EKUAZIO DIFERENTZIAL ARRUNTAK

adb. $F = m \cdot a$

$-Kx = m \cdot \frac{dv}{dt}$

$-Kx = m \frac{d}{dt}\left(\frac{dx}{dt}\right)$

$-Kx = m \frac{d^2x}{dt^2}$

$-Kx = mx'' \longrightarrow$ x-ren 2. deribatua

$mx'' + Kx = 0$

* Bigarren ordenako ekuazio diferentziala

1) SOLUZIOAK

adb. $x' + x = 0 \longrightarrow x(t) = e^{-t}$ (soluzio partikularra)

$x'(t) = -e^{-t}$

$-e^{-t} + e^{-t} = 0$

* Beraz, $x(t) = e^{-t}$ soluzioetako bat da, baina ez bakarra!

$x(t) = \underline{K} e^{-t}$ (soluzio orokorra)

↳ edozein zenbaki erreal

- soluzio partikularrak ateratzeko lehenik soluzio orokorrak lortu behar ditugu

ARIKETAK $t \to x(t) = x$

a) $tx' = 2x(t) \to tx' - 2x = 0$

$x(t) = 5t^2 \to x'(t) = 10t$

$tx' - 2x = 0 \to t \cdot 10t - 2 \cdot 5t = 10t^2 - 10t^2 = 0$

d) $x'' + x = 0$

$x(t) = 3\sin t - 4\cos t \to$ soluzio partikular bat

$x'(t) = 3\cos t + 4\sin t$

$x''(t) = -3\sin t + 4\cos t$

$(-3\sin t + 4\cos t) + (3\sin t - 4\cos t) = 0$

2 ALDAGAI BANANDUETAKO EKUAZIO DIFERENTZIALAK

- $x' = f(t,x) \longrightarrow$ FORMA NORMALA

$x' + 3x + t = 0$

$x' = 3x - t \to f(t,x)$

(1)• $x' = f(t,x) \longrightarrow$ ALDAGAI BANANDUA

$x' = g(x)\, h(x)$

$x' = 3t \cdot x$

- $x' = tx + x$

$x' = \underbrace{t}_{g(x)}\underbrace{(x+1)}_{h(x)}$

(1) - Faktorizatu

(2) - Bi aldeak dt-rekin biderkatu

(3) - Funtzio bat beste aldera zatitzen pasa

(4) - Bi emaitzen integrala atera

$x' = g(x)\, h(x)$

(2) $x' dt = g(t)\, h(x)\, dt$

$dx = g(t)\, h(x)\, dt$

(3) $\frac{1}{h(x)} dx = g(t)\, dt$

(4) $\int \frac{1}{h(x)} dx = \int g(t)\, dt + K$

ARIKETAK

(3) c) $\begin{cases} x' = tx^2 + t \\ x(0) = 0 \end{cases}$ → Forma normala

$x' = t(x^2+1)$

$x\,dt = t(x^2+1)dt$

$dx = t(x^2+1)dt$

$\frac{1}{x^2+1}dx = t\,dt$

$\int \frac{1}{x^2+1}dx = \int t\,dt$

$\operatorname{arctg} x = \frac{t^2}{2} + K$ → soluzio orokorra

(*) soluzio hau ez da betetzen, baldintzetako bat $x(0)=0$ delako

soluzio partikularra:

$t_0 = 0 \qquad x_0 = x(t_0) = 0$

$x(0) = 0 \rightarrow x_0$

$0 \rightarrow t_0$

$\operatorname{arctg}(0) = \frac{0^2}{2} + K \longrightarrow \underline{\underline{K=0}}$ $\boxed{\operatorname{arctg} x = \frac{t^2}{2}}$

b) $\begin{cases} \cot g\, x \cos^{-2} t \quad x' = -tg\, t \sin^{-2} x \\ x(0) = \frac{\pi}{4} \end{cases}$

x bakandu lehenengo funtziotik

$x' = -\underbrace{\frac{tg\, t}{\cos^{-2} t}}_{g(x)} \underbrace{\frac{\sin^2 x}{\cot g\, x}}_{h(x)}$ $\qquad tg = \frac{\sin t}{\cos t}$

$\cot g = \frac{\cos}{\sin}$

$x' = -\frac{\sin t}{\cos^3 t} \frac{\sin^3 x}{\cos x}$

$\frac{1}{\sin^2 x} = \operatorname{cosec} x$

② $x' dt = -\frac{\sin t}{\cos^3 t} \frac{\sin^3 x}{\cos x} dt$

$\frac{1}{\cos^2 t} = \sec t$

③ $\frac{\cos x}{\sin^3 x} dx = -\frac{\sin t}{\cos^3 t} dt$

④ $\int \frac{\cos x}{\sin^3 x} dx = -\int \frac{\sin t}{\cos^3 t} dt = \int \overset{f'(x)}{\cos x}\, (\overset{f(x)}{\sin x})^{\overset{n}{-3}} dx = -\int \overset{g'(t)}{\sin t}\, (\overset{g(t)}{\cos t})^{-3} dt$

$\frac{(\sin x)^{-2}}{-2} = \frac{(\cos t)^{-2}}{-2} + K$

$\frac{\operatorname{cosec}^2 x}{-2} = \frac{\sec^2 t}{-2} + K \longrightarrow \operatorname{cosec}^2 x = \sec^2 t + K'$

solutio orokorra

solutio partikularra:

$t_0 = 0 \qquad x_0 = x(0) = \frac{\pi}{4}$

$\operatorname{cosec}^2 \left(\frac{\pi}{4}\right) = \sec^2 (0) + K'$

$\operatorname{cosec}^2 \left(\frac{\pi}{4}\right) = \frac{1}{\sin^2\left(\frac{\pi}{4}\right)} = \frac{1}{\left(\frac{\sqrt{2}}{2}\right)^2} = \underline{2}$

$\sec^2(0) = \frac{1}{\cos^2(0)} = 1^2 = \underline{1}$

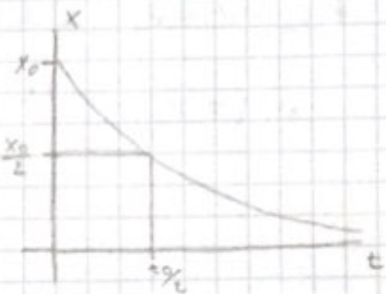
$2 = 1 + K' \longrightarrow \underline{K' = 1}$

APLIKAZIOAK

① DESINTEGRAZIO ERRADIAKTIBOA

· Une bakoitzean, atomoen kopuruaren aldakuntza-tasa une horretan dauden atomoen kopuruarekiko proportzionala da.

$$\boxed{x' = -Kx}$$



Problemak planteatzeko:

$$\begin{cases} x' = -Kx \cdot 1 \\ x(0) = x_0 \end{cases}$$

$\underset{dx}{\underset{\parallel}{x'dt}} = -Kx \cdot 1 \longrightarrow \int \frac{1}{x} dx = \int -K dt$

$\ln|x| = -Kt + K$ ● ez da beharrezkoa balio absolutuak jartzea atomo kopurua beti positibo delako!

$e^{\ln x} = e^{-Kt+K}$

$X = e^{-Kt} \, e^{K} \longrightarrow \boxed{x(t) = K_1 \cdot e^{-Kt}}$ → soluzio orokorra

soluzio partikularra:

$t_0 = 0 \qquad x_0 = x(0) = 0$

$x_0 = K_1 \cdot e^{-K \cdot 0} \longrightarrow K_1 = x_0$

$\boxed{x(t) = x_0 \cdot e^{-Kt}}$ → soluzio partikularra

(K) Proportzionaltasun konstantea kalkulatzeko:

$x(t_1) = x_1$

$x_1 = x_0 \cdot e^{-Kt_1} \longrightarrow \frac{x_1}{x_0} = e^{-Kt_1}$

$\ln\left(\frac{x_1}{x_0}\right) = \ln e^{-Kt_1} \longrightarrow \ln\left(\frac{x_1}{x_0}\right) = -Kt_1$ $(\ln e)$ 1

$\boxed{K = -\frac{1}{t_1}\ln\left(\frac{x_1}{x_0}\right)} > 0 \longrightarrow x_1 < x_0$ delako

K soluzio partikularrean ordeztuz:

$x(t) = x_0 \cdot e^{-Kt}$ eta $K = -\frac{1}{t_1}\ln\left(\frac{x_1}{x_0}\right)$

$x(t) = x_0$ esp $\frac{1}{t_1}$ ln

② POPULAZIOAREN HAZKUNDEA (sinplea)

- Une bakoitzean, populazioaren aldakuntza-tasa une horretako populazioarekiko proportzionala da

$\boxed{x' = Kx}$

Problemak planteatzeko

$$\begin{cases} x' = Kx \\ x(0) = x_0 \end{cases}$$

$\underset{dx}{x'dt} = Kx \longrightarrow \int \frac{1}{x}\,dx = \int K\,dt$

$\ln|x| = Kt + K \longrightarrow e^{\ln x} = e^{Kt+K}$ * x beti positiboa da

$x = e^{Kt}\, e^{K} \longrightarrow \boxed{x(t) = K_1\, e^{Kt}} \longrightarrow$ soluzio orokorra

$x_1 > x_0$

soluzio partikularra:

$t_0 = 0 \qquad x(t_0) = x_0$

$x_0 = k_1 e^{k \cdot 0} \longrightarrow k_1 = x_0$

$\boxed{x(t) = x_0 \cdot e^{kt}} \longrightarrow$ soluzio partikularra

K atera

$x(t_1) = x_1$

$x_1 = x_0 \cdot e^{kt} \longrightarrow \frac{x_1}{x_0} = e^{kt}$

$\ln\left(\frac{x_1}{x_0}\right) = \ln e^{kt} \longrightarrow \ln\left(\frac{x_1}{x_0}\right) = kt \ln e$

$\boxed{K = \frac{1}{t_1} \ln\left(\frac{x_1}{x_0}\right)} > 0 \rightarrow x_1 > x_0$ delako

ARIKETA

Desintegrazio erradiaktiboa kalkulatu

$\frac{x_0}{2}$ eta $t_{1/2}$ puntuan

$\frac{x_0}{2} = x_0 e^{-kt_{1/2}} \longrightarrow \frac{1}{2} = e^{-kt_{1/2}}$

$\ln\left(\frac{1}{2}\right) = \ln e^{-kt_{1/2}} \longrightarrow \ln\left(\frac{1}{2}\right) = -K t_{1/2}$

$T_{1/2} = -\frac{\ln\left(\frac{1}{2}\right)}{K} = \frac{\ln\left(\frac{1}{2}\right)^{-1}}{K} = \frac{\ln(2)}{K}$

$\boxed{t_{1/2} = \frac{\ln 2}{K}} \longrightarrow$ erdi-bizitza

3,4,8,9 ariketak ez

③ HOZTE - LEGEA

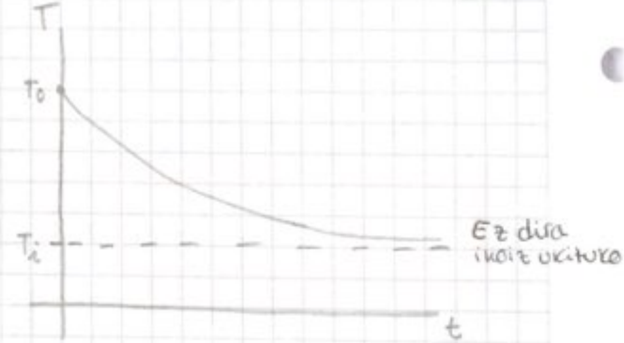
- Une bakoitzean objektuak duen tenperaturaren aldakuntza-tasa $(T-T_1)$ kendurarekiko proportzionala da

$$\boxed{T' = -K(T-T_i)} \qquad (T-T_i) > 0$$

T → objektuak une horretan duen tenperatura

T_i → inguruneak duen tenperatura (konstantea

* objektua eta ingurunearen arteko tenperatura aldea gero eta handiagoa izan orduan eta handiagoa izango da hozte abiadura



Problemak ebazteko:

$$\begin{cases} T' = -K(T-T_0) \\ T(0) = T_0 \end{cases}$$

$$T'dt = -K(T-T_i)\,dt$$
$$\| \quad dT$$

$$\int \frac{1}{T-T_i}\,dT = \int -K\,dt \rightarrow \ln(T-T_i) = -Kt + K$$

$$e^{\ln(T-T_i)} = e^{-Kt+K} \rightarrow T-T_i = e^{-Kt}\,e^{K}$$

$$\boxed{T-T_i = K_1\,e^{-Kt}} \rightarrow \text{soluzio orokorra}$$

solozio partikularra:

$t_0 = 0 \qquad T(t_0) = T_0$

$T_0 = T_i + K_1 e^{-K0} \rightarrow K = T-T_i$

$$\boxed{T(t) = T_i + (T_0 - T_i)\,e^{-Kt}} \rightarrow \text{soluzio partikularra}$$

$t = 0 \rightarrow T(0) = T_0$

$t \rightarrow +\infty \rightarrow T \rightarrow T_i$

④ EREDU LOGISTIKOA (Malthus)

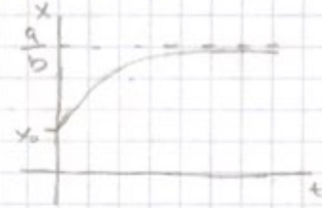
- Populazio hazkundearena orokortzen du

$$\boxed{x' = ax - bx^2}$$

- Baldintzak kontuan hartzen ditu, baliabide finitoena alegia.
- a eta b konstante erreal positiboak dira

Problema planteatzean

$$\begin{cases} x' = \underbrace{ax - bx^2}_{=g(x)} \\ x(t_0) = x_0 \end{cases}$$



$$x' dt = (ax - bx^2) \cdot dt$$

$$\frac{1}{ax-bx^2}\, dx = dt \longrightarrow \int \frac{1}{ax-bx^2}\, dx = \int dt = K$$

ZATIKETA

Faktorizatu $\quad ax - bx^2 = x(a - bx)$

Ekuazioa $\quad \dfrac{1}{ax-bx^2} = \dfrac{A}{x} + \dfrac{B}{a-bx} = \dfrac{A(a-bx) + Bx}{x(a-bx)}$

$x = 0 \longrightarrow 1 = Aa \longrightarrow A = \frac{1}{a}$

$x = \frac{a}{b} \longrightarrow 1 = B\frac{a}{b} \longrightarrow B = \frac{b}{a}$

Integratu $\quad \displaystyle\int \frac{1/a}{x}\, dx + \int \frac{b/a}{a-bx}\, dx$

$\frac{1}{a} \ln|x| + \frac{b}{a} \cdot -\frac{1}{b} \int \frac{-b}{a-bx}\, dx$

$\frac{1}{a} \ln(x) - \frac{1}{a} \ln(a - bx) + K = \frac{1}{a} \ln\left(\frac{x}{a-bx}\right)$

$$\boxed{\frac{1}{a} \ln\left(\frac{x}{a-bx}\right) = t + K} \longrightarrow \text{soluzio orokorra}$$

solozio partikularra

$t_0 = 0 \qquad x(t_0) = x_0$

$$\frac{1}{a}\ln\left(\frac{x_0}{a-bx_0}\right) = \cancel{t_0} + K \longrightarrow K\text{-ren tokian ordezkatu}$$

$$\frac{1}{a}\ln\left(\frac{x}{a-bx}\right) = t + \frac{1}{a}\ln\left(\frac{x_0}{a-bx_0}\right)$$

X bakandu ...

$$\frac{1}{a}\ln\left(\frac{x}{a-bx}\right) - \frac{1}{a}\ln\left(\frac{x_0}{a-bx_0}\right) = t$$

$$\frac{1}{a}\ln\left(\frac{\frac{x}{a-bx}}{\frac{x_0}{a-bx_0}}\right) = t \longrightarrow \ln\left(\frac{x(a-bx_0)}{x_0(a-bx)}\right) = at$$

oso interesgarria

$$\frac{x(a-bx_0)}{x_0(a-bx)} = e^{at} \longrightarrow x(a-bx_0) = \underbrace{x_0(a-bx)e^{at}}_{ax_0e^{at} - bx_0xe^{at}}$$

$$x(a-bx_0) + bx_0xe^{at} = ax_0e^{at}$$

$$\boxed{x(t) = \frac{ax_0e^{at}}{bx_0e^{at} - bx_0 + a}} \longrightarrow \text{solozio partikularia}$$

t infinitura doanean ..

$$\lim_{t\to\infty}\frac{ax_0e^{at}}{bx_0e^{at} - bx_0 + a} = \lim_{t\to\infty}\frac{ax_0\infty}{bx_0\infty \underbrace{-bx_0+a}}$$

* bx_0+a zenbakiak oso txikiak direnez ∞-dren alboan 0 bezala kontsideratuko ditugu

$$bx_0+a \approx 0 \qquad \lim_{t\to\infty}\frac{ax_0\infty}{bx_0\infty} = \boxed{\lim_{t\to\infty}\frac{a}{b}}$$

⑤ ERREAKZIO KIMIKOAK

$$A + B \longrightarrow C$$

$t_0 = 0$	a_0	b_0	0
	$\downarrow a$	$\downarrow b$	$\downarrow a+b$
t	$(a_0 - a)$	$(b_0 - b)$	$(a+b)$

$X = a + b \longrightarrow X(t)$ = produktuaren masa t unean (C)

- Produktuaren masaren aldakuntza tasa biderkurarekiko $(a_0 - a)(b_0 - b)$ proportzionala da une bakoitzean

$$X' = K(a_0 - a)(b_0 - a)$$

$$A + B \longrightarrow C$$

M	N	$M+N$	
$\frac{M}{M+N}$	$\frac{N}{M+N}$	$1 \longrightarrow 1g$ produktuan	
$\frac{MX}{M+N}$	$\frac{NX}{M+N}$	$X \longrightarrow Xg$ produktuan	

$$a(t) = \underbrace{\frac{M}{M+N}}_{\alpha} X \qquad b(t) = \underbrace{\frac{N}{M+N}}_{1-\alpha} X$$

$$X(t) = K(a_0 - \alpha X)(b_0 - (1-\alpha)X)$$

$$dX = K(a_0 - \alpha X)(b_0 - (1-\alpha)X)\, dt$$

$$\int \frac{1}{(a_0 - \alpha X)(b_0 (1-\alpha)X} = \int K\, dt$$

$$\frac{1}{(a_0 - \alpha X)(b_0 (1-\alpha)X} = \frac{A}{a_0 - \alpha X} + \frac{B}{b_0 (\alpha - 1)X}$$

$$X = \frac{a_0}{\alpha} \longrightarrow 1 = A\left[b_0 - (1-\alpha)\frac{a_0}{\alpha}\right] \longrightarrow \frac{\alpha}{\alpha b_0 - (1-\alpha)a_0} = A$$

$$X = \frac{b_0}{1-\alpha} \longrightarrow 1 = B\left(a_0 - \left(\alpha \frac{b_0}{1-\alpha}\right)\right) \longrightarrow \frac{1-\alpha}{a(1-\alpha) - \alpha b_0} = B$$

$\longrightarrow$

$$\int \frac{1}{(a_0-\alpha x)(b_0-(1-\alpha)x)}dx = \int \frac{\frac{\alpha}{\alpha b_0-(1-\alpha)a_0}}{(a_0-\alpha x)} + \frac{\frac{1-\alpha}{a_0(1-\alpha)-\alpha b_0}}{(b_0-(1-\alpha)x)}dx$$

$$\parallel dx = \underbrace{\frac{1}{\alpha b_0-(1-\alpha)x}}_{\text{Konstantea}} \int \frac{\alpha}{a_0-\alpha x}dx \; + $$

→ barruan uztea komeni da kalkulu gutxiago egiteko

$$+\frac{1}{a_0(1-\alpha)-\alpha b_0}\int \frac{1-\alpha}{b_0-(1-\alpha)x}dx =$$

$$\frac{-1}{\alpha b_0-(1-\alpha)x}\int \frac{-\alpha \rightarrow g'(x)}{\underbrace{a_0-\alpha x}_{g(x)}}dx + \frac{-1}{a_0(1-\alpha)-\alpha b_0}\int \frac{-(1-\alpha)}{b_0-(1-\alpha)x}dx$$

$$= \frac{-1}{\underbrace{\alpha b_0-(1-\alpha)a_0}_{C}} \ln(a_0-\alpha x) - \frac{-(1-\alpha)}{\underbrace{a_0(1-\alpha)+b_0}_{-C}} \ln(b_0-(1-\alpha))$$

$C \longleftrightarrow -C$ Kontrakoak dira

$$-\frac{1}{C}\ln(a_0-\alpha x) + \frac{1}{C}\ln[b_0-(1-\alpha)x]$$

$$= \frac{1}{C}\left[\ln[b_0-(1-\alpha)x] - \ln(a_0-\alpha x)\right]$$

$$= \boxed{\frac{1}{C}\ln\left(\frac{[b_0-(1-\alpha)x]}{(a_0-\alpha x)}\right) = Kt + K} \longrightarrow \text{soluzio orokorra}$$

soluzio partikularra ateratzen ...

