CHAPTER 2. FLUID STATICS

1. Pressure on a point. Pascal´s law
2. The basic hydrostatic equation
3. Measurement of pressure
4. Forces on surfaces: plane and curved surfaces
5. Mechanics of buoyant bodies and submerged bodies

1. Pascal´s law: pressure on a point, isotropy

- ✓ Isotropy in the definition of pressure
- ✓ Proof: force balance in a differential wedge of fluid

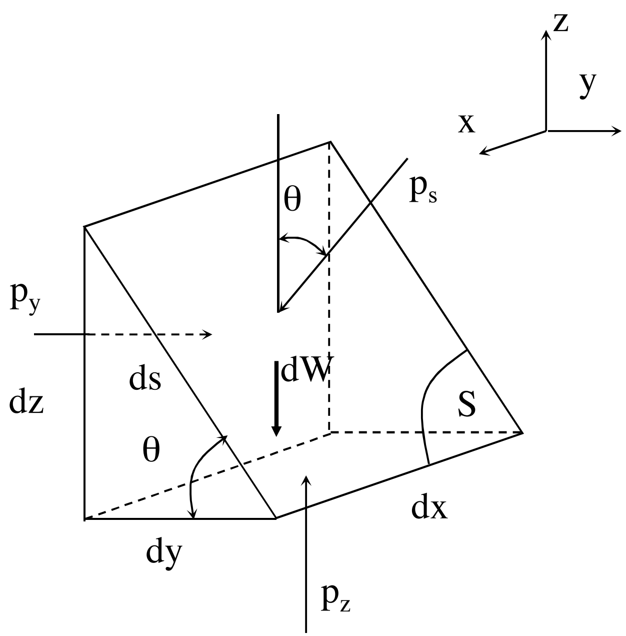


Figura 1.28 Differential wedge of fluid

- Conclusion:

$$p_s = p_x = p_y = p_z = p$$

2. The basic hydrostatic equation

- The basic hydrostatic equation*
- Application to obtain the pressure field *: (1) constant density (liquids)
- Application to obtain the pressure field *: (2) variable density

2. 1. The basic hydrostatic equation

- ✓ Concept: $\rho\vec{F} = \vec{\nabla}p$ ⇒ $\rho(f_x\hat{i} + f_y\hat{j} + f_z\hat{k}) = \left(\frac{\partial p}{\partial x}\hat{i} + \frac{\partial p}{\partial y}\hat{j} + \frac{\partial p}{\partial z}\hat{k}\right)$

- ✓ Deduction. Force balance:

  In a differential, static cube, centred in a point P(x,y,z) with a known pressure of p=p(x,y,z), downward vertical gravity

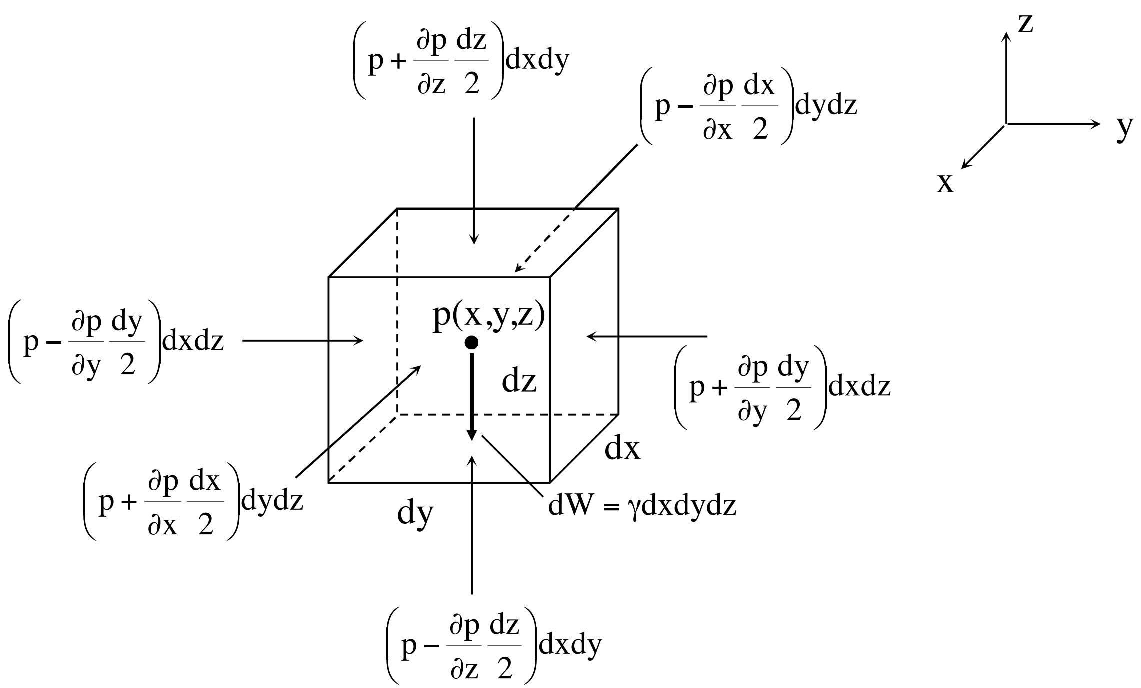


Figure 2.1 Infinitesimal element of fluid

2.1. The basic hydrostatic equation

✓ Conclusions of the force balance

- The basic hydrostatic equation:

$$\left.\begin{array}{l} -\dfrac{\partial p}{\partial x} = 0 \\ -\dfrac{\partial p}{\partial y} = 0 \\ -\dfrac{\partial p}{\partial z} - \gamma = 0 \end{array}\right\} \quad \boxed{-\dfrac{dp}{dz} = \gamma}$$

- Isobaric line: force of the external field is normal to any isobaric line

- Free surface, isobaric surfaces

2.2. Application to obtain the pressure field: (1) constant density (liquids)

✓ Incompressible fluid, integration of the basic law

$$-\frac{dp}{dz} = \gamma \quad \Rightarrow \quad p = p_0 - \gamma z$$

✓ Use of heights and depths:

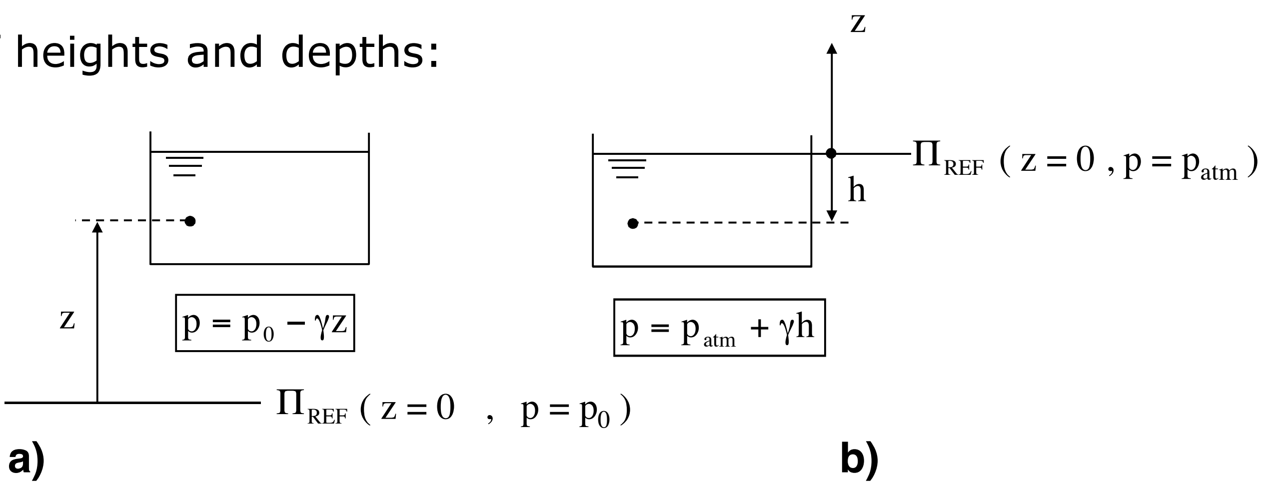


Figure 2.4 Hydrostatic equation in case of constant density.
a) General reference plane; **b)** Reference plane in the free surface

2.2. Application to obtain the pressure field: (1) constant density (liquids)

✓ Pressure, exclusive dependence on depth

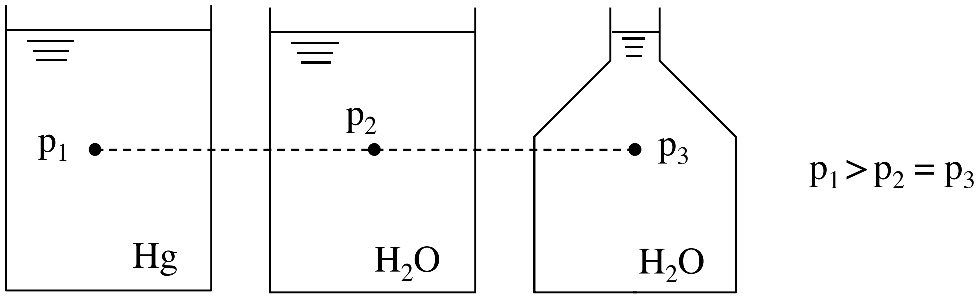
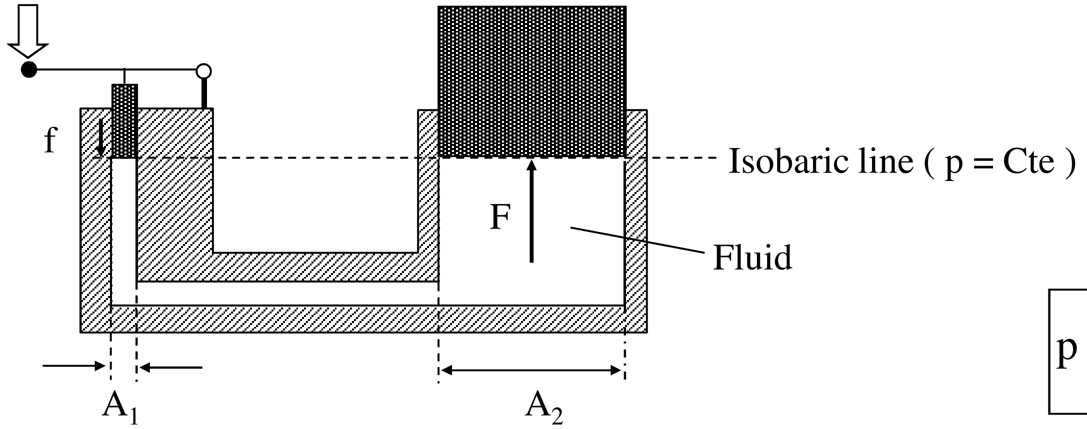


Figure 2.5 Pressure into tanks with different shapes and containing different liquids

✓ Principle of communicating vessels



$$p = Cte = \frac{f}{A_1} = \frac{F}{A_2}$$

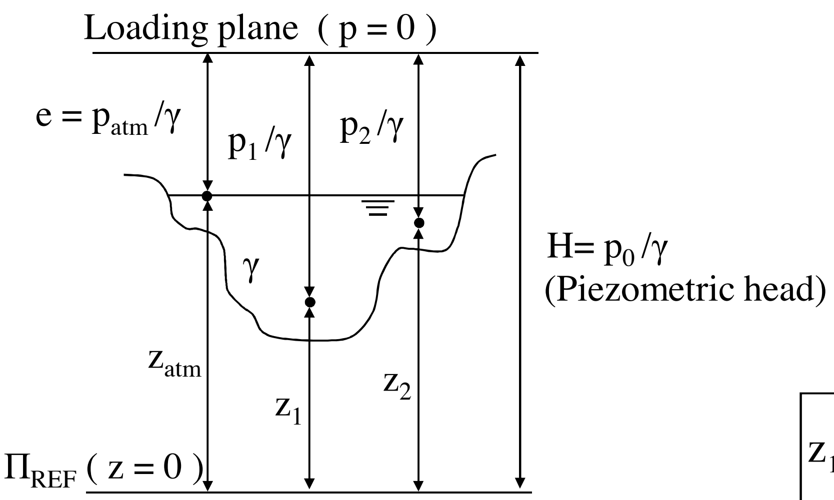
Figure 2.6 Principle of communicating vessels

2.2. Application to obtain the pressure field: (1) constant density (liquids)

✓ Pressures´ law given in heights

$$p = p_0 - \gamma z \quad \Rightarrow \quad \frac{p_0}{\gamma} = z + \frac{p}{\gamma} = H = Cte$$

("piezometric head")



$$z_1 + \frac{p_1}{\gamma} = z_2 + \frac{p_2}{\gamma} = z_{atm} + \frac{p_{atm}}{\gamma} = H = Cte$$

Figura 2.7 Piezometric head constancy

2.2. Application to obtain the pressure field: (1) constant density (liquids)

✓ Equivalent liquid column

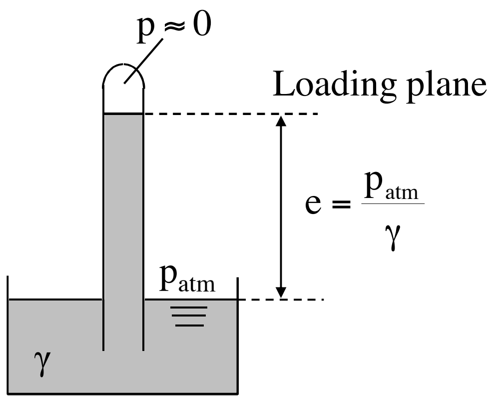


Figura 2.8 Mercury barometer

2.2. Application to obtain the pressure field: (2) variable density (gases)

✓ Compressible fluid, integration of the basic law

$$-\frac{dp}{dz} = \gamma \quad \Rightarrow \quad p = p_0 - \int_{z0}^{z} \gamma dz = p_0 - g\int_{z0}^{z} \rho dz$$

