CHAPTER 9. INCOMPRESSIBLE VISCOUS FLOW

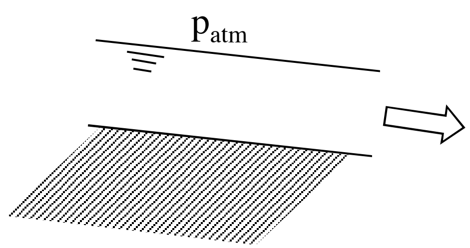
1. Introduction
2. Equations of fluid flow
3. Energy loss (headloss)
4. Energy head diagram
5. Pipe characterization
6. Fluid transport through open channels
7. General steady-state non uniform motion equation in open flows
8. Steady-state uniform motion in open flows

Gustavo A. Esteban - 2016

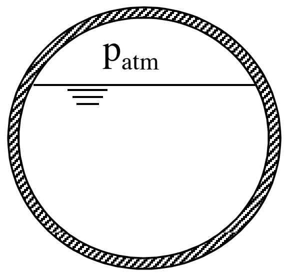
1. Introduction

✓ Different forms of fluid transport

- Open-channel flow



CHANNEL

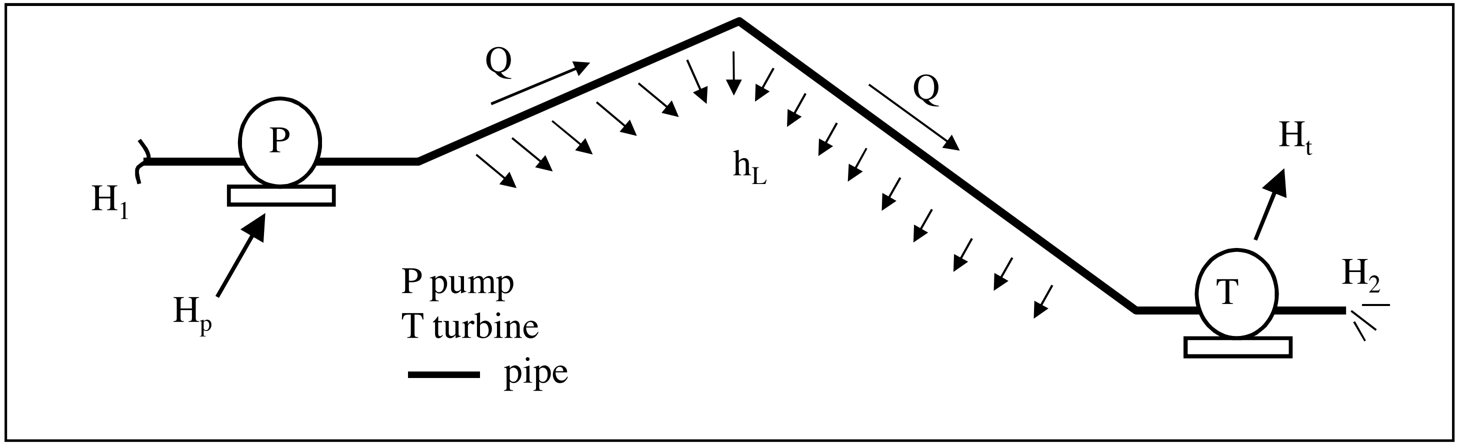


PARTIALLY FILLED PIPE
(sewage water)

- Pressurized transport

2. Equations of fluid flow: transport by gravity and by pumping

- ✓ General equations



- Mechanical energy equation

$$z_1 + \frac{p_1}{\gamma} + \alpha_1 \frac{U_1^{\,2}}{2g} + H_p = z_2 + \frac{p_2}{\gamma} + \alpha_2 \frac{U_2^{\,2}}{2g} + H_t + h_L$$

$$\alpha = \frac{\iint_A U^3 dA}{U_{avg}^{\;3} A}$$

$$H = z + \frac{p}{\gamma} + \alpha \frac{U^2}{2g}$$

- Continuity equation $Q = Ct. = A_1U_1 = A_2U_2$

- Power $P = \gamma QH$

2. Equations of fluid flow: transport by gravity and by pumping

- ✓ Dissipated power

$$\dot{E}_{loss} = \gamma Q h_L$$

- ✓ Power of a pump

$$\left|\dot{W}_p\right| = \gamma Q H_p$$

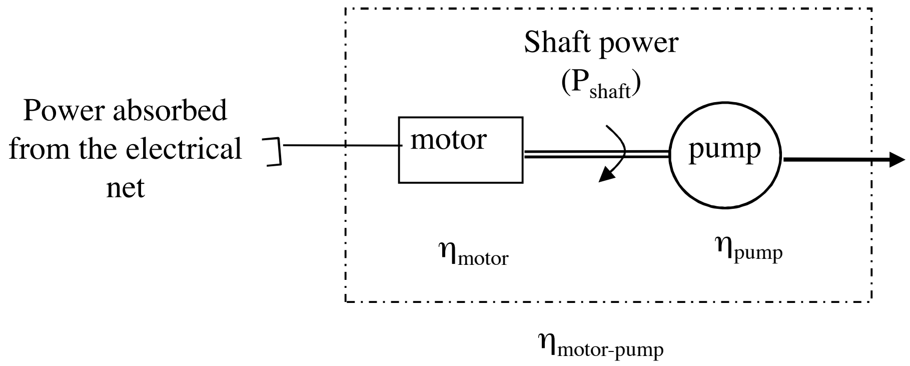
- ✓ Power of a turbine

$$\dot{W}_t = \gamma Q H_t$$

2. Equations of fluid flow: transport by gravity and by pumping

✓ Efficiencies

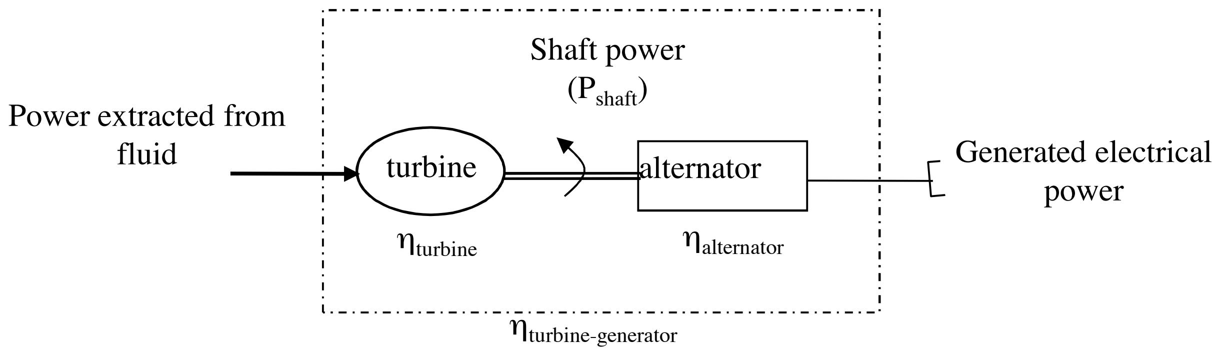
- Pump



$$\eta_{pump} = \frac{\left|\dot{W}_p\right|}{P_{shaft}} = \frac{\gamma Q H_p}{P_{shaft}}$$

$$\eta_{motor} = \frac{P_{shaft}}{P_{electrical}}$$

- Turbine

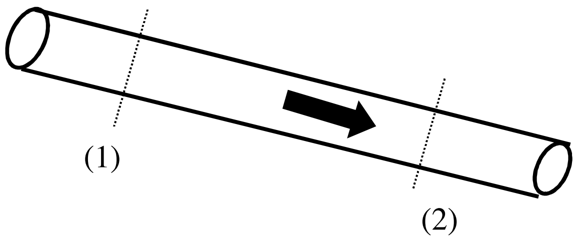


$$\eta_{turbine} = \frac{P_{shaft}}{\dot{W}_t} = \frac{P_{shaft}}{\gamma Q H_t}$$

$$\eta_{alternator} = \frac{P_{electrical}}{P_{shaft}}$$

2. Equations of fluid flow: transport by gravity and by pumping

✓ Fluid transport by gravity



- Continuity equation

$$Q = Ct. = A_1 U_1 = A_2 U_2$$

- Energy equation

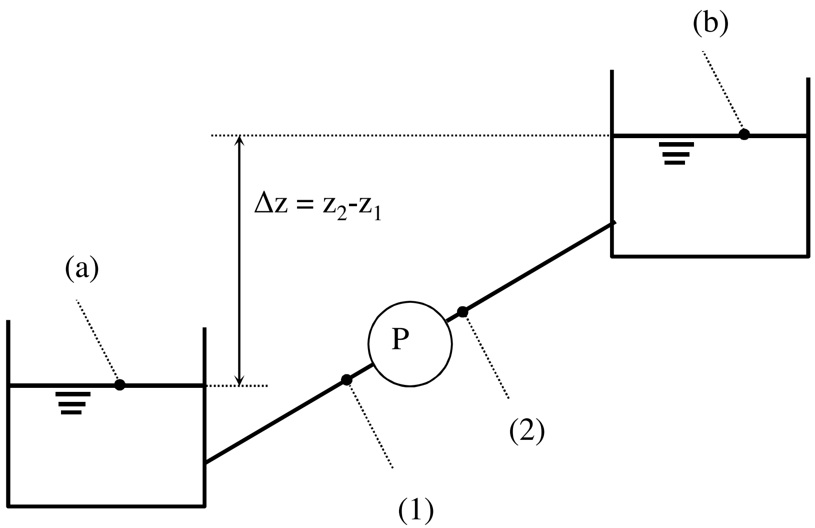
$$H_1 = H_2 + \Delta H_{1\text{-}2}$$

$$H = z + \frac{p}{\gamma} + \frac{U^2}{2g}$$

$$\Delta H_{1\text{-}2} = h_f + h_s$$

2. Equations of fluid flow: transport by gravity and by pumping

✓ Fluid transport by pumping



- Continuity equation

$$Q = Ct. = A_1U_1 = A_2U_2$$

- Energy equation

Between (a) and (1): $H_a = H_1 + \Delta H_{a-1}$
Between (1) and (2): $H_1 + H_p = H_2$
Between (2) and (b): $H_2 = H_b + \Delta H_{2-b}$

$$H_a+H_p= H_b+\Delta H_{a-1}+\Delta H_{2-b}= H_b+\Delta H_{a-b}$$

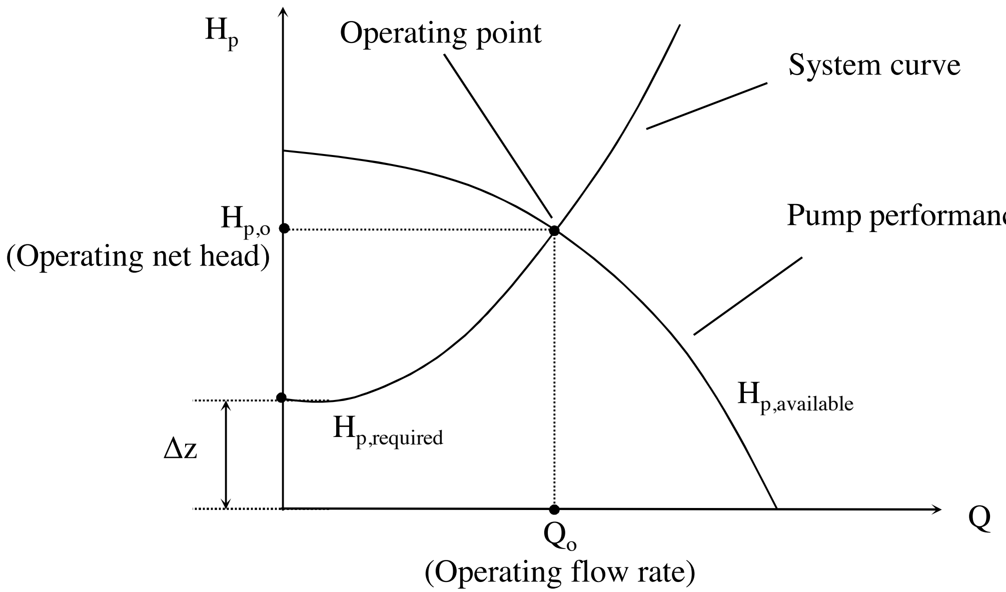
$$H_p= \Delta z+\Delta H_{a-b}$$ **System curve**

2. Equations of fluid flow: transport by gravity and by pumping

- ✓ Fluid transport by pumping

Pump performance curve $H_p = A - BQ - CQ^2$

System curve $H_p = \Delta z + \Delta H_{a\text{-}b}$



3. Energy losses

✓ Classification

- Continuous energy losses (known as well as major losses or frictional losses)

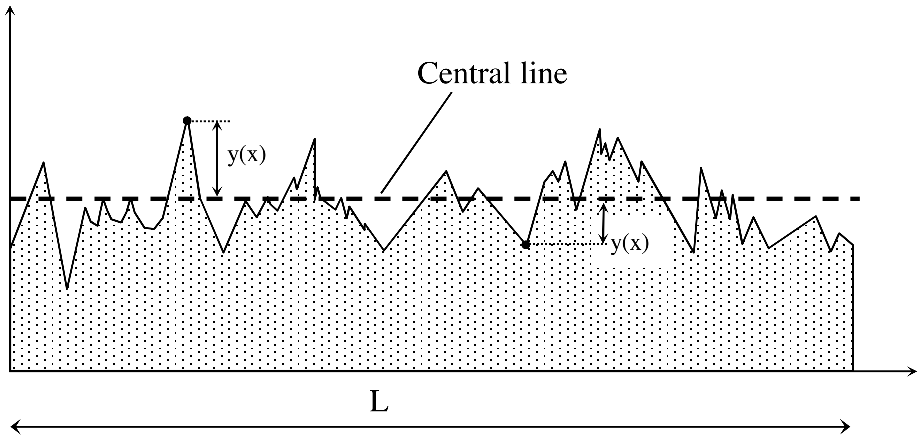
- Local energy losses (known as well as minor losses or secondary losses)

3. Energy losses

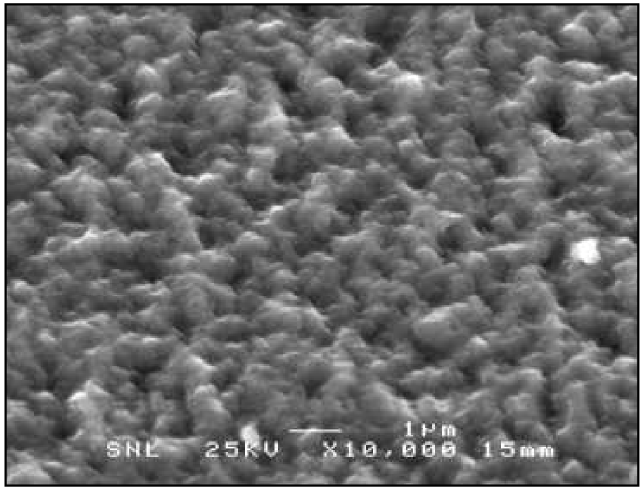
- ✓ Major energy losses
  - Dependence

$$h_f = \Phi(\rho, \mu, L, D, \varepsilon, U)$$

  - Roughness



$$\varepsilon = \frac{1}{L}\int_0^L |y(x)|\,dx$$



3. Energy losses

✓ Major energy losses

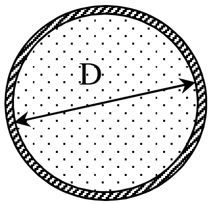
- Calculation

$$h_f = J\,L$$

J: unitary head loss (m/m)
L: pipe length

- Hydraulic radius

$$R_H = \frac{\text{Cross-sectional area}}{\text{Wetted perimeter}} = \frac{A}{p}$$



$$R_H = \frac{\pi D^2/4}{\pi D} = \frac{D}{4}$$

- Hydraulic diameter

$$D_H = 4R_H$$

3. Energy losses

✓ Major energy losses. Calculation

- Chezy

$$U = C\sqrt{R_H J}$$

C: Chezy coefficient
U: average velocity in the pipe or channel (m/s)
R_H: hydraulic radius (m)
J: unitary energy losses (m/m)

Field of application:

- Pipes and channels in general
- Turbulent flow

- Darcy - Weisbach

$$h_f = f \frac{L}{D} \frac{U^2}{2g}$$

f: friction factor or Darcy factor (dimensionless)
D: internal diameter of the pipe (m)
L: length of the pipe (m)
U: average velocity of the fluid (m/s)
g: acceleration of gravity (m/s^2)

Field of application:
- Turbulent flow (applicable to laminar flow too, in which case the friction factor must be calculated by the particular analytical expression f = 64/Re)
- Valid for any kind of pipe, non-circular section included ($D = D_H = 4R_H$) and any type of material (absolute roughness)
- Any type of fluid

