1.1 PROBABILITATEA

$P = \frac{A.K}{K.P}$ $\quad P(\text{gutxienez } 1) = 1 - P(\text{ezertxo bat})$

$P(A \cap B) = P(A) \cdot P(B)$

$P(A \cap B) = P(A) \cdot P(B/A)$

$P(A \cup B) = P(A) + P(B) [- P(A \cap B)]$

$P(A/B) = \frac{P(A \cap B)}{P(B)}$

$P(B) = P(A \cap B) + P(A^c \cap B)$

$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A) \cdot P(B/A)}{P(B)}$

2 ZORIZKO ALDAGAIA DIMENTSIO 1-n

- BANAKETA DISKRETUAK:

· PROBABILITATE FUNTZIOA: $P(x) \begin{cases} 0 \le P(x) \le 1 \\ \sum P(x_i) = 1 \end{cases}$

· BANAKETA FUNTZIOA: $F(x) \begin{cases} F(-\infty) = 0 \\ F(\infty) = 1 \end{cases}$

$$\begin{bmatrix} P(a < x < b) = F(b) - F(a) - P(b) \\ P(a \le x \le b) = F(b) - F(a) + P(a) \\ P(a \le x < b) = F(b) - F(a) + P(a) - P(b) \\ P(a < x \le b) = F(b) - F(a) \end{bmatrix}$$

- BANAKETA JARRAITUAK:

· DENTSITATE FUNTZIOA: $f(x) \begin{cases} f(x) \ge 0 \\ \int_{-\infty}^{\infty} f(x)\,dx = 1 \end{cases}$

· BANAKETA FUNTZIOA: $F(x) \begin{cases} F(-\infty) = 0 \\ F(\infty) = 1 \\ F(x) = \int_{-\infty}^{x} f(x)dx \end{cases}$

$$\begin{bmatrix} P(x < a) = P(x \le a) = F(a) = \int_{-\infty}^{a} f(x)\,dx \\ P(x = a) = \emptyset \end{bmatrix}$$

- BANAKETA JARRAITU ETA UNIFORMEA

$f(x) = \begin{cases} \frac{1}{b-a} & a \le x \le b \\ \emptyset & b.k. \end{cases}$ $\quad m_x = \frac{a+b}{2}$

$\sigma_x^2 = \frac{(b-a)^2}{12}$

3 ZORIZKO ALDAGAIA 2 DIMENTSIOTAN

- DENTSITATE FUNTZIOA: $\begin{cases} f(x,y) \ge \emptyset \\ \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\,dx\,dy = 1 \end{cases}$

- BANAKETA FUNTZIOA: $F(x,y) = \int_{-\infty}^{x}\int_{-\infty}^{y} f(x,y)\,dx\,dy$

- BANAKETA MARGINALAK:

$f(x) = \int_{-\infty}^{\infty} f(x,y)\,dy$ (→ LIMS Y) $\quad f(y) = \int_{-\infty}^{\infty} f(x,y)\,dx$ (→ LIMS X)

- BANAKETA BALDINTZATUAK:

$f(x/y) = \frac{f(xy)}{f(y)}$ $\quad f(y/x) = \frac{f(xy)}{f(x)}$

- INDEPENDENTZIA

$f(xy) = f(x)\,f(y)$ $\quad F(xy) = F(x)\,F(y)$

4 MULTZOEN TEOREMA

GUZTIAK	ORDENAREKIN	ORDENIK GABE
$P_m = m!$	$V_{n,m} = \frac{n!}{(n-m)!}$	$C_{n,m} = \binom{n}{m} = \frac{n!}{m!(n-m)!}$
$PR_n^{r_1,r_2,r_3} = \frac{n!}{r_1!\,r_2!\,r_3!}$	$VR_{n,m} = n^m$	$CR_{n,m} = \binom{n+m-1}{m} = \frac{(n+m-1)!}{m!(n-1)!}$

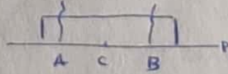
5 MOMENTUAK

$m = E[x] \begin{cases} \sum x_i P(x_i) \\ \int_{-\infty}^{\infty} x f(x)\,dx \end{cases}$ $\quad \sigma^2 \begin{cases} \sum (x - m_x)^2 P(x) \\ \int_{-\infty}^{\infty} (x - m_x)^2 f(x)\,dx \end{cases} \rightarrow \begin{bmatrix} \sigma^2 = E[x^2] - (E[x])^2 \\ \sigma^2 = E[x^2] - m_x^2 \end{bmatrix}$ $\begin{bmatrix} \alpha_1 = E[x] = m \\ \alpha_{2,m} = \sigma^2 \end{bmatrix}$