$t_0 = 0 \qquad x_{(t_0)} = x(0) = 0$

$$\begin{cases} x' = K(a_0-\alpha x)(b_0-(1-\alpha)x) \\ x(0) = 0, \end{cases}$$

$\frac{1}{C}\ln\left(\frac{b_0}{a_0}\right) = K$ orduan,

$$\frac{1}{C}\ln\left(\frac{[b_0-(1-\alpha)x]}{(a_0-\alpha x)}\right) = Kt + \frac{1}{C}\ln\left(\frac{b_0}{a_0}\right)$$

$$C\cdot\frac{1}{C}\ln\left(\frac{[b_0-(1-\alpha)x]}{(a_0-\alpha x)}\right) = CKt + C\frac{1}{C}\ln\left(\frac{b_0}{a_0}\right)$$

$$\ln\left(\frac{b_0-(1-\alpha)x}{a_0-\alpha x}\right)-\ln\left(\frac{b_0}{a_0}\right)=cKt=$$

$$=\boxed{\ln\left[\frac{a_0[b_0-(1-\alpha)x]}{b_0(a_0-\alpha x)}\right]=cKt} \rightarrow \text{oso erabilgarria}$$

$$=\frac{a_0[b_0-(1-\alpha)x]}{b_0(a_0-\alpha)x}=e^{cKt}\rightarrow$$

$$=a_0[b_0-(1-\alpha)x]=b_0(a_0-\alpha x)e^{cKt}$$

$$=a_0b_0-\underline{a_0(1-\alpha)x}=\underbrace{(a_0b_0-\alpha b_0 x)e^{cKt}}_{a_0b_0e^{cKt}-\underline{\alpha b_0 x e^{cKt}}}$$

(*) Bestaldera

$$=\alpha b_0 x e^{cKt}-a_0(1-\alpha)x=a_0b_0e^{cKt}-a_0b_0$$

$$=x(\alpha b_0 e^{cKt}-a_0(1-\alpha))=a_0b_0(e^{cKt}-1)$$

$$\rightarrow \boxed{x(t)=\frac{a_0b_0(e^{cKt}-1)}{\alpha b_0 e^{cKt}-a_0(1-\alpha)}}$$

cK kalkulatzeko...

$$x(t_1)=x_1$$

$$\boxed{cK=\frac{1}{t_1}\ln\left[\frac{a_0[b_0-(1-\alpha)x]}{b_0(a_0-\alpha x)}\right]}$$

Limitea $t\rightarrow\infty$ denean...

$$\lim_{t\to\infty}x(t)=\lim_{t\to\infty}\frac{a_0b_0(e^{cKt}-1)}{\alpha b_0 e^{cKt}-a_0(1-\alpha)}$$

Baldintza $cK>0$

$$e^{cKt}\xrightarrow{t\to\infty}+\infty$$

$$\lim_{t\to\infty}\frac{a_0}{\alpha}$$

Baldintza $cK<0$

$$e^{cKt}\xrightarrow{t\to\infty}0$$

$$\lim_{t\to\infty}\frac{a_0b_0+1}{+a_0(1-\alpha)}=\lim_{t\to\infty}\frac{b_0+1}{1-\alpha}$$

! (*) Bi limite daude, erreakzio batean mugatzaile 2 egon daitezkeelako

Zeintzuk diren masak jakiteko
edo negatzailea jakiteko

Baldin $CK>0$

$$\boxed{\lim_{t\to\infty} x(t) = \frac{a_0}{\alpha}}$$

* $\lim_{t\to\infty} (a_0 - a(t)) = \lim_{t\to\infty} (a_0 - \alpha x(t)) = a_0 - \alpha \lim x(t) =$

$= a_0 - \cancel{\alpha} \frac{a_0}{\cancel{\alpha}} = 0$

* $\lim_{t\to\infty} (b_0 - b(t)) = \lim [b_0 - (1-\alpha)x(t)] = b_0 - (1-\alpha) \lim_{t\to\infty} x(t) =$

$= b_0 - (1-\alpha)\frac{a_0}{\alpha} > 0$

Baldin $CK<0$

$$\boxed{\lim_{t\to\infty} x(t) = \frac{b_0}{1-\alpha}}$$

* $\lim_{t\to\infty} (a_0 - a(t)) = \lim_{t\to\infty} (a_0 - \alpha x(t)) = a_0 - \alpha \lim_{t\to\infty} x(t) =$

$= a_0 - \alpha \frac{b_0}{1-\alpha} > 0$

* $\lim_{t\to\infty} (b_0 - b(t)) = \lim [b_0 - (1-\alpha)x(t)] = b_0 - (1-\alpha) \lim_{t\to\infty} x(t) =$

$= b_0 - \cancel{(1-\alpha)} \frac{b_0}{\cancel{1-\alpha}} = 0$

EKUAZIO DIFERENTZIAL LINEALAK

$$\boxed{x' = a(t)x + b(t)}$$

! x bakarrik agertu behar da
$\frac{1}{x}$ x^2 ~~sin x~~

$$x' e^{-\int a(t)dt} \underbrace{- a(t) x e^{-\int a(t)dt}} = b(t) e^{-\int a(t)dt}$$

$$\left(e^{-\int a(t)dt}\right)' = -a(t) e^{-\int a(t)dt}$$

$$= \left(x e^{-\int a(t)dt}\right)' = b(t) e^{-\int a(t)dt}$$

$$= x e^{-\int a(t)dt} = K + \int b(t) e^{-\int a(t)dt}$$

$$\boxed{= x(t) = e^{\int a(t)dt} \left[K + \int b(t) e^{-\int a(t)dt}\right]} \rightarrow \text{soluzio orokorra}$$

ARIKETAK

5 a) $\begin{cases} tx' - x = 1 \rightarrow tx' = x + t \rightarrow x' = \underbrace{\frac{1}{t}}_{=a(t)} x + \underbrace{t}_{=b(t)} \\ x(1) = 2 \end{cases}$

$a(t) = \frac{1}{t}$ $\quad b(t) = 1$

$$x(t) = e^{\int \frac{1}{t} dt} \left[K + \int 1 e^{-\int \frac{1}{t} dt} dt\right] =$$

$$x(t) = e^{\ln t} \left[K + \int 1 e^{-\ln t} dt\right] \rightarrow x(t) = t \left[K \int e^{\ln t^{-1}} dt\right]$$

$$x(t) = t \left[K + \int t^{-1} dt\right] \rightarrow x(t) = t \left[K + \ln t\right] \rightarrow \text{soluzio orokorra}$$

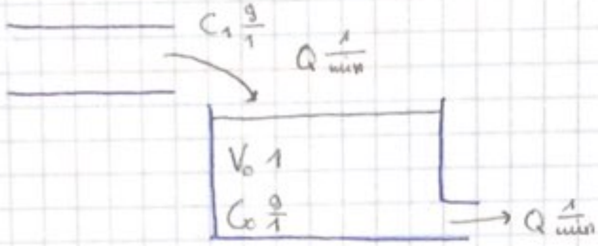
$t_0 = 1 \qquad x_0 = x(1) = 2$

$2 = 1(K + \ln 1) \rightarrow 2 = K$

$x(t) = t(2 + \ln t) \rightarrow x(t) = 2t + t \ln t$

APLIKAZIOAK

(6) DEPOSITUA



$$X_0 = C_0 V_0$$

$\Delta t \longrightarrow \Delta X$ Denbora tarte batean gatzaren masa aldatuko da

$$\Delta X = x(t + \underbrace{\Delta t}_{h}) - x(t)$$

ΔX = sartzen dena − ateratzen dena

$$\Delta X = C_1 \cdot Q \cdot h - \frac{x(t)}{V_0} \cdot Q \cdot h$$

$\frac{g}{l}$ $\frac{l}{min}$ min — kontzentrazioa konstante hartu — $\frac{l}{min}$ min

$$\Delta X = x(t+h) - x(t) = \frac{x(t)}{V_0} \cdot Q \cdot h - C_1 \cdot Q \cdot h$$

$$\frac{x(t+h) - x(t)}{h} = -\frac{x(t)}{V_0} \cdot Q + C_1 Q$$

→ h da aldagaia $[t, t+h]$

$$\frac{x(t+h) - x(t)}{h} = -\frac{Q}{V_0} \cdot x(t) + C_1 Q$$

→ Deribatua $\lim_{h\to 0} \frac{f(x_0+h) - f(x_0)}{h} = f'(x_0)$

$$x(t) = \lim_{h\to 0} \frac{x(t+h) - x(t)}{h} = -\frac{Q}{V_0} x(t) + C_1 Q$$

$$X'(t) = \underbrace{-\frac{Q}{V_0}}_{=a(t)} X(t) + \underbrace{C_1 Q}_{=b(t)}$$

EDL adierazpena: $X(t) = e^{-\int a(t)dt}\left[K + \int e^{-\int a(t)dt} dt\right]$

Gure adierazpena ordezkatuz...

$$x(t) = e^{-\int \frac{Q}{V_0} dt}\left[K + \int e^{\int \frac{Q}{V_0} dt} \, dt\right]$$

$$= e^{-\frac{Q}{V_0}t}\left[K + C_1 \cancel{Q} \frac{V_0}{\cancel{Q}} \int \frac{Q}{V_0} e^{\frac{Q}{V_0}t} \, dt\right]$$

$$= e^{-\frac{Q}{V_0}t}\left[K + C_1 V_0 e^{\frac{Q}{V_0}t}\right] = \boxed{K e^{-\frac{Q}{V_0}t} + C_1 V_0} \longrightarrow \text{Orokorra}$$

$X(0) = C_0 V_0 \longrightarrow$ hasierako masa t_0 denean $t_0 = 0$

$$C_0 V_0 = K e^{-\frac{Q}{V_0}0} + C_1 V_0 \longrightarrow C_0 V_0 = K + C_1 V_0$$

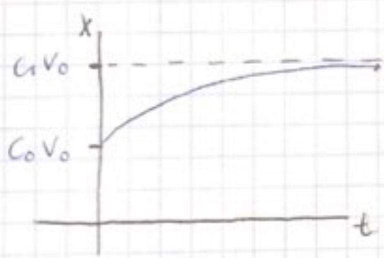
$$K = (C_0 - C_1)V_0$$

$$\boxed{x(t) = C_1 V_0 + (C_0 + C_1) V_0 e^{-\frac{Q}{V_0}t}} \longrightarrow \text{Partziala}$$

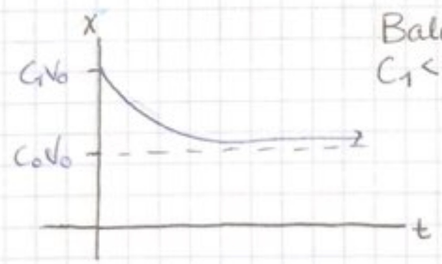
Limitea

$$\lim_{t\to\infty} x(t) = \lim_{t\to\infty}\left[C_1 V_0 + (C_0 + C_1) V_0 e^{\overbrace{-\frac{Q}{V_0}t}^{-\infty}}\right] \qquad e^{-\infty} = 0$$

$$\lim_{t\to\infty} x(t) = \lim_{t\to\infty} C_1 V_0$$



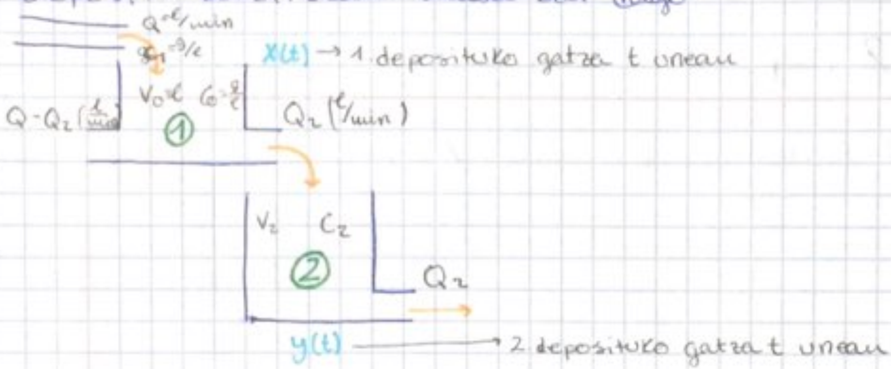
Baldin $C_1 > C_0$



Baldin $C_1 < C_0$

7 DEPOSITU 2

- Deposito bakoitzean masa bat dago



$$\Delta y = y(t+h) - y(t)$$

$$\Delta y = \frac{x(t)}{V_0} \cdot Q_2 \cdot h - \frac{y(t)}{V_2} \cdot Q_2 \cdot h$$

$$\frac{g}{\ell} \quad \frac{\ell}{min} \quad min \qquad \frac{g}{\ell} \quad \frac{\ell}{min} \quad min$$

$$y(t+h) - y(t) = \frac{Q_2}{V_0} x(t)\, h - \frac{Q_2}{V_2} y(t) \cdot h$$

$$\frac{y(t+h) - y(t)}{h} = \frac{Q_2}{V_0} x(t) - \frac{Q_2}{V_2} y(t)$$

$$y'(t) = \lim_{h \to 0} \frac{y(t+h) - y(t)}{h} = \frac{Q_2}{V_0} x(t) - \frac{Q_2}{V_2} y(t)$$

$$y'(t) = -\frac{Q_2}{V_2} y(t) + \frac{Q_2}{V_0} x(t)$$

soluzioak ...... ⟶

1. Depposituko soluzio partikularra:

$$\boxed{x(t) = c_1 V_0 + (c_0 - c_1) V_0 e^{-\frac{Q}{V_0} t}}$$

2. Depposituko soluzio partikularra:

1. Baldin, $(V_0 Q_2 - V_2 Q) \neq 0$,

$$y(t) = c_1 V_2 + \left[V_2 (c_1 - c_2) - \frac{(c_0 - c_1) Q_2 V_0 V_2}{V_0 Q_2 - V_2 Q} \right] e^{-\frac{Q_2}{V_2} t} + \frac{(c_0 - c_1) Q_2 V_0 V_2}{V_0 Q_2 - V_2 Q} e^{-\frac{Q}{V_0} t}$$

2. Baldin $V_0 Q_2 - V_2 Q = 0$

$$y(t) = c_1 V_2 + \left(V_2 (c_1 - c_2) + (c_0 - c_1) Q_2 t \right) e^{-\frac{Q_2}{V_2} t}$$

ALDAGAI BANANDUETAKO EKUAZIO BIHURGARRIAK

Aldagai bananduetako ek. diferentzialak	1. ordenako ek. diferentzial linealak
$x' = f(t,x)$	$x' = a(t)x + b(t)$
$x' = h(t)\ g(x)$	$x(t) = e^{\int a(t)\,dt}\left[K + \int b(t)\, e^{-\int a(t)\,dt}\, dt\right]$

- Kasu orokorra
- Ekuazio diferentzial homogeneoak
- Bernoulliren ekuazio diferentzialak

$$x' = f(t,x)$$

Aldagaiak $(t,x) \longrightarrow (t,u) \longrightarrow x'$

- Aldagai aldaketa: $x \to u$
- Deribatuak kalkulatu $(t,x) \longrightarrow (t,u)$

$$x' \qquad u' = \frac{du}{dt}$$

- Ekuazio diferentziala kalkulatu
- Aldagai aldaketa egin

⊛ Kasu bakoitzean aldagai aldaketa ezberdin bat egin beharko da

ADIBIDEA

$$\begin{cases} 4t^2 x x' = 2 + 3t x^2 \longrightarrow x' = \dfrac{2+3tx^2}{4t^2 x} \\ x(1) = 0 \end{cases}$$

! Ez da lineala

! Ez da aldagai banandutako ek. diferentziala

Aldagai aldaketa: $x = \dfrac{u}{t^{1/2}} \longrightarrow x \cdot t^{1/2} = u$

- Deribatu: $x' = \dfrac{u' t^{1/2} - u \frac{1}{2} t^{-1/2}}{t}$

- x-ak ordezkatu dagokion tokietan: $x' = \dfrac{2+3tx^2}{4t^2 x}$

$$\frac{u' \cdot t^{1/2} - u \frac{1}{2} t^{-1/2}}{t} = \frac{2 + 3t \left(\frac{u}{t^{1/2}}\right)^2}{4t^2 \frac{u}{t^{1/2}}} \qquad \to \frac{u^2}{t} \qquad \to t^{3/2}$$

$$\frac{u' t^{1/2} - u \frac{1}{2} t^{-1/2}}{t} = \frac{2 + 3u^2}{4t^{3/2} u}$$

$$\frac{u' t^{1/2}}{t} - \frac{1}{2} \frac{u t^{-1/2}}{t} = \frac{2}{4} \frac{1}{t^{3/2} u} + \frac{3}{4} \frac{u^2}{t^{3/2} u}$$

$$u' t^{-1/2} - \frac{1}{2} u t^{-3/2} = \frac{1}{2} t^{-3/2} u^{-1} + \frac{3}{4} t^{-3/2} u$$

$$u' - \frac{1}{2} u \underbrace{\left(t^{-3/2} t^{1/2}\right)}_{t^{-1}} = \frac{1}{2} u^{-1} \underbrace{\left(t^{-3/2} t^{1/2}\right)}_{t^{-1}} + \frac{3}{4} u \underbrace{\left(t^{-3/2} \cdot t^{1/2}\right)}_{t^{-1}}$$

$$u' - \frac{1}{2} u t^{-1} = \frac{1}{2} u^{-1} t^{-1} + \frac{3}{4} u t^{-1}$$

$$u' = \frac{1}{2} u^{-1} t^{-1} + \frac{3}{4} u t^{-1} + \frac{1}{2} u t^{-1}$$

$$u' = \underbrace{t^{-1}}_{h(x)} \underbrace{\left(\frac{1}{2} u^{-1} + \frac{3}{4} u + \frac{1}{2} u\right)}_{g(x)}$$

$$u' = t^{-1} \left(\frac{5}{4} u + \frac{1}{2} u^{-1}\right) \longrightarrow u' = \frac{1}{t} \left(\frac{5u^2 + 2}{4}\right)$$

• Ekuazio diferentziala askatu

$$\underbrace{u' \, dt}_{dx} = \frac{1}{t}\left(\frac{5u^2+2}{4u}\right) dt$$

$$\frac{4u}{2+5u^2}\, du = \frac{1}{t}\, dt$$

① $\int \frac{1}{t}\, dt = \ln t$

② $\int \frac{4u}{2+5u^2}\, du = \frac{4}{10}\int \frac{10u}{2+5u^2}\, du = \frac{2}{5}\ln(2+5u^2)$

$$\frac{2}{5}\ln(2+5u^2) = \ln t + K$$

• Aldagai aldaketa: $u = x\,t^{1/2} \longrightarrow u^2 = x^2 t$

$$\frac{2}{5}\ln(2+5x^2t) = \ln t + K \quad \text{soluzio orokorra}$$

- soluzio partikularra

$t_0 = 1 \qquad x_0 = x_0(t_0) = x(1) = 0$

$$\frac{2}{5}\ln(2+5\cdot 0\cdot 1) = \underbrace{\ln 1}_{0} + K$$

$$\frac{2}{5}\ln(2) = K$$

$$\frac{2}{5}\ln(2+5tx^2) = \ln t + \frac{5}{2}\ln 2$$

$$\frac{2}{5}\ln(2+5tx^2) - \frac{2}{5}\ln 2 = \ln t$$

$$\frac{2}{5}\left[\ln(2+5tx^2) - \ln 2\right] = \ln t$$

$$\frac{2}{5}\ln\left[\frac{2+5tx^2}{2}\right] = \ln t$$

$$\frac{2}{5}\ln\left[1+\frac{5}{2}tx^2\right] = \ln t \quad \text{soluzio partikularra}$$

Ariketa

7 $\begin{cases} 4t^2 x x' = 2 - 3tx^2, \rightarrow x' = \dfrac{2-3tx^2}{4t^2x} \\ x(1) = 0 \end{cases}$

- Aldagai aldaketa $x = \dfrac{u}{t^{3/4}} \rightarrow u = x t^{3/4}$

- Deribatu $x' = \dfrac{u' \cdot t^{3/4} - u \frac{3}{4} t^{-1/4}}{t^{3/2}} = u' t^{-3/4} - \frac{3}{4} u t^{-7/4}$

- x-ak ordezkatu:

$$u' t^{-3/4} - \frac{3}{4} u t^{-7/4} = \frac{2 - 3t \frac{u^2}{t^{3/2}}}{4t^2 \frac{u}{t^{3/4}}}$$

$$u' t^{-3/4} - \frac{3}{4} u t^{-7/4} = \frac{2 - 3u^2 t^{-1/2}}{4u t^{5/4}}$$

$$u' t^{-3/4} = \frac{2 - 3u^2 t^{-1/2}}{4u t^{5/4}} + \frac{3}{4} u t^{-7/4}$$

$$u' = \frac{(2 - 3u^2 t^{-1/2})}{4u t^{5/4}} t^{3/4} + \frac{3}{4} u t^{-7/4} t^{3/4} \rightarrow t^{-1}$$

$$u' = \frac{2t^{-1/2}}{4u} - \frac{3u^{\cancel{2}} t^{-1/2}}{\cancel{4u}} t^{-1/2} + \frac{3}{\cancel{4}} u t^{-1}$$

$$u' = \frac{\cancel{2} t^{-1/2}}{{}_2\cancel{4}u} \longrightarrow u' = \frac{t^{-1/2}}{2u} = \underbrace{t^{-1/2}}_{h(x)} \underbrace{\frac{1}{2u}}_{g(x)}$$

- Ekuazio diferentziala

$$u' dt = t^{-1/2} \frac{1}{2u} dt \rightarrow 2u\,du = t^{-1/2} dt$$

$$u^2 = \frac{t^{-1/2+1}}{-\frac{1}{2}+1} + K \rightarrow u^2 = \frac{t^{1/2}}{1/2} + K$$

- Aldagai aldaketa: $u = x t^{3/4} \rightarrow u^2 = x^2 t^{3/2}$

$$x^2 t^{3/2} = 2t^{1/2} + K$$

$$x^2 = \frac{2t^{1/2} + K}{t^{3/2}} \quad \text{soluzio orokorra}$$

SOLUZIO PARTIKULARRA

$t_0 = 1 \quad X_0 = x(t_0) = x(1) = 0$

$0 = \frac{2 \cdot 1^{3/2} + K}{1^{3/2}} \longrightarrow 0 = 2 + K \longrightarrow \underline{\underline{K = -2}}$

$\underline{\underline{x^2 = \frac{2t^{3/2} - 2}{t^{3/2}}}} \longrightarrow$ soluzio partikularra

EKUAZIO DIFERENTZIAL HOMOGENEOAK

$x' = f(t, x)$

- $f(Kt, Kx) = f(t, x)$ baldin $K \in \mathbb{R}\{0\}$

- BALDINTZA hau betetzen bada HOMOGENEOA da

Adb.

