CHAPTER 4. THE ENERGY EQUATION

1. The energy equation
2. The mechanical energy equation. Definition of energy loss and headloss
3. Bernoulli equation in steady-state. Piezometric head constancy
4. Concepts related to power and efficiency

1. The energy equation

1. General equation of conservation of energy*
2. Simplified forms of the energy equation*
3. Average velocity – Coriolis coefficient*
4. Application to a streamline*

1.1. General equation of conservation of energy

✓ Word statement:

{ Rate of energy transfer to the system } = { Rate of energy change (increase) in the system }

- Intrinsic energy:

  - Kinetic energy
  - Potential energy
  - Internal energy
  - Chemical energy
  - Nuclear energy

- Energy transfer:

  - Heat
  - Work

  } Signs agreement

1.1. General equation of conservation of energy

✓ Formulation:

$$\frac{dQ}{dt} - \frac{dW}{dt} = \frac{dE_{sys}}{dt}$$

- Intrinsic energy:

  - Kinetic energy
  - Potential energy
  - Internal energy

$$b = e = \tilde{u} + \frac{U^2}{2} + gz$$

$$E_{sys} = \iiint_{V_{sys}} \rho(\tilde{u} + \frac{U^2}{2} + gz)dV$$

- Transport theorem:

$$\frac{dB_{sys}}{dt} = \frac{d}{dt}\iiint_{Vc} \rho b dV + \oiint_{Ac} b\rho\vec{U}d\vec{A}$$

$$\dot{Q} - \dot{W} = \frac{d}{dt}\iiint_{Vc} \rho(\tilde{u} + \frac{U^2}{2} + gz)dV + \oiint_{Ac} \rho(\tilde{u} + \frac{U^2}{2} + gz)\vec{U}_r d\vec{A}$$

1.1. General equation of conservation of energy

✓ Analysis of the types of heat (Course: Thermal technology in the 3rd year)

- Conduction
- Convection
- Radiation

✓ Analysis of the types of work

- Shaft work
- Work done by shear-stresses at the control surface
- Work done by pressure at the control surface:
  - Flux work
  - Deformation work

1.1. General equation of conservation of energy

✓ Analysis of the types of works

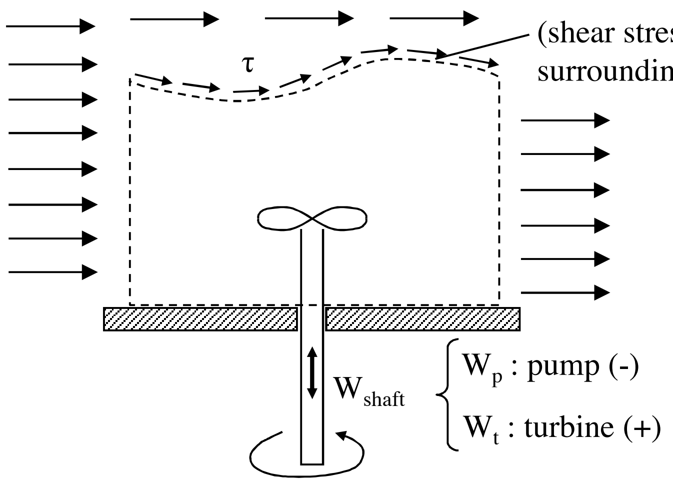


Figure 4.1 Analysis of the types of mechanical work

- Shaft work: $\dot{W}_{shaft} = M \cdot \omega$

- Work done by shear stresses:

$$\dot{W}_{shear} = -\oiint_{A_c} \vec{U}(\vec{\tau} dA)$$

- Work done by pressure:

- Flux work: $\dot{W}_{flux} = \oiint_{A_c} \vec{U}_r p d\vec{A}$

- Deformation work: $\frac{dW_D}{dt} = \dot{W}_D = \oiint_{A_c} \vec{U}_C p d\vec{A}$

