \quad (B) \quad 0$

$x = a \quad Pa/2$

Problema 7.13.





$V_A + V_B = 5 + 10a$

$V_B \cdot 2\cancel{a} - 5\cancel{a} - 10a\,3\cancel{a} = 0$

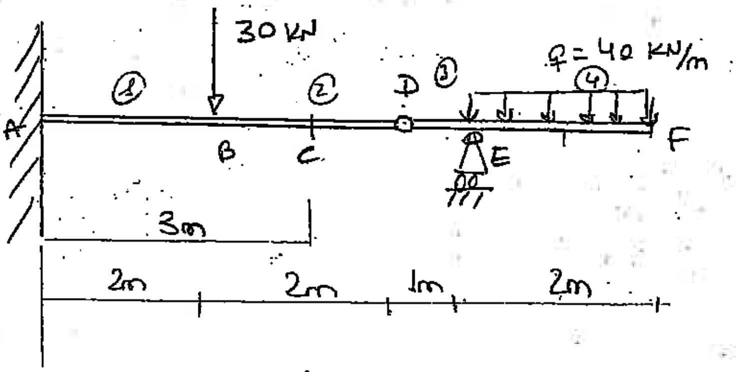
$\boxed{V_B = \dfrac{5+30a}{2}}$

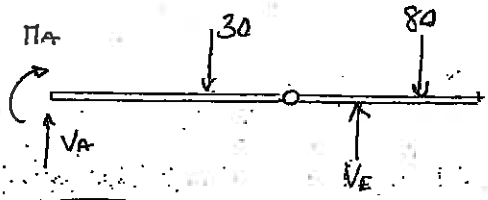
$V_A = \dfrac{10+20a-5-30a}{2}$

$\boxed{V_A = \dfrac{5-10a}{2}}$

única dificultad en letras.

Problema 7.14.



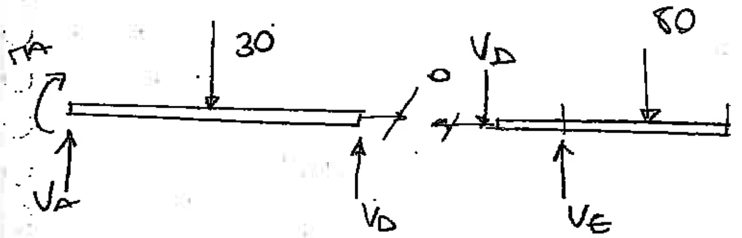


$\boxed{h = 1}$

$\boxed{V_A + V_E = 110}$

$-M_A - 60 + V_E \cdot 5 - 480 = 0$

$\boxed{M_A = 5 V_E - 540}$



$V_A + V_D = 30$

$V_D\,4 - M_A - 60 = 0$

$V_E = V_D + 80$

$V_E - 160 = 0$

$\boxed{V_E = 160 \text{ KN}}$

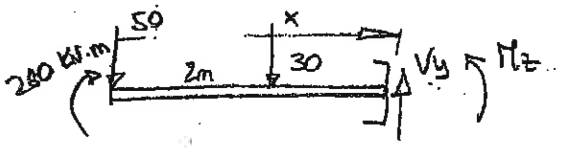
$\boxed{V_D = 80 \text{ KN}}$

$\boxed{A = -50 \text{ KN}}$

$\boxed{M_A = 260 \text{ KN·m}}$

Análisis de los esfuerzos interiores análogo:

estudio ②



$\boxed{V_y = 80\ kN}$ cte en ②

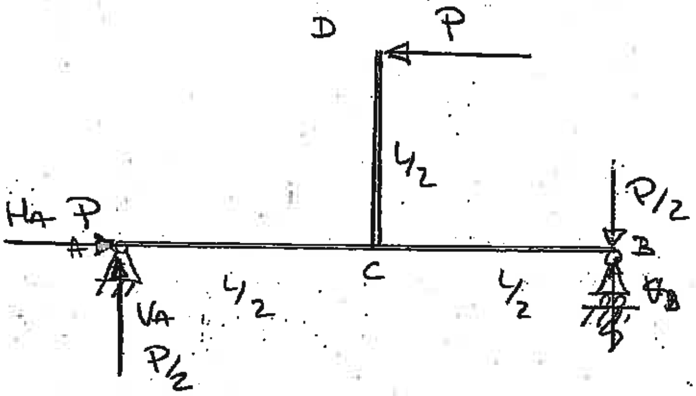
$\underline{V_C = 80\ kN}$

$30(x-2) + 50x - 260 + M_z = 0$

$M_z = 260 - 50x - 30(x-2)$

$M_z\ (C)\ x = 3 \rightarrow \underline{M_C = 80\ kN \cdot m}$

Problema 7.15



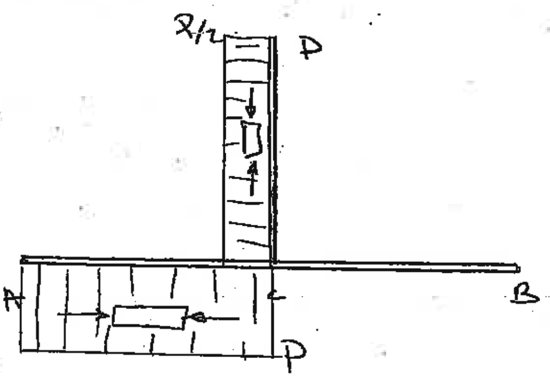
$\boxed{H_A = P}$

$\boxed{V_A + V_B = 0}$

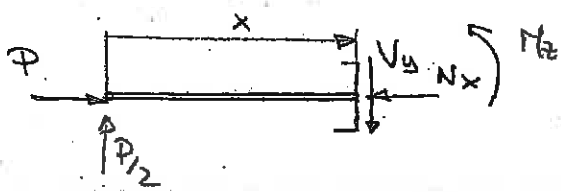
$V_B \cdot L + P\,L/2 = 0$

$\left\{ \begin{array}{l} V_B = -P/2 \\ V_A = P/2 \end{array} \right\}$

$\boxed{N_x}$



$\boxed{AC}$



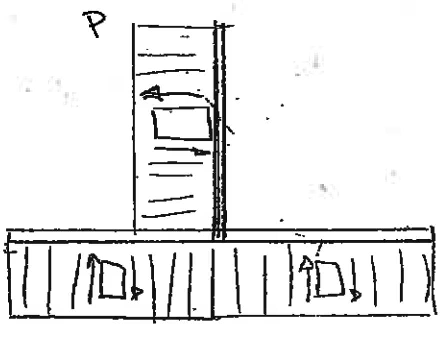
$\boxed{N_x = P}$

$\boxed{V_y = P/2}$

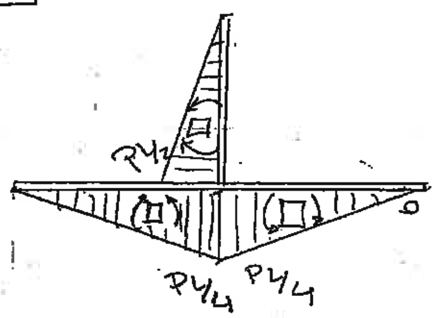
$M_z - P/2\,x = 0$

$\boxed{M_z = \frac{Px}{2}}$

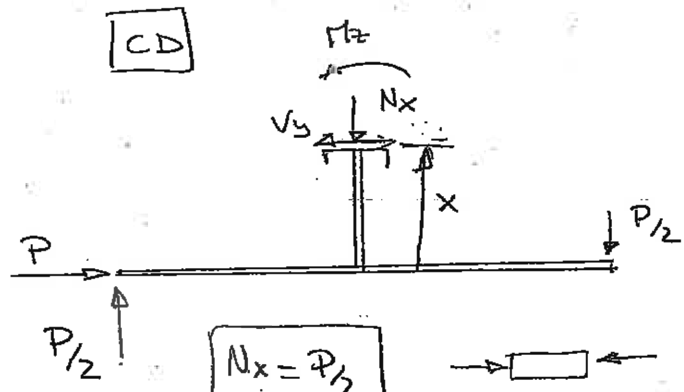
$\boxed{V_y}$



$\boxed{M_z}$



$\boxed{CD}$



$\boxed{N_x = P/2}$

$\boxed{V_y = P}$

$$M_z + P_x - \frac{P}{2}\cdot \frac{L}{2} - \frac{P}{2}\frac{L}{2} = 0$$

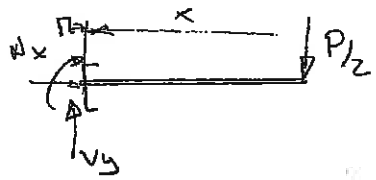
$$M_z = 2\cdot\frac{P\cdot L}{4} - P_x$$

$$\boxed{M_z = \frac{P\cdot L}{2} - P_x}$$

$x=0 \rightarrow PL/2$

$x = L/2 \rightarrow 0$

$\boxed{CB}$



$N_x = 0$

$\boxed{V_y = P/2}$

$-M_z - P/2\,x = 0$

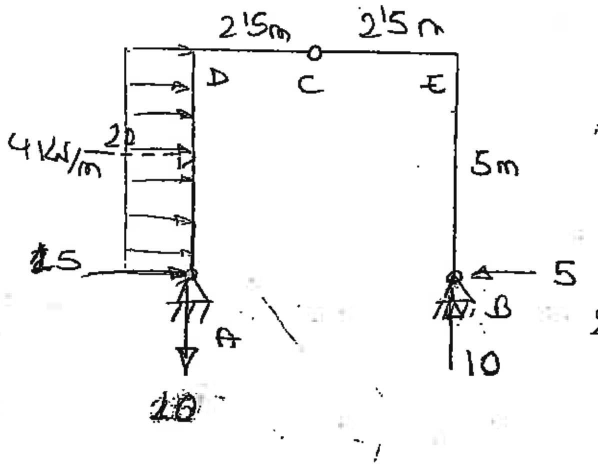
$M_z = -P/2\, x$

$\boxed{M_z = P/2\, x}$

$x=0 \rightarrow 0$

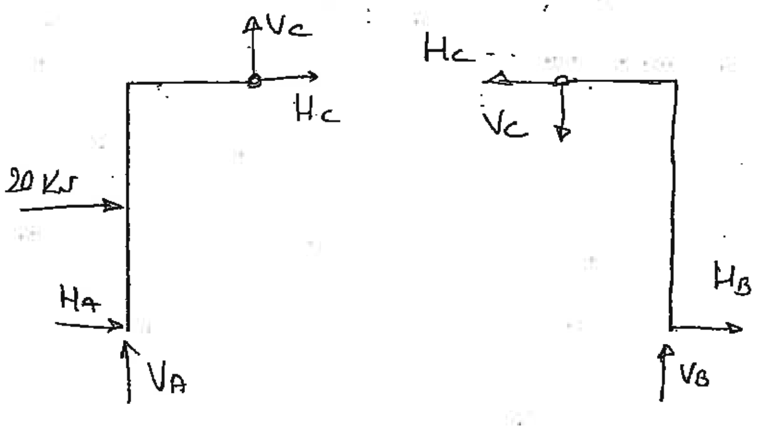
$x = L/2 \quad PL/4$

Problema 7.16



Estructura hiperestática $h=1$

↳ En rótula,



$$\begin{cases} H_A + H_C + 20 = 0 \\ V_A + V_C = 0 \\ -50 - H_C \cdot 5 + V_C \cdot 2'5 = 0 \end{cases} \qquad \begin{cases} H_B - H_C = 0 \\ V_B - V_C = 0 \\ V_C \cdot 2'5 + H_C \cdot 5 = 0 \end{cases}$$

$2H_C + V_C = 0 \qquad V_C = -2H_C \qquad \boxed{H_B = -5}$

$-50 - 5H_C - 5H_C = 0 \qquad \boxed{V_B = 10}$

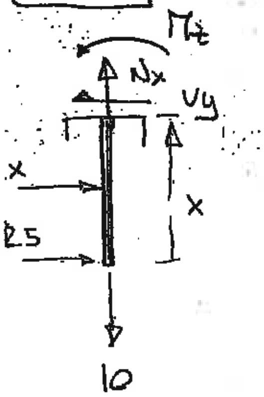
$-50 - 10H_C = 0$

$H_C = -\frac{50}{10} \qquad \boxed{H_C = -5} \qquad \boxed{V_C = +10}$

en KN

$H_A = -20 - H_C = -20 + 5 = -15 \qquad \boxed{H_A = 25} \qquad \boxed{V_A = -10}$

AD

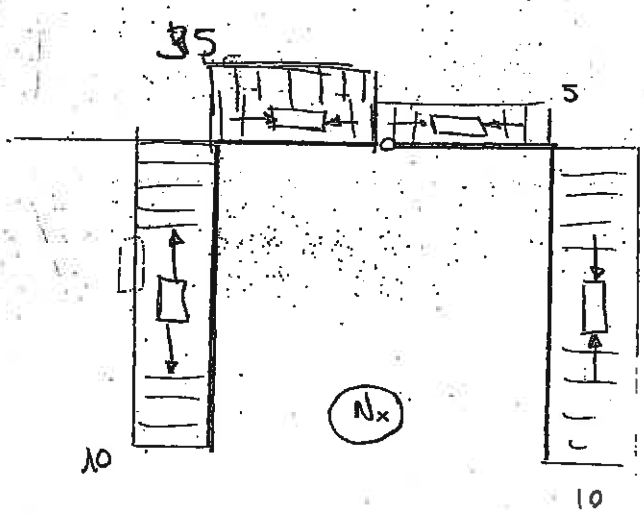


$\boxed{N_x = 10}$

$\boxed{V_y = 25 + 4x}$

$x = 0 \rightarrow 25$

$x = 5 \rightarrow 35$



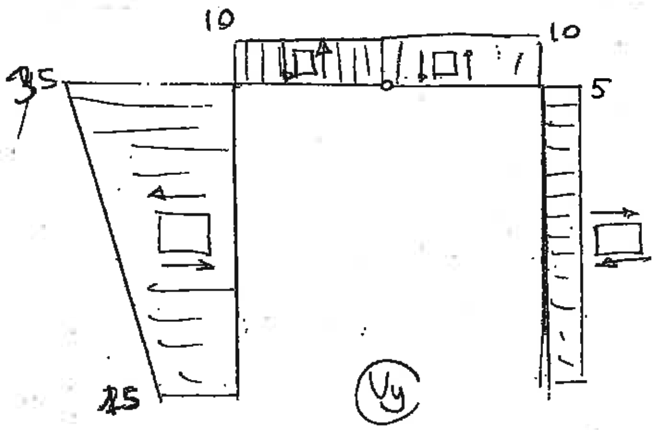
$M_t + \frac{4x^2}{2} + 25x = 0$

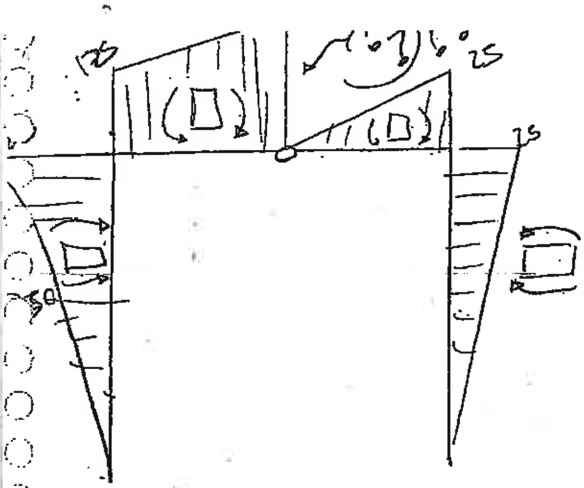
$M_t = \frac{4x^2}{2} + 25x$

$x = 0 \rightarrow 0$

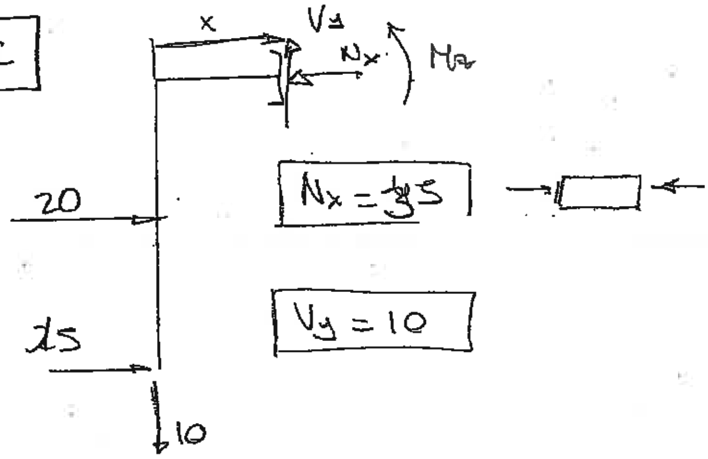
$x\,2'5 \rightarrow 750$

$x = 5 \rightarrow 125$





DC



$\boxed{N_x = 35}$

$\boxed{V_y = 10}$

$M_z + 10\cdot x + 20\cdot 2'5 + 15\cdot 5 = 0$

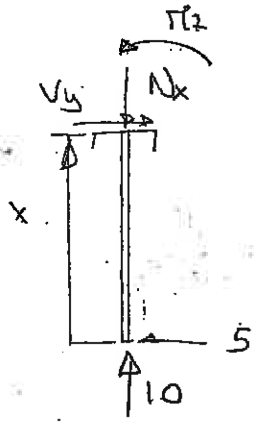
$M_z = -125 - 10x$

$\boxed{M_z = 125 + 10x}$

$x=0 \to 125$

$x=2'5 \to 150$

BC



$\boxed{N_x = 10}$

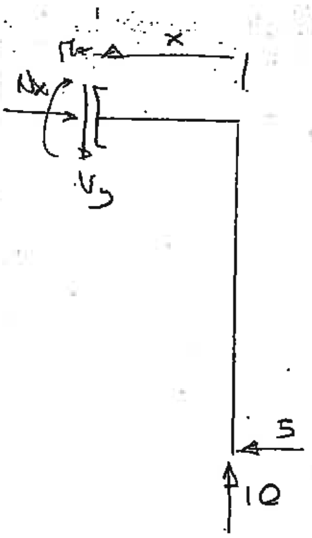
$\boxed{V_y = 5}$

$M_z - 5\cdot x = 0 \quad \boxed{M_z = 5x}$

$x=0 \to 0$

$x=5 \to 25$

EC



$\boxed{N_x = 5}$

$\boxed{V_y = 10}$

$-M_z + 10x - 5\cdot 5 = 0$

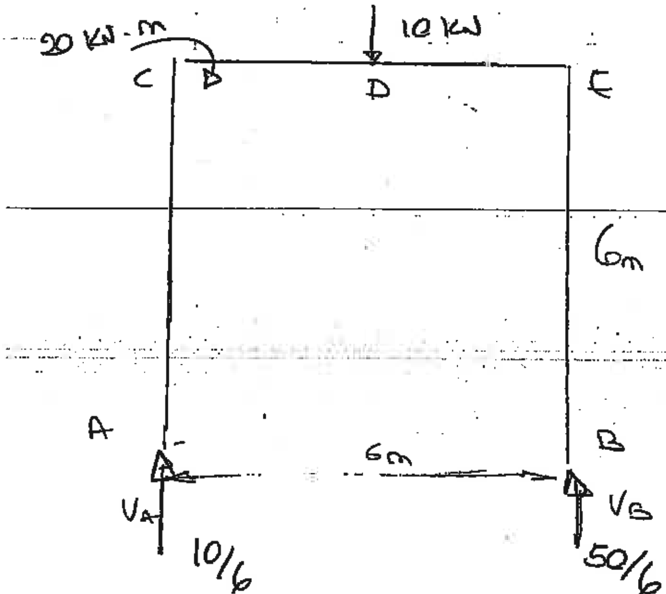
$M_z = 10x - 25$

$x=0 \to -25$

$x=2'5 \to 0$

$x=0'25$

Problema 7.18.