✓ Major energy losses. Calculation

- Poiseuille

$$J = \frac{32\mu U}{\gamma D^2}$$

μ: dynamic viscosity (Pa·s)
γ: specific weight (N/m^3)
U: average velocity (m/s)
D: internal diameter of the pipe (m)

Field of application:

- Laminar flow
- Pipes with a circular section

✓ Extension of Darcy-Weisbach equation to laminar regime

$$J = \frac{32\mu U}{\gamma D^2}$$ **POISEUILLE**

$$J = f \frac{1}{D} \frac{U^2}{2g}$$ **DARCY - WEISBACH**

$$f = 64 \frac{\mu}{\rho D U} = \frac{64}{Re}$$

3. Energy losses

✓ Major energy losses. Calculation

- Manning

$$J = \frac{n^2 U^2}{\sqrt[3]{R_H^{\ 4}}}$$

$$U = \frac{1}{n} R_H^{\ 2/3} J^{1/2}$$

$$Q = \frac{1}{n} A R_H^{\ 2/3} J^{1/2}$$

n: Manning coefficient
U: average velocity (m/s)
R_H: hydraulic radius (m)

Field of application:

- Water transport in turbulent regime

- Proof

- Manning $$C = \frac{R^{1/6}}{n}$$

- Chezy $$U = C\sqrt{RJ}$$

⇒

$$U = \frac{1}{n} R^{2/3} J^{1/2}$$

$$Q = \frac{1}{n} A R^{2/3} J^{1/2}$$

3. Energy losses

✓ Major energy losses. Calculation

- Hazen-Wiliams

$$U = 0{,}355 C D^{0.63} J^{0.54}$$
$$U = 0{,}85 C R_H^{0.63} J^{0.54}$$

C: Hazen-Williams coefficient
U: average velocity (m/s)
R_H: hydraulic radius (m)
D: diameter (m)

Field of application:

- Ducts of pressurized water
- Non very rough ducts
- It can not be applied in laminar regime or in complete turbulence
- Very useful in the solution of pipeline networks, it does not have the iterative problems of Darcy-Weisbach equation

3. Energy losses

✓ Major energy losses. Calculation

- Friction coefficient

  - White - Colebrook

$$\frac{1}{\sqrt{f}} = -2\log\left(\frac{\varepsilon}{3{,}71D} + \frac{2{,}51}{Re\sqrt{f}}\right)$$

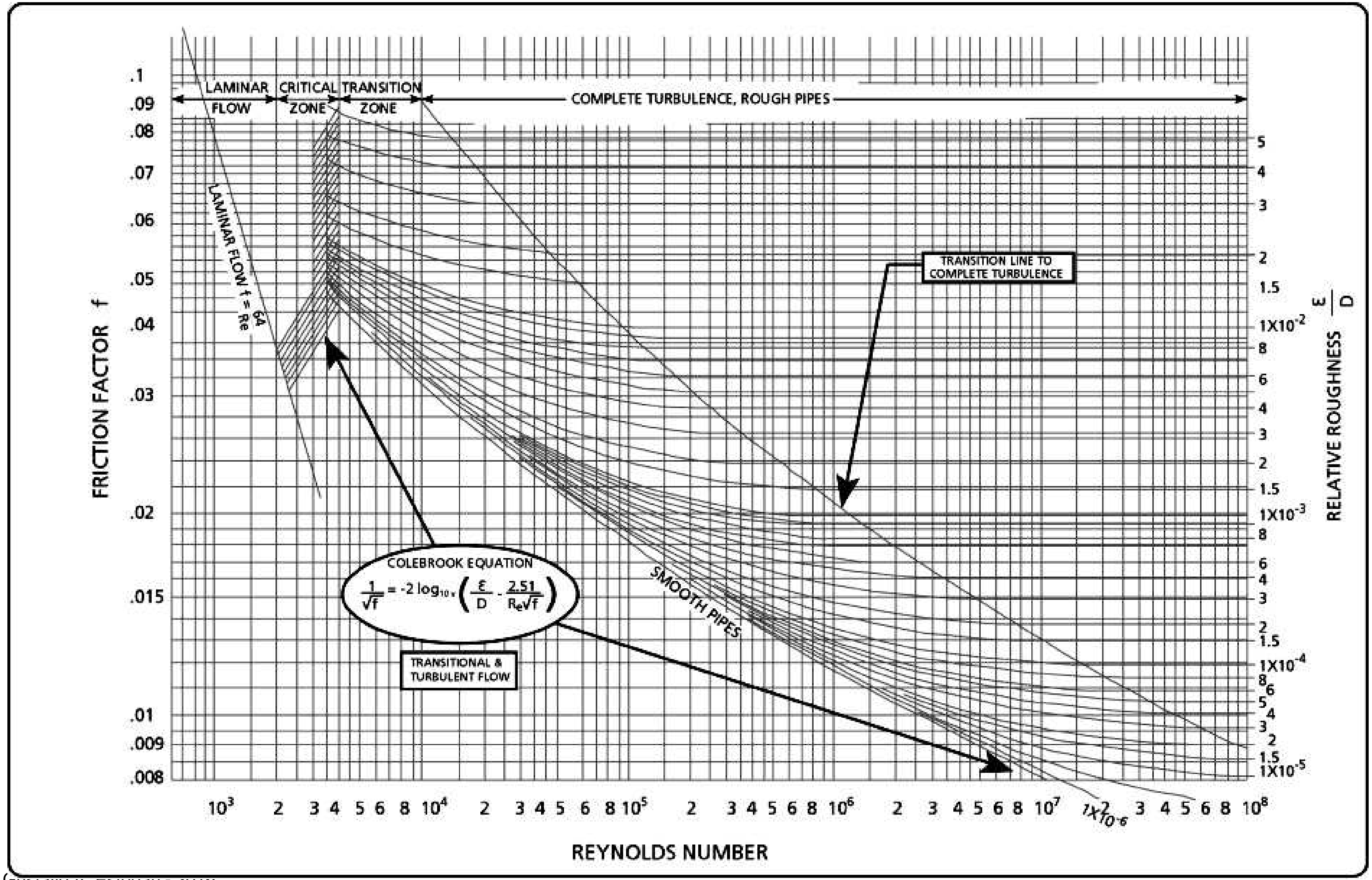
  - Prabhata K. Swamme y Akaland K. Jain (PSAK) (1976)

$$\frac{1}{\sqrt{f}} = -2\log\left[\left(\frac{\varepsilon}{3{,}71D}\right) + \frac{5{,}74}{Re^{0{,}9}}\right]$$

  - Haaland (1982)

$$\frac{1}{\sqrt{f}} - 1{,}8\log\left[\left(\frac{\varepsilon}{3{,}7D}\right)^{1{,}11} + \frac{6{,}9}{Re}\right]$$

3. Energy losses



3. Energy losses

✓ Minor losses

- Concept
- Calculation

1. Loss coefficient (or resistance coefficient) "K"

$$h_s = K \frac{U^2}{2g}$$

2. Equivalent length "L_e"

$$h_s = f \frac{L_e}{D} \frac{U^2}{2g}$$

3. Discharge coefficient

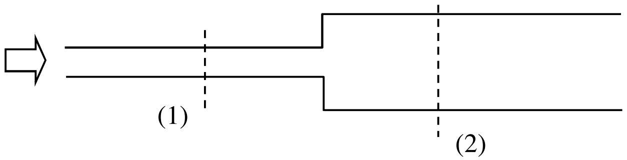
$$Q = C_d A \sqrt{2g\Delta H} = C_d A \sqrt{2 \frac{\Delta P}{\rho}}$$

$$\Delta H = \frac{\Delta p}{\gamma} = h_s$$

$$K = 1/C_d^2$$

3. Energy losses

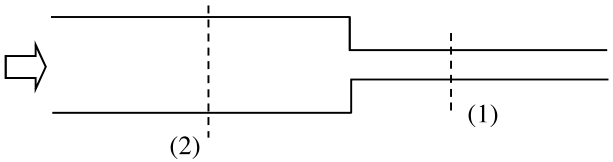
- ✓ Sudden expansion (Borda – Carnot):



$$h_s = K_1 \frac{U_1^2}{2g} = K_2 \frac{U_2^2}{2g} = \frac{(U_1 - U_2)^2}{2g}$$

$$\beta = \frac{D_1}{D_2} \qquad K_1 = (1-\beta^2)^2 \qquad K_2 = \frac{K_1}{\beta^4}$$

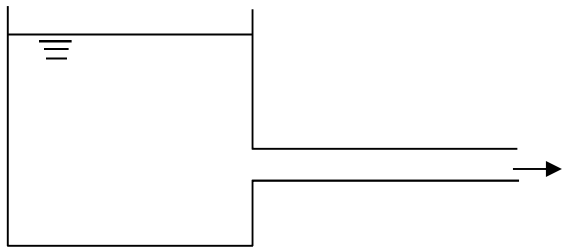
- ✓ Sudden contraction:



$$h_s = K_1 \frac{U_1^2}{2g} = K_2 \frac{U_2^2}{2g}$$

$$\beta = \frac{D_1}{D_2} \qquad K_1 = 0{,}5(1-\beta^2) \qquad K_2 = \frac{K_1}{\beta^4}$$

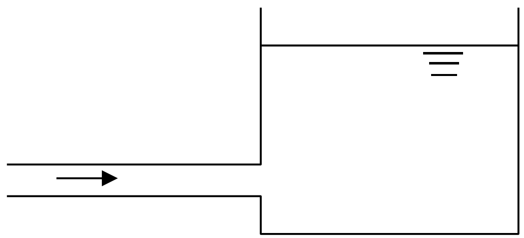
✓ Pipe inlet



K = 0,78 (reentrant)
K = 0,5 (sharp-edged)
K = 0 (smooth inlet, rounded)

$$h_s = K\frac{U^2}{2g}$$

✓ Pipe exit



$$h_s = \frac{U^2}{2g}$$

K = 1

3. Pipe accessories and valves

- ✓ Calculation

$$h_s = K\frac{U^2}{2g}$$

$$K_{max} - K_{min}$$

Kmax for pipes with small diameter and Kmin with pipes with large diameter

$$K = cte \cdot f_T$$

f_T=f friction coefficient in complete turbulence, independent from Reynolds number

$$h_s = f\frac{L_e}{D}\frac{U^2}{2g}$$

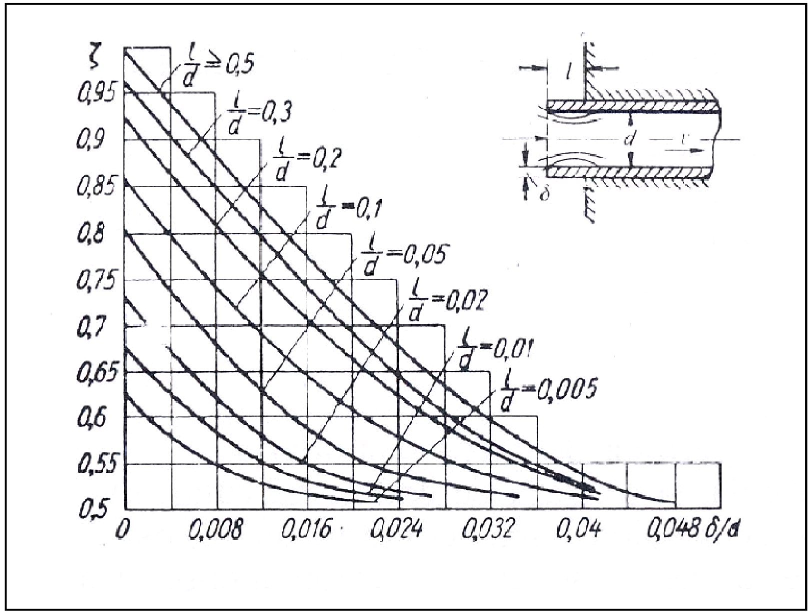
$$Q = C_d A\sqrt{2\frac{\Delta P}{\rho}} = C_d A\sqrt{2g\Delta H}$$

- ✓ Examples

3. Pipe accessories and valves

✓ Examples

1. Pipe inlet. Reentrant



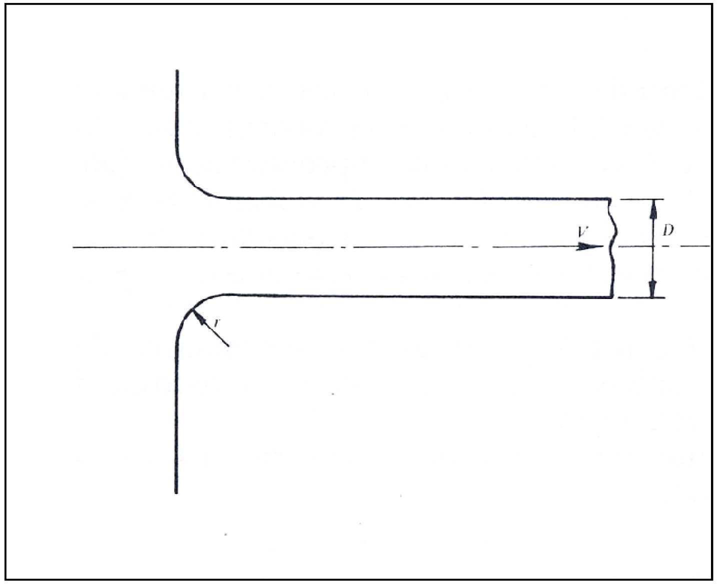
(Taken from C. Mataix)

Loss coefficient "K=ζ"

3. Pipe accessories and valves

✓ Examples

2. Pipe inlet. Rounded inlet



(Taken from C. Mataix)

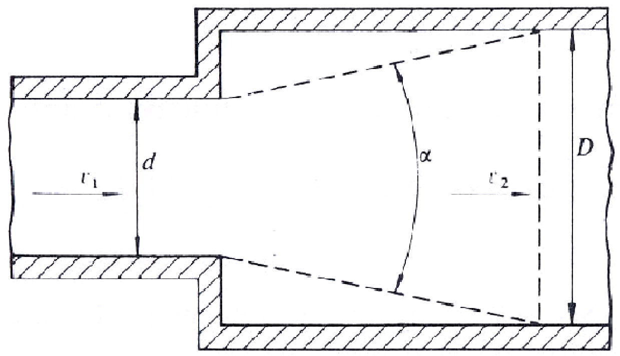
r/D	0	0,02	0,04	0,08	0.12	0,16	> 0,2
ζ	0,5	0,37	0.26	0,15	0,09	0,06	< 0,03

Loss coefficient "K=ζ"

3. Pipe accessories and valves

- ✓ Examples

3. Gradual expansion



$$H_r = m\frac{(U_1 - U_2)^2}{2g} = m\left[1-\left(\frac{d}{D}\right)^2\right]^2 \frac{U_1^2}{2g}$$

$$m\left[1-\left(\frac{d}{D}\right)^2\right]^2 = K_1$$

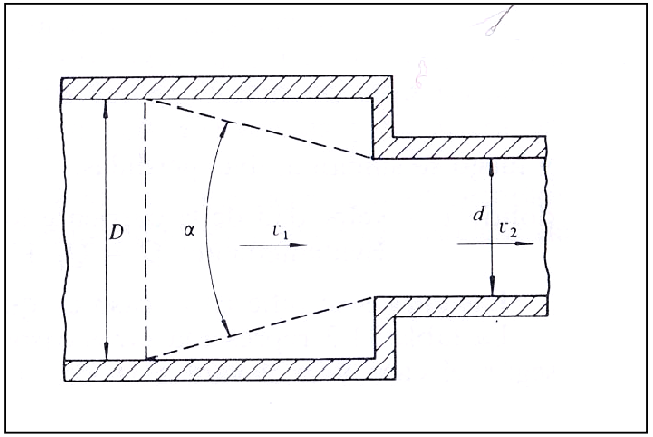
(Taken from C. Mataix)