1.1. General equation of conservation of energy

- ✓ Energy equation

$$\dot{Q} - \dot{W} = \frac{d}{dt}\iiint_{V_c} \rho(\tilde{u} + \frac{U^2}{2} + gz)dV + \oiint_{A_c} \rho(\tilde{u} + \frac{U^2}{2} + gz)\vec{U}_r d\vec{A}$$

$$\Downarrow$$

$$\dot{Q} - (\dot{W}_{shaft} + \dot{W}_{shear}) - \oiint_{A_c} p\vec{U}_r d\vec{A} - \oiint_{A_c} p\vec{U}_C d\vec{A} = \frac{d}{dt}\iiint_{V_c} \rho(\tilde{u} + \frac{U^2}{2} + gz)dV + \oiint_{A_c} \rho(\tilde{u} + \frac{U^2}{2} + gz)\vec{U}_r d\vec{A}$$

$$\Downarrow$$

$$\dot{Q} - (\dot{W}_{shaft} + \dot{W}_{shear}) - \oiint_{A_c} p\vec{U}_C d\vec{A} = \frac{d}{dt}\iiint_{V_c} \rho(\tilde{u} + \frac{U^2}{2} + gz)dV + \oiint_{A_c} \rho(\tilde{u} + \frac{p}{\rho} + \frac{U^2}{2} + gz)\vec{U}_r d\vec{A}$$

"Integral approach of the General Equation of Energy"

1.1. General equation of conservation of energy

- ✓ Energy equation

$$\dot{Q} - (\dot{W}_{shaft} + \dot{W}_{shear}) - \oiint_{A_c} p\vec{U}_C d\vec{A} = \frac{d}{dt}\iiint_{V_c} \rho(\tilde{u} + \frac{U^2}{2} + gz)dV + \oiint_{A_c} \rho(\tilde{u} + \frac{p}{\rho} + \frac{U^2}{2} + gz)\vec{U}_r d\vec{A}$$

- ✓ Steady-state regime and rigid and fixed control volume:

$$\dot{Q} - (\dot{W}_{shaft} + \dot{W}_{shear}) = \oiint_{A_c} \rho(\tilde{u} + \frac{p}{\rho} + \frac{U^2}{2} + gz)\vec{U} d\vec{A}$$

- ✓ Finite number of inlet and outlet, uniform flow:

$$\dot{Q} - (\dot{W}_{shaft} + \dot{W}_{shear}) = \sum_{out} q_m (\tilde{u} + \frac{p}{\rho} + \frac{U^2}{2} + gz) - \sum_{in} q_m (\tilde{u} + \frac{p}{\rho} + \frac{U^2}{2} + gz)$$

1.3. Average velocity – Coriolis' coefficient

- ✓ Average velocities and properties

- ✓ Kinetic energy correction coefficient or Coriolis' coefficient α

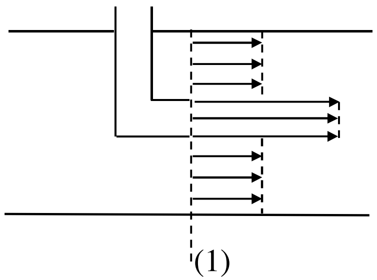


Figure 4.2 Jet injection inside a pipe

$$\dot{E}_k = \iint_A \rho \frac{U^3}{2} dA \neq \rho \frac{U_{av}{}^3}{2} A$$

$$\dot{E}_k = \alpha \cdot \rho \frac{U_{av}{}^3}{2} A = \alpha \cdot q_m \frac{U_{av}{}^2}{2}$$

$$\alpha = \frac{\iint_A U^3 dA}{U_{av}{}^3 A}$$

- ✓ Finite number of inlets and outlets:

$$\dot{Q} - (\dot{W}_{shaft} + \dot{W}_{shear}) = \sum_{out} q_m (\tilde{u} + \frac{p}{\rho} + \alpha \frac{U_{av}{}^2}{2} + gz) - \sum_{in} q_m (\tilde{u} + \frac{p}{\rho} + \alpha \frac{U_{av}{}^2}{2} + gz)$$

