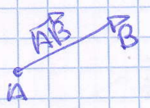
GEOMETRIA

BEKTOREAK PLANOAN:

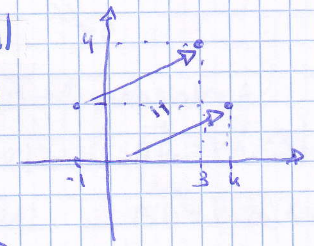
Bektore finkoa:



Ezaugarriak:
- Modulua $|\vec{AB}|$
- Norabidea
- Noranzkoa

$A(x_1, y_1)$

$B(x_2, y_2)$



$\left.\begin{array}{l} B(-1,2) \\ B(3,4) \end{array}\right\} (3-(-1)), (4-2) = 4, 2$

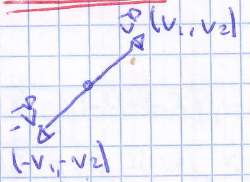
bektore finkoa

bektore askea

Modulua: $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2} = \sqrt{20}$

$\vec{AB} = \vec{B} - \vec{A} = (x_2 - x_1, y_2 - y_1)$

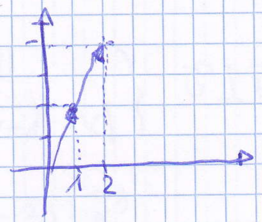
$\vec{v}$-ren aurkakoa $= -\vec{v} = (-v_1, -v_2)$



$\vec{u}(u_1, u_2)$ eta $v(v_1, v_2)$ = paraleloak: $\vec{u} \parallel \vec{v} \rightarrow \frac{u_1}{v_1} = \frac{u_2}{v_2}$

$\vec{u}(1,2)$

$\vec{v}(2,4)$



$\vec{u}(u_1, u_2)$ eta $\vec{v}(v_1, v_2)$ perpendikularrak: $\vec{u} \perp \vec{v}$

$\vec{u} \cdot \vec{v} = 0 \Leftrightarrow u_1 \cdot v_1 + u_2 \cdot v_2 = 0$

biderketa eskalarra

Bektorea: $(2,3) \rightarrow$ Perpendikularra: $(3,-2), (-3,2)$

$\left.\begin{array}{l} \vec{u}(u_1, u_2) \\ \vec{v}(v_1, v_2) \end{array}\right\}$

Batuketa: $\vec{u} + \vec{v} = (u_1 + v_1 ; u_2 + v_2)$

Kenketa: $\vec{u} - \vec{v} = (u_1 - v_1, u_2 - v_2)$

Biderketa bider (zenbaki eskalarra)

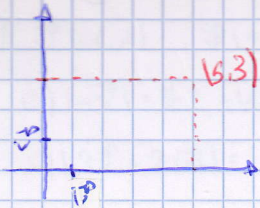
$d\vec{u} = d(u_1, u_2) = (du_1, du_2) \quad d \in \mathbb{R}$

Biderketa eskalarra:

$\vec{u} \cdot \vec{v}$

$\left.\begin{array}{l} \vec{u}(u_1, u_2) \\ \vec{v}(v_1, v_2) \end{array}\right\} \vec{u} \cdot \vec{v} = u_1 \cdot v_1 + u_2 \cdot v_2 \in \mathbb{R}$

100. orrialdea (teoria)



$\vec{u}(1,0)$, $\vec{v}(0,1)$ → oinarri kanonikoa

Bi bektore, $\vec{u}, \vec{v}$

- linealki menpekoak dira proportzionalak badira (paraleloak)
- linealki askeak dira proportzionalak ez badira (ez paraleloak)

$\vec{u}, \vec{v}$ } oinarri bat osatzen dute:

1) Askeak

2) Edozein bektore bi horien konbinazio lineal bezala jar daiteke $\exists a, b \in \mathbb{R}$ $\vec{w} = a \cdot \vec{u} + b \cdot \vec{v}$