$\boxed{H_A = 0}$

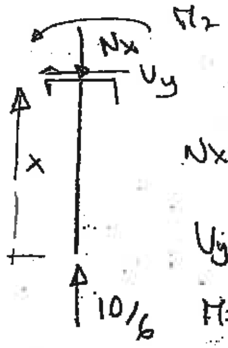
$V_A + V_B = 10$

$V_B \cdot 6 - 10 \cdot 3 - 20 = 0$

$V_B = \dfrac{20+30}{6} = \dfrac{50}{6}$

$V_A = 10 - 50/6 = \dfrac{60-50}{6} = \dfrac{10}{6}$

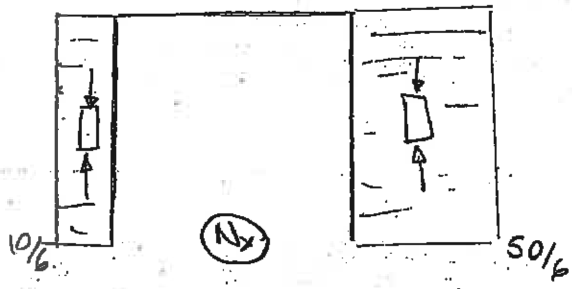
AC



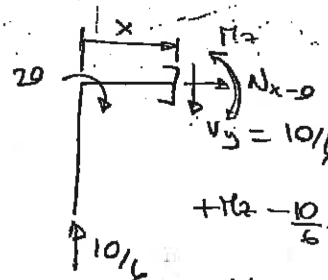
$N_x = 10/6$

$V_y = 0$

$M_z = 0$

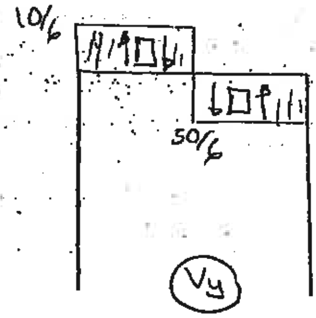


CD

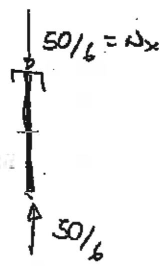


$+M_z - \frac{10}{6}x - 20 = 0$

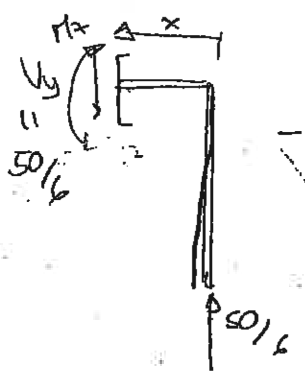
$M_z = 20 + \frac{10}{6}x \quad \begin{cases} x=0 \to 20 \\ x=3 \to 25 \end{cases}$



BE



ED

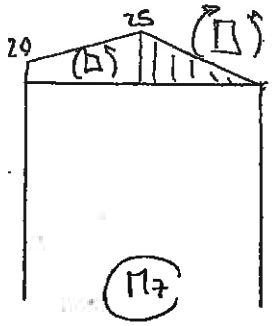


$N_x = 0$

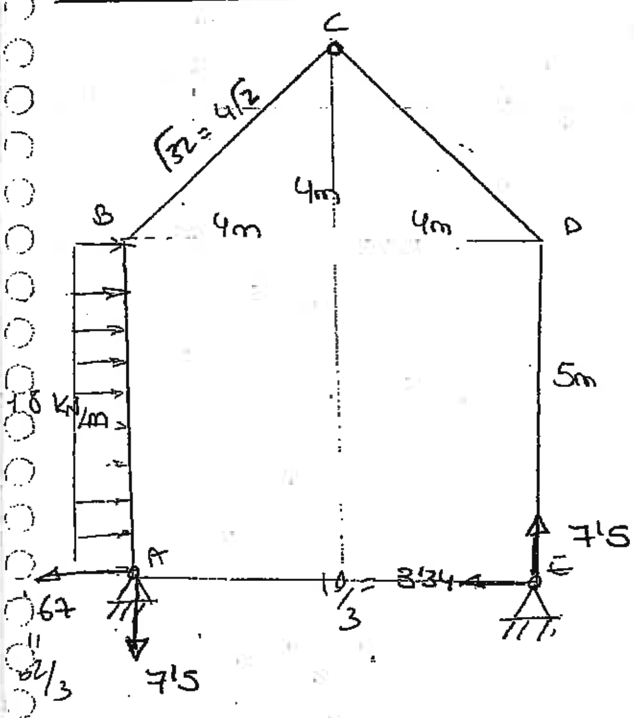
$-M_z + \frac{50}{6}x = 0$

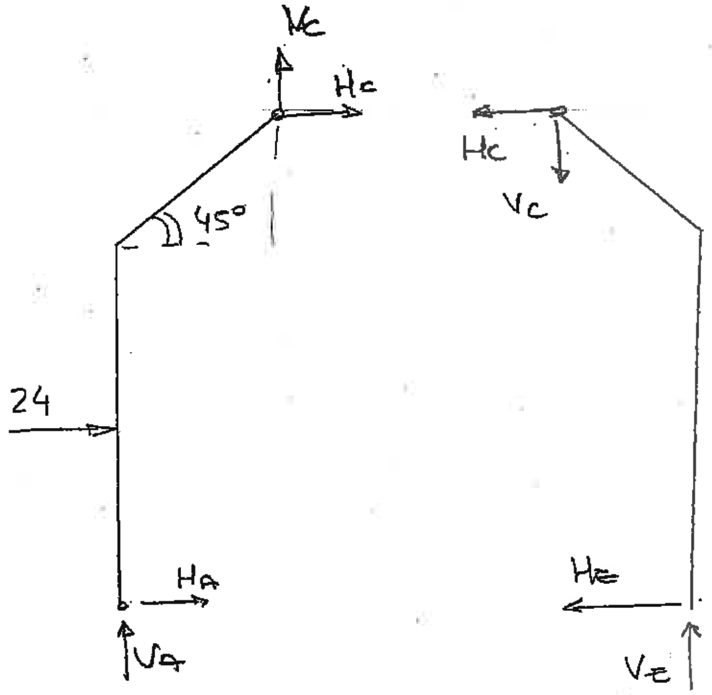
$M_z = \frac{50}{6}x \quad x=0 \to 0$

$x=3 \to 25$



Problema 7.19.





$$\begin{cases} H_A + 24 + H_C = 0 \\ V_A + V_C = 0 \\ -24 \cdot \frac{5}{2} - H_C \cdot 9 + V_C 4 = 0 \end{cases}$$

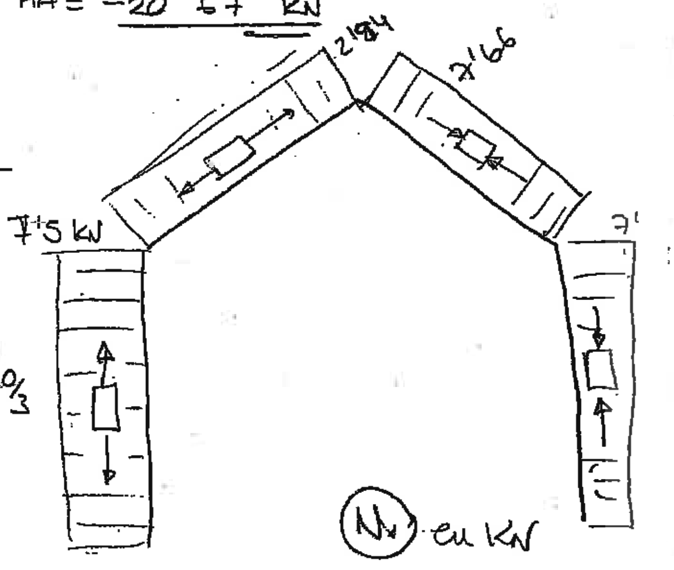
$$\begin{cases} H_C + H_E = 0 \\ V_E - V_C = 0 \\ H_C \cdot 9 + V_C \cdot 4 = 0 \end{cases} \qquad H_E = \underline{3'34 \text{ kN}} \qquad V_E = \underline{7'5 \text{ kN}}$$

$$H_C = \frac{-4V_C}{9} = -3'34$$

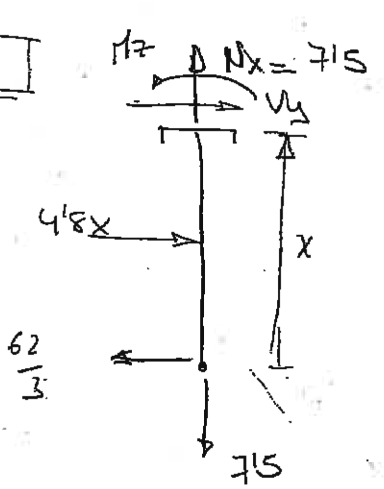
$$-60 + 4V_C + 4V_C = 0$$

$$-60 + 8V_C = 0 \qquad V_C = 60/8 = 30/4 = 15/2 = \underline{7'5 \text{ kN}}$$

$V_A = \underline{-7'5 \text{ kN}}$ $\qquad H_A = -24 + 3'34$ $\qquad H_A = \underline{-20'67 \text{ kN}}$



AB



$$\boxed{V_y = \frac{62}{3} - 4'8x}$$

$x = 0 \rightarrow \frac{62}{3}$

$x = 5 \rightarrow -3'33 = -\frac{10}{3}$

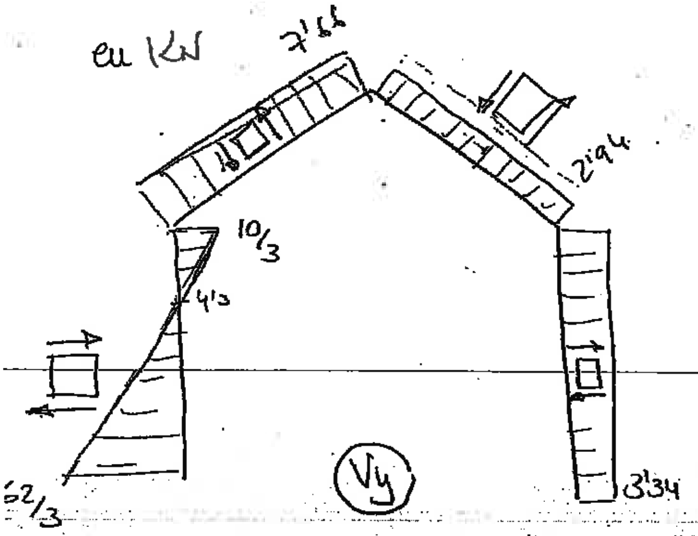
$x = 4'3055$

$$M_z + 4'8\frac{x^2}{2} - \frac{62}{3}x = 0 \qquad \boxed{M_z = \frac{62x}{3} - 4'8\frac{x^2}{2}}$$

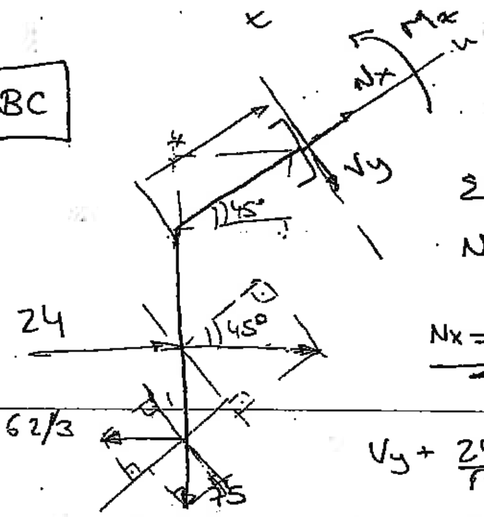
$x = 0 \rightarrow 0$

$x = 2'5 \rightarrow -36'67$

$x = 4'3055 \rightarrow 44'49$



BC



$\Sigma F_N = 0$

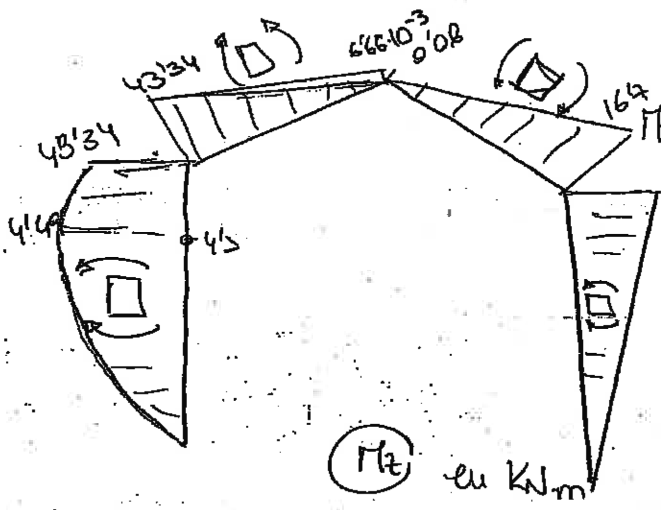
$N_x + 24/\sqrt{2} - \frac{20'67}{\sqrt{2}} - \frac{7'5}{\sqrt{2}}$

$\underline{N_x = 2'94}$

$V_y + \frac{24}{\sqrt{2}} - \frac{62}{3\sqrt{2}} + \frac{7'5}{\sqrt{2}} = 0$

$\underline{V_y = -7'66}$

$M_z + \frac{7'5\,x}{\sqrt{2}} + 24\,(2'5 + x/\sqrt{2}) - \frac{62}{3}\,(5 + x/\sqrt{2}) = 0$



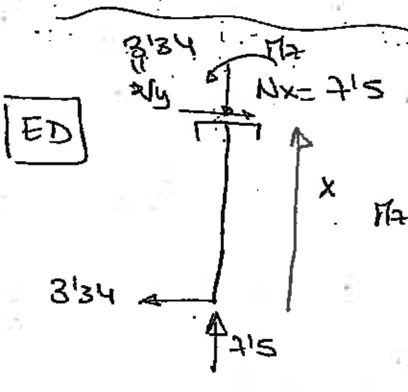
$M_z = \left(-\frac{7'5}{\sqrt{2}} - \frac{24}{\sqrt{2}} + \frac{62}{3\sqrt{2}}\right)x - 24\cdot 2'5 + \frac{62\cdot 5}{3}$

$M_z = 43'34 + 7'66\,x$

$x = 0 \rightarrow 43'33$

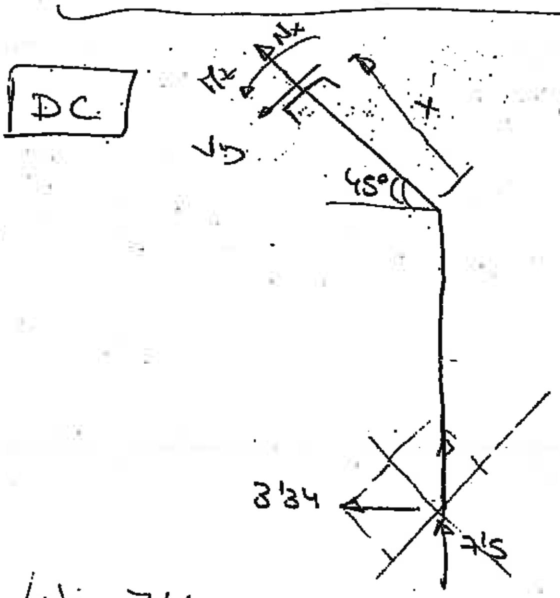
$x = 4\sqrt{2} \rightarrow 6'66\cdot 10^{-3}$

ED



$M_z = 3'34\,x$

DC



$\Sigma F_N = 0 \quad N_x + 3'34/\sqrt{2} + 7'5/\sqrt{2} = 0 \qquad \underline{N_x = 7'66}$

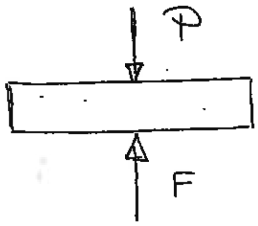
$V_y + 3'34/\sqrt{2} - 7'5/\sqrt{2} = 0 \qquad \underline{V_y = 2'94}$

$M_z + x/\sqrt{2}\;7'5 - 3'34\,(5 + x/\sqrt{2}) = 0 \qquad \underline{M_z = 16'7 - 2'94\,x}$

$x = 0 \rightarrow 16'7$

PROBLEMAS TEMA 8: ESFUERZO AXIAL SIMPLE

Problema 8.1.



$\boxed{F = P}$ Repartido entre los 2 muelles

$F = K_1 \cdot L_1' + K_2 \cdot L_2' = P$

$F_1 = K_1 \cdot L_1'$ $\quad \boxed{F_1 + F_2 = P}$

$F_2 = K_2 \cdot L_2'$

Los 2 juntos se alargan:

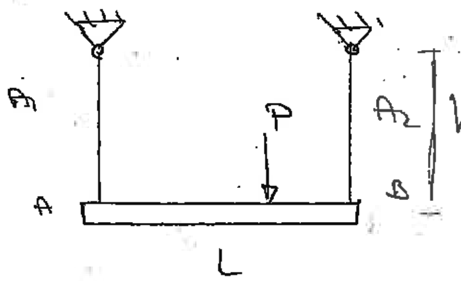
$P = L'(K_1 + K_2) \longrightarrow L' = \dfrac{P}{(K_1 + K_2)} = L_1' + L_2'$

$F_1 = \dfrac{K_1 \cdot P}{(K_1 + K_2)}$ $\quad F_2 = \dfrac{K_2 \cdot P}{(K_1 + K_2)}$

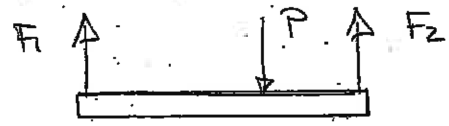
Problema 8.2.

AB → no se puede deformar

x?



1º) Equilibrio



$\boxed{P = F_1 + F_2}$

$F_2 \cdot L - P \cdot x = 0$ $\quad \boxed{F_2 = \dfrac{Px}{L}}$

2º) Compatibilidad de def. !

↳ para q se mantengan en total, $\delta_1 = \delta_2$

3º) Ley de comportamiento, Axial simple (cables)

$\delta_1 = \delta_2$ $\quad \delta_1 = \cancel{\dfrac{P \cdot h}{E \cdot A_1} = \dfrac{P \cdot h}{E \cdot A_2}}$

$\delta_1 = \dfrac{F_1 \cdot \cancel{h}}{\cancel{E} \cdot A_1} = \dfrac{F_2 \cdot K}{\cancel{E} \cdot A_2}$

$F_1 = F_2 \cdot \dfrac{A_1}{}$

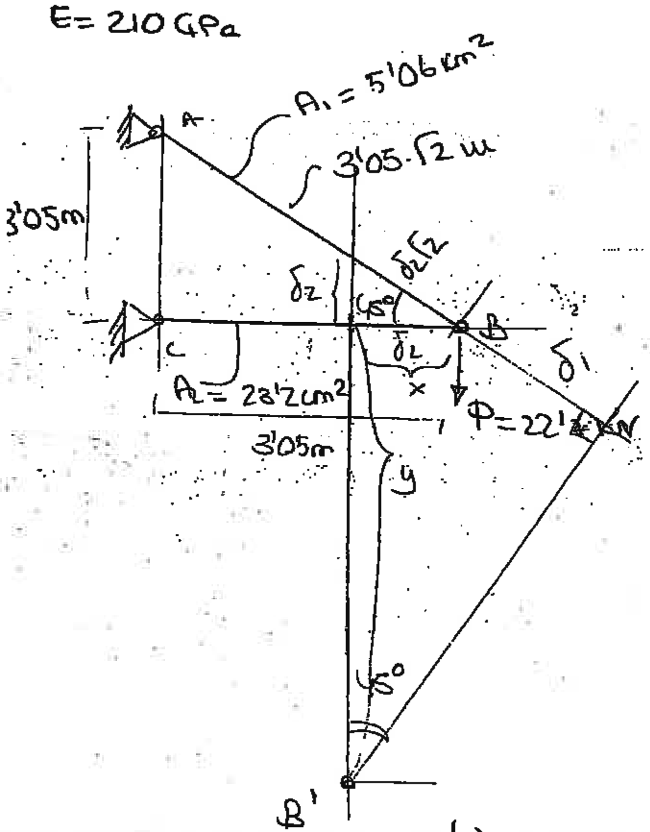
$$P = \frac{P x}{L}\frac{A_1}{A_2} + \frac{P x}{L} \qquad \frac{L}{x} = \left(\frac{A_1}{A_2}+1\right)$$

$$\cancel{P} = \cancel{P}\frac{x}{L}\left(\frac{A_1}{A_2}+1\right) \qquad x = \frac{L}{\left(\frac{A_1}{A_2}+1\right)}$$

$$x = L\left[\frac{A_2}{A_1+A_2}\right]$$

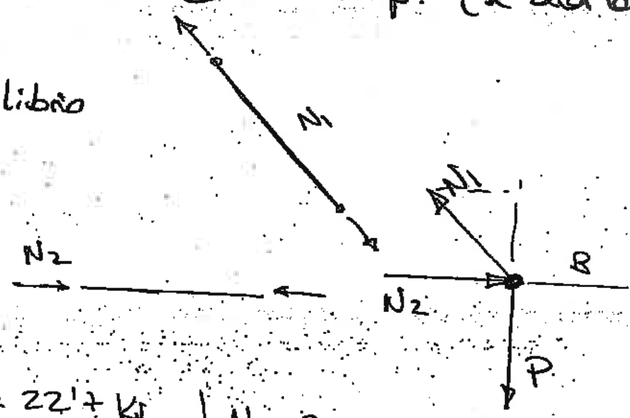
Problema 8.3.

$E = 210\ GPa$



Hipótesis → ① a tracción (se alarga)
② a comp. (se acorta)

a) Equilibrio



$$\frac{N_1}{\sqrt{2}} = 22'7\ kN \qquad \underline{N_1 = 32'10\ kN}$$

$$\frac{N_1}{\sqrt{2}} = N_2 \qquad \underline{N_2 = 22'7\ kN}$$

Problema isostático

b) Compat. de def.

$x = \delta_2$

$$\frac{1}{\sqrt{2}} = \frac{\delta_1 + \delta_2\sqrt{2}}{y+\delta_2}$$

$$(y+\delta_2) = \frac{\delta_1 + \delta_2\sqrt{2}}{1/\sqrt{2}}$$

$$y = \frac{\delta_1+\delta_2\sqrt{2}}{1/\sqrt{2}} - \delta_2$$

c) Ley de Comportamiento

$$\delta_1 = \frac{N_1 \cdot l_1}{E_1 A_1} = \frac{32100 \cdot 3'05\sqrt{2}}{210\cdot 10^9 \cdot 5'06\cdot 10^{-4}} = 1'3030\ mm$$

$$\delta_2 = \frac{22'700 \cdot 3'05}{210\cdot 10^9 \cdot 23'2\cdot 10^{-4}} = \underline{0'1421\ mm}$$

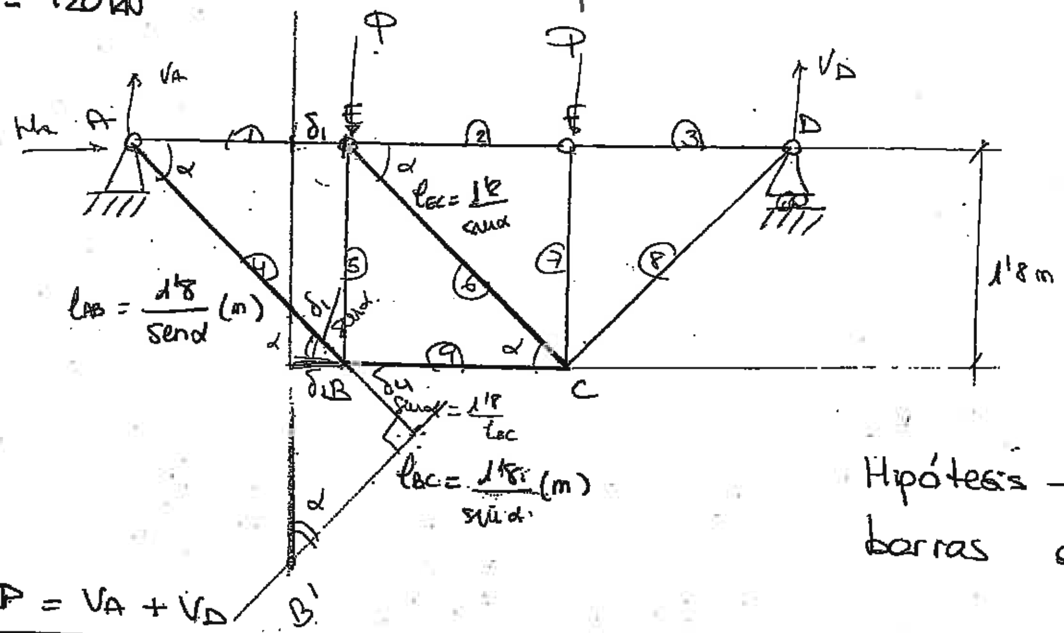
$$y = 2'9848\ mm$$

Problema 8.4.

