

8. GAIA

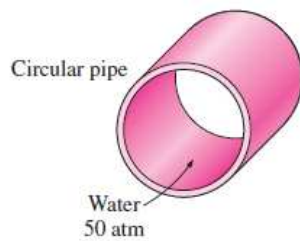
BARNEKO KONBEKZIO BEHARTUA

8.0 - HELBURUAK

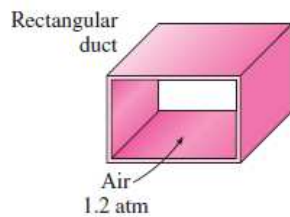
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- Barne-fluxuaren **batez besteko abiadura** lortu, abiadura-profilean oinarrituta, eta **batez besteko tenperatura** lortu, tenperatura-profilean oinarrituta
- Barne-fluxuko **fluxu-eskualdeen ulermen** bisuala eduki, hala nola sarrera-eskualdearena eta erabat garatutako fluxu-eskualdearena, eta sarrera-luzera hidrodinamikoa eta termikoa kalkulatu
- Gainazal-tenperatura konstantea eta gainazaleko bero-fluxuaren baldintza konstanteak dituen hodi batean doan fluido bat **berotzeko eta hozteko prozesua aztertu**, eta **batez besteko tenperatura-diferentzia logaritmikoarekin** lan egin
- **Fluxu laminar** erabat garatuen abiadura-profilaren, presio-jaitsieraren, marruskadura-faktorearen eta Nusselten zenbakiaren **erlazio analitikoa lortu**
- **Fluxu turbulentu** erabat garatuen marruskadura-faktorea eta Nusselten zenbakia kalkulatu, erlazio enpirikoez baliatuta, eta presio-jaitsiera eta bero-transferentziaren abiadura kalkulatu

Barne-fluxuan, fluidoa hodiaren barne-gainazalek inguratua dago



← **Fluido LIKIADOA denean**

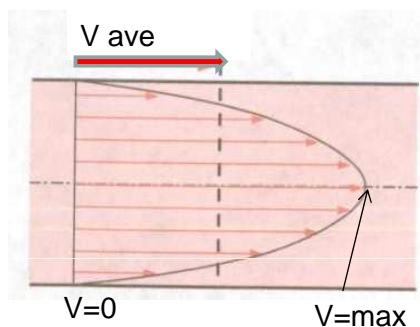


← **Fluido GASA denean**



8.2 – BATEZ BESTEKO ABIADURA ETA TENPERATURA

BATEZ BESTEKO ABIADURA (V_{AVE})

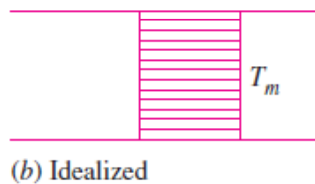
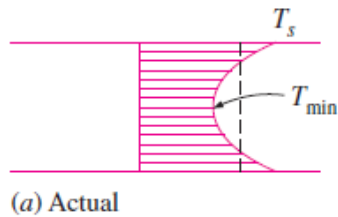


MASAREN kontserbazio-printzipioa

$$\dot{m} = \rho V_{prom} A_c = \int_{A_c} \rho u(r) dA_c$$

$$V_{prom} = \frac{2}{R^2} \int_0^R u(r) r dr$$

$u(r)$: abiadura - profila

BATEZ BESTEKO TENPERATURA (T_m)ENERGIAREN kontserbazio-printzipioa

$$\dot{E}_{\text{fluido}} = \dot{m} c_p T_m = \int_{A_c} \rho C_p T(r) u(r) dA_c$$

$$T_m = \frac{2}{V_{\text{prom}} R^2} \int_0^R T(r) u(r) r dr$$

8.2 – BATEZ BESTEKO ABIADURA ETA TENPERATURA

FLUXU LAMINARRAK ETA TURBULENTUAK HODIETAN

$$\text{Re} = \frac{\rho V_{\text{prom}} D}{\mu} = \frac{V_{\text{prom}} D}{\nu}$$

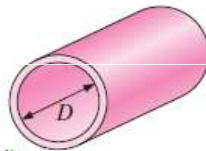
$$\nu = \frac{\mu}{\rho} \quad \text{Biskositate zinematikoa}$$