<u>Gases. Standard atmosphere (I.S.A.)</u>

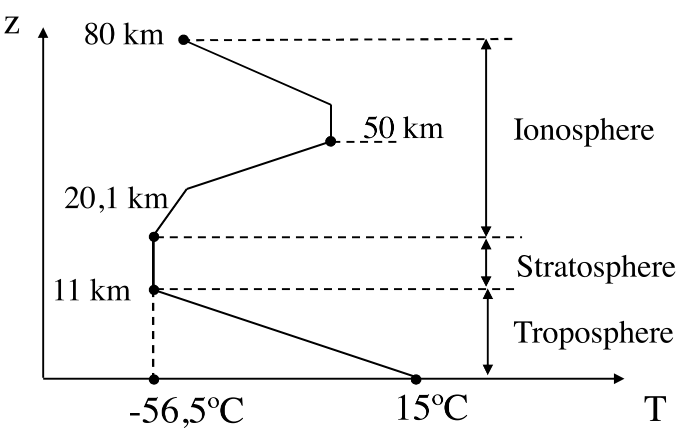


Figure 2.10 Temperature evolution with height "z" in the earth´s atmosphere

$$-\frac{dp}{dz} = \gamma$$

$$T = T_0 - Bz$$

$$p = \gamma R' T$$

$$\Rightarrow \quad p = p_0 \left(1 - \frac{B}{T_0} z\right)^{\frac{1}{BR'}}$$

3. Measurement of pressure

Measurement of pressure:

1. Piezometer (static pressure probe)*
2. U tube manometer*
3. Differential manometer*

3.1. Piezometer

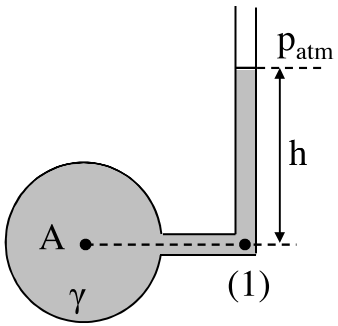
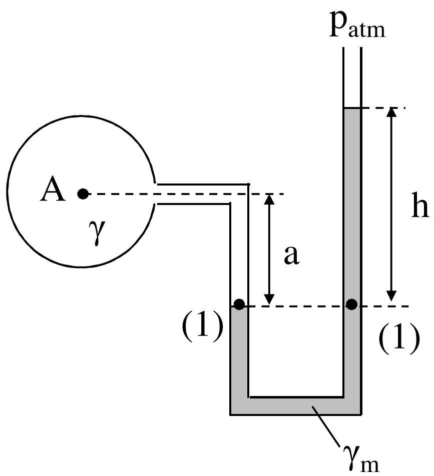


Figure 2.11 Piezometer

$$p_A = \gamma h$$

(gauge pressure)

3.2. U tube manometer



$$p_A = \gamma_m h - \gamma a$$

(gauge pressure)

Figure 2.12 U tube manometer

3.3. Differential manometer

✓ Differential manometer: U tube

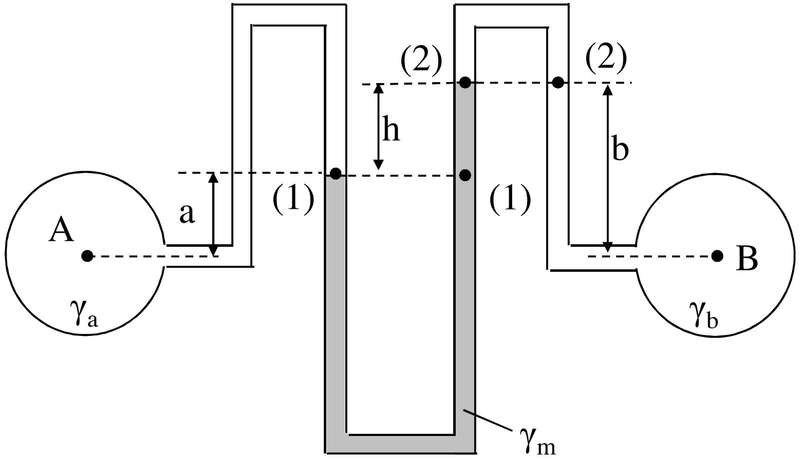


Figure 2.14. Two regions connected by a U tube differential manometer

$$p_A - p_B = \gamma_m h + \gamma_a a - \gamma_b b$$

$$p_A - p_B = h(\gamma_m - \gamma)$$

"Equation of the differential manometer"

• "Mnemonic manometer rule":

$$p_A = p_B - \gamma_b b + \gamma_m h + \gamma_a a$$

From (B): Go up to (2) | Go down to (1) | Go down to (A)