$f(t,x) = \frac{t^2 - x^2}{t^2 + x^2}$

$f(Kt, Kx) = \frac{(Kt)^2 - (Kx)^2}{(Kt)^2 - (Kx)^2} = \frac{K^2(t^2 - x^2)}{K^2(t^2 + x^2)} = f(t, x)$

Homogeneoa da

$f(t,x) = \sin\left(\frac{t}{x}\right) e^{\frac{x}{t}}$

$f(Kt, Kx) = \sin\left(\frac{Kt}{Kx}\right) e^{\frac{Kx}{Kt}} = f(t, x)$

Homogeneoa da

! $\frac{x}{t}$ edo $\frac{t}{x}$ dagoenean HOMOGENEOAK dira !

- Funtzio homogeneoetako aldagai aldaketa

$\boxed{u = \frac{x}{t}}$ BETI $\longrightarrow x = ut$

- Deribatua $x' = u't + u$

- Ordezkatu $x' = u't + u \quad \boxed{t = 1} \quad \boxed{x = u}$

$u't + u = f(u) \longrightarrow u' = \frac{f(u) - u}{t}$

- Ekuazioa askatu: $u' = \frac{f(u) - u}{t}$

$u' dt = \frac{1}{t} (f(u) - u) dt$

$\frac{1}{f(u) - u} du = \frac{1}{t} dt$

Ariketa

$$\begin{cases} (t^2 - 2x^2)\,dt + 2tx\,dx = 0 & dx \to x'\,dt \\ x(1) = 0 \end{cases}$$

$$(t^2 - 2x^2)\cancel{dt} + 2txx'\cancel{dt} = 0$$

$$x' = \underbrace{\frac{-(t^2 - 2x^2)}{2tx}}_{f(t,x)}$$

- Homogeneoa da? $f(t,x) = f(kt, kx)$

$$f(kt, kx) = \frac{-[(kt)^2 - 2(kx)^2]}{2(kt)(kx)} = \frac{-\cancel{k^2}(t^2 - 2x^2)}{\cancel{k^2}\,2tx} = f(t,x)$$

Homogeneoa da!

- Ordezkatu: $\frac{1}{f(u) - u}\,du = \frac{1}{t}\,dt$

$$f(u) = \frac{-(1 - 2u^2)}{2u}$$

$$\frac{1}{-\frac{(1-2u^2)}{2u} - u}\,du = \frac{1}{t}\,dt \longrightarrow \frac{1}{\frac{-1+2u^2 - 2u^2}{}}\,du = \frac{1}{t}\,dt$$

$$-2u\,du = \frac{1}{t}\,dt \longrightarrow \int -2u\,du = \int \frac{2u}{1/t}\,dt$$

$$-u^2 = \ln t + K$$

- Aldagai aldaketa $u = \frac{x}{t}$

$$-\left(\frac{x}{t}\right)^2 = \ln t + K \longrightarrow x^2 = -t^2(\ln t + K) \text{ soluzio orokorra}$$

- Soluzio partikularra:

$$t_0 = 1 \qquad x(1) = 0$$

$$0^2 = -1(\ln 1 + K) \longrightarrow 0 = -K \longrightarrow K = 0$$

$$x^2 = -t^2 \ln t \text{ soluzio partikularra}$$

! Ez kalkulatu erro karratua bestela bi soluzio izan ditzakegu eta agian ez dira biak zuzenak

8 $\begin{cases} (t^2 + x^2) + t x x' = 0 \\ x(1) = 1 \end{cases}$

$x' = \frac{-(t^2+x^2)}{t x}$

- Homogeneoa?

$f(Kt, Kx) = \frac{-[(Kt)^2 + (Kx)^2]}{Kt\, xK} = \frac{-K^2(t^2+x^2)}{K^2\, t x} = f(x,t)$

Bai, homogeneoa da

- Aldagai aldaketa: $u = \frac{x}{t}$
- Ordezkatu $\frac{1}{f(u) - u} du = \frac{1}{t} dt$

$f(u) = \frac{-(1+u^2)}{u}$

$\frac{1}{\frac{-(1+u^2)}{u} - u} du = \frac{1}{t} dt \longrightarrow \frac{1}{\frac{-1-u^2-u^2}{u}} du = \frac{1}{t} dt$

$\frac{u}{-1-2u^2} du = \frac{1}{t} dt \longrightarrow \frac{1}{4}\int \frac{\overbrace{4u}^{g'(u)}}{\underbrace{-1-2u^2}_{g(u)}} du = \frac{1}{t} dt$

$-\frac{1}{4} \ln|-1-2u^2| = \ln t + K$

- Aldagai aldaketa

$-\frac{1}{4} \ln\left|-1-2\frac{x^2}{t^2}\right| = \ln t + K$ solozio orokorra

solozio partikularra

$t_0 = 1 \qquad x_0 = x(t_0) = x(1) = -1$

$-\frac{1}{4} \ln\left|-1-2\frac{(-1)^2}{1^2}\right| = \ln 1 + K$

$-\frac{1}{4} \ln 3 = K$

$-\frac{1}{4} \ln\left|-1-2\frac{t^2}{x^2}\right| = \ln t - \frac{1}{4}\ln 3$

$-\frac{1}{4} \ln\left|-1-2\frac{t^2}{x^2}\right| + \frac{1}{4}\ln 3 = \ln t$

$\frac{1}{4}\left[\ln 3 - \ln\left|-1-2\frac{t^2}{x^2}\right|\right] = \ln t \rightarrow \frac{1}{4}\ln\left(\frac{3}{\left|-1-2\frac{t^2}{x^2}\right|}\right) = \ln t$

$\left(\frac{3}{\left|-1-2\frac{t^2}{x^2}\right|}\right)^{1/4} = t$ solozio partikularia

d) $\begin{cases} t\sin\left(\frac{x}{t}\right)x' = x\sin\left(\frac{x}{t}\right) + t \\ x(1) = \frac{\pi}{2} \end{cases}$

$$x' = \frac{x\sin\left(\frac{x}{t}\right)+t}{t\sin\left(\frac{x}{t}\right)} \longrightarrow f(t,x) = \frac{x\sin\left(\frac{x}{t}\right)+t}{t\sin\left(\frac{x}{t}\right)}$$

- Homogeneoa da? $f(t,x) = f(kt,kx)$

$$f(kt,kx) = \frac{kx\sin\left(\frac{kx}{kt}\right)+kt}{kt\sin\left(\frac{kx}{kt}\right)} = \frac{\cancel{k}\left(x\sin\left(\frac{x}{t}\right)\right)}{\cancel{k}\left(t\sin\left(\frac{x}{t}\right)\right)} = f(t,x)$$

- Aldagai aldaketa: $\frac{1}{f(u)-u}du = \frac{1}{t}dt$

$x = u \quad t = 1$

$$f(u) = \frac{u\cdot\sin u + 1}{\sin u}$$

$$\frac{1}{\frac{u\cdot\sin u+1}{\sin u} - u}du = \frac{1}{t}dt \longrightarrow \frac{1}{\frac{u\sin u + 1 - u\sin u}{\sin u}}du = \frac{1}{t}dt$$

$$\sin u\, du = \frac{1}{t}dt \longrightarrow \int \sin u\, du = \int \frac{1}{t}dt$$

$$-\cos u = \ln t + K$$

- Aldagai aldaketa $u = \frac{x}{t}$

$-\cos\left(\frac{x}{t}\right) = \ln t + K$ solozio orokorra

solozio partikularra

$t_0 = 1 \quad x_0 = x(t_0) = x(1) = \frac{\pi}{2}$

$$-\cos\left(\frac{\pi/2}{1}\right) = \ln 1 + K \rightarrow 0 = K$$

$-\cos\left(\frac{x}{t}\right) = \ln t$ solozio partikularra

e) $\begin{cases} \left(e^{\frac{t}{x}}+1\right) + e^{\frac{t}{x}}\left(1-\frac{t}{x}\right)x' = 0 \\ x(1) = 0 \end{cases}$

$$x' = \frac{-\left(e^{\frac{t}{x}}+1\right)}{e^{\frac{t}{x}}\left(1-\frac{t}{x}\right)}$$

- Homogeneoa da? $f(t,x) = f(kt,kx)$

$$f(kt,kx) = \frac{-\left(e^{\frac{\cancel{k}t}{\cancel{k}x}}+1\right)}{e^{\frac{\cancel{k}t}{\cancel{k}x}}\left(1-\frac{\cancel{k}t}{\cancel{k}x}\right)} = f(t,k)$$

- Aldagai aldaketa: $t=1 \quad x=u$

$$f(u) = \frac{-\left(e^{1/u}+1\right)}{e^{1/u}\left(1-\frac{1}{u}\right)}$$

- Ordezkatu: $\frac{1}{f(u)-u}\,du = \frac{1}{t}\,dt$

$$\frac{1}{\frac{-\left(e^{1/u}+1\right)}{e^{1/u}\left(1-\frac{1}{u}\right)} - u}\,du = \frac{1}{t}\,dt$$

$$\frac{e^{1/u}\left(1-\frac{1}{u}\right)}{-\left(e^{1/u}+1\right) - u\left[e^{1/u}\left(1-\frac{1}{u}\right)\right]}\,du = \frac{1}{t}\,dt$$

$$\frac{e^{1/u}\left(1-\frac{1}{u}\right)}{-e^{1/u}-1-e^{1/u}(u-1)}\,du = \frac{1}{t}\,dt$$

$$\frac{e^{1/u}\left(1-\frac{1}{u}\right)}{\cancel{-e^{1/u}}-1-ue^{1/u}+\cancel{e^{1/u}}}\,du = \frac{1}{t}\,dt \longrightarrow \frac{e^{1/u}\left(1-\frac{1}{u}\right)}{-ue^{1/u}-1}\,du = \frac{1}{t}\,dt$$

$h(u) = -1-ue^{1/u}$

$h'(u) = -\left[e^{1/u} + u\left(\frac{-1}{u^2}\right)e^{1/u}\right] \longrightarrow h'(u) = -\left[e^{1/u} - \frac{1}{u}e^{1/u}\right]$

$h'(u) = -e^{1/u}\left(1-\frac{1}{u}\right)$

- Integratu

$$\int \frac{\overbrace{-e^{1/u}\left(1-\frac{1}{u}\right)}^{h'(u)}}{\underbrace{-1-ue^{1/u}}_{h(u)}}\,du = \int \frac{1}{t}\,dt$$

$$-\ln|-1-ue^{1/u}| = \ln t + K$$

u beti os denez $\ln(1+ue^{1/u})$

- Ordezkatu $u=\frac{x}{t}$

$$-\ln\left(1+\frac{x}{t}e^{\frac{t}{x}}\right) = \ln t + K$$

$$K_1 = \ln\left(1+\frac{x}{t}e^{\frac{t}{x}}\right) + \ln t$$

$$K_1 = \ln(t+xe^{t/x}) \longrightarrow e^{K_1} = t + xe^{t/x}$$

$\underline{K_2 = t + xe^{t/x}}$ → soluzio orokorra

soluzio partikularra:

$t_0 = 1 \qquad x(1) = 1$

$\underline{K_2 = 1 + e}$

$\underline{t + xe^{t/x} = 1 + e}$ → soluzio partikularra

BERNOULLIREN EK. DIFERENTZIALAK

$x' = a(t)x + b(t)x^n$ $\quad$ $a(t)$, $b(t)$ jarraituak

$n=0 \rightarrow$ lineala

$n=1 \rightarrow$ aldagai banandueetako ek diferentziala

Aldagai aldaketa egin behar da. $\boxed{u = x^{1-n}}$

$u = x^{1-x} \rightarrow \underline{x = u^{\frac{1}{1-n}}} \rightarrow \underline{x' = \frac{1}{1-n} u^{\frac{1}{1-n}-1} \cdot u'}$

$x' = a(t)x + b(t)x^n$

Aldagai aldaketa

$$\frac{1}{1-n} u^{\frac{1-1+n}{1-n}} u' = a(t)\, u^{\frac{1}{1-n}} + b(t)\, u^{\frac{n}{1-n}}$$

$$\frac{1}{1-n} u^{\frac{n}{1-n}} u' = a(t)\, u^{\frac{1}{1-n}} + b(t)\, u^{\frac{n}{1-n}}$$

$$u^{\frac{n}{1-n}} u' = (1-n)\,a(t)\, u^{\frac{1}{1-n}} + (1-n)\,b(t)\, u^{\frac{n}{1-n}}$$

$$u' = (1-n)\,a(t) \underbrace{u^{\frac{1}{1-n}}\, u^{-\frac{n}{1-n}}}_{u^{\frac{1-n}{1-n}} = u} + (1-n)\,b(t) \cancel{u^{\frac{n}{1-n}}}\, \cancel{u^{-\frac{n}{1-n}}}$$

$$\boxed{u' = \underbrace{(1-n)\,a(t)}_{a(t)}\, u + \underbrace{(1-n)\,b(t)}_{b(t)}}$$

ARIKETA

6- a) $x' = -\frac{1}{t}x + t^3x^3$

$a(t) = -\frac{1}{t}$ $\quad b(t) = t^3$ $\quad n = 3$ $\quad u = x^{-2}$

- Aldagai aldaketa: $u' = (1-n)a(t)u + (1-n)b(t)$

$u' = \underbrace{-2\left(-\frac{1}{t}\right)}_{a(t)} u + \underbrace{(-2)t^3}_{b(t)} \rightarrow u' = \frac{2}{t}u + (-2)t^3$

- Ekuazioa askatu: $u' =$

$u(t) = e^{\int \frac{2}{t}dt}\left[K + \int (-2)t^3 e^{-\int \frac{2}{t}dt} dt\right]$

$u(t) = e^{2\ln t}\left[K - 2\int t^3 e^{-2\ln t} dt\right]$

$u(t) = t^2\left[K - 2\int t^3 t^{-2} dt\right]$

$u(t) = t^2\left[K - 2\int t\, dt\right] \rightarrow u(t) = t^2\left[K - t^2\right]$

$u(t) = Kt^2 - t^4$

aldagai aldaketa: $x^{-2} = Kt^2 - t^4 \rightarrow x^2 = \frac{1}{Kt^2 - t^4}$

b) $x' - \cos t\, x = \frac{1}{2}\sin(2t)\, x^2$

$x' = \frac{1}{2}\sin(2t)x^2 + \cos t\, x$

$a(t) = \cos t$ $\quad b(t) = \frac{1}{2}\sin(2t)$ $\quad n = 2$ $\quad u = x^{-1}$

- Aldagai aldaketa: $u' = (1-n)a(t)u + (1-n)b(t)$

$u' = -(\cos t)u - \left(\frac{1}{2}\sin(2t)\right)$

- Ekuazioa askatu

$u(t) = e^{\int -(\cos t)dt}\left[K + \int -\left(\frac{1}{2}\sin(2t)\right) e^{-\int -(\cos t)dt} dt\right]$

$u(t) = e^{-\sin t}\left[K - \frac{1}{2}\int \underbrace{\sin(2t)}_{2\sin t\cos t} e^{\sin t} dt\right]$

$u(t) = e^{-\sin t}\left[K - \frac{1}{2}\int \underbrace{2\sin t\cos t\, e^{\sin t}}_{I} dt\right]$

$$I = \int \sin t \cos t \, e^{\sin t} \, dt$$

$$w = \sin t \longrightarrow dw = \cos t \, dt$$

$$dv = \cos t \, e^{\sin t} \, dt \longrightarrow v = e^{\sin t}$$

$$I = \sin t \, e^{\sin t} - \int \underbrace{e^{\sin t} \cos t}_{v = e^{\sin t}} \, dt$$

$$u(t) = e^{-\sin t} \left[K - \left(\sin t \, e^{\sin t} - e^{\sin t} \right) \right]$$

$$x^{-1}(t) = e^{-\sin t} \left[K - \left(\sin t \, e^{\sin t} - e^{\sin t} \right) \right]$$

EKUAZIO DIFERENTZIAL ZEHATZAK

$$P(t,x)\,dt + Q(t,x)\,dx = 0$$

EK diferentzial zehatza izateko baldintza:

$$\frac{\partial P}{\partial x}(t,x) = \frac{\partial Q}{\partial t}(t,x)$$

Adb.

$$\overbrace{3x}^{P(t,x)}\,dt \underbrace{-(t^2+x^2)}_{Q(t,x)}\,dx = 0$$

Definizioa:

$df = f'(t)\,dt$

$dx = x'(t)\,dt$

Demagun, $u(t,x)$

$$du = \underbrace{\frac{\partial u}{\partial t}}_{P(t,x)}\,dt + \overbrace{\frac{\partial u}{\partial x}}^{Q(t,x)}\,dx$$

EBAZTEKO:

1. Egiaztatu $\frac{\partial P}{\partial x}(t,x) = \frac{\partial Q}{\partial t}(t,x)$

2. Ekuazio sistema planteatu

$$\begin{cases} \frac{\partial u}{\partial t}(t,x) = P(t,x) \\ \frac{\partial u}{\partial x}(t,x) = Q(t,x) \end{cases}$$

6. h(x) ordezkatu

3. Integratu:

1- $u(t,x) = \int P(t,x)\,dt + h(x)$

2- $u(t,x) = \int Q(t,x)\,dx + h(t)$

$\int P(t,x)dt + h(x)$

$\int Q(t,x)dx + h(t)$

4. Deribatu partziala kalkulatu

$$1 - \underbrace{\frac{\partial u}{\partial x}(t,x)}_{Q(t,x)} = \frac{\partial}{\partial x}\left(\int P(t,x)\,\underline{dt}\right) + h'\underline{(x)}$$

$$1 - Q(t,x) = \frac{\partial}{\partial x}\left(\int P(t,x)\,dt\right) + h'(x)$$

5. h(x) kalkulatu

ARIKETA

10 a) $\begin{cases} \overbrace{(9t^2 + x - 1)}^{P(t,x)}\,dt + \overbrace{(4x + t)}^{Q(t,x)}\,dx = 0 \\ x(0) = 1 \end{cases}$

$P(t,x) = 9t^2 + x - 1$

$Q(t,x) = 4x + t$

1. Deribatu partzialak

$\frac{\partial P}{\partial x}(t,x) = 1 = \frac{\partial Q}{\partial t}(t,x) = 1 \rightarrow$ zehatza

2. Ekuazio sistema

$\begin{cases} \frac{\partial u}{\partial t}(t,x) = P(t,x) = 9t^2 + x - 1 \\ \frac{\partial u}{\partial x}(t,x) = Q(t,x) = 4x + t \end{cases}$

3. Integrala x-rekiko

konstantea

$u(t,x) = \int (4x + t)\,dx + \underline{h(t)}$ kte

$u(t,x) = 2x^2 + tx + h'(t)$

4. h(t) kalkulatu (Deribatu partziala t-rekiko)

$\frac{\partial u}{\partial t}(t,x) = x + h'(t)$

$9t^2 + \cancel{x} - 1 = \cancel{x} + h'(t) \rightarrow h'(t) = 9t^2 - 1$

$h(t) = \int 9t^2 - 1\,dt \rightarrow h(t) = 3t^3 - t + \underline{K_1}$ kte

5. h(t) ordezkatu

$2x^2 + tx + 3t^3 - t + K_1 = K_2$

$\underline{2x^2 + tx + 3t^3 - t = K_3} \rightarrow$ soluzio orokorra

- soluzio partikularra:

$t_0 = 0 \qquad x(0) = 1$

$2 \cdot 1^2 + 0 \cdot 1 + 3 \cdot 0^3 - 0 = K_3 \longrightarrow \underline{K_3 = 2}$

$\underline{2x^2 + tx + 3t^3 - t = 2} \rightarrow$ soluzio partikularra

b) $\begin{cases} \left(\frac{x}{t}+6t\right)dt + (\ln t - 2)\, dx = 0 \\ x(1) = 2 \end{cases}$

$P(t,x) = \frac{x}{t} + 6t$

$Q(t,x) = \ln t - 2$

1- $\frac{\partial P}{\partial x}(t,x) = \frac{1}{t} = \frac{\partial Q}{\partial t}(t,x) = \frac{1}{t}$ → zehatza da

2- $\begin{cases} \frac{\partial u}{\partial t} = P(t,x) = \frac{x}{t} + 6t \\ \frac{\partial u}{\partial x} = Q(t,x) = \ln t - 2 \end{cases}$

3- $u(t,x) = \int\left(\frac{x}{t} + 6t\right) dt + h(x)$

$= x\int\frac{1}{t}\,dt + 3\int 2t\, dt + h(x)$

$= x\ln t + 3t^2 + h(x)$

4.- $\frac{\partial u}{\partial x}(t,x) = \ln t + h'(x)$

$Q(t,x) = \ln t + h'(x) \rightarrow \ln t - 2 = \ln t + h'(x)$

$h'(x) = -2$

5- $h'(x) = \int -2\,dx \rightarrow h(x) = -2x + k_1$

6- $x\ln t + 3t^2 - 2x + k_1 = k_2$

$x\ln t + 3t^2 - 2x = k_3$ → soluzio orokorra

soluzio partikularra

$t_0 = 1 \qquad x(1) = 2$

$2\ln 1 + 3\cdot 1^2 - 4 = k \rightarrow k = -1$

$2\ln t + 3t^2 - 2x = -1$ → soluzio partikularra

c) $\begin{cases} (at + bx)\,dt = -(bt + cx)\,dx \\ x(1) = 2 \end{cases}$

$\underbrace{(at + bx)}_{P(t,x)}dt + \underbrace{(bt + cx)}_{Q(t,x)}dx = 0$

$P(t,x) = at + bx \qquad Q(t,x) = bt + cx$

1. Egiaztatu

$\frac{\partial P}{\partial x}(t,x) = b \quad = \quad \frac{\partial Q}{\partial t}(t,x) = b \rightarrow$ zehatza da

2. Sistema

$\begin{cases} \frac{\partial u}{\partial t} = P(t,x) = at + bx \\ \frac{\partial u}{\partial x} = Q(t,x) = bt + cx \end{cases}$

3. Integratu (t-rekiko)

$u(t,x) = \int (at + bx)\,dt + h(x)$

$u(t,x) = \frac{at^2}{2} + bxt + h(x)$

4. Deribatu partziala (x-rekiko)

$\frac{\partial u}{\partial x} = \frac{at^2}{2} + bxt + h'(x)$

$Q(t,x) = bt + h'(x)$

$\cancel{bt} + cx = \cancel{bt} + h'(x) \rightarrow h'(x) = cx$

5. h(x) Kalkulatu

$h'(x) = \int cx\,dx \rightarrow h(x) = \frac{cx^2}{2} + K_1$

6. h(x) ordezkatu

$\frac{at^2}{2} + bxt + \frac{cx^2}{2} = K \rightarrow$ soluzio orokorra

Soluzio orokorra

$t_0 = 1 \qquad x(1) = 2$

$\frac{a}{2} + 2b + 2c = K$

$\frac{at^2}{2} + bxt + \frac{cx^2}{2} = \frac{a}{2} + 2b + 2c$

$at^2 + 2bxt + cx^2 = a + 4b + 4c$ ✓ → soluzio partikularra

EDA. ZEHATZETARA BIHURGARRIAK

$$P(t,x)dt + Q(t,x)dx = 0$$

Baldintza: $\frac{\partial P}{\partial x}(t,x) \neq \frac{\partial Q}{\partial t}(t,x)$

$\mu(t,x)$: Faktore integratzailea

Faktore integratzailea aplikatuz

$$\mu(t,x)P(t,x)dt + \mu(t,x)Q(t,x)dx = 0 \longrightarrow \text{ZEHATZA}$$

Zehatza bada...

$$\frac{\partial}{\partial x}(\mu(t,x)P(t,x)) = \frac{\partial}{\partial t}(\mu(t,x)Q(t,x))$$

$$\frac{\partial \mu}{\partial x}P + \mu\frac{\partial P}{\partial x} = \frac{\partial \mu}{\partial t}Q + \mu\frac{\partial Q}{\partial t}$$

$$\frac{\partial \mu}{\partial x}P - \frac{\partial \mu}{\partial t}Q = \mu\frac{\partial Q}{\partial t} - \mu\frac{\partial P}{\partial x}$$

$$\frac{\partial \mu}{\partial x}P - \frac{\partial \mu}{\partial t}Q = \mu\left(\frac{\partial Q}{\partial t} - \frac{\partial P}{\partial x}\right) \rightarrow \mu = \frac{\frac{\partial \mu}{\partial x}P - \frac{\partial \mu}{\partial t}Q}{\frac{\partial Q}{\partial t} - \frac{\partial P}{\partial x}}$$

1. Kasua: $\mu = \mu(t)$

$$\mu = \frac{-Q\frac{\partial \mu}{\partial t}}{\frac{\partial Q}{\partial t} - \frac{\partial P}{\partial x}} \rightarrow -\frac{\frac{\partial Q}{\partial t} - \frac{\partial P}{\partial x}}{Q} = \underbrace{\frac{1}{\mu}\frac{d\mu}{dt}}_{\text{aldagai banandutako ek.}}$$

Aldagai banandutako ek.: $-\frac{\frac{\partial Q}{\partial t} - \frac{\partial P}{\partial x}}{Q}dt = \frac{1}{\mu}d\mu$

$$\int -\frac{\frac{\partial Q}{\partial t} - \frac{\partial P}{\partial x}}{Q}dt = \ln\mu$$

$$\boxed{\mu(t) = e^{\int -\frac{\frac{\partial Q}{\partial t} - \frac{\partial P}{\partial x}}{Q}}}$$

- Ordezkatu faktore integratzailea

! t bakarrik edo konstante denean

2. Kasua: $\mu = \mu(x)$

$$\mu = \frac{\frac{d\mu}{dx} \cdot P}{\frac{\partial Q}{\partial t} - \frac{\partial P}{\partial x}} \rightarrow \frac{\frac{\partial Q}{\partial t} - \frac{\partial P}{\partial x}}{P} = \overbrace{\frac{1}{\mu} \frac{d\mu}{dx}}^{\text{aldagai banandutetako ek.}}$$

Aldagai banandutetako ek.: $\dfrac{\frac{\partial Q}{\partial t} - \frac{\partial P}{\partial x}}{P} dt = \dfrac{1}{\mu} d\mu$

$$\ln \mu = \int \frac{\frac{\partial Q}{\partial t} - \frac{\partial P}{\partial x}}{P} dx$$

$$\boxed{\mu(x) = e^{\int \frac{\frac{\partial Q}{\partial t} - \frac{\partial P}{\partial x}}{P} dx}}$$

- Ordezkatu faktore integratzaileak

! x bakarrik dagoenean edo konstante denean

ARIKETA

Adb. 8

$$(tx^2 + t^2x^2 + 3)\,dt + t^2x\,dx = 0$$

$$P(t,x) = tx^2 + t^2x^2 + 3$$

$$Q(t,x) = t^2x$$

$$\frac{\partial P}{\partial x}(t,x) = 2tx + 2t^2x$$

$$\frac{\partial Q}{\partial t}(t,x) = 2tx$$

- Desberdinak direnez, Ez da zehatza

1. Faktore integratzailea

2 kasua: $$\frac{\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial t}}{P} = \frac{2tx - (2tx + 2t^2x)}{t^2 + t^2x^2 + 3} \longrightarrow \frac{-2t^2x}{t^2 + t^2x^2 + 3}$$

1 kasua: $$\frac{\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial t}}{Q} = \frac{2tx - (2tx + 2t^2x)}{t^2x} \longrightarrow \frac{-2t^2x}{t^2x} = 2$$

Beraz, $\mu(t) = e^{\int \frac{\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial t}}{Q}\,dt}$

$$\mu(t) = e^{\int 2\,dt} \longrightarrow \mu(t) = e^{2t}$$

2. Formulan faktore integratzailea sartu.

$$\mu(t,x)\,P(t,x) + \mu(t,x)\,Q(t,x) = 0$$

$$\underbrace{e^{2t}(tx^2 + t^2x^2 + 3)}_{P_1(t,x)}\,dt + \underbrace{e^{2t}\,t^2}_{Q_1(t,x)}\,dx = 0$$

3. Baldintza betetzen dela baieztatu

$$\frac{\partial P_1}{\partial x}(t,x) = e^{2t}(2tx + 2t^2x)$$

$$\frac{\partial Q_1}{\partial t}(t,x) = x(2e^{2t}t^2 + e^{2t}2t) = e^{2t}(2tx + 2t^2x)$$

- Berdinak direnez, ZEHATZA da

4. Sistema ebatzi

$$\begin{cases} \frac{\partial u}{\partial t} = P_1(t,x) = e^{2t}(tx^2 + t^2x^2 + 3) \\ \frac{\partial u}{\partial x} = Q_1(t,x) = e^{2t}t^2x \end{cases}$$

5. Integrala:

kte

$$u(t,x) = \int e^{2t}t^2 x\,dx + h(t)$$

$$= e^{2t}t^2\frac{x^2}{2} + h(t)$$

6. Deribatu partziala

$$\frac{\partial u}{\partial t}(t,x) = \frac{x^2}{\cancel{2}}(2e^{2t}t^2 + e^{2t}\cancel{2}t) + h'(t)$$

7. h(t) kalkulatu:

$$e^{2t}(\cancel{tx^2} + \cancel{t^2x^2} + 3) = \cancel{x^2t^2e^{2t}} + \cancel{x^2te^{2t}} + h'(t)$$

$$3e^{2t} = h'(t)$$

$$h(t) = \int 3e^{2t}\,dt \longrightarrow h(t) = \frac{3}{2}e^{2t} + K_1$$

8. Ordezkatu:

$$u(t,x) = e^{2t}t^2\frac{x^2}{2} + \frac{3}{2}e^{2t}$$

$$u(t,x) = \frac{1}{2}e^{2t}t^2x^2 + \frac{3}{2}e^{2t} + K_1 = K_2$$

$$u(t,x) = \frac{1}{2}e^{2t}t^2x^2 + \frac{3}{2}e^{2t} = K \longrightarrow$$ soluzio orokorra

$$\boxed{\dot{x} = f(t,x)}$$

- $f(t,x) = g(t)\,h(x) \longrightarrow$ Aldagai banandueta koa
- $f(t,x) = f(kt, kx) \longrightarrow$ Homogeneoa
- $f(t,x) = a(t)x + b(t) \longrightarrow$ Lineala
- $f(t,x) = a(t)x + b(t)x^n \longrightarrow$ Bernoulli $n \neq 0,1$
- $f(t,x) = -\frac{P(t,x)}{Q(t,x)} \longrightarrow$ Zehatza

ESTATISTIKA

- Estatistika deskribatzailea
- Probabilitatea. Zorizko aldagaiak
- Inferentzia estatistikoa

ESTATISTIKA DESKRIBATZAILEA

· Populazio bat aztertzeko lagin bat hartzen da eta honek ezaugarri komunak dituzte

Datuak: x_i Maiztasuna: n_i

Maiztasun erlatiboa: $f_i = \frac{n_i}{n}$ $(0 \le f_i \le 1)$

Ehunekoa: $100 f_i = 100 \frac{n_i}{n}$

Maiztasun absolutu metatua: $N_i = \sum_{j=1}^{i} n_j$

Maiztasun erlatibo metatua: $F_i = \sum_{j=1}^{i} f_j = \frac{N_i}{n}$

Ehuneko metatua: $100 F_i = 100 \frac{N_i}{n}$

Klaseak	x_i	n_i	f_i	$100f_i$	N_i	F_i
$[1,3)$	$x_1 = \frac{a_1+a_2}{2} = 2$	100	$\frac{100}{200} = 0{,}5$	50	100	0,5
$[3,5)$	$x_2 = \frac{a_2+a_3}{2} = 4$	50	0,25	25	150	0,75
$[5,7)$	$x_3 = \frac{a_3+a_4}{2} = 6$	25	0,125	12,5	175	0,875
$[7,9)$	$x_4 = \frac{a_4+a_5}{2} = 8$	25	0,125	12,5	200	1
		$\sum_{i=1}^{4} n_i = 200$	4=1	4=100	4=625	

Batez besteko aritmetikoa

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{k} x_i n_i = \sum_{i=1}^{k} x_i \frac{n_i}{n} = \sum_{i=1}^{k} x_i f_i \qquad \text{non } n = \sum_{i=1}^{k} n_i$$

• Muturreko kasuetan batez bestekoak ez du balio

mediana (Me) eta moda (Mo) erabiltzen da

↳ Ezker eta eskuinean datu kopuru bera
(n bikoitia bada, $(n-1)/2$)

↳ Gehien errepikatzen dena
(maiztasun handieneko balioa

POSIZIO ESTATISTIKOAK

• 100 zati berdinetan banatzen dutenei pertzentil deritze

Aldagaiaren balio bati P_α edo α pertzentila esaten zaio

eta ezkerrean $\% \alpha$ uzten du eta eskuinean datuen

$\% (100-\alpha)$ $100F_i$ -ri behatu behar zaio eta α bertako balioekin alderatu

↳ n bikoitia denean, ez dago 50. pertzentila

$P_{50} = \frac{P_{49} - P_{51}}{2}$ egin behar da $\rightarrow 100F_i = P_\alpha$ denean

• 10 zati berdinetan banatzen zaienei dezil deritze

Aldagaien balio bati D_β edo β dezila esaten zaio balioen

$\% 10\beta$ ezkerrean uzten du $D_\beta = P_{10\beta}$

$D_1 = P_{10}$ $D_2 = P_{20}$ ... ↳ $100F_i = P_{10\beta}$ denean batez bestekoa

• 4 zati berdinetan banatzen dutenei kuartila deritze

$Q_1 = P_{25}$ $Q_2 = P_{50}$ $Q_3 = P_{75}$

↳ $100F_i = Q$ denean batez bestekoa

SAKABANATZE ESTATISTIKOA

• Laginaren balio handiena eta txikienaren arteko diferentziari heina deritzo (R)

• 1 eta 3 kuartilen arteko kendura kuartilen arteko heina da (RI)

• Aldagaien balioen eta batezbestekoaren arteko diferentziei batezbestekoarekiko desbideratze deritzo.

Bariantza desbideratzearen karratuen batazbestekoa

$$\boxed{s^2 = \frac{1}{n}\sum_{i=1}^{k}(x_i-\bar{x})^2 n_i = \sum_{i=1}^{k}(x_i-\bar{x})^2 f_i = \frac{1}{n}\sum_{i=1}^{k}x_i^2 n_i - \bar{x}^2}$$

DESBIDERATZE TIPIKOA

$$\boxed{s = \sqrt{\frac{1}{n}\sum_{i=1}^{k}x_i^2 n_i - \bar{x}^2} = \sqrt{s^2}}$$

• Kuasibariantza $\boxed{\hat{s}^2 = \frac{1}{n-1}\sum_{i=1}^{k}(x_i-\bar{x})^2 n_i} = \frac{n}{n-1}s^2$

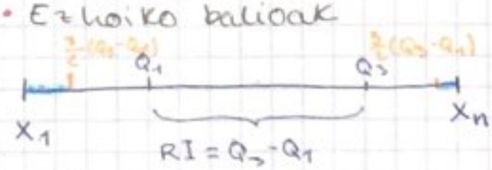
$\boxed{\hat{s}^2 > s^2}$

KUASIDESBIDERATZE TIPIKOA

$$\boxed{\hat{s} = \sqrt{\hat{s}^2}}$$

• Aldakuntza koefizientea $\boxed{AK = \frac{s}{|\bar{x}|}}$

• Ezohiko balioak



$\frac{3}{2}(Q_3-Q_1)$

→ Ez hoiko balioa: Q_1 baino txikiagoa bada, eta datu horren eta Q_1-en arteko distantzia $\frac{3}{2}(Q_3-Q_1)$ baino handiagoa da

→ Ezhoiko balioa Q_3 baino handiagoa bada, eta datu horren eta Q_3-ren arteko distantzia $\frac{3}{2}(Q_3-Q_1)$ baino handiagoa da.

DATU TALDEKATUAK

- Batez bestekoa, desbideratze tipikoa, kuasidesbideratze tipikoa bariantza, kuasi bariantza kalkulatzeko klase markak aplikatu behar dira
- Mediana, pertzentilak, dezilak, eta kuartilak kalkulatzeko klaseak aplikatu behar dira
- Moda kalkulatzeko klase modala erabiliko dugu.
  - Klaseak: $[a_i, a_{i+1})$ $\quad [a_1, a_2)$
  - Klase markak: x_i $\quad x_1 = \dfrac{a_1 + a_2}{2}$

ARIKETAK

1

$[a_i, a_{i+1})$	x_i	n_i
$[0, 20)$	$\frac{0+20}{2} = 10$	15
$[20, 40)$	$\frac{20+40}{2} = 30$	n_2
$[40, 60)$	$\frac{40+60}{2} = 50$	15
$[60, 80)$	$\frac{60+80}{2} = 70$	16

a)

$$n = \sum_{i=1}^{k} n_i \longrightarrow 15 + n_2 + 15 + 16 = 46 + n_2 \qquad \bar{x} = 35$$

$$35 = \frac{1}{46+n_2}(10 \cdot 15 + 30\, n_2 + 50 \cdot 15 + 70 \cdot 16)$$

$$35(46 + n_2) = 30n_2 + 2020 \longrightarrow 5n_2 = 2020 - 1610$$

$$n_2 = 82$$

b) $n_2 = 16 \qquad S^2 = \frac{1}{n}\sum_{i=1}^{k} x_i^2 n_i - \bar{x}^2$

$$n = \sum_{i=1}^{k} n_i \longrightarrow n = 62 \qquad \bar{x} = \frac{1}{n}\sum_{i=1}^{k} x_i \cdot n_i \longrightarrow \bar{x} = \frac{2500}{62}$$

$$S^2 = \frac{1}{62}(10^2 \cdot 15 + 30^2 \cdot 16 + 50^2 \cdot 15 + 70^2 \cdot 16) - \left(\frac{2500}{62}\right)^2$$

$$S^2 = 499{,}52$$

$$S = \sqrt{S^2} \longrightarrow S = 22{,}35$$

3

x_i	n_i	N_i	$100F_i$
1	1	1	$\frac{1}{16}\cdot 100 = 6,25$
2	2	$1+2=3$	$\frac{3}{16}\cdot 100 = 18,75$
3	3	$1+2+3=6$	$\frac{6}{16}\cdot 100 = 37,75$
4	2	8	$\frac{8}{16}\cdot 100 = 50$
5	3	11	$\frac{11}{16}\cdot 100 = 68,75$
6	3	14	$\frac{14}{16}\cdot 100 = 87,5$
8	1	15	$\frac{15}{16}\cdot 100 = 93,75$
9	1	16	$\frac{16}{16}\cdot 100 = 100$

$$n = \sum_{i=1}^{k} n_i = 16$$

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{k} x_i n_i = 4,5$$

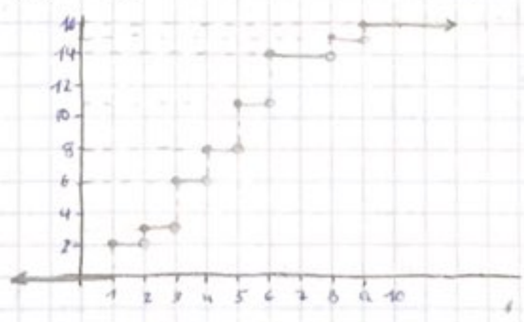
$$s^2 = \frac{1}{n}\sum_{i=1}^{k} x_i^2 n_i - \bar{x}^2 = 4,5$$

$$\hat{s}^2 = \frac{n}{n-1} s^2 = 4,8$$

$Q_1 = P_{25} \longrightarrow 18,75 < \alpha = 25 < 37,5 \qquad \frac{3+3}{2} = 3$

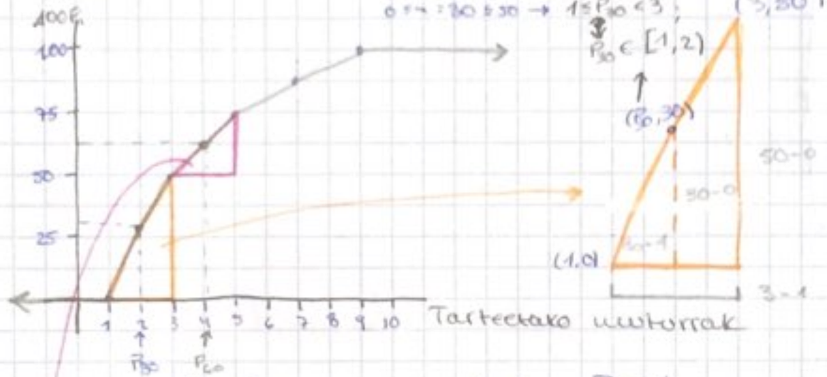
$Q_2 = P_{50} \longrightarrow 37,5 < \alpha = 50 \leq 50 \qquad \frac{4+5}{2} = 4,5$

$Q_3 = P_{75} \longrightarrow 68,75 < \alpha = 75 < 87,5 \qquad \frac{6+6}{2} = 6$

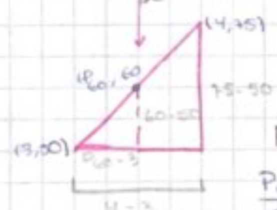


KLASEAK	x_i	n_i	N_i	$100F_i$
[1, 3)	2	100	100	50
[3, 5)	4	50	150	75
[5, 7)	6	25	175	87,5
[7, 9)	8	25	200	100

$0 \le \alpha = 30 \le 50 \rightarrow 1 \le P_{30} < 3$; $\Updownarrow$ $P_{30} \in [1,2)$



$$P_{30} = 1 + \frac{30-0}{50-0}(3-1) \longleftarrow \frac{3-1}{50-0} = \frac{P_{30}-1}{30-0} \longrightarrow P_{30} = 2,2$$



$\underbrace{50}_{[a_1,a_i) \text{ tarteko datuen \%}} \le \alpha = 60 \le 75$

$P_{60} \in [3,5)$ $\Updownarrow$ $a_i = 3 \le P_{60} < 5 = a_{i+1}$

$$P_{60} = 3 + \frac{60-50}{75-50}(5-3)$$

$$\frac{P_{60}-3}{60-50} = \frac{5-3}{75-50} \longrightarrow P_{60} = 3,8$$

- Maiztasun absolutuekin

$$\boxed{P_\alpha = a_i + \frac{\frac{\alpha n}{100} - N_{i-1}}{n_i}(a_{i+1} - a_i)}$$

- Maiztasun erlatiboekin

$$\boxed{P_\alpha = a_i + \frac{\frac{\alpha}{100} - F_{i-1}}{f_i}(a_{i+1} - a_i)}$$

- Ehunekoekin

$$\boxed{P_\alpha = a_i + \frac{\alpha - [a_1, a_i) \text{ tarteko datuen \%}}{\underbrace{i \text{ klaseko datuen \%}}_{\%a_{i+1} - \%a_i}}(a_{i+1} - a_i)}$$

Adb.: $P_\alpha = 8,4$ α?

