Frogatu ondoko deribazioak zuzenak direla Gentzenen sistema naturalean

1. $\forall x\ (Px \rightarrow Qx);\ \forall x\ (Qx \rightarrow Rx) \vdash \forall x\ (Px \rightarrow Rx)$
2. $\forall x\ (Px \wedge Qx) \vdash \forall x\ Px \wedge \forall x\ Qx$
3. $\forall x\ (Px \rightarrow Qx);\ \exists x\ \neg Qx \vdash \exists x\ \neg Px$
4. $\forall x \forall y\ Rxy \vdash \forall x \forall y\ (Rxy \wedge Ryx)$
5. $\forall x\ (Px \rightarrow Qx);\ \forall x\ Px \vdash \forall x\ Qx$
6. $\exists x\ (Px \wedge Qx);\ \forall x\ (Qx \rightarrow \neg Rx) \vdash \exists x\ (Px \wedge \neg Rx)$
7. $\exists x\ (Px \wedge Qx) \vdash \exists x\ Px \wedge \exists x\ Qx$
8. $\forall x\ (Px \rightarrow Qx);\ \exists x\ Px \vdash \exists x\ Qx$
9. $\forall x\ (Px \vee \neg Qx);\ (\forall x\ Px \vee \forall x\ \neg Qx) \rightarrow \neg \exists x\ Fx;\ Fa \vdash \exists x\ (Px \wedge Qx)$
10. $\exists x \neg Px \vdash \neg \forall x Px$
11. $\forall x\ (Rx \rightarrow \neg Qx);\ \forall x\ (Px \rightarrow Qx) \vdash \forall x\ (Px \rightarrow \neg Rx)$
12. $\neg \exists x\ (Px \wedge \neg Rx) \rightarrow \neg \forall x\ (Qx \rightarrow Px);\ \forall x\ (Px \rightarrow Rx) \vdash \exists x\ \neg(\neg Qx \vee Px)$
13. $\forall x\ (Px \rightarrow Qx);\ \exists x\ \neg Qx \vdash \exists x\ \neg Px$
14. $\forall x\ (Px \vee (Qx \rightarrow \neg Fx));\ \exists x\ (Fx \wedge Qx);\ \exists x\ (\neg Px \wedge Fx) \vdash \exists x \exists y\ (Px \wedge \neg Qy)$
15. $\forall x\ (Px \vee Qx);\ \exists x\ \neg Qx \vdash \exists x\ Px$
16. $\forall x\ (Px \rightarrow Qx);\ \exists x\ (Px \wedge Fx) \vdash \exists x\ (Qx \wedge Fx)$
17. $\forall x\ (Pc \rightarrow Qx) \vdash Pc \rightarrow \forall x\ Qx$
18. $\exists x\ (Pc \wedge Qx) \vdash Pc \wedge \exists x\ Qx$
19. $\forall x\ (Px \rightarrow Qx);\ \forall x\ (Qx \rightarrow \neg Rx);\ \forall x\ [(Px \wedge Sx) \rightarrow Rx];\ \exists x\ Px \vdash \neg \forall x\ (Qx \rightarrow Sx)$
20. $\forall x\ (Px \rightarrow Qx);\ \forall x\ (Qx \rightarrow \neg Rx);\ \forall x\ [(Px \wedge Sx) \rightarrow Rx] \vdash \forall x\ [(Px \rightarrow \neg(Qx \rightarrow Sx)]$
21. $\forall x\ (Px \rightarrow \neg Qx);\ \exists x\ (Qx \wedge Rx) \vdash \exists x\ (\neg Px \wedge Rx)$
