CHAPTER 5. INVISCID INCOMPRESSIBLE FLUID FLOW

1. Application of Bernoulli equation to an incompressible fluid in steady-state regime:

    1.1. Measurement of pressure and velocity

    1.2. Measurement of volumetric flow rate in pressurized ducts

1.1. Application of Bernoulli to the measurement of pressure and velocity

✓ Bernoulli equation

$$\gamma z + p + \frac{1}{2}\rho U^2 = Cte = p_T$$

- Hydrostatic pressure
- Static pressure
- Dynamic pressure
- Total pressure or stagnation pressure

$$z + \frac{p}{\gamma} + \frac{U^2}{2g} = Cte = h_T$$

- Elevation head
- Pressure head or static head
- Velocity head
- Piezometric head
- Total head

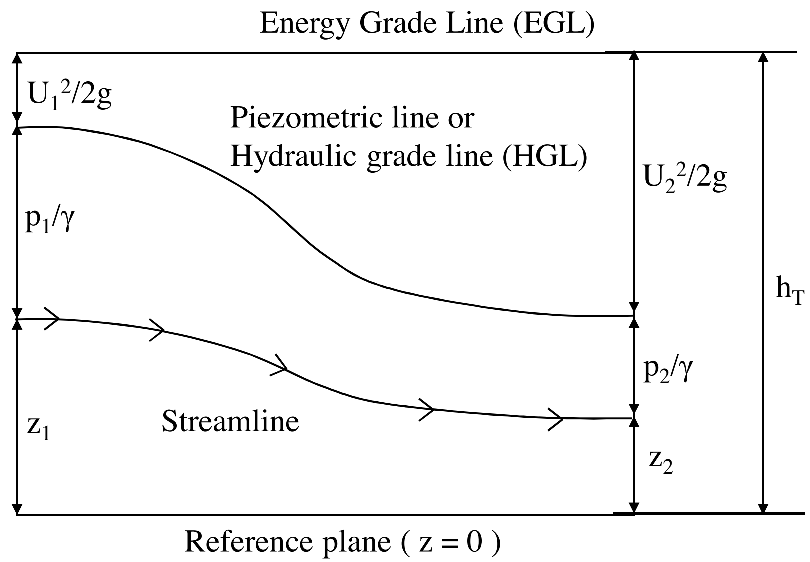


Figure 4.4 Energy heads diagram

1.1. Application of Bernoulli to the measurement of pressure and velocity

✓ Pitot tube or total pressure probe

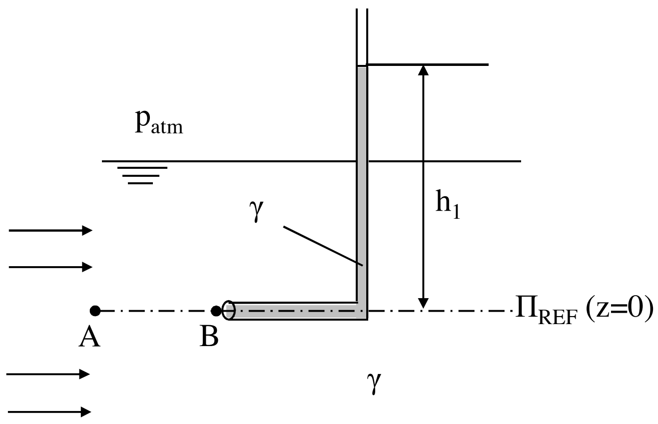


Figure 5.1 Pitot tube

$$\frac{p_A}{\gamma} + \frac{U_A^2}{2g} = h_1$$

$$(\gamma h_1) = p_A + \frac{1}{2}\rho U_A^2$$

1.1. Application of Bernoulli to the measurement of pressure and velocity

- ✓ Piezometer or static pressure probe

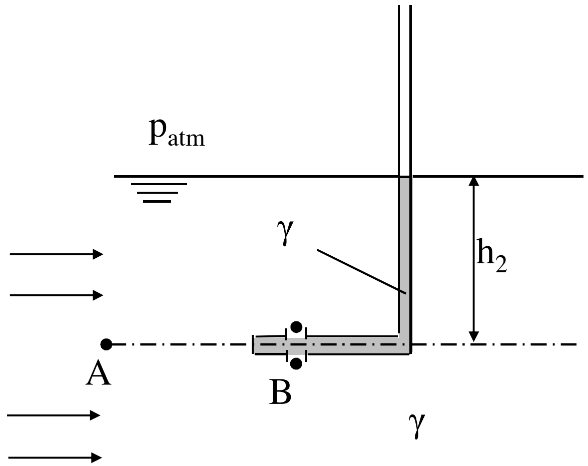


Figure 5.2 Piezometer

$$\frac{p_A}{\gamma} = h_2$$

$$(\gamma h_2) = p_A$$

1.1. Application of Bernoulli to the measurement of pressure and velocity

- ✓ Prandtl tube (or Pitot-static probe)