Tensió admissible $\sigma_W = 100$ MPa

$P = 120$ kN

} → secciones de AB, BC, EC



Hipótesis → todas las barras a tracción

$2P = V_A + V_D$

$\boxed{H_A = 0}$

$V_D \cdot 3 \cancel{\frac{1'8}{\sin\alpha}} - P \cdot 2 \cdot \cancel{\frac{1'8}{\sin\alpha}} - P \cdot \cancel{\frac{1'8}{\sin\alpha}} = 0 \qquad 3V_D - 3P = 0$

$\boxed{V_D = P}$

$\boxed{V_A = P}$

A)

$$\begin{cases} N_1 + N_4 \cdot \cos\alpha = 0 \\ P = N_4 \sin\alpha \longrightarrow \boxed{N_4 = \frac{P}{\sin\alpha}} \end{cases}$$

a tracción

$\boxed{N_1 = + \frac{P\cos\alpha}{\sin\alpha}}$ a compresión

E)

$$\begin{cases} -P - N_5 - N_6 \sin\alpha = 0 \\ \frac{P\cos\alpha}{\sin\alpha} + N_2 + N_6 \cos\alpha = 0 \end{cases}$$

3 incóg

B)

$$\begin{cases} +\frac{P}{\sin\alpha} \cdot \cos\alpha = N_9 \\ N_5 = -P \frac{\cancel{\sin\alpha}}{\cancel{\sin\alpha}} \end{cases}$$

$\boxed{N_9 = \frac{P\cos\alpha}{\sin\alpha}}$ a tracción

$\boxed{N_5 = P}$ a compresión

↳ E) $N_6 \sin\alpha = -P - (-P) = 0$ $\boxed{N_6 = 0}$ Barra

$N_2 = -\frac{P\cos\alpha}{\sin\alpha}$ $\boxed{N_2 = \frac{P\cos\alpha}{\sin\alpha}}$ a com

Ya tengo esfuerzos que necesito: isostático

$tg\alpha = \frac{1'8}{2'4} \Rightarrow \alpha = 36'86989°$

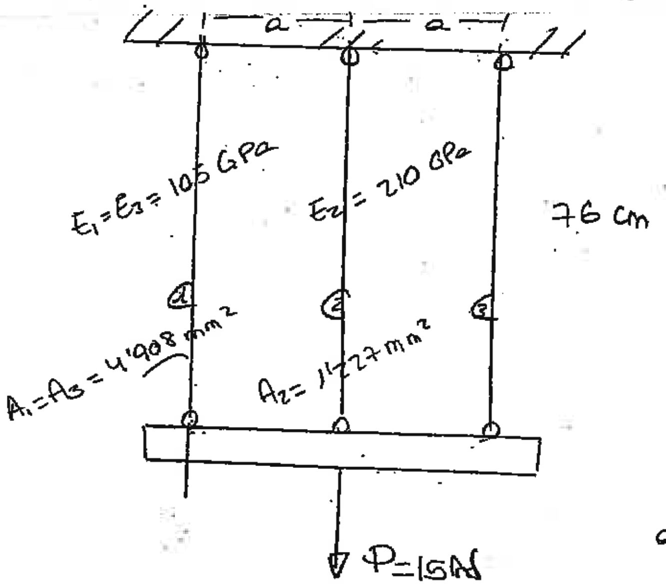
$N_4 = \frac{P}{sen\alpha} = 200\ kN$

$P = 120\,000\ N$

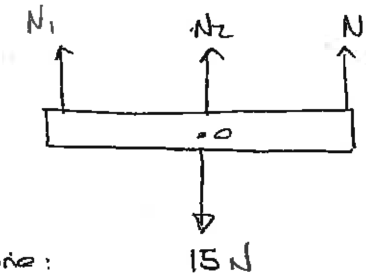
$\sigma_w = \frac{N_4}{A_4} \longrightarrow A_4 = 0'002\ m^2 = \underline{\underline{20\ cm^2}}$

$N_9 = \frac{P}{tg\alpha} = 160\ kN \longrightarrow \sigma_w = \frac{N_9}{A_9} \longrightarrow A_9 = 0'0016\ m^2 = \underline{\underline{16\ cm^2}}$

Problema 8.5.



1) ¿Cómo se reparte la carga del péndulo entre los distintos redondos:



Hipótesis tracción

a) Equilibrio:

$$\boxed{N_1 + N_2 + N_3 = 15}$$

$h = 1$

$\Sigma M_O = 0 \qquad N_3 \cdot \alpha - N_1 \alpha = 0 \qquad \boxed{N_3 = N_1}$

b) Compatibilidad de def: (péndulo rígido)

$\delta_1 = \delta_3 = \dfrac{N_1 \cdot L}{EA}$ ⊖ Como se alarga igual y no se puede def. ⇓ $\underline{\delta_1 = \delta_2 = \delta_3}$

c) Ley de comportamiento:

$$\left.\begin{array}{l} \delta_1 = \delta_3 = \dfrac{N_1 \cdot 0'76}{105 \cdot \cancel{10^6} \cdot 4'908 \cdot \cancel{10^{-6}}} \\[2ex] \delta_2 = \dfrac{N_2 \cdot 0'76}{210 \cdot \cancel{10^6} \cdot 1'227 \cdot \cancel{10^{-6}}} \end{array}\right\}$$

$$\frac{N_1}{515'34} = \frac{N_2}{257'67}$$

$N_2 = N_1 \cdot 0'5$

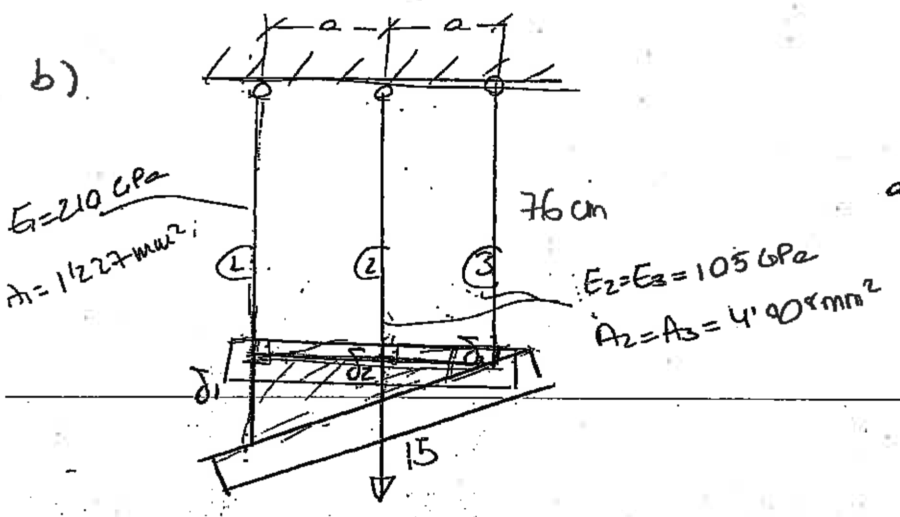
$N_2 = \dfrac{N_1}{2}$

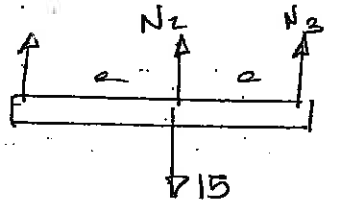
$$N_1 + \frac{N_1}{2} + N_1 = 15$$

$$\frac{5}{2} N_1 = 15$$

$$\left\{\begin{array}{l} N_1 = 6 \text{ N} \\ N_3 = 6 \text{ N} \\ N_2 = 3 \text{ N} \end{array}\right.$$

b)





Hipótesis tracción

a)

$$\begin{cases} \boxed{N_1+N_2+N_3=15} \\ N_3 \cdot a - N_1 a = 0 \end{cases}$$

$$\boxed{N_3 = N_1}$$

$$\boxed{2N_1 + N_2 = 15}$$

b) Compatibilidad de def. → Alineación:

$$\frac{\delta_1-\delta_3}{2\cancel{a}} = \frac{\delta_2-\delta_1}{\cancel{a}} \qquad \boxed{\delta_1 - \delta_3 = 2(\delta_2-\delta_1)}$$

c) Ley de comportamiento:

$$\delta_1 = \frac{N_1\ 0'76}{210 \cdot \cancel{10^9} \cdot 1'227 \cdot \cancel{10^{-6}}}$$

$$\delta_2 = \frac{N_2 \cdot 0'76}{105 \cdot 4'908 \cdot 10^3}$$

$$\delta_3 = \frac{N_3 \cdot 0'76}{105 \cdot 4'908 \cdot 10^3} = \frac{N_1\ 0'76}{105\ 4'908 \cdot 10^3}$$

$$\frac{N_1 \cdot \cancel{0'76}}{210 \cdot 1'227 \cdot 10^3} - \frac{N_1\ \cancel{0'76}}{105\ 4'908 \cdot 10^3} = 2 \cdot \frac{N_2\ 0'76}{105 \cdot 4'908}$$

$$2\left(\frac{N_2 \cdot \cancel{0'76}}{105 \cdot 4'908 \cdot 10^3} - \frac{N_1\ \cancel{0'76}}{210 \cdot 1'227 \cdot \cancel{10^3}}\right)$$

$N_2 = \ldots N_1$

$3'88 \cdot \cancel{10^{-3}}\ N_1 - 1'94 \cdot \cancel{10^{-3}}\ N_1 =$

$2\,(1'94 \cdot \cancel{10^{-3}}\ N_2 - 3'88 \cdot \cancel{10^{-3}}\ N_1) \to$

$-5'82\ N_1 + 3'88\ N_2 = 0$

$\boxed{N_2 = 1'5\ N_1}$ $\quad N_1 = 6\ N_1$

$2N_1 + 1'5 N_1 = 15$

$\boxed{N_1 = 4'28\ N = N_3}$

$\boxed{N_2 = 6'42\ N}$

$N_3 = \ldots$

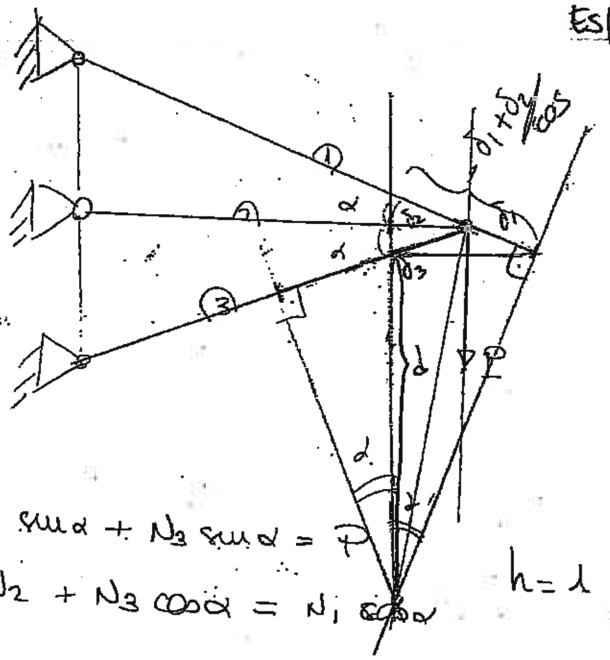
$\delta_1 = 0'0000126\ m = 1'26 \cdot 10^{-3}\ cm$ (acero)

$\delta_2 = 9'46 \cdot 10^{-6}\ m = 0'946 \cdot 10^{-3}\ cm$ centro

$\delta_3 = 6'31\ 10^{-6}\ m = 0'631\ 10^{-3}\ cm$

Problema 8.6

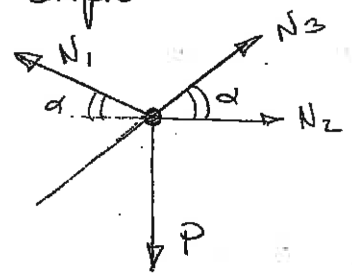
Esfuerzos en cada barra



Hipótesis

1 tracción
2, 3 compresión

a) Equilibrio



$$\begin{cases} N_1 \sin\alpha + N_3 \sin\alpha = P \\ N_2 + N_3 \cos\alpha = N_1 \cos\alpha \end{cases}$$

$h = l$

$$N_3 = \frac{P - N_1 \sin\alpha}{\sin\alpha}$$

$$N_2 + \frac{P\cos\alpha}{\sin\alpha} - N_1 \cos\alpha = N_1 \cos\alpha$$

b) Compatibilidad de def.

$$\sin\alpha = \frac{\delta_1 + \delta_2/\sin\alpha}{d + 2\cdot\frac{\delta_2}{\sin\alpha}\cdot\cos\alpha} = \frac{\delta_3 - \delta_2/\sin\alpha}{d}$$

$$d\left(\delta_1 + \frac{\delta_2}{\sin\alpha}\right) = \left(\delta_3 - \frac{\delta_2}{\sin\alpha}\right)\left(d + 2\,\delta_2 \frac{\cos\alpha}{\sin\alpha}\right)$$

$$\delta_1 d + \frac{\delta_2}{\sin\alpha} d = \delta_3 d - \frac{\delta_2}{\sin\alpha} d + 2\delta_2\delta_3 \frac{\cos\alpha}{\sin\alpha} - 2\,\delta_2^2 \frac{\cos\alpha}{\sin^2\alpha}$$

$$d\left(\delta_1 - \delta_3 + 2\frac{\delta_2}{\sin\alpha}\right) = 2\delta_2 \frac{\cos\alpha}{\sin\alpha}\left(\delta_3 - \frac{\delta_2}{\sin\alpha}\right)$$

$$\cancel{\sin\alpha} = \frac{2\,\frac{\delta_2}{\cos}\cdot\cancel{\sin\alpha}}{\delta_1 + \delta_2/\sin\alpha} \qquad \delta_1 + \frac{\delta_2}{\sin\alpha} = 2\frac{\delta_2}{\cos\alpha}$$

$$\delta_1 = \frac{2\delta_2}{\cos\alpha} - \frac{\delta_2}{\sin\alpha} = \delta_2\left(\frac{2}{\cos\alpha} - \frac{1}{\sin\alpha}\right)$$

c) Ley de comportamiento $\qquad \ell_1 \rightarrow \ell_2 = \ell_1 \cos\alpha$

$\ell_1 = \ell_3$

$$\begin{cases} \delta_1 = \dfrac{N_1 \ell_1}{EA} \\ \delta_2 = \dfrac{N_2 \cdot \ell_1 \cos\alpha}{EA} \\ \delta_3 = \dfrac{N_2 \ell_1}{EA} \end{cases}$$

$$\cancel{\frac{N_1 \cancel{\ell_1}}{\cancel{EA}} = 2 \cdot \frac{N_2 \cancel{\ell_1 \cos\alpha}}{\cancel{EA}} \cos\alpha - \frac{N_2 \cancel{\ell_1 \cos\alpha}}{\cancel{EA}} \cdot \frac{1}{\sin\alpha}}$$

$$\cancel{N_1 = 2 N_2 \cos^2\alpha - N_2 \frac{\cos\alpha}{\sin\alpha}}$$

$$\cancel{N_1 = N_2 \cos\alpha \left(2\cos\alpha - \frac{1}{\sin\alpha}\right)}$$

$$\cancel{N_2 + P\frac{\cos\alpha}{\sin\alpha} - 2N_2\cos^2\alpha\left(2\cos\alpha - \frac{1}{\sin\alpha}\right) = 0}$$

$$\cancel{N_2\left[1 - 2\cos^2\alpha\left(2\cos\alpha - \frac{1}{\sin\alpha}\right)\right] = -P\frac{\cos\alpha}{\sin\alpha}}$$

$$\cancel{N_2 = \frac{P\cos\alpha/\sin\alpha}{\left[2\cos^2\alpha\left(2\cos\alpha - \frac{1}{\sin\alpha}\right) - 1\right]}}$$

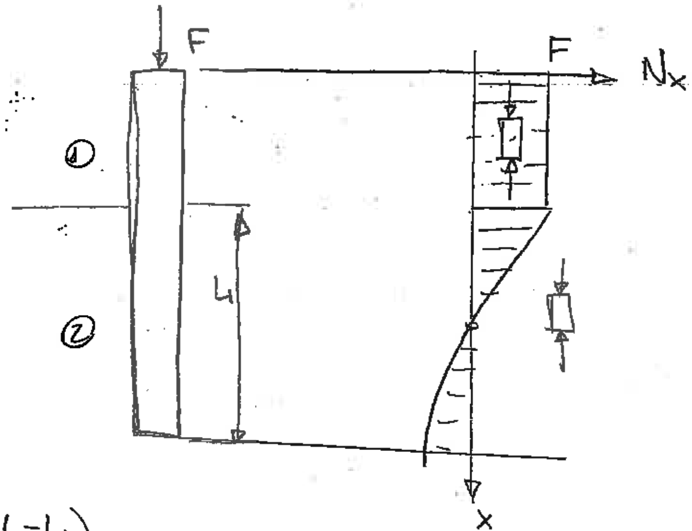
$$\frac{N_1 \cancel{\ell_1}}{\cancel{EA}} = 2\frac{N_2 \cancel{\ell_1}}{\cancel{EA}} - \frac{N_2 \cancel{\ell_1} \cos\alpha}{\cancel{EA} \sin\alpha} \qquad N_1 = N_2\left(2 - \frac{\cos\alpha}{\sin\alpha}\right)$$

$$N_2 + P\frac{\cos\alpha}{\sin\alpha} - 2N_2\cos\alpha\left(2 - \frac{\cos\alpha}{\sin\alpha}\right) = 0$$

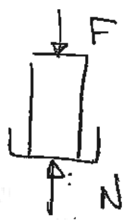
? No me da el resultado

Problema 8.7.

1)

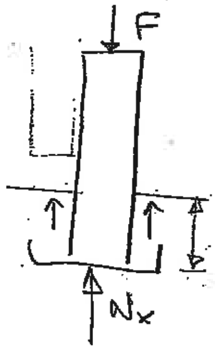


① $0 \leq x \leq (L-L_1)$



$N_x = F$ (compresión)

② $(L-L_1) \leq x \leq L$ Llamo x a $x-(L-L_1)$

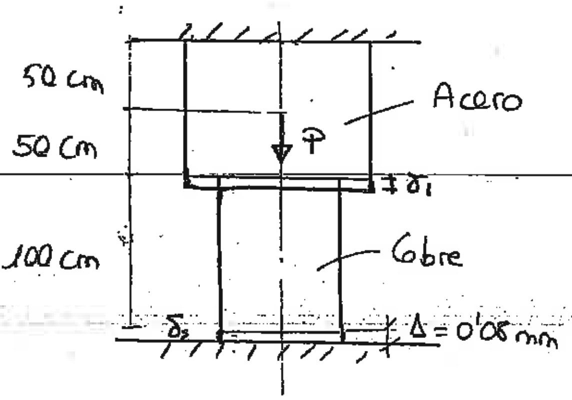


Esfuerzo de roz. $\int_0^x K x^2 dx = K \frac{x^3}{3}$ DUDA

$F - K\frac{x^3}{3} - N_x = 0$ $\boxed{N_x = F - Kx^3/3}$ a compresión

$N_x = F(1 - Kx^3/3)$

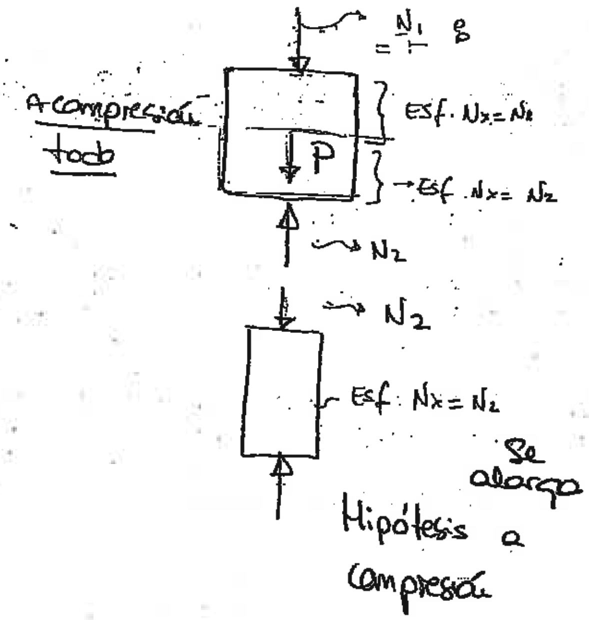
Problema 8.8.



Tensiones en las ≠ parte de la barra debido a P y a ~~ΔT = 20ºC~~

Hipótesis → ambos se alargan y después trabajan a compresión

a) Equilibrio



$N_1 + P = N_2 \qquad P = 1'5 \cdot 10^6\ N$

b) Compat. de def.

$|\delta_1| + |\delta_2| = 0'08\ mm$ $\left\{ \begin{array}{l} \delta_1 \text{ elongación acero } \left\{ \begin{array}{l} \delta_{11} \\ \delta_{12} \end{array} \right. \\ \delta_2 \text{ elongación cobre} \end{array} \right.$

c) Ley de comportamiento:

$$\delta_{11} = -\frac{N_1 \, e_{11}}{E_{11} \, A_{11}} + \alpha \, e_{11} \, \Delta T$$

$$\delta_{11} = -\frac{N_1 \cdot 0'5}{210 \cdot 10^9 \cdot 200 \cdot 10^{-4}} + 10 \cdot 10^{-6} \cdot 0'5 \cdot 20$$

$$\delta_{11} = -1'19 \cdot 10^{-10} N_1 + 1 \cdot 10^{-4}$$

$$\delta_{12} = -\frac{N_2 \cdot 0'5}{210 \cdot 10^9 \cdot 200 \cdot 10^{-4}} + 10 \cdot 10^{-6} \cdot 0'5 \cdot 20 = -1'19 \cdot 10^{-10} N_2 + 1 \cdot 10^{-4}$$

$$\delta_2 = \frac{-N_2 \cdot 1}{105 \cdot 10^9 \cdot 100} + 16 \cdot 10^{-6} \cdot 1 \cdot 20 = -9'52 \cdot 10^{-10} N_2 + 3'2 \cdot 10^{-4}$$

$$-1'19 \cdot 10^{-10} N_1 + 1'19 \cdot 10^{-10} N_2 + 2 \cdot \cancel{10^{-4}} + 9'52 \cdot 10^{-10} N_2 + 3'2 \cdot \cancel{10^{-4}} = 0'8 \cdot \cancel{10^{-4}}$$

$$-1'19 \cdot 10^{-6} N_1 + 10'71 \cdot 10^{-6} N_2 = -4'4 \qquad N_1 = N_2 - 1'5 \cdot 10^6$$

$$-1'19 \cdot 10^{-6} \cdot (N_2 - 1'5 \cdot 10^6) + 10'71 \cdot 10^{-6} N_2 = -4'4$$

$-11'9\cdot 10^{-6}\, N_2 = \cancel{11'615} = 6'186$

N_2 a compresión

a tracción

$N_2 = +\cancel{5}19\,747'89$ N hipótesis ✓

$-980\,252'1$ N hipótesis ✗

$N_1 = +1\,719\,747'899$ N

$\sigma = \frac{N}{A}$

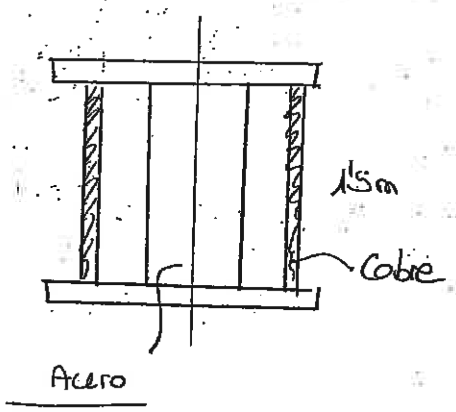
N_1 a tracción

Tensiones

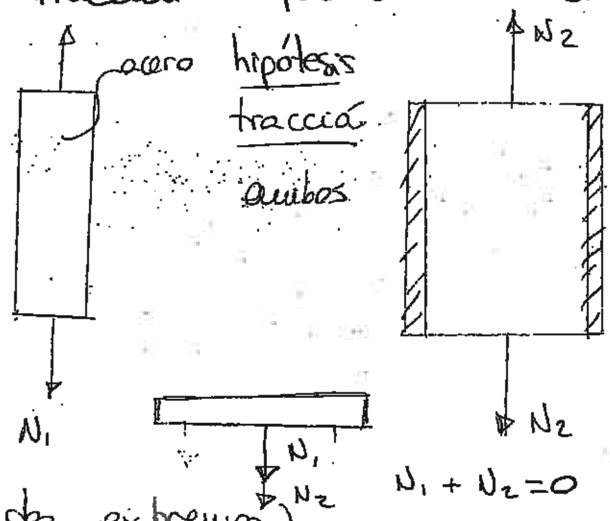
→ Acero
- $-49'01$ MPa → $49'01$ MPa a tracción
- ~~85'98 MPa~~ (N_1)
- ~~10'98 MPa~~ (N_2)
- $25'98$ MPa → $25'98$ ~~MPa~~ MAL a compresión

→ Cobre : ~~21'97 MPa~~ $51'9$ MPa → $51'9$ MPa a compresión

Problema 8.9.