22. $\exists x\ (Px \wedge \neg Qx);\ \forall x\ (Px \rightarrow Rx) \vdash \exists x\ (Qx \vee Rx)$
23. $\forall x\ (Fx \wedge \neg Gx);\ \forall x\ (Hx \rightarrow Gx) \vdash \exists x\ (Fx \wedge \neg Hx)$
24. $\forall x\ [(Hx \wedge Rx) \leftrightarrow Cx];\ \forall x\ [Mx \rightarrow (Rx \wedge \neg Hx)];\ \forall x\ [Lx \rightarrow (Rx \wedge \neg Hx)];\ \forall x\ [Dx \rightarrow (\neg Rx \wedge Hx)]$
    $\vdash \forall x\ [(Mx \vee Lx \vee Dx) \rightarrow \neg Cx]$
25. $\forall x\ (Rx \rightarrow Px);\ \forall x\ (Px \rightarrow \neg Sx);\ \forall x\ [(Rx \wedge Qx) \rightarrow Sx] \vdash \forall x\ [Rx \rightarrow \neg(Px \rightarrow Qx)]$
26. $\forall x\ (Fx \wedge \neg Gx);\ \forall x\ (Hx \rightarrow Gx) \vdash \forall x\ (Fx \wedge \neg Hx)$
27. $\forall x\ (Rx \rightarrow Qx);\ \exists x\ (Px \wedge \neg Qx) \vdash \exists x\ (Px \wedge \neg Rx)$
28. $\forall x\ \neg(Rx \wedge Qx);\ \exists x\ (Rx \wedge Px) \vdash \exists x\ (Px \wedge \neg Qx)$
29. $\forall x\ (Qx \rightarrow \neg Px);\ \forall x\ (Rx \rightarrow Qx) \rightarrow \exists x\ (Qx \wedge Px) \vdash \exists x\ (Rx \wedge \neg\ Qx)$
30. $\neg \exists x\ (Px \wedge \neg Rx) \rightarrow \neg \forall x\ (Qx \rightarrow Px);\ \forall x\ (Px \rightarrow Rx) \vdash \exists x\ \neg(\neg Qx \vee Px)$
31. $\forall x\ [(Px \wedge Qx) \rightarrow \neg Rx];\ \exists x\ (Px \wedge Qx \wedge Sx) \vdash \exists x\ (Px \wedge Sx \wedge \neg Rx)$
32. $\exists y \forall x\ Rxy \vdash \exists y\ Ryy$
33. $\forall x\ (Px \rightarrow Qx) \vdash \forall x \forall y\ [(Px \wedge Ryx) \rightarrow (Qx \wedge Ryx)]$
34. $\forall x \forall y\ [[(Px \wedge Py) \wedge \exists z\ (Pz \wedge Rxz \wedge Ryz)] \rightarrow Sxy];\ \forall x \forall y\ [(Px \wedge Py \wedge Rxy) \rightarrow Ryx]$
    $\vdash \forall x \forall y\ [(Px \wedge Py \wedge \neg Sxy) \rightarrow \neg \exists z\ (Pz \wedge Rzx \wedge Rzy)]$
35. $\vdash \exists x\ (Px \rightarrow \forall y\ Py)$
36. $Pc \vdash \forall x\ (x{=}c \rightarrow Px)$
37. $\forall x\ (x{=}c \rightarrow Px) \vdash Pc$
38. $Pc \vdash \exists x\ (x{=}c \wedge Px)$
39. $\exists x\ (x{=}c \wedge Px) \vdash Pc$
40. $\forall x \exists y\ Rxy;\ \forall x \forall y\ (Rxy \rightarrow \forall z\ Rzy) \vdash \exists y \forall x\ Rxy$