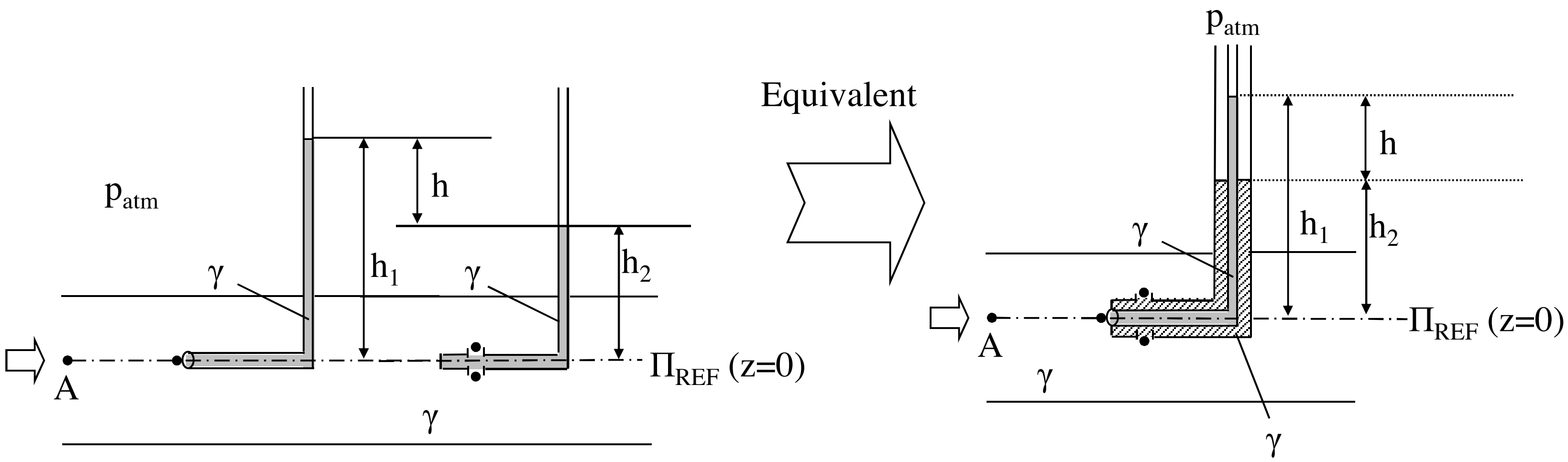


Figure 5.3 The combination of a Pitot tube and a piezometer forms a Prandtl tube

$$h = \frac{U_A^{\,2}}{2g}$$

$$(\gamma h) = \frac{1}{2}\rho U_A^{\,2}$$

$$U_A = \sqrt{2gh}$$

1.1. Application of Bernoulli to the measurement of pressure and velocity

- ✓ Prandtl tube. Use of differential manometer

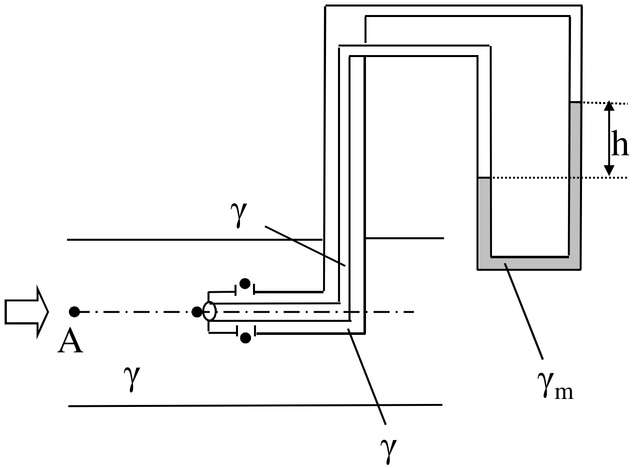
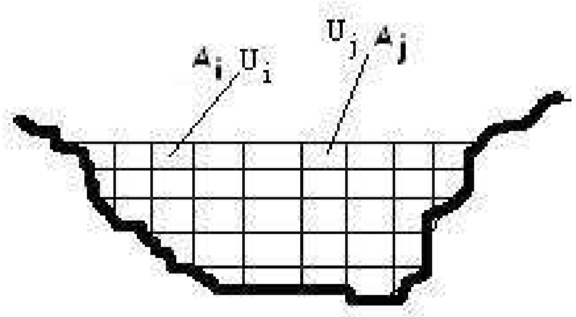