~~Tracción~~ Esfuerzo axial simple



$N_1 + N_2 = 0$

$N_2 = -N_1$

Compatibilidad def : (rígidamente unidos extremos)

$\delta_1 = \delta_2$

Ley de comportamiento

$$\delta_1 = \frac{N_1\cdot 1'5}{210\cdot 10^9\cdot 10\cdot 10^{-4}} + 12'5\cdot 10^{-6}\cdot 1'5\cdot 70 = 7'14\cdot 10^{-\cancel{9}6} N_1 + 1'3125\cdot \cancel{10^{-3}}$$

sólo simplifica al igualar!

$$\delta_2 = \frac{N_2\cdot 1'5}{120\cdot 10^9\cdot 12\cdot 10^{-4}} + 17\cdot 10^{-6}\cdot 1'5\cdot 70 = 1'0416\cdot 10^{-\cancel{8}5} N_2 + 1'785\cdot \cancel{10^{-3}}$$

$$7'14 \cdot 10^{-6} N_1 + 1'3125 = 10'416 \cdot 10^{-6} N_2 + 1'785$$

$$7'14 \cdot 10^{-6} N_1 + 10'416 \cdot 10^{-6} (+N_1) = 0'4725$$

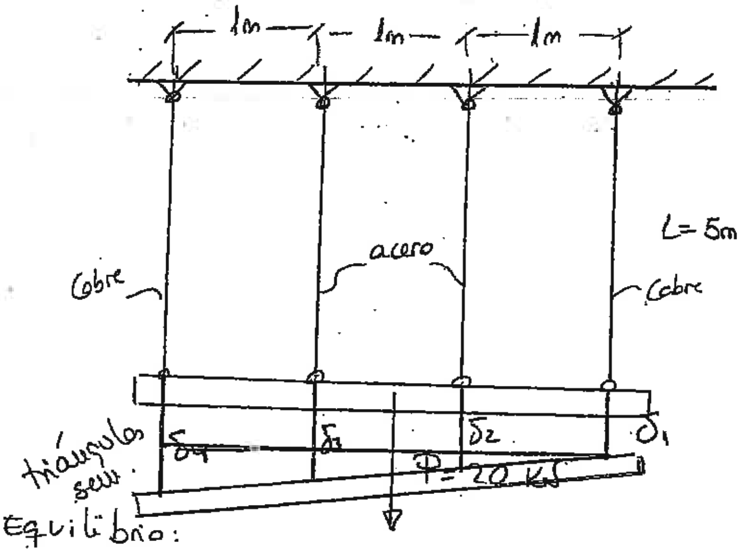
$$17'556 N_1 \cdot 10^{-6} = 0'4725$$

$$\begin{cases} N_1 = 26913'87 \text{ N} \\ N_2 = -26913'87 \text{ N} \end{cases}$$

$$\left.\begin{array}{l} \sigma_{ac} = 26'9138 \text{ MPa} \\ \text{(tracción)} \\ \\ \sigma_{cobre} = 22'428 \text{ MPa} \\ \text{(compresión)} \end{array}\right\} \checkmark$$

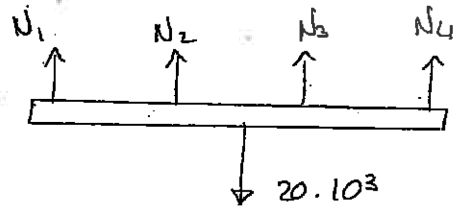
$$\delta_1 = \delta_2 = 1'5 \cdot 10^{-3} \text{ m} = \underline{0'15 \text{ cm}}$$

Problema 8.10.



$\Delta T = 20°C$

Hipótesis → todas a tracción



a) Equilibrio:

$N_1 + N_2 + N_3 + N_4 = 20 \cdot 10^3$ (1)

$N_2 + N_3 \cdot 2 + N_4 \cdot 3 = 20 \cdot 10^3 \cdot 1'5$ (2)

$h = 2$

2 ecs.

b) Compatibilidad de deformaciones. (∴ viga rígida)

$$\frac{\delta_4 - \delta_1}{3} = \frac{\delta_3 - \delta_1}{2} \qquad \frac{\delta_3 - \delta_1}{2} = \frac{\delta_2 - \delta_1}{1} \qquad \frac{\delta_4 - \delta_1}{3} = \frac{\delta_2 - \delta_1}{1}$$

c)

$$\delta_1 = \frac{N_1 e}{E_c \cdot A} + \alpha_{c} l_c \Delta T = \frac{N_1 \cdot 5}{120 \cdot 10^9 \cdot 5 \cdot 10^{-4}} + 17 \cdot 10^{-6} \cdot 5 \cdot 20 = 8'34 \cdot 10^{-8} N_1 + 1'7 \cdot 10^{-}$$
$$= 1'68 \cdot 10^{-3} m = 0'168$$

$$\delta_2 = \frac{N_2 \cdot 5}{210 \cdot 10^9 \cdot 5 \cdot 10^{-4}} + 12 \cdot 10^{-6} \cdot 5 \cdot 20 = 4'76 \cdot 10^{-8} N_2 + 1'2 \cdot 10^{-3} = 0'168 \text{ cm}$$

$$\delta_3 = \frac{N_3 \cdot 5}{210 \cdot 10^5 \cdot 5} + 12 \cdot 10^{-6} \cdot 5 \cdot 20 = 4'76 \cdot 10^{-8} N_3 + 1'2 \cdot 10^{-3} = 0'168 \text{ cm}$$

$$\delta_4 = \frac{N_4 \cdot 5}{120 \cdot 10^5 \cdot 5} + 17 \cdot 10^{-6} \cdot 5 \cdot 20 = 8'34 \cdot 10^{-8} N_4 + 1'7 \cdot 10^{-3} = 0'168 \text{ cm}$$

Todas las barras se estiran igual

$$\frac{8'34 \cdot 10^{-8} (N_4 - N_1)}{3} = \frac{4'76 \cdot 10^{-8} N_3 - 8'34 \cdot 10^{-8} N_1 - 0'5 \cdot 10^{-3}}{2}$$

$$16'68 \cdot 10^{-8} (N_4 - N_1) = 14'28 \cdot 10^{-8} N_3 - 25'02 \cdot 10^{-8} N_1 - 1'5 \cdot 10^{-3}$$

$$16'68 \cdot 10^{-5} N_4 + \cancel{9'17} \, 8'34 \cdot 10^{-5} N_1 - 14'28 \cdot 10^{-5} N_3 = -1'5 \cdot \cancel{10^{-3}} \quad (3)$$

$$\frac{8'34\cdot 10^{-8}(N_4-\cancel{N_1})}{3} = \frac{4'76\cdot 10^{-8} N_2 - 8'34\cdot\cancel{10^{-8}} N_1 - 0'5\cdot 10^{-3}}{1}$$

$$8'34\cdot 10^{\cancel{-8}\,-5} N_4 + 16'68\cdot 10^{-5} N_1 - 14'28 N_2 10^{-5} = -1'5\cdot\cancel{10^{-3}} \qquad (4)$$

$$\begin{cases} N_2 = 0'584\, N_4 + 1'168\, N_1 + 10.504'2 \\ N_3 = 1'168\, N_4 + \overset{0'5840}{\cancel{2'92}}\, N_1 + 10504'2 \end{cases}$$

$$N_1 + (0'584\,\cancel{N_4} + 1'168\, N_1 + 10504'2) + (1'\cancel{168}\, N_4 + 2\cancel{'92}\overset{0'5840}{}N_1 + 10504'2) + \cancel{N_4} = 20$$

$$\overset{}{\underset{2'752}{\cancel{5'088}}} N_1 + 2'752\, N_4 + 1008'4 = \ldots$$

$$(0'584\,\cancel{N_4} + 1'168\,\cancel{N_1} + 105\cancel{04'2}) + (2'336\,\cancel{N_4} + 5'8\overset{1'168}{\cancel{4}}\,N_1 + 21\,\cancel{008'4}) + 3\cancel{N_4} = 30\,000$$

$$5'92\, N_4 + \overset{2'336}{\cancel{7'008}} N_1 + 1512'6 = 0$$

$$N_4 = \frac{-1512'6 - \cancel{2'336}\, N_1}{5'92}$$

$$\cancel{5'088 N_1} - 0'4\cancel{648}(1512'6 - 7'008 N_1) + 1008'4$$

$$8'3453\, N_1 = -\cancel{305'34}$$

MAL $N_1 = -36'58\ N$

$$2'752\, N_1 - 0'4648\,(1512'6 + 2'336 N_1) + 1008'4 = 0$$

$$1'66\, N_1 + 305'34 = 0 \qquad N_1 = -183'94\ N$$

$$N_4 = \frac{-1512'6 - 2'336\cdot N_1}{5'92} = -182'92\ N$$

$N_2 = 10182'53\ N$

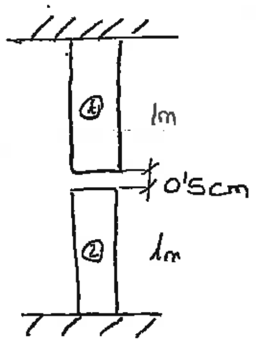
$N_3 = 10183'12\ N$

(cobre)

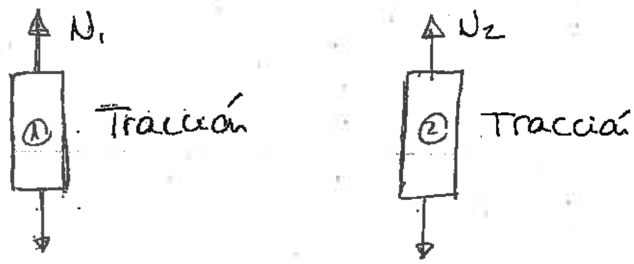
Cobre $\begin{cases} N_1 = 183'94\,N \text{ a compresión} \\ N_4 = 182'92\,N \text{ a compresión} \end{cases}$

acero $\begin{cases} N_2 = 10\,182'53\,N \text{ a tracción} \\ N_3 = 10183'12\,N \text{ a tracción} \end{cases}$

Problema 8.11.



Hipótesis



1º) ΔT ? / producir contacto entre barras sin presión

Barras no trabajan → se dilatan libremente

$\delta_1 + \delta_2 = 0'005$ (m) $\left\{ \begin{array}{l} \delta_1 = \alpha_1 e_1 \Delta T \\ \delta_2 = \alpha_2 e_2 \Delta T \end{array} \right.$ $175 \cdot 10^{-7} \cdot 1 \cdot \Delta T + 125 \cdot 10^{-7} \cdot 1 \cdot \Delta T = 0'005$

$\Delta T = 166'67\,°C$

2º) ΔT ? / fluencia en una de las barras

Se han juntado y se tocan → hipótesis; ambas comprimidas.

a) Equilibrio.

$\boxed{N_1 = N_2 = N}$

$\sigma_1 = \frac{N}{A_1} = \frac{N}{A_2} = \frac{N}{A} = \sigma$

$\sigma_{fluencia} = \underset{(1)}{200}$ MPa

b) Compat. Def.

$\delta_1 + \delta_2 = 0'005$

c) Ley de comport.

$\delta_1 = -\frac{N \cdot e_1}{E_1 A_1} + \alpha_1 e_1 \Delta T$

alargamiento

$\delta_2 = -\frac{N \cdot e_2}{E_2 A_2} + \alpha_2 e_2 \cdot \Delta T$

$\delta_1 = -\frac{N \cdot 1}{10 \cdot 10^9 \cdot 20 \cdot 10^{-4}} + 175 \cdot 10^{-7} \cdot 1 \cdot \Delta T$; $-5 \cdot 10^{-8} N + 175 \cdot 10^{-7} \Delta T$

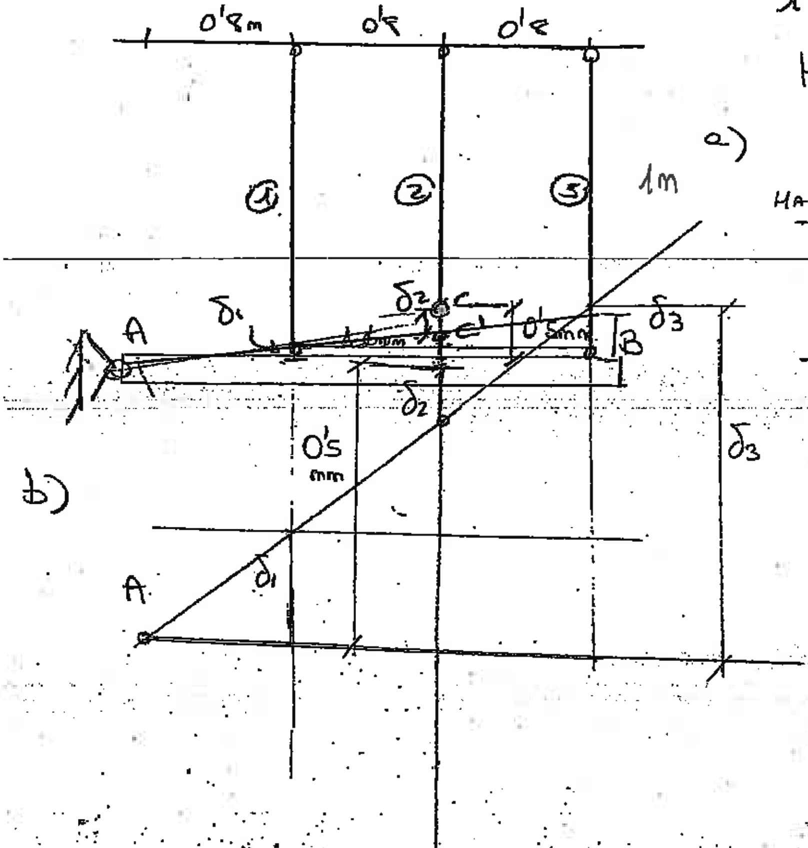
$\delta_2 = -\frac{N \cdot 1}{200 \cdot 10^9 \cdot 20 \cdot 10^{-4}} + 125 \cdot 10^{-7} \Delta T$ $\quad -2'5 \cdot 10^{-9} N + 125 \cdot 10^{-7} \Delta T = 0'005$

$3 \cdot 10^{-5} \Delta T - 0'021 = 0'005$

$\sigma = 200 \cdot 10^6 \rightarrow N = 400000$ N

$\Delta T = 866'67\,°C$

Problema 8.12.



1°) Tens. montaje en barras

Hipótesis $\begin{cases} ② \text{ Tracció} - \\ ①, ③ \text{ Compressió} - \end{cases}$

a)

V_A N_1 N_2 N_3 H_A $\quad H_A = 0$

$N_1 - N_2 + N_3 - V_A = 0$

$-N_1 \cdot 0'8 + N_2 \cdot 1'6 - N_3 \cdot 2'4 = 0$

$-N_1 + 2N_2 - 3N_3 = 0 \; (1) \; h=2$

$$\begin{cases} \dfrac{\delta_1}{0'8} = \dfrac{0'5 \cdot 10^{-3} - \delta_2}{1'6} \\ \dfrac{\delta_1}{0'8} = \dfrac{\delta_3}{2'4} \end{cases}$$

c) $\delta_1 = \dfrac{N_1 \cdot l_i}{E_i A_i} = \dfrac{N_1}{200 \cdot 10^9 \cdot 20 \cdot 10^{-4}} = N_1 \cdot 2'5 \cdot 10^{-9}$

$\delta_2 = \dfrac{N_2 \cdot}{200 \cdot 10^5 \, 20} = N_2 \cdot 2'5 \cdot 10^{-9} \cdot 0'9995 \, m$ error: en vez de 1 es (pequeño)

$\delta_3 = N_3 \cdot 2'5 \cdot 10^{-9}$

$2'5 \cdot 10^{-9} N_1 \cdot 1'6 = 0'8 \, (0'5 \cdot 10^{-3} - 2'5 \cdot 10^{-9} N_2)$

$4 \cdot 10^{-9} N_1 = 0'5 \cdot 10^{-3} - 2'5 \cdot 10^{-9} N_2 \quad (2)$

$\cancel{2'5 \cdot 10^{-9}} N_1 \cdot 2'4 = 0'8 \cdot N_3 \cdot \cancel{2'5 \cdot 10^{-9}}$

$\boxed{N_1 = 0'34 \, N_3} \quad \boxed{N_3 = 3 N_1}$

$N_2 = \dfrac{0'5 \cdot 10^{-3} - 4 \cdot 10^{-9} N_1}{2'5 \cdot 10^{-9}}$

$\boxed{N_2 = 200 \cdot 10^3 - 1'6 \cdot N_1}$

$\cancel{-N_1} + 2\,(200 \cdot 10^3 - \cancel{1'6 N_1}) - \cancel{9 N_1} = 0$

$+13'2 \, N_1 = + 400 \cdot 10^3$

$N_1 = 30'303 \cdot 10^3 \; N$

$\hookrightarrow \sigma_1 = 15'15 \, MPa$ a compresió

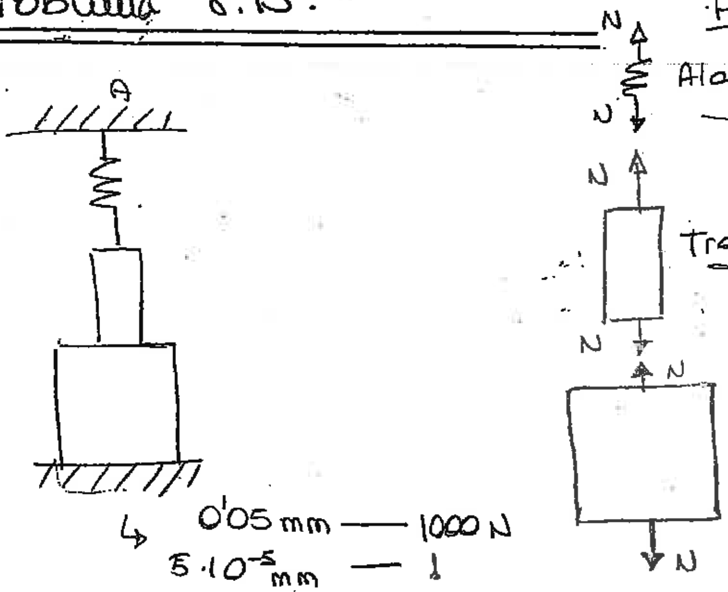
$N_2 = 0'51515'15 \, N \rightarrow \sigma_2 = 75'75 MPa$ (a tracció)

$N_3 = 90909'09 \, N \rightarrow \sigma_3 = 45'45 \, MPa$ (a compres.)

2°) → Igual pero ec. de equilibr... $\oplus$ $\ominus$

20) Exam. Visitar al aplicar P = 700000 N →

Problema 8.13.



Hipótesis:

a) $\boxed{1\ GPa = \frac{kN}{mm^2}}$

$$10^9 \frac{N}{m^2} = \frac{10^3\ N}{10^{-6}\ m^2}$$

↳ 0'05 mm —— 1000 N
$5\cdot10^{-5}$ mm —— 1
$= 5\cdot10^{-8}$ m/N

b) $(\delta_3 + \delta_2 + \delta_1)\, m \cdot \frac{1\ N}{5\cdot10^{-8}\ m} = N$

$(\delta_3 + \delta_2 + \delta_1) \cdot \frac{1}{0'05} = N$

$\left(\frac{N}{50} + \delta_2 + \delta_1\right) = 0'05\ N$

$\left(\frac{N}{50} + \frac{N}{40} + \frac{N}{60} + \frac{2}{0'2}\right) = 0'05\ N$

Sobra un cero ???