$(5,3) = 5(1,0) + 3(0,1) = 5\vec{u} + 3\vec{v}$

Oinarri ortogonala: $\square + \vec{u} \perp \vec{v}$

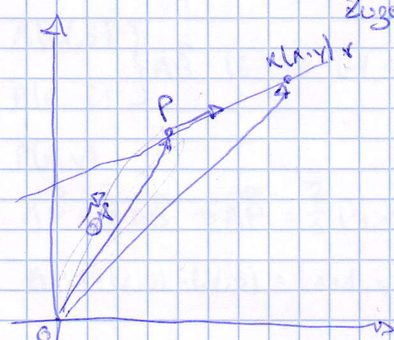
" normala: $\square + (|\vec{u}| = |\vec{v}| = 1)$

" ortonormala: $\square + \vec{u} \perp \vec{v} + |\vec{u}| = |\vec{v}| = 1$

Zuzenaren ekuazioa:

$y = \underbrace{m}_{\text{malda}}x + \underbrace{n}_{\text{ordenatuaren jatorria}}$

$m = \frac{\Delta y}{\Delta x}$



Zuzen bat ezagutzeko: · 2 puntu
· 1 puntu + norabidea

$P(p_1, p_2)$

$\vec{v}(v_1, v_2)$ ↓ zuzenaren bektore zuzentzailea

Ek. bektoriala:

$\boxed{\vec{OX} = \vec{OP} + d\vec{v}}$ $d \in \mathbb{R}$

$(x,y) = (p_1, p_2) + d(v_1, v_2)$

$(x+y) = (p_1 + dv_1, p_2 + dv_2)$

$$\left.\begin{aligned} x &= p_1 + dv_1 \\ y &= p_2 + dv_2 \end{aligned}\right\} d \in \mathbb{R}$$

Ek. parametrikoak

$$\boxed{\frac{x - p_1}{v_1} = \frac{y - p_2}{v_2}} \quad \begin{cases} d = \frac{x - p_1}{v_1} \leftarrow x = p_1 + dv_1 \\ d = \frac{y - p_2}{v_2} \leftarrow y = p_2 + dv_2 \end{cases}$$

Ek. jarraitua

$\frac{x - p_1}{v_1} = \frac{y - p_2}{v_2} \rightarrow (x - p_1) \cdot v_2 = (y - p_2) \cdot v_1 \rightarrow v_2 x - v_2 p_1 = v_1 y - v_1 p_2 \rightarrow v_2 x - v_1 y - v_2 p_1 + p_2 v_1 = 0$

$\boxed{Ax + By + C = 0}$

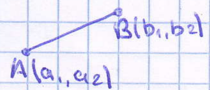
Ek. orokorra edo inplizitua

Ek. esplizitua

$$\begin{aligned} By &= -Ax - C \\ y &= \underbrace{\frac{-A}{B}}_{\text{malda}}x \underbrace{- \frac{C}{B}}_{\text{jatorria}} \end{aligned}$$

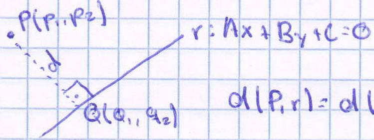
Distantziak:

• Bi punturen arteko distantzia



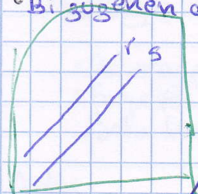
$d(A,B) = |\vec{AB}| =$

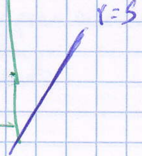
• Puntu batetik zuzen baterako distantzia:



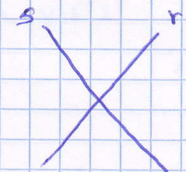
$$d(P,r) = d(P,Q) = |PQ| = \frac{|Ap_1 + Bp_2 + C|}{\sqrt{A^2+B^2}}$$

• Bi zuzenen arteko distantzia:





$d(r,s) = 0$



$d(r,s) = 0$