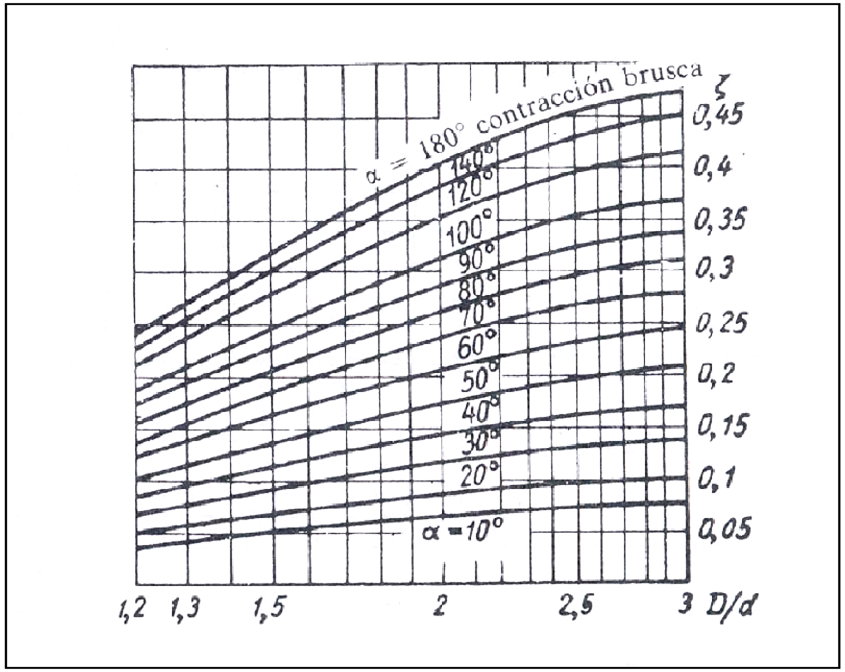
α°	2,5	5	7,5	10	15	20	25	30
m	0,18	0,13	0,14	0,16	0,27	0,43	0,62	0,81

3. Pipe accessories and valves

✓ Examples

4. Gradual contraction





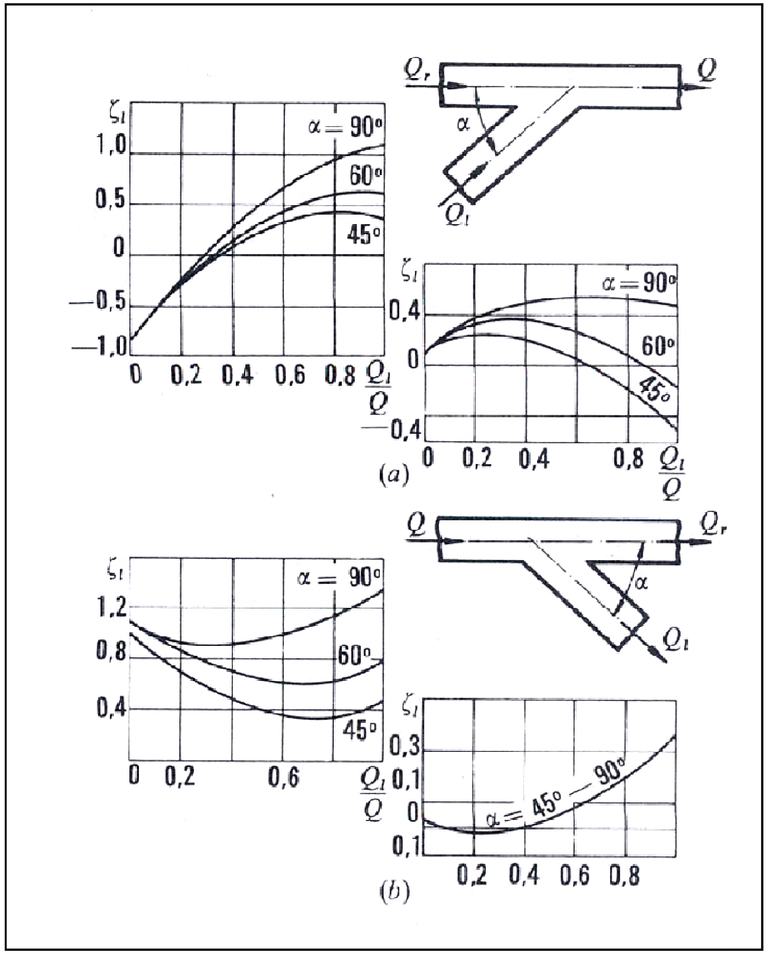
Loss coefficient "K=ζ"

(Taken from C. Mataix)

3. Pipe accessories and valves

- ✓ Examples

5. Tees (convergent and divergent pipes)



Lateral and straight flow rate:

$$Q = Q_l + Q_s$$

$$h_{s,l} = \zeta_l \frac{U^2}{2g}$$

$$h_{s,s} = \zeta_s \frac{U^2}{2g}$$

$$h_s = h_{s,l} + h_{s,s}$$

Loss coefficient "K=ζ"

(Taken from C. Mataix)

3. Pipe accessories and valves

✓ Examples

6. Tees (convergent and divergent pipes)

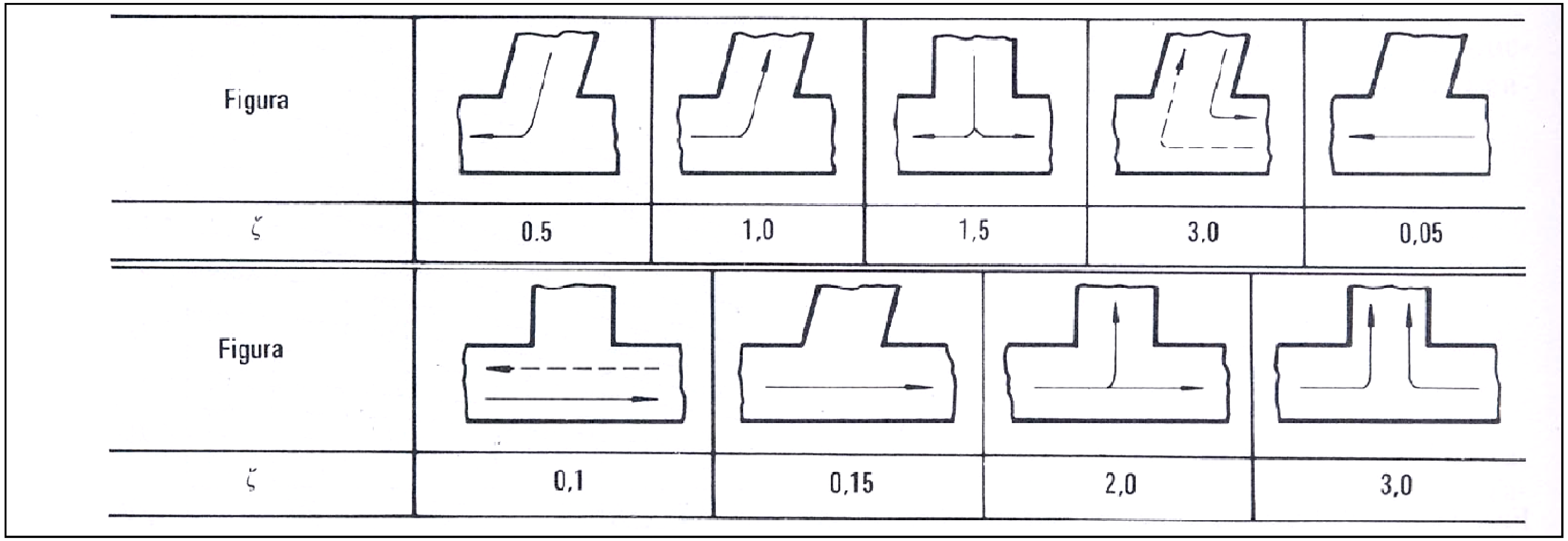


Figura					
ζ	0.5	1,0	1,5	3.0	0,05

Figura				
ζ	0,1	0,15	2,0	3,0

(Taken from C. Mataix)

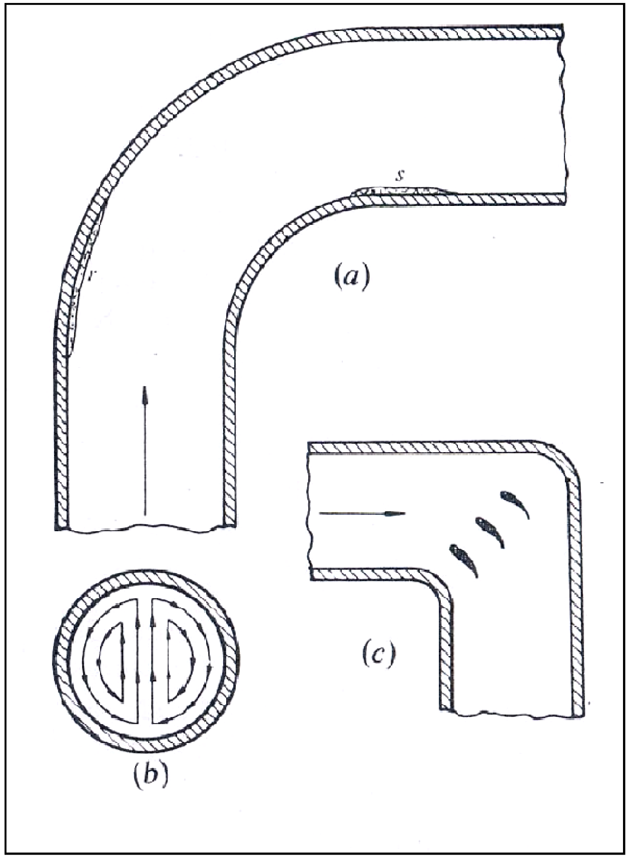
$$h_s = \zeta \frac{U^2}{2g}$$

Loss coefficient "K=ζ"

3. Pipe accessories and valves

✓ Examples

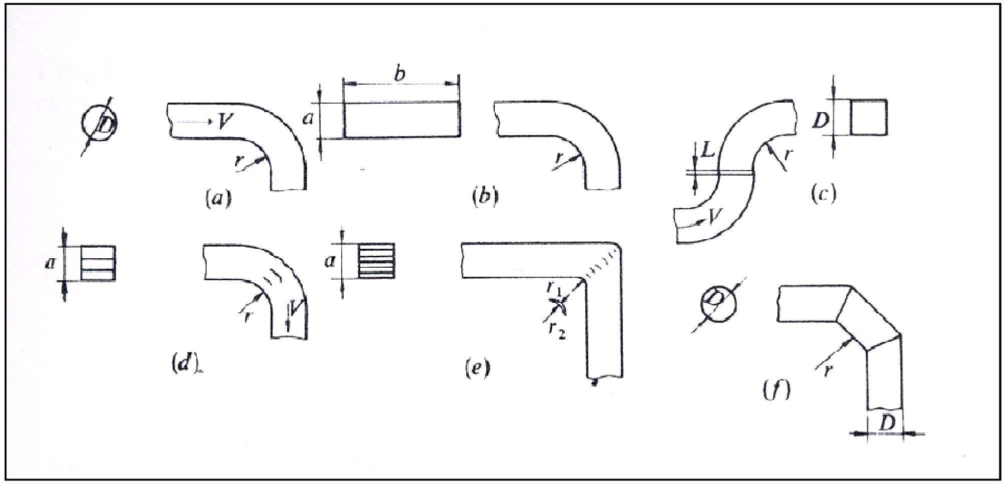
7. Bend (guiding profiles)



(Taken from C. Mataix)

- ✓ Examples

8. Bends



(Taken from C. Mataix)

Loss coefficient "K=ζ"

(a) $\frac{r}{D} = 0$ | 0,25 | 0,5 | 1,0

$\frac{r}{D}$	0	0,25	0,5	1,0
ζ	0,8	0,4	0,25	0,16

(b)

$\frac{r}{a}$	$\frac{b}{a} = 1$	$\frac{b}{a} = 2$	$\frac{b}{a} = 3$	$\frac{b}{a} = 4$
0	$\zeta = 1{,}0$	$\zeta = 0{,}9$	$\zeta = 0{,}8$	$\zeta = 0{,}73$
0,25	0,4	0,4	0,39	0,32
0,5	0,2	0,2	0,19	0,16
1,0	0,13	0,13	0,13	0,10

(c) $L = 0$ $\quad L = D$

$\zeta = 0{,}62$ $\quad \zeta = 0{,}68$

(d)

N.º de álabes =	1	2	3
$\frac{r}{a}$ =	0,25	0,2	0,15
ζ =	0,15	0,12	0,10

(e) $r_1 = \frac{r_2}{2} = \frac{a}{b}$ $\qquad \zeta = 0{,}1$

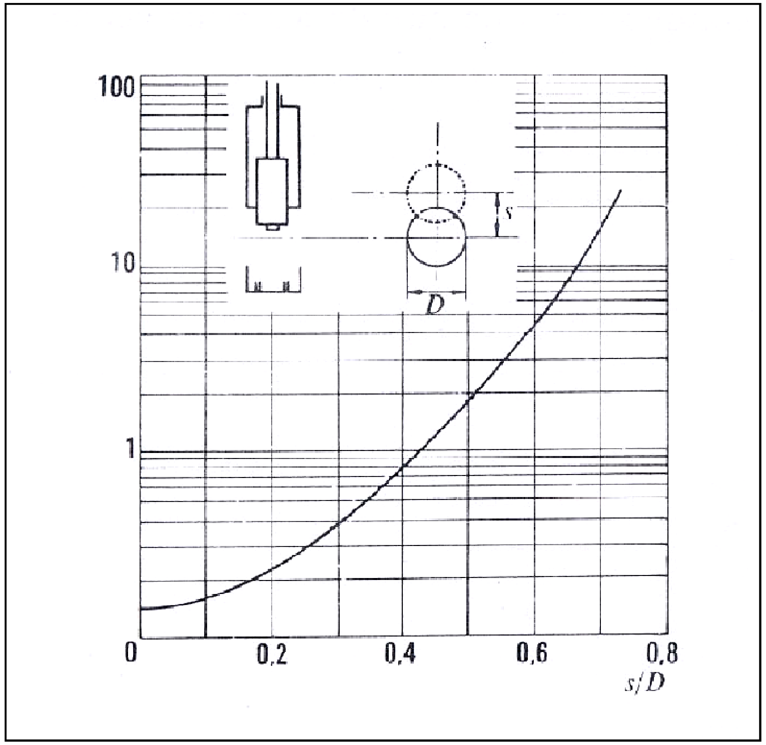
(f)

$\frac{r}{D}$ =	0,25	0,5	1
ζ (codo de 3 piezas)	0,8	0,4	0,3
ζ (codo de 5 piezas)	0,5	0,3	0,2

3. Pipe accessories and valves

✓ Examples

9. Gate valve



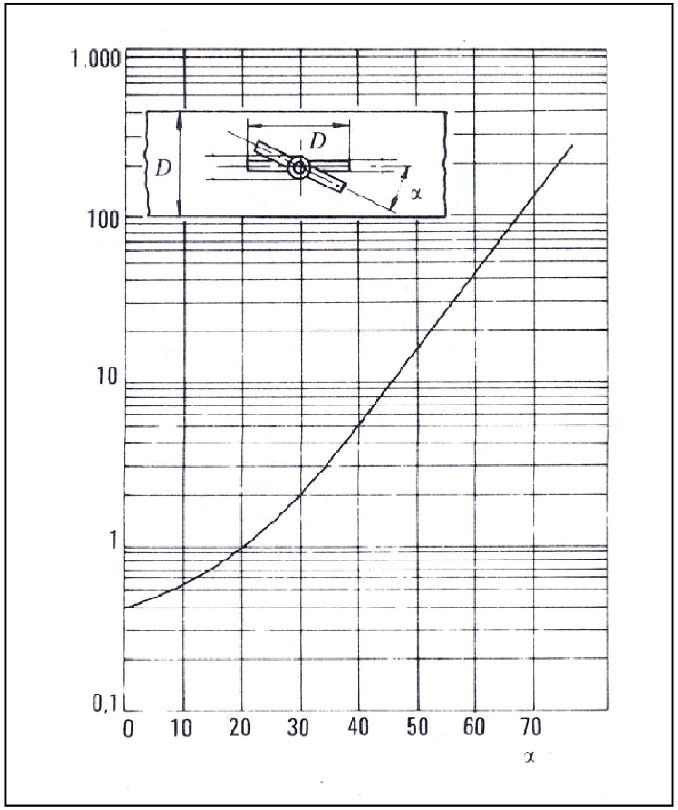
Loss coefficient "K=ζ"

(Taken from C. Mataix)

3. Pipe accessories and valves

✓ Examples

10. Butterfly valve



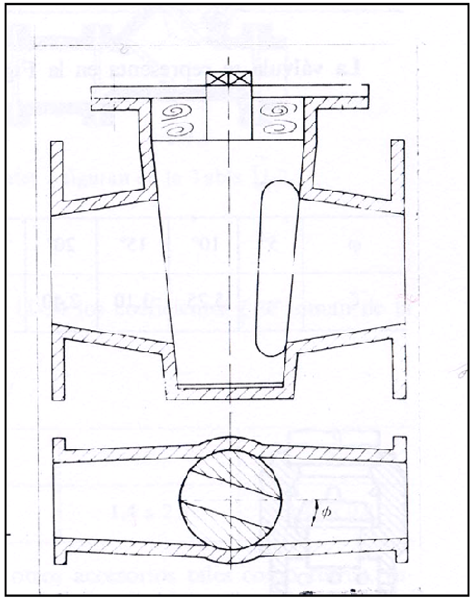
(Taken from C. Mataix)