1.4. Application to a streamline

✓ Energy equation in a streamline

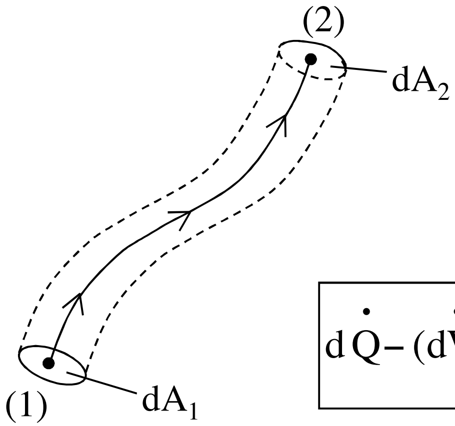


Figure 4.3 Streamline and differential streamtube

$$d\dot{Q} - (d\dot{W}_{shaft} + d\dot{W}_{shear}) = dq_{m2}(\tilde{u}_2 + \frac{p_2}{\rho_2} + \frac{U_2^2}{2} + gz_2) - dq_{m1}(\tilde{u}_1 + \frac{p_1}{\rho_1} + \frac{U_1^2}{2} + gz_1)$$

Specific properties: $q = \frac{d\dot{Q}}{dq_m}$ $\quad w_{shaft} = \frac{d\dot{W}_{shaft}}{dq_m}$ $\quad w_{shear} = \frac{d\dot{W}_{shear}}{dq_m}$

$$q - (w_{shaft} + w_{shear}) = (\tilde{u}_2 - \tilde{u}_1) + \left(\frac{p_2}{\rho_2} - \frac{p_1}{\rho_1}\right) + \left(\frac{U_2^2}{2} - \frac{U_1^2}{2}\right) + (gz_2 - gz_1)$$

$$\left(\tilde{u}_1 + \frac{p_1}{\rho_1} + \frac{U_1^2}{2} + gz_1\right) + q - (w_{shaft} + w_{shear}) = \left(\tilde{u}_2 + \frac{p_2}{\rho_2} + \frac{U_2^2}{2} + gz_2\right)$$

2. The mechanical energy equation. Definition of energy loss and headloss

- ✓ The mechanical energy equation

$$q-(w_{shaft}+w_{shear})=(\tilde{u}_2-\tilde{u}_1)+\left(\frac{p_2}{\rho_2}-\frac{p_1}{\rho_1}\right)+\left(\frac{U_2^{\,2}}{2}-\frac{U_1^{\,2}}{2}\right)+(gz_2-gz_1)$$

$$-(w_{shaft}+w_{shear})-\left[(\tilde{u}_2-\tilde{u}_1)+\int_1^2 p\,dv-q\right]-\int_1^2\frac{dp}{\rho}=\left(\frac{U_2^{\,2}}{2}-\frac{U_1^{\,2}}{2}\right)+(gz_2-gz_1)$$

$$-(w_{shaft}+w_{shear})-\int_1^2\frac{dp}{\rho}-(gh_L)=\left(\frac{U_2^{\,2}}{2}+gz_2\right)-\left(\frac{U_1^{\,2}}{2}+gz_1\right)$$

Mechanical energy equation for a steady-state flow along a streamline

- $-(w_{shaft}+w_{shear})$: Work to machinery or shear stresses
- $-\int_1^2\frac{dp}{\rho}$: Work by pressure forces
- (gh_L): Mechanical energy loss
- $\left(\frac{U_2^{\,2}}{2}+gz_2\right)-\left(\frac{U_1^{\,2}}{2}+gz_1\right)$: Change of specific mechanical energy of the fluid