Frogatu ondoko deribazioak zuzenak direla Gentzenen sistema naturalean

1.

1.	$\forall x\,(Px \rightarrow Qx)$	Prem
2.	$\forall x\,(Qx \rightarrow Rx)$	Prem
3.	$Pa \rightarrow Qa$	∀A 1
4.	$Qa \rightarrow Ra$	∀A 2
5.	$Pa \rightarrow Ra$	Irag. 3,4
6.	$\forall x\,(Px \rightarrow Rx)$	∀S 5

2.

1.	$\forall x\,(Px \wedge Qx)$	Prem
2.	$Pa \wedge Qa$	∀A 1
3.	Pa	KA 2
4.	Qa	KA 2
5.	$\forall x\,Px$	∀S 3
6.	$\forall x\,Qx$	∀S 4
7.	$\forall x\,Px \wedge \forall x\,Qx$	KS 5,6

3.

1.	$\forall x\,(Px \rightarrow Qx)$	Prem
2.	$\exists x\,\neg Qx$	Prem
3.	$Pa \rightarrow Qa$	∀A 1
4.	$\neg Qa$	Hip.
5.	$\neg Pa$	MT 3,4
6.	$\exists x\,\neg Px$	∃S 5
7.	$\exists x\,\neg Px$	∃A 2, 4-6

4.

1.	$\forall x \forall y\,Rxy$	Prem
2.	$\forall y\,Ray$	∀A 1
3.	Rab	∀A 2
4.	$\forall y\,Rby$	∀A 1
5.	Rba	∀A 4
6.	$Rab \wedge Rba$	KS 3,5
7.	$\forall y\,(Ray \wedge Rya)$	∀S 6
8.	$\forall x \forall y\,(Rxy \wedge Ryx)$	∀S 7

5.

1.	$\forall x\,(Px \rightarrow Qx)$	Prem
2.	$\forall x\,Px$	Prem
3.	$Pa \rightarrow Qa$	∀A 1
4.	Pa	∀A 2
5.	Qa	MP 3,4
6.	$\forall x\,Qx$	∀S 5

6.

1.	$\exists x\,(Px \wedge Qx)$	Prem
2.	$\forall x\,(Qx \rightarrow \neg Rx)$	Prem
3.	$Qa \rightarrow \neg Ra$	∀A 2
4.	$Pa \wedge Qa$	Hip
5.	Pa	KA 4
6.	Qa	KA 4
7.	$\neg Ra$	MP 3,6
8.	$Pa \wedge \neg Ra$	KS 5,7
9.	$\exists x\,(Px \wedge \neg Rx)$	∃S 8
10.	$\exists x\,(Px \wedge \neg Rx)$	∃A 1, 4-9

7.

1.	$\exists x\,(Px \wedge Qx)$	Prem
2.	$Pa \wedge Qa$	Hip
3.	Pa	KA 2
4.	Qa	KA 2
5.	$\exists x\,Px$	∃S 3
6.	$\exists x\,Qx$	∃S 4
7.	$\exists x\,Px \wedge \exists x\,Qx$	KS 5,6
8.	$\exists x\,Px \wedge \exists x\,Qx$	∃A 1, 2-7

8.

1.	$\forall x\,(Px \rightarrow Qx)$	Prem
2.	$\exists x\,Px$	Prem
3.	$Pa \rightarrow Qa$	∀A 1
4.	Pa	Hip
5.	Qa	MP 3,4
6.	$\exists x\,Qx$	∃S 5
7.	$\exists x\,Qx$	∃A 2, 4-6

9.

1.	$\forall x\,(Px \vee \neg Qx)$	Prem
2.	$(\forall x\,Px \vee \forall x\,\neg Qx) \rightarrow \neg \exists x\,Fx;$	Prem
3.	Fa	Prem
4.	$\exists x\,Fx$	∃S 3
5.	$\neg(\forall x\,Px \vee \forall x\,\neg Qx)$	MT 2,4
6.	$\neg \forall x\,Px \wedge \neg \forall x\,\neg Qx$	DM 5
7.	$\neg \forall x\,\neg Qx$	KA 6
8.	$\exists x\,Qx$	Def ∀ 7
9.	Qb	Hip
10.	$Pb \vee \neg Qb$	∀A 1
11.	Pb	SD 9, 10
12.	$Pb \wedge Qb$	KS 9, 11
13.	$\exists x\,(Px \wedge Qx)$	∃S 12
14.	$\exists x\,(Px \wedge Qx)$	∃A 8, 9-13

10.

1.	$\exists x \neg Px$	Prem
2.	$\neg Pa$	Hip
3.	$\forall x\,Px$	Hip
4.	Pa	∀A 3
5.	$Pa \wedge \neg Pa$	KS 2,4
6.	$\neg \forall x\,Px$	Abs, 2-5
7.	$\neg \forall x Px$	∃A 1, 2-6

11.

