CHAPTER 3. FUNDAMENTAL CONCEPTS USED IN FLUID FLOW ANALYSIS

1. Display of the fluid flow
2. Description and classification of fluid flow
3. The boundary layer concept
4. Concepts: streamtube, volumetric flow rate, mass flow rate
5. Reynolds´ transport theorem
6. The continuity equation

1. Display of the fluid flow

1. Field of application of kinematics. Lagrangian and Eulerian description.

2. Display of the fluid flow

   - ✓ Streamline*
   - ✓ Pathline or trajectory*
   - ✓ Streakline*

1.1. Field of application of kinematics. Lagrangian and Eulerian description

- Kinematics
- Flow field

$$\boxed{\vec{U} = u(x,y,z,t)\hat{i} + v(x,y,z,t)\hat{j} + w(x,y,z,t)\hat{k}}$$

- Eulerian description
- Lagrangian description

1.2. Display of the fluid flow

✓ Streamline

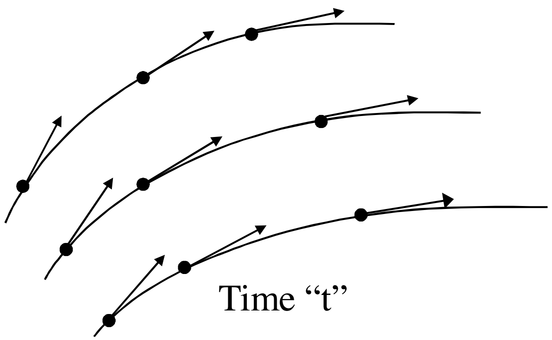


Figure 3.1 Streamlines

- Definition
- They cannot cut each other
- Analytical concept
- Line defined in a precise moment or time

1.2. Display of the fluid flow

✓ Pathline or trajectory

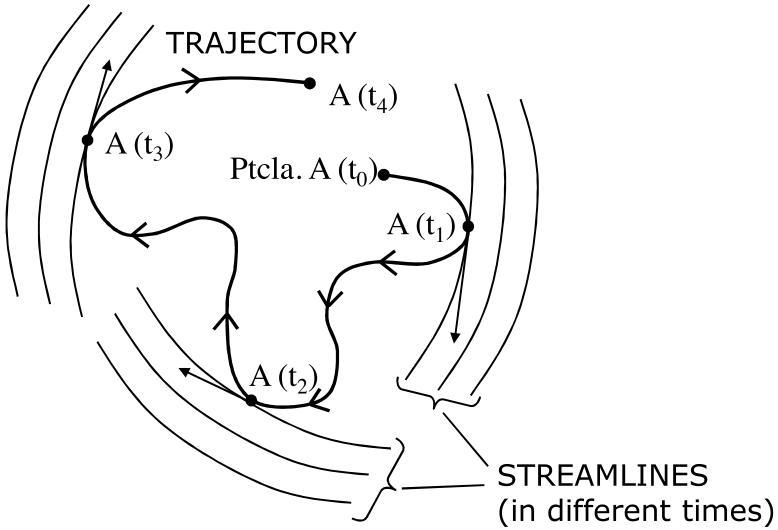


Figure 3.2 Trajectory

- Definition
- Observable in laboratory
- Line developed with the passage of time