$$(gh_L)=\left[(\tilde{u}_2-\tilde{u}_1)+\int_1^2 p\,dv-q\right]$$

Mechanical energy loss

2. The mechanical energy equation. Definition of energy loss and headloss

- ✓ Mechanical energy equation. Incompressible flow

$$\frac{p_1}{\rho} + \frac{U_1^2}{2} + gz_1 - (w_{shaft} + w_{shear}) = \frac{p_2}{\rho} + \frac{U_2^2}{2} + gz_2 + gh_L$$

$$(gh_L) = \left[\underbrace{(\tilde{u}_2 - \tilde{u}_1)} - \underbrace{q}\right]$$

$$\left\{\begin{matrix}\text{Gain of internal}\\ \text{energy}\end{matrix}\right\} + \left\{\begin{matrix}\text{Heat given to the}\\ \text{surroundings}\end{matrix}\right\}$$

3. Bernoulli equation

✓ Bernoulli equation in steady-state regime

$$-(w_{shaft} + w_{shear}) - \int_1^2 \frac{dp}{\rho} - (gh_L) = \left(\frac{U_2^{\,2}}{2} + gz_2\right) - \left(\frac{U_1^{\,2}}{2} + gz_1\right)$$

- ✓ Flow without any friction (inviscid fluid)
- ✓ Flow without any heat transfer (neither addition nor extraction)
- ✓ Flow without shaft work between (1) and (2)

$$\int_1^2 \frac{dp}{\rho} + \frac{1}{2}(U_2^{\,2} - U_1^{\,2}) + g(z_2 - z_1) = 0$$

"**Bernoulli equation in a compressible fluid**" (steady-state regime) valid for a streamline.

