

3 ZINEMATIKA

$$\vec{\Delta v} = \vec{v}(t + \Delta t) - \vec{v}(t)$$

DESPLAZAMENDUA

$$\vec{v}_{\text{bko}} = \frac{\Delta \vec{r}}{\Delta t} \quad \vec{v} = \lim_{\Delta t \rightarrow 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$$

ABIADURA

$$\vec{a}_{\text{bb}} = \frac{\Delta \vec{v}}{\Delta t} \quad \vec{a} = \lim_{\Delta t \rightarrow 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}$$

AZELERAZIOA

4. ABIADURAREN ETA AZELERAZIOAREN OSAGAIAK INTRINTSEKOAK

$$a_t = \frac{dv}{dt}$$

AZELERAZIO TANGENZIALA

$$a_n = \frac{v^2}{R}$$

AZELERAZIO NORMALA

$$a^2 = a_n^2 + a_t^2$$

AZELERAZIOA

HIGIDURA ZUZENA

- H2U

$$x = x_0 + v \cdot t \quad \begin{cases} v = k \cdot t \\ a = 0 \end{cases}$$

- H.2.U.A

$$v = v_0 + a \cdot t$$

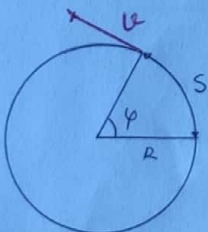
$$x = x_0 + v_0 \cdot t + \frac{1}{2} a \cdot t^2$$

$$v^2 - v_0^2 = 2a \cdot (x - x_0)$$

$$[a = k \cdot t]$$

$$v = \sqrt{v_0^2 + 2gh}$$

HIGIDURA ZIRKULARRA



$$s = \varphi \cdot R$$

$$\omega = \frac{d\varphi}{dt} \quad (\text{rad/s})$$

ABIADURA ANGELARRA

$$\alpha = \frac{d\omega}{dt} \quad (\text{rad/s}^2)$$

AZELERAZIO ANGELARRA

$$v = \omega \cdot R$$

$$[a_t = \alpha \cdot R]$$

$$a_n = \frac{v^2}{R} = \omega^2 \cdot R$$

$$a_t = \frac{(\omega \cdot R)}{t}$$

$$[\vec{a} = \vec{a}_n + \vec{a}_t]$$

H.2.m.k.U

$$\boxed{\varphi = \varphi_0 + \omega \cdot t} \quad [\omega = k\omega \quad \alpha = 0]$$

$$\boxed{s = s_0 + v \cdot t}$$

H.2.m.k.U.A.

$$\boxed{\omega = \omega_0 + \alpha \cdot t}$$

$$\boxed{\varphi = \varphi_0 + \omega_0 t + \frac{1}{2} \alpha \cdot t^2}$$

$$\boxed{\omega^2 - \omega_0^2 = 2\alpha \cdot (\varphi - \varphi_0)} \quad [\alpha = k\alpha]$$

$$\boxed{s = s_0 + v_0 \cdot t + \frac{1}{2} a t^2}$$

6. HIGIDURA ERLATIBODUN ERREFERENTZIA SISTEMAK

$$\left[\begin{array}{l} \vec{r} = \vec{R} + \vec{r}' \\ \vec{v} = \vec{V} + \vec{v}' \\ \vec{a} = \vec{A} + \vec{a}' \end{array} \right]$$

$$\left[\begin{array}{l} v = v' \\ a = a' \end{array} \right]$$

$$\left[\begin{array}{l} x = x' + vt' \\ y = y' \end{array} \right]$$

$$\left[\begin{array}{l} v = v' \\ a = a' \end{array} \right]$$

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$$\left[\begin{array}{l} v = v' \\ a = a' \end{array} \right]$$

$$\left[\begin{array}{l} x = x' + vt' \\ y = y' \end{array} \right]$$

4 MEKANIKA

NEWTONEN LEGEAK

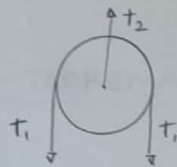
$$\left\{ \begin{array}{l} 1: \text{BALDIN } \vec{F}_{\text{TOT}} = 0 \quad \text{ORDUAN } \vec{v} = kx \\ 2: \vec{F}_{\text{TOT}} = m \cdot \vec{a} \\ 3: \vec{F}_{12} = -\vec{F}_{21} \end{array} \right.$$

3- INDARREN ADIBIDEAK

① PISUA $\vec{P} = m \cdot \vec{g}$

② MALGUKIAREN INDARRAK $F = kx$

③ TENTSIOA SOKETAN



$$T_2 = 2T_1$$

④ INDAR NORMALA



$$N \perp d$$

⑤ MARRUSKA DURA INDARRA

$$F_R \leq \mu \cdot N \quad \left[\begin{array}{l} \text{IRRIST. (=)} \\ \text{EZ. IRR. (<)} \end{array} \right]$$

4- ERREFERENTZIA SISTEMA AZELERATUAK

$$\vec{F}_{\text{TOT}} = m \cdot \vec{a}$$

SISTEMA INERTZIALETAN

$$\vec{F}_{\text{TOT}} - m \vec{A} = m \vec{a} \quad (\vec{A} \text{ inertziazetiko})$$

SISTEMA EZ-INERTZIALA

5] PARTIKULAREN DINAMIKA OROKORRA

1.- HELBURU NAGUSIA

$$\boxed{\vec{v} - \vec{v}_0 = \int_{t_0}^t \frac{\vec{F}}{m} \cdot dt} \quad \boxed{\vec{r} - \vec{r}_0 = \int_{t_0}^t \vec{v} \cdot dt}$$

2.- PARTIKULA BATEN TEOREMAK: MOMENTU LINEALA

$$\boxed{\vec{p} = m \cdot \vec{v} \text{ (kg} \cdot \text{m/s)}} \quad \boxed{\vec{F} = \frac{d\vec{p}}{dt}}$$