1.2. Display of the fluid flow

✓ Streakline

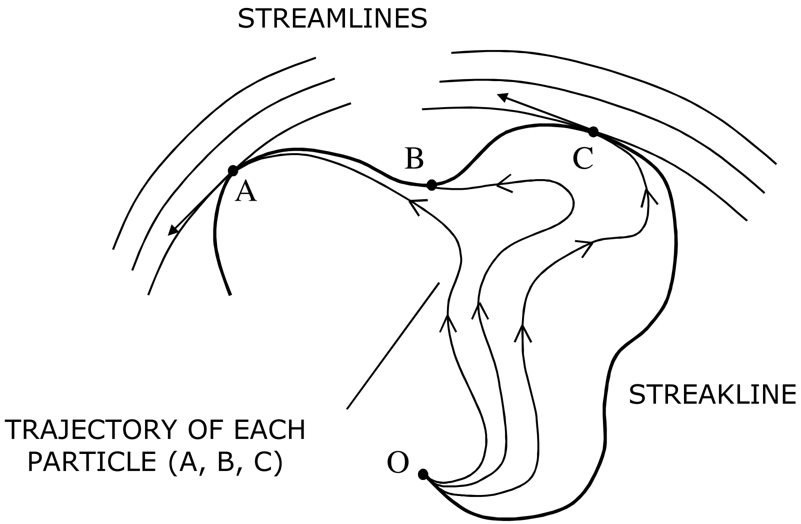


Figure 3.3 Streakline

- Definition
- Observable in laboratory
- Line modified with the passage of time

2. Description and classification of fluid flow

✓ Depending on the field of application

- External flow
- Internal flow

✓ Depending on the compressibility of the fluid

- Incompressible fluid flow
- Compressible fluid flow

✓ Depending on the evolution with time and space*

- Steady-state regime
- Variable regime

2. Description and classification of fluid flow

✓ Depending on the evolution with time and space

- Steady-state regime

$$\vec{U} = f(x,y,z) \qquad \rho = f(x,y,z) \qquad p = f(x,y,z) \quad , \ldots$$

$$\frac{\partial \vec{U}}{\partial t} = 0 \qquad \frac{\partial \rho}{\partial t} = 0 \qquad \frac{\partial p}{\partial t} = 0 \quad , \ldots$$

<u>Uniform steady-state regime</u>

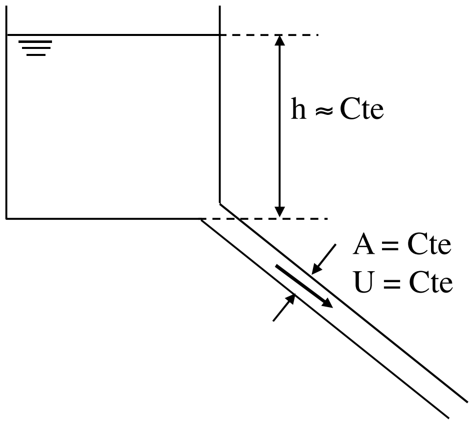


Figure 3.5 Uniform steady-state regime

<u>Varied steady-state regime</u>

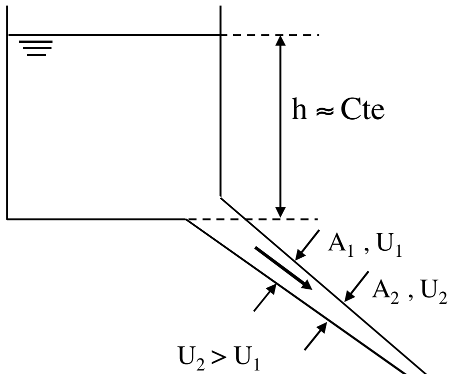


Figure 3.6 Varied steady-state regime

2. Description and classification of fluid flow

✓ Depending on the evolution with time and space

- Variable regime (non steady-state)

$$\vec{U} = f(x,y,z,t) \qquad \rho = f(x,y,z,t) \qquad p = f(x,y,z,t) \quad , \ldots$$

$$\frac{\partial \vec{U}}{\partial t} \neq 0 \qquad \frac{\partial \rho}{\partial t} \neq 0 \qquad \frac{\partial p}{\partial t} \neq 0 \quad , \ldots$$

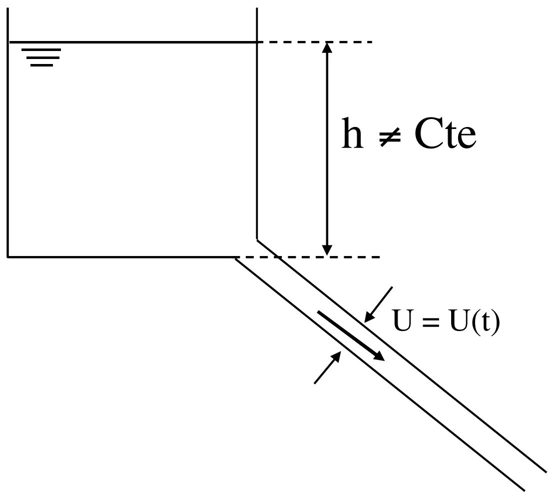


Figure 3.7 Variable regime

<u>"Pseudo uniform" variable regime</u>

2. Description and classification of fluid flow

✓ Depending on the viscous behaviour of the fluid

- Inviscid flow
- Viscous flow

<u>Laminar regime</u>

- Layers
- Streamlines
- Shock-absorbing effect of viscosity
- Parabolic velocity profile
- Energy loss