Figure 5.4 Prandtl tube connected to a U tube

$$h(\gamma_m - \gamma) = \frac{1}{2}\rho U_A^{\;2}$$

1.2. Application of Bernoulli to the measurement of flow rate

- ✓ Prandtl tube. Measurement of flow rate



$$Q = \iint_A U \cdot dA \quad \Rightarrow \quad Q = \sum U_i \cdot A_i$$

Figure 5.5 Division of a channel to measure the flow rate

- ✓ Prandtl tube. Velocity coefficient

$$U_{real} = C_v U_{theoretical} = C_v \sqrt{2gh}$$

1.2. Application of Bernoulli to the measurement of flow rate

- ✓ Venturi

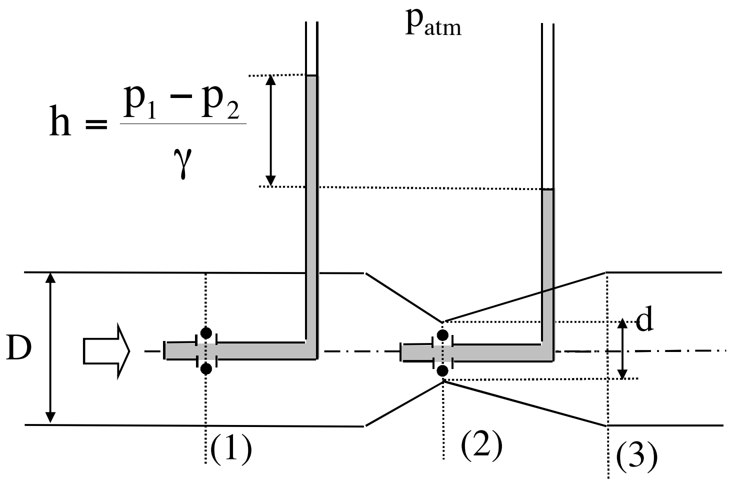


Figure 5.7 Venturi meter

$$U_2 = \sqrt{\frac{2gh}{\left(1 - \frac{d^4}{D^4}\right)}} = \sqrt{\frac{2gh}{\left(1 - \beta^4\right)}}$$

$$Q = A_2 U_2 = \frac{A_2}{\sqrt{1 - \beta^4}} \sqrt{2gh}$$

1.2. Application of Bernoulli to the measurement of flow rate

✓ Venturi

- Headloss: Velocity coefficient

$$U_r = C_v U_t$$

$$U_{2,real} = C_v \sqrt{\frac{2gh}{\left(1 - \frac{d^4}{D^4}\right)}}$$

- Vena contracta: Contraction coefficient

$$A_r = C_c A_t$$

$$\beta_r^2 = C_c \beta^2$$

$$Q_r = A_r U_r = C_v \frac{C_c A_t}{\sqrt{1 - C_c^2 \beta^4}} \sqrt{2gh}$$

$$Q_r = C_d \frac{A_t}{\sqrt{1-\beta^4}} \sqrt{2gh} \iff Q_r = C_d E A_t \sqrt{2gh}$$

1.2. Application of Bernoulli to the measurement of flow rate

✓ Nozzle

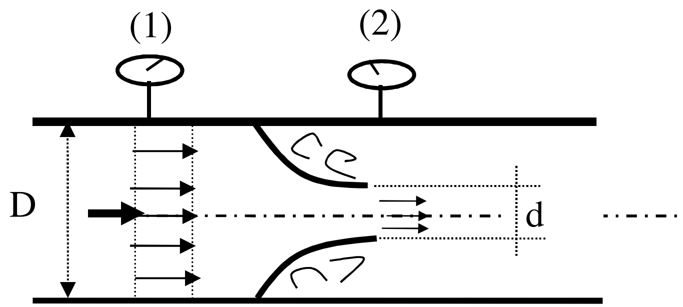


Figure 5.9 Nozzle inside a pipe

✓ Orifice meter or diaphragm

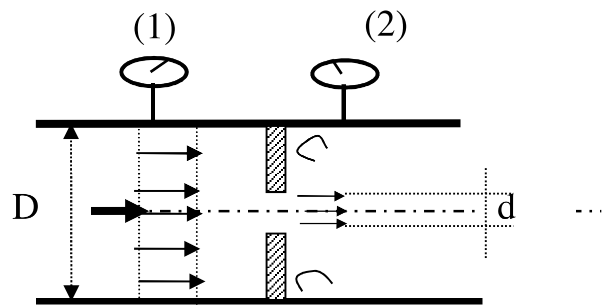


Figure 5.10 Diaphragm inside a pipe