MOMENTU LINEALA MOMENTU LINEALAREN TEOREMA

$$\boxed{I = \Delta \vec{p} = \vec{F} \cdot \Delta t} \quad \boxed{\vec{p} \text{ kontserbazioa / } \vec{p} = k t e}$$

IMPULTSOA, BULKADA BALDIN: $\vec{F}_{\text{tot}} = 0$

2.2 MOMENTU ANGELUARRAREN TEOREMA

$$\boxed{\vec{L} = \vec{r} \times \vec{p}} \quad [L = r \cdot p \cdot \sin \theta \rightarrow \vec{v} \parallel \vec{p}]$$

MOMENTU ANGELUARRA $\theta = 0$

- PUNTU JAKIN BATEAN -

$$\boxed{M_0 = \frac{d\vec{L}_0}{dt} = \vec{r} \times \vec{F}_{\text{tot}}}$$

MOMENTU ANGELUARRAREN TMA

- TORTSIO MOMENTUA PUNTU JAKIN BATEAN -

$$[\vec{L} \text{ kontserbazioa / } \vec{L}_0 = k t e]$$

BALDIN: $\vec{M}_0 = 0$

2.3.- PARTIKULA BATEN ENERGIAREN TEOREMA

$$\boxed{dW = \vec{F} \cdot d\vec{r}} \quad \boxed{W = \int_{r_1}^{r_2} \vec{F} \cdot d\vec{r}}$$

INFINITESIMALA FINITUA

$$\boxed{W = \Delta E_z} \quad \boxed{W = P \cdot t}$$

$$\boxed{P = \frac{dW}{dt} = \vec{F} \cdot \vec{v}} \quad (J/s = \text{kg} \cdot \text{m}^2/\text{s}^3 = \text{Watt})$$

POTENTZIA

$$\boxed{P = \frac{dE_z}{dt}}$$

3. POSIZIOAREN MENDERKO INDARRAK

F KONTSERBAKORRA

$$\Delta E_p = -W \quad \left[\Delta E_p = -\int \vec{F} \cdot d\vec{r} = E_p(r_2) - E_p(r_1) \right] \quad [dE_p = \vec{F}_{\text{con}} \cdot d\vec{r}]$$

• MALGUKIEN INDAR ELASTIKOA

$$E_p(x) = \frac{kx^2}{2}$$

$$\Delta E_p = -\int_{x_0}^x kx \cdot dx$$

• GRABITATEA

$$E_p(y) = mg \cdot y$$

$$\Delta E_p = -\int_{y_0}^y m \cdot g \cdot dy$$

- KONTSERBAZIOA -

$$\begin{cases} W_{\text{tot}} = -\Delta E_p \\ W_{\text{tot}} = \Delta E_z \end{cases} \Rightarrow \Delta E_z = -\Delta E_p \quad \begin{cases} \Delta E_z + \Delta E_p = 0 \\ E_z + E_p = K E = E_{\text{MEK}} \end{cases}$$

• BALDIN

$$F_{\text{tot}} = F_k + F_{\text{EK}}$$

$$\Delta E_z = W_{\text{tot}} = W_k + W_{\text{EK}}$$

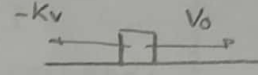
$$\downarrow$$
$$-\Delta E_p$$

$$\Delta E_z + \Delta E_p = W_{\text{EK}} \Rightarrow W_{\text{EK}} = E_{\text{MEK}}$$

6- ABIADURAREN MENPERO INDARRA

$$\vec{F}(\vec{v}) = m \cdot \vec{a}$$

DIMENTSIO BAKARRA $\left\{ \begin{array}{l} -k \cdot v^n = m \cdot \frac{dv}{dt} \end{array} \right.$



INDAR Kte.

$$v = v_0 \cdot e^{-(K/m) \cdot t}$$

$$x = \left(1 - e^{-(K/m)t} \right) \cdot \frac{m v_0}{K}$$

$$\left[\begin{array}{l} t=0 \rightarrow x=0 \\ t=\infty \rightarrow x = \frac{m v_0}{K} \end{array} \right]$$

$$x = - \frac{m}{K} (v - v_0)$$

6 HIGIDURA OSZILAKORRA

1. SARRERA

$$\boxed{f = \frac{1}{T}} \quad (Hz) \quad T(s)$$

$$\boxed{\omega = 2\pi f = \frac{2\pi}{T}}$$

- MAIZTASUN ANGELUARRA -

$$\boxed{F = -kx}$$

- INDAR BERRESKURARAILER -

2. ADIERAZ PEN MATEMATIKOA

$$F = m \cdot a \rightarrow \boxed{-kx = m \cdot a = F \quad [a = \ddot{x}] \quad \ddot{x} = -\frac{k}{m} x}$$

$$\boxed{[H.H.S\text{-}ren \text{ EKUAZIOA}]: \quad \ddot{x} + c'x = 0}$$

• non $\omega_0 = \sqrt{\frac{k}{m}}$

$$\boxed{x = A \cdot \sin(\omega_0 t + \delta)}$$

$$\boxed{\dot{x} = v = A \cdot \omega_0 \cdot \cos(\omega_0 t + \delta)}$$

$$\boxed{\ddot{x} = a = \underbrace{-A \cdot \omega_0^2}_{\max} \cdot \sin(\omega_0 t + \delta)}$$

- HASIERAKO FASEA

$$\boxed{\delta = \arctan\left(\frac{x_0 \cdot \omega_0}{v_0}\right)}$$

- ANPLITUDEA

$$\boxed{A = \frac{\sqrt{v_0^2 + x_0^2 \omega_0^2}}{\omega_0}}$$

- PERIODOA

$$\boxed{T = \frac{2\pi}{\omega_0} = 2\pi \sqrt{\frac{m}{k}}}$$

- Gerak harmonis H.H.S. a.k

1) PENDULUA

$$\begin{cases} \ddot{\theta} + \omega_0^2 \theta = 0 \\ \omega_0 = \sqrt{\frac{g}{l}} \end{cases}$$