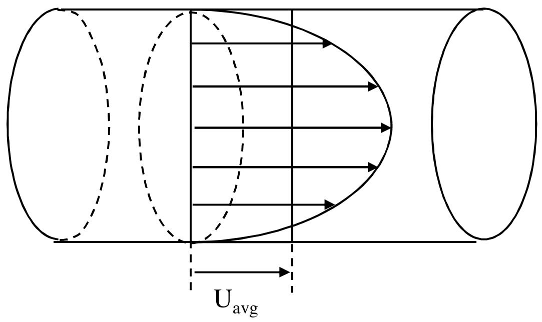


Figure 3.8 Velocity profile in laminar regime

2. Description and classification of fluid flow

✓ Depending on the viscous behaviour of the fluid

- Viscous flow

Turbulent regime

- Messy, chaotic
- Dissipation and additional shearing forces
- Turbulence
- Logarithmic velocity profile
- Energy loss

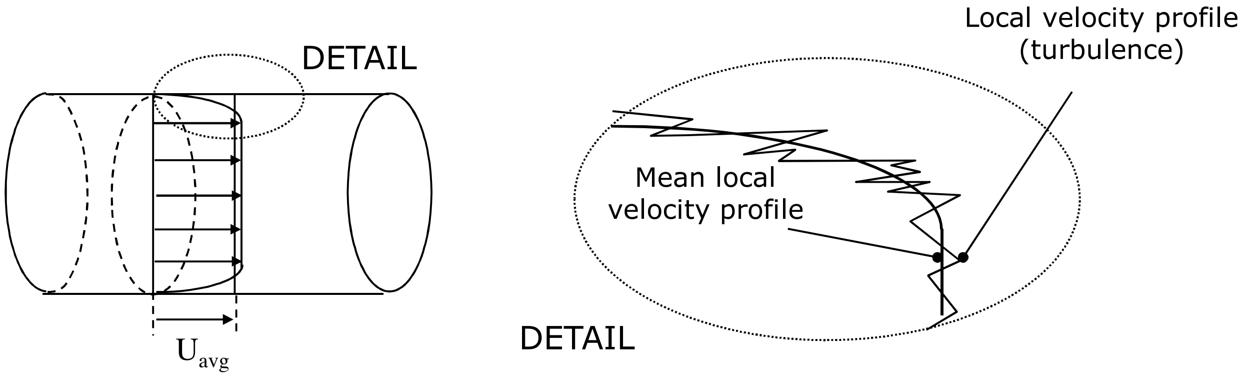


Figure 3.9 Velocity profile in turbulent regime

2. Description and classification of fluid flow

✓ Experience of Osborne Reynolds

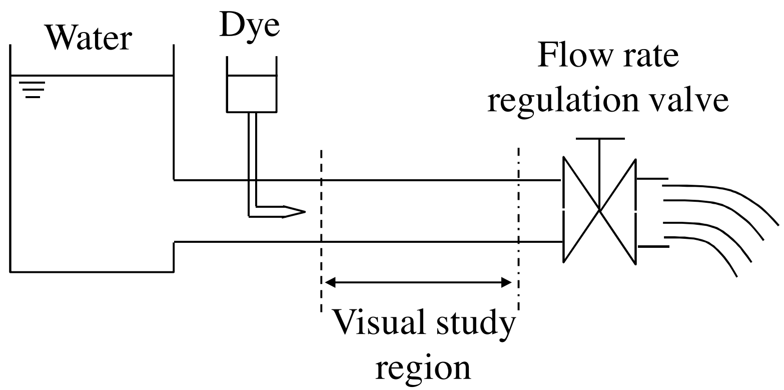
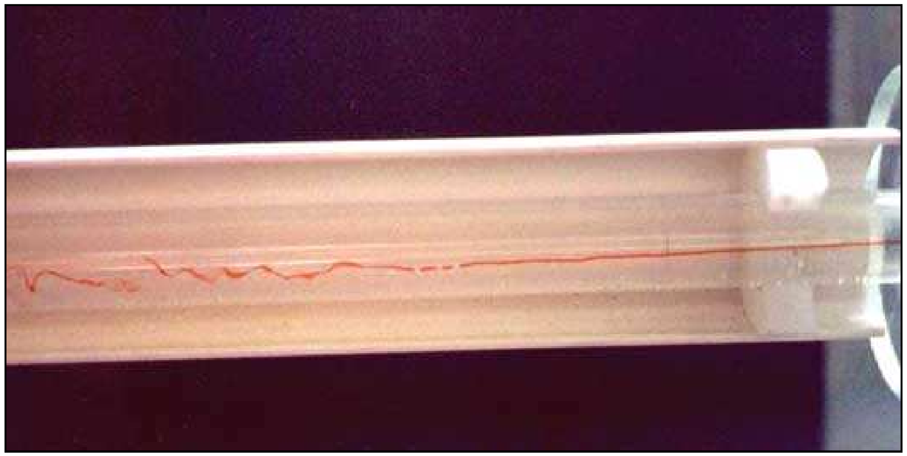
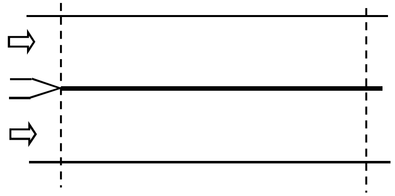