$\alpha_{k,c} = E[x-c]^k \begin{cases} \sum (x_i - c)^k P(x_i) \\ \int_{-\infty}^{\infty} (x-c)^k f(x)\,dx \end{cases}$ $\quad \alpha_{k,0} = \alpha_k = E[x^k] \begin{cases} \sum x^k P(x_i) \\ \int_{-\infty}^{\infty} x^k f(x)\,dx \end{cases}$

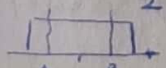
· STEINER: $\alpha_{2,c} = \sigma^2 + (m-c)^2$

· TCHEBYSCHEV:



$\begin{bmatrix} P(x \in \text{barruan}) \ge 1 - 1/K^2 \\ P(x \in \text{kanpoan}) \le 1/K^2 \end{bmatrix}$

1) $C = \frac{A+B}{2}$ $\quad$ 2) $C = m$?



$A = C - K\sqrt{\alpha_{2,c}}$

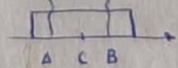
$B = C + K\sqrt{\alpha_{2,c}}$

$C \neq m$ bada

3) $\alpha_{2,c} = \sigma^2 + (m-c)^2$

4) K

5) $P(x \in \text{barruan})$ edo $P(x \in \text{kanpoan})$



$A = m - K\sigma$

$B = m + K\sigma$

$C = m$ bada

3) K

4) $P(x \in \text{barruan})$ edo $P(x \in \text{kanpoan})$

KASU BEREZIA

- Prob eman
- tartea eskatu
- $\boxed{C = m}$ suposatu

6 BANAKETA DISKRETUAK

- BINOMIALA: $x \sim B(n, p)$ $[q = 1-p]$

$$P(x) = \binom{n}{x} p^x q^{n-x} \qquad m = np \qquad \sigma^2 = npq$$

- KONBOLUZIOA: $x \sim B(n_x, p)$ $y \sim B(n_y, p)$

$$w = x + y \Rightarrow [w \sim B(n_x + n_y, p)]$$

- POISSON: $x \sim P(\lambda)$ $P(x) = \dfrac{e^{-\lambda} \cdot \lambda^x}{x!}$

$m = \lambda$ $\sigma^2 = \lambda$

- KONBOLUZIOA: $x \sim P(\lambda_x)$ $y \sim P(\lambda_y)$

$$w = x + y \Rightarrow [w \sim P(\lambda_x + \lambda_y)]$$

- POISSONI ELKARTUTAKO BANAKETAK

·EXPONENTZIALA: (JARRAITUA)

[gertaketa denbora, ez dauka memoriarik]

$$x \sim e(a) \quad \begin{cases} f(x) = a\, e^{-ax} \\ F(x) = 1 - e^{-ax} \end{cases} \quad \boxed{a = 1/\lambda} \quad \left[\sigma^2 = \frac{1}{a^2}\right]$$

7 BANAKETA JARRAITUAK

- NORMALA: $x \sim N(m, \sigma)$ $[\sigma = \sqrt{\sigma^2}]$

$E(x) = m$ $Var(x) = \sigma^2$ $[x = m + \sigma z]$ — TIPIFIKATU —

1.- $y = ax + b \Rightarrow y \sim N(am_x + b, a\sigma_x)$

2.- $x \sim N(m_x, \sigma_x)$ $y \sim N(m_y, \sigma_y)$

$w = x - y$ $w \sim N(m_x - m_y, \sqrt{\sigma_x^2 + \sigma_y^2})$

8 LIMITE ZENTRALAREN TEOREMA

- BINOMIAL-POISSON: $[n \geq 20 \quad p \leq 0{,}05]$

$$x \sim B(n, p) \longrightarrow x \sim P(np)$$

- POISSON-NORMAL: $\left[\begin{matrix} n \geq 100 \text{ l.z.t.} \\ \lambda > 5 \end{matrix}\right]$

$$x \sim P(\lambda) \longrightarrow x \sim N(\lambda, \sqrt{\lambda})$$

- BINOMIAL-NORMAL: $\left[\begin{matrix} n \geq 100 \text{ l.z.t.} \\ np > 5 \text{ eta } nq > 5 \end{matrix}\right]$

$$x \sim B(n, p) \longrightarrow x \sim N(np, \sqrt{npq})$$

$$\boxed{\begin{matrix} P_B \rightarrow P_N \\ P_P \rightarrow \end{matrix} \quad \pm 0{,}5}$$