3.3. Differential manometer

✓ Differential manometer: inverted U tube

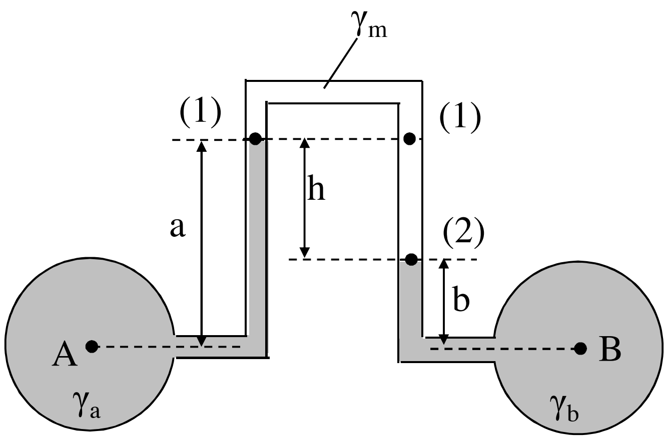


Figure 2.16 Inverted U tube differential manometer

$$p_A = p_B - \gamma_b b - \gamma_m h + \gamma_a a$$

$$p_A - p_B = h\ (\gamma - \gamma_m)$$

3.6. Other gauges

✓Bourdon manometer

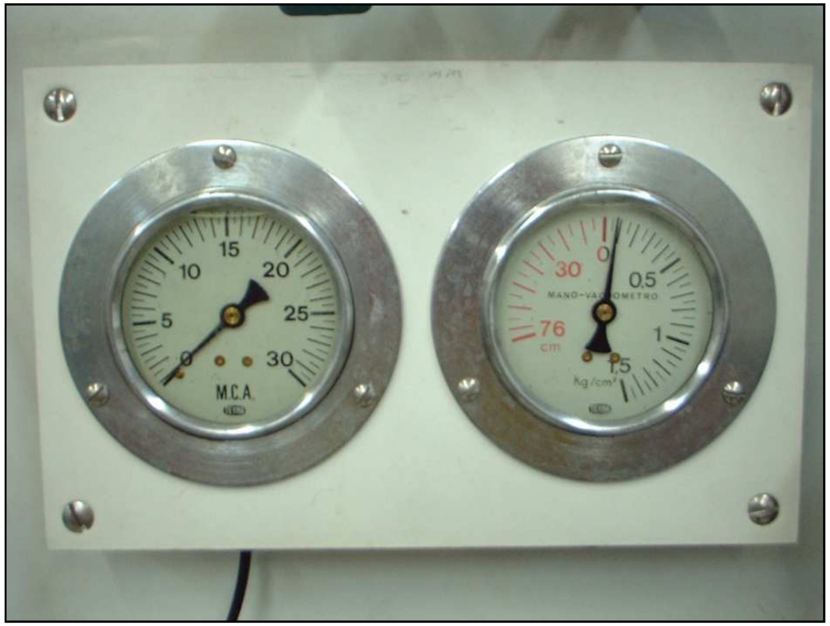


Figure 2.17 Bourdon manometer, available at the Fluid Mechanics Laboratory of the Faculty of Engineering in Bilbao. On the left hand side a typical manometer, on the right hand side manometer / vacuum meter

Figure 2.18 Details of the interior of a Bourdon manometer

4. Forces on surfaces

Forces on surfaces:

1. Plane surfaces*

2. Curved surfaces*

4.1. Forces on surfaces: plane surfaces

✓ CASE 1:

- Pressure forces caused by the atmosphere
- Pressure forces caused by gases
- Pressure forces caused by liquids on isobaric surfaces (parallel to free surface)

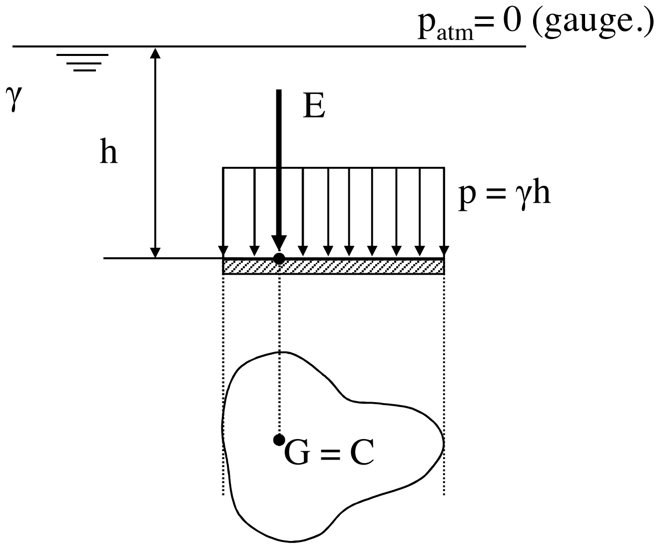


Figure 2.20 Uniform pressure distribution

- Resulting hydrostatic force

$$E = \iint_A p\,dA = pA$$

- Point of application

4.1. Forces on surfaces: plane surfaces

✓ CASE 2:

- Force by a liquid on an inclined surface

Method of the formula

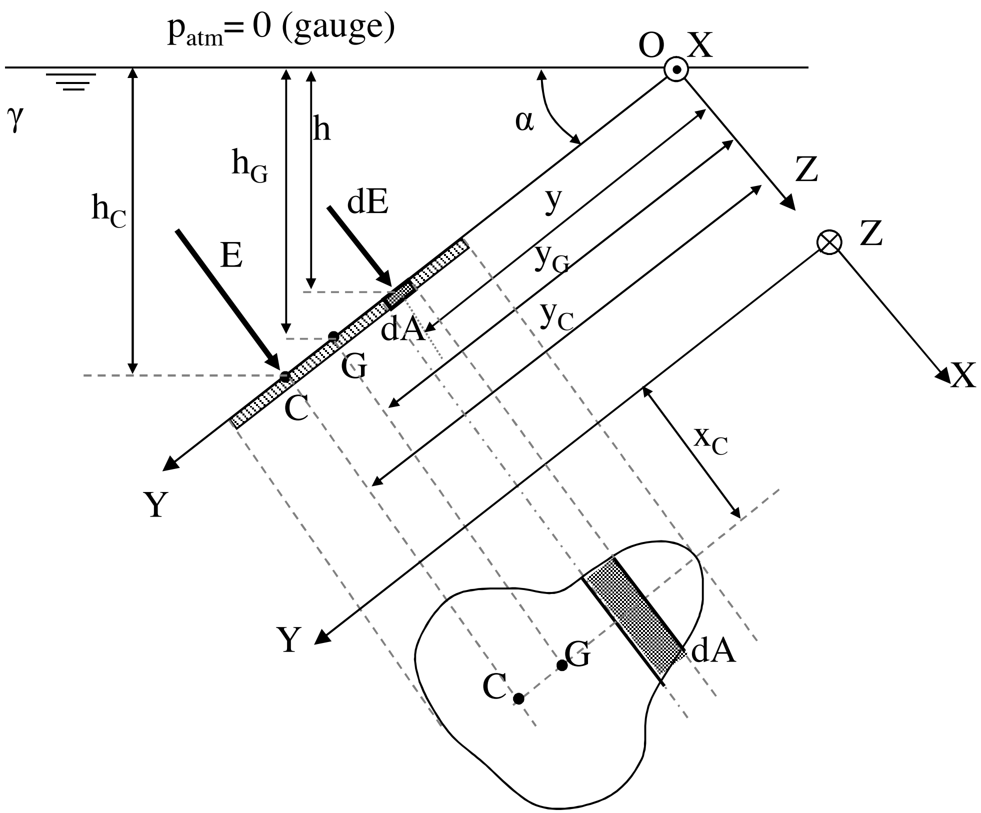


Figure 2.21 Hydrostatic pressure distribution on a plane plate

- **Force**

$$E = \gamma h_G A = p_G A$$

- **Point of application**

$$y_C = \frac{I_{xG}}{y_G A} + y_G$$

✓ CASE 2:

- Force by a liquid on an inclined surface

Method of the prism

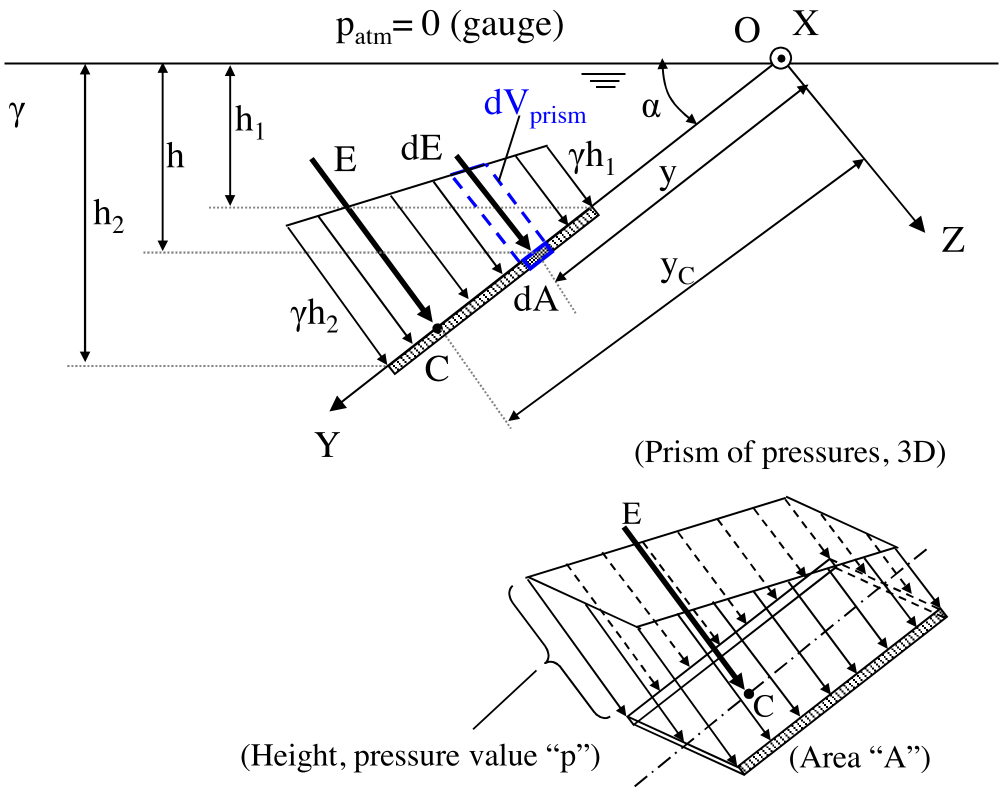


Figure 2.23 Method of the prism of pressures

- **Force**

$$E = V_{prism}$$

- **Point of application**

$$y_C = y_{G,prism}$$

4.2. Forces on surfaces: curved surfaces

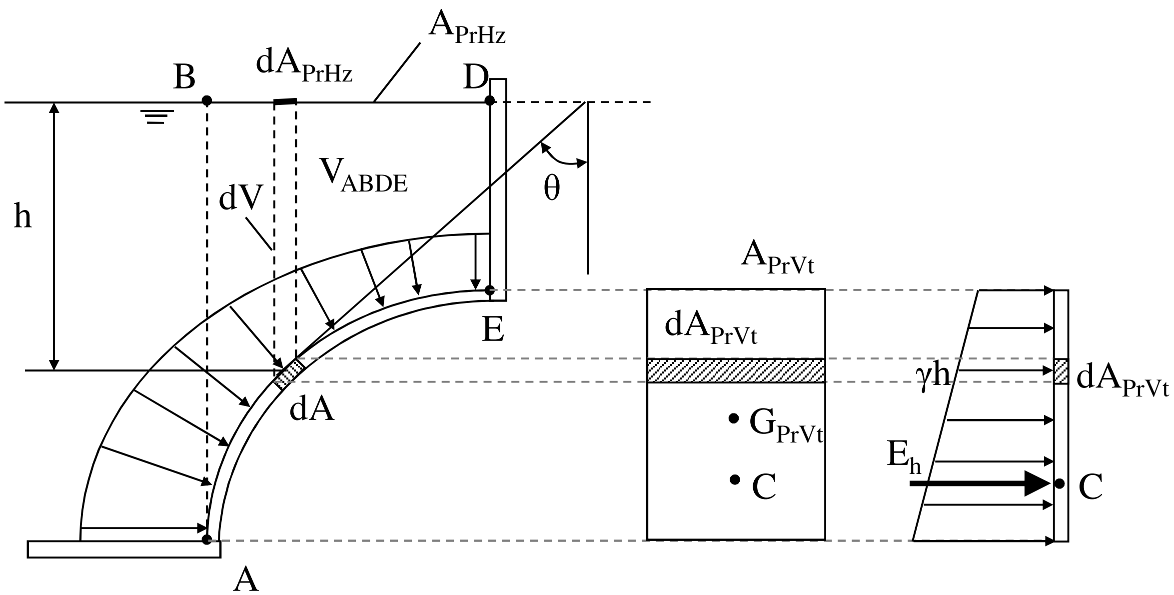


Figure 2.24 Forces on curved surfaces

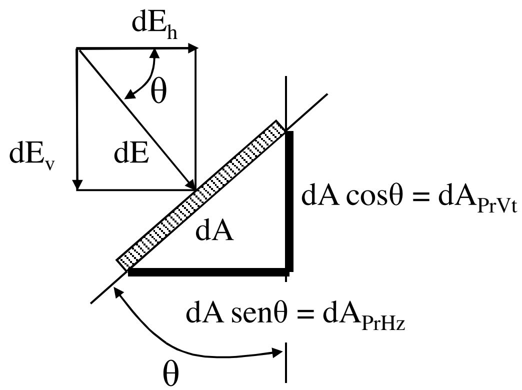


Figure 2.25 Force on a differential element of area

- Horizontal force:

$$E_h = \gamma h_{G,Pr\,Vt} A_{Pr\,Vt}$$

- Point of application

- Vertical force:

$$E_v = \gamma V_{ABDE}$$

- Point of application

4.2. Forces on surfaces: curved surfaces

✓ Curved three-dimensional surface

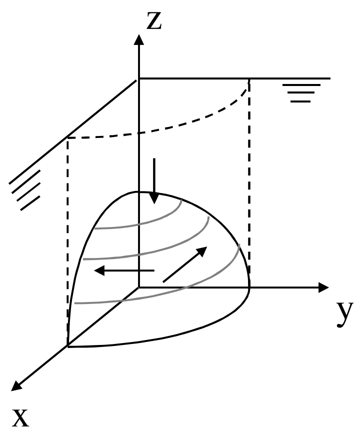


Figure 2.26 Curved three-dimensional surface

✓ There is not any fluid above

$$E_v = \gamma V_{ABDE}$$

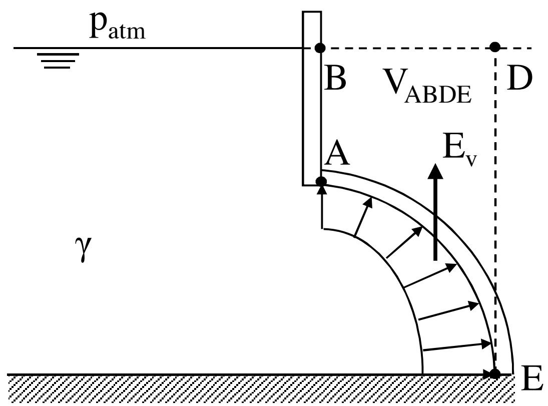
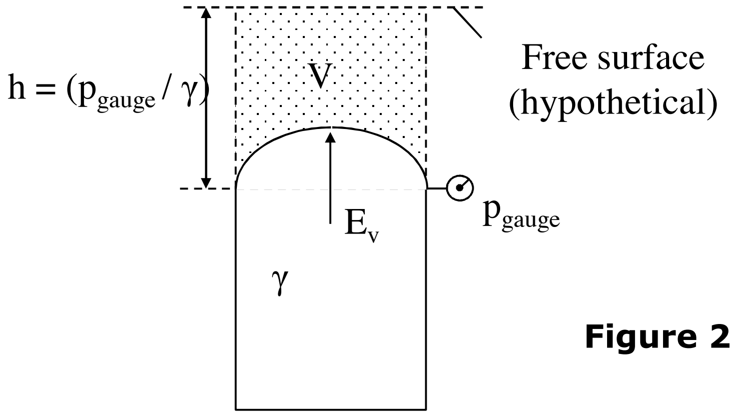


Figure 2.27 Fluid below the surface

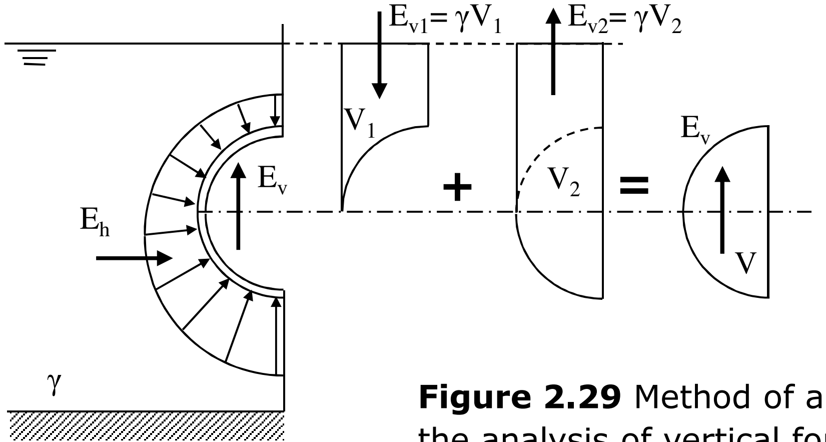
✓ Pressurized containers



$$E_v = \gamma V$$

Figure 2.28 Pressurized container

✓ Vertical force, addition – subtraction of volumes



$$E_v = \gamma V$$

Figure 2.29 Method of addition / subtraction of volumes for the analysis of vertical forces

5. Mechanics of buoyant and submerged bodies

1. Mechanics of buoyant and submerged bodies

   ✓ Equilibrium and stability of submerged bodies*

   ✓ Equilibrium and stability of partially submerged bodies*

     - Calculation of the metacentric radius*
     - Effect of an internal liquid mass*
     - Restoring Couple*

5. Mechanics of buoyant and submerged bodies

- ✓ 1st Archimedes' principle

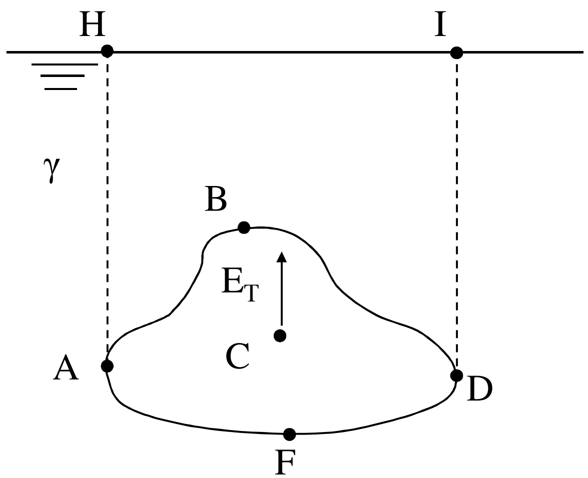
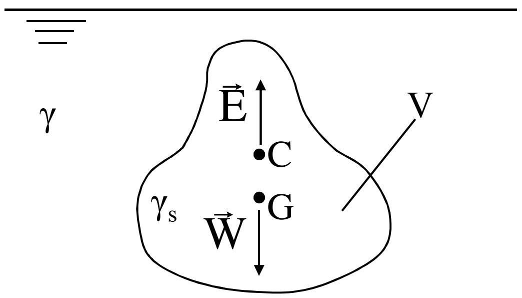


Figure 2.31 Body submerged in a fluid with a specific weight γ

$$\uparrow E_T = E_{AFD} - E_{ABD} = \gamma(V_{AFDIH} - V_{ABDIH}) = \gamma V_{ABDF}$$

- ✓ Equilibrium of totally submerged bodies



$E = W$; Equilibrium situation ($\rho = \rho_s$)
$E > W$; The body floats ($\rho > \rho_s$)
$E < W$; The body sinks ($\rho < \rho_s$)