FigurE 3.10 Experiment by Osborne-Reynolds



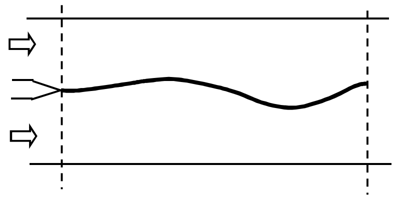
(http://www.flometrics.com)

1- Low flow rates



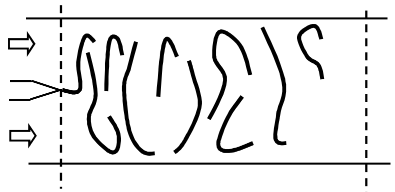
- The streakline is a straight line
- LAMINAR REGIME

2- Medium flow rates



- The streakline is wavy, it contains some disturbances
- TRANSITORY REGIME

3- High flow rate



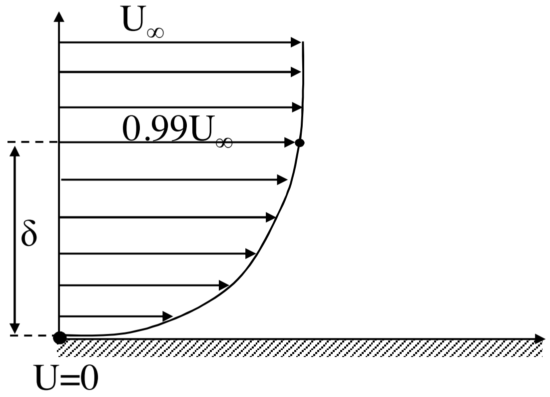
- The streakline losses its continuity, the turbulence of the fluid flow provokes the dilution of the tracer
- TURBULENT REGIME

3. The boundary layer concept: external and internal flow

- ✓ Motive

  - Deficiencies in the Euler equations to describe the behaviour of real flows
  - It joins together the applicability of Euler´s equations with the obligatory nature of the Navier-Stokes´ equations in some specific regions of the fluid

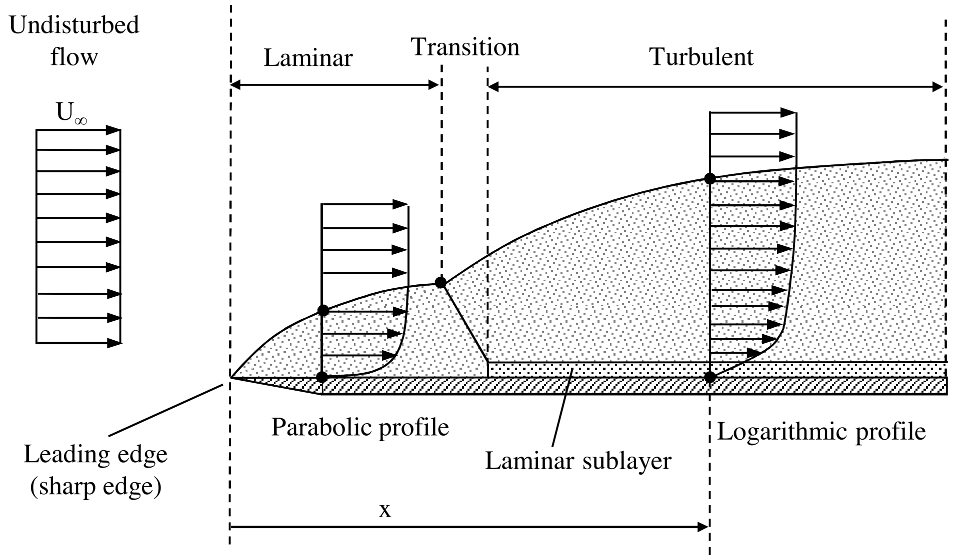
- ✓ Definition



(Thickness of the boundary layer)