- $\text{Re} < 2.300$ fluxu laminarra: fluido oso biskosoak edo abiadura baxuak
- $\text{Re} > 10.000$ fluxu turbulentoa



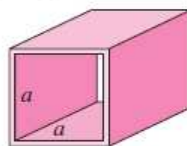
Circular tube:

$$D_h = \frac{4(\pi D^2/4)}{\pi D} = D$$



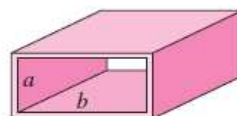
Square duct:

$$D_h = \frac{4a^2}{4a} = a$$



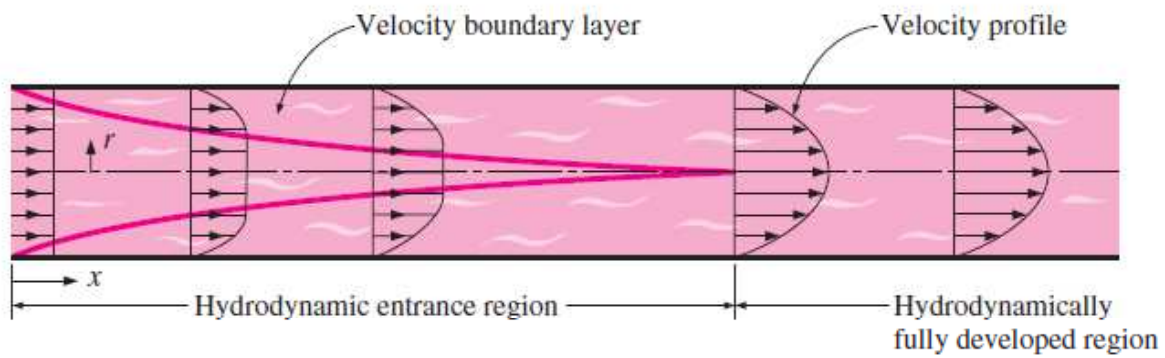
Rectangular duct:

$$D_h = \frac{4ab}{2(a+b)} = \frac{2ab}{a+b}$$

**Diametro Hidraulikoa**

$$D_h = \frac{4A_c}{p}$$

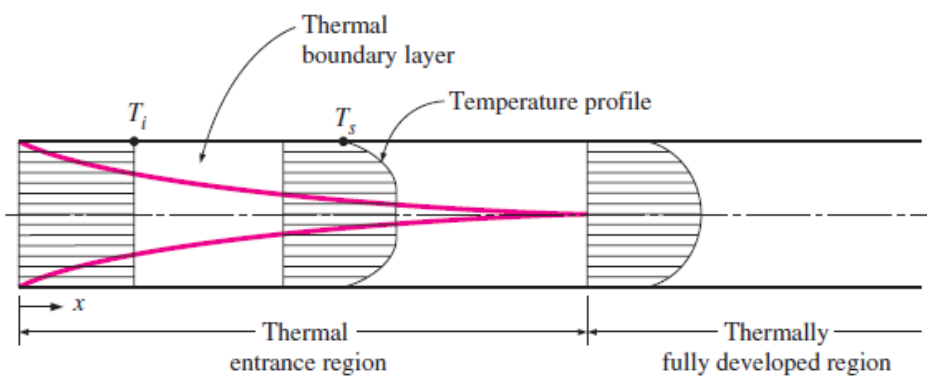
HODI BATEKO MUGALDE-GERUZA ABIADURAREN GARAPENA



Hidrodinamikoki erabat
garatutako eskualdea

$$\frac{\partial u(r, x)}{\partial x} = 0 \rightarrow u = u(r)$$

HODI BATEKO MUGALDE-GERUZA TERMIKOAREN GARAPENA

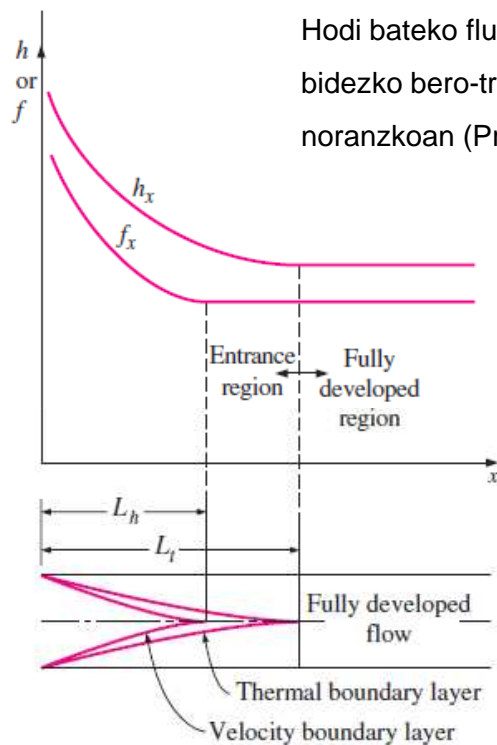


Termikoki erabat
garatutako eskualdea

$$\frac{\partial}{\partial x} \left[\frac{T_s(x) - T(r, x)}{T_s(x) - T_m(x)} \right] = 0$$

Gainazaleko bero-
fluxua

$$\dot{q} = \dot{q}_s A = h_x (T_s - T_m) = k \left. \frac{\partial T}{\partial r} \right|_{r=R}$$



8.3 – SARRERA-ESKUALDEA

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SARRERA-LUZERAK EXPERIMENTALAK

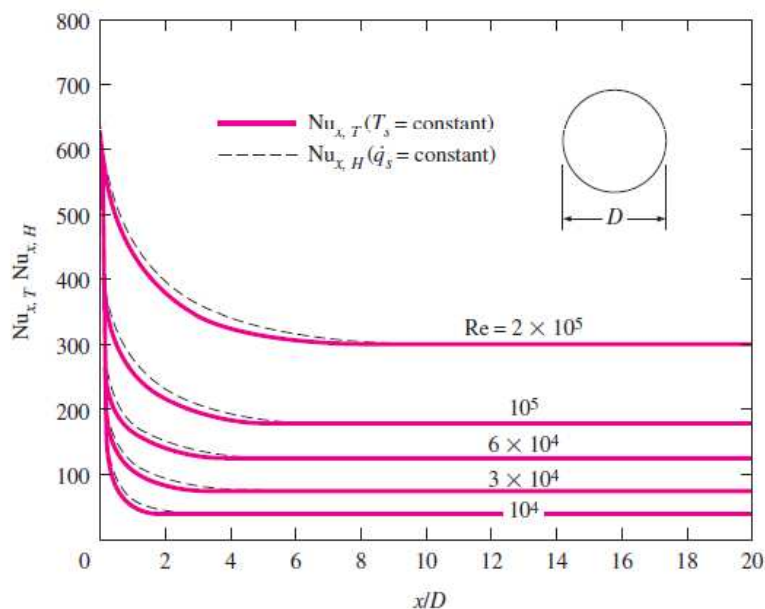
Fluxu Laminarra

$$L_{h,\text{laminar}} \approx 0,05 \text{ Re} D$$

$$L_{t,\text{laminar}} \approx 0,05 \text{ Re Pr } D = \text{Pr } L_{h,\text{laminar}}$$

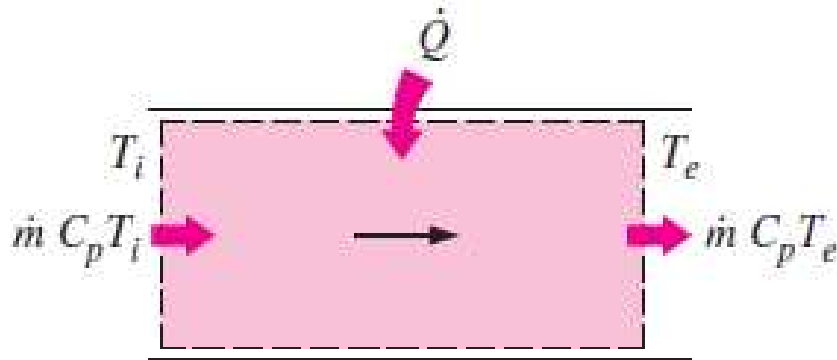
Fluxu turbulentua

$$L_{h,\text{turbulent}} \approx L_{t,\text{turbulent}} \approx 10D$$



Fluxu turbulentua duen hodi batean zeharreko Nusselten zenbaki lokalaren aldaketa