$0'2 = -0'01167\ N$

$N = -171'42$ KN

$N = -17'137$ kN

c) $K_3 \cdot \delta_3 = N \qquad \delta_3 = \frac{N}{50}$

$\delta_2 = \frac{N \cdot \ell_2}{E_2 A_2} = \frac{N \cdot 1000}{20 \cdot 2000} = \frac{N}{40}$

$\delta_1 = \frac{N \cdot \ell_1}{E_1 A_1} + \alpha \cdot \ell_1 \Delta T =$

$\frac{N}{60} + 2\cdot10^{-5}\cdot1000\cdot100$

$\delta_1 = \frac{N}{60} + \frac{2}{0'2}$

$N = 171'42\ N$ a compresión

tomando $\alpha = 2\cdot10^{-6}$ ✓

$\sigma_1 = -28'57$ MPa

$-2'856$

$\sigma_2 = -85'71$ MPa

$-8'571$

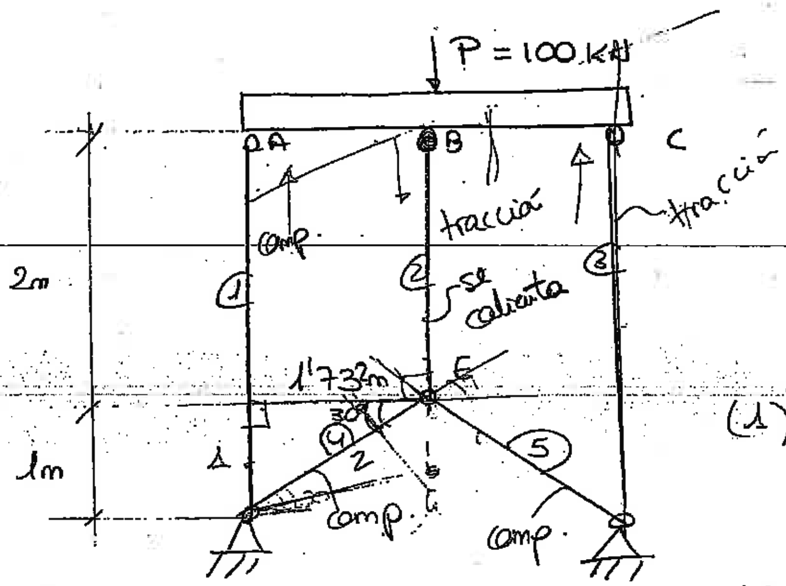
$\delta_3 = -3'42$ mm → $-0'34$ mm

$\delta_2 = -4'28$ mm → $-0'428$ mm

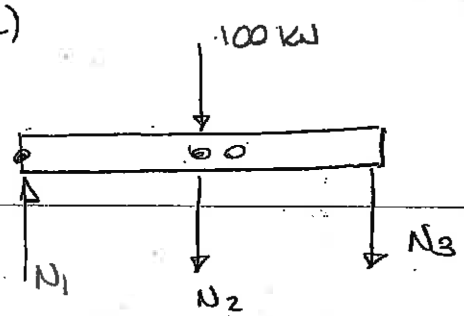
$\delta_1 = -$ → $1'71$ mm ✓

$\delta_B = 0'942$ mm

Problema 8.14.



a)

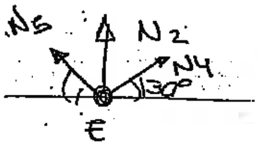


(1) $\boxed{N_1 = 100 + N_2 + N_3}$ (kN)

$-N_1 \cdot 1'732 - N_3 \cdot 1'732 = 0$

(2) $\boxed{N_1 = -N_3}$

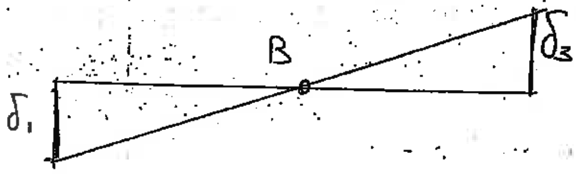
$\mathrm{tg}\,30 = \dfrac{1}{d}$



$N_4 \dfrac{\sqrt{3}}{2} = N_5 \dfrac{\sqrt{3}}{2}$ $\boxed{N_4 = N_5}$ (3)

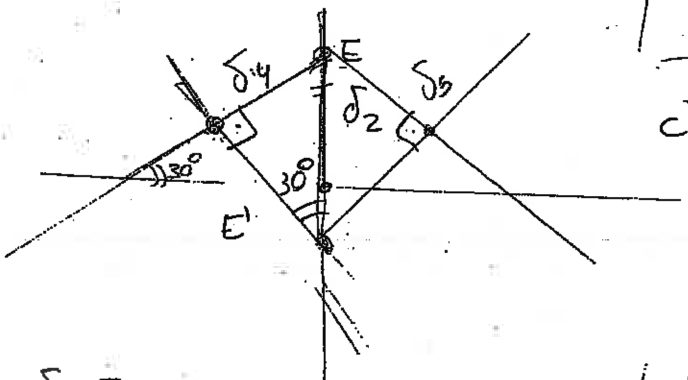
$\boxed{N_2 + \dfrac{N_5}{2} + \dfrac{N_4}{2} = 0} \longrightarrow N_2 + N_4 = 0$ $\boxed{N_2 = -N_4}$ (3)

b)



$\boxed{\delta_1 = \delta_3}$

$\boxed{\delta_4 = \delta_5 = \dfrac{\delta_2}{2}}$



$\delta_1 = \delta_3 \Rightarrow$ $\boxed{N_1 = -N_3}$ ✓

$\delta_4 = \delta_5$ $\boxed{N_4 = N_5}$ ✓

$\cancel{\delta_4} = 0'05\, N_4 = \dfrac{0'05}{2} N_2 + \dfrac{2'4}{2}$

$0'05\, N_4 = -0'025\, N_4 + 1'2$

c)

$\delta_1 = \dfrac{N_1 \cdot 3000}{200 \cdot 200} = 0'075\, N_1$

$\delta_2 = \dfrac{N_2 \cdot 2000}{200 \cdot 200} + 2000 \cdot 10^{-5} \cdot 120$

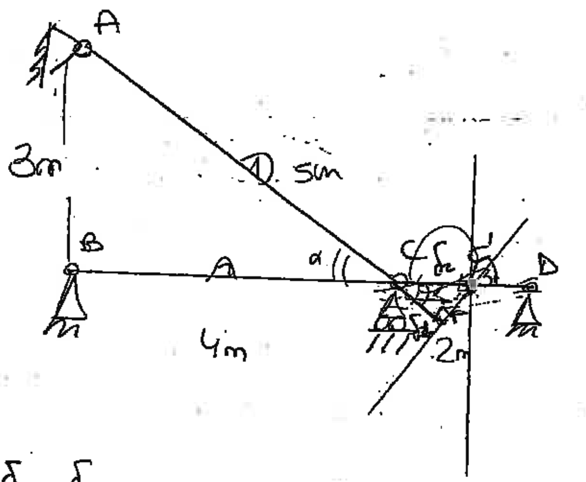
$\delta_2 = 0'05 N_2 + 2'4$

$\delta_3 = \dfrac{N_3 \cdot 2000}{200 \cdot 200} = 0'075\, N_3$

$\delta_4 = \dfrac{N_4 \cdot 2000}{200 \cdot 200} = 0'05\, N_4$

$\delta_5 = \dfrac{N_5\, 2000}{200 \cdot 200} = 0'05\, N_5$

Problema 8.15

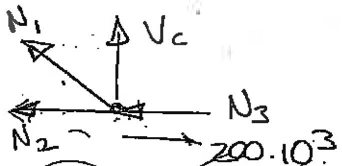


C ③ compresión ③

② tracció

① tracció

a)



$$\boxed{N_2 + N_3 + N_1 \cdot \frac{4}{5} = 200 \cdot 10^3}$$

$$V_c = N_1 \frac{3}{5} = 22'132 \text{ kN}$$

b) $\delta_2 = \delta_3$

$$\cos\alpha = \frac{\delta_1}{\delta\delta_2} = \frac{4}{5}$$

c) $$\delta_1 = \frac{N_1 e_1}{E_1 A_1} = \frac{N_1 \cdot 5}{200 \cdot 10^9 \cdot 12 \cdot 10^{-4}} = 2'0834 \cdot 10^{-8} N_1$$

$$\delta_2 = \frac{N_2 e_2}{E_2 A_2} \cancel{+ \alpha e_2 \Delta T} = \frac{N_2 \cdot 4}{200 \cdot 10^5 \cdot 12} \cancel{- 10^{-5} \cdot 4 \cdot 25} = 1'67 \cdot 10^{-8} N_2 \cancel{+ 1 \cdot 10^{-3}}$$

$$= 0'615 \text{ mm} = \delta_3$$

comp   comp   dilata

$$\delta_3 = \frac{N_3 \cdot 2}{200 \cdot 10^5 \, 12} - \alpha' e_3 \Delta T = 8'34 \cdot 10^{-9} N_3 - 10^{-5} \cdot 2 \cdot 25$$

$$= 8'34 \cdot 10^{-9} N_3 - 5 \cdot 10^{-4}$$

$$1'67 \cdot 10^{-8} N_2 \cancel{+ 1 \cdot 10^{-3}} = 2'0834 \cdot 10^{-8} N_3$$

$$1'67 \cdot 10^{-8} N_2 = 8'34 \cdot 10^{-9} N_3 - 5 \cdot 10^{-4}$$

$$N_3 = 0'8015 N_2 - 48000$$

$$N_3 = 2 N_2 + 60000$$

$$5 \left(1'67 \cdot 10^{-8} \cdot N_2 - \cancel{1 \cdot 10^{-3}} \right) = 4 \cdot 2'0834 \cdot 10^{-8} N_1$$

$$\cancel{N_1 = 1'0019 N_2 - 60000} \qquad \cancel{N_1 = N_2} \qquad N_1 = 1'00196 \; N_2$$

$$\cancel{N_2 + 2N_2 + 60000 + 0'8 \; 0'8015 \; N_2 = 200\,000}$$

$$\cancel{N_2 + 0'8015 \; N_2 - 48000 + 0'80152 \; N_2 - 48000 = 200\,000}$$

$$\cancel{N_2 = 1137149 \text{ Pa}} \qquad N_2 = 36\,827 \longrightarrow \sigma_2 = 30'68 \text{ MPa}$$

Hipótesis ✓

$$\sigma_1 = 30'74 \text{ MPa}$$

$$\sigma_2 = 94'76 \text{ MPa}$$

$$\sigma_3 = 111'37 \text{ MPa}$$

$EA = 200 \cdot 10^6 \, kPa \cdot 1'2 \cdot 10^{-3} \, m^2 = 240 \cdot 10^3 \, kN$

$$\frac{N_2 L_2}{EA} = \frac{N_3 L_3}{EA} - \alpha L_3 \Delta T$$

(1) $0'8 \cdot \frac{N_2 L_2}{\cancel{EA}} = \frac{N_1 L_1}{\cancel{EA}}$ $\quad$ $3'2 N_2 = 5 N_1$ $\boxed{N_1 = 0'64 N_2}$

(2) $N_3 = 200 - 0'8 \cdot (0'64 N_2) - N_2$

$$\frac{4 \cdot N_2}{240 \cdot 10^3} - \frac{2 N_3}{240 \, 10^3} + 10^{-5} \cdot 25 \cdot 2 = 0$$

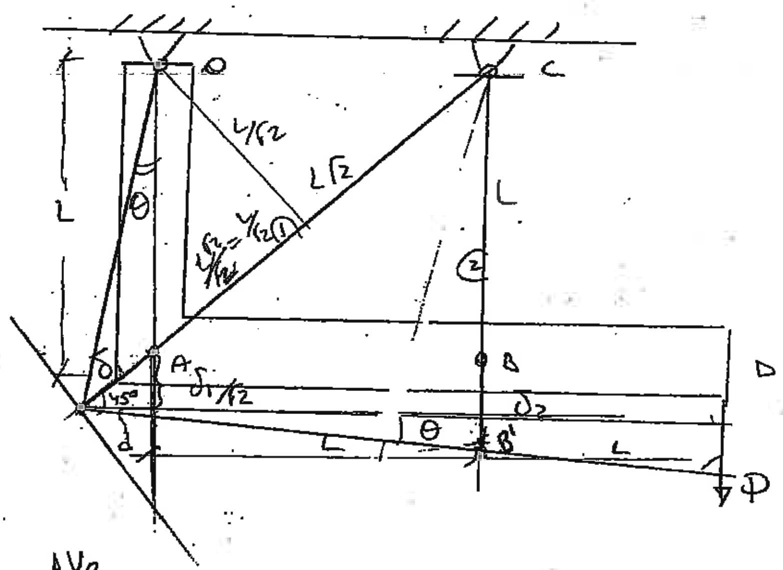
$$\frac{4 N_2}{240 \cdot 10^3} - \frac{2}{240 \, 10^3} \cdot (200 - 1'512 N_2) + 5 \cdot 10^{-4} = 0$$

$N_2 = -155'25 \, kN$

$N_3 = 283'54 \, kN$ --- ✓

$N_1 = -35'36 \, kN$

Problema 8.18



$L = 200$ cm

$E = 200$ GPa

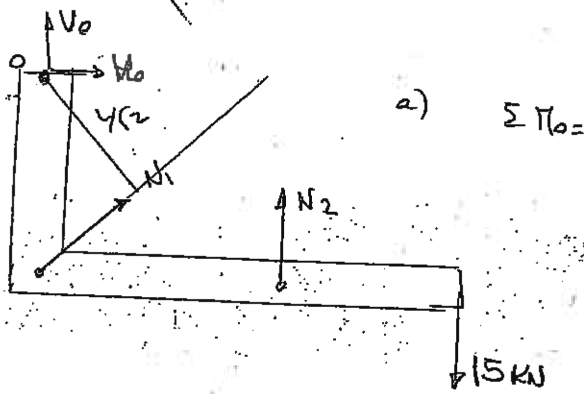
$P = 15$ kN

$A_1 = 2\,cm^2$

$A_2 = 2\sqrt{2}\ cm^2$

1) N_1, N_2 ?

2) Descenso D

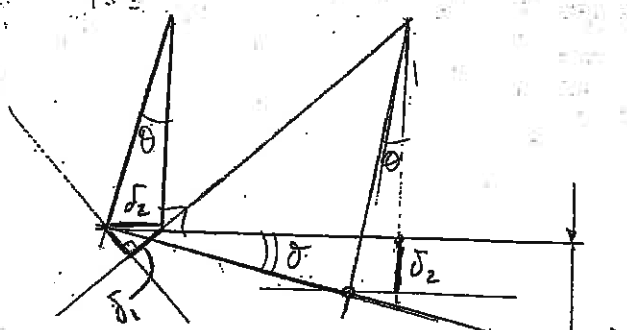


a) $\Sigma M_O = 0$

$$N_1 \cdot \frac{\cancel{L}}{\sqrt{2}} + N_2 \cancel{L} - 15 \cdot 2\cancel{L} = 0$$

$$\frac{N_1}{\sqrt{2}} + N_2 - 30 = 0 \quad (1)$$

b) d



$$\boxed{\delta_1 = \frac{\delta_2}{\sqrt{2}}}$$

$$\frac{N_1 L}{E A_1} = \frac{N_2 L}{E A_2 \sqrt{2}} \quad (2)$$

Descenso D $\rightarrow$ $tg\,\theta = \dfrac{\delta_2}{L} = \dfrac{\delta_D}{2L}$

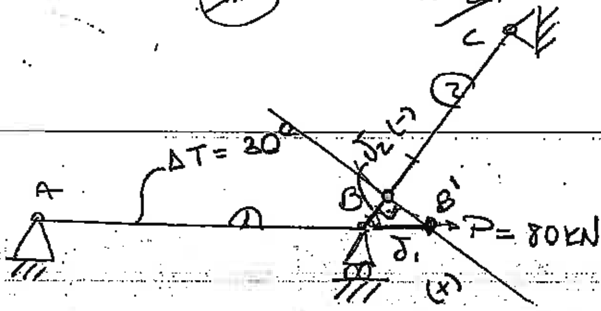
Problema 8.16.

$A = 2 cm^2$ $\quad E \cdot A = 200 \cdot 10^6 \frac{kN}{m^2} \cdot 2 \cancel{cm^2} \cdot \frac{1 m^2}{10^4 cm^2} = 400 \cdot 10^2 kN = 40000 \, kN$

$L = 600 cm$

$E = 200 GPa$

$\alpha = 10^{-5} \, °C^{-1}$

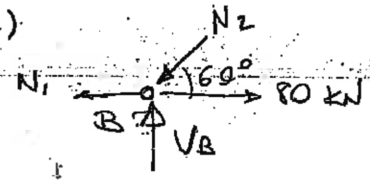


Hipótesis

1 tracció

2 compresió

a)



b) $\delta_1 \cos 60 = \delta_2$

$\frac{\delta_1}{2} = \delta_2 \qquad \delta_1 = 2\delta_2$

(1) $N_1 + \frac{N_2}{2} = 80$

$N_1 = 80 - \frac{N_2}{2}$

c) (cm, kN)

$$\begin{cases} \delta_1 = \frac{N_1 L}{EA} + \alpha L \Delta T = N_1 \cdot \frac{600}{40000} + 10^{-5} \cdot 600 \cdot 30 \\ \delta_2 = \frac{N_2 L}{EA} = N_2 \cdot \frac{600}{40000} \end{cases}$$

$\delta_1 = 1'08$ cm

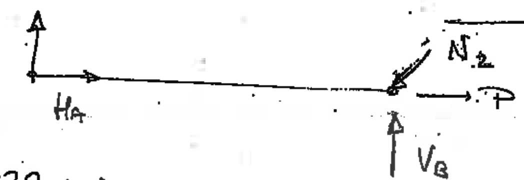
$0'015 N_1 + 0'18 = 0'03 N_2$

$0'015 \left(80 - \frac{N_2}{2}\right) + 0'18 = 0'03 N_2$

$N_2 = 39'42$ kN → a comp.

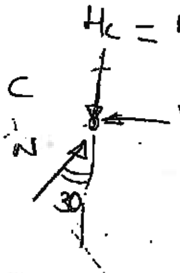
$N_1 = 60'28$ kN → a tracció

$V_B = \frac{N_2 \sqrt{3}}{2} = 34'138$ kN



$\Sigma M_B = 0 \quad V_A = 0$

$H_A + 80 = \frac{N_2}{2} \qquad H_A = -60'29$ kN



$H_C = \frac{N_2 \sqrt{3}}{2} = 34'13$ kN

$V_C = \frac{N_2}{2} = 19'71$ kN

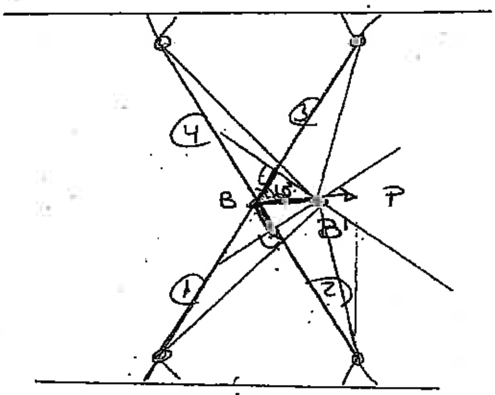
$R_C = 39'42$ kN

Problema 8.19.

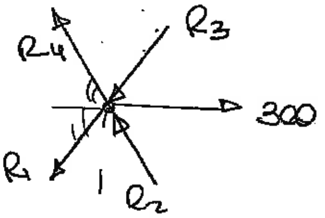
L, E, A_i ($i=1,2,3,4$)

Φ 1º) Esfuerzos barras

2º) Desp. B.



a) Equilibrio



$$300 = \frac{R_4}{2} + \frac{R_1}{2} + \frac{R_3}{2} + \frac{R_2}{2} \quad (1)\ R_1 + R_2 + R_3 + R_4 = 600\,k$$

$$R_4\frac{\sqrt{3}}{2} - R_1\frac{\sqrt{3}}{2} + R_2\frac{\sqrt{3}}{2} - R_3\frac{\sqrt{3}}{2} = 0 \qquad R_1 + R_3 = R_2 + R_4 \quad (2)$$

b) $\delta_1 = \delta_3$

$\delta_2 = \delta_4$

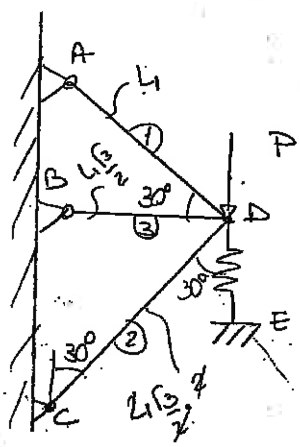
c) $$\frac{R_1 L}{EA_1} = \frac{R_3 L}{EA_3} \quad (3)$$

$$\frac{R_2 L}{EA_2} = \frac{R_4 L}{EA_4} \quad (4)$$

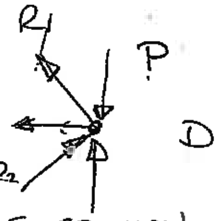
$$\text{Desp. } B = \frac{\delta_1}{\cos 60^\circ} = 2\frac{R_1 L}{EA_1}$$

Problema 8.20.

(!!!) NO HAY QUE SUPONER QUE LA REACCIÓN DE UNA BARRA INDEFORMABLE SEA NULA



a)



no conoc. su comport. ← R_3

Teus. barras ?

Desp. D ?

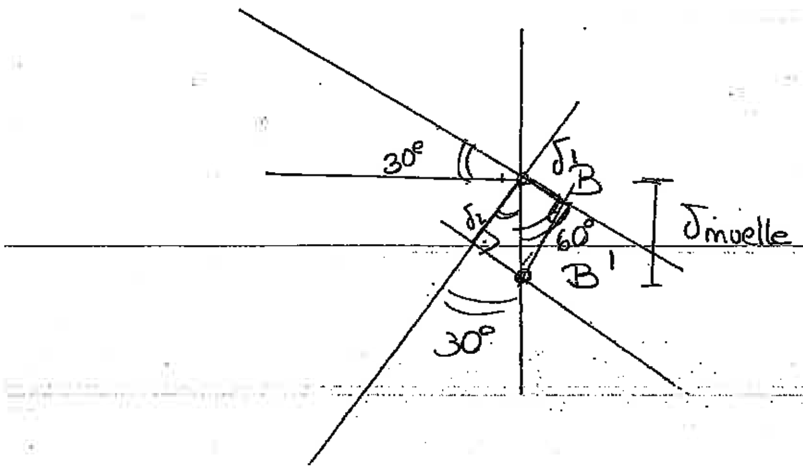
Planteamos eq. vertical

$$\frac{R_1}{2} - 10 + V_D + R_2\frac{\sqrt{3}}{2} = 0 \quad (1)$$

(muelle)

Hipótesis { muelle compr. ①, ③ tracción ② comp.

b) Compat. deformaciones:



$$\delta_{muelle} = \frac{\delta_2}{\cos 30°}$$

$$\delta_{muelle} = \frac{\delta_1}{\cos 60°}$$

c) $\delta_1 = \frac{R_1 \cdot L_1}{E_1 A_1}$

$\delta_2 = \frac{R_2 L_2 \sqrt{3}}{E_2 A_2}$

$\delta_3 = 0$ ✓

$\delta_{muelle} = \frac{R_1 L_1}{E_1 A_1} \cdot 2$

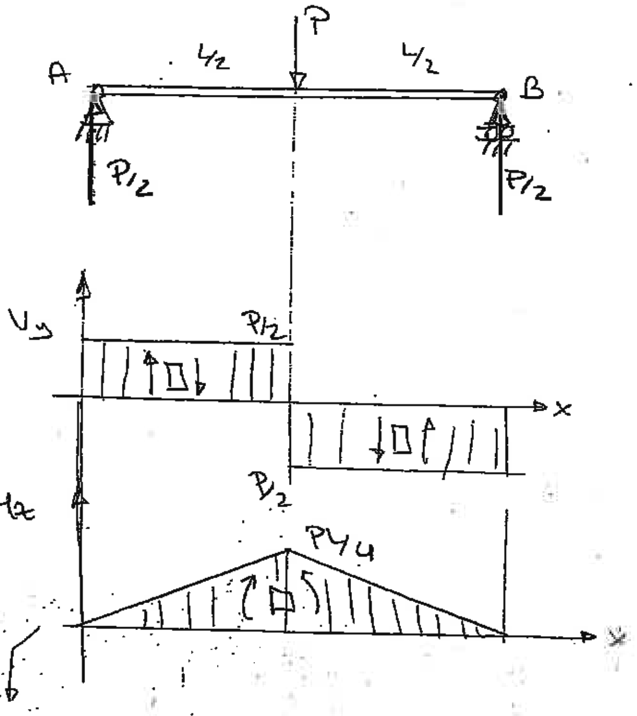
$$\frac{R_2 \cancel{\sqrt{3}L}}{E_2 A_2} \cdot \frac{\cancel{2}}{\cancel{\sqrt{3}}} = \frac{R_1 \cancel{L}}{E_1 A_1} \cdot \cancel{2}$$

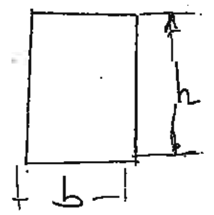
$$R_1 = \frac{E_1 A_1}{E_2 A_2} \cdot R_2 \quad (2) \quad R_1 = 1'5\, R$$

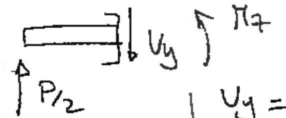
(3) $V_D = K \cdot x = 300\ kN/m \cdot \frac{R_1 L}{E_1 A_1} \cdot 2$

Problemas Tema 10: Teoría General de la Flexión. Tensiones (I)