9 LAGINKETA TEORIA

$$\bar{x} = \frac{\sum_{i=0}^{n} x_i}{n} \qquad S^2 = \frac{\sum_{i=0}^{n} (\bar{x} - x_i)^2}{n} \qquad S = \sqrt{S^2}$$

$$f = \frac{x}{n}$$

- LAGIN-MEDIAREN BANAKETA: (BATAZBESTEKOAREN GAIN INTERFERENTZIAK)

a) σ^2 EZAGUNA

$$\bar{x} \sim N(m, \sigma/\sqrt{n}) \quad [\bar{x} = m + \frac{\sigma}{\sqrt{n}} z]$$

b) σ^2 EZEZAGUNA

1.- $n > 30$

$$\bar{x} \sim N(m, S/\sqrt{n}) \quad [\bar{x} = m + \frac{S}{\sqrt{n}} z]$$

2.- $n \leq 30$

$$\bar{x} \sim t(n-1) \quad [\bar{x} = m + \frac{S}{\sqrt{n-1}} t]$$

- LAGIN MEDIA-KONPARAKETAREN BANAKETA: (σ^2 berdina)

$$t = \frac{(\bar{x} - \bar{y}) - (m_x - m_y)}{\sqrt{\frac{1}{n_x} + \frac{1}{n_y}} \sqrt{\frac{n_x S_x^2 + n_y S_y^2}{n_x - n_y - 2}}} \qquad t \sim t(n_x + n_y - 2)$$

$$\boxed{F_\alpha(a, b) = \frac{1}{F_{1-\alpha}(b, a)}}$$

(BARIANTZAREN GAIN INTERFERENTZIAK)

· LAGIN-BARIANTZAREN BANAKETA

$$w = \frac{n S^2}{\sigma^2} \qquad w \sim \chi(n-1)$$

· LAGIN-BARIANTZEN KONPARAKETAREN BANAKETA

$$F = \frac{n_x S_x^2 (n_y - 1)\, \sigma_y^2}{n_y S_y^2 (n_x - 1)\, \sigma_x^2} \qquad F \sim F(n_x - 1, n_y - 1)$$

11.1 ERRORE MOTAK

COGEMOS DATO DEL CONTRASTE

$$P(\text{1. ERROREA}) = P(H_0 \text{ baztertu} / H_0 \text{ beteta}) = \alpha$$

$$P(\text{2. ERROREA}) = P(H_0 \text{ onartu} / H_0 \text{ EZ bete}) = \beta$$

COGEMOS NUEVO DATO

FROGAREN POTENTZIA: $1 - \beta$

10 KONFIANTZA TARTEEN BIDEZ BALIOZTAPENA

- MEDIA:

- σ^2 EZAGUNA: $P(\bar{x} - z_{\alpha/2}\, \sigma/\sqrt{n} < m < \bar{x} + z_{\alpha/2}\, \sigma/\sqrt{n}) = 1-\alpha$
- σ^2 EZEZAGUNA: $\boxed{n<30}$ $P(\bar{x} - t_{\alpha/2}\, S/\sqrt{n-1} < m < \bar{x} + t_{\alpha/2}\, S/\sqrt{n-1}) = 1-\alpha$ $[t - t(n-1)]$

  $\boxed{n \geq 30}$ $P(\bar{x} - z_{\alpha/2}\, S/\sqrt{n} < m < \bar{x} + z_{\alpha/2}\, S/\sqrt{n}) = 1-\alpha$

- MEDIA DIFERENTZIA:
  - σ^2 EZAGUNAK: $P\left((\bar{x}-\bar{y}) - z_{\alpha/2}\sqrt{\frac{\sigma_x^2}{n_x}+\frac{\sigma_y^2}{n_y}} < m_x - m_y < (\bar{x}-\bar{y}) + z_{\alpha/2}\sqrt{\frac{\sigma_x^2}{n_x}+\frac{\sigma_y^2}{n_y}}\right) = 1-\alpha$
  - σ^2 EZEZAGUNAK, BAINA BERDINAK:

    $$P\left((\bar{x}-\bar{y}) - t_{\alpha/2}\sqrt{\frac{n_x S_x^2 + n_y S_y^2}{n_x+n_y-2}}\sqrt{\frac{1}{n_x}+\frac{1}{n_y}} < m_x - m_y < (\bar{x}-\bar{y}) + t_{\alpha/2}\sqrt{\frac{n_x S_x^2 + n_y S_y^2}{n_x+n_y-2}}\sqrt{\frac{1}{n_x}+\frac{1}{n_y}}\right) = 1-\alpha$$
  - σ^2 BAT EZEZAGUNA BESTEA EZ:

    $$P\left((\bar{x}-\bar{y}) - z_{\alpha/2}\sqrt{\frac{S_x^2}{n_x}+\frac{S_y^2}{n_y}} < m_x - m_y < (\bar{x}-\bar{y}) + z_{\alpha/2}\sqrt{\frac{S_x^2}{n_x}+\frac{S_y^2}{n_y}}\right) = 1-\alpha$$
  - DIFERENTZIAREN MEDIA:

    $m_d = m_x - m_y$ $\quad \bar{d} = \frac{\sum_{i=0}^{n} d_i}{n}$ $\quad S_d^2 = \frac{\sum_{i=0}^{n} (d_i - \bar{d})^2}{n}$

    $$P\left(\bar{d} - z_{\alpha/2}\frac{S_d}{\sqrt{n}} < m_x - m_y < \bar{d} + z_{\alpha/2}\frac{S_d}{\sqrt{n}}\right) = 1-\alpha$$

- BARIANTZA

- $P\left(\frac{nS^2}{\chi^2_{\alpha/2}} < \sigma^2 < \frac{nS^2}{\chi^2_{1-\alpha/2}}\right) = 1-\alpha$ $\quad [\chi^2 \sim \chi^2(n-1)]$
- $P\left(\frac{n_y S_y^2 (n_x-1)}{n_x S_x^2 (n_y-1)} F_{1-\alpha/2} < \frac{\sigma_y^2}{\sigma_x^2} < \frac{n_y S_y^2 (n_x-1)}{n_x S_x^2 (n_y-1)} F_{\alpha/2}\right) = 1-\alpha$

  $[F \sim F((n_x-1),(n_y-1))]$

- PROPORTZIOA

- KOLEKTIBO BINOMIALA: $P\left(f - z_{\alpha/2}\sqrt{\frac{f(1-f)}{n}} < p < f + z_{\alpha/2}\sqrt{\frac{f(1-f)}{n}}\right) = 1-\alpha$ $\quad [f = \frac{x}{n}]$
- PROPORTZIO DIF., K.B. $\rightarrow$ tendentzia NORMALA:

  $$P\left((f_x - f_y) + z_{\alpha/2}\sqrt{\frac{f_x(1-f_x)}{n_x}+\frac{f_y(1-f_y)}{n_y}} < p_x - p_y < (f_x - f_y) - z_{\alpha/2}\sqrt{\frac{f_x(1-f_x)}{n}+\frac{f_y(1-f_y)}{n}}\right) = 1-\alpha$$

- TASA

$$P\left(\bar{x} - z_{\alpha/2}\sqrt{\frac{\bar{x}}{n}} < \lambda < \bar{x} + z_{\alpha/2}\sqrt{\frac{\bar{x}}{n}}\right) = 1-\alpha$$

III ALDERAKETA-HIPOTESIA

- MEDIA:

• σ^2 EZAGUNA: $\boxed{n > 30}$ $z = \dfrac{\bar{x} - m}{\sigma/\sqrt{n}}$ $Ha \begin{cases} m > m_0 \\ m < m_0 \\ m \neq m_0 \end{cases} \rightarrow \begin{cases} z > z_\alpha \\ z < z_{1-\alpha} \\ |z| > z_{\alpha/2} \end{cases}$

$\boxed{n < 30}$ $t = \dfrac{\bar{x} - m}{\sigma/\sqrt{n-1}}$ $[t(n-1)]$ $Ha \begin{cases} m > m_0 \\ m < m_0 \\ m \neq m_0 \end{cases} \rightarrow \begin{cases} t > t_\alpha \\ t < t_{1-\alpha} \\ |t| > t_{\alpha/2} \end{cases}$

• σ^2 EZEZAGUNA: $t = \dfrac{\bar{x} - m}{s/\sqrt{n-1}}$ $[t(n-1)]$ $Ha \begin{cases} m > m_0 \\ m < m_0 \\ m \neq m_0 \end{cases} \rightarrow \begin{cases} t > t_\alpha \\ t < t_{1-\alpha} \\ |t| > t_{\alpha/2} \end{cases}$