Loss coefficient "K=ζ"

3. Pipe accessories and valves

✓ Examples

11. Plug valve



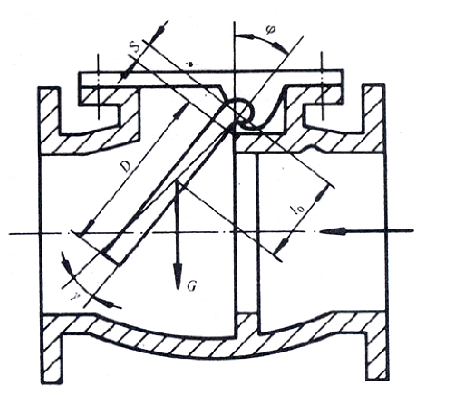
(Taken from C. Mataix)

φ	5°	10°	15°	20°	25°	30°	40°	45°	50°	60°	65°	70°	90°
ζ	0,05	0,29	0,75	1,56	3,10	5,47	17,3	31.2	52,6	206	486	—	∞

Loss coefficient "K=ζ"

- ✓ Examples

12. Check valve (non-return valve)



(Taken from C. Mataix)

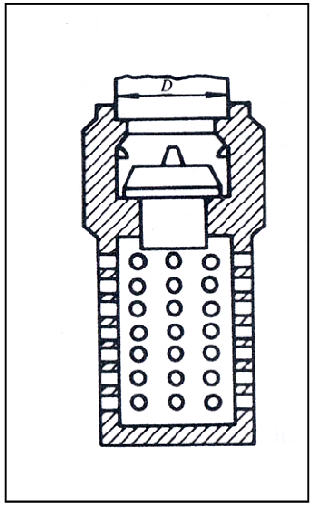
φ	5°	10°	15°	20°	25°	30°	40°	50°	60°	65°	70°	90°
ζ	—	5,25	3,10	2,40	2,10	2,0	1,85	1,80	1,55	1,2	—	∞

Loss coefficient "K=ζ"

3. Pipe accessories and valves

✓ Examples

13. Strainer foot valve (pump inlet)



D mm	ζ	D mm	ζ
40	12,0	200	5,2
50	10,0	250	4,4
65	8,8	300	3,7
80	8,0	350	3,4
100	7,0	400	3,1
125	6,5	450	2,8
150	6,0	500	2,5

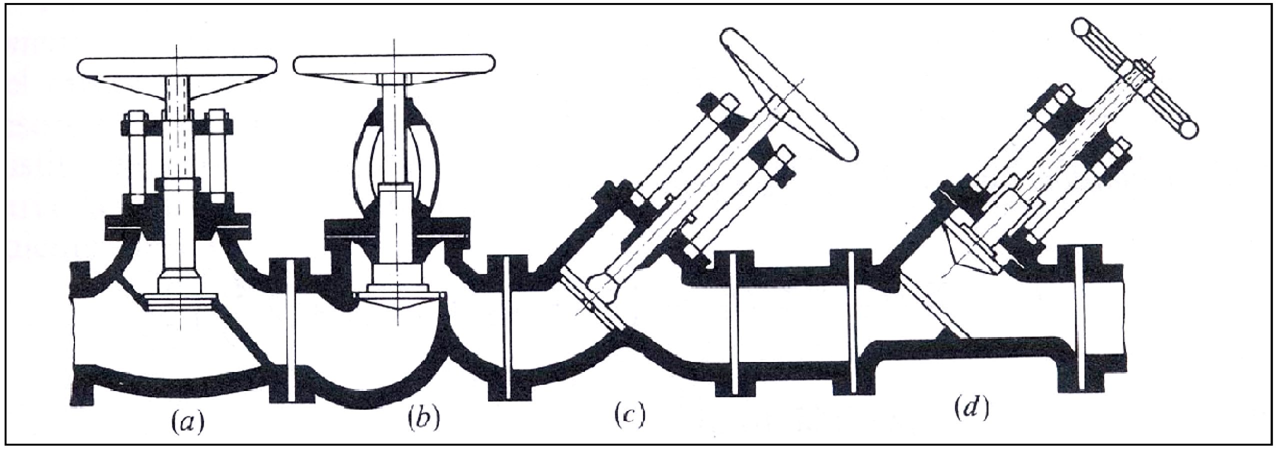
(Taken from C. Mataix)

Loss coefficient "K=ζ"

3. Pipe accessories and valves

✓ Examples

14. Seat valves



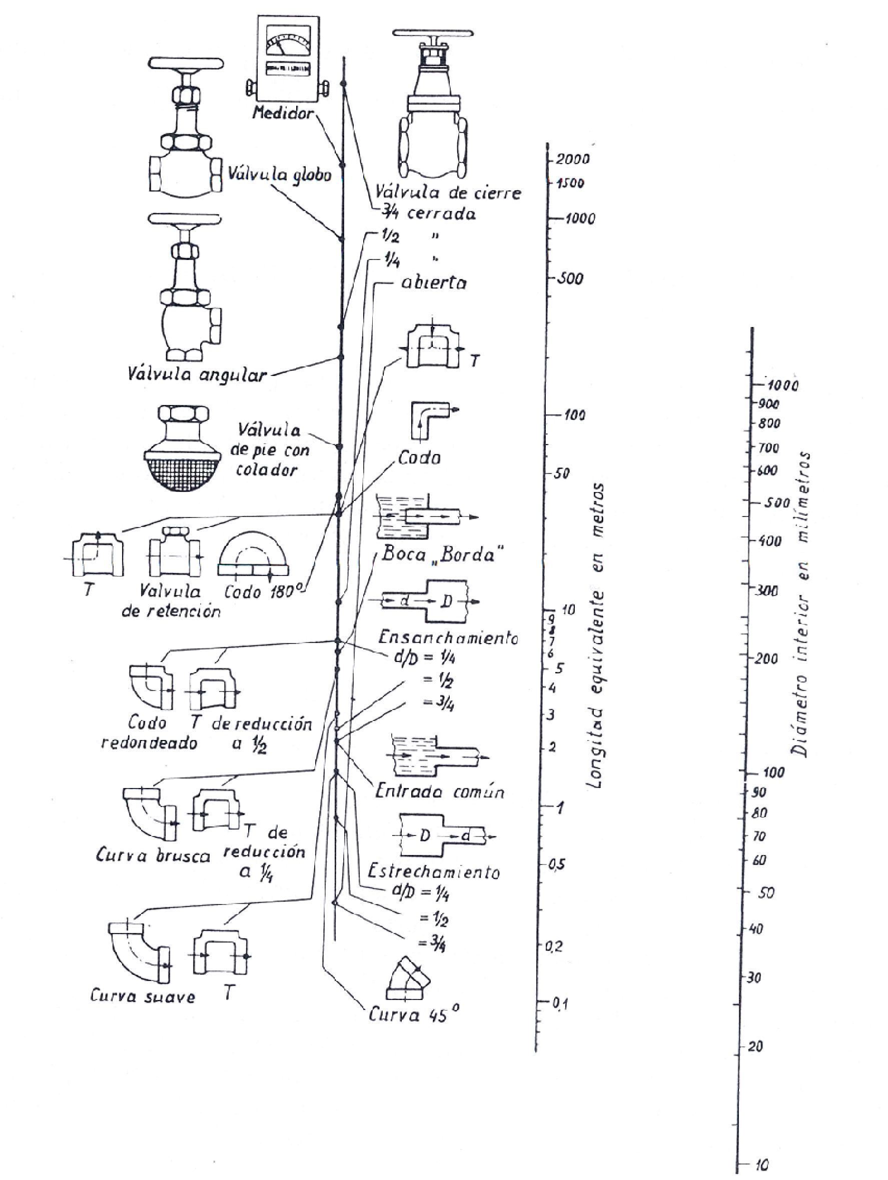
(Taken from C. Mataix)

Esquema	*a*	*b*	*c*	*d*
ζ	2,9	2,0 a 2,7	1,4 a 2,5	0,44 a 0,8

Loss coefficient "K=ζ"

3. Pipe accessories and valves

✓ Minor energy losses nomograph



Equivalent length "L_e"

$$h_s = f \frac{L_e}{D} \frac{U^2}{2g}$$

(Taken from C. Mataix)

4. Energy head diagram

- ✓ Bernoulli equation

$$z + \frac{p}{\gamma} + \frac{U^2}{2g} = Cte = h_T$$

- ✓ Energy heads diagram

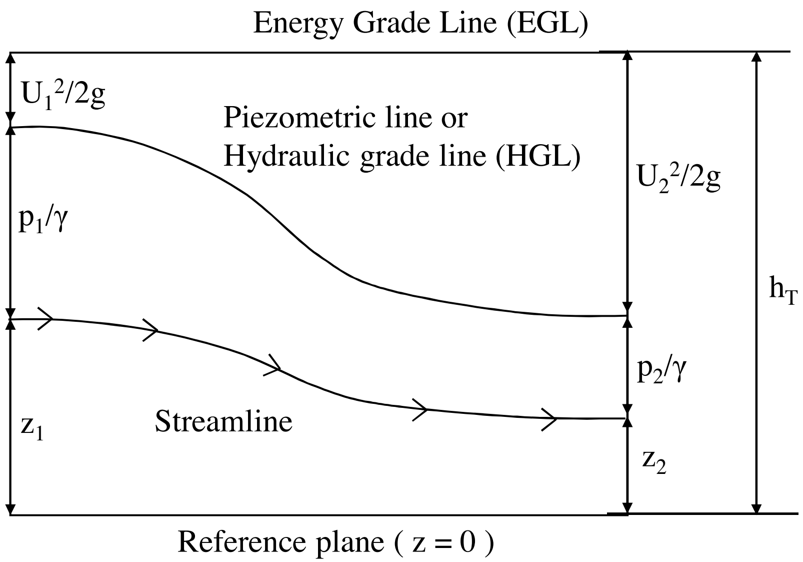


Figure 4.4 Energy head diagram

4. Energy head diagram

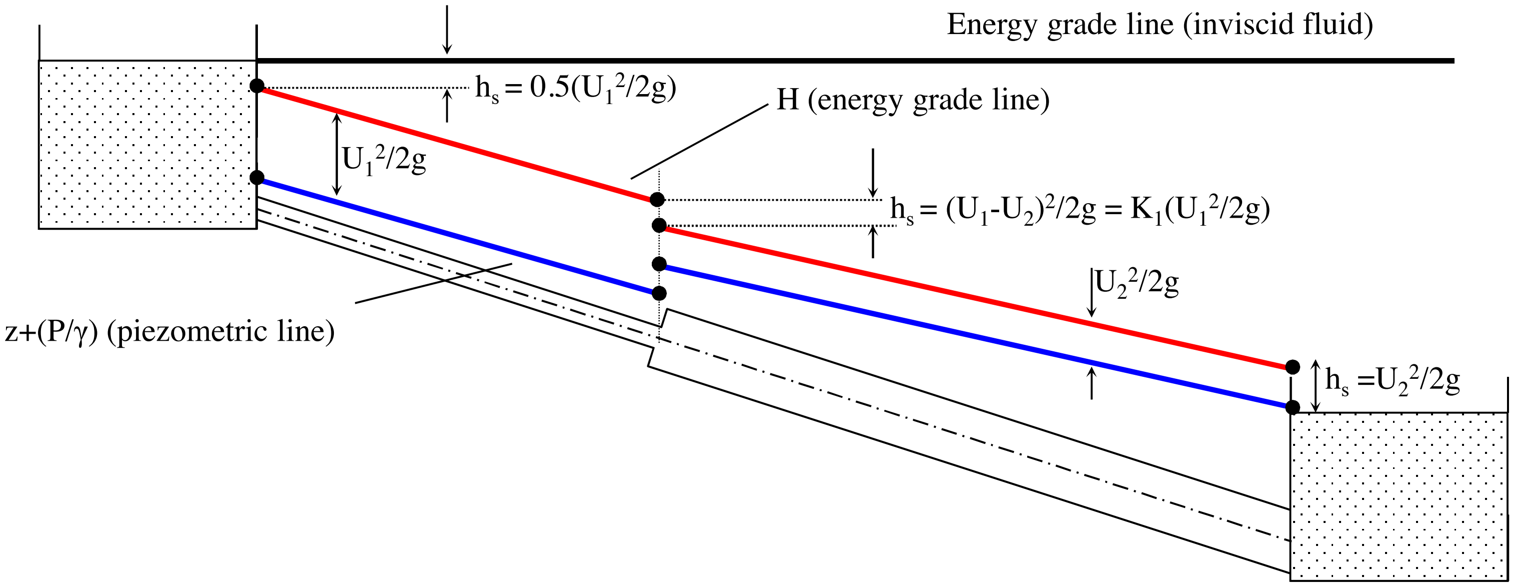


Figure 9.14 Energy heads diagram and piezometric lines. Pipe and sudden expansion