- Incompressible fluid

$$gz + \frac{p}{\rho} + \frac{U^2}{2} = Cte$$

- ✓ Alternative forms of Bernoulli equation

$$gz + \frac{p}{\rho} + \frac{U^2}{2} = Cte$$

$$\gamma z + p + \frac{1}{2}\rho U^2 = Cte = p_T$$

$$z + \frac{p}{\gamma} + \frac{U^2}{2g} = Cte = h_T$$

{Elevation head} {Pressure head} {Velocity head} {Total head}

- ✓ Energy heads diagram

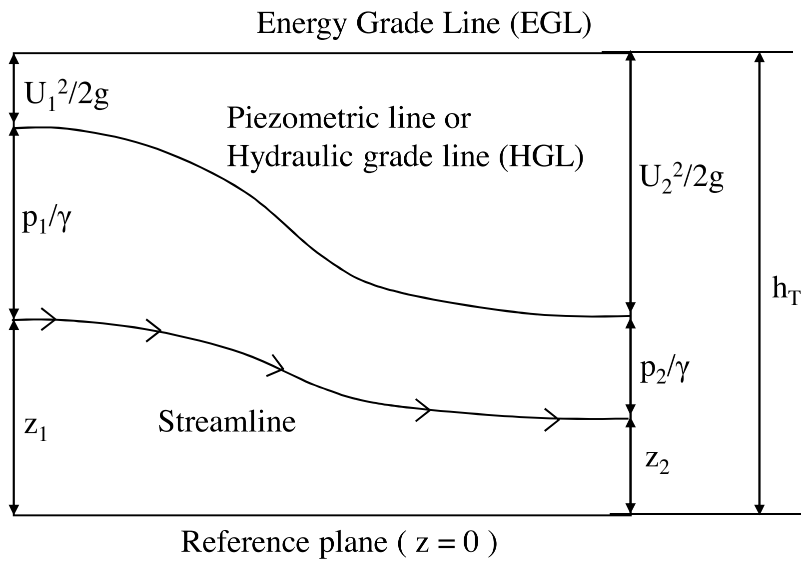


Figure 4.4 Energy heads diagram

3. Piezometric head constancy

✓ Piezometric head constancy

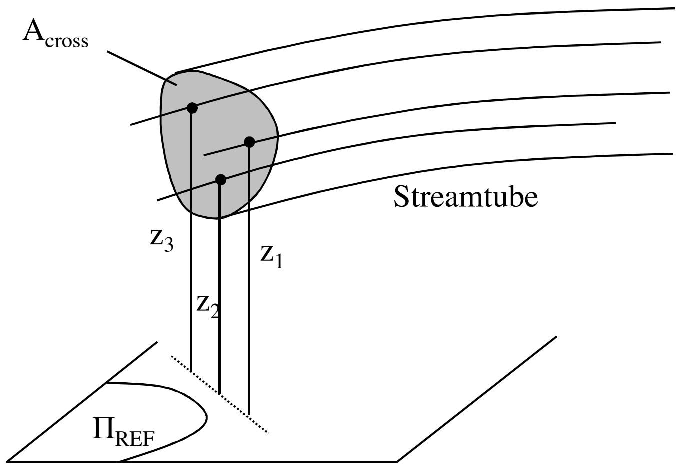


Figure 4.5 Piezometric head constancy

$$\frac{p_1}{\gamma} + z_1 = \frac{p_2}{\gamma} + z_2 = \frac{p_3}{\gamma} + z_3 = Cte$$

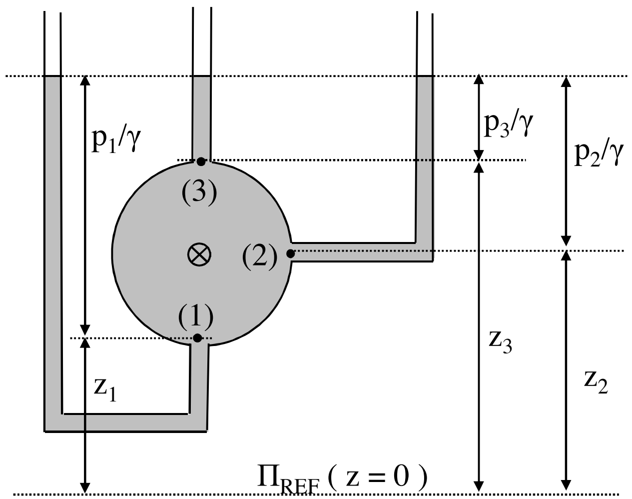


Figure 4.6 Manometers connected to a pipe

3. Bernoulli equation

✓ Generalization to a streamtube of:

- Bernoulli equation

$$z_1 + \frac{p_1}{\gamma} + \alpha_1 \frac{U_1^2}{2g} = z_2 + \frac{p_2}{\gamma} + \alpha_2 \frac{U_2^2}{2g} = h_{T,section}$$

- Steady-state mechanical energy equation

$$z_1 + \frac{p_1}{\gamma} + \alpha_1 \frac{U_1^2}{2g} + H_p = z_2 + \frac{p_2}{\gamma} + \alpha_2 \frac{U_2^2}{2g} + H_t + h_L$$

H_p: Net head of the pump

H_t: Net head of the turbine

h_L: Headloss

4. Definition of power terms

✓ Power of a finite stream

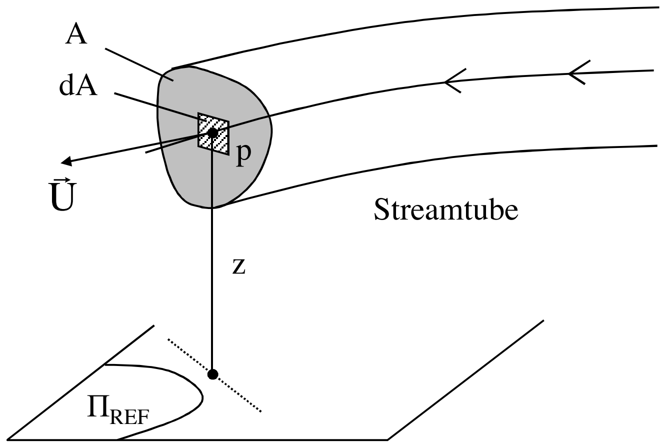


Figure 4.8 Streamtube

$$h_T = z + \frac{p}{\gamma} + \frac{U^2}{2g}$$

$$\dot{E} = \gamma Q\left(z + \frac{p}{\gamma} + \alpha\frac{U_{av}^{\ 2}}{2g}\right) = \gamma Q h_{T,section}$$