$a_i = 7 \le P_\alpha = 8,4 < 9 = a_{i+1} \longrightarrow \underbrace{87,5}_{[a_1, a_i) \text{ tarteko datuen \%}} \le \alpha \le 100$

$$8,4 = 7 + \frac{\alpha - 87,5}{100 - 87,5}(9-7) \longrightarrow \alpha = 87,5 + \frac{(8,4-7)\,12,5}{2}$$

Bi ALDAGAIREN BATERAKO ANALISIA

- (X, Y) bi dimentsioko aldagaien bidez adierazten dira
  ↳ Planoan irudikatzean, puntu hodeia edo sakabanatze diagrama izango dugu
- Bi aldagaien batera bestekoari grabitate zentro $(\bar{x}, \bar{y})$ deritzo

$$\boxed{\bar{x} = \frac{1}{n}\sum_{i=1}^{k} x_i \, n_{i\cdot}} \quad \text{x-en bazter maiztasunak}$$

$$\boxed{\bar{y} = \frac{1}{n}\sum_{j=1}^{\ell} y_j \, n_{\cdot j}} \quad \text{y-ren bazter maiztasunak}$$

$X \backslash Y$	y_1	y_2	y_ℓ	$n_{i\cdot}$
x_1	n_{11} +	n_{12} +	$n_{1\ell}$	$n_{1\cdot}$
x_2	n_{21} +	n_{22} +	$n_{2\ell}$	$n_{2\cdot}$
x_k	n_{k1} +	n_{k2} +	$n_{k\ell}$	$n_{k\cdot}$

x-en bazter maiztasunak

BAZTER MAIZTASUNAK

$$n_{i\cdot} = n_{i1} + n_{i2} + \dots + n_{i\ell} = \boxed{\sum_{j=1}^{\ell} n_{ij}}$$

$$n_{\cdot j} = n_{1j} + n_{2j} + \dots + n_{kj} = \boxed{\sum_{i=1}^{k} n_{ij}}$$

BAZTER BARIANTZA

$$S_x^2 = \frac{1}{n}\sum_{i=1}^{k} (x_i - \bar{x})^2 n_{i\cdot} = \boxed{\frac{1}{n}\sum_{i=1}^{k} x_i^2 n_{i\cdot} - \bar{x}^2}$$

$$S_y^2 = \frac{1}{n}\sum_{j=1}^{\ell} (y_j - \bar{y})^2 n_{\cdot j} = \boxed{\frac{1}{n}\sum_{j=1}^{\ell} y_j^2 n_{\cdot j} - \bar{y}^2}$$

KOBARIANTZA

$$S_{xy} = \frac{1}{n}\sum_{i=1}^{k}\sum_{j=1}^{\ell} (x_i - \bar{x})(y_j - \bar{y}) n_{ij}$$

$$\boxed{= \frac{1}{n}\sum_{i=1}^{k}\sum_{j=1}^{\ell} x_i y_j n_{ij} - \bar{x}\,\bar{y}}$$

Letra larriak → aldagaiak
letra xeheak → balioak

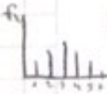
ZORIZKO ALDAGAIAK

$X \sim P(\lambda)$ Poisson

$X \sim B_{(n,p)}$ Binomiala

Zorizko aldagaiak

Diskretua
(aldagaiaren balioaren kopurua
→ finitua
→ infinitu zenbagarria

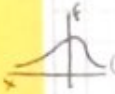
Probabilitate funtzioa $f(x_i) = P(X = x)$ 

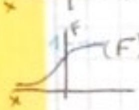
Banaketa funtzioa $F(x) = \sum_{x_i \le x} P(X = x)$ 

Batez bestekoa
$\mu_X = E[X] = \sum_i x_i f(x_i) = \sum_i x_i P(X = x)$

Bariantza
$\sigma_x^2 = V[X] = \sum x_i^2 f(x_i) - \mu_x^2$
$= \sum_i x_i^2 P(X = x) - \mu_x^2$

Jarraitua
(aldagaiaren balioak tarte batean definitu-ta daudenean

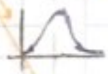
 (f) Dentsitate funtzioa

 (F) Banaketa funtzioa
$F(x) = \int_{-\infty}^{x} f(x)\,dx$

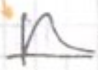
Batez bestekoa
$\mu_x = E[X] = \int_{-\infty}^{\infty} x f(x)\,dx$

Bariantza
$\sigma_x^2 = V[X] = \int_{-\infty}^{\infty} x^2 f(x)\,dx - \mu_x^2$

Normala $X \sim N_{(n,p)}$ 

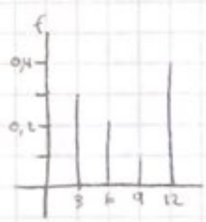
 Pearson $X \sim \chi^2$

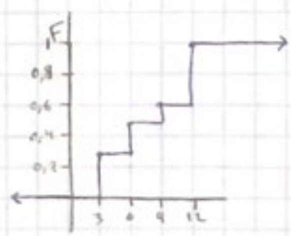
Student $X \sim t_n$ 

 Fischer-Snedecor $X \sim F_{(m,n)}$

(4)

x_i	$P_i(X = x)$
3	0,3
6	0,2
9	0,1
12	0,4





$$F(x) = \begin{cases} 0 & x < 3 \\ 0{,}3 & 3 \le x < 6 \\ 0{,}5 & 6 \le x < 9 \\ 0{,}6 & 9 \le x < 12 \\ 1 & 12 \le x \end{cases}$$

+0,3 +0,2 +0,1 +0,4

$\mu_x = E[X] = \sum_{i=1}^{k} x_i p_i = 7{,}8$

$\sigma_x^2 = V[X] = \sum_{i=1}^{k} (x_i - \mu_x)^2 p_i = 14{,}76$

$\sigma_x = \sqrt{14{,}76}$

bateraezinak

$P(3 < X \le 9) = P(x = 6) \cup P(x = 9)$

$P(x = 6) + P(x = 9) \rightarrow 0{,}2 + 0{,}1 = 0{,}3$

edo

$P(3 < x \le 9) = P(X = 9) - P(x \le 3)$

$0{,}6 - 0{,}3 = 0{,}3$

BANAKETA BINOMIALA

- Zoriazko aldagai diskretua
- Bi emaitza
  - ↳ Arrakasta ⟶ P
  - ↳ Porrota ⟶ Q

  $P + Q = 1$
- Gertaerak elkarrekiko independenteak izatea

$$\boxed{X \sim B(n, p)}$$

$$\boxed{P(X=k) = \binom{n}{k} p^k q^{n-k}}$$ ⟶ Biderketa elkarrekiko independenteak direlako egiten da biderketa.

$$PR_{n,\, k;\, n-k} = C_{k,\, n-k}$$

- Batezbestekoa $\mu_x = E[X] = np$
- Bariantza $\sigma_x^2 = V[X] = npq$

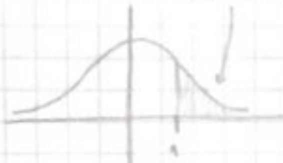
$$\boxed{Y = n - X \sim B(n, q)} \qquad \boxed{P(Y = n-k) = \binom{n}{n-k} q^{n-k} p^k}$$

- $P(Y = n-k) = P(X = k)$ izango da!

$$\binom{n}{k} = \frac{n!}{k!\,(n-k)!}$$

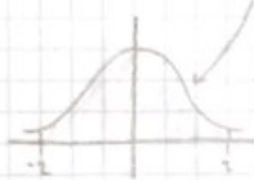
(18) $X = \{\text{Garbigailuen iraupena}\}$ $\quad X \sim N(60,9)$

a) $P(X>69) \xrightarrow{z=\frac{x-\mu}{\sigma}} P\left(Z>\frac{69-60}{9}\right) = P(Z>1)$



$P(Z>1) = 1 - \overbrace{P(Z\le 1)}^{F(1)} \rightarrow 1-0,8413 = 0,1587$

b) $P(42<X<78) \xrightarrow{\text{tipifikatu}} P\left(\frac{42-60}{9}<X<\frac{78-60}{9}\right) = P(-2<X<2)$

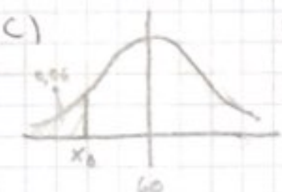


$P(-2<Z<2) = P(Z\le 2) - P(Z\le -2)$

$= P(Z\le 2) - [1-P(Z\le 2)] = 2\overbrace{P(Z\le 2)}^{F(2)} - 1$

$= 2\cdot 0,9772 - 1 = 0,9544$

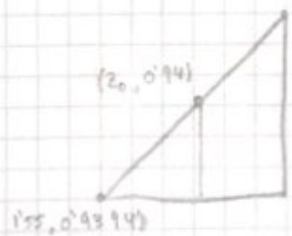
c)



$P(X\le x_0) = 0,06 \rightarrow P\left(Z\le \frac{x_0-60}{9}\right) = 0,06$

$= P\left(Z \ge -\frac{x_0-60}{9}\right) = 1 - P\left(Z\le -\frac{x_0-60}{9}\right) = 0,94$

Taulan begiratu behar dugu z_0 balioa aurkitzeko, baina 0'94 taulan ez dagoenez tales erabili behar dugu



$z_0 \rightarrow \frac{z_0 - 1'55}{0'94 - 0'9394} = \frac{1'56 - 1,55}{0'9406 - 0'9394} \rightarrow z_0 = 1,555$

$-\frac{x_0-60}{9} = 1,555 \rightarrow x_0 = 46,005$

DISKRETUA		JARRAITUA
$X\sim B(n,p)$	$\longrightarrow$	$X \sim N(\mu,\sigma)$

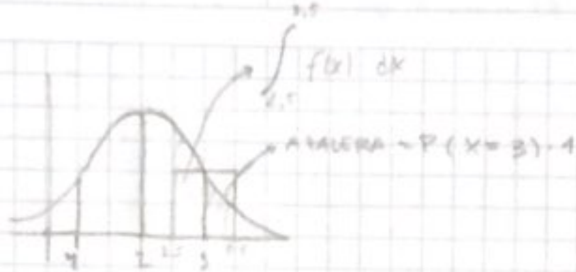
① $n\ge 30$, $0,1<p<0,9$ $\quad \mu = np \quad \sigma^2 = npq$

② $n\ge 30$ $\quad p<0,1$ $\quad np\ge 5$ $\quad \mu = np \quad \sigma^2 = npq$

③ $n\ge 30$ $\quad q<0,1$ $\quad nq\ge 5$ $\quad \mu = np \quad \sigma^2 = npq$

$X\sim P(\lambda)$ $\qquad\qquad X\sim N(\mu,\sigma)$

① $\lambda \ge 5$



$P(X=k) \simeq P(k-0{,}5 \leq X \leq k+0{,}5)$

(16) $X = \{$ aurpegia ateratzen duen aldi kopurua, 7 aldiz botatzean $\}$

$p = 0{,}5 \qquad X \sim B(7, 0'5)$

$q = 0{,}5 \qquad P((X=3) \cup (X=4)) = P(X=3) + P(X=4)$

$= 0{,}2734 + 0{,}2734 = \underline{\underline{0{,}5468}}$

b) $Y = \{$ aurpegia ateratzen den aldi kop, 7.000 aldiz botatzean $\}$

$p = 0{,}5 \qquad Y \sim B(7000, 0'5) \qquad P = (3000 \leq X \leq 4000)$

$n = 7000 \geq 30 \qquad \mu = np = 3500$

$0{,}1 < p = 0{,}5 < 0{,}9 \qquad \sigma^2 = npq = 1750$

$Y \sim N(3500, \sqrt{1750})$

$P(3000 \leq X \leq 4000) \xrightarrow{\text{jarraitutasuna zuzendu}} P(3000 - 0'5 \leq Y \leq 4000 + 0'5)$

$P(2999'5 \leq X \leq 4000'5) \xrightarrow{\text{tipifikatu}} P(-11'96 \leq Z \leq 11'96)$

Balio oso handia denez $\underline{\underline{1}}$ da

Banaketa binomiala erabiltzean aldagaiak hainbat balio hartu ditzake
Baina POISSONENean asko daude, beraz beti osagarria hartu behar da

(11) $X = \{$ alaben kop., 10 seme-alaba izanik $\}$

$Y = \{$ semeen kop. " " " $\}$

$Y = n - X \qquad n = 10$

$X \sim B(10, 0{,}55) \qquad Y \quad B(10, 0{,}45)$

$p = 0{,}55 \qquad\qquad p = 0{,}45$
$q = 0{,}45 \qquad\qquad q = 0{,}55$

a) $P(X=0) = P(Y=10) = \underline{0{,}0003}$

b) $P(X \le 8) = P(Y \ge 2) = 1 - P(Y < 2) = 1 - P((Y=0) \cup (Y=1))$

$= 1 - [P(Y=0) + P(Y=1)] = 1 - (0{,}0025 + 0{,}0207) = \underline{0{,}9768}$

c) $P(X=5) = P(Y=5) = \underline{0{,}2340}$

d) $P(X \ge 3) = P(Y \le 7) = 1 - P(Y > 7) = 1 - P((Y=10) \cup (Y=8) \cup (Y=9))$

$= 1 - [P(Y=10) + P(Y=9) + P(Y=8)] = 1 - (0{,}0229 + 0{,}0042 + 0{,}0003) = \underline{0{,}9726}$

e) $P(X=10) = P(Y=0) = 0{,}0025$

(13) $\%\ 0{,}1 \longrightarrow p = 0{,}001$

$n = 2000$

a) $X = \{$ gaixoen kop. 2000 biztanleko herri batean $\}$

$X \sim B(2000, 0{,}001) \qquad n = 2000 \ge 30$
$\downarrow \qquad\qquad p = 0{,}001 < 0{,}1$
$\underline{X \sim P(2)} \qquad np = 2000 \cdot 0{,}001 = 2 \le 5$

b) $P(X=0) = \underline{0{,}1353}$

c) $P(X \le 3) = P((X=0) \cup (X=1) \cup (X=2) \cup (X=3))$

$= P(X=0) + P(X=1) + P(X=2) + P(X=3) = 0{,}1353 + 0{,}2707 + 0{,}2707 + 0{,}1804$

$\underline{= 0{,}8571}$

d) $P(X \ge 3) = 1 - P(X < 3) = 1 - P((X=0) \cup (X=1) \cup (X=2))$

$= 1 - [P(X=0) + P(X=1) + P(X=2)] = 1 - (0{,}1353 + 0{,}2707 + 0{,}2707) = \underline{0{,}3236}$

10. $X = \{$Pertsona batoitzaren adina$\}$

$X \sim N(32, 8)$

a) $P(x < 18) \xrightarrow{\text{tipifikatu}} P\left(z < \frac{18-32}{8}\right) = P(z < -1,75)$

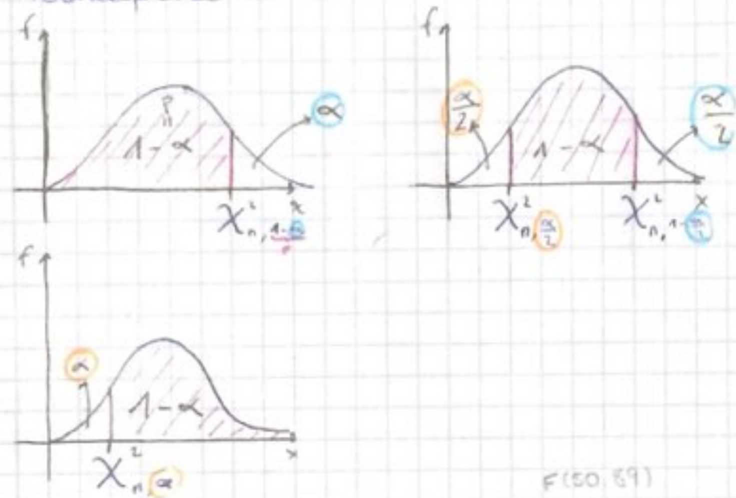
$= P(z \geq 1,75) = 1 - P(z \leq 1,75) = 1 - 0,9599 = 0,0401$

b) $P(x > 60) = p$

$2 \cdot 10^6 \cdot p = 400$

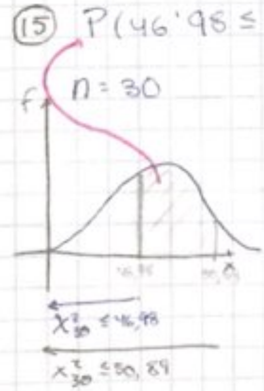
PEARSON-en BANAKETA

- n askatasun-graduko Pearsonen aldagaia χ^2_n
- Batez bestekoa $\mu = n$
- Desbideratze tipikoa $\sigma = \sqrt{2n}$
- Irudikapena



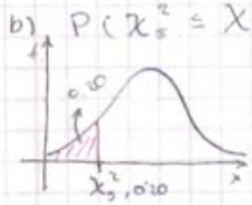
(15) $P(46'98 \le \chi^2_{30} \le 50,89) = \overbrace{P(\chi^2_{30} \le 50,89)}^{F(50,89)} - \overbrace{P(\chi^2_{30} \le 46,98)}^{F(46,98)}$

$= 0,99 - 0,975 = 0,015$



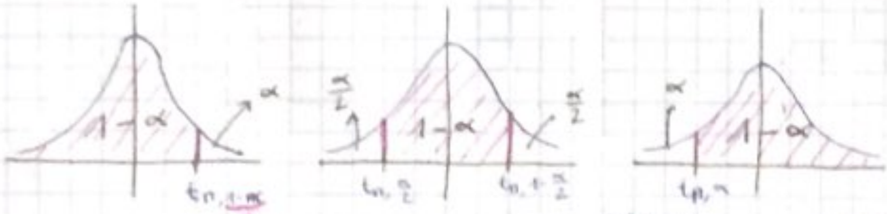
b) $P(\chi^2_5 \le \chi^2_{5,0'20}) = 0.20$

$\chi^2_{5,0'20} = 2.343$



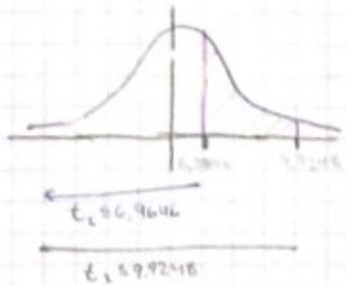
STUDENT-en BANAKETA

- n askatasun graduko student-en aldagaia t_n
- Batez bestekoa $\mu = 0$
- Desbideratze tipikoa $\sigma = \sqrt{\dfrac{n}{n-2}}$
- Irudikapena