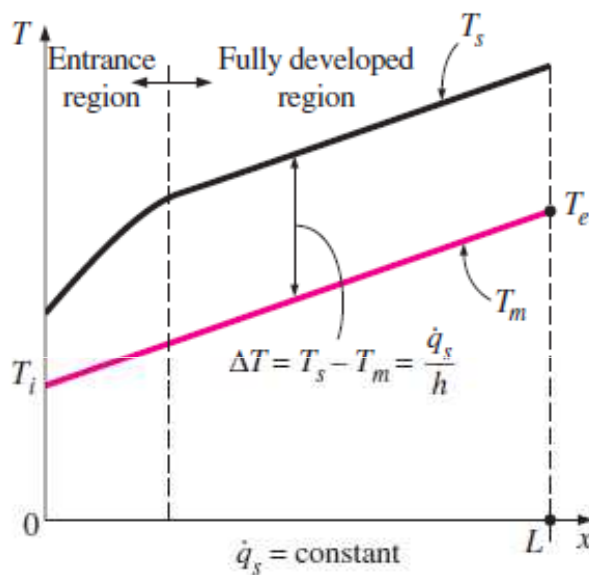
Hodi bateko fluido baten fluxu geldikorraren energia-kontserbazioaren ekuazioa



$$\dot{Q} = \dot{m} c_p (T_e - T_i) \quad (\text{W})$$

$$\dot{q}_s = h_x (T_s - T_m) \quad (\text{W/m}^2)$$

8.4 – ANALISI TERMIKO OROKORRA

GAINAZALEKO BERO-FLUXU KONSTANTEA ($\dot{q}_s = \text{CONSTANTE}$)

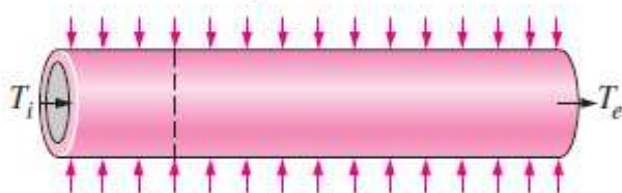
$$\dot{Q} = \dot{q}_s A_s = \dot{m} c_p (T_e - T_i) \quad (\text{W})$$

$$T_e = T_i + \frac{\dot{q}_s A_s}{\dot{m} C_p}$$

$$\dot{q}_s = h(T_s - T_m) \rightarrow T_s = T_m + \frac{\dot{q}_s}{h}$$

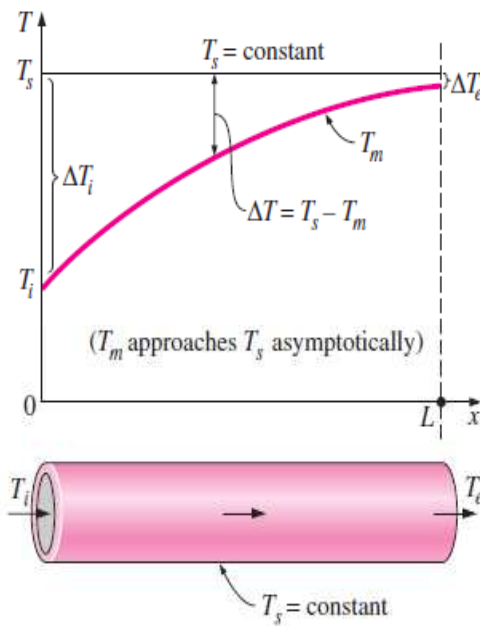
$$T_s - T_m = cte$$

$$\frac{\partial T}{\partial x} = \frac{dT_s}{dx} = \frac{dT_m}{dx} = \frac{2\dot{q}_s p}{\rho V_{prom} C_p R} = cte$$



Tutu zirkularra:

$$p = 2\pi R \quad \text{y} \quad \dot{m} = \rho V_{prom} A_c$$

GAINAZAL-TENPERATURA KONSTANTEA ($T_s = \text{CONSTANTE}$)

$$\dot{Q} = hA_s \Delta T_{prom} = hA_s (T_s - T_m)_{prom} \quad (\text{W})$$

$$\Delta T_{prom} \approx \Delta T_{ma} = \frac{\Delta T_i - \Delta T_e}{2} = \frac{(T_s - T_i) + (T_s - T_e)}{2}$$