3. The boundary layer concept: external and internal flow

✓ External flow



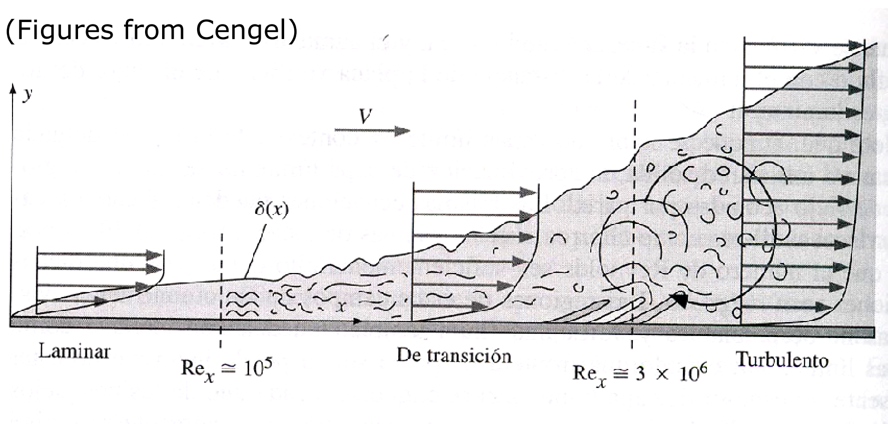
(Boundary layer in a plane plate)

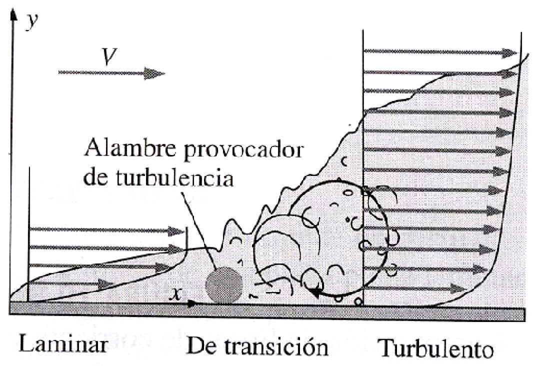
- Transition from laminar to turbulent:

$$Re_x = \frac{U_\infty x}{\nu} = 5 \cdot 10^5 \div 10^6$$

- Early transition, source of turbulence

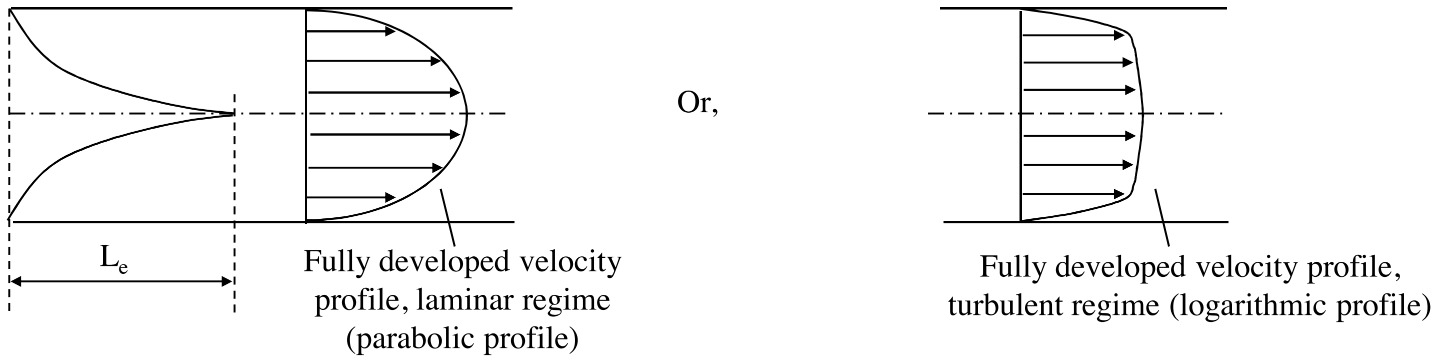
(Figures from Cengel)





3. The boundary layer concept: external and internal flow

✓ Internal flow



(Development of the boundary layer in a closed duct)