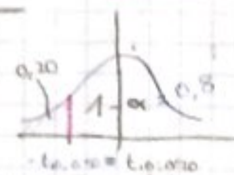
(15) $P(6{,}9646 < t_2 < 9{,}9248) = \overbrace{P(t_2 \leq 9{,}9248)}^{F(9{,}9248)} - \overbrace{P(t_2 \leq 6{,}9646)}^{F(6{,}9646)}$

$= 0{,}995 - 0{,}99 = \underline{0{,}004}$



$t_{10;0,20}$

$n = 10$



$F(t_{10;0,20}) = 0{,}20 \rightarrow P(t_{10} \leq t_{10;0,20}) = 0{,}20$

FISHER - SNEDECOR - en BANAKETA

- (m, n) askatasun-graduko Fisher-Snedecoren aldagaia $F_{(m,n)}$

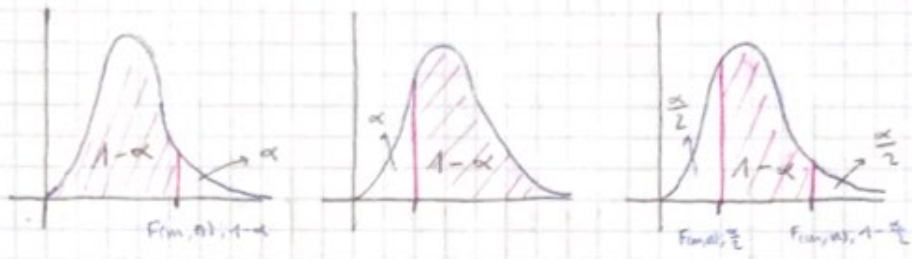
Aurrena / 1 probable

$F(n_1, n_2)$

- Bataz bestekoa $\mu = \frac{n}{n-2}$
- Bariantza: $\sigma^2 = \frac{2n^2(m+n-2)}{m(n-2)^2(n-4)}$

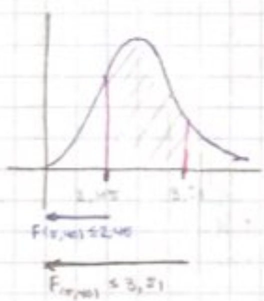
$$F_{(m,n)} = \frac{\frac{\chi^2_{m}}{m}}{\frac{\chi^2_n}{n}}$$

- Irudikapena



(15) $P(2{,}45 \leq F_{(5,40)} \leq 3{,}51) = P(F_{(5,40)} \leq 3{,}51) - P(F_{(5,40)} \leq 2{,}45)$ barnean

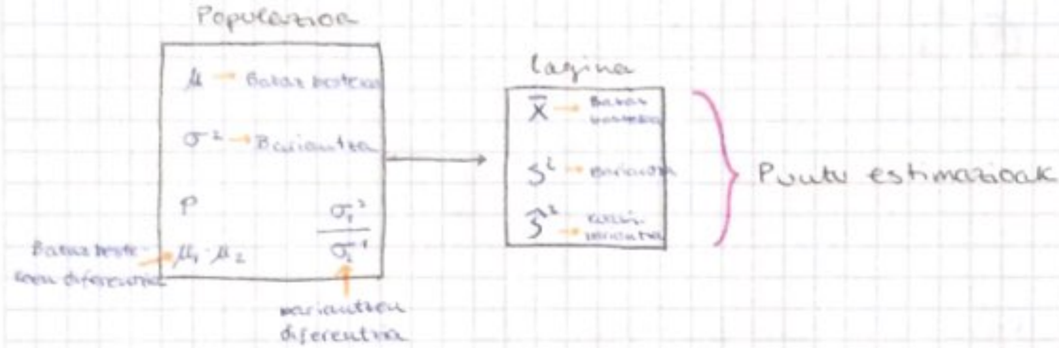
$= 0{,}99 - 0{,}95 = \underline{0{,}04}$



PROPIETATEA: $\boxed{F_{(m,n);p} = \frac{1}{F_{(n,m);1-p}}}$

b) $F_{(9,4);0{,}05} = \frac{1}{F_{(4,9);0{,}95}} = \frac{1}{3{,}63} = \underline{0{,}275}$

INFERENTZIA : HIPOTESI TESTAK



- Aldagaiak adierazteko letra LARRIAK erabiltzen dira

$\bar{X}$ $\quad S^2 \quad \hat{P}$ → Zorizko aldagaiak (estimatzaileak zentratzaileak)

- ALDAGAIAK EZ dira elkarrekiko INDEPENDENTEAK

$$D = X - Y \qquad d_i = x_i - y_i \qquad \bar{d} = \frac{1}{n}\sum_{i=1}^{n} d_i$$

$$S_d^2 = \frac{1}{n}\sum_{i=1}^{n} d_i^2 - \bar{d}^2 \qquad \hat{S}_d^2 = \frac{n}{n-1} S_d^2$$

- ALDAGAIAK elkarrekiko INDEPENDENTEAK dira:

Fischer-en banaketa erabiltzen da

OINARRIZKO KONTZEPTUAK

1. ↳ HIPOTESI NULUA H_0: $\mu = \mu_0$, $\mu \geq \mu_0$, $\mu \leq \mu_0$ ($\sigma^2 = \sigma_0^2$)

   ↳ " ALTERNATIBOA H_a: $\mu > \mu_0$, $\mu \neq \mu_0$, $\mu < \mu_0$ ($\sigma^2 \neq \sigma_0^2$)

2. T kalkulatu $\frac{\bar{X} - \mu_0}{\frac{\sigma}{\sqrt{n}}} \sim N(0,1)$

3. Estimazio puntuak kalkulatu (horretarako suposatzen dugu hipotesi nulua betetzen dela)

$$Z_0 = \frac{\bar{X} - \mu_0}{\frac{\sigma}{\sqrt{n}}} \qquad X_0^2 = \frac{(n-1)\hat{S}^2}{\sigma_0^2}$$

4. Onarpen eremua $A_{1-\alpha} : [-Z_{1-\frac{\alpha}{2}}, Z_{1-\frac{\alpha}{2}}]$
   ↳ NULUA ONARTU

   Eremu kritikoa $C_\alpha : (-\infty, -Z_{1-\frac{\alpha}{2}}) \cup (Z_{1-\frac{\alpha}{2}}, \infty)$
   ↳ ALTERNATIBOA ONARTU

α: Esangura maila

5. Gure puntuak ez dira naturan esanguratsuak hipotesi NULUA baztertzeko

   Gure puntuak naturan esanguratsuak dira hipotesi ALTERNATIBOA onartu

* Enuntziatuan ez bada esangura mailarik aipatzen

$\alpha = 0.10$, $\alpha = 0.05$ eta $\alpha = 0,01$ aztertu behar dira

ARIKETAK

POPULAZIOA

(4) $\mu = 1000$ $\alpha = 0,05$

LAGINA

$n = 5$

$x_1 = 995$ $x_2 = 992$ $x_3 = 1005$

$x_4 = 998$ $x_5 = 1000$

* Isats biko hipotesi testa

(1) $H_0 \rightarrow \mu = 1000$

$H_a \rightarrow \mu \neq 1000$

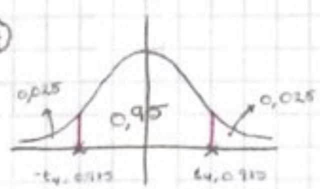
(2) $\dfrac{\overline{X} - \mu}{\frac{S}{\sqrt{n}}} = \dfrac{\overline{X} - \mu}{\frac{s}{\sqrt{n-1}}} \sim t_{n-1}$

(3) Hipotesi nulua onartuz, $\dfrac{\overline{X} - 1000}{\frac{s}{2}} \sim t_4$

$t_0 = \dfrac{\overline{x} - 1000}{\frac{s}{2}}$ $\overline{X} = 998$ $s^2 = 19,6$

$t_0 = \dfrac{998 - 1000}{\frac{\sqrt{19,6}}{2}} \longrightarrow t_0 = -0,9035 \rightarrow$ puntu estimatzea

(4)



$\alpha = 0,05 \longrightarrow \frac{\alpha}{2} = 0,025$

$1 - \alpha = 0,95$ $1 - \frac{\alpha}{2} = 0,975$

$t_{4,0975} = 2,7764$ PUNTU KRITIKOA

$A_{0.95} = [-2,7764 , 2,7764]$

$C_{0,05} = (-\infty, -2.7764) \cup (2,7764, \infty)$

$t_0 \in A_{0,95}$

(5) Gure datuak ez dira nahiko esanguratsuak hipotesi NULUA baztertzeko. Puntu estimatzea onarpen eremuan dagoelako

> bada isatsa ESKUINEAN

< bada isatsa EZKERREAN

Isats bakarrekoa da < edo > denean
Isats bikoa ≠ denean

$t_{n-1,\,1-\alpha}$ → PUNTU KRITIKOA

⑥ POPULAZIOA

$\mu = 1260$ $\quad \sigma^2$ ezezaguna

LAGINA

$n = 4$

$x_1 = 1269 \quad x_2 = 1271 \quad x_3 = 1263 \quad x_4 = 1265$

α txikia bada, eremu kritikoa txikia izango da, eta puntu estimazioa seguruenik A eremuan egongo da

① - $H_0: \mu = 1260$

$H_a: \mu > 1260$

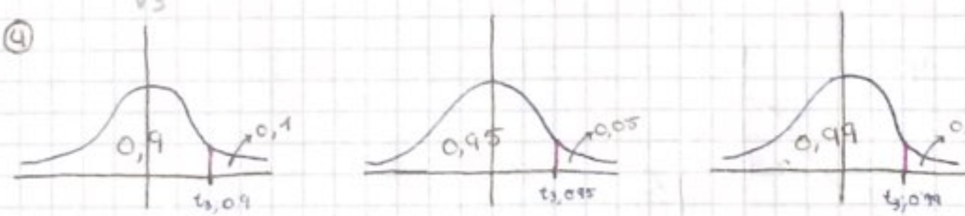
② - $\dfrac{\bar{x} - \mu}{\frac{S}{\sqrt{n}}} = \dfrac{\bar{x} - \mu}{\frac{s}{\sqrt{n-1}}} \sim t_{n-1}$

③ - Hipotesi nulua onartuz $\dfrac{\bar{x} - 1260}{\frac{s}{\sqrt{3}}} \sim t_3$

$t_0 = \dfrac{\bar{x} - 1260}{\frac{s}{\sqrt{3}}} \qquad \bar{x} = 1267 \qquad s^2 = 10$

$t_0 = \dfrac{1267 - 1260}{\frac{\sqrt{10}}{\sqrt{3}}} \longrightarrow t_0 = 3,834$ → Puntu estimazioa

④



$t_{3,0.9} = 1,6377 \qquad t_{3,0.95} = 2,3534 \qquad t_{3,0.99} = 4,5407$

$A_{0,9} = [-1,6377;\,1,6377]$

$C_{0,1} = (-\infty, -1,6377] \cup (1,6377;\,\infty)$

$A_{0.95} = [-2,3534,\,2,3534]$

$C_{0,05} = (-\infty, -2,3534] \cup (2,3534,\,\infty)$

$A_{0,99} = [-4,5407,\,4,5407]$

$C_{0,01} = (-\infty, -4,5407] \cup (4,5407,\,\infty)$

⑤ Estimazio puntua eremu kritikoaren barnean dago, beraz, puntuak nahiko esanguratsuak dira hipotesi nulua baztertzeko $t \in C_{0,1}$

HIPOTESI ALTERNATIBOA

Aurreko bera

HIPOTESI ALTERNATIBOA

$t \in C_{0,05}$

Estimazio puntua eremu onartuaren barnean dago, beraz, puntuak ez dira nahikoa esanguratsuak hipotesi nulua baztertzeko

HIPOTESI NULUA $t \in A_{0,99}$

$\alpha = 0,05$

9) X = {Odoleko kolesterol maila tratamenduaren aurretik}
Y = { " " " " ondoren}

Pertsona berak direnez Ez dira elkarrekiko INDEPENDENTEAK

$X \sim N(\mu_1, \sigma_1)$ $\qquad Y \sim N(\mu_2, \sigma_2)$

1-
$H_0 = \mu_1 - \mu_2 = 0$
$H_a = \mu_1 - \mu_2 > 0$

ESTIMATZAILEA

2- $D = X - X \longrightarrow \overline{D} = \overline{X} - \overline{Y}$ $\qquad \dfrac{\overline{D} - (\mu_1 - \mu_2)}{\frac{Sd}{\sqrt{n-1}}} \sim t_{n-1}$

$d_i = x_i - y_i$

$d_1 = 8 \qquad d_2 = 11 \qquad d_3 = -1 \quad \dots$

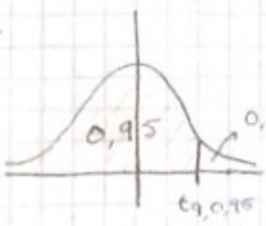
$\overline{d} = \frac{1}{n}\sum_{i=1}^{n} d_i \qquad S_d^2 = \frac{1}{n}\sum_{i=1}^{n} d_i^2 - \overline{d}^2 \qquad \hat{S}_d^2 = \frac{n}{n-1} Sd^2$

3- PUNTU ESTIMAZIOA

$t_0 = \dfrac{\overline{d} - 0}{\frac{\hat{S}_d}{\sqrt{10}}} \qquad \overline{d} = 7.4 \qquad \hat{S}_d^2 = 112.0444$

$t_0 = \dfrac{7.4}{\sqrt{\frac{112.0444}{10}}} = 2.2107$

4-



$t_{9;0,95} = 1,8331$

$A_{0,95} = [-1,8331, 1,8331]$

$C_{0,05} = (-\infty, -1.8331) \cup (1,8331, \infty)$

5- Puntu estimazioa eremu kritikoaren $t_0 \in C_{0,05}$ barnean dagoenez gure datuak nahiko esanguratsuak dira hipotesi nulua baztertzeko HIPOTESI ALTERNATIBOA

(11) $X = \{$A herriko heldu bakoitzaren altuera$\}$ $X \sim N(\mu_1, \sigma_1)$

$Y = \{$B " " " "$\}$ $Y \sim N(\mu_2, \sigma_2)$

- Lagina:

$\bar{X} = 170$ $\bar{Y} = 175$ $n_1 = 80$

$\hat{S}_x = 20$ $\hat{S}_y = 5$ $n_2 = 100$

1-

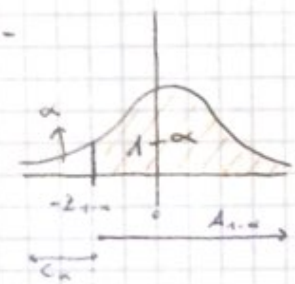
$H_0: \mu_1 - \mu_2 = 0$

$H_a: \mu_1 - \mu_2 < 0$ → Isatsa ezkerrean

2- $$\frac{\bar{X} - \bar{Y} - (\mu_1 - \mu_2)}{\sqrt{\frac{\hat{S}_x^2}{n_x} + \frac{\hat{S}_y^2}{n_2}}} \overset{asb}{\sim} N(0,1)$$ ESTIMATZAILEA

3- PUNTU ESTIMAZIOA: $$z_0 = \frac{170 - 175 - 0}{\sqrt{\frac{20^2}{80} + \frac{15^2}{100}}} = -1.86$$

4-



$z_{0.90} = 1,282$

$z_{0.95} = 1.645$

$z_{0.99} = 2,327$

$A_{0.90} = [-1.282, 1.282]$

$C_{0.10} = (-\infty, -1.282) \cup (1.282, \infty)$ ALTERNATIBOA

$A_{0.95} = [-1.645, 1.645]$

$C_{0.05} = (-\infty, -1.645) \cup (1.645, \infty)$ ALTERNATIBOA

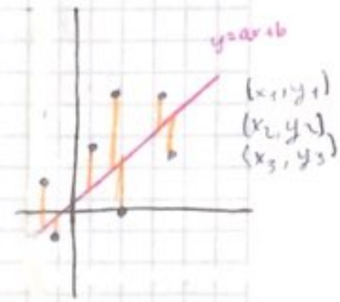
$A_{0.99} = [-2,327, 2,327]$

$C_{0.01} = (-\infty, -2.327) \cup (2.327, \infty)$ NULUA

$z_0 \in A_{0.99}$ $z_0 \in C_{0.05} \in C_{0,10}$

ERREGRESIOAK

ERREGRESIO LINEALA



(X,Y) Bi aldagaien arteko erlazioa jakiteko zuzen bat marrazten da. Zuzen egokiak pare horietara ahalik eta ondoen egokituko da. Minimo karratuen bidezko doikuntza metodoa erabiliz.

Funtzio honi batazbesteko errore koadratikoa deritzo

$$\boxed{BEK(a,b) = \frac{1}{n}\sum_{i=1}^{n}\left[y_i - (ax_i + b)\right]^2}$$

x ezaguna denean:

$$\begin{cases} \sum_{i=1}^{n} y_i = a_n + b\sum_{i=1}^{n} x_i \\ \sum_{i=1}^{n} x_i y_i = a\sum_{i=1}^{n} x_i + b\sum_{i=1}^{n} x_i^2 \end{cases} \longrightarrow \boxed{y - \bar{y} = \frac{S_{xy}}{S_x^2}(x - \bar{x})}$$

x gaineko Y-ren erregresio lineala

Horren malda, $b_{xy} = \frac{S_{xy}}{S_x^2}$ da, → x gaineko Y-ren koefizientea

$a = \bar{y} - \frac{S_{xy}}{S_x^2}\bar{x}$

$$\boxed{x - \bar{x} = \frac{S_{xy}}{S_x^2}(y - \bar{y})}$$ → y ezaguna denean, x ateratzeko

Y gaineko X-ren erregresio lineala

Horren malda $b_{xy} = \frac{S_{xy}}{S_x^2}$ da → Y gaineko X-en koefizientea

Kobariantza		Erregresio-koefizienteak			Erregresio zuzenak
$S_{xy} < 0$	$\longleftrightarrow$	$b_{xy} < 0$	$\longleftrightarrow$	$b_{xy} < 0$	Beherakorra
$S_{xy} > 0$	$\longleftrightarrow$	$b_{xy} > 0$	$\longleftrightarrow$	$b_{xy} > 0$	Gorakorra
$S_{xy} = 0$	$\longleftrightarrow$	$b_{xy} = 0$	$\longleftrightarrow$	$b_{xy} = 0$	Elkarrekiko perpendikularrak

- $S_x \neq 0$ $(S_y \neq 0)$, $y = \bar{y}$ $(x = \bar{x})$ ⟶ zuzen bakarra
- $S_x = 0$

ERREGRESIO LINEALA

- Erregresio - zuzenen eta hodei puntuen arteko doikuntza maila zehazteko korrelazio linealaren koefizientea edo korrelazio koefizientea erabiltzen da

$$\boxed{r = \frac{S_{xy}}{\sqrt{S_x^2 \, S_y^2}}}$$

- Korrelazio koefizientearen tarte batzuek ezaugarri propioak dituzte

r	aldagaien arteko erlazioa	zuzenaren joera	korrelazioa	aurreikuspen fidagarritasuna
1	funtzionala	↗	sendoa	% 100
(0,5 , 1)	zorizkoa	↗	sendoa	% 100r
(0 , 0,5)	zorizkoa	↗	ahulea	% 100r
0	puntuak sakabanatuta		ez dago	0
(-0,5 , 0)	zorizkoa	↘	ahulea	% 100|r|
(-1 , -0,5)	zorizkoa	↘	sendoa	% 100|r|
-1	funtzionala	↘	sendoa	% 100

- $r = \pm 1$ denean $y - \bar{y} = x - \bar{x}$

ARIKETA

(19)

$x \backslash y$	1	2	3	4	5	6	7	$n_{i\cdot}$
10	0	0	0	0	0	0	1	1
20	0	0	0	0	0	1	0	1
30	0	0	0	0	1	0	0	1
40	0	0	0	1	0	0	0	1
50	0	0	1	0	0	0	0	1
60	0	1	0	0	0	0	0	1
70	1	0	0	0	0	0	0	1
$n_{\cdot j}$	1	1	1	1	1	1	1	

$\bar{x} = \frac{1}{2}\sum_{i=1}^{n} x_i \, n_{i\cdot}$ ez da derrigorra kontutan hartu behar 1 direlako

$S_x^2 = \frac{1}{n}\sum_{i=1}^{n} x_i^2 \cancel{n_{i\cdot}} - \bar{x}^2$

$\bar{y} = \frac{1}{2}\sum_{j=1}^{n} y_j \cancel{n_{\cdot j}}$ $\quad S_y^2 = \frac{1}{n}\sum_{j=1}^{n} y_j^2 \cancel{n_{\cdot j}} - \bar{y}^2$

$S_{xy} = \frac{1}{n}\sum_{i=1}^{n}\sum_{j=1}^{n} x_i \, y_j \, n_{ij} - \bar{x}\cdot\bar{y} = \frac{1}{7}(10\cdot 7\cdot 1 + 20\cdot 6\cdot 1 + 30\cdot 5\cdot 1$