4. Energy head diagram

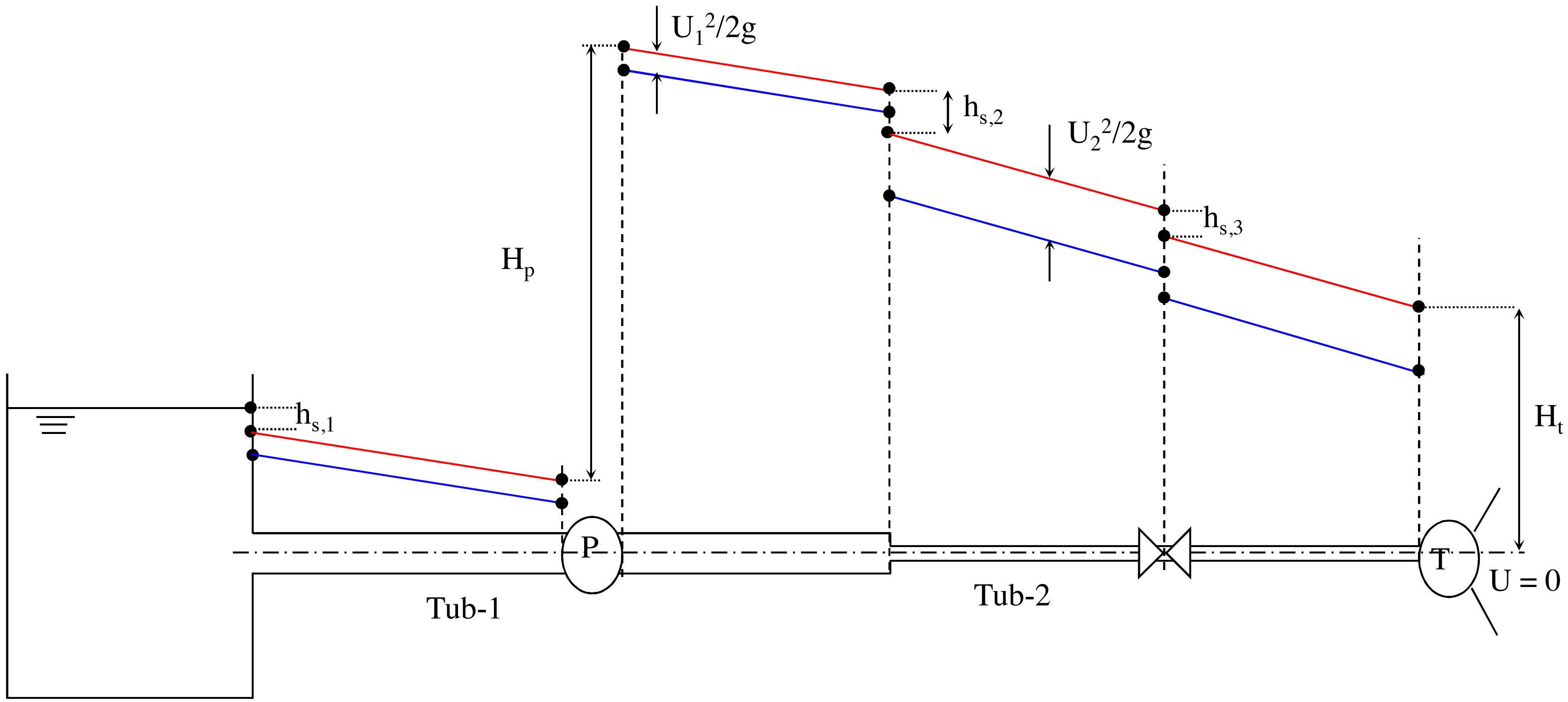
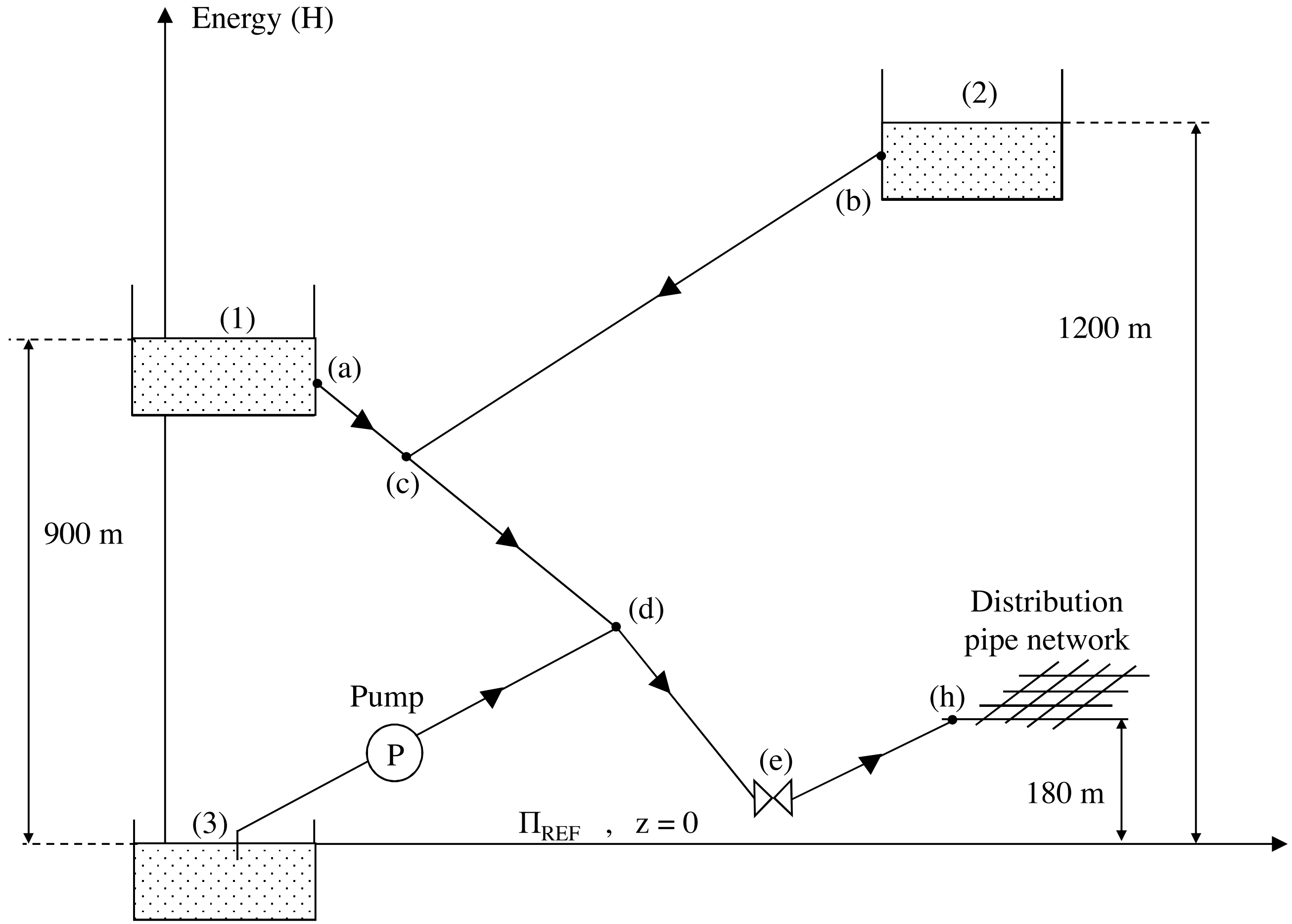


Figure 9.15 Energy heads diagram and piezometric lines. Pump and turbine

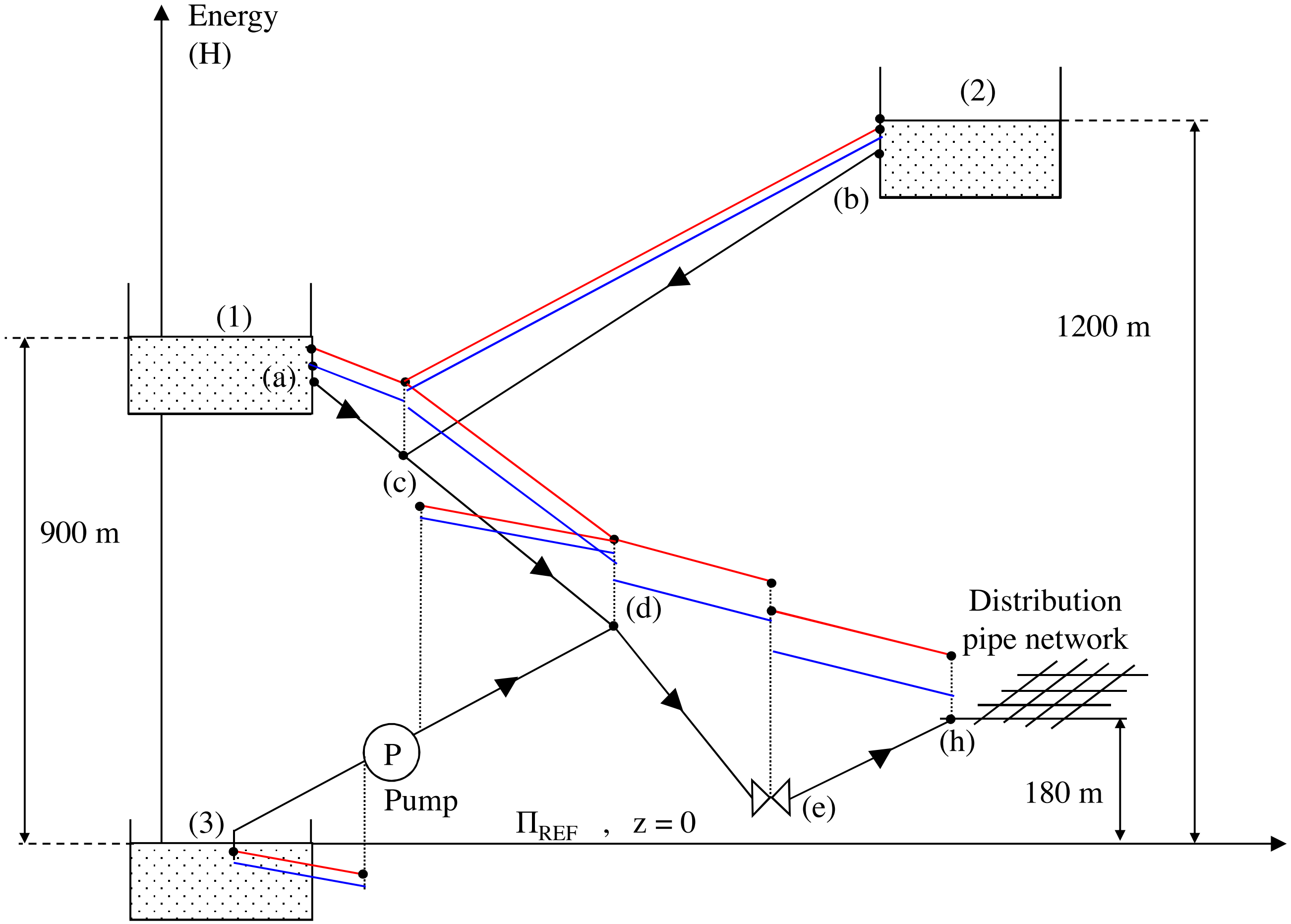
4. Energy head diagram

PRACTICAL EXAMPLE



4. Energy head diagram

PRACTICAL EXAMPLE

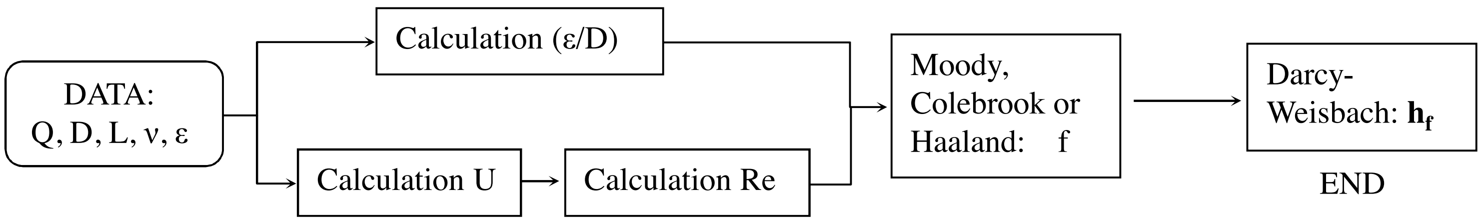


5. Pipe characterization

✓ Types of fluid flow problems

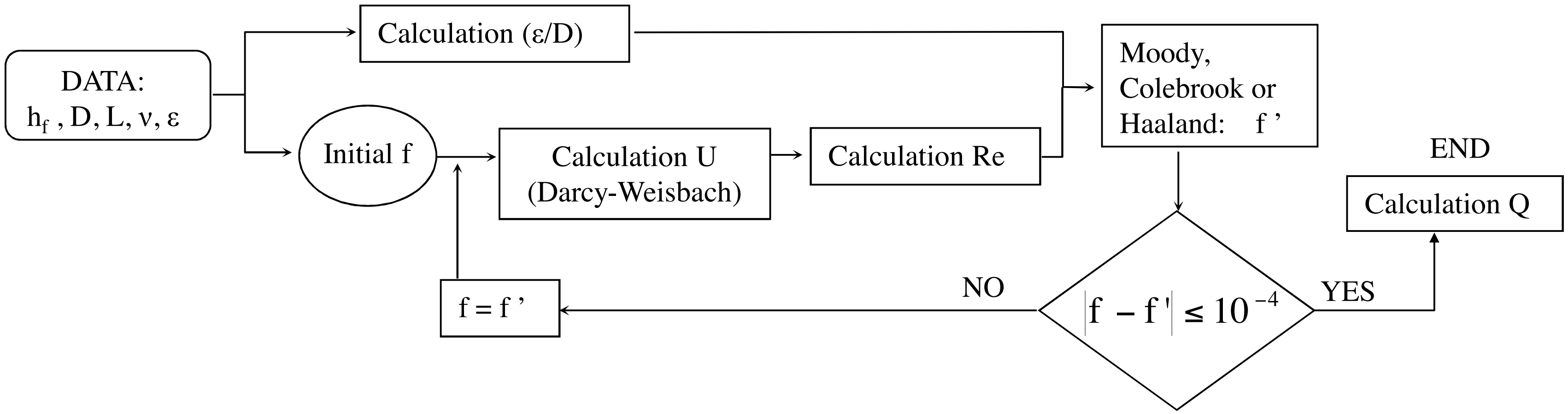
Type	Data	Objective (unknown)	
I	L, ν, ε, **Q, D**	h_f	Direct problem
II	L, ν, ε, **h_f, D**	Q	Inverse problem (requires iteration)
III	L, ν, ε, **h_f, Q**	D	Inverse problem (requires iteration)

- TYPE I: (analysing or checking a duct – energy consumption) hf



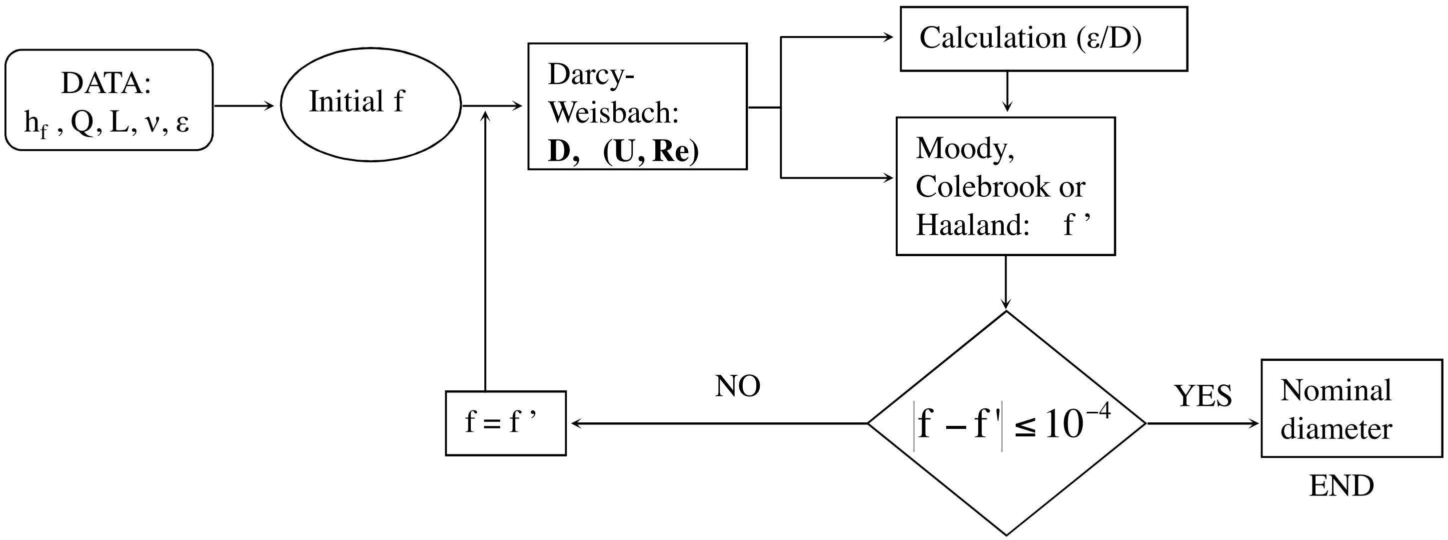
5. Pipe characterization

- TYPE II: (analysing or checking a duct – supply) Q



5. Pipe characterization

- TYPE III:(duct design) D



5. Optimum / economical diameter

✓ Transport by gravity

D↑ ➔ A↑ ➔ U↓ ; Problems related to deposition and drag of solid particles.

D↓ ➔ A↓ ➔ U↑ ; water hammer phenomenon, which can be associated to cavitation.

Recommendation:
0,5 m/s<U<1,5 m/s

✓ Transport by pumping

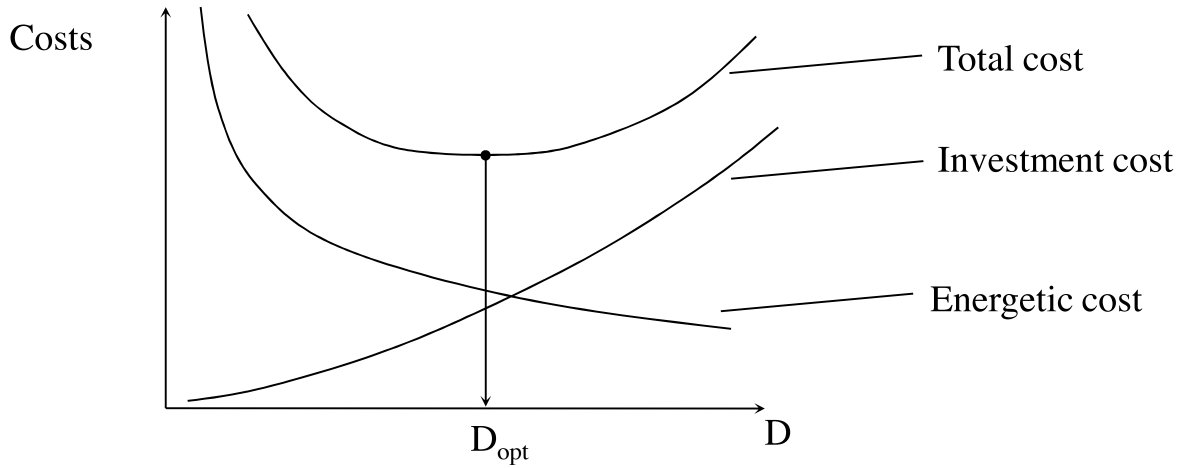
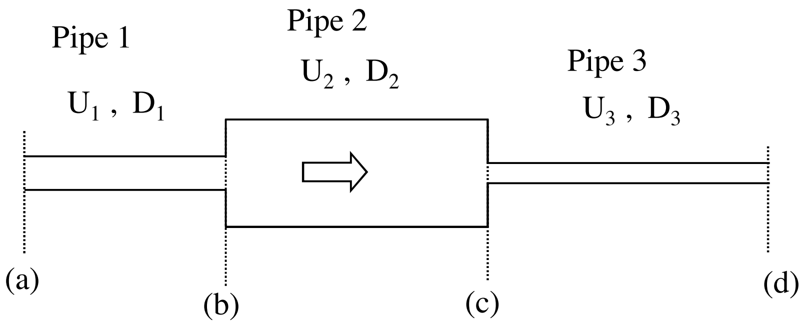


Figure 9.16 Optimum – economical diameter

5. Pipe systems. Resolution methods

- ✓ Pipes connected in series



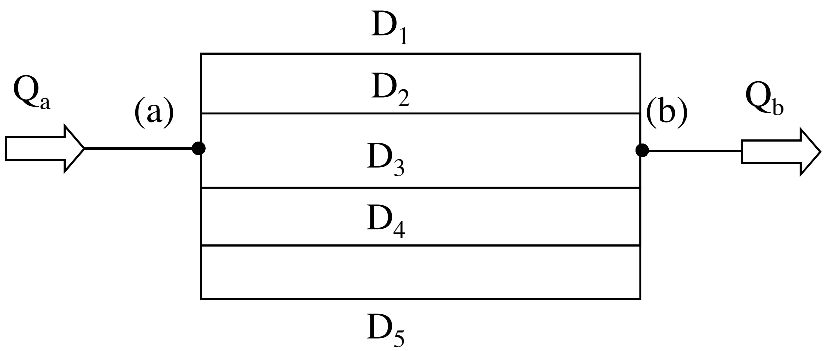
$$Q = Ct. = A_1 U_1 = A_2 U_2 = A_3 U_3$$

$$\Delta H_{a-d} = \Delta H_{a-b} + \Delta H_{b-c} + \Delta H_{c-d}$$

"*In a system of pipes connected in series every individual pipe transports the same flow rate and the total head loss is the sum of the head losses in individual pipes*".