- Entrance length: $\dfrac{L_e}{D} = f\,(Re_D)$

    - Laminar regime: $\dfrac{L_e}{D} = 4.4\,Re_D^{\,1/6}$

    - Turbulent regime: $\dfrac{L_e}{D} = 0.06\,Re_D$

4. Concepts

1. Streamtube*
2. Volumetric and mass flow rate*
3. Average velocity*

4.1. Streamtube

- ✓ Definition
- ✓ Usefulness

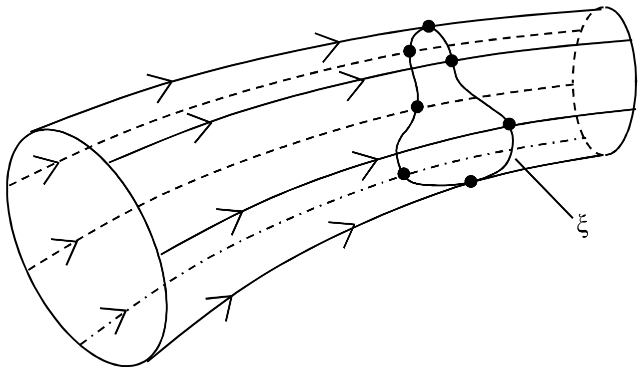


Figure 3.15 Streamtube

4.2. Volumetric and mass flow rate

- ✓ Definition
- ✓ Calculation

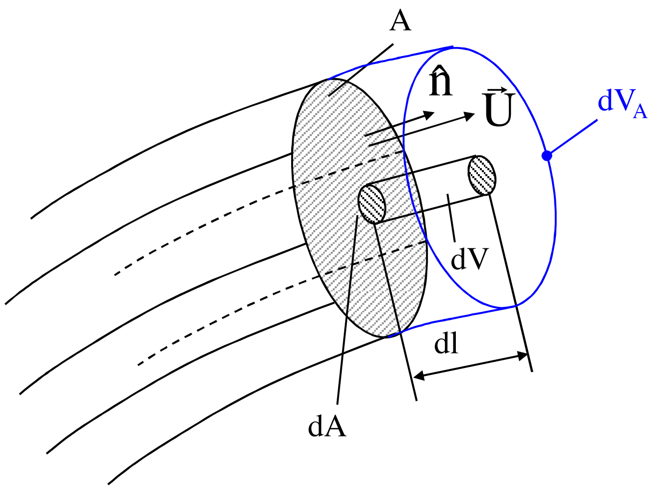


Figure 3.16 Volumetric and mass flow rate

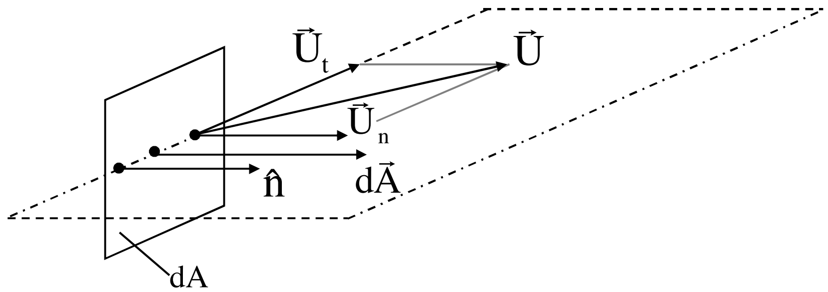


Figure 3.17 Situation wherein streamlines are not perpendicular to the area

$$Q = \iint_A U dA$$

$$q_m = \iint_A \rho U dA$$

$$Q = \iint_A \vec{U} d\vec{A}$$

$$q_m = \iint_A \rho \vec{U} d\vec{A}$$

4.3. Average velocity

✓ Definition

$$U_{av} = \frac{\iint_A U dA}{A}$$

$$Q = U_{av} A$$

$$q_m = \rho U_{av} A$$

- But:

$$q_m = \iint_A \rho U dA = (\rho U)_{av} A \neq \rho_{av} U_{av} A = \left(\frac{\iint_A \rho dA}{A} \right) \left(\frac{\iint_A U dA}{A} \right) A$$

5. Reynolds' transport theorem

✓ Study of the behaviour of a fluid. Fundamental laws:

- Three-dimensional space. <u>Control volume</u> (Euler)
- Fluid mass. <u>System</u> (Lagrange)

✓ Detail level

- Differential approach. Differential volumes in each point
- Integral approach. Finite control volume