2) GERACHINAS

$$\begin{cases} \ddot{x} + \omega_0^2 x = 0 \\ \omega_0 = \sqrt{\frac{k}{m}} \end{cases}$$

- Energia

$$E_{mek} = E_p^{max} = E_k^{max} = \frac{1}{2} m A^2 \omega_0^2 = \frac{1}{2} k A^2$$

$$\begin{cases} E_p = \frac{1}{2} k x^2 \\ E_k = \frac{1}{2} m \dot{x}^2 \end{cases}$$

$$(b + j\omega) \cdot \omega \cdot A = x$$

$$(b + j\omega) \cdot \omega \cdot A = v = \dot{x}$$

$$(b + j\omega) \cdot \omega \cdot A = \ddot{x}$$

$$\frac{m}{2} \dot{x}^2 = \frac{1}{2} k x^2$$

$$\frac{m \dot{x}^2}{2} = \frac{k x^2}{2}$$

$$\left(\frac{m \dot{x}^2}{2} - \frac{k x^2}{2} \right) = 0$$

7 PARTIKULA SISTEMEN DINAMIKA

2. PARTIKULA SISTEMEN TMA-K

MOMENTU LINEALA:

$$\vec{F}_{TOT}^{KAN} = \frac{d\vec{p}_{TOT}}{dt}$$

⊖

$$\vec{F}_{1 \rightarrow 2} + \vec{F}_{2 \rightarrow 1} + \vec{F}_1^{KAN} + \vec{F}_2^{KAN}$$

BARNEKOA

⊖

BALDIN

$$\vec{F}^{KAN} = 0$$

$$\vec{p} = k t e$$

MOMENTU ANGELUARRA:

$$\vec{L} = \vec{r} \times \vec{p} \quad M = \vec{r} \times \vec{F}$$

$$M = \frac{dL}{dt}$$

$$\vec{M}^{BAR} = 0$$

BALDIN

$$\vec{M}^{KAN} = 0$$

$$\vec{L} = k t e$$

$$\vec{M}_1 + \vec{M}_2 = \frac{d(\vec{L}_1 + \vec{L}_2)}{dt}$$

$$\Downarrow$$

$$M_1^{KAN} + M_1^{BAR} + M_2^{KAN} + M_2^{BAR}$$

$$\Downarrow$$

$$\vec{M}^{KAN}$$

ENERGIA:

$$\Delta E_2 = W_{BAR} + W_{KAN}$$

$$\Delta E_2 = W$$

3. TAKKAK

$$\vec{p}_{TOT} = k t e$$

$$v_2' - v_1' = -e(v_2 - v_1)$$

$e =$ ITZULDE KOEF.]

$$m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2'$$

$$W_{BAR} = \frac{1}{2} m_1 v_1'^2 + \frac{1}{2} m_2 v_2'^2 - \frac{1}{2} m_1 v_1^2 - \frac{1}{2} m_2 v_2^2$$

$$\rightarrow \boxed{0} = \text{TALKA ELASTIKOA}$$

- $e = 1$ TALKA ELASTIKOA
- $0 < e < 1$ TALKA INELASTIKOA
- $0 = e$ TALKA PLASTIKOA

$\boxed{< 0}$ = TALKA PARZIALKI INELASTIKOA

$\boxed{v_1' = v_2'}$ = TALKA PLASTIKOA

4. MASA ZENTROA

$$\vec{R}_{M2} = \frac{\sum_{i=1}^n m_i \vec{r}_i}{M}$$

$$\vec{V}_{M2} = \frac{\sum_{i=1}^n m_i \vec{v}_i}{M}$$

$$\vec{F}_{KAN} = \frac{d\vec{P}}{dt} = M \frac{d\vec{V}}{dt} = M \vec{A}_{M2}$$

5. DESKONPOSAKETA

$$\begin{aligned} \vec{P} &= \sum_{i=1}^n m_i \vec{v}_i = M \cdot \vec{V}_{M2} \\ \vec{L} &= \sum_{i=1}^n \vec{r}_i \times m_i \vec{v}_i = \vec{R}_{M2} \times M \vec{V}_{M2} + \vec{L}' \\ E_z &= \sum_{i=1}^n \frac{1}{2} m_i \vec{v}_i^2 = \frac{1}{2} M \cdot \vec{V}_{M2}^2 + E_z' \\ M_{KAN}^{M2} &= \frac{d\vec{L}'}{dt} \end{aligned}$$

8 SOLIDO ZURRUNAREN DINAMIKA (ERROTAZIOA)

2- SOLIDO ZURRUNAREN ESTATIKA

$$\vec{F}_{\text{tot}} = \sum_{i=1}^n \vec{F}_i = 0$$

$$\vec{F}_{\text{KAN}} = M \cdot \vec{A}_{mz} = \frac{d\vec{p}}{dt}$$

$$\begin{cases} \sum F_x = 0 \\ \sum F_y = 0 \\ \sum F_z = 0 \end{cases}$$

$$\vec{M}_{\text{KAN}} = \sum_{i=1}^n \vec{M}_i = 0 \quad \left[\vec{M}_{\text{KAN}} = \frac{dL}{dt} \right]$$

INDAR BATEAN
MOMENTUA PIR
JAKIN BATEAN

$$\vec{M} = \vec{r} \times \vec{F}$$

$$M = F \cdot d$$

3- SOLIDO ZURRUNAREN ERROTAZIOAREN DINAMIKA ARDAZ FINKO BATEAN MODUAN

$$dL_0 = \vec{r} \times d\vec{p}$$

$$L_{0z} = \int r^2 dm = W \cdot I$$

INERTZIA MOMENTUA

$$I = \int r^2 dm \quad (\text{kg} \cdot \text{m}^2)$$

SOLIDOAREN ERROTAZIOA
DINAMIKA EKVAZIOA

$$\vec{M} = I \cdot \vec{\alpha}$$

SIMETRIA BADU EDO \geq ARDAZ. INGURUAN

$$L_x = L_y = 0 \quad \vec{L} = L_z$$

[INERTZI TAUOLA]

4- INERTZIA ARDAZ NAGUSIAK - STEINER TMA

$$I_p > I_{mz}$$

$$I_p = I_{mz} + M \cdot d^2$$

ARDAZ PARALELOEN TMA.