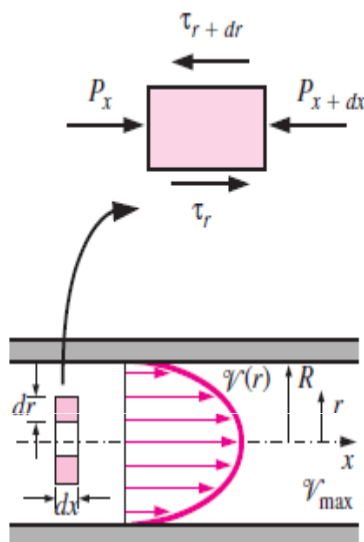
Energia Balantzea:

$$\dot{m} c_p dT_m = h(T_s - T_m) dA_s$$

$$T_e = T_s - (T_s - T_i) \exp(-hA_s / \dot{m} c_p)$$

$$\Delta T_{ln} = \frac{(T_s - T_e) - (T_s - T_i)}{\ln[(T_s - T_e) / (T_s - T_i)]} = \frac{\Delta T_e - \Delta T_i}{\ln(\Delta T_e / \Delta T_i)}$$

8.5 – FLUXU LAMINARRA HODIETAN



Indar-balantzeak:

$$(2\pi r dr P)_x - (2\pi r dr P)_{x+dx} + (2\pi r dx \tau)_r - (2\pi r dx \tau)_{r+dr} = 0$$

Lim $dr, dx \rightarrow 0$, ordezkapena $\tau = -\mu(du/dr)$

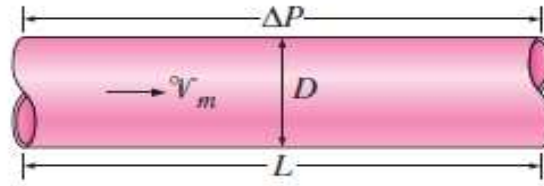
$$\frac{dP}{dx} = -\frac{2\tau_w}{R} \quad \text{berrantolatuz eta bi aldiz integratuz}$$

$$u(r) = \frac{1}{4\mu} \left(\frac{dP}{dx} \right) r^2 + C_1 \ln r + C_2$$

$$\left. \begin{aligned} u(r) &= -\frac{R^2}{4\mu} \left(\frac{dP}{dx} \right) \left(1 - \frac{r^2}{R^2} \right) \\ V_{prom} &= \frac{2}{R^2} \int_0^R u(r) r dr = -\frac{R^2}{8\mu} \left(\frac{dP}{dx} \right) \end{aligned} \right\} \quad \begin{aligned} u(r) &= 2V_{prom} \left(1 - \frac{r^2}{R^2} \right) \\ \text{non } r=0 &\rightarrow u_{max} = 2V_{prom} \end{aligned}$$

PRESIO-GALERA (ΔP)

$$\frac{dP}{dx} = \frac{P_2 - P_1}{L}$$



Fluxu Laminarra

$$\Delta P = P_1 - P_2 = \frac{8\mu L V_{prom}}{R^2} = \frac{32\mu L V_{prom}}{D^2}$$

Presio Galera

$$\Delta P_L = f \frac{L}{D} \frac{\rho V_{prom}^2}{2}$$

f = Darcy-ren
marruskadure
faktorea

$$f = \frac{8\tau_w}{\rho V_{ave}^2}$$

Hodi
zirkularra,
Laminarra

$$f = \frac{64\mu}{\rho D V_{ave}} = \frac{64}{Re}$$

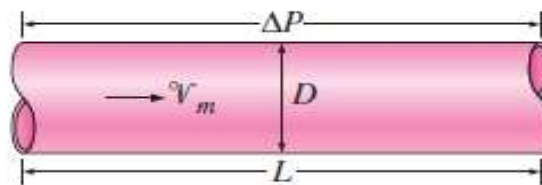
8.5 – FLUXU LAMINARRA HODIETAN

KARGA-GALERA (h_L