Figure 2.32 Equilibrium of submerged bodies

- ✓ Stability of totally submerged bodies

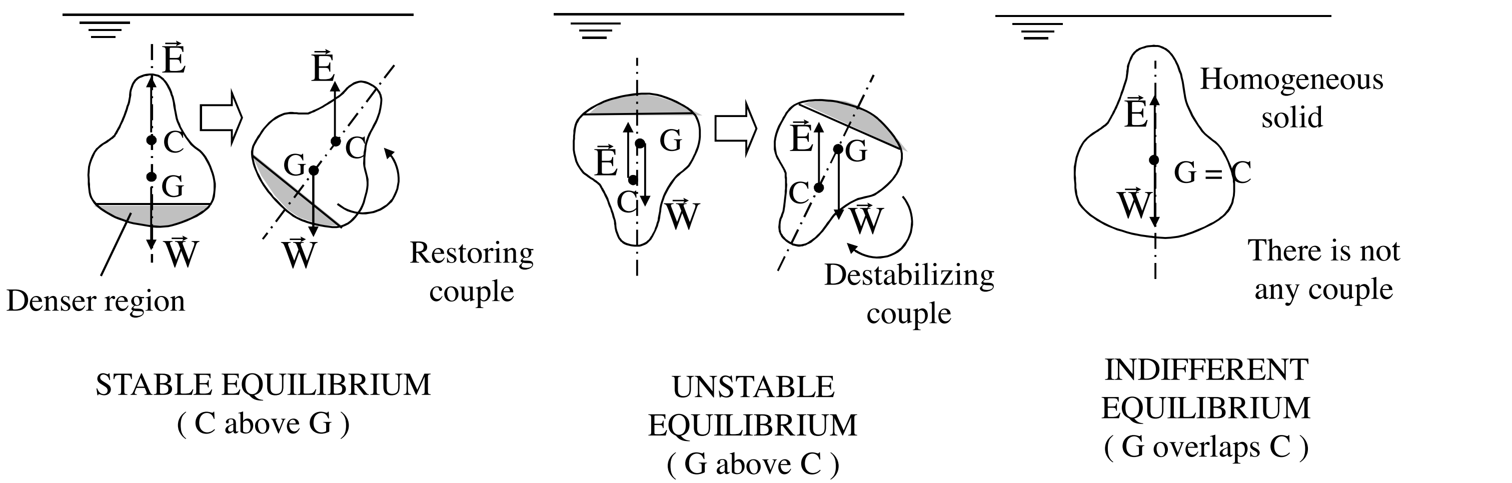


Figure 2.33 Study of the stability of submerged bodies

5. Mechanics of buoyant and submerged bodies

✓ Equilibrium of partially submerged bodies

- 2nd Archimedes´ principle:

$$W = \gamma_s V = E = \gamma V_{submerged}$$

✓ Stability of partially submerged bodies

- C above G:

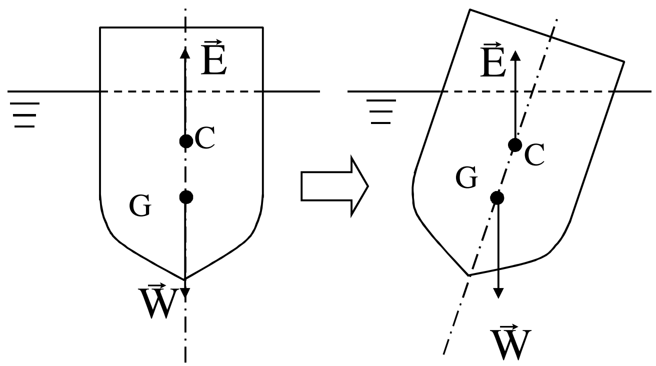


Figure 2.34 Stable equilibrium in buoyant bodies

✓ Stability of partially submerged bodies

- G above C:

Stability

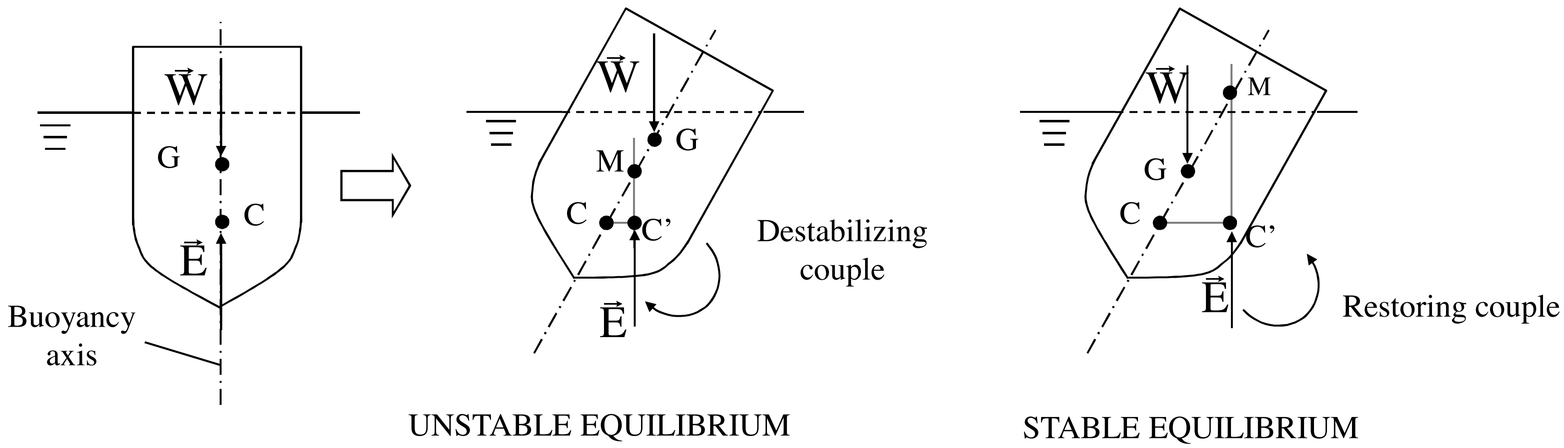


Figure 2.36 Stability of buoyant bodies in case C is below C

✓ Stability of partially submerged bodies

Nomenclature

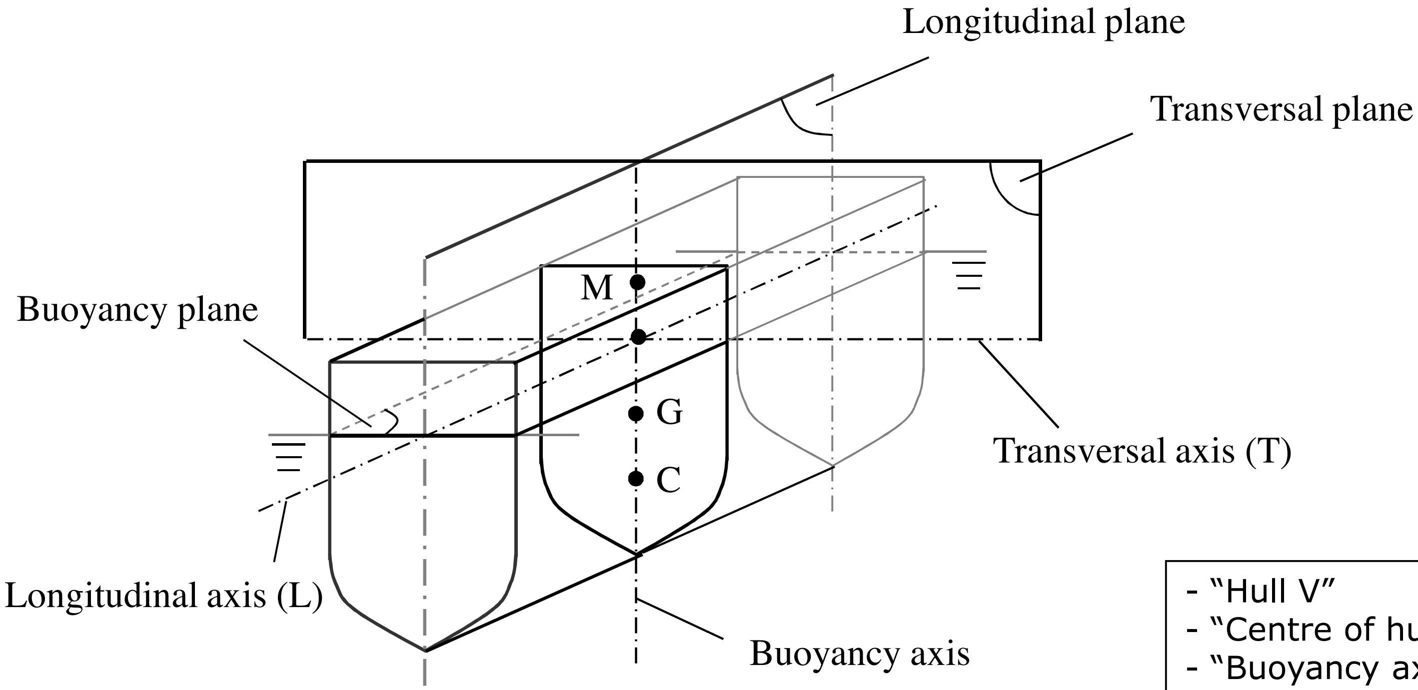


Figure 2.35 Nomenclature in problems dealing with buoyancy

- "Hull V"
- "Centre of hull C"
- "Buoyancy axis"
- "Buoyancy (line) plane"
- "Longitudinal plane"
- "Transversal plane"
- "Longitudinal axis (L)"
- "Transversal axis (T)"
- "Rocking"
- "Pitching"

5. Mechanics of buoyant and submerged bodies

✓ Stability of partially submerged bodies

- G below C: calculation of the position of the metacentre M

Resultant couple by the buoyant force (after rocking)	=	Initial couple by the buoyant force (before rocking)	+	Resultant couple by the modifications

$$\int_{abOcd} x\,dE = \gamma\theta \iint_A x^2\,dA = \gamma\theta I_L$$

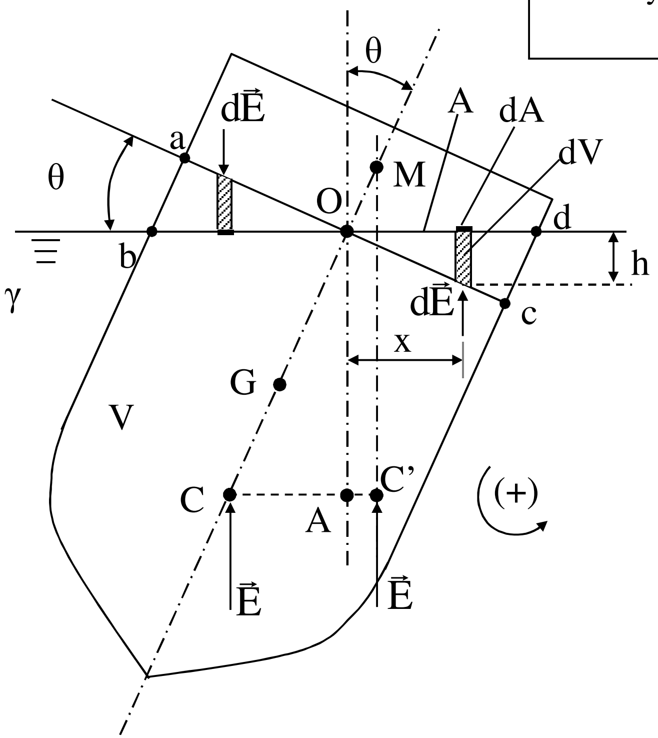
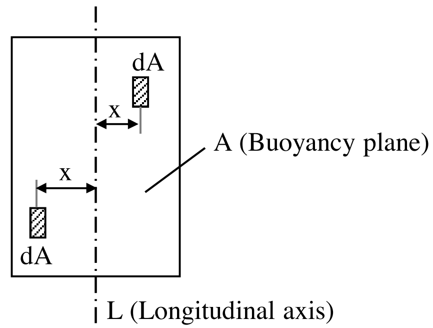


Figure 2.37 Calculation of the position of the metacentre with small heel angles in the buoyant body



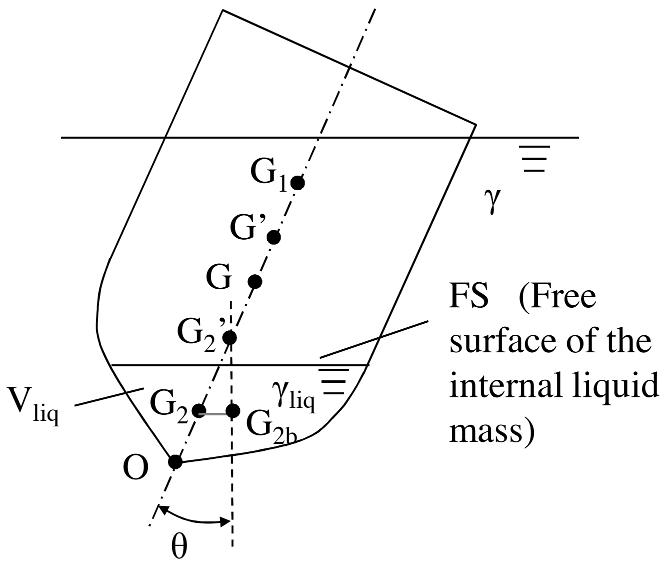
- Metacentric radius:

$$\overline{CM} = \frac{I_L}{V}$$

$$I_L = \iint_A x^2\,dA$$

✓ Stability of partially submerged bodies

- Internal liquid mass



$$\overline{GG'} = \frac{\sum \gamma_{liq} I_{FS,L}}{\sum W}$$

Figure 2.39 Stability in case of internal liquid mass inside the buoyant body

5. Mechanics of buoyant and submerged bodies

✓ Stability of partially submerged bodies

- Restoring couple

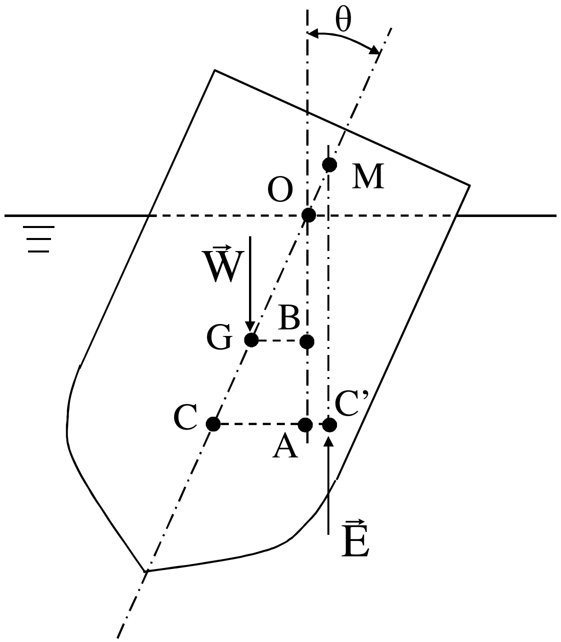


Figure 2.42 Restoring couple

$$M = W \times \overline{GM} \times \theta$$