5. Pipe systems. Resolution methods

- ✓ Pipes connected in parallel



$$Q_a = Q_b = Q_1 + Q_2 + Q_3 + Q_4 + Q_5$$

$$H_a = H_b + \Delta H_{a\text{-}b}$$

$$\Delta H_{a-b} = f_1 \frac{L_1}{D_1}\frac{U_1^2}{2g} = f_2 \frac{L_2}{D_2}\frac{U_2^2}{2g} = f_3 \frac{L_3}{D_3}\frac{U_3^2}{2g} = f_4 \frac{L_4}{D_4}\frac{U_4^2}{2g} = f_5 \frac{L_5}{D_5}\frac{U_5^2}{2g}$$

"In a system of pipes connected in parallel the pressure drop between junctions and thus the head loss in each individual pipe must be the same".

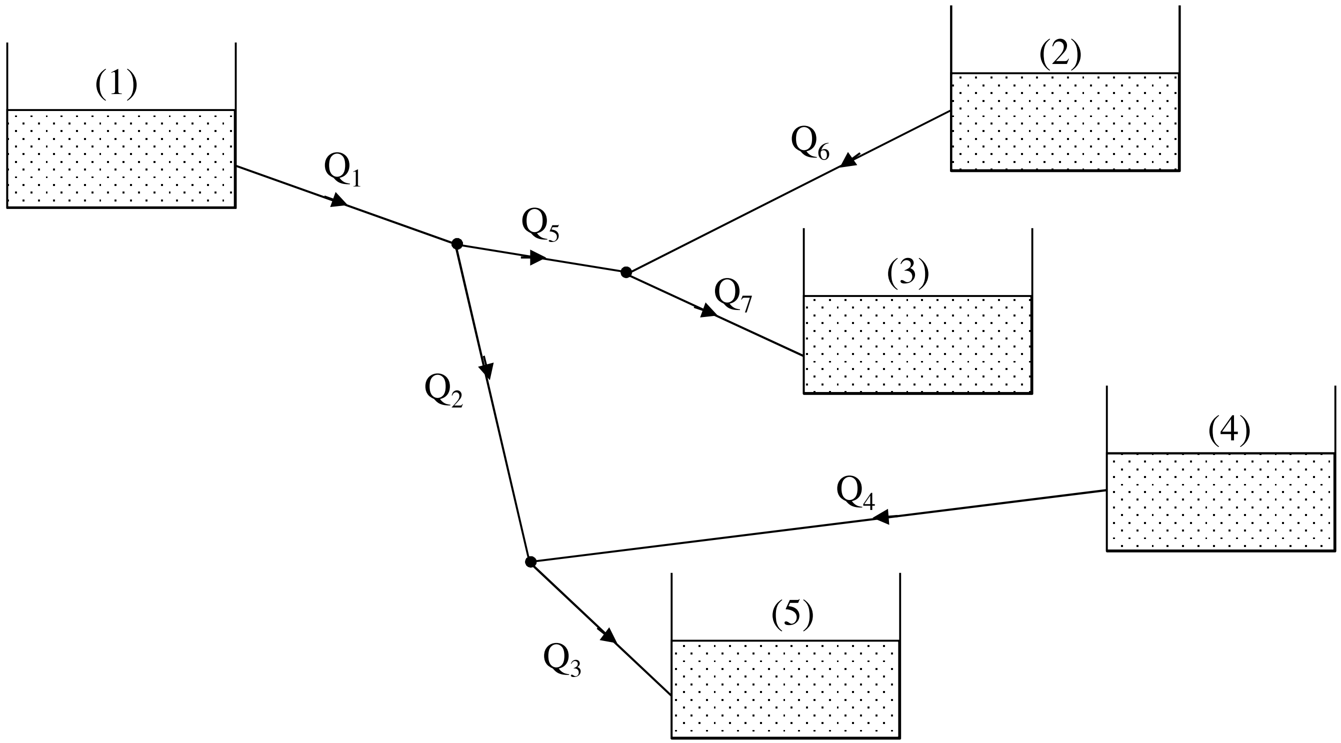
5. Pipe systems. Resolution methods

✓ Distribution networks

- Data
- Unknowns
- Types of network
    1. BRANCHED
    2. MESHED

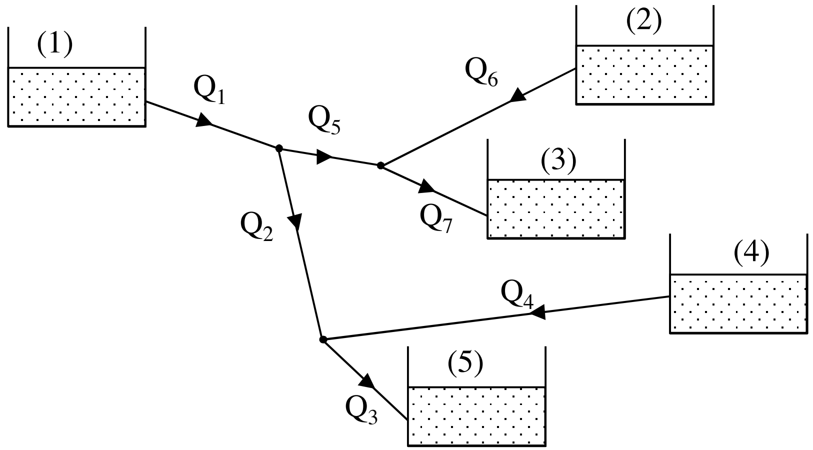
5. Pipe systems. Resolution methods

- ✓ Branched network



5. Pipe systems. Resolution methods

✓ Branched networks



- Continuity equation (junctions):

$$\Sigma Q_{in} = \Sigma Q_{out}$$

$$Q_1 = Q_2 + Q_5$$
$$Q_3 = Q_2 + Q_4$$
$$Q_7 = Q_5 + Q_6$$

- Energy equation (branches):

$H_{initial} = H_{final} + K'Q^2$ (coincident direction)
$H_{initial} = H_{final} - K'Q^2$ (opposite direction)

$$K' = \frac{8}{\pi^2 g} \frac{fL}{D^5} \qquad K' = \frac{8}{\pi^2 g} \frac{K}{D^4}$$

(1) to (5) $H_1 = H_5 + K_1'Q_1^2 + K_2'Q_2^2 + K_3'Q_3^2$
(1) to (2) $H_1 = H_2 + K_1'Q_1^2 + K_5'Q_5^2 - K_6'Q_6^2$
(1) to (4) $H_1 = H_4 + K_1'Q_1^2 + K_2'Q_2^2 - K_4'Q_4^2$
(1) to (3) $H_1 = H_3 + K_1'Q_1^2 + K_5'Q_5^2 + K_7'Q_7^2$

7 EQUATIONS WITH 7 UNKNOWNS

5. Pipe systems. Resolution methods

✓ Meshed networks

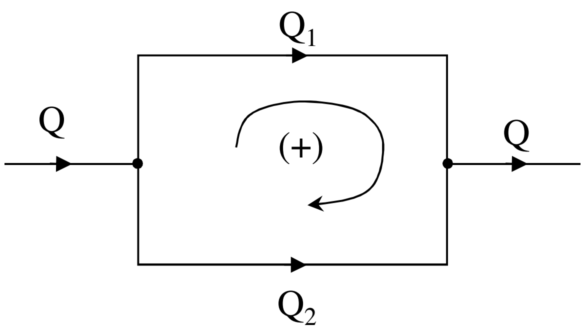
- Continuity equation ((n-1) junctions):

$$\Sigma Q_{in}=\Sigma Q_{out}$$

- Energy equation (meshes):

$$\sum K_i^{'} Q_i^2 = 0$$

The simplest mesh, parallel system:

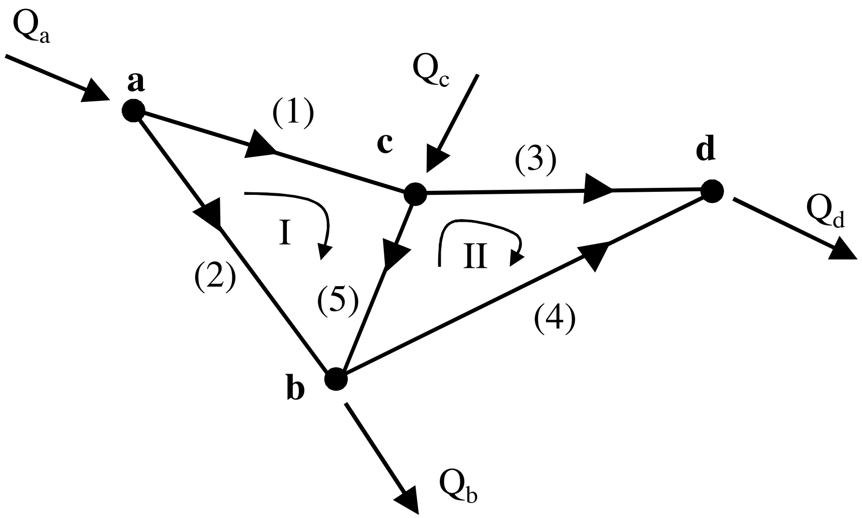


$$Q = Q_1+Q_2$$

$$K_1'Q_1^2-K_2'Q_2^2 = 0$$

5. Pipe systems. Resolution methods

✓ Meshed networks



- Continuity equation ((n-1) junctions):

Node a)	$Q_1 + Q_2 = Q_a$
Node b)	$Q_5 + Q_2 - Q_4 = Q_b$
Node d)	$Q_3 + Q_4 = Q_d$

- Energy equation (meshes):

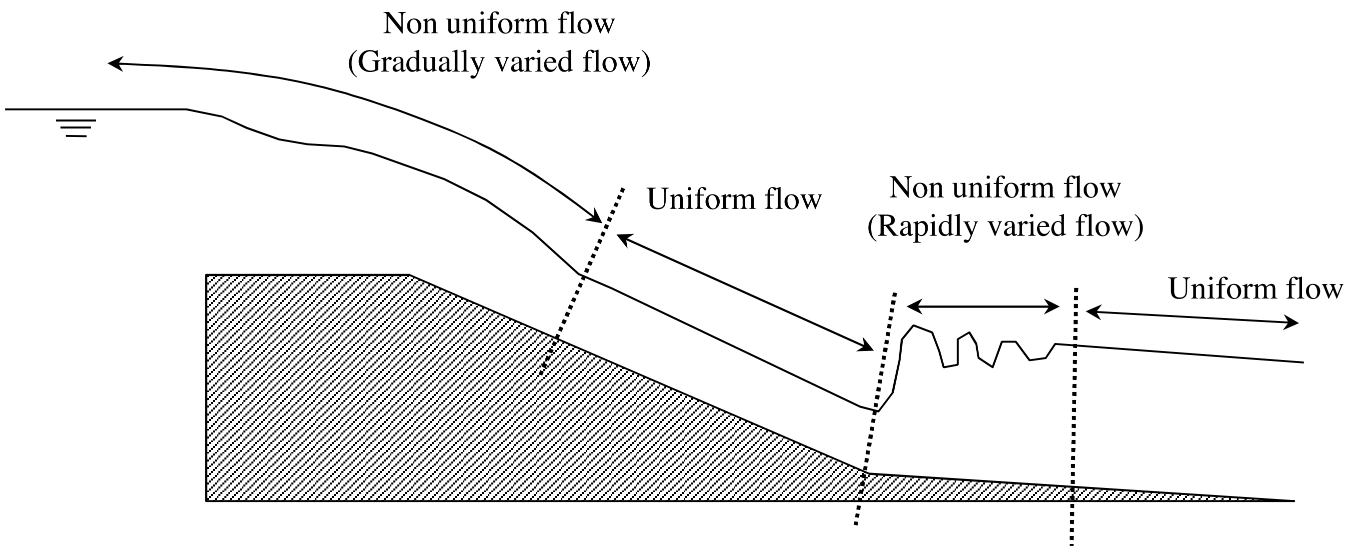
Loop I)	$K'_1 Q_1^2 + K'_5 Q_5^2 - K'_2 Q_2^2 = 0$
Loop II)	$K'_3 Q_3^2 - K'_4 Q_4^2 - K'_5 Q_5^2 = 0$

6. Classification of motion

- ✓ According to viscous behaviour

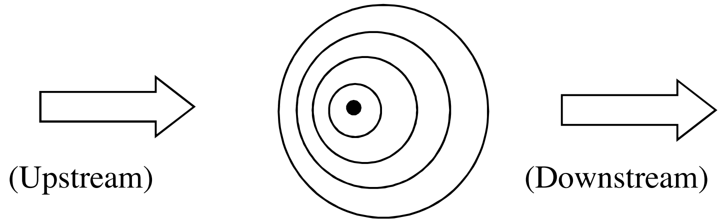
- ✓ According to the variation of physical properties with time

- ✓ According to the variation of physical properties with the length of the channel



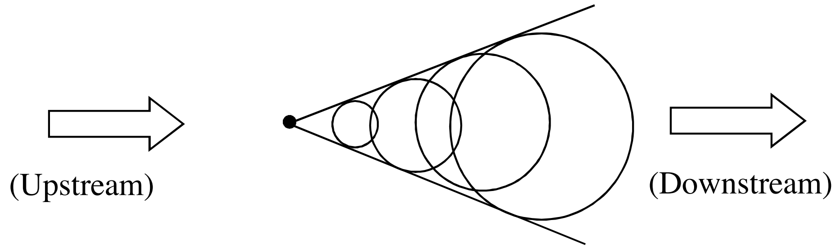
6. Classification of motion

✓ According to Froude number



Free surface top view: the waves that are caused by a pulsating source (black dot) move upstream and downstream.

Figure 9.22 Disturbance in subcritical flow (slow flow)



Free surface top view: the waves that are caused by a pulsating source (black dot) move only downstream.