5. Reynolds' transport theorem

✓ Deduction

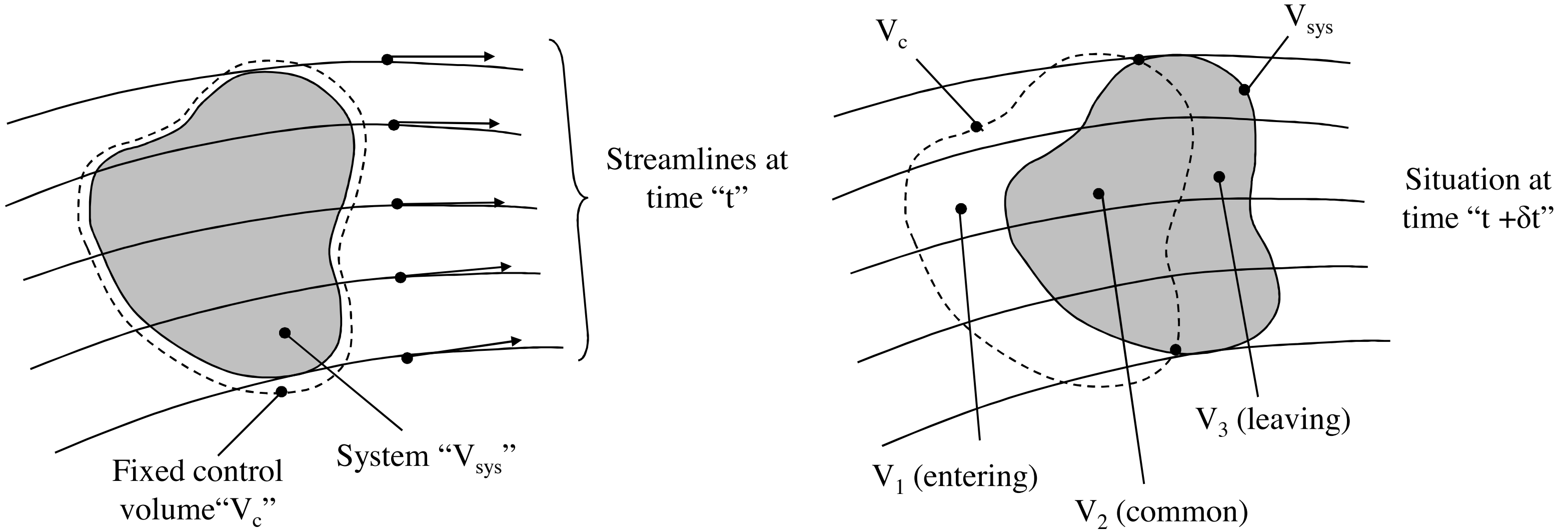


Figure 3.18 Relationship between system and control volume

- "Extensive" generic property "B". Specific (intensive) property "b"

$$B_{sys} = \iiint_{Vsys} \rho b dV$$

5. Reynolds' transport theorem

✓ Deduction

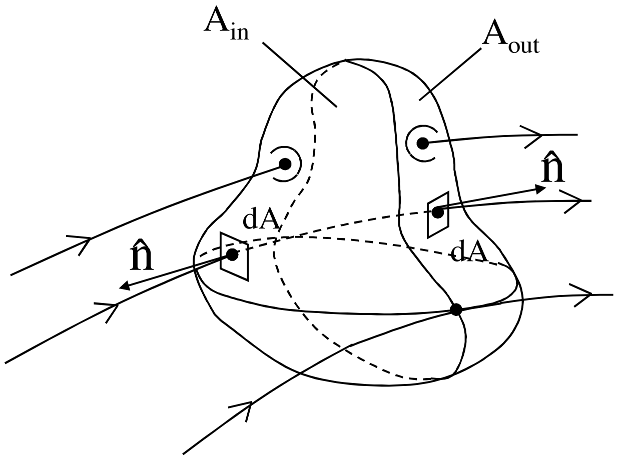
$$\frac{dB_{sys}}{dt} = \frac{dB_{Vc}}{dt} + \dot{B}_{out} - \dot{B}_{in}$$

- "Accumulation rate" or "local variation term"
- "Leaving flux" or "leaving convective variation term"
- "Entering flux" or "entering convective variation term"

✓ Word statement

5. Reynolds' transport theorem

✓ Deduction



$$\frac{dB_{sys}}{dt} = \frac{d}{dt}\iiint_{Vc} \rho b dV + \oiint_{Ac} b\rho\vec{U}d\vec{A}$$

Figure 3.19 Fluxes into and out of a control volume

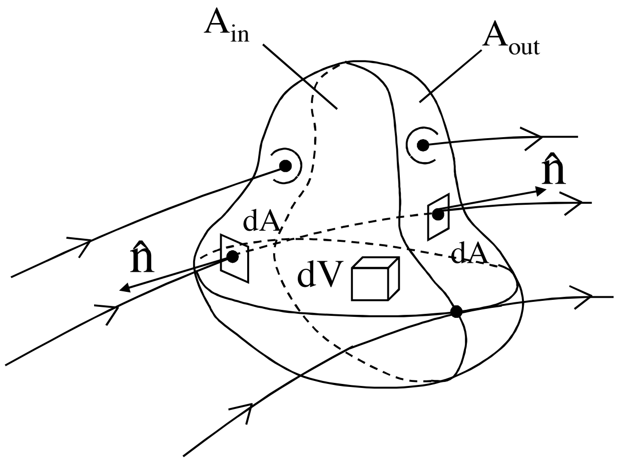
B = m	Mass	Mass conservation or continuity equation
B = E	Energy	Energy conservation, 1st law of Thermodynamics
$B = \vec{M} = m\vec{U}$	Momentum	Conservation of momentum Euler's 1st theorem
$B = \vec{H} = \vec{r} \wedge m\vec{U}$	Angular momentum	Conservation of angular momentum (Euler´s 2nd theorem).