$\bar{x} = 40$ $\quad S_x^2 = 400$ $\quad \bar{y} = 4$ $\quad S_y^2 = 4$

$S_x = 20$ $\quad S_y = 2$ $\quad S_{xy} = -40$

$r = \frac{S_{xy}}{\sqrt{S_x^2 \cdot S_y^2}} \longrightarrow r = -1$

* Diagonaleko balioak soilik 1 badira $r = \pm 1$ izango da!

b) $y - \bar{y} = \frac{S_{xy}}{S_x^2}(x - \bar{x}) \longrightarrow y = -\frac{1}{10}x + 8$

$x - \bar{x} = \frac{S_{xy}}{S_y^2}(y - \bar{y}) \longrightarrow x = -\frac{1}{10}y + 8$

* $r = \pm 1$ denean bi formulekin emaitza bera lortzen da!

c) $x = 65$

$\hat{y} = -\frac{1}{10}65 + 8 = 1,5$

(16) $n = 40$

a)

$X \backslash Y$	0	1	2	3	4	5	$n_{i\cdot}$
[0,2)	2	0	0	0	0	0	2
[2,4)	2	2	0	0	0	0	4
[4,6)	4	4	4	2	0	0	14
[6,8)	2	2	2	2	2	0	10
[8,10)	0	2	2	2	2	2	10
$n_{\cdot j}$	10	10	8	6	4	2	$n = 40$

$[a_i, a_{i+1})$	x_i	$n_{i\cdot}$	$N_{i\cdot}$
[0,2)	$\frac{0+2}{2} = 1$	2	$2+0 = 2$
[2,4)	$\frac{2+4}{2} = 3$	4	$2+4 = 6$
[4,6)	$\frac{4+6}{2} = 5$	14	$6+14 = 20$
[6,8)	7	10	30
[8,10)	9	10	40

$n = 40$

y_i	$n_{\cdot i}$	$N_{\cdot j}$
0	10	10
1	10	20
2	8	28
3	6	34
4	4	38
5	2	40

$n = 40$

b)

$\bar{x} = \frac{1}{n}\sum_{i=1}^{k} x_i n_{i\cdot} = 6,1$

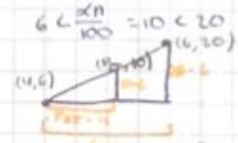
$S_x^2 = \frac{1}{n}\sum_{i=1}^{k} x_i^2 n_{i\cdot} - \bar{x}^2 = 4,99$

$Q_1 = P_{25} \to \alpha = 25$

$\frac{\alpha n}{100} = \frac{25 \cdot 40}{100} = 10$

$4 < Q_1 = P_{25} < 6$

$6 < \frac{\alpha n}{100} = 10 < 20$



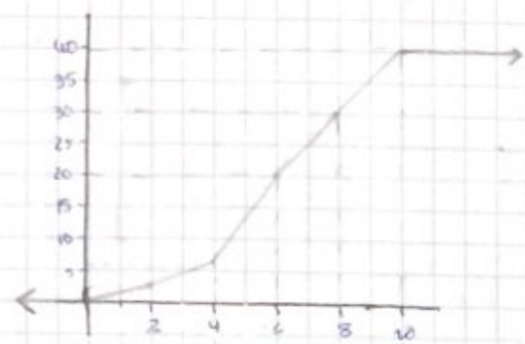
$\frac{P_{25}-4}{10-6} = \frac{6-4}{20-6} \to P_{25} = 4,57$

$Q_2 = P_{50} = D_5 = M_e$

$\frac{\alpha n}{100} = 20$ $N_i = 20$ ez dagoke, dato taldekatuko balio handiena

$Q_2 = 6$

$Q_3 = P_{75}$ $\frac{\alpha n}{100} = 30 \to Q_3 = 8$



Maiztasun absolutu metatuen poligonoa (datu taldekatuekin)

d) $\bar{y} = \frac{1}{n} \sum_{j=1}^{l} y_j n_{\cdot j} = 1,75$

$s_y^2 = \frac{1}{n} \sum_{j=1}^{l} y_j^2 n_{\cdot j} - \bar{y}^2 = 2,1875$

$Q_1 = P_{25} \longrightarrow$ ez ditugunez datu taldekatuak kalkoa da maiztasun absolutu metatuak begiratzea

$\boxed{\frac{\alpha n}{100} = 10}$

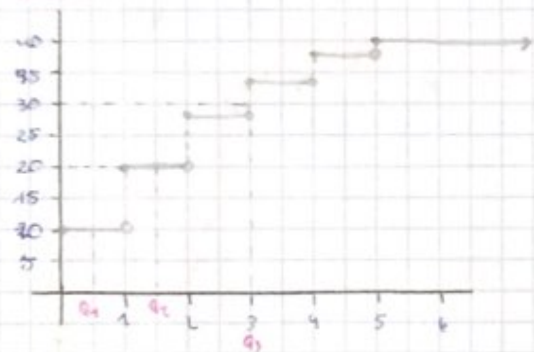
$P_{25} = 0,5 \longrightarrow \begin{matrix} 0 - 10 \\ 1 - 20 \end{matrix}$ beraz, $\frac{0+1}{2} = 0,5$

$Q_2 = P_{50} \longrightarrow \frac{\alpha n}{100} = 20$

$P_{50} = 1,5 \longrightarrow \begin{matrix} 1 - 20 \\ 2 - 28 \end{matrix}$ beraz, $\frac{1+2}{2} = 1,5$

$Q_3 = P_{70} \longrightarrow \frac{\alpha n}{100} = 30$

$P_{75} = 3 \longrightarrow \begin{matrix} 2 - 28 \\ 3 - 34 \end{matrix}$ beraz, $\frac{3+3}{2} = 3$



e) $S_{xy} = \frac{1}{n} \sum_{i=1}^{k} \sum_{j=1}^{l} x_i y_j n_{ij} - \bar{x} \cdot \bar{y}$

$S_{xy} = \frac{1}{40} (1 \cdot 0 \cdot 2 + 3 \cdot 0 \cdot 2 + 3 \cdot 1 \cdot 2 + 5 \cdot 0 \cdot 4 + 5 \cdot 1 \cdot 4 + 5 \cdot 2 \cdot 4 + 5 \cdot 3 \cdot 2$

$+ 7 \cdot 0 \cdot 2 + 7 \cdot 1 \cdot 2 + 7 \cdot 2 \cdot 2 + 7 \cdot 3 \cdot 2 + 7 \cdot 4 \cdot 2 + 9 \cdot 1 \cdot 2 + 9 \cdot 2 \cdot 2 + 9 \cdot 3 \cdot 2$

$+ 9 \cdot 4 \cdot 2 + 9 \cdot 5 \cdot 2) - 6,1 \cdot 1,75 = 1,9750$

$\sum_{i=1}^{k} \sum_{j=1}^{l} x_i y_j n_{ij} \longrightarrow$ matrizeen bidez kalkula daiteke

$$\begin{bmatrix} 1 & 3 & 5 & 7 & 9 \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 & 0 & 0 & 0 \\ 2 & 2 & 0 & 0 & 0 & 0 \\ 4 & 4 & 4 & 2 & 0 & 0 \\ 2 & 2 & 2 & 2 & 2 & 0 \\ 0 & 2 & 2 & 2 & 2 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$$

$r = \frac{S_{xy}}{S_x S_y} = 0,5978$

✻ Lehenengo 4 zifra dezimalak hartu!

(14)

$X \backslash Y$	$[5,7)$	$[7,10)$	$[10,12)$	$n_{i\cdot}$
$[20,25)$	5	5	3	13
$[25,35)$	2	4	6	12
$[35,40)$	1	4	4	9
$n_{\cdot j}$	8	13	13	34

a) $P_\alpha = 27$

$$P_\alpha = a_i + \frac{\frac{\alpha n}{100} - N_{(i-1)}}{n_{i\cdot}}(a_{i+1} - a_i)$$

$$27 = 25 + \frac{\frac{\alpha 34}{100} - 13}{12}(35-25)$$

$24 = \left(\frac{\alpha 34}{100} - 13\right) \cdot 10 \longrightarrow 2,4 = \frac{\alpha 34}{100} - 13$

$15,4 = \frac{\alpha 34}{100} \longrightarrow 1540 = \alpha 34 \longrightarrow \alpha = 45,27$

$100 - 45,27 = \%54,7$ uoztu zen ✓

b) $\underline{\underline{X}}$ $\quad \bar{x} = \frac{1}{n}\sum_{i=1}^{k} x_i n_{i\cdot} = \frac{1}{34}(22,5 \cdot 13 + 30 \cdot 12 + 37,5 \cdot 9) = 29,12$ ✓

$[a_i, a_{i+1})$	x_i	$n_{i\cdot}$	N_i
$[20,25)$	22,5	13	13
$[25,35)$	30	12	25
$[35,40)$	37,5	9	34

$n = 34$

$s_x^2 = \frac{1}{n}\sum_{i=1}^{k} x_i^2 n_{i\cdot} - \bar{x}^2 = \frac{1}{34}(22,5^2 \cdot 13 + 30^2 \cdot 12 + 37,5^2 \cdot 9) - 29,12^2 = 35,62$ ✓

$\underline{\underline{Y}}$

$[a_j, a_{j+1})$	y_j	$n_{\cdot j}$	N_j
$[5,7)$	6	8	8
$[7,90)$	8,5	13	21
$[10,12)$	11	13	34

$n = 34$

$\bar{y} = \frac{1}{n}\sum_{j=1}^{l} y_j n_{\cdot j} = \frac{1}{34}(6 \cdot 8 + 8,5 \cdot 13 + 11 \cdot 13) = 8,87$ ✓

$s_y^2 = \frac{1}{n}\sum_{j=1}^{l} y_j^2 n_{\cdot j} - \bar{y}^2 = \frac{1}{34}(6^2 \cdot 8 + 8,5^2 \cdot 13 + 11^2 \cdot 13) - 8,87^2 = 3,73$ ✓

Kobariantza

$$S_{xy} = \frac{1}{n}\sum_{i=1}^{k}\sum_{j=1}^{l} x_i y_j n_{ij} - \bar{x}\cdot\bar{y} = \frac{1}{34}(22{,}5\cdot 6\cdot 5 + 22{,}5\cdot 8{,}5\cdot 5 +$$

$$22{,}5\cdot 11\cdot 3 + 30\cdot 6\cdot 2 + 30\cdot 8{,}5\cdot 4 + 30\cdot 11\cdot 6 + 37{,}5\cdot 6\cdot 1 +$$

$$37{,}5\cdot 8{,}5\cdot 4 + 37{,}5\cdot 11\cdot 4) - 29{,}12\cdot 8{,}87 = 3{,}08$$ ✓

c) $$r = \frac{S_{xy}}{\sqrt{S_x^2\cdot S_y^2}} = \frac{3{,}08}{\sqrt{35{,}48\cdot 3{,}68}} = 0{,}27$$

$$y - \bar{y} = \frac{S_{xy}}{S_x^2}(x - \bar{x}) \longrightarrow y - 8{,}87 = \frac{3{,}08}{35{,}62}(x - 29{,}12)$$

$$y = 0{,}086x + 6{,}35$$ ✓

d) $r = 0{,}27$ beraz datuen fidagarritasuna $100\cdot r = 26{,}75$

Neurritzoa da. $|r| < 0{,}5$ denez ez du zentzurik

(4)

x_i	n_i	N_i	Ehunekoa $\frac{n_i}{n}\cdot 100$
2	4	4	10
4	6	10	15 $\longrightarrow \frac{n_i}{n}\cdot 100 = \alpha \rightarrow \frac{6}{n}\cdot 100 = 15$
6	8	18	20
8	6	24	15
10	12	36	30
12	4	40	10

$n = 40$

$n_i = 40$

$Q_2 = P_{50} = D_5 = M_e \longrightarrow \frac{\alpha n}{100} = \frac{50\cdot 40}{100} = 20$

$M_e = 8 \longrightarrow \frac{8+8}{2} = 8$

(5) $F_1 = f_1$ $\quad F_i = F_{i-1} + f_i \Rightarrow f_i = F_i - F_{i-1}$

$\frac{n_i}{n} = f_i \longrightarrow n_i = f_i n$

$F_i = \frac{N_i}{n} \longrightarrow N_i = F_i n$

$[a_i, a_{i+1})$	x_i	n_i	N_i	F_i	f_i
[75, 85)	$\frac{75+85}{2} = 80$	10	10	0,1	0,1
[85, 95)	90	15	25	0,25	0,15
[95, 105)	100	20	45	0,45	0,2
[105, 115)	110	25	70	0,7	0,25
[115, 125)	120	20	90	0,9	0,2
[125, 135)	130	10	100	1	0,1

$n = 100$

b) $\bar{x} = \frac{1}{n}\sum_{i=1}^{k} x_i n_i \longrightarrow \bar{x} = 103$

⑨

x_i	$f_{i\cdot}$
1	0,2
2	0,3
3	0,3
4	0,2
	$f=1$

$\bar{x} = \frac{1}{n}\sum_{i=1}^{k} x_i\, n_{i\cdot} \longrightarrow \bar{x} = \sum_{i=1}^{k} x_i \frac{n_{i\cdot}}{n}$ $f_{i\cdot}$

$\bar{X} = 2,5$

f. simetrikoa izateagatik $\bar{x}$ erdiko balioa da

$s_x^2 = \sum_{i=1}^{k} x_i^2 f_{i\cdot} - \bar{x}^2 \longrightarrow s_x^2 = 1,05$

y_j	$f_{\cdot j}$
2	0,2
4	0,4
6	0,4
8	0
	$f=1$

$\bar{y} = \frac{1}{n}\sum_{j=1}^{l} y_j\, n_{\cdot j} \longrightarrow \bar{y} = \sum_{j=1}^{l} y_j \frac{n_{\cdot j}}{n}$

$\bar{y} = 4,4$

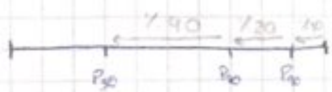
$s_y^2 = \sum_{j=1}^{l} y_j^2 f_{\cdot j} - \bar{y}^2 \longrightarrow s_y^2 = 2,24$

b) $s_{xy} = \sum_{i=1}^{k}\sum_{j=1}^{l} x_i y_j f_{ij} - \bar{x}\cdot\bar{y} \longrightarrow s_{xy} = 0$

c) $x - \bar{x} = 0$; $y - \bar{y} = 0$

d) $r = 0$ Ez dago erlaziorik

②



$[a_i, a_{i+1})$	x_i	n_i	N_i	$100F_i$
$[0,3)$	$\frac{0+3}{2} = 1,5$	5	5	10
$[3,5)$	$\frac{3+5}{2} = 4,5$	22	27	54
$[5,7)$	$\frac{5+7}{2} = 6,5$	18	45	90
$[7,9)$	$\frac{7+9}{2} = 8,5$	5	50	100
$[9,10)$	$\frac{9+10}{2} = 9,5$	0	50	100

$n=50$

$P_{90} = 7$ $P_{70} = ?$ $\underset{100F_{i-1}}{54} < \alpha = 70 < \underset{100F_i}{90}$ $\underset{a_i}{5} < P_{70} < \underset{a_{i+1}}{7}$

$P_{70} = 5 + \frac{70-54}{90-54}(7-5) = 5,\bar{8}$

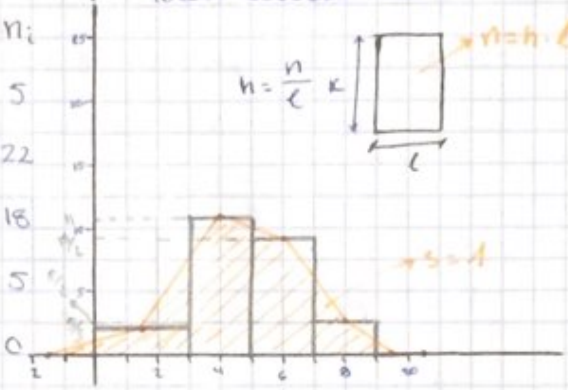
$\underset{100F_{i-1}}{10} < \alpha = 30 < \underset{100F_i}{54}$ edo $\underset{a_i}{3} < P_{30} < \underset{a_{i+1}}{5}$ $P_{30} = 3 + \frac{30-10}{54-10}(5-3) = 3,\bar{9}$

→

	$[a_i, a_{i+1})$	x_i	n_i
$l=3 \leftarrow$	$[0,3)$	1,5	5
$l=2 \leftarrow$	$[3,5)$	4,5	22
$l=2 \leftarrow$	$[5,7)$	6,5	18
$l=2 \leftarrow$	$[7,9)$	8,5	5
$l=1 \leftarrow$	$[9,10)$	9,5	0

$n=50$

HISTOGRAMA
(Datu taldekatuak)

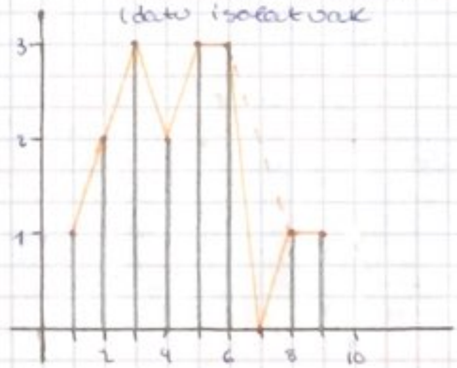


BARRA-DIAGRAMA
(datu isolatuak

③

x_i	n_i
1	1
2	2
3	3
4	2
5	3
6	3
8	1
9	1

$n=16$



ERREGRESIOAK

ERREGRESIO LINEALA

Kasu honetan, $ax+b$ itxurako zuzen bat aukeratzen da, jasotako datuetara doitzeko; a eta b parametroen balioak datuen araberakoak dira.

$$y-\bar{y}=\frac{S_{xy}}{S_x^2}(x-\bar{x}) \quad \text{edo} \quad x-\bar{x}=\frac{S_{xy}}{S_y^2}(y-\bar{y})$$

ERREGRESIO LOGARITMIKOA

Kasu honetan, $a\ln x+b$ itxurako kurba bat aukeratzen da, a eta b parametroen balioak datuen araberakoak dira

$$\boxed{y=a\ln x+b}$$

$$(x_i, y_i) \downarrow (\ln x_i, y_i)=(X_i, y_i)$$

$$X=\ln x \quad y=y \qquad y=aX+b$$

$$a=\frac{n\sum_{i=1}^{n}X_i y_i-\sum_{i=1}^{n}X_i\sum_{i=1}^{n}y_i}{n\sum_{i=1}^{n}X_i^2-\left(\sum_{i=1}^{n}X_i\right)^2} \qquad X_i=\ln x_i$$

$$\boxed{b=\frac{1}{n}\sum_{i=1}^{n}y_i-\frac{a}{n}\sum_{i=1}^{n}x_i}$$

- Errore tipikoa

ERREGRESIO ESPONENTZIALA

$y = be^{ax}$ $\longrightarrow$ $Y = AX + B$

$\ln y = \ln(be^{ax}) \longrightarrow \ln y = \ln b + \ln e^{ax}$

$\ln y = \ln b + ax \longrightarrow \ln y = ax + \underbrace{\ln b}_{=B}$

$x = x$ $\quad Y = \ln y$ $\quad B = \ln b$

ERREGRESIO POTENTZIALA

$y = bx^a \longrightarrow Y = AX + B$

$\ln y = \ln(bx^a) \longrightarrow \ln y = \ln b + \ln x^a$

$\ln y = a \ln x + b$

$(x, y) \longrightarrow (\underbrace{\ln x}_{X}, \underbrace{\ln y}_{Y})$

$B = \ln b \longrightarrow b = e^B$

$$a = \frac{n\sum_{i=1}^{n} X_i Y_i - \sum_{i=1}^{n} X_i \sum_{i=1}^{n} Y_i}{n\sum_{i=1}^{n} X_i^2 - \left(\sum_{i=1}^{n} X_i\right)^2} \longrightarrow \boxed{B = \frac{1}{n}\sum_{i=1}^{n} Y_i - \frac{a}{n}\sum_{i=1}^{n} X_i}$$

ERREGRESIO HIPERBOLIKOA

$y = \frac{a}{x} + b \longrightarrow X = \frac{1}{x} \qquad y = y \qquad y = aX + b$

$(x_i, y_i) \longrightarrow (\frac{1}{x}, y_i) \qquad (x, y) \qquad (\underbrace{X}_{\frac{1}{x}}, y)$

$$a = \frac{n\sum_{i=1}^{n} \overset{1/x_i}{X_i} y_i - \sum_{i=1}^{n} \overset{1/x_i}{X_i} \sum_{i=1}^{n} y_i}{n\sum_{i=1}^{n} \underset{1/x_i^2}{X_i^2} - \left(\sum_{i=1}^{n} X_i\right)^2} \qquad \boxed{b = \frac{1}{n}\sum_{i=1}^{n} y_i - \frac{a}{n}\sum_{i=1}^{n} X_i}$$