1.	$\forall x\,(Rx \rightarrow \neg Qx)$	Prem
2.	$\forall x\,(Px \rightarrow Qx)$	Prem
3.	$Ra \rightarrow \neg Qa$	∀A 1
4.	$Pa \rightarrow Qa$	∀A 2
5.	Pa	Hip
6.	Qa	MP 4,5
7.	$\neg Ra$	MT 3,6
8.	$Pa \rightarrow \neg Ra$	DT 4-7
9.	$\forall x\,(Px \rightarrow \neg Rx)$	∀S 8

12.

1.	$\neg \exists x\,(Px \wedge \neg Rx) \rightarrow \neg \forall x\,(Qx \rightarrow Px)$	Prem
2.	$\forall x\,(Px \rightarrow Rx)$	Prem
3.	$Pa \rightarrow Ra$	∀A 2
4.	$\neg\,(Pa \wedge \neg Ra)$	Def. → 3
5.	$\forall x\,\neg(Px \wedge \neg Rx)$	∀S 4
6.	$\neg \exists x\,(Px \wedge \neg Rx)$	Def. ∀ 5
7.	$\neg \forall x\,(Qx \rightarrow Px)$	MP 1,6
8.	$\exists x\,\neg(Qx \rightarrow Px)$	Def ∀ 5
9.	$\neg(Qa \rightarrow Pa)$	Hip
10.	$Qa \wedge \neg Pa$	Def → 9
11.	$\neg(\neg Qa \vee Pa)$	DM 10
12.	$\exists x\,\neg(\neg Qx \vee Px)$	∃S 11
13.	$\exists x\,\neg(\neg Qx \vee Px)$	∃A 8, 9-12

13.

1.	$\forall x\,(Px \rightarrow Qx)$	Prem
2.	$\exists x\,\neg Qx$	Prem
3.	$Pa \rightarrow Qa$	∀A 1
4.	$\neg Qa$	Hip
5.	$\neg Pa$	MT 3,4
6.	$\exists x\,\neg Px$	∃S 5
7.	$\exists x\,\neg Px$	∃A 2, 4-6

14.

1.	∀x (Px ∨ (Qx → ¬Fx))	Prem
2.	∃x (Fx ∧ Qx)	Prem
3.	∃x (¬Px ∧ Fx)	Prem
4.	Fa ∧ Qa	Hip
5.	¬(Qa → ¬Fa)	Def ∧ 4
6.	Pa ∨ (Qa → ¬Fa)	∀A 1
7.	Pa	SD 5,6
8.	¬Pb ∧ Fb	Hip
9.	¬Pb	KA 8
10.	Fb	KA 8
11.	Pb ∨ (Qb → ¬Fb)	∀A 1
12.	Qb → ¬Fb	SD 9, 11
13.	¬Qb	MT 10, 12
14.	Pa ∧ ¬Qb	KS 7, 13
15.	∃y (Pa ∧ ¬Qy)	∃S 14
16.	∃y (Pa ∧ ¬Qy)	∃A 3, 8-15
17.	∃x∃y (Px ∧ ¬Qy)	∃S 16
18.	∃x∃y (Px ∧ ¬Qy)	∃A 2, 4-17

15.

1.	∀x (Px ∨ Qx)	Prem
2.	∃x ¬Qx	Prem
3.	Pa ∨ Qa	∀A 1
4.	¬Qa	Hip
5.	Pa	SD 3,4
6.	∃x Px	∃S 5
7.	∃x Px	∃A 2, 4-6

16.

1.	∀x (Px → Qx)	Prem
2.	∃x (Px ∧ Fx)	Prem
3.	Pa → Qa	∀A 1
4.	Pa ∧ Fa	Hip
5.	Pa	KA 4
6.	Qa	MP 3,5
7.	Fa	KA 4
8.	Qa ∧ Fa	KS 6,7
9.	∃x (Qx ∧ Fx)	∃S 8
10.	∃x (Qx ∧ Fx)	∃A 2, 4-9

17.

1.	∀x (Pc → Qx)	Prem
2.	Pc → Qa	∀A 1
3.	Pc	Hip
4.	Qa	MP 2,3
5.	∀x Qx	∀S 4
6.	Pc → ∀x Qx	DT 2-5

18.