4. Definition of power terms

- ✓ Dissipated power

$$\dot{E}_{lost} = \gamma Q h_L$$

- ✓ Power of a pump

$$\left|\dot{W}_p\right| = \gamma Q H_p$$

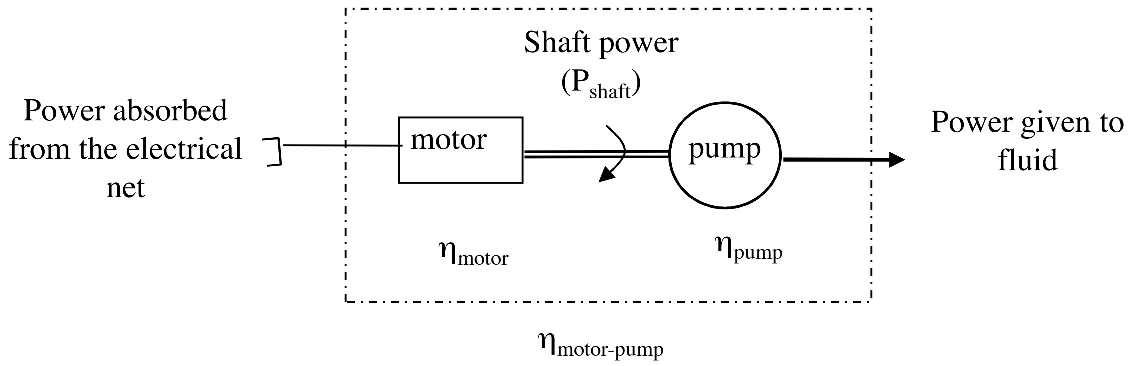
- ✓ Power of a turbine

$$\dot{W}_t = \gamma Q H_t$$

4. Definition of power terms

✓ Efficiencies

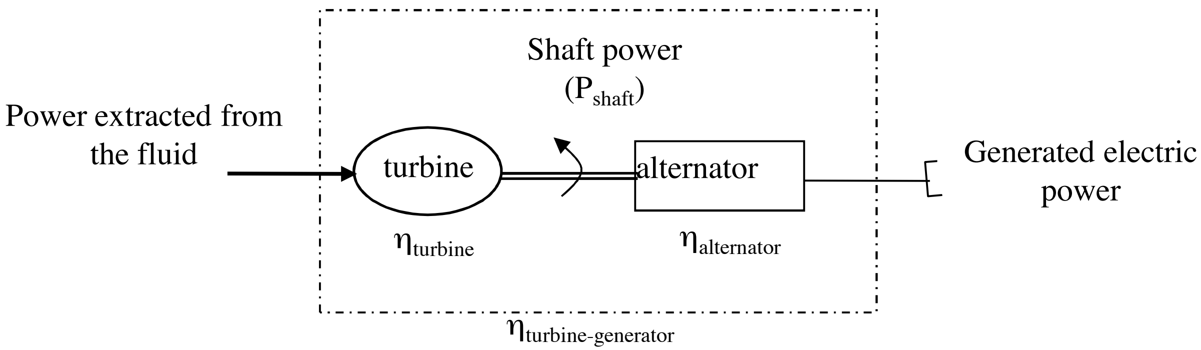
- Pump



$$\eta_{pump} = \frac{|\dot{W}_p|}{P_{shaft}} = \frac{\gamma Q H_p}{P_{shaft}}$$

$$\eta_{motor} = \frac{P_{shaft}}{P_{electric}}$$

- Turbine



$$\eta_{turbine} = \frac{P_{shaft}}{\dot{W}_t} = \frac{P_{shaft}}{\gamma Q H_t}$$

$$\eta_{alternator} = \frac{P_{electric}}{P_{shaft}}$$