5- ERROTAZIOA, ENERGIA ZUZENAREKIKO

$$E_2 = \frac{1}{2} I_p \omega^2$$

ERROTABEN
DAGGENEAN
(Mz)

$$E_2 = \frac{1}{2} I_p \omega^2$$

ERROTAZIO
HUTSA

$$E_2 = \frac{1}{2} I_{mz} \cdot \omega^2 + \frac{1}{2} M V_{mz}^2$$

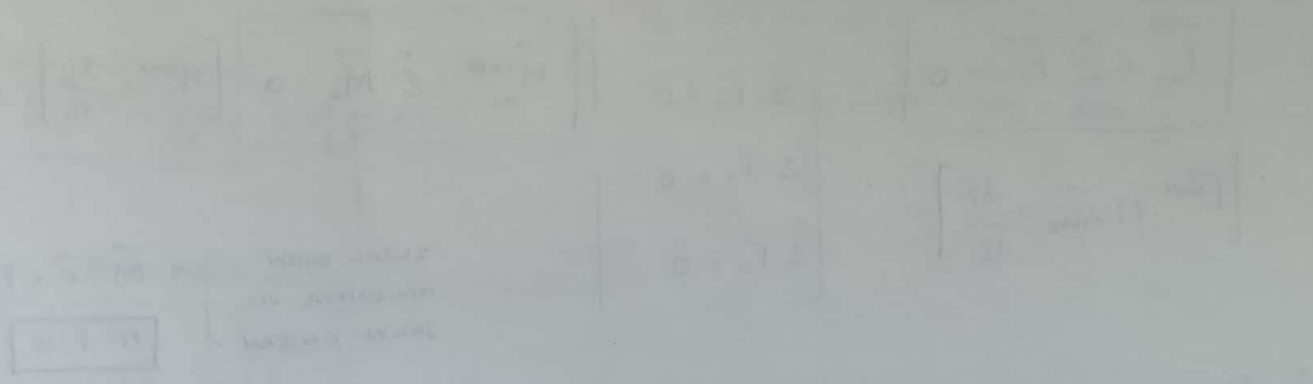
POTENTZIA:

$$P = F \cdot v = M \omega$$

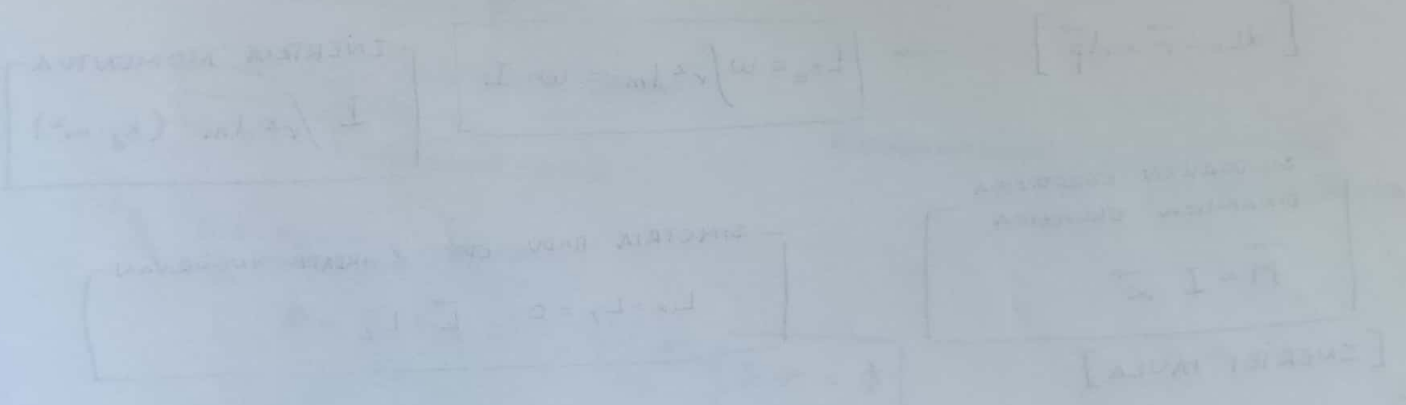
ENERGIA POTENZIALA

$$E_p = g \cdot z_{mz} \cdot M$$

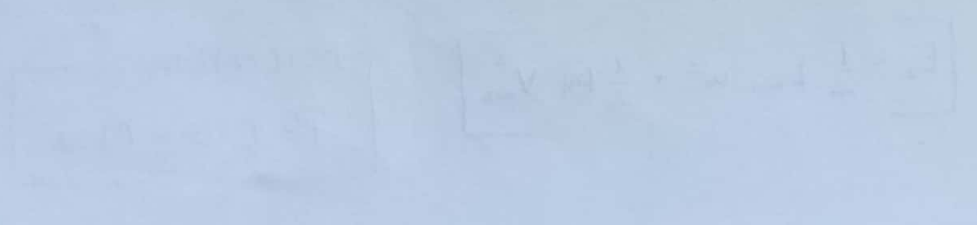
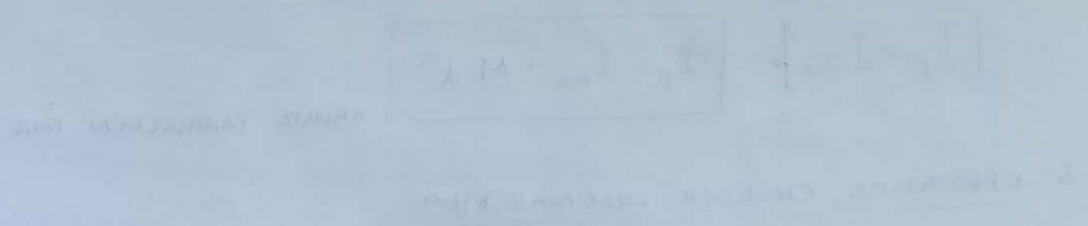
$$z_{mz} = \frac{1}{M} \int z \, dm$$



2. Para determinar el centro de masa de un cuerpo extenso, se divide en elementos diferenciales de masa \$dm\$.



1. Se divide el cuerpo en elementos diferenciales de masa \$dm\$.



10 FLUIDOAK

- DENTSITATEA

$$\rho = \frac{m}{V} \quad \left(\frac{\text{kg}}{\text{m}^3} \right) \quad \left[\text{URA: } 1 \frac{\text{g}}{\text{cm}^3} \right]$$

- PRESIOA

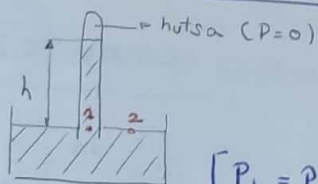
$$d\vec{F} = P \cdot d\vec{s} \quad \left[P = \frac{dF}{ds} \quad \left(\frac{\text{N}}{\text{m}^2} = \text{Pa} \right) \right] \quad \left[P = \frac{F}{S} \right]$$

3- FLUIDOEN ESTATIKA

$$P_{\text{atm}} = 1,013 \cdot 10^5 \text{ Pa} \quad \left[P = P_{\text{atm}} + \rho g z \right]$$

4- APLIKAZIOAK

4.1- MERKURIOZKO BAROMETROA



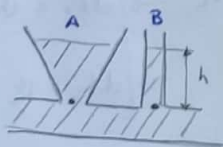
$$[P_1 = P_2]$$

$$P_{\text{atm}} = \rho g h$$

GAIKINAN DUENA DA PRESIOA

$$\left[\begin{array}{l} P_1 = \rho g h \\ P_2 = P_{\text{atm}} \end{array} \right]$$

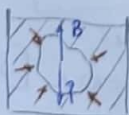
4.2- ONZI KOMUNIKATUAK



$$P_A = P_B$$

$$\Leftrightarrow [h_A = h_B]$$

4.3- ARKIMIDESEAN PRINTZIOA



• LIKIDOA: $[P = B]$

• SOLIDOA: $[B = mg = \rho_s \cdot V \cdot g] \Rightarrow$

$$\left[\begin{array}{l} B < P_s \text{ HONDORATU} \\ B = P_s \text{ OREKAN} \\ B > P_s \text{ GAINAZALERA} \end{array} \right]$$

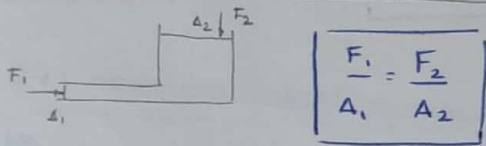
- FLOTAZIOA

- $V_m =$ BOLUMEN MURGILDUA
 - $V =$ BOLUMEN OSOA
- } SOLIDO

- $B' =$ BULTZADA
- OARDEKATUTAKO LINDOAREN PISUA $= V_m \cdot \rho_f \cdot g = B'$
- $P_s =$ SOLIDOAREN PISUA $= V \cdot \rho_s \cdot g$

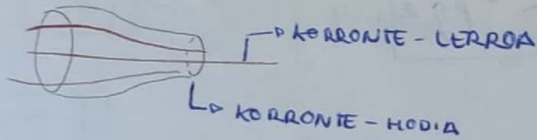
$$V \cdot \rho_s = V_m \cdot \rho_f \Leftrightarrow [B' = P_s]$$

4.4. PASCALEN PRINTZIPIOA



$$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$

5.- FLUIDOEN DINAMIKA



MARRUSKADURA EZ \rightarrow [MORMAK BISKOSITATEA]

5.1. FLUXUA (EMARIA)

$$\phi = \frac{dV}{dt} = \left(\frac{m^3}{s}\right) = \frac{s \cdot v \cdot dz}{dt} = [S \cdot v = \phi]$$

5.2. JARRAITASUN ECUAZIOA



$$dV_1 = S_1 \cdot dl_1 = S_1 \cdot v_1 \cdot dt \rightarrow [dl_1 = v_1 \cdot dt]$$

$$\rightarrow [dl_2 = v_2 \cdot dt]$$

• FLUIDOAKONPRIMAEZINA BADA: $[dv_1 = dv_2]$

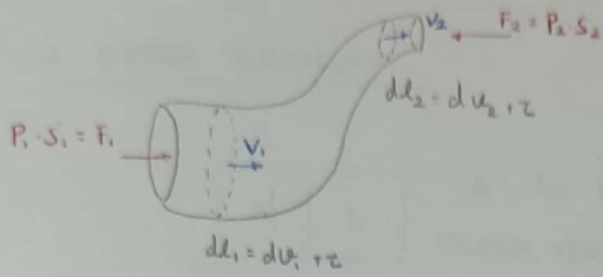
$$S_1 \cdot dl_1 = S_2 \cdot dl_2 \rightarrow S_1 \cdot v_1 \cdot dt = S_2 \cdot v_2 \cdot dt$$