1.	∃x (Pc ∧ Qx)	Prem
2.	Pc ∧ Qa	Hip
3.	Pc	KA 2
4.	Qa	KA 2
5.	∃x Qx	∃S 4
6.	Pc ∧ ∃x Qx	KS 3, 5
7.	Pc ∧ ∃x Qx	∃A 1, 2-6

19.

1.	∀x (Px → Qx)	Prem
2.	∀x (Qx → ¬Rx)	Prem
3.	∀x [(Px ∧ Sx) → Rx]	Prem
4.	∃x Px	Prem
5.	Pa → Qa	∀A 1
6.	Qa → ¬Ra	∀A 2
7.	(Pa ∧ Sa) → Ra	∀A 3
8.	Pa	Hip
9.	Qa	MP 5,8
10.	¬Ra	MP 6,9
11.	¬(Pa ∧ Sa)	MT 7,9
12.	¬Pa ∨ ¬Sa	DM 11
13.	¬Sa	SD 8, 12
14.	Qa ∧ ¬Sa	KS 9, 13
15.	¬(Qa → Sa)	Def ∧ 14
16.	∃x ¬(Qx → Sx)	∃S 15
17.	∃x ¬(Qx → Sx)	∃A 4, 8-15
18.	¬∀x (Qx → Sx)	Def ∀ 17

20.

1.	∀x (Px → Qx)	Prem
2.	∀x (Qx → ¬Rx)	Prem
3.	∀x [(Px ∧ Sx) → Rx]	Prem
4.	Pa → Qa	∀A 1
5.	Qa → ¬Ra	∀A 2
6.	(Pa ∧ Sa) → Ra	∀A 3
7.	Pa	Hip
8.	Qa	MP 4,7
9.	¬Ra	MP 5, 8
10.	¬(Pa ∧ Sa)	MT 6, 9
11.	¬Pa ∨ ¬Sa	DM 10
12.	¬Sa	SD 7, 11
13.	Qa ∧ ¬Sa	KS 8,12
14.	¬(Qa → Sa)	Def ∧ 13
15.	Pa → ¬(Qa → Sa)	DT 7-14
16.	∀x [(Px → ¬(Qx → Sx)]	∀S 15

21.

1.	∀x (Px → ¬Qx)	Prem
2.	∃x (Qx ∧ Rx)	Prem
3.	Pa → ¬Qa	∀A 1
4.	Qa ∧ Ra	Hip
5.	Qa	KA 4
6.	¬Pa	MT 3, 5
7.	Ra	KA 4
8.	¬Pa ∧ Ra	KS 6,7
9.	∃x (¬Px ∧ Rx)	∃S 8
10.	∃x (¬Px ∧Rx)	∃A 2, 4-9

22.

1.	∃x (Px ∧ ¬Qx)	Prem
2.	∀x (Px → Rx)	Prem
3.	Pa → Ra	∀A 2
4.	Pa ∧ ¬Qa	Hip
5.	Pa	KA 4
6.	Ra	MP 3,5
7.	Qa ∨ Ra	DS 6
8.	∃x (Qx ∨ Rx)	∃S 7
9.	∃x (Qx ∨ Rx)	∃A 1, 4-8

23.

1.	∀x (Fx ∧ ¬Gx)	Prem
2.	∀x (Hx → Gx)	Prem
3.	Fa ∧ ¬Ga	∀A 1
4.	Ha → Ga	∀A 2
5.	¬Ga	KA 3
6.	¬Ha	MT 4, 5
7.	Fa	KA 3
8.	Fa ∧ ¬Ha	KS 6, 7
9.	∃x (Fx ∧ ¬Hx)	∃S 8

24.