Figure 9.23 Disturbance in supercritical flow (fast flow)

7. General steady-state non uniform motion equation in open flows

✓ "Specific energy" at a cross section

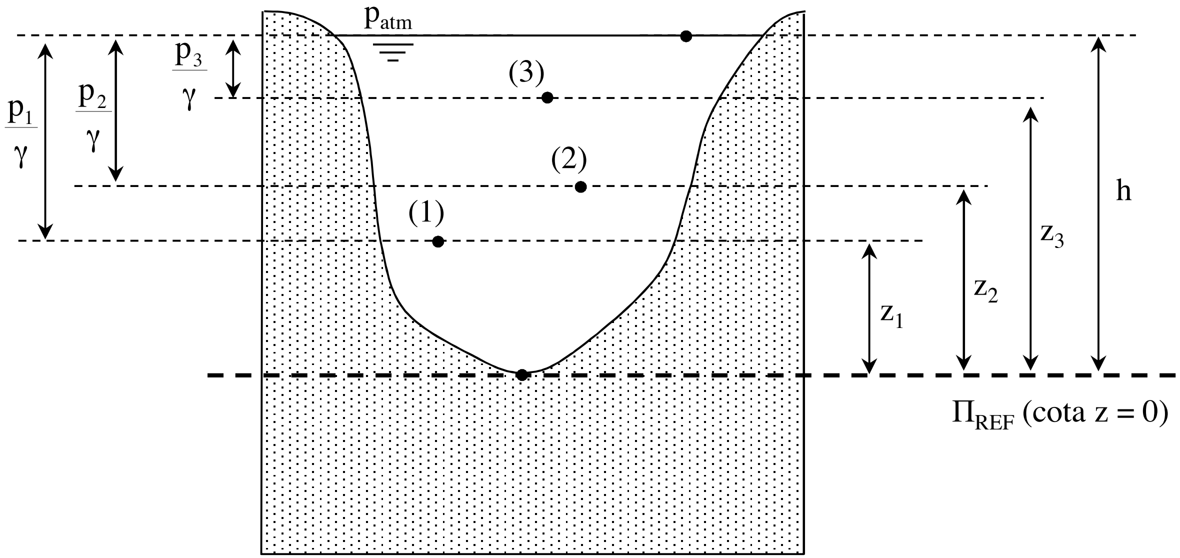


Figure 9.25 Calculation of the specific energy in a channel

$$E_i = z_i + \frac{p_i}{\gamma} + \frac{U_i^2}{2g} \quad \Rightarrow \quad E_i = h + \frac{U_i^2}{2g} \quad \Rightarrow \quad E = h + \alpha \frac{U^2}{2g}$$

Specific energy

7. General steady-state non uniform motion equation in open flows

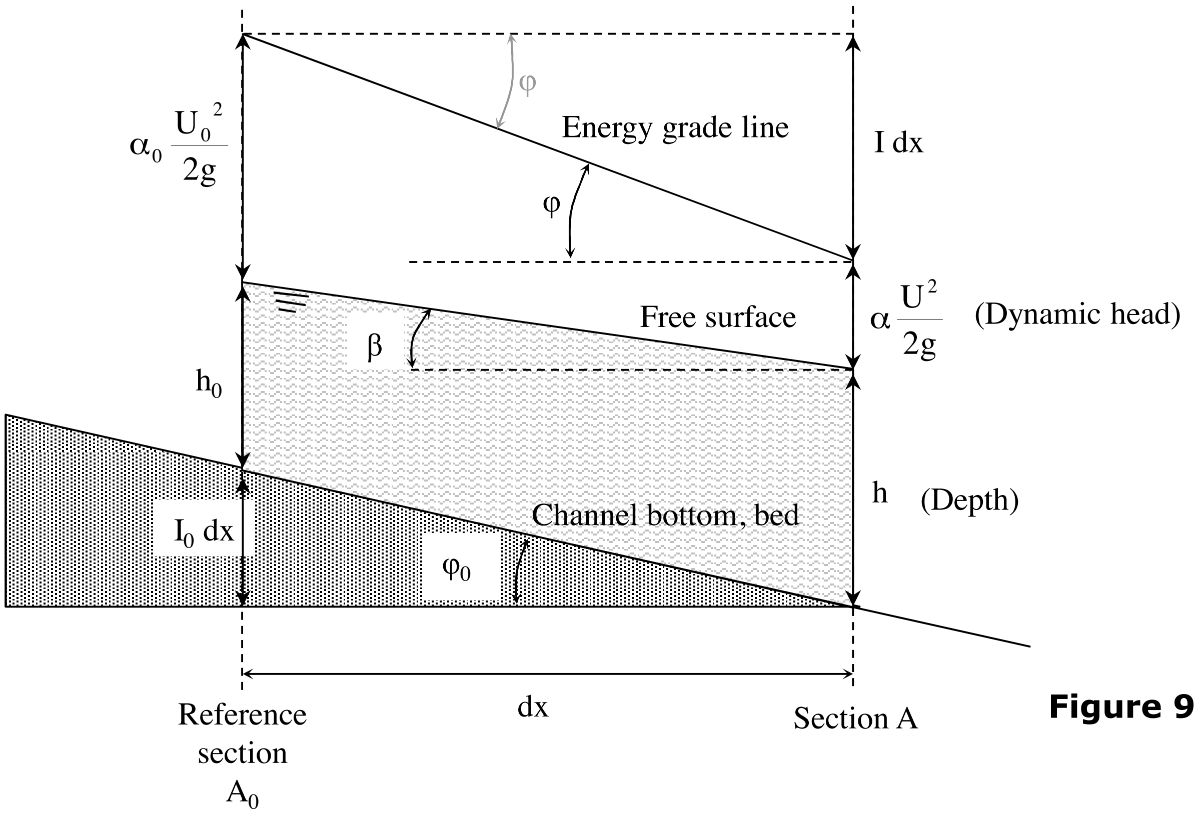


Figure 9.24 Differential length along the channel

φ: angle of the line energy with the horizontal
I = tg φ : slope of the energy line (friction slope), (fluid energy decrease per unit length "x") equals to the unitary head loss.
φ_0 : angle of the bottom with the horizontal
I_0 = tg φ_0 : bottom slope
β : angle of the free surface (or piezometric line) with the horizontal
J = tg β : free surface slope

7. General steady-state non uniform motion equation in open flows

✓ Working hypothesis

- Small slopes

$$I_0 = tg\ \varphi_0 = \sin \varphi_0 = \varphi_0$$
$$I = tg\ \varphi = \sin \varphi = \varphi$$
$$J = tg\ \beta = \sin \beta = \beta$$

Valid for $I_0 < 0{,}1$ and/or $\varphi_0 < 5{,}7^o$

- Steady-state regime

- Smooth variation of the slopes

7. General steady-state non uniform motion equation in open flows

✓ Infinitesimal length of the channel

$$h_0 + \alpha_0 \frac{U_0^{\,2}}{2g} + I_0 dx = h + \alpha \frac{U^2}{2g} + I dx$$

⇓

$$\frac{d}{dx}\left(h + \alpha \frac{U^2}{2g}\right) + (I - I_0) = 0$$

⇓

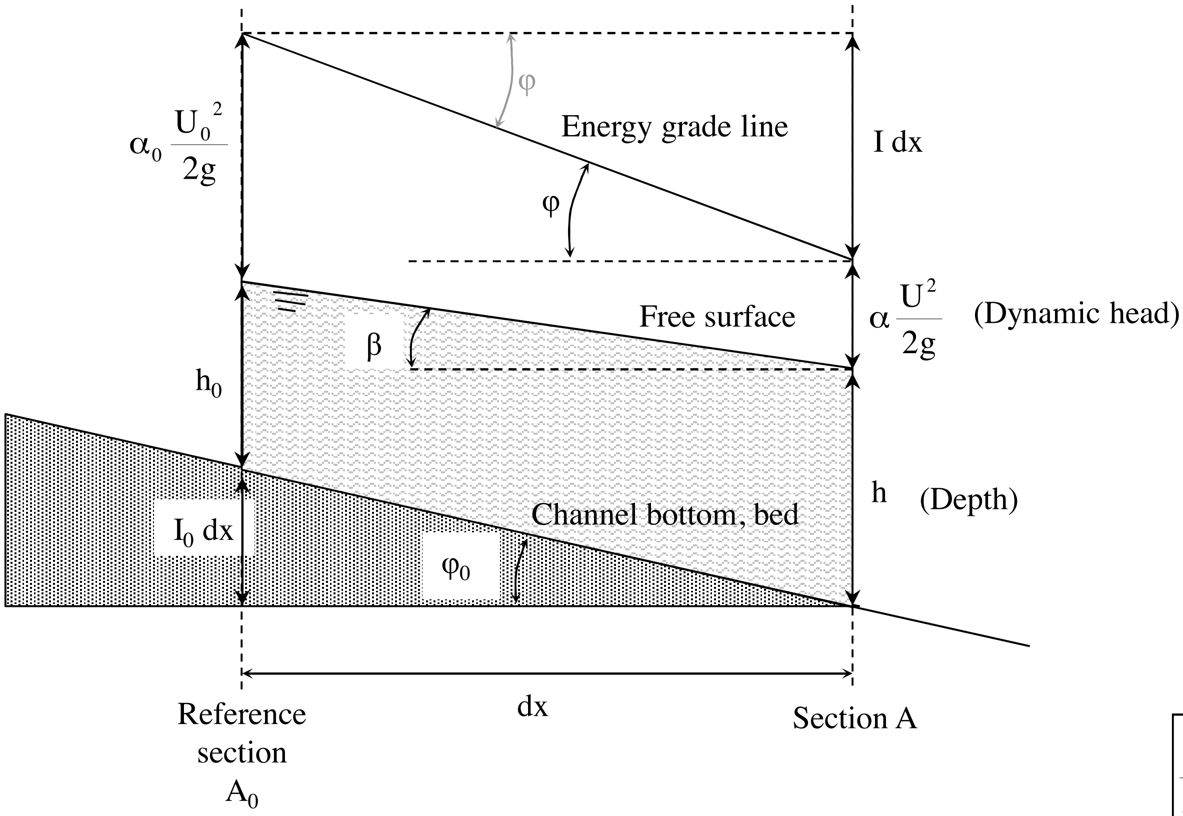
$$\frac{dE}{dx} = I_0 - I$$

8. Steady-state uniform motion in open flows

1. Motion equation
2. Chezy formula
3. Best hydraulic cross section
4. Acceptable average velocity
5. Velocity distribution
6. Specific energy and alternate depths

8.1. Motion equation

✓ Steady-state non uniform regime



$$\frac{d}{dx}\left(h + \alpha\frac{U^2}{2g}\right) + (I - I_0) = 0$$

8.1. Motion equation

✓ Steady-state regime

$$\frac{d}{dx}\left(h + \alpha\frac{U^2}{2g}\right) + (I - I_0) = 0$$

$$\frac{dE}{dx} = I_0 - I$$

✓ Steady-state uniform regime

$$\frac{d}{dx}\left(\alpha\frac{U^2}{2g}\right) = 0$$

$$\frac{dh}{dx} = 0$$

$\Rightarrow$

$$\frac{dE}{dx} = 0 = I_0 - I$$

$$I = I_0$$

$$\beta = \varphi_0 \quad ; \quad tg\beta = tg\varphi_0 \quad ; \quad J = I_0$$

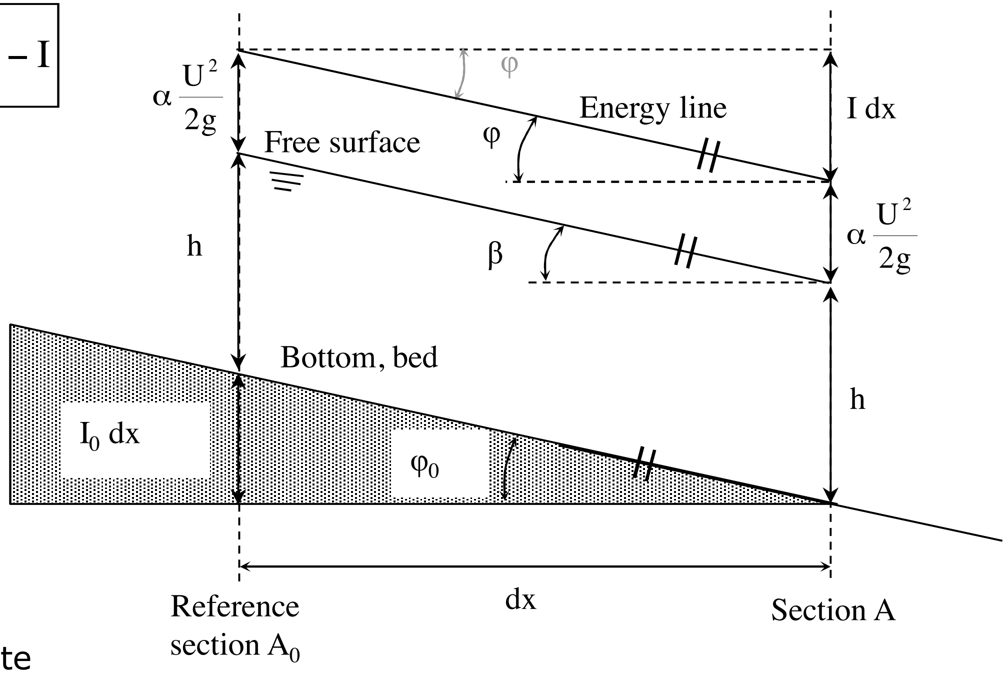


Figure 9.27 Steady-state uniform motion in channels

8.2. Chezy formula

- ✓ Equation

$$U = C\sqrt{RJ}$$

$$C = \sqrt{\frac{2g}{C_f\left(\frac{\varepsilon}{R}, Re\right)}}$$

- ✓ Proof

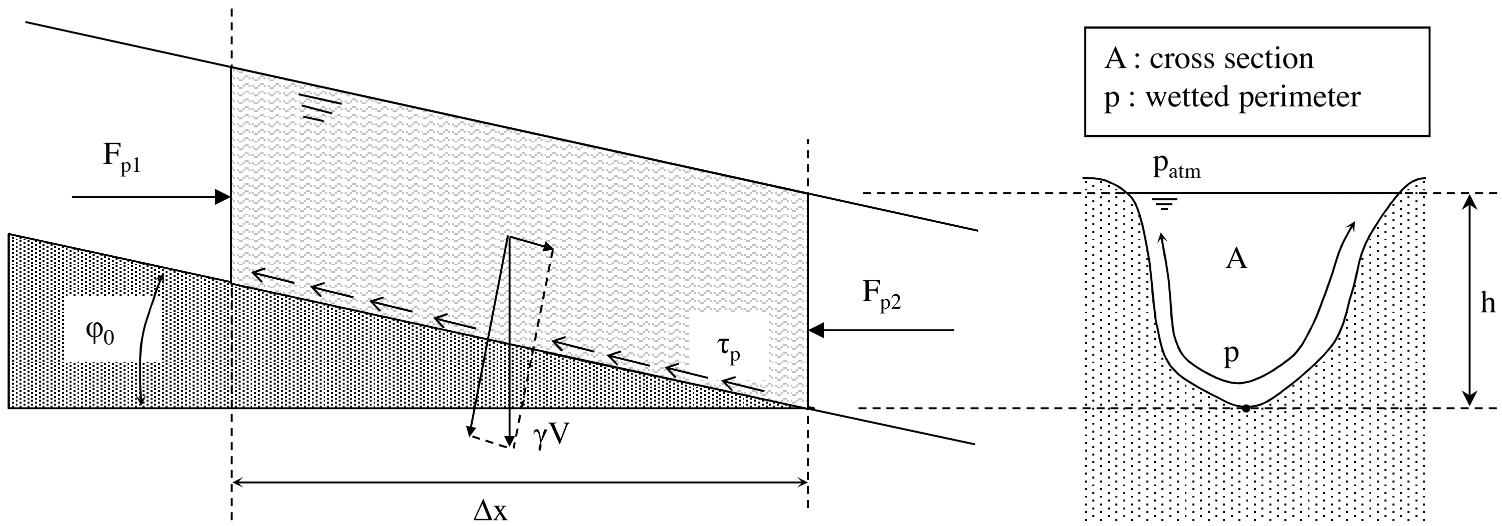


Figure 9.28 Proof of Chezy formula in open channels

8.2. Manning formula

✓ Calculation of the Chezy coefficient

- Manning

$$C = \frac{R^{1/6}}{n}$$

$$U = C\sqrt{RJ}$$

$$U = \frac{1}{n} R^{2/3} J^{1/2}$$

$$Q = \frac{1}{n} A R^{2/3} J^{1/2}$$

n "Manning roughness coefficient"

Tables with constant values of "n" are usually used (it ranges form n = 0,01 for glass up to n = 0,15 for trees in a river bed); though there exists an additional dependence of n on hydraulic radius and Reynolds number.