$$[S_1 \cdot v_1 = S_2 \cdot v_2]$$

• KONPRIMAGARRIA BADA: $[dw_1 = dw_2]$

FLUXUAREN KONSERBAZIOA $[Sv = kw]$

S.3.- BERNOULLI-REN EKUAZIOA (ENERGIAREN KONTSERBAZIOA)



$$\Delta E_2 = W_{tot} = W_p + W_a = F_1 dl_1 - F_2 dl_2 + (-\Delta E_p)$$

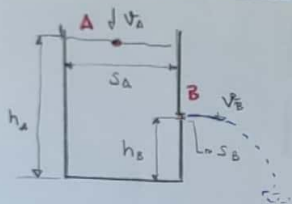
$$\left[\Delta E_2 = \frac{1}{2} dm_2 v_2^2 - \frac{1}{2} dm_1 v_1^2 \right]$$

$$\boxed{P + \frac{1}{2} \rho v^2 + \rho g h = kte} \quad \text{BERNOULLI-REN EKUAZIOA}$$

G.- APLIKAZIOAK

G.1.- TORRICELLI-REN FORMULA

(CONTZIA HUSTU)

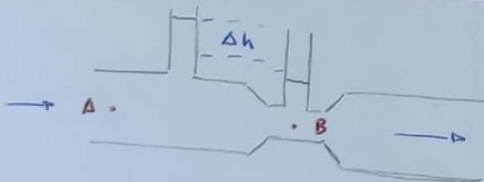


$$\left[\begin{array}{l} v_A \cdot S_A = S_B \cdot v_B \\ P_A = P_B = P_{atm} \\ P + \frac{1}{2} \rho v^2 + \rho g h = kte \Rightarrow v_B^2 = 2g(h_a - h_b) \end{array} \right]$$

$$\boxed{v_B = \sqrt{2gh}}$$

ZULOTXO TXIKIA : $S_B \ll S_A \Rightarrow v_B \gg v_A$

G.2.- VENTURI EFEXTUA

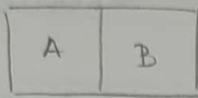


$$\left[\begin{array}{l} S_A \cdot v_A = S_B \cdot v_B \\ P_A - P_B = \rho g \Delta h \\ P + \frac{1}{2} \rho v^2 + \rho g h = kte \end{array} \right] \rightarrow \rho g \Delta h = \frac{1}{2} \rho \left(\frac{S_A^2}{S_B^2} - 1 \right) v_A^2$$

$$\boxed{v_A = \sqrt{\frac{2g \Delta h}{\left(\frac{S_A}{S_B}\right)^2 - 1}}}$$

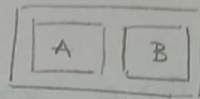
II TERMO: 1 PRT2P.

2.- OREKA TERMIKO A



HORMA DIA TERMIKO A

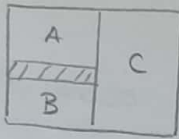
A eta B
OREKA TERMIKO A



HORMA ADIABATIKA

A eta B
E2 OREKA TERMIKOAN

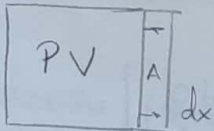
3.- "ZERO" PRINZIPIOA



$\left. \begin{array}{l} A \text{ eta } C \text{ OREKAN} \\ B \text{ eta } C \text{ OREKAN} \end{array} \right\} \Rightarrow A \text{ eta } B \text{ OREKAN}$

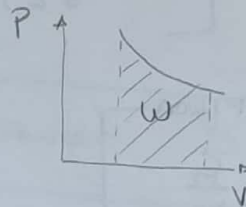
[TEMPERATURA ESKALAK \rightarrow KELVIN]
 [GAS IDEALAK $PV = nRT$]

4.- LAN HIDROSTATIKOA



$$dW = F \cdot dx = P \cdot A \cdot dx = P \cdot dV$$

$$W = \int_{V_h}^{V_a} P dV$$



5.- BARNE ENERGIA

$$E = E_z + E_p$$

MOLEKULA

$$U = \sum_{i=1}^n E_{z_i}$$

MOLEKULA
GUSTIEN
BATURA

MOLEKULEN HIGIDURA ASKATASUN GRADU BAKOITZEKO

$$U = \frac{3}{2} nRT$$

(MONOATOMIKO) [3 askatasun gradu]

$$U = \frac{5}{2} nRT$$

(DIATOMIKO) [5 askatasun gradu]

6- TERMODINAMIKAREN 1. PRINTZIPIOA

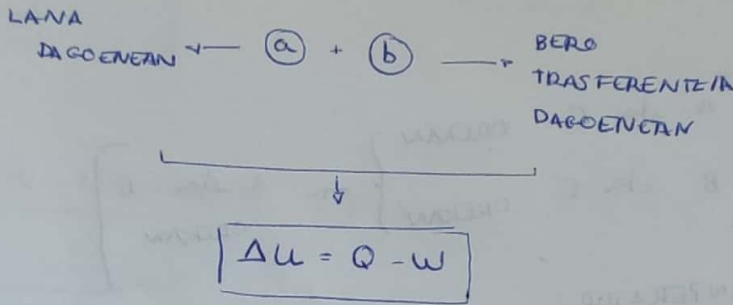
a) $[\Delta U = -W]$

BERO TRASFERENDIARIK EZ \Rightarrow NORMA ADIABATIKOAK

b) $[\Delta U = Q]$

$Q > 0$ BEROA XURGATU

$Q < 0$ BEROA EMAN



7- BERO AHALMENA

$$C = \frac{Q}{T_c - T_h} = \frac{Q}{\Delta T}$$

→

$$\frac{dQ}{dT} = C$$

$\left(\frac{J}{K}\right) \left(\frac{cal}{K}\right)$

(A) [V kte] $C_v = \left(\frac{dQ}{dT}\right)$ (B) [P kte] $C_p = \left(\frac{dQ}{dT}\right)$