ERREGRESIO MICHAELARRA

$$y = \frac{bx}{x+a} \longrightarrow Y = AX + B \qquad \frac{1}{y} = \frac{a}{b}\cdot\frac{1}{x} + \frac{1}{b}$$

$$\frac{1}{y} = \frac{x+a}{bx} \qquad \frac{1}{y} = \frac{x}{bx} + \frac{a}{bx}$$

$$(x, y) \longrightarrow (X, Y) \qquad \left(\frac{1}{x}, \frac{1}{y}\right)$$

$$(x_i, y_i) \longrightarrow \left(\frac{1}{x_i}, \frac{1}{y_i}\right)$$

$$A = \frac{n\sum_{i=1}^{n} \overset{1/x_i}{X_i}\,\overset{1/y_i}{Y_i} - \sum_{i=1}^{n} \overset{1/x_i}{X_i} \sum_{i=1}^{n} \overset{1/y_i}{Y_i}}{n\sum_{i=1}^{n} \underset{1/x_i^2}{X_i^2} - \left(\sum_{i=1}^{n} \underset{1/x_i}{X_i}\right)^2}$$

$$\boxed{b = \frac{1}{n}\sum_{i=1}^{n} \overset{1/y_i}{Y_i} - \frac{A}{n}\sum_{i=1}^{n} \underset{1/x_i}{X_i}}$$

ERREGRESIO LOGISTIKOA

$$y = \frac{1}{1+be^{ax}} \longrightarrow Y = AX + B$$

$$(x_i, y_i) \longrightarrow \left(x_i, \ln\left(\frac{1}{y_i} - 1\right)\right)$$

$$\frac{1}{y} = 1 + be^{ax} \longrightarrow \frac{1}{y} - 1 = be^{ax} \qquad \ln\left(\frac{1}{y} - 1\right) = \ln\left(be^{ax}\right)$$

$$\ln\left(\frac{1}{y} - 1\right) = \ln b + \ln e^{ax} \longrightarrow \ln\left(\frac{1}{y} - 1\right) = \ln b + ax$$

$$\boxed{0 < y_i < 1} \qquad (x, y) \rightarrow (X, Y) =$$

$$\boxed{\frac{1}{y} > 1}$$

(a negatiboa zein positiboa izan daiteke)

$$B = \ln b \qquad \boxed{b = e^{B} > 0}$$

ERREGRESIO PARABOLIKOA

$f(x) = ax^2 + bx + c$

Errore tipikoa: $\sqrt{\frac{1}{n}\sum_{i=1}^{n}(f(x_i) - y_i)^2}$

$$\begin{cases} a_{11}x + a_{12}y + a_{13}z = b_1 \\ a_{21}x + a_{22}y + a_{23}z = b_2 \\ a_{31}x + a_{32}y + a_{33}z = b_3 \end{cases}$$

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

$$\begin{matrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{matrix}$$

Kasu guztietan $|A| \neq 0$

$$x = \frac{\begin{vmatrix} b_1 & a_{12} & a_{13} \\ b_2 & a_{22} & a_{23} \\ b_3 & a_{32} & a_{33} \end{vmatrix}}{|A|} \qquad y = \frac{\begin{vmatrix} a_{11} & b_1 & a_{13} \\ a_{21} & b_2 & a_{23} \\ a_{31} & b_3 & a_{33} \end{vmatrix}}{|A|}$$

$$z = \frac{\begin{vmatrix} a_{11} & a_{12} & b_1 \\ a_{21} & a_{22} & b_2 \\ a_{31} & a_{32} & b_3 \end{vmatrix}}{|A|}$$

(7) $\bar{x} = 1,5$ $\bar{y} = 2,7$ $S_x = 0,2$

$S_y = 1,5$ $r = 0,8$

a) $r = \frac{S_{yx}}{S_y S_x} \longrightarrow 0,8 = \frac{S_{xy}}{1,5 \cdot 0,2} \longrightarrow S_{xy} = 0,24$

x-ren gaineko y-ren erregresio zuzena

$y - \bar{y} = \frac{S_{yx}}{S^2_x}(x - \bar{x}) \longrightarrow y - 2,7 = \frac{0,24}{0,04}(x - 1,5)$

$y = 6x - 6,3$ $y = 6 \cdot 2 - 6,3 \longrightarrow y = 5,7$

b) y-ren gaineko x-en erregresio zuzena

$x - \bar{x} = \frac{S_{xy}}{S^2_y}(y - \bar{y}) \longrightarrow x - 1,5 = \frac{0,24}{1,5^2}(y - 2,7)$

$x = 0,16y - 1,068$ $x = 0,16 \cdot 2 - 1,068 \longrightarrow y = 1,42\bar{5}\bar{3}$

(10)

$n = 4$

x_i	$n_{i\cdot}$
0	1
2	1
4	1
6	1

y_j	$n_{\cdot j}$
4	1
8	1
12	1
16	1

a) $\bar{x} = \frac{1}{n}\sum_{i=1}^{k} x_i n_{i\cdot} = 3$

$S^2_x = \frac{1}{n}\sum_{i=1}^{k} x_i^2 n_{i\cdot} - \bar{x}^2 = 5$

$\bar{y} = \frac{1}{n}\sum_{j=1}^{l} y_j n_{\cdot j} = 10$

$S^2_y = \frac{1}{n}\sum_{j=1}^{l} y_j^2 n_{\cdot j} - \bar{y}^2 = 20$

$S_{xy} = \frac{1}{n}\sum_{i=1}^{k}\sum_{j=1}^{l} x_i y_j n_{ij} - \bar{x}\,\bar{y} = 10$

$r = \frac{S_{xy}}{\sqrt{S^2_x S^2_y}} = 1$

X/Y	4	8	12	16
0	1	0	0	0
2	0	1	0	0
4	0	0	1	0
6	0	0	0	1

b) Aldagaien arteko korrelazioa oso sendoa da, %100eko fidagarritasuna du eta erregresio pakar aproposa

a) $(x - \bar{x}) = \frac{S_{xy}}{S^2_y}(y - \bar{y})$

$x - 3 = \frac{10}{20}(6 - 10) \longrightarrow x = 1$

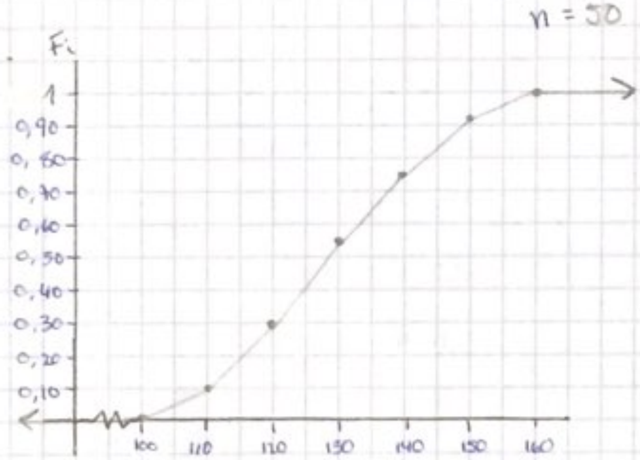
$(y - \bar{y}) = \frac{S_{xy}}{S^2_x}(x - \bar{x})$

$y - 10 = \frac{10}{5}(5 - 3) \longrightarrow y = 14$

(12)

$[a_i, a_{i+1})$	x_i	n_i	N_i	f_i	F_i
[100, 110)	105	5	5	0,1	0,1
[110, 120)	115	10	15	0,2	0,3
[120, 130)	125	12	27	0,24	0,54
[130, 140)	135	11	38	0,22	0,76
[140, 150)	145	8	46	0,16	0,92
[150, 160)	155	4	50	0,08	1
		$n = 50$		$\sum f_i = 1$	

b)



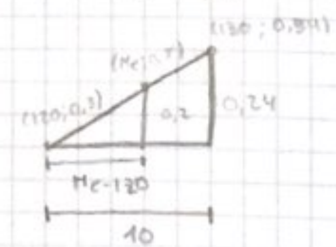
c) $\bar{x} = \frac{1}{n}\sum_{i=1}^{k} x_i n_i = 128,8$

$S_x^2 = \frac{1}{n}\sum_{i=1}^{k} x_i^2 n_i - \bar{x}^2 = 203,56$

$P_{50} = Q_2 = D_5 = Me \longrightarrow F_i$ erabiltzen badugu $\frac{\alpha}{100} = 0,5$

$P_\alpha = a_i + \frac{\frac{\alpha}{100} - F_{i-1}}{f_i}(a_{i+1} - a_i)$

$P_{50} = 120 + \frac{0,5 - 0,3}{0,54 - 0,3}(130 - 120) = 128,\overline{3}$



$\frac{Me - 120}{0,2} = \frac{10}{0,24} \rightarrow Me = 128,\overline{3}$

(11) $\sum x_i = 1400$ $\quad \sum y_r = 10000$ $\quad \overset{S_{xy}}{\sum\sum x_i y_r} = 358000$

$\sum x_i^2 = 50000$ $\quad \sum y_r^2 = 2600000$

$\bar{x} = \frac{1}{n} \sum x_i n_{i.} = \frac{1400}{40} = 35$

$\bar{y} = \frac{1}{n} \sum y_r n_{.r} = \frac{10000}{40} = 250$

$S_{xy} = \frac{1}{n} \sum\sum x_i y_r n_{ij} - \bar{x}\,\bar{y} = \frac{358000}{40} - 35 \cdot 250 = 200$

$S_x^2 = \frac{1}{n} \sum x_i^2 n_{i.} - \bar{x}^2 = \frac{50000}{40} - 35^2 = 25$

$S_y^2 = \frac{1}{n} \sum y_i^2 n_i - \bar{y}^2 = \frac{2600000}{40} - 250^2 = 2500$

$r = \frac{S_{xy}}{\sqrt{S_x^2 S_y^2}} = \frac{200}{\sqrt{25 \cdot 2500}} = 0,8$

Bi aldagaien arteko erlazioa zorizkoa da, 2 zuzen gorakor daude, korrelazioa 0. sendoada eta aurreikuspen altua, beraz merezi du egitea

(13)

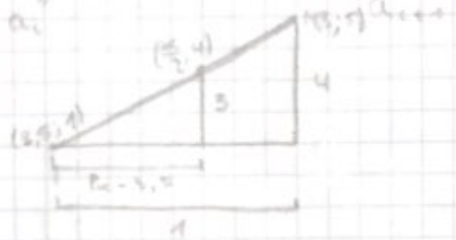
$[a_i, a_{i+1})$	x_i	n_i	$\frac{\alpha n}{100} \downarrow N_i$	$f_i \frac{n_i}{n}$	$\frac{\alpha}{100} \downarrow F_i$	$\alpha \downarrow 100F_i$	$100f_i$
[2,5; 3,5)	3	1	1	0,02	0,02	2	2
[3,5; 4,5)	4	4	5	0,08	0,1	10	8
[4,5; 5,5)	5	5	10	0,1	0,2	20	10
[5,5; 6,5)	6	9	19	0,18	0,38	38	18
[6,5; 7,5)	7	12	31	0,24	0,62	62	24
[7,5; 8,5)	8	9	40	0,18	0,8	80	18
[8,5; 9,5)	9	5	45	0,1	0,9	90	10
[9,5; 10,5)	10	4	49	0,08	0,98	98	8
[10,5; 11,5)	11	1	50	0,02	1	100	2
		n=50		$f_i = 1$			

c) $P_\alpha = 14$ $\qquad P_\alpha = a_i + \dfrac{\frac{\alpha n}{100} - N_{i-1}}{n_i}(a_{i+1} - a_i)$

$$14 = 3,5 + \frac{\frac{\alpha}{2} - 1}{4}(4,5 - 3,5)$$

$$0,5 = \frac{\alpha/2 - 1}{4} \cdot 1 \longrightarrow 2 = \frac{\alpha}{2} - 1 \longrightarrow 6 = \alpha$$

$a_i\ 3,5 < P_\alpha < 4,5\ a_{i+1}$ $\qquad 1 < \frac{\alpha n}{100} < 5$



$$\frac{\frac{\alpha}{2} - 3,5}{3} = \frac{1}{4}$$

$$\frac{\alpha}{2} - 3,5 = \frac{3}{4}$$

$P_{64} = \dfrac{20 + 30}{2} = 25 \longrightarrow$ Bitarteko balioa kalkulatu behar da α-ak 100%-ko balio batekin bat egiten duelako

PROBABILITATEA

(12) $P(A_1) = 0,4$ $\quad P(A_2) = 0,9$ $\quad A_1 \cup A_2 = E$

$P(A_1 \cup A_2) = P(E) = 1$

a) $P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1 \cap A_2)$

$1 = 0,4 + 0,9 - P(A_1 \cap A_2) \rightarrow \underline{P(A_1 \cap A_2) = 0,3}$

b) $P(\bar{A}_1 \cap \bar{A}_2) = P(\overline{A_1 \cup A_2}) \rightarrow 1 - P(A_1 \cup A_2) = P(\bar{A}_1 \cap \bar{A}_2)$

$P(\bar{A}_1 \cap \bar{A}_2) = 0$

c) $P(\bar{A}_1 \cup \bar{A}_2) = P(\overline{A_1 \cap A_2}) \rightarrow 1 - P(A_1 \cap A_2) = P(\bar{A}_1 \cup \bar{A}_2)$

$P(\bar{A}_1 \cup \bar{A}_2) = 0,7$

$P(A_1 \cap A_2)$ 0,3	$P(A_1 \cap \bar{A}_2)$ 0,1	$P(A_1) = 0,4$
$P(\bar{A}_1 \cap A_2)$ 0,6	$P(\bar{A}_1 \cap \bar{A}_2)$ 0	$P(\bar{A}_1) = 1-0,4 = 0,6$
$P(A_2) = 0,9$	$P(\bar{A}_2) = 1-0,9 = 0,1$	$P(E) = 1$

$P(A_1 \cup A_2) = 1$

INDEPENDENTEAK $\quad P(A \cap B) = P(A)\,P(B)$

13

$P(A_1 \cap A_2) = 0,4$	$0,2 = P(A_1 \cap \bar{A}_2)$
$P(\bar{A}_1 \cap A_2) = 0,4$	$b = P(\bar{A}_1 \cap \bar{A}_2)$

$P(A_1) = 0,6$

$P(\bar{A}_1) = 0,4$

$P(A_2) = 0,8$ $\qquad P(\bar{A}_2) = 0,2$

d) $P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1 \cap A_2) = 1$

e) $P(A_1 \cap \bar{A}_2) + P(\bar{A}_1 \cap A_2) = P((A_1 \cap \bar{A}_2) \cup (\bar{A}_1 \cap A_2)) = 0,4 + 0,2 = 0.6$

17 $\quad m = 5 \qquad \{A, B, G, T, Z\} \qquad n = 5$

a) ERREPIKATUZKO ALDAKUNTZA: $VR_{m,n} = m^n$

$VR_{5,5} = 5^5 = \underline{3125}$

b) $x = \{$Pentapeptidoak bost aminoazidoak edukitzea$\}$

$P(x) = \dfrac{x}{\text{aukera guztiak}}$

$x \rightarrow$ PERMUTAZIO ARRUNTA: $P_m = m!$

$A \rightarrow 1 \qquad G \rightarrow 1$

$B \rightarrow 1 \qquad T \rightarrow 1 \qquad Z \rightarrow 1$ $\quad$ beraz $m = 5$

Aldeko kasuen kopurua $P_5 = 5! = 120$

$P(x) = \dfrac{120}{3125} \longrightarrow \underline{P(x) = \dfrac{24}{625}}$

(18) $\{a, n, b\}$ $m = 3$ $n = 5 \to$ esperimentua 5 aldiz errepikatu

Ordena kontuan

· Emaitzak errepikatu daitezke

> Errepikatuzko aldakuntzak

$VR_{m,n} = m^n \to VR_{3,5} = 3^5 = 243$

b) $x = \{2n, 2a, b\}$

$n \to 2$ $a \to 2$ $b \to 1$ $m = 5$ 5 aldiz errepikatu balioak

· Ordena kontuan

· Errepikatu egiten dira

> Errepikatuzko permutazioak

$PR_{m_1, m_2, m_3} = \frac{m!}{m_1!\, m_2!\, m \ldots} \to PR = \frac{5!}{2!\,2!\,1!} = 30$

$P(x) = \frac{30}{243} = \frac{10}{81}$

(4) · Ordenak axola du

· Errepikatu daiteke

> Errepikatuzko permutazioak

$PR_{m, m_1, m_2 \ldots} = \frac{m!}{m_1!\, m_2! \ldots}$

$x_1 + x_2 + x_3 + x_4 \leq 6$

$\overbrace{1,1,1}^{3}, \overset{1}{2}$

1	1	1	1	4	$m_1 = 1$
1	1	1	2	5	$m_2 = PR_{4;3,1} = \frac{4!}{3!\,1} = 4$
1	1	1	3	6	$m_3 = PR_{4;3,1} = \frac{4!}{3!\,1!} = 4$
1	1	2	2	6	$m_4 = PR_{4;2,2} = \frac{4!}{2!\,2!} = 6$

Aldeko kasuen kop: $1 + 4 + 4 + 6 = 15 = m_1 + m_2 + m_3 + m_4$

$\{1, 2, 3, 4, 5, 6\}$ $m = 6$ $n = 4$

$VR_{6;4} = 6^4 = 1296$

$P(x) = \frac{15}{1296} = \frac{5}{432}$

(9) $m = 6$ $n = 12$

Probabilitate bereko kasuen kop

Errepikatuzko aldakuntzak $VR_{m,n} = m^n$

$VR_{6,12} = 6^{12} = 2176782336$

Aldeko kasuen kopurua:

- Ordena kontuan
- Errepikapen jakina

$\Big\rangle$ Errepikatuzko permutazioak

$PR_{m, m_i, m_j, \dots} = \frac{m!}{m_i!\, m_j! \dots}$

$1 \rightarrow 2$ $4 \rightarrow 2$

$2 \rightarrow 2$ $5 \rightarrow 2$ $m = 12$

$3 \rightarrow 2$ $6 \rightarrow 2$

$$PR_{12,2,2,2,2,2,2} = \frac{12!}{2!\,2!\,2!\,2!\,2!\,2!} = 7484400$$

$$P(x) = \frac{\frac{12!}{(2!)^6}}{6^{12}} = 0{,}003438$$

(7) $m = 49$ $n = 6$

- Ordenak ez du axola
- Ez dira errepikatzen

$\Big\rangle$ konbinazio arruntak

$$C_{49,6} = \binom{49}{6} = \frac{49!}{6!\,(49-6)!} = 49 \boxed{nCr} 6$$

Aldeko kasuen kopurua (8 zenbaki aukeratu) 20160

$m = 8$ $n = 6$ $C_{8,6} = 28$

$x = \{ \dots \}$ $P(x) = \frac{28}{13983816} = 2 \cdot 10^{-6}$

BAYES-EN TEOREMA

	0,0156	0,0044	
	$P(A \cap B)$	$P(A \cap \bar{B})$	0,02 = $P(A)$
0,0588 =	$P(\bar{A} \cap B)$	$P(\bar{A} \cap \bar{B})$ = 0,9212	0,98 = $P(\bar{A})$
	$P(B)$ = 0,0744	$P(\bar{B})$ = 0,9256	

(14) $A = \{$Minbizia izatea$\}$

$B = \{$Minbizia diagnostikatzea$\}$

$P(A) = 0,02$

$P(B/A) = 0,78$ $\quad\quad P(B/\bar{A}) = 0,06$

$\quad\quad\quad\quad\quad\quad\quad \hookrightarrow P(\bar{A} \cap B) = P(\bar{A})\,P(B/\bar{A}) = 0,06 \cdot 0,98 = 0,0588$

$\downarrow$

$P(A \cap B) = P(A)\,P(B/A) = 0,02 \cdot 0,78 = 0,0156$

a) $P(B) = P(B \cap E) = P(B \cap (A \cup \bar{A}))$

$= P\big((B \cap A) \cup (B \cap \bar{A})\big) = P(B \cap A) + P(B \cap A)$

$= P(A)\,P(B/A) + P(\bar{A})\,P(B/\bar{A}) = 0,0744$

b) $P(A/B) = \dfrac{P(A \cap B)}{P(B)} = 0,2907$

c) $P(\bar{A}/B) = \dfrac{P(\bar{A} \cap B)}{P(B)} = 0,7903$

d) $P(\bar{B}) = 0,9256$

e) $P(A/\bar{B}) = \dfrac{P(A \cap \bar{B})}{P(\bar{B})} =$

(6)

	I Inkomunikoa	$\bar{I}$ Inkomunikoa ez	GUZTIRA
E Emakumea	$P(E \cap I)$ 0,008	$P(E \cap \bar{I})$ 0,512	0,52
Gizona $\bar{E}$	$P(\bar{E} \cap I)$ 0,063	$P(\bar{E} \cap \bar{I})$ 0,416	0,48
GUZTIRA	$0,071\bar{3}$	$0,928\bar{6}$	1

a) $P(I) = 0,071\bar{3}$

b) $P(\bar{I}/E) = \dfrac{P(\bar{I} \cap E)}{P(E)} = 0,9846$

c) $P(I/\bar{E}) = \dfrac{P(I \cap \bar{E})}{P(\bar{E})} = 0,1319$