1. ∀x [(Hx ∧ Rx) ↔ Cx] Prem
2. ∀x [Mx → (Rx ∧ ¬Hx)] Prem
3. ∀x [Lx → (Rx ∧ ¬Hx)] Prem
4. ∀x [Dx → (¬Rx ∧ Hx)] Prem
5. (Ha ∧ Ra) ↔ Ca ∀A 1
6. Ca → (Ha ∧ Ra) BBA 5
7. Ma → (Ra ∧ ¬Ha) ∀A 2
8. La → (Ra ∧ ¬Ha) ∀A 3
9. Da → (¬Ra ∧ Ha) ∀A 4
10. Ma ∨ La ∨ Da Hip
11. Ma Hip
12. Ra ∧ ¬Ha MP 7,11
13. ¬Ha KA 12
14. ¬Ha ∨ ¬Ra DS 13
15. ¬(Ha ∧ Ra) DM 14
16. ¬Ca MT 6, 15
17. La Hip
18. Ra ∧ ¬Ha MP 8, 17
19. ¬Ha KA 18
20. ¬Ha ∨ ¬Ra DS 19
21. ¬(Ha ∧ Ra) DM 20
22. ¬Ca MT 6, 21
23. Da Hip
24. ¬Ra ∧ Ha MP 9, 23
25. ¬Ra KA 24
26. ¬Ha ∨ ¬Ra DS 25
27. ¬(Ha ∧ Ra) DM 26
28. ¬Ca MT 6, 27
29. ¬Ca DA 10, 11-16, 17-22, 23-28
30. (Ma ∨ La ∨ Da) → ¬Ca DT 10-29
31. ∀x [(Mx ∨ Lx ∨ Dx) → ¬Cx] ∀S 30

25.

1. ∀x (Rx → Px) Prem
2. ∀x (Px → ¬Sx) Prem
3. ∀x [(Rx ∧ Qx) → Sx] Prem
4. Ra → Pa ∀A 1
5. Pa → ¬Sa ∀A 2
6. (Ra ∧ Qa) → Sa ∀A 3
7. Ra Hip
8. Pa MP 4, 7
9. ¬Sa MP 5, 8
10. ¬(Ra ∧ Qa) MT 6, 9
11. ¬Ra ∨ ¬Qa MP 10
12. ¬Qa SD 7, 11
13. Pa ∧ ¬Qa KS 8, 12
14. ¬(Pa → Qa) Def ∧ 13
15. Ra → ¬(Pa → Qa) DT 7-14
16. ∀x [Rx → ¬(Px → Qx)] ∀S 15

26.

1. ∀x (Fx ∧ ¬Gx) Prem.
2. ∀x (Hx → Gx) Prem
3. Fa ∧ ¬Ga ∀A 1
4. Ha → Ga ∀A 2
5. ¬Ga KA 3
6. ¬Ha MT 4,5
7. Fa KA 3
8. Fa ∧ ¬Ha KS 6,7
9. ∀x (Fx ∧ ¬Hx) ∀S 8

27.

1. ∀x (Rx → Qx) Prem
2. ∃x (Px ∧ ¬Qx) Prem
3. Ra → Qa ∀A 1
4. Pa ∧ ¬Qa Hip
5. Pa KA 4
6. ¬Qa KA 4
7. ¬Ra MT 3, 6
8. Pa ∧ ¬Ra KS 5,8
9. ∃x (Px ∧ ¬Rx) ∃S 8
10. ∃x (Px ∧ ¬Rx) ∃A 2, 4-9

28.

1. ∀x ¬(Rx ∧ Qx) Prem
2. ∃x (Rx ∧ Px) Prem
3. ¬(Ra ∧ Qa) ∀A 1
4. ¬Ra ∨ ¬Qa DM 3
5. Ra ∧ Pa Hip
6. Ra KA 5
7. Pa KA 5
8. ¬Qa SD 4,6
9. Pa ∧ ¬Qa KS 7,8
10. ∃x (Px ∧ ¬Qx) ∃S 9
11. ∃x (Px ∧ ¬Qx) ∃A 2, 5-10

29.

1. ∀x (Qx → ¬Px) Prem
2. ∀x (Rx → Qx) → ∃x (Qx ∧ Px) Prem
3. ¬∃x (Rx ∧ ¬Qx) Hip
4. ∀x ¬(Rx ∧ ¬Qx) Def ∃ 3
5. ¬(Ra ∧ ¬Qa) ∀A 4
6. Ra → Qa Def ∧ 5
7. ∀x (Rx → Qx) ∀S 6
8. ∃x (Qx ∧ Px) MP 2,5
9. Qa → ¬Pa ∀A 1
10. Qa ∧ Pa Hip
11. Qa KA 8
12. Pa KA 8
13. ¬Pa MP 7,9
14. Pa ∧ ¬Pa KS 10,11
15. ∃x (Px ∧ ¬Px) ∃S 12
16. ∃x (Px ∧ ¬Px) ∃A 6, 8-13
17. ∃x (Rx ∧ ¬Qx) Abs. 3-14

30.