- Other options:
  - ✓ Kutter: $$C = \frac{100\sqrt{R}}{m + \sqrt{R}}$$ **"m" Kutter roughness coefficient**
  - ✓ Bazin: $$C = \frac{87}{\left(1 + \frac{\gamma}{R^{1/2}}\right)}$$ **"γ" Bazin roughness coefficient**

8.3. Best hydraulic cross section

- Chezy – Manning:

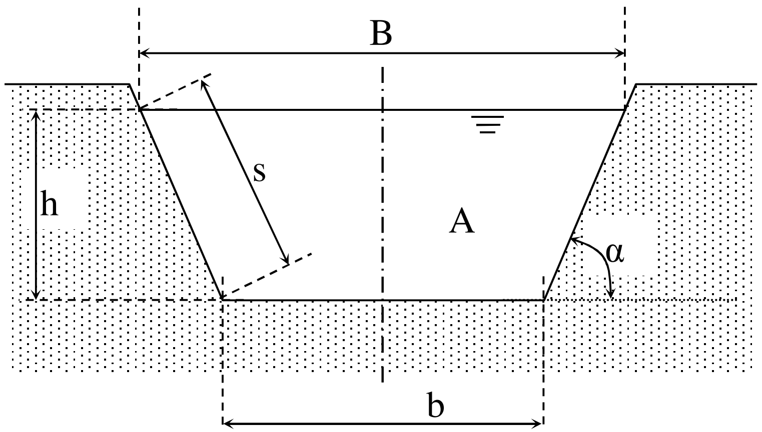
$$Q = \frac{1}{n} A R^{2/3} J^{1/2}$$

MAXIMUM FLOW RATE Q, for a given cross-sectional area A and slope J

- Maximum hydraulic radius, minimum wetted perimeter:

$$\uparrow R = \frac{A}{\downarrow p}$$

- TRAPEZOIDAL CROSS SECTION



$$\mathbf{p = f\,(h,\alpha)}$$

$$p = f(h,\alpha) = \frac{A}{h} - h \cdot ctg\alpha + \frac{2h}{sen\alpha}$$

8.3. Best hydraulic cross section

✓ TRAPEZOIDAL CROSS SECTION

- CASE 1: trapezoid angle known — $\mathbf{p = f(h)}$
- CASE 2: trapezoid angle unknown — $\mathbf{p = f(h,\alpha)}$

- CASE 1: trapezoid angle known — $\mathbf{p = f(h)}$

MAXIMIZE

$$p = f(h) = \frac{A}{h} - h \cdot ctg\alpha + \frac{2h}{sen\alpha}$$

$$\frac{dp}{dh} = 0 \Rightarrow h = \sqrt{\frac{A sen\alpha}{2 - \cos\alpha}}, \quad B = 2s, \quad h_1 = h$$

$$\Rightarrow R = \frac{h}{2}$$

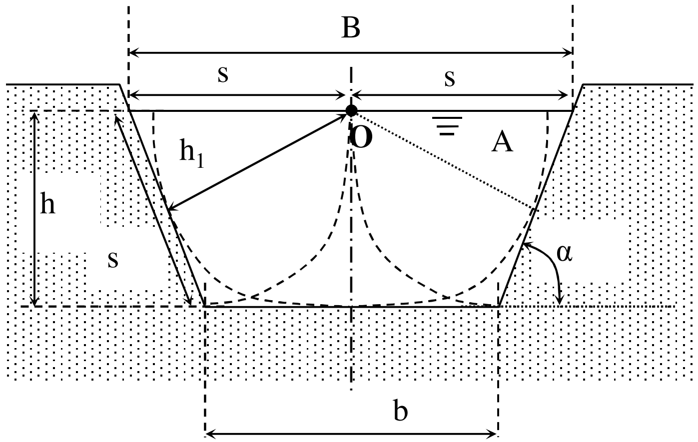


Figure 9.30 Trapezoidal cross section circumscribed into a circumference of radius equaling to the depth "h"

8.3. Best hydraulic cross section

- CASE 1: trapezoid angle known

$$\mathbf{p = f\,(h)}$$

RECTANGULAR CHANNEL: $\alpha = \frac{\pi}{2}$

$$h = \frac{b}{2} = s$$

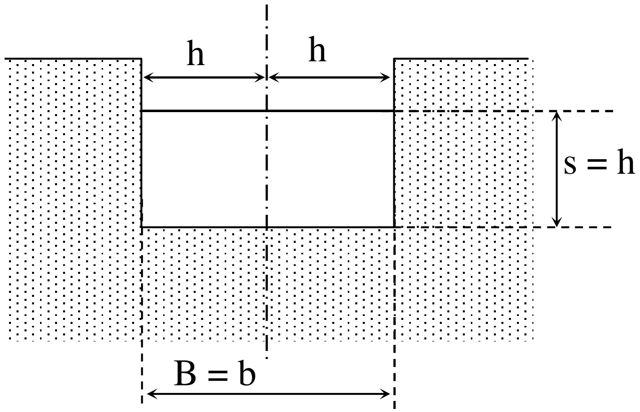


Figure 9.31 Rectangular channel

8.3. Best hydraulic cross section

- CASE 2: trapezoid angle unknown

$$\mathbf{p = f\,(h,\alpha)}$$

MAXIMIZE

$$p = f(h) = \frac{A}{h} - h \cdot ctg\alpha + \frac{2h}{sen\alpha} \quad \Rightarrow \quad \frac{\partial p}{\partial h} = 0 \qquad \frac{\partial p}{\partial \alpha} = 0$$

$$\frac{\partial p}{\partial h} = 0 \quad \Rightarrow \quad h = \sqrt{\frac{A sen\alpha}{2 - \cos\alpha}} \qquad B = 2s \qquad h_1 = h$$

(The same as before)

$$\frac{\partial p}{\partial \alpha} = 0 \quad \Rightarrow \quad \alpha = \frac{\pi}{3} = 60^o$$

$$h = \frac{\sqrt{A}}{1{,}3161}$$

$$b = s$$

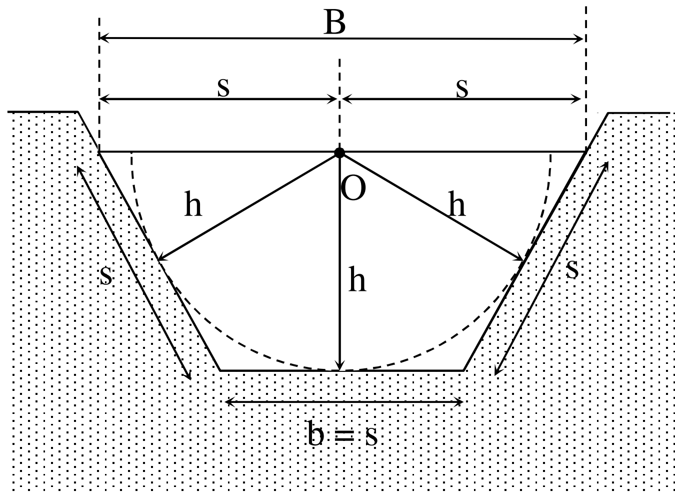


Figure 9.32 Trapezoidal cross section when the trapezoid angle is not known beforehand. Inscribed circumference of radius “h”

8.4. Acceptable average velocity. Velocity distribution

✓ Acceptable average velocity

- High velocity consequences
- Small velocity consequences
- Definition

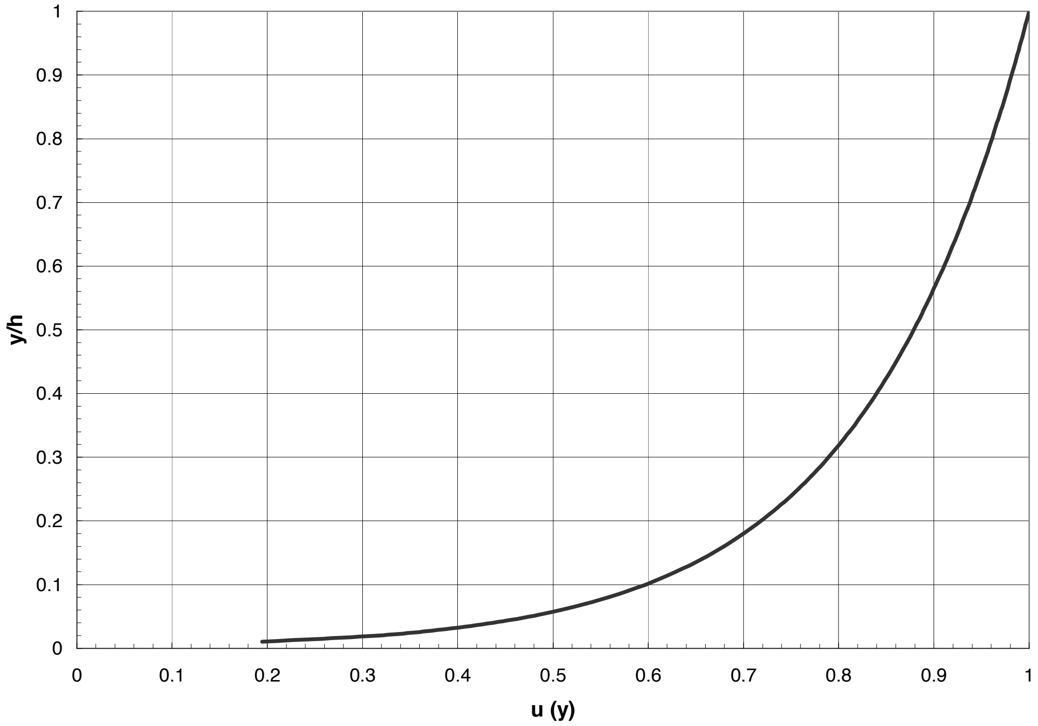
Channel covered with concrete:	- Straight lengths: $U_{máx} = 6$ m/s, usually: $U < 3$ m/s - Branches: $U < 1$ m/s or 1,5 m/s - $U_{min} = 0{,}5$ m/s

8.4. Acceptable average velocity. Velocity distribution

- ✓ Velocity distribution. Vanoni law

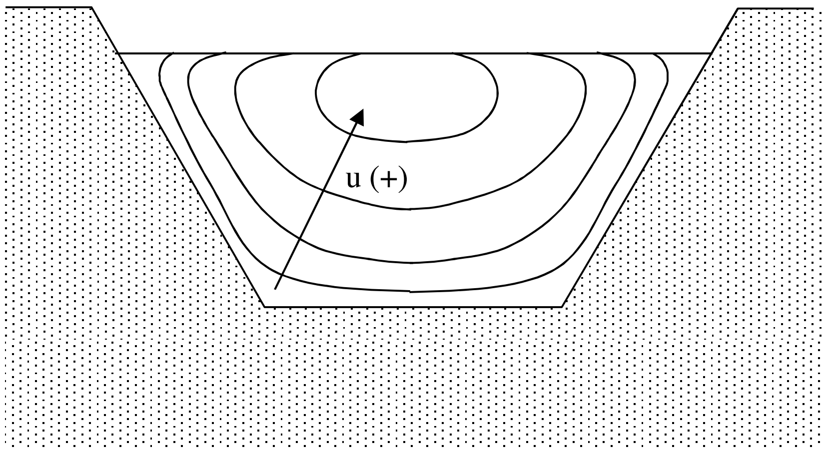
$$\frac{u(y) - u_{max}}{\sqrt{ghJ}} = \frac{2{,}3}{K} \log \frac{y}{h}$$

u(y) : velocity as a function of depth “y”
K: Von Karman constant (K = 0,40)



8.4. Acceptable average velocity. Velocity distribution

- ✓ Velocity distribution. Influence of lateral slopes and air



- ✓ Velocity distribution. Experimental measurement of the average velocity

- ✓ Velocity distribution. Correction coefficients

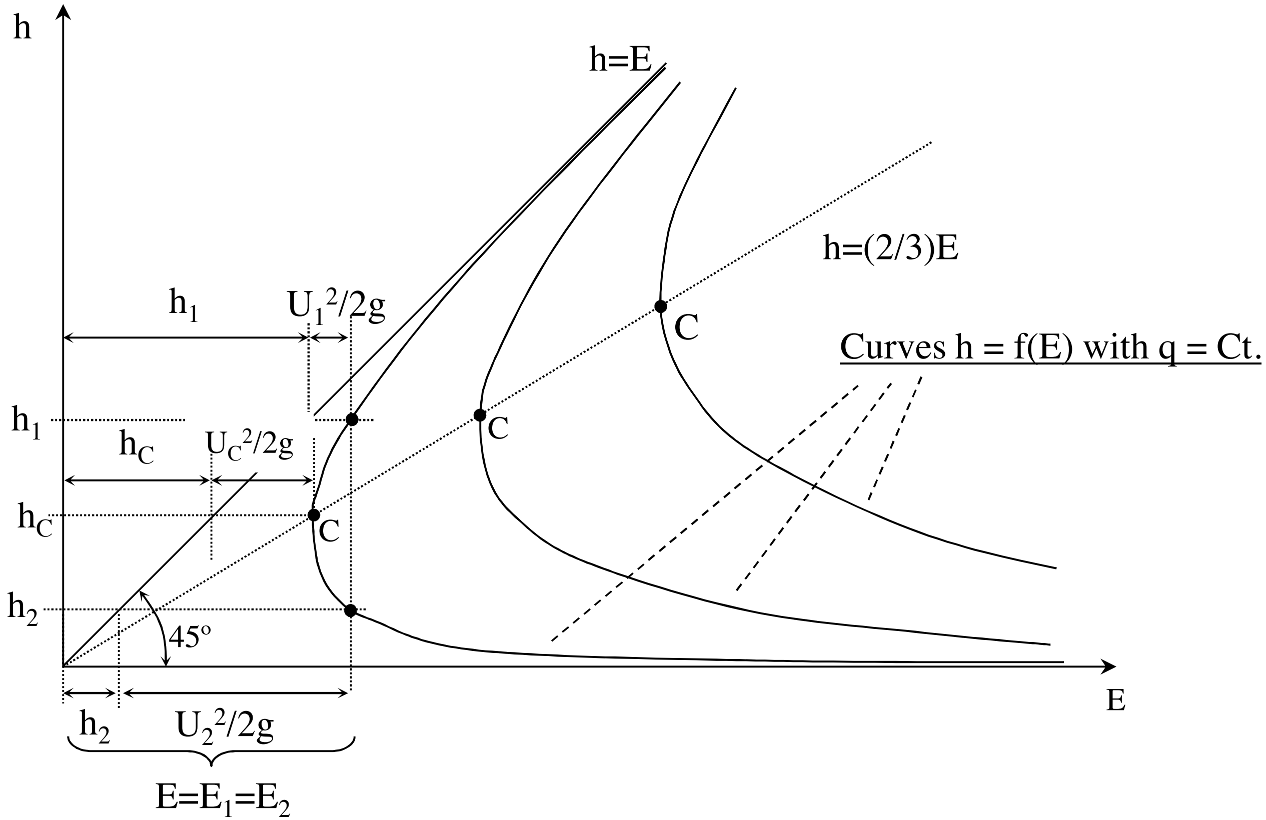
8.6. Alternate depths

✓ Specific energy. Rectangular channel

$$E = h + \frac{U^2}{2g}$$

$$Q = (bh)U \qquad q = \frac{Q}{b}$$

$$E = h + \frac{q^2}{2gh^2} \Rightarrow (E-h)h^2 = \frac{q^2}{2g} = Ct$$



8.6. Alternate depths

✓ Slopes

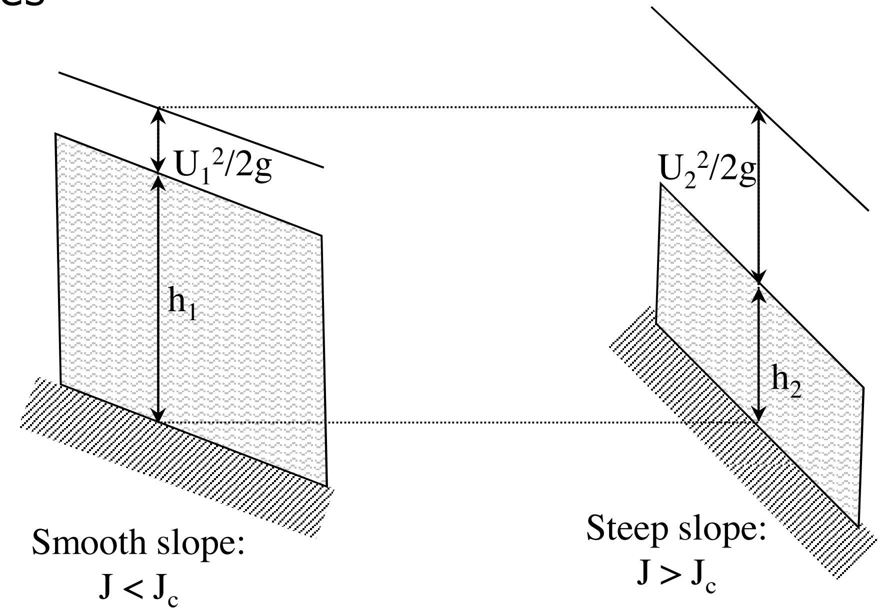


Figure 9.36 Alternate depths: subcritical and supercritical depths

✓ Critical conditions (C)

$$\frac{dE}{dh} = 0 \Rightarrow h_C = \left(\frac{q^2}{g}\right)^{1/3}$$ **Critical depth**

$$Fr_C = \frac{U_C}{\sqrt{gh_C}} = 1$$ **Critical Froude number**

$$E = h + \frac{q^2}{2gh^2}$$

$$U_C = \sqrt{gh_C}$$ **Critical velocity**

$$E_C = \frac{3h_C}{2}$$ **Critical energy**