Problema 1

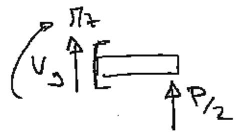




① 

$V_y = P/2$

$M_z = \frac{P}{2}x$

② 

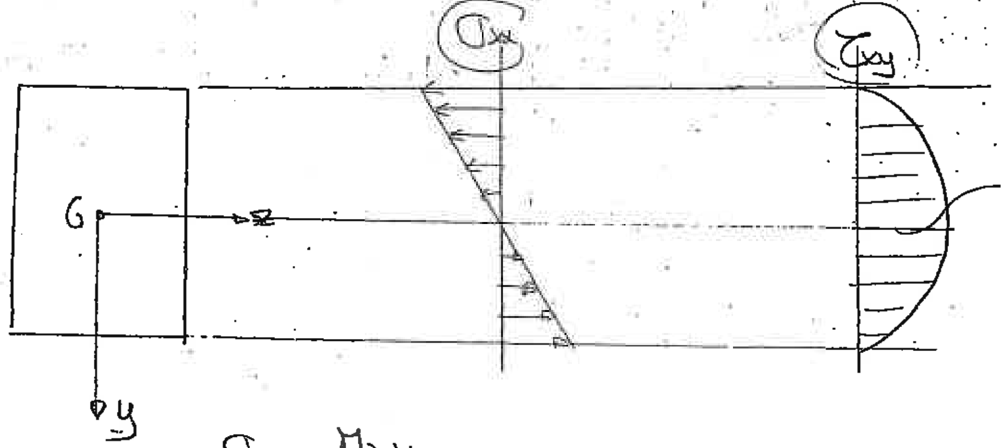
$V_y = -P/2$

$-M_z + \frac{P}{2}x = 0 \qquad M_z = \frac{P}{2}x$

$x = 0\ (B) \rightarrow 0$

$x = L/2\ (C) \rightarrow PL/4$

M_z máximo : $PL/4$



$\tau_{xy} = \frac{3}{2}\,\frac{P/2}{b \cdot h}$ (τ_{xy} máx

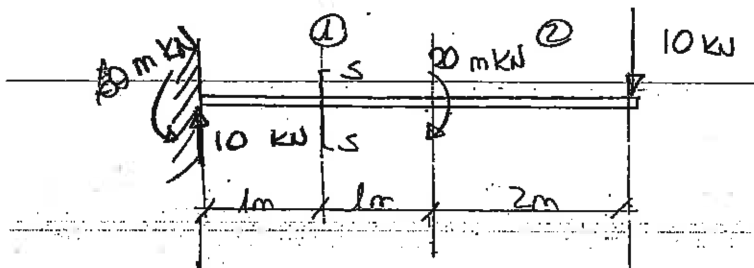
$$\sigma_{xx} = \frac{M_z\, y}{I_z}$$

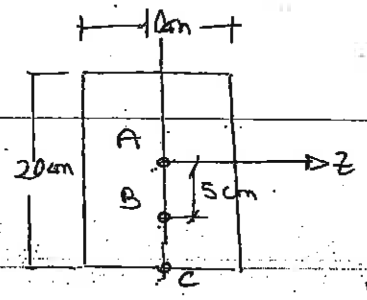
$$\sigma_{xx}\ \text{máx} = \frac{M_z\, y_{máx}}{I_z} = \frac{PL/4 \cdot h/2}{\frac{1}{12}\, b h^{3}} = \frac{PL}{8} \cdot \frac{12}{bh^2} = \frac{3}{2}\,\frac{PL}{bh^2}$$

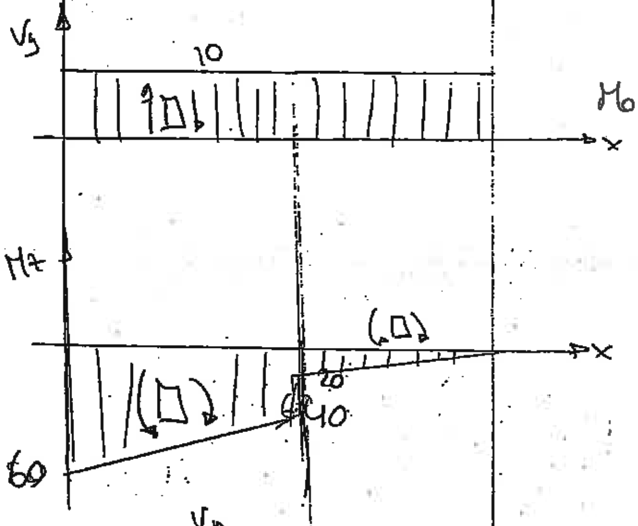
$$\frac{\sigma_{xx}\ \text{máx}}{\tau_{xy}\ \text{máx}} = \frac{\cancel{\frac{3}{2}} PL / \cancel{b} h^2}{\cancel{\frac{3}{2}}\, P/2 \cdot 1/\cancel{b} h} = \frac{L}{h} \cdot \frac{2}{1} = \frac{2L}{h}$$

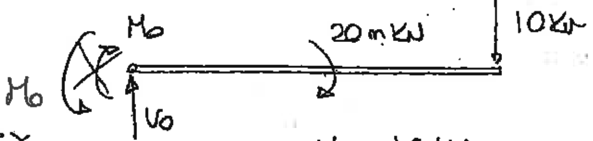
Problema 10.2.

Estados tensionales en A, B, C de sección s-s y representar los círculos de Mohr:









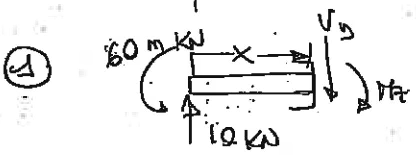
$V_0 = 10$ KN

$-20 - M_0 - 10 \cdot 4 = 0$

$M_0 = -20 - 40 = -60$

$M_0 = 60$ m·KN

①



$V_y = 10$ KN

$60 - 10x - M_z = 0$

$M_z = 60 - 10x$

$x = 0 \rightarrow 60$

$x = 2 \rightarrow 40$ $\quad x = 1 \rightarrow 50$

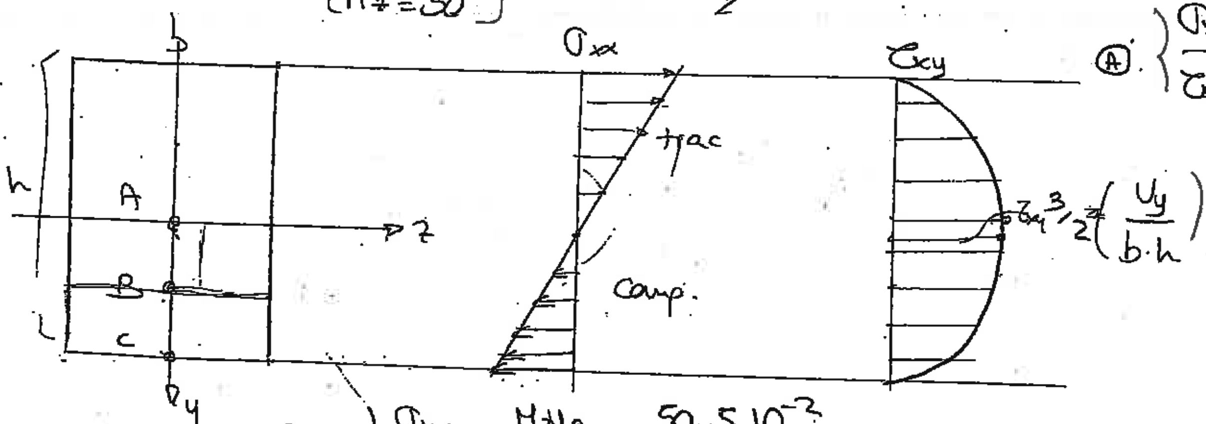
②

$V_y = 10$

$M_z = 10y$ $\quad \begin{cases} y = 0 \rightarrow 0 \\ y = 2 \rightarrow 20 \end{cases}$

Sección S-S: $\begin{cases} V_y = 10 \\ M_z = 50 \end{cases}$

Ⓒ $\begin{cases} \tau_{xy} = 0 \\ \sigma_{xx} = \dfrac{50 \cdot 0'1}{0'1 \cdot 0'2^3} \cdot 12 = 75\,000 \text{ KN/m}^2 \end{cases}$ a comp.



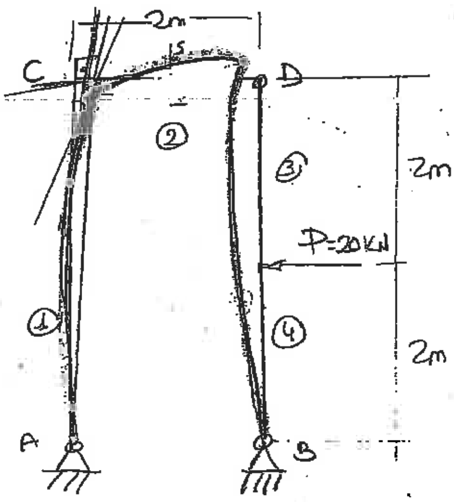
Ⓐ $\begin{cases} \sigma_{xx} = 0 \\ \tau_{xy} = \dfrac{3}{2} \cdot \dfrac{10 \cdot 10^3}{(10 \cdot 20) \cdot 10} \end{cases}$

$\tau_{xy} = 0'75$ MPa

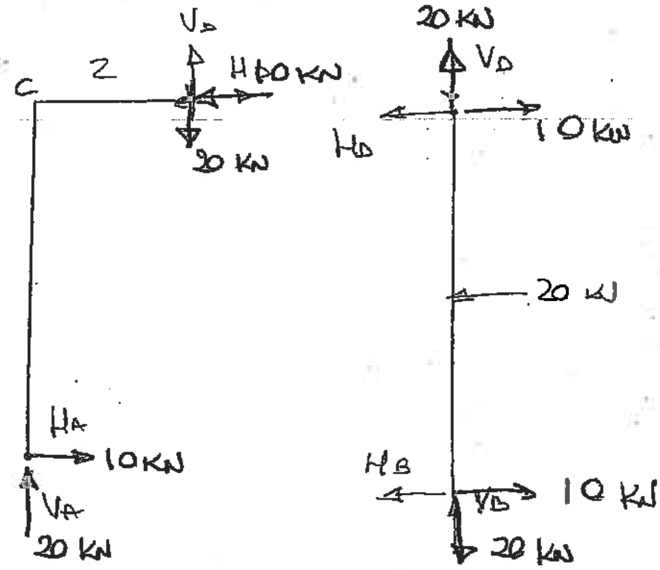
$\tau_{xy} = 750$ KN/m² ✓

Ⓑ ✓ $\begin{cases} \sigma_{xx} = \dfrac{M_z y_B}{I_z} = \dfrac{50 \cdot 5 \cdot 10^{-2}}{0'1 \cdot 0'2^3/12} = 37\,500 \text{ KN/m}^2 \text{ a compresión} \\ \tau_{xy} = V_y \cdot Q_z \;\; 10 \cdot [0'1 \cdot (0'1 - 0'05) \cdot 0'075] \ldots \end{cases}$

Problema 10.5.



1)



$V_D = -20$ kN

$V_A + V_D = 0$

$H_A + H_D = 0 \quad H_A = 10$ kN

$H_A \cdot 4 = V_A \cdot 2$

$V_A = \frac{40}{2} = 20$ kN

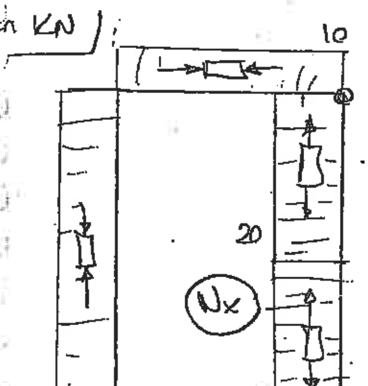
$20 + H_B + H_D = 0 \qquad H_D = -20 - (-10) = -10$ kN

$V_B - V_D = 0 \qquad V_B = -20$ kN

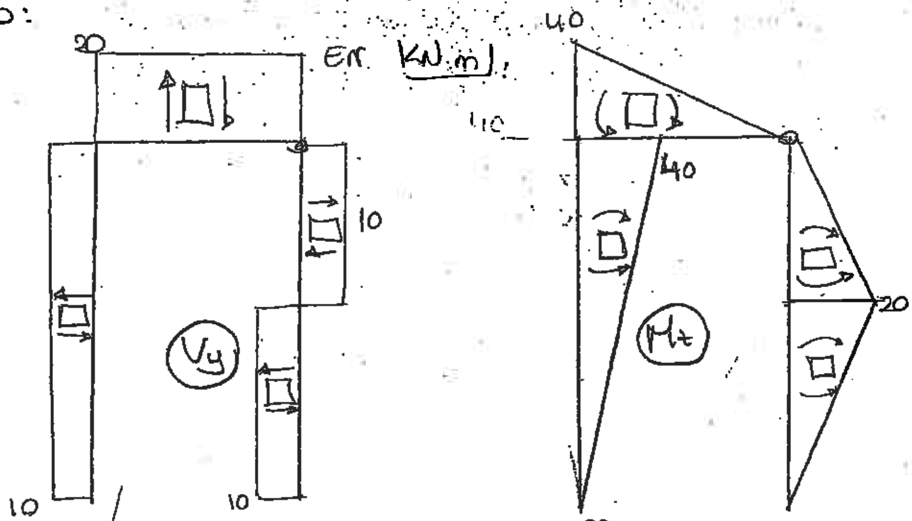
$-20 \cdot 2 - H_B \cdot 4 = 0 \rightarrow H_B = -10$ kN

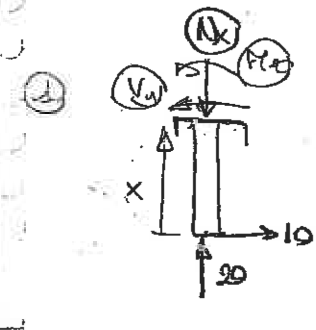
2) Diagramas de esfuerzos:

(en KN):



En KN·m):





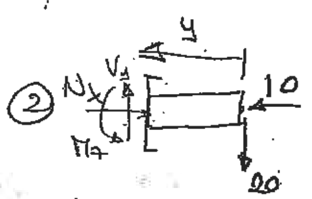
$N_x = 20$ kN

$V_y = 10$ kN

$M_z = -10x$

$x = 0 \rightarrow 0$

$x = 4 \rightarrow -40$

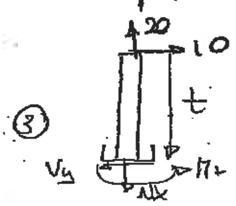


$N_x = 10$

$V_y = 20$

$M_z - 20y = 0$

$M_z = 20y$



$N_x = 20$

$V_y = 10$

$M_z = 10t$

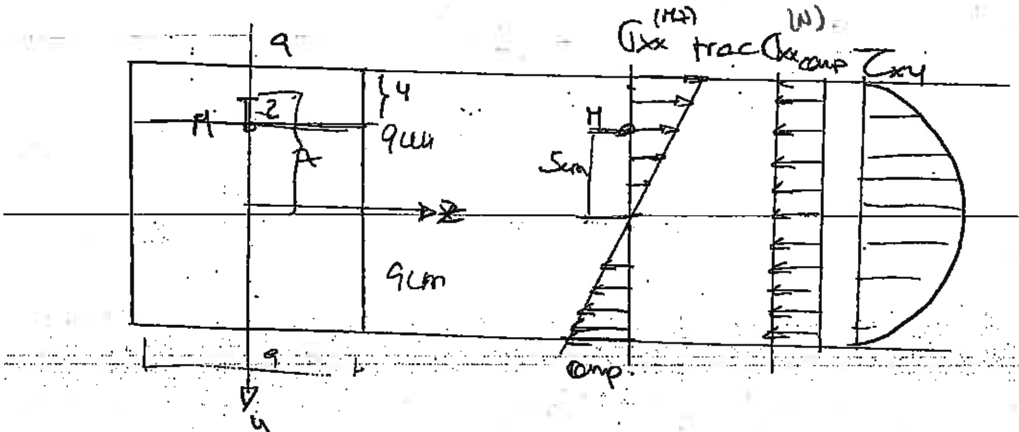
$V_y = 10$

$M_z = -10u$

$M_z = 10 \times 2 = 20$

Secció S-S

$$\begin{cases} V_y = 20 \text{ kN} \\ M_z = 20 \text{ kNm} \end{cases}$$



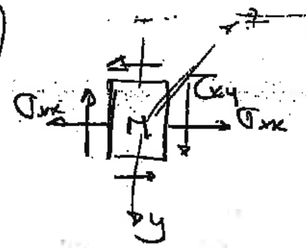
$X(22'86, -1'28)$

$Y(0, 1'28)$

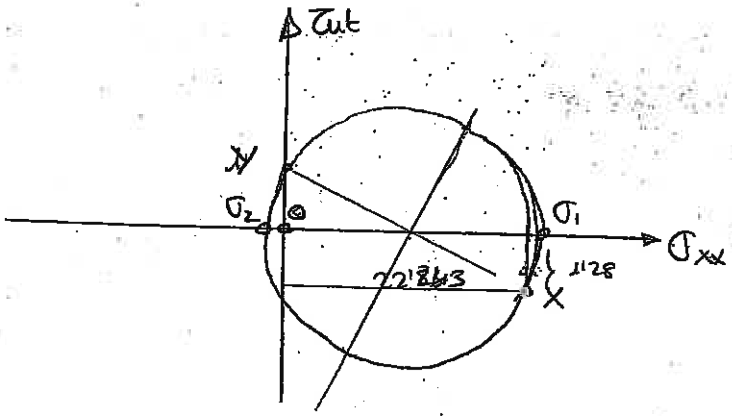
$$\sigma_{xx}^{(N)} = N_x/A = \frac{10}{0'18 \cdot 0'09} = 0'617 \text{ MPa a comp.}$$

$$\sigma_{xx}^{(M_z)} = \frac{M_z \cdot y_M}{I_z} = \frac{20 \cdot 0'05}{\frac{1}{12} \cdot 0'09 \cdot 0'18^3} = 22'862 \text{ MPa a tracció}$$

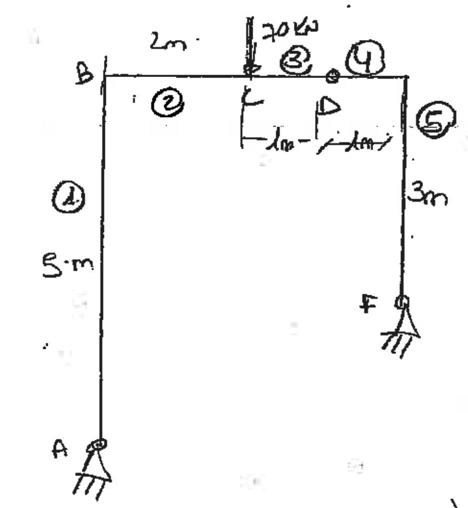
$\rightarrow \sigma_{xx}(\text{tracc}) = 22'24$

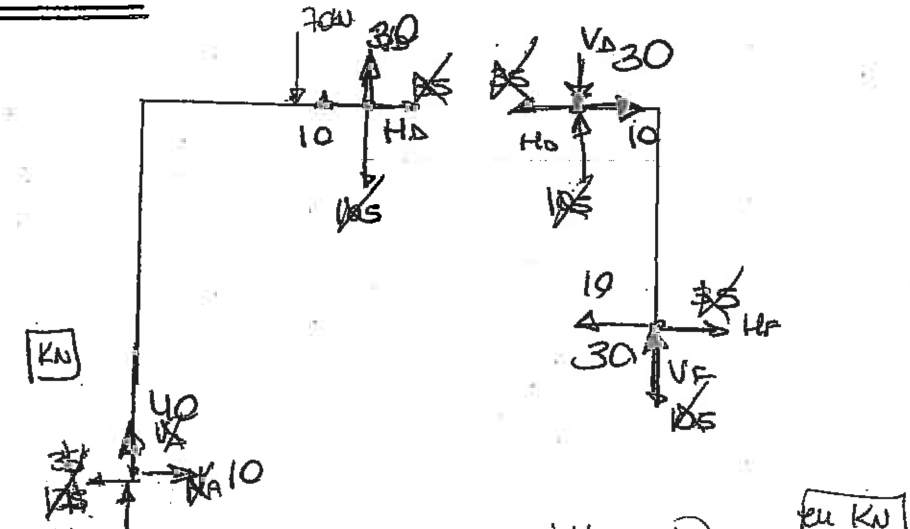


$$\tau_{xy} = \frac{V_y Q_z}{b I_z} = \frac{20 \cdot (4 \cdot 9 \cdot 7) \cdot 10^{-6}}{9 \cdot 10^{-2} \cdot \frac{1}{12} \cdot 0'09 \cdot 0'18^3} = 1'280 \text{ MPa}$$



Problema 10.6.





en kN

$$\begin{cases} H_A + H_D = 0 \\ V_A + V_D - 70 = 0 \\ -70 \cdot 2 + V_D \cdot 3 - H_D \cdot 5 = 0 \end{cases}$$

$$\begin{cases} V_F - V_D = 0 \\ H_F - H_D = 0 \\ V_D \cdot 1 + H_D \cdot 3 = 0 \end{cases}$$

$V_D = -H_D 3$

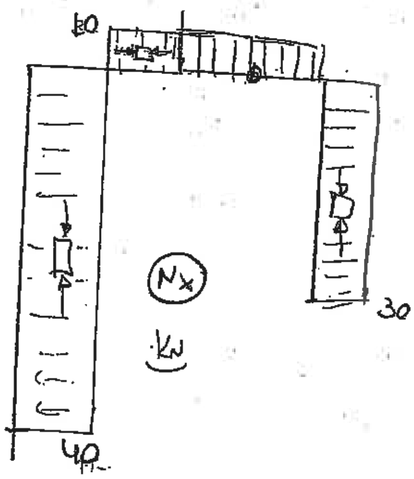
$-140 + 3(-H_D \cdot 3) - H_D \cdot 5 = 0$

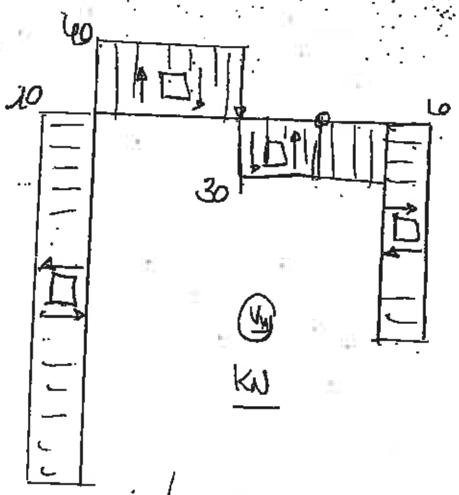
$V_D = +30$ kN

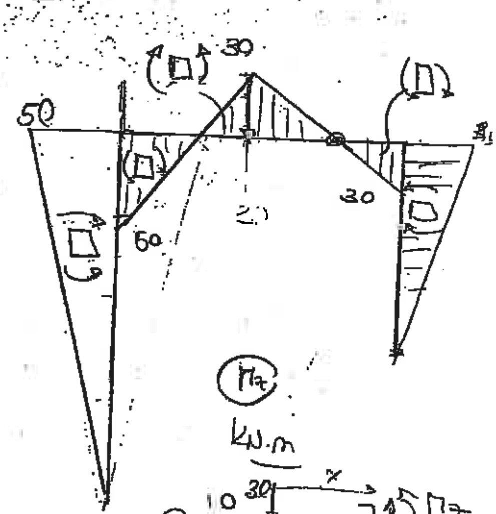
$-140 + 9H_D - 5H_D = 0$ $\quad -140 + 14H_D = 0$ $\quad H_D = 10$ kN