1. ¬∃x (Px ∧ ¬Rx) → ¬∀x (Qx → Px) Prem
2. ∀x (Px → Rx) Prem
3. ¬∃x ¬(Px → Rx) Def ∀ 2
4. ¬∃x (Px ∧ ¬Rx) Def ∧ 3
5. ¬∀x (Qx → Px) MP 2,4
6. ∃x ¬(Qx → Px) Def ∀ 5
7. ∃x ¬(¬Qx ∨ Px) Def → 6

31.

1. ∀x [(Px ∧ Qx) → ¬Rx] Prem
2. ∃x (Px ∧ Qx ∧ Sx) Prem
3. Pa ∧ Qa ∧ Sa Hip
4. (Pa ∧ Qa) → ¬Ra ∀A 1
5. Pa ∧ Qa KA 3
6. ¬Ra MP 4,5
7. Pa ∧ Sa KA 3
8. Pa ∧ Sa ∧ ¬Ra KS 6,7
9. ∃x (Px ∧ Sx ∧ ¬Rx) ∃S 8
10. ∃x (Px ∧ Sx ∧ ¬Rx) ∃A 2, 3-9

32.

1. ∃y∀x Rxy Prem
2. ∀x Rxa Hip
3. Raa ∀A 2
4. ∃y Ryy ∃S 3
5. ∃y Ryy ∃A 1, 2-4

33.

1.	∀x (Px → Qx)	Prem
2.	Pa ∧ Rba	Hip
3.	Pa	KA 2
4.	Pa → Qa	∀A 1
5.	Qa	MP 3,4
6.	Rba	KA 2
7.	Qa ∧ Rba	KS 5,6
8.	(Pa ∧ Rba) → (Qa ∧ Rba)	DT 2-7
9.	∀y [(Pa ∧ Rya) → (Qa ∧ Rya)]	∀S 8
10.	∀x∀y [(Px ∧ Ryx) → (Qx ∧ Ryx)]	∀S 9

34.

1.	∀x∀y[[(Px∧Py) ∧ ∃z(Pz ∧ Rxz ∧ Ryz)] → Sxy]	Prem
2.	∀x∀y [(Px ∧ Py ∧ Rxy) → Ryx]	Prem
3.	¬∀x∀y[(Px∧Py∧¬Sxy) → ¬∃z(Pz∧Rzx∧Rzy)]	Hip
4.	∃x∃y[(Px∧Py∧¬Sxy) ∧∃z(Pz∧Rzx∧Rzy)]	Def ∀,→ 3
5.	∃y[(Pa∧Py∧¬Say) ∧∃z(Pz∧Rza∧Rzy)]	Hip
6.	(Pa ∧ Pb ∧ ¬Sab) ∧ ∃z (Pz ∧ Rza ∧ Rzb)	Hip
7.	Pa ∧ Pb ∧ ¬Sab	KA 6
8.	∃z (Pz ∧ Rza ∧ Rzb)	KA 6
9.	¬Sab	KA 6
10.	∀y[[(Pa∧Py) ∧ ∃z(Pz ∧ Raz ∧ Ryz)] → Say]	∀A 1
11.	[(Pa ∧ Pb) ∧ ∃z (Pz ∧ Raz ∧ Rbz)] → Sab	∀A 10
12.	¬[(Pa ∧ Pb) ∧ ∃z (Pz ∧ Raz ∧ Rbz)]	MT 9,11
13.	¬(Pa ∧ Pb) ∨ ¬∃z (Pz ∧ Raz ∧ Rbz)	DM 12
14.	Pa ∧ Pb	KA 7
15.	¬∃z (Pz ∧ Raz ∧ Rbz)	SD 13,14
16.	∀z ¬(Pz ∧ Raz ∧ Rbz)	Def ∃ 15
17.	∀z (¬Pz ∨ ¬Raz ∨ Rbz)	DM 16
18.	Pc ∧ Rca ∧ Rcb	Hip
19.	Pc ∧ Rca	KA 18
20.	Pa	KA 7
21.	Pc ∧ Pa ∧ Rca	KS 19,20
22.	(Pc ∧ Pa ∧ Rca) → Rac	∀A 2
23.	Rac	MP 19, 20
24.	Pc ∧ Rcb	KA 16
25.	Pb	KA 12
26.	Pc ∧ Pb ∧ Rcb	KS 22, 23
27.	∀y [(Pa ∧ Py ∧ Ray) → Rya]	∀A 2
28.	(Pc ∧ Pb ∧ Rcb) → Rbc	∀A 27
29.	Rbc	MP 26, 28
30.	¬Pc ∨ ¬Rac ∨ ¬Rbc	∀A 17
31.	¬Pc ∨ ¬Rac	SD 29, 30
32.	¬Pc	SD 23, 31
33.	Pc	KA 18
34.	Pc ∧ ¬Pc	KS 32, 33
35.	∃x (Px ∧ ¬Px)	∃S 31
36.	∃x (Px ∧ ¬Px)	∃A 8, 18-35
37.	∃x (Px ∧ ¬Px)	∃A 5, 6-36
38.	∃x (Px ∧ ¬Px)	∃A 4, 5-37
39.	∀x∀y[(Px∧Py∧¬Sxy)→¬∃z(Pz∧Rzx∧Rzy)]	Abs,3-34