• MEDIA - DIF.:

- σ^2 EZAGUNAK: $z = \dfrac{(\bar{x}-\bar{y}) - (m_x - m_y)}{\sqrt{\dfrac{\sigma_x^2}{n_x} + \dfrac{\sigma_y^2}{n_y}}}$ $Ha \begin{cases} m_x - m_y > d_0 \\ m_x - m_y < d_0 \\ m_x - m_y \neq d_0 \end{cases} \rightarrow \begin{cases} z > z_\alpha \\ z < z_{1-\alpha} \\ |z| > z_{\alpha/2} \end{cases}$

- σ^2 EZEZAGUNAK, BERDINAK: $t = \dfrac{(\bar{x}-\bar{y}) - (m_x - m_y)}{\sqrt{\dfrac{n_x s_x^2 + n_y s_y^2}{n_x + n_y - 2}}\sqrt{\dfrac{1}{n_x} + \dfrac{1}{n_y}}}$ $[t(n_x + n_y - 2)]$

$Ha \begin{cases} m_x - m_y > d_0 \\ m_x - m_y < d_0 \\ m_x - m_y \neq d_0 \end{cases} \rightarrow \begin{cases} t > t_\alpha \\ t < t_{1-\alpha} \\ |t| > t_{\alpha/2} \end{cases}$

- DIFERENTZIAREN MEDIA: σ^2 EZEZAGUNAK

$t = \dfrac{\bar{d} - d_0}{S_d/\sqrt{n-1}}$ $[t(n-1)]$ $Ha \begin{cases} d > d_0 \\ d < d_0 \\ d \neq d_0 \end{cases} \rightarrow \begin{cases} t > t_\alpha \\ t < t_{1-\alpha} \\ |t| > t_{\alpha/2} \end{cases}$

- BARIANTZA:

- $\chi^2 = \dfrac{nS^2}{\sigma^2}$ $\chi^2(n-1)$ $Ha \begin{cases} \sigma^2 > \sigma_0 \\ \sigma^2 < \sigma_0 \\ \sigma^2 \neq \sigma_0 \end{cases} \rightarrow \begin{cases} \chi^2 > \chi^2_\alpha \\ \chi^2 < \chi^2_{1-\alpha} \\ |\chi^2| > \chi^2_{\alpha/2} \end{cases}$

- DIF.: $F = \dfrac{n_x S_x^2 (n_y - 1)}{n_y S_y^2 (n_x - 1)} \sim F[(n_x - 1), (n_y - 1)]$ $Ha \begin{cases} \frac{\sigma_x^2}{\sigma_y^2} > 1 \\ \frac{\sigma_x^2}{\sigma_y^2} < 1 \\ \frac{\sigma_x^2}{\sigma_y^2} \neq 1 \end{cases} \rightarrow \begin{cases} F > F_\alpha \\ F < F_{1-\alpha} \\ |F| > F_{\alpha/2} \end{cases}$

- PROPORTZIOA:

- BINOMIAL: $[np_0 > 10 \quad n(1-p_0) > 10]$ $z = \dfrac{f - p_0}{\sqrt{p_0 q_0 / n}}$ $[f = \frac{x}{n}]$ $Ha \begin{cases} p > p_0 \\ p < p_0 \\ p \neq p_0 \end{cases} \rightarrow \begin{cases} z > z_\alpha \\ z < z_{1-\alpha} \\ |z| > z_{\alpha/2} \end{cases}$

- DIF.: $z = \dfrac{(f_x - f_y) - d_0}{\sqrt{\dfrac{f_x(1-f_x)}{n_x} + \dfrac{f_y(1-f_y)}{n_y}}}$ $[n_x p_x > 5 \quad n_x(1-p_x) > 5]$ $Ha \begin{cases} p_x - p_y > d_0 \\ p_x - p_y < d_0 \\ p_x - p_y \neq d_0 \end{cases} \rightarrow \begin{cases} z > z_\alpha \\ z < z_{1-\alpha} \\ |z| > z_{\alpha/2} \end{cases}$

- TASA

- POISSON: $z = \dfrac{\bar{x} - \lambda_0}{\sqrt{\bar{x}/n}}$ $Ha \begin{cases} \lambda > \lambda_0 \\ \lambda < \lambda_0 \\ \lambda \neq \lambda_0 \end{cases} \rightarrow \begin{cases} z > z_\alpha \\ z < z_{1-\alpha} \\ |z| > z_{\alpha/2} \end{cases}$