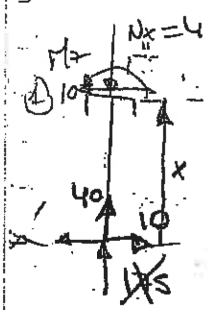
$V_A = 70 - 30 = 40$ kN $\qquad H_A = -10$ kN

$V_F = 30$ kN $\qquad H_F = 10$ kN





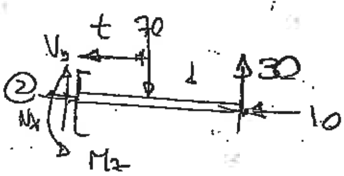




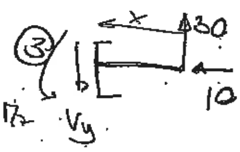
$M_z - 35x = 0$

$M_z = -10x$

$\left| \begin{array}{l} x = 0 \to 0 \\ x = 5 \to -50 \end{array} \right.$

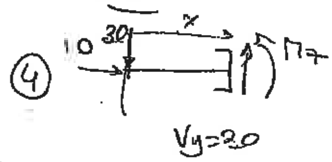


$-M_z + 30(1 + t) - 70t = 0$

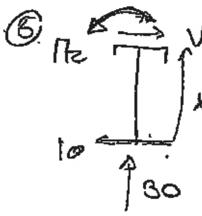


$M_z = -30x$

(D) -30



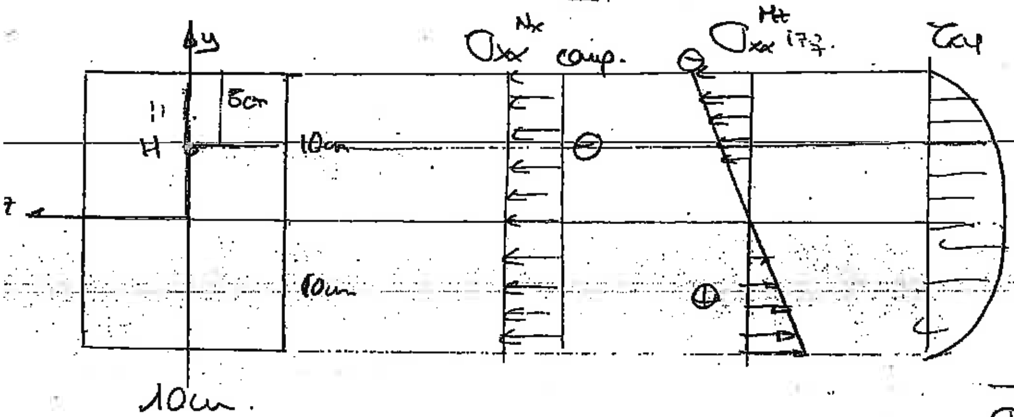
$M_z = -30x$



$M_z = 10x$

$M_z = 70t - 30 - 30t = 40t - 30 = 0 \to t = 2$

Sección C $\left\{ \begin{array}{l} N_x = 10\,\text{kN compresión} \\ V_y \text{ izq} = 40\,\text{kN} \quad // \quad V_y \text{ drcha} = 30\,\text{kN} \\ M_t = 80\,\text{kNm} \end{array} \right.$



$\sigma_{xx}^{(Nx)} = \dfrac{10}{200 \cdot 10^{-4}} = 500 \dfrac{kN}{m^2}$ (compr.)

$\sigma_{xx}^{(Mt)} = \dfrac{M_t \cdot y_H}{I_z} = \dfrac{30 \cdot 0'05}{\frac{1}{12} \cdot 10 \cdot 20^3 \cdot 10^{-8}}$ (comp.)

$= 22.500\ kN/m^2$

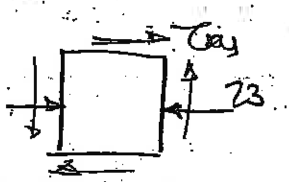
σ_{xx} (comp) $= 23000 \dfrac{kN}{m^2} = 23$

$\tau_{xy} = \dfrac{V_y \cdot Q_z}{b \cdot I_z}$

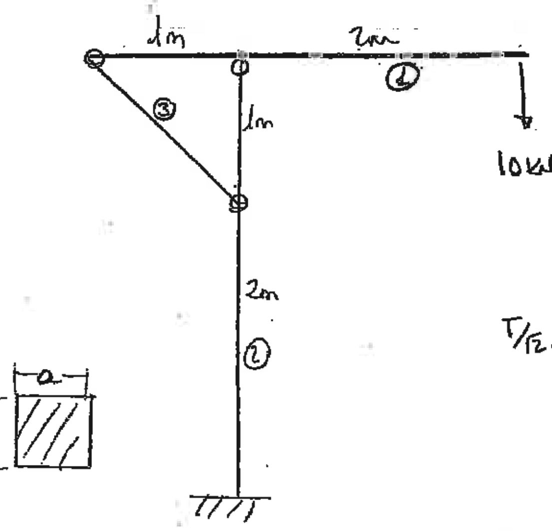
Izq: $\tau_{xy} = \dfrac{40 \cdot (5 \cdot 10 \cdot 7'5)\ (10^{-6})}{0'1 \cdot \frac{1}{12} \cdot 10 \cdot 20^3 \cdot 10^{-8}} = 2'25$ MPa

drcha: $\tau_{xy} = 1'687$ MPa

$\boxed{\text{MPa}}$

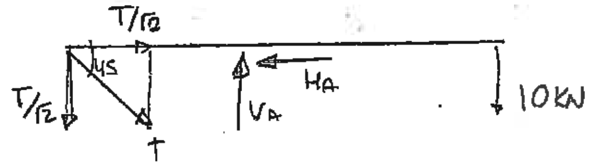


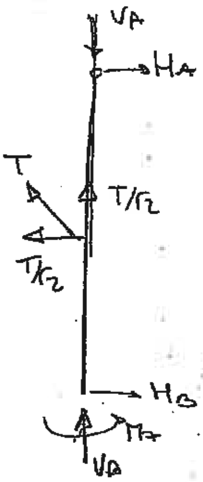
Problema 10.17.



$\sigma_{adm} = 140\,MPa$

Barras a flexión, despreciar tens. debidas a otros esf.





$H_A = T/\sqrt{2} = \underline{20\ kN}$

$V_A = 10 + T/\sqrt{2} = \underline{30\ kN}$

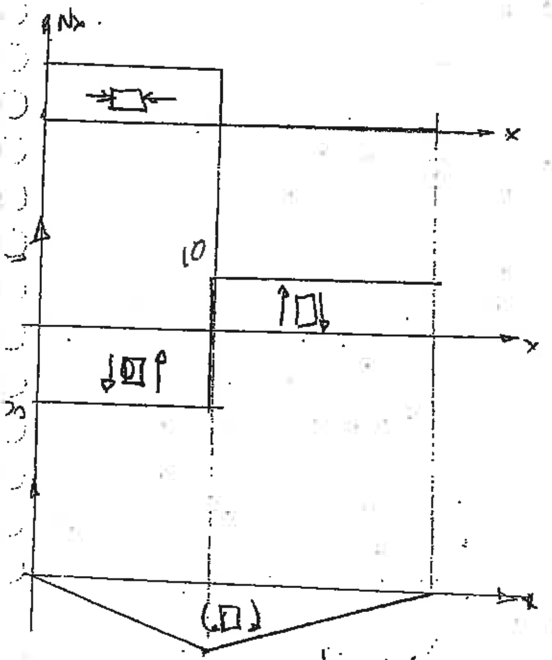
$\Sigma M_A = 0 \quad T/\sqrt{2} \cdot 1 = 10 \cdot 2 \rightarrow T = 20\sqrt{2}$

$H_B + H_A = T/\sqrt{2} \rightarrow \boxed{H_B = 0}$

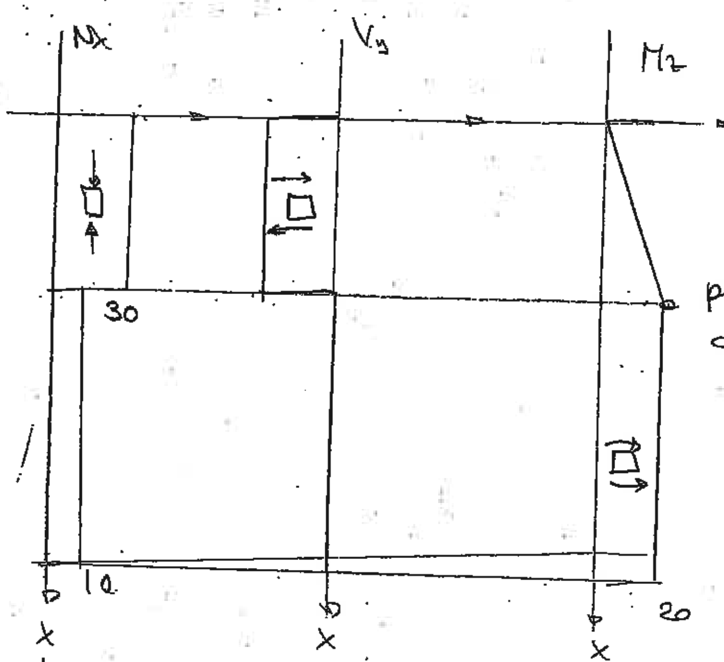
$V_B + T/\sqrt{2} = V_A \rightarrow V_B = 10\ kN$

$M_B = H_A 3 - T/\sqrt{2} \cdot 2 = 60 - 40 = 20\ kN\cdot m$

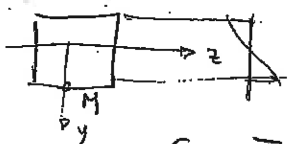
Barra 1



Barra 2



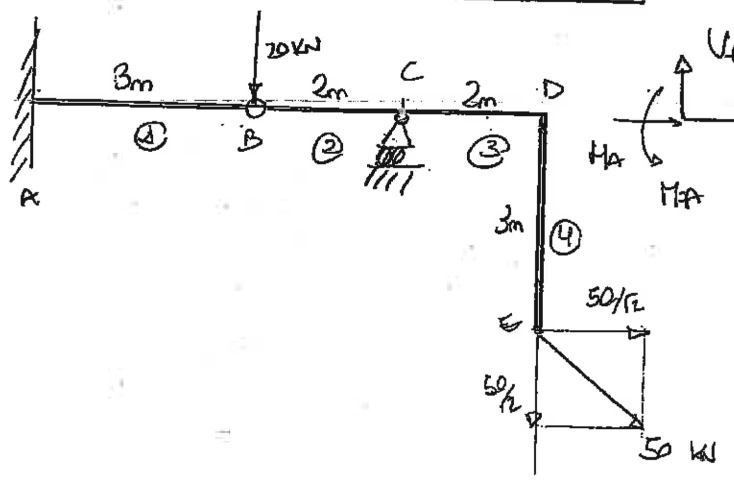
ambos $M_z = 20\ kN\cdot m$ $\quad \sigma_{xx} = \frac{M_y \cdot z}{I_z}$ de barras ① y ②

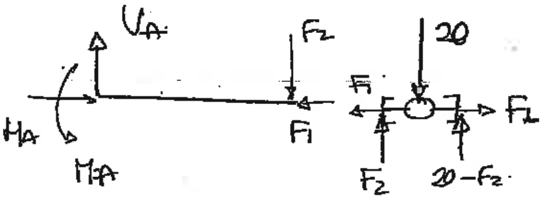


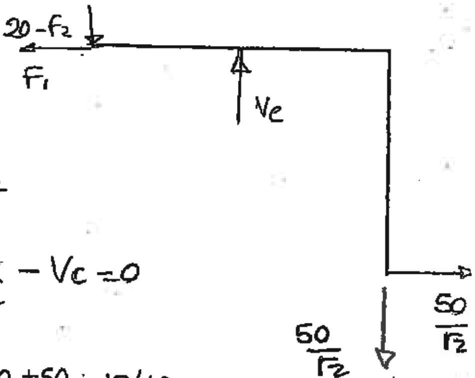
$\sigma_{xx\,max} = \frac{20 \cdot a/2}{\frac{1}{12} \cdot 2a \cdot a^3} = \frac{120}{a^3} = 140 \cdot 10^3$ $\quad \boxed{a = 0'094\ m}$

③ $\rightarrow \quad \sigma_x = \frac{T}{} = 140 \cdot 10^3 \quad a = \sqrt{\frac{20\sqrt{2}}{}} = 0'0142\,m$

Problema 10.13.







$$F_1 = \frac{50}{\sqrt{2}} =$$

$$20 - F_2 + \frac{50}{\sqrt{2}} - V_C = 0$$

$$\Sigma M_C = 0: \quad V_C \cdot 2 - \frac{50}{\sqrt{2}} \cdot 4 + \frac{50}{\sqrt{2}} \cdot 3 = 0$$

$$F_2 = 20 + \frac{50}{\sqrt{2}} - 17'67$$

$$V_C = \frac{25}{\sqrt{2}}(4+3) = 25/\sqrt{2} = 17'67\ \text{kN}$$

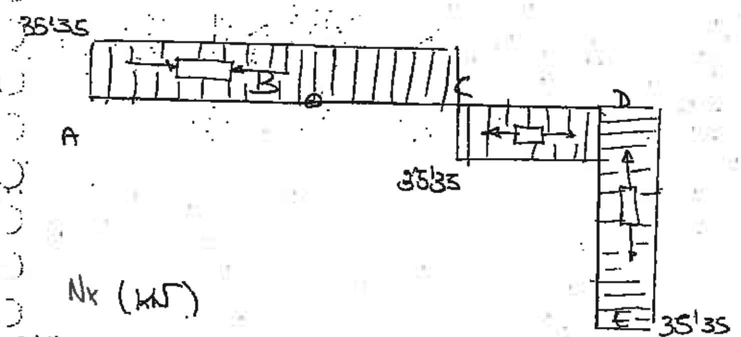
$$F_2 = 37'68$$

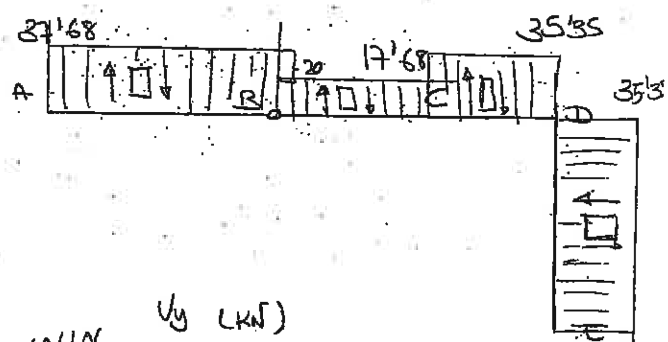
$H_A = F_1 = 50/\sqrt{2}$ kN ,

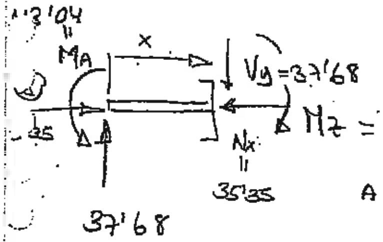
$$M_A = F_2 \cdot 3 = 113'04\ \text{kN·m}$$

$H_A = 35'35$ kN

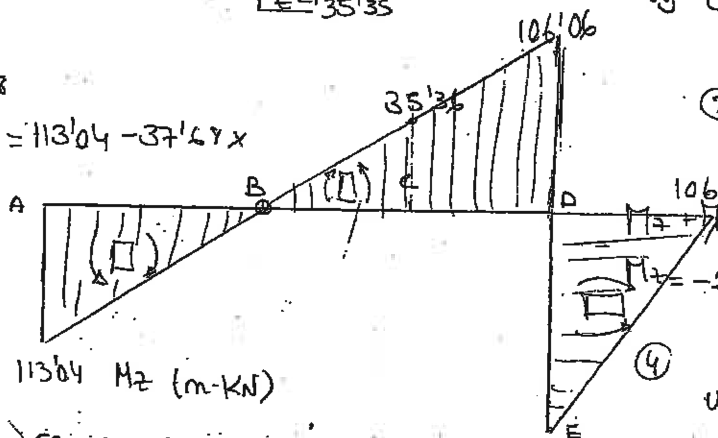
$V_A = 37'68$ kN

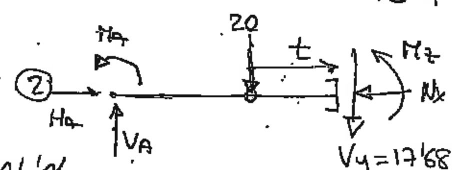






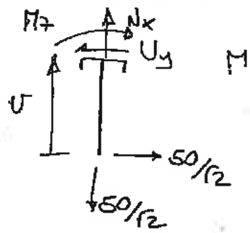
$M_z = 113'04 - 37'68x$



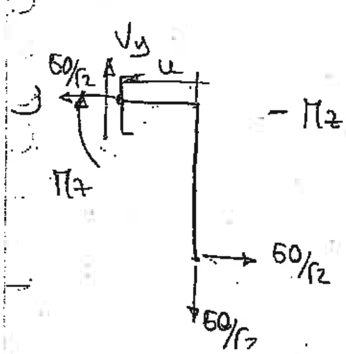


$$M_z + \cancel{M_A} + 20t - \cancel{V_A}(\cancel{3}+t) = 0$$

$$M_z = -20t + 37'68 \cdot t = 17'68t$$



$$M_z = 35'35u$$

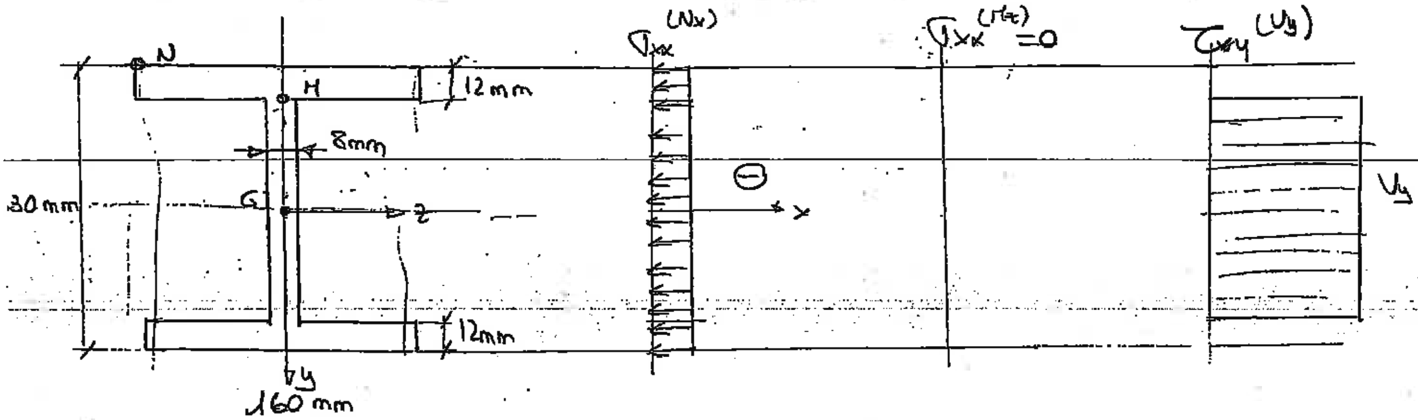


$$-M_z - \cancel{\frac{50}{\sqrt{2}}u} + \frac{60}{\sqrt{2}} \cdot 3 = 0$$

$$M_z = 106'06 - 35'35u$$

2º) Sección B

$$\begin{cases} N_x = 35'35 \text{ kN (Compresión)} \\ V_y = 37'68 \text{ kN} \\ M_z = 0 \end{cases}$$



Punto M

$$\sigma_{xx}^{(N_x)} = \frac{35'35}{[2\times 12\times 160 + 8\times(330-24)]\cdot 10^{-6}} = 5'621 \text{ MPa (Compresión)}$$

$$\tau_{xy} = \frac{V_y Q_z}{b I_z} \rightarrow \tau_{xy} = \frac{V_y}{b_1 h_1} = \frac{37'68}{(8\cdot 306)\cdot 10^{-6}} = 15'39 \text{ MPa}$$

τ_{xy} M σ_{xx}

Punto N

$$\sigma_{xx}^{(N_x)} = 5'62 \text{ MPa (Compresión)}$$

$$\tau_{xy} = 0$$

Sección C $\begin{cases} N_x = 35'35 \text{ (Comp).} \\ V_y = \cancel{17'68}\ 33'35 \\ M_z = 35'36 \end{cases}$

$\sigma_{xx}^{(N)} = 5'621$ MPa (Comp)

$\sigma_{xx}^{(M_z)} \Rightarrow$

Punto M $\sigma_{xx}^{M_z} = \frac{M_z\, y_m}{I_z} = \frac{\overset{35'35}{\cancel{17'68}} \cdot 0'153}{1'16\cdot 10^{-4}\ m^4} = 23'319$ Comp.

$$I_z = \left(\frac{1}{12}\cdot 160\cdot 330^3 - 2\times 2\times \frac{1}{3}\cdot 76\cdot 153\right)\cdot 10^{-12} = 4'79\cdot 10^{-4}\ m^4$$

Punto M:

~~$\sigma_{xx} = 28'94$ MPa~~

$$I_z = \frac{1}{12}\cdot 0'16\cdot 0'33^3 - 2\cdot \frac{1}{12}\cdot 0'076\cdot (0'306)^3 = 1'16\cdot 10^{-4}\ m^4$$

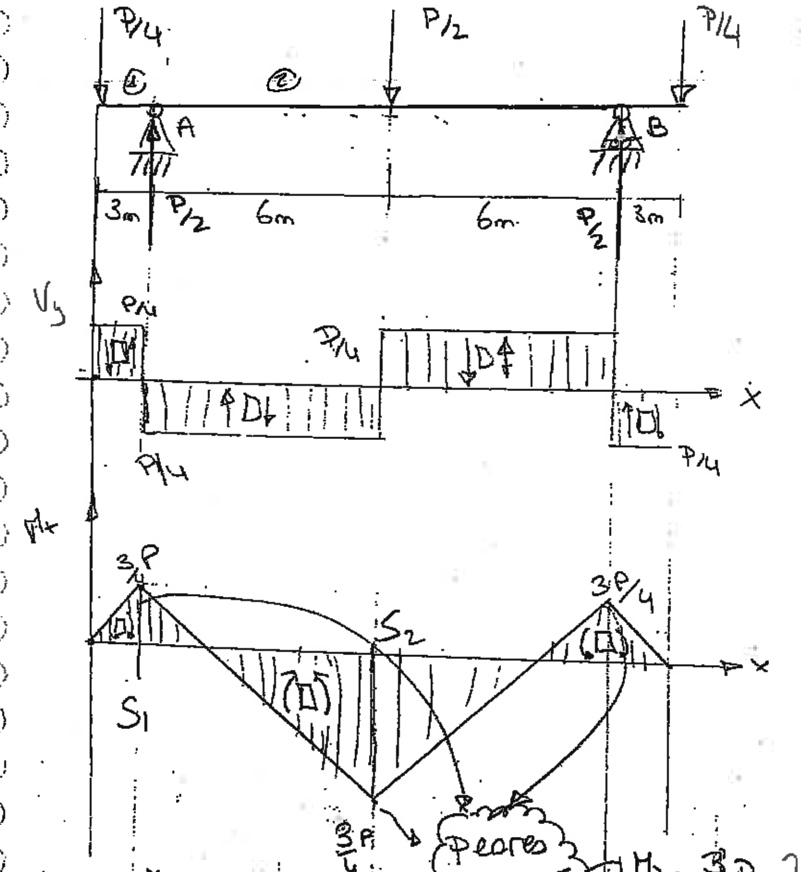
$\sigma_{xx} = 46'631$

$$\tau_{xy} = \frac{V_y}{b_1 h_1} = \frac{35'35}{2'448\cdot 10^{-3}} = 14'44 \text{ MPa}$$

Punto N $\sigma_x = \frac{35'35\cdot 0'165}{1'16\cdot 10^{-4}} + 5'621 = 50'287$ Comp

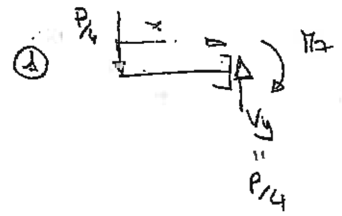
$\tau_{xy} = 0 \rightarrow 15'39$ MPa Esf. desgarre m. máx τ_{xy} máx:

Problema 10.14.