- BERO ESPEZIFIKOA

$$C = \frac{\dot{Q}}{m} \quad C = \frac{\dot{Q}}{h \text{ kcal}}$$

(A) [V kte]

$$C_v = \frac{3}{2} nR \quad (\text{MONOATOMIKOA})$$

$$C_v = \frac{5}{2} nR \quad (\text{DIATOMIKOA})$$

(B) [P kte]

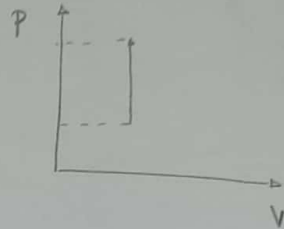
$$C_p = \frac{5}{2} nR \quad (\text{MONOATOMIKOA})$$

$$C_p = \frac{7}{2} nR \quad (\text{DIATOMIKOA})$$

8. PROZESU KUASIESTATIKOAK GAS IDEALETAN

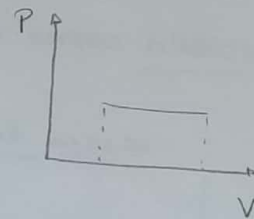
PROZESU ISOKOROAK (V Kte)

$$\left. \begin{array}{l} Q = C_v \cdot \Delta T \\ \Delta U = C_v \cdot \Delta T \\ W = 0 \end{array} \right\} \Delta U = Q - W \quad \boxed{\Delta U = Q}$$



PROZESU ISOBAROAK (P Kte)

$$\left. \begin{array}{l} W = P \cdot \Delta V \\ Q = C_p \cdot \Delta T \\ \Delta U = C_v \cdot \Delta T \end{array} \right\} \begin{array}{l} \Delta U = Q - W \\ C_p = C_v + nR \end{array}$$

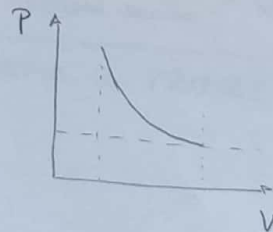


$\Delta V > 0$ ESPANTSIO $\Delta V < 0$ KONPRESIO

PROZESU ISOTERMIAKOAK (T Kte)

$$\left. \begin{array}{l} \Delta U = 0 \\ W = nRT \ln \left(\frac{V_a}{V_h} \right) \\ Q = nRT \ln \left(\frac{V_a}{V_h} \right) \end{array} \right\}$$

$\frac{V_a}{V_h} > 1$ ESPANTSIO $\frac{V_a}{V_h} < 1$ KONPRESIO

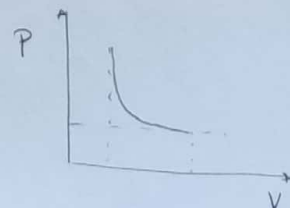


PROZESU ADIABATIKOAK

$$\left. \begin{array}{l} Q = 0 \\ \delta W = \frac{nRT}{V} dV \\ W = -C_v \Delta T \\ \Delta U = C_v \Delta T \end{array} \right\}$$

$PV^\gamma = kTe$ INDIZE ADIABATIKO

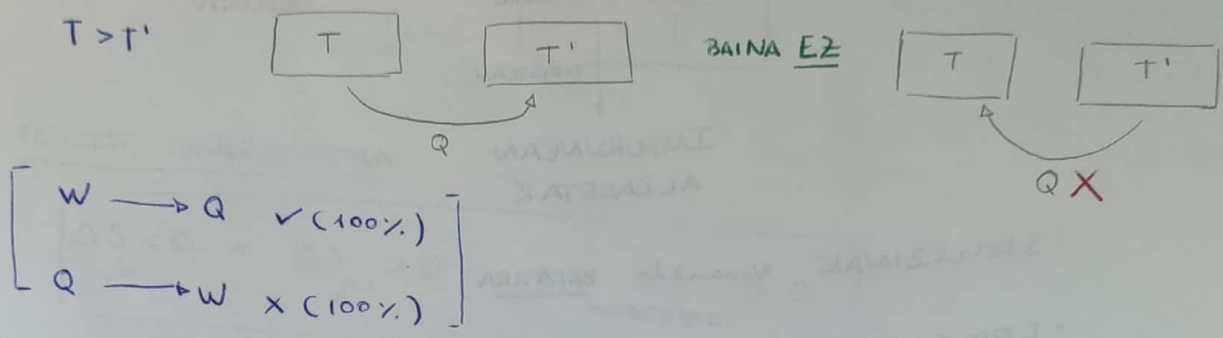
$\gamma = \frac{C_p}{C_v}$ $\gamma = 5/3$ (MONO) $\gamma = 7/5$ (DIAT.)