6. Continuity equation

✓ Equation of conservation of mass (or continuity)

$$\frac{dm_{sys}}{dt} = 0$$

✓ Reynolds' transport theorem



$$\frac{dB_{sys}}{dt} = \frac{d}{dt}\iiint_{Vc} \rho b dV + \oiint_{Ac} b\rho \vec{U} d\vec{A}$$

- Property: mass

$$B_{sys} = m_{sys} = \iiint_{V_{sys}} b\rho dV = \iiint_{V_{sys}} \rho dV$$

b = 1

$$\frac{dm_{sys}}{dt} = \frac{d}{dt}\iiint_{V_c} \rho dV + \oiint_{A_c} \rho \vec{U} d\vec{A} = 0$$

⟺

$$\frac{dm_{Vc}}{dt} + \dot{m}_{out} - \dot{m}_{in} = 0$$

6. Continuity equation

✓ Application to a streamtube (steady-state regime)

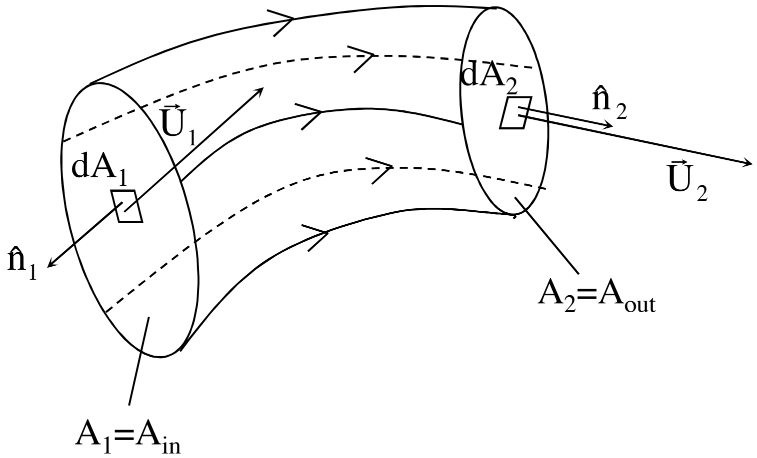


Figure 3.20 Application of the continuity equation to a streamtube

$$\dot{m}_{out} - \dot{m}_{in} = \oint\!\!\oint_{A_c} \rho \vec{U} d\vec{A} = 0$$

$$q_{m1} = q_{m2} \qquad \rho_1 U_{av1} A_1 = \rho_2 U_{av2} A_2$$

Incompressible fluid: $U_{av1} A_1 = U_{av2} A_2$ $\quad Q_1 = Q_2$

6. Continuity equation

- ✓ Uniform velocity profile:

Compressible fluid:

$$q_m = \rho U A = Cte$$

Incompressible fluid:

$$Q = UA = Cte$$

Several inlets and outlets:

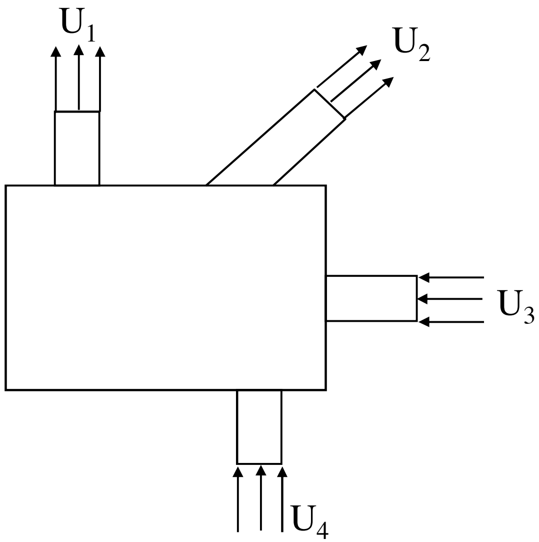


Figure 3.21 Control volume with several inlets and outlets

$$\sum q_{m,in} = \sum q_{m,out}$$

Incompressible fluid:

$$\sum Q_{in} = \sum Q_{out}$$