35.

1.	¬∃x (Px → ∀y Py)	Hip
2.	∀x ¬(Px → ∀y Py)	Def ∃ 1
3.	¬(Pa → ∀y Py)	∀A 2
4.	Pa ∧ ¬∀y Py	Def → 3
5.	¬∀y Py	KA 4
6.	Pa	KA 4
7.	∀y Py	∀S 6
8.	∀y Py ∧ ¬∀y Py	KS 5, 7
9.	∃x (Px → ∀y Py)	Abs. 1-8

36.

1.	Pc	Prem
2.	a=c	Hip
3.	Pa	LL 1,2
4.	a=c → Pa	DT 2-3
5.	∀x (x=c → Px)	∀S 4

37.

1.	∀x (x=c → Px)	Prem
2.	c=c	Err.
3.	c=c → Pc	∀A 1
4.	Pc	MP 3,4

38.

1.	Pc	Prem
2.	c=c	Err.
3.	¬∃x (x=c)	Hip
4.	∀x (x≠c)	Def ∀ 3
5.	c≠c	∀A 4
6.	c=c ∧ c≠c	KS 2,5
7.	∃x x=c	Abs. 3-6
8.	a=c	Hip
9.	Pa	LL 1,8
10.	a=c ∧ Pa	KS 8,9
11.	∃x (x=c ∧ Px)	∃S 10
12.	∃x (x=c ∧ Px)	∃A 7, 8-11

39.

1.	∃x (x=c ∧ Px)	Prem.
2.	¬Pc	Hip
3.	a=c	Hip
4.	¬Pa	LL 2,3
5.	a=c → ¬Pa	DT 3,4
6.	¬(a=c ∧ Pa)	Def → 5
7.	∀x ¬(x=c ∧ Px)	∀S 6
8.	¬∃x (x=c ∧ Px)	Def ∀ 7
9.	∃x (x=c ∧ Px) ∧ ¬∃x (x=c ∧ Px)	KS 1,8
10.	Pc	Abs 2-9

40.

1.	∀x∃y Rxy	Prem
2.	∀x∀y (Rxy → ∀z Rzy)	Prem
3.	∃y Ray	∀A 1
4.	Rab	Hip
5.	∀y (Ray → ∀z Rzy)	∀A 2
6.	Rab → ∀z Rzb	∀A 5
7.	∀z Rzb	MP 4,5
8.	∀x Rxb	AA 6
9.	∃y∀x Rxy	∃S 7
10.	∃y∀x Rxy	∃A 3, 4-8