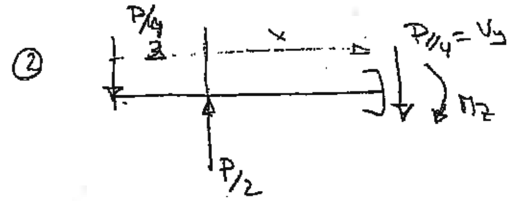


Tens. máximas adm $\begin{cases} \sigma_t = 2\ MPa \\ \sigma_c = 15\ MPa \end{cases}$

$2\cdot\frac{P}{4} = \frac{P}{2} \rightarrow \frac{P}{2}+\frac{P}{2}=P$



$M_z = \frac{P}{4}x$

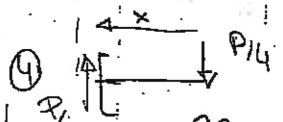


$-M_z + \frac{P}{4}(3+x) - \frac{P}{2}x = 0$

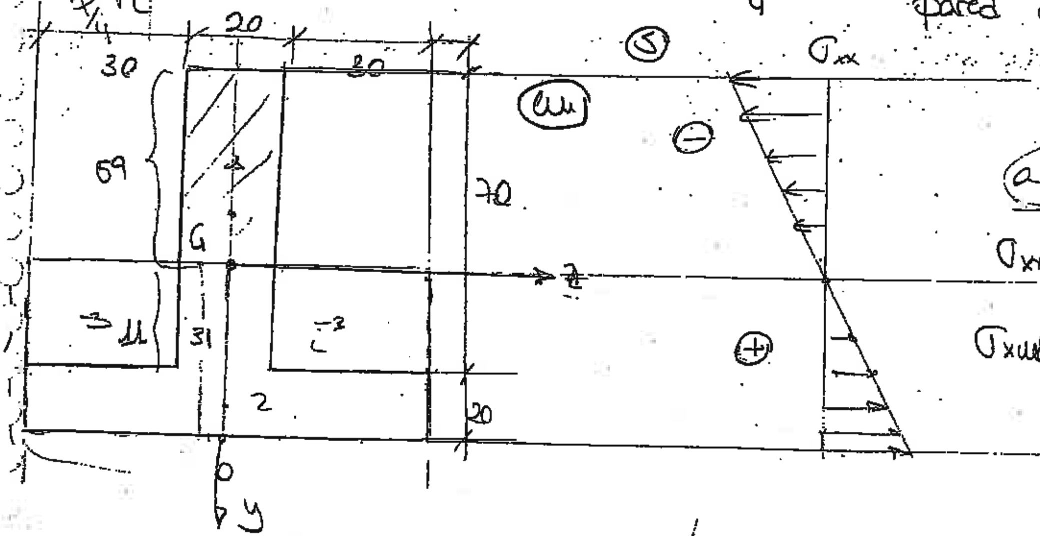
$M_z = \frac{3P}{4} + \frac{P}{4}x - \frac{P}{2}x = \frac{3P}{4} - \frac{P}{4}x$

$x=6 \longrightarrow \frac{3P}{4} - \frac{6P}{4} = -\frac{3P}{4}$

peores secciones $\left\{ M_z = \frac{3}{4}P \quad V_y = \frac{P}{4} \right\}$



No es de pared delgada $\Rightarrow$ τ_{xy} no se tiene en cuenta.



a compresión

$\sigma_{xx\,m\acute{a}x} = \frac{M_z\, y_{m\acute{a}x}}{I_z} =$

$\sigma_{xx\,m\acute{a}x} = \frac{\frac{3P}{4}\cdot 0'59}{0'02137} \leq 15\ MPa$

$P = 724'4\ kN$

y_G: $20\cdot 80\cdot 10 + 70\cdot 20\cdot 55 = (20\cdot 80 + 70\cdot 20)\, y_G$

$y_G = 31$

a tracción

$\sigma_{xx\,m\acute{a}x} = \frac{\frac{3P}{4}\cdot 0'31}{0'02137} \leq 2\ MPa$

$P = \boxed{183'82\ kN}$

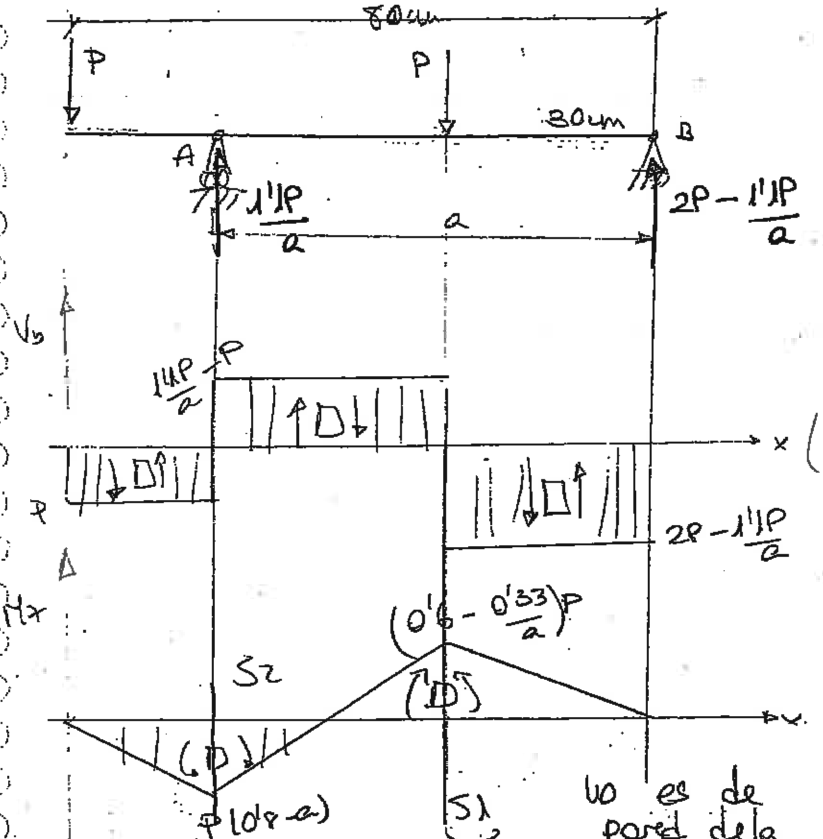
$I_z = \left(\frac{1}{3}\cdot 20\cdot 59^3 + \frac{1}{3}\cdot 80\cdot 31^3 - 2\cdot\frac{1}{3}\cdot 30\cdot 11^3\right)\cdot 10^{-8}$ — $cm^4 \rightarrow (2\times 4)$

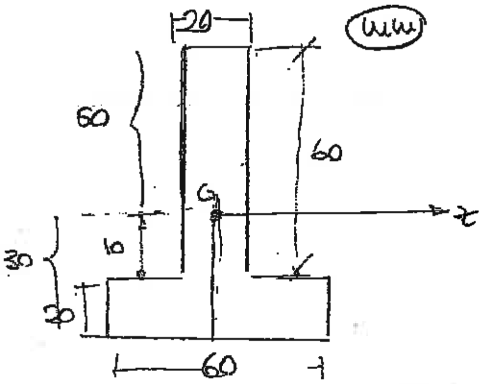
$= 0'02137\ m^4$

S_1 tracc $\rightarrow \frac{3P/4\cdot 0'59}{0'02137} \leq 2 \rightarrow 96'587\ kN$

comp $\rightarrow \frac{3P}{4}\cdot 0'31 \leq 15 \rightarrow 1380'20\ kN$

Problema 10.15.





$$\left(\cancel{60\cdot 20}\cdot 10 + \cancel{60\cdot 20}\, 50\right) = y_G \cdot 2 \cdot \cancel{60\cdot 20}$$

$$y_G = 30$$

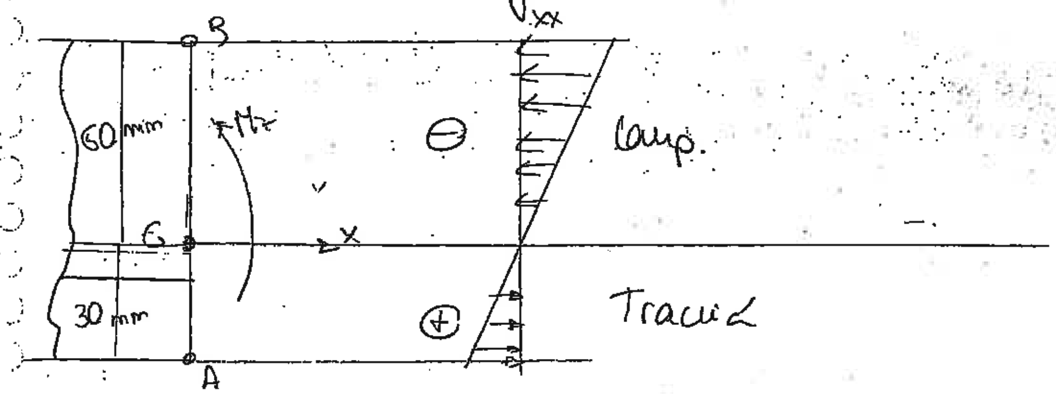
$$I_z = \left(\frac{1}{3}\cdot 20\cdot 50^3 + \frac{1}{3}\cdot 60\cdot 30^3 - 2\cdot\frac{1}{3}\cdot 20\cdot 10^3\right)\cdot 10^{-12} =$$

$$= 1'36\cdot 10^{-6}\ m^4$$

no es de pared delg.

$\hookrightarrow M_{máx} = \left(0'6 - \frac{0'33}{a}\right)P$

Tens. admisibles: $\oplus$ 100 MPa, $\ominus$ 300 MPa



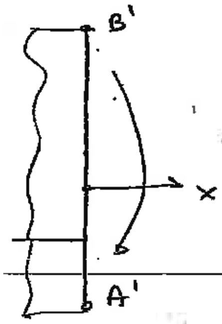
Punto A: tracción $\sigma_x = 100 = \dfrac{M_z y_A}{I_z} = \dfrac{0'03}{1'36\cdot 10^{-6}}\left(0'6 - \dfrac{0'33}{a}\right)P$

$$13235'29\,P - \frac{7279'4\,P}{a} = 100 \qquad (1)$$

Punto B: Comp. $\sigma_x = 300 = \dfrac{0'05}{1'36\cdot 10^{-6}}\left(0'6 - \dfrac{0'33}{a}\right)P = 22058'8\,P - \dfrac{12132'}{a}$

$$22058'8P - \frac{12132'3P}{a} = 300 \qquad (2)$$

Secció S2



Pt B' tracció $\sigma_x = 100 = \frac{M_z y_{B'}}{I_z}$

punt B' comp. $\sigma_y = 300 = \frac{M_z y_{A'}}{I_z}$

M_z major ?

punt A $M_z = \frac{100 I_z}{y_A} = 3333'3\, I_z$ (y_A 0'03)

punt B $M_z = \frac{300 I_z}{y_B} = 6000\, I_z$

punt A' $M_z = \frac{300 I_z}{y_{A'}} = 10\,000\, I_z$ → major M_z

2 ecs. 2 incòg.

punt B' $M_z = \frac{100 I_z}{y_{B'}} = 2000\, I_z$

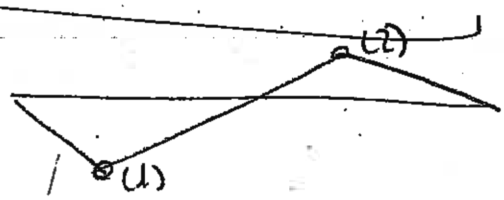
B: $22\,058'8\,P - \frac{12\,132'3\,P}{a} = 300$ (1)

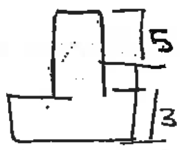
A' $300 = \frac{0'03}{1'36 \cdot 10^{-6}} (0'8 - a) P \rightarrow 300 = 17\,647'058\,P - 22\,058'8\,aP$ (2)

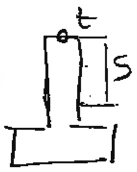
$a = 71'6$ cm

$P = 32'53$ kN

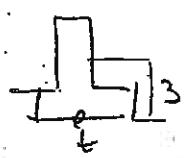
A tracció aquesta menys,





① 

$\sigma_x^t = \frac{M_1 5}{I_z}$

② 

$\sigma_x^t = \frac{M_2 \cdot 3}{I_z}$

$\frac{M_1}{M_2} = \frac{3}{5}$ relació de màx. aprofitament del mat.

$\frac{P(80-a)}{60P - \frac{3300P}{a}} = \frac{3}{5}$ $a = 71'6$ cm

$\sigma_x^t = 100 \cdot \frac{N}{} \cdot \frac{1\,kN}{} \cdot 10^3 mm^2 = 10$ kN

① $\frac{P(80-71'6) \cdot 5}{}$ ⇒ $35'53$ kN

$$\varepsilon_{xy} = \frac{1+\nu}{E} \cdot \tau_{xy} = \frac{1+0'25}{200 \cdot 10^3} \cdot (-10) = -0'0000625$$

$$\varepsilon_{xz} = \varepsilon_{yz} = 0$$

$$[D_{ij}] = 10^{-5} \begin{bmatrix} 20 & -6'25 & 0 \\ -6'25 & -5 & 0 \\ 0 & 0 & -5 \end{bmatrix}$$

$$\begin{vmatrix} 20-\varepsilon & -6'25 & 0 \\ -6'25 & -5-\varepsilon & 0 \\ 0 & 0 & -5-\varepsilon \end{vmatrix} = 0$$

$$(25 + \varepsilon^2 + 10\varepsilon)(20-\varepsilon) + 6'25^2(5-\varepsilon)$$

$$500 - 25\varepsilon + 20\varepsilon^2 - \varepsilon^3 + 200\varepsilon - 10\varepsilon^2$$

$$-39'065\varepsilon + 195'3125 = 0$$

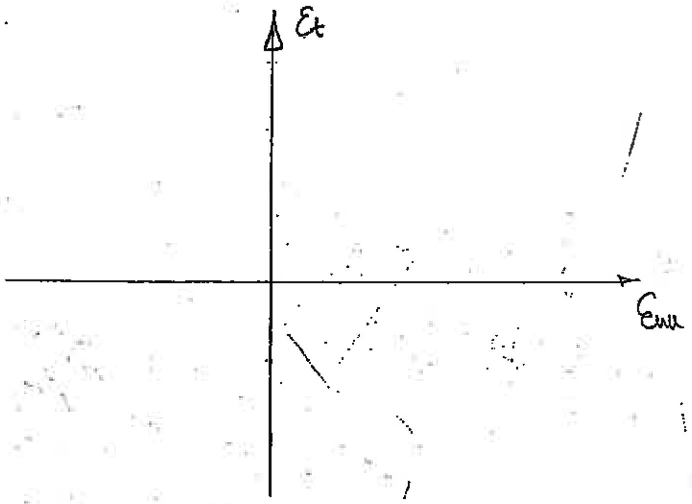
$$-\varepsilon^3 + 10\varepsilon^2 + 135'935\varepsilon + 695'3125 = 0$$

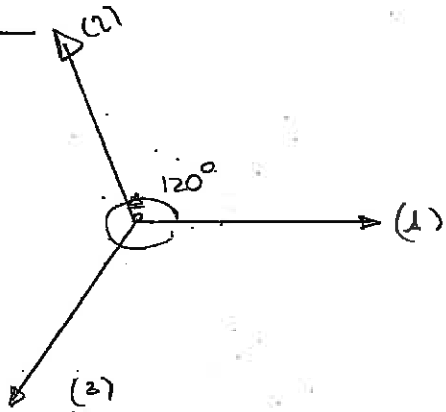
DUDA

$$\varepsilon_1 = 2'148 \cdot 10^{-4} = 21'48 \cdot 10^{-5}$$
$$\varepsilon_2 = -0'6476 \cdot 10^{-4} = -6'476 \cdot 10^{-5}$$
$$\varepsilon_3 = -0'5 \cdot 10^{-4} = -5 \cdot 10^{-5}$$

Problema 5.6.

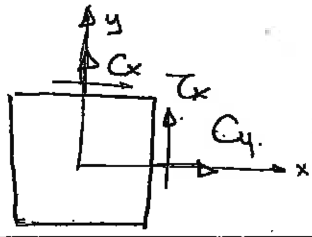
$E = 68'9$ GN/m² $\nu = 0'31$





¿ DUDA

Problema 5.7.



1) Expresión de τ_{xy}

Ecs. de equilibrio:

$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} + \phi_x = 0$$

$$0 + \frac{\partial \tau_{xy}}{\partial y} + 0 + 0 = 0$$

$$\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \tau_{yz}}{\partial z} + \phi_y = 0$$

$$\frac{\partial \tau_{xy}}{\partial y} = 0 \qquad \boxed{\tau_{xy} = cte}$$

$$\frac{\partial \tau_{xy}}{\partial x} + 0 + 0 + 0 = 0 \qquad \boxed{\tau_{xy} = cte}$$

2) Deformaciones ?

Deducir por integración u, v $\left\{ \begin{array}{l} (z=0) \\ \text{en } O: \; u = v = \frac{\partial u}{\partial y} = 0 \end{array} \right.$

$$\begin{cases} \varepsilon_{xx} = \frac{1}{E}\left[c_y - \nu c_x \right) = \frac{c}{E}(y - \nu x) \\ \varepsilon_{yy} = \frac{1}{E}\left[c_x - \nu c_y \right) = \frac{c}{E}(x - \nu y) \\ \varepsilon_{zz} = -\frac{\nu c}{E}(x+y) \end{cases}$$

$$\begin{cases} \gamma_{xy} = 2\varepsilon_{xy} = 2 \cdot \frac{1+\nu}{E} \cdot \tau_{xy} = \cancel{2} \frac{K}{\cancel{2}G} = \frac{K}{G} \\ \gamma_{xz} = 0 \\ \gamma_{yz} = 0 \end{cases}$$

$$\varepsilon_{xx} = \frac{\partial u}{\partial x} \qquad u = \int \frac{c}{E}(y - \nu x)\, dx = \frac{c}{E}\left[yx - \frac{\nu}{2} x^2 \right]$$

3) Alargamiento OC con estado de def. plano $\rightarrow \vec{n} = (1/\sqrt{2}, 1/\sqrt{2})$

$\rightarrow \vec{\varepsilon}_n \cdot \vec{n} = \underline{\varepsilon_{nn}}$

Problema 5.8.

Def. plana : $\sigma_x = ay^2$, $\sigma_y = -ax^2$, $\tau_{xy}=0$

¿Desplazamiento? → Condición $O(0,0) \rightarrow u = v = 0$

$$\begin{cases} \varepsilon_{xx} = \frac{1}{E}\left[ay^2 - \nu(-ax^2)\right] = \frac{a}{E}(y^2 + \nu x^2) \\ \varepsilon_{yy} = \frac{1}{E}\left[-ax^2 - \nu(ay^2)\right] = -\frac{a}{E}(x^2 + \nu y^2) \\ \varepsilon_{zz} = \frac{1}{E}\left(-\nu(ay^2 + ax^2)\right] = -\frac{a\nu}{E}(y^2 - x^2) \end{cases}$$

$$\varepsilon_{xy} = \frac{\tau_{xy}}{2G} = 0 \qquad \varepsilon_{xz} = \varepsilon_{yz} = 0$$

$$\frac{\partial u}{\partial x} = \varepsilon_{xx} \rightarrow u = \int \varepsilon_{xx}\,dx = \int \frac{a}{E}(y^2 + \nu x^2)\,dx = \frac{a}{E}\left(y^2x + \frac{\nu x^3}{3}\right) +$$

$$x, y = 0 \rightarrow u = 0 \rightarrow \text{cte} = 0$$

$$u = \frac{a}{E}\left(y^2x + \nu\frac{x^3}{3}\right) \quad \text{Mal}$$

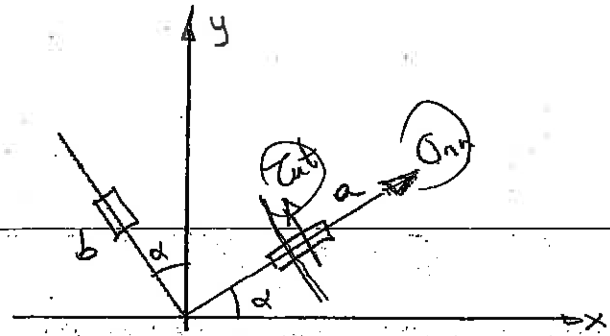
Problema 5.9.

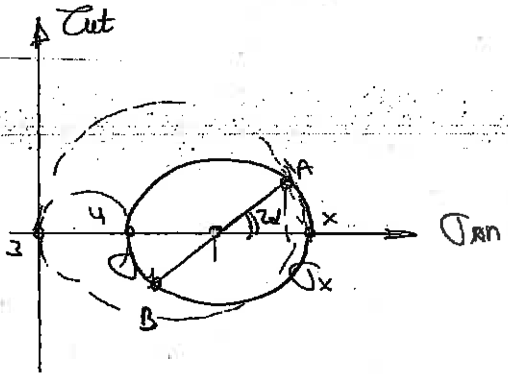
Igual que el 6.8 DUDA

Problema 5.10.

Tracción biaxial. $\left\} \begin{matrix} \sigma_x \\ \sigma_y \end{matrix} \right.$

Tensiones ppales.





$$\begin{cases} \sigma_{nn_A} = \underbrace{\dfrac{\sigma_x+\sigma_y}{2} + \dfrac{\sigma_x-\sigma_y}{2}\cdot\cos 2\alpha} \\[2ex] \tau_{nt_A} = \dfrac{\sigma_x-\sigma_y}{2}\,\sin 2\alpha \end{cases}$$