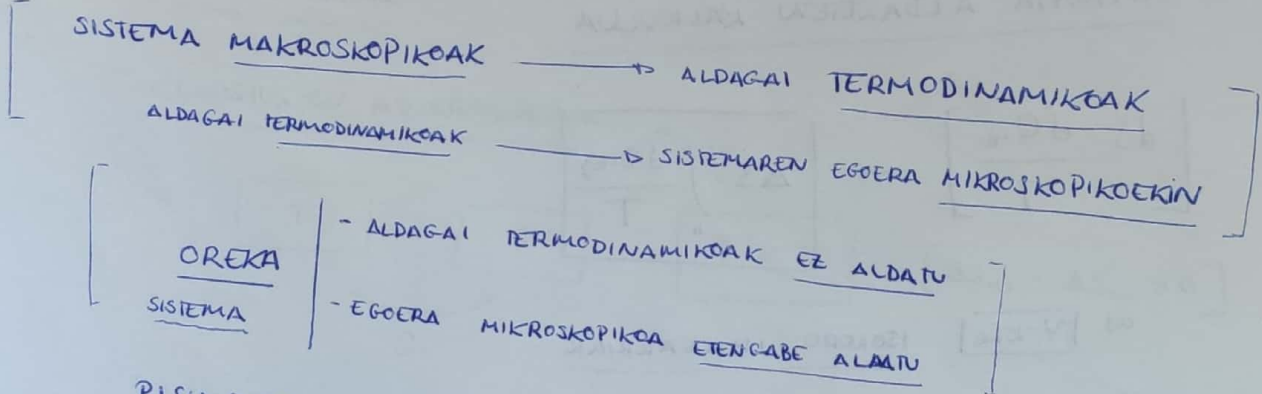
$\Delta U > 0$ $\Delta T > 0$

[12] TERMO. 2. PRINTZIPIOA

- 1. PRINTZIPIOA



2. ENTROPIA: 2. PRINTZIPIOA



PISU ESTATIKOA Ω : $\Omega = \Omega(V, T)$ [EGOERA FUNTZIO]

"EGOERA MAKROSKOPIKO BATEN ZENBAT EGOERA MIKROSKOPIKO"

ENTROPIA: $S = k_B \cdot \ln \Omega$

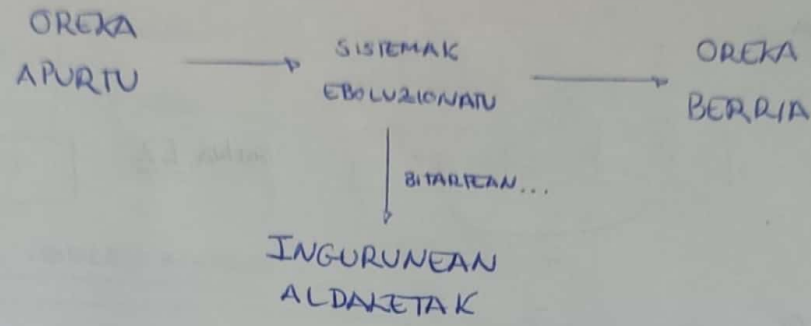
\rightarrow BOLZMAN kte

$\Delta S_{\text{SISTEMA ISOLATUA}} > 0$

TERMO: 2. PRNTZP.

- $\Delta S > 0 \rightarrow$ PROZESU POSIBLEA
- $\Delta S < 0 \rightarrow$ PROZESUA EZ POSIBLE

3- PROZESU IZULGARRIAK ETA IZULEZINAK



- IZULEZINAK: Nonantzko BAKARRA
- IZULGARRIAK: BI Nonantzko

4- ENTROPIA ALDAKETA KALKULUA

$$\left[ds = \frac{dQ_{irz.}}{T} \right] \rightarrow \left[\Delta S = \int_h^a \frac{dQ_{irz.}}{T} \right]$$

a) $V = kTe$ ISOKORO IZULGARRIAK

XURGATUTAKO BERDA $\left[dQ_{irz.} = C_v dt \right] \rightarrow \left[\Delta S = C_v \ln \frac{T_a}{T_h} \right]$

b) $P = kTe$ ISOBARO IZULGARRIAK

XURGATUTAKO BERDA $\left[dQ_{irz.} = C_p dt \right] \rightarrow \left[\Delta S = C_p \ln \frac{T_a}{T_h} \right]$

c) $T = kTe$ ISOTERMO IZULGARRIAK

$$\left[dQ_{irz.} = nR T \frac{dV}{V} \right] \rightarrow \left[\Delta S = nR \ln \frac{V_a}{V_h} \right]$$

d) ADIABATIKO IZULGARRIAK

$$\left[dQ_{irz.} = 0 \right] \quad \left[\Delta S = 0 \right]$$

e) FOKU BAT

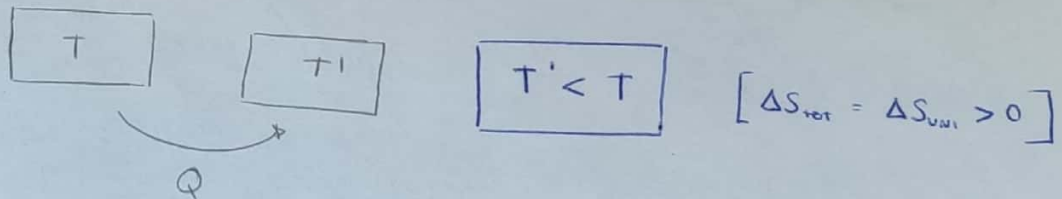
$$C \rightarrow \infty \begin{bmatrix} -Q \text{ aldatu} \\ -T = kTe \end{bmatrix} \Rightarrow \boxed{\Delta S = \frac{Q_{\text{fokv}}}{T}}$$

- PROZESU IRUZELINETAN

$$\boxed{\Delta S_{\text{fokv}} < 0 + \Delta S_{\text{gas}} > 0 = \Delta S_{\text{UNIBERTSOA}} > 0}$$

5: BESTE ADIERAZPENAK

- CLAUSIUS-EN ADIERAZPENA:



- KELVIN PLANK-EN ADIERAZPENA

$$\boxed{\Delta S_{\text{tot}} = \Delta S_T = \frac{Q_{\text{xur}}}{T} = \frac{-Q}{T} < 0} \quad \text{BATERAZINA!}$$

6: MOTORE TERMIKOAK

$$\boxed{\Delta U_{\text{motor}} = Q_{\text{xur}} - Q'_{\text{eman}} - W_{\text{transfer}} = 0} \quad \boxed{W = Q - Q'}$$

- ETERINA

$$\boxed{\eta = \frac{W}{Q_{\text{xur}}} \leq 1}$$

- CARNOT

$$\boxed{\eta_{\text{max}} = 1 - \frac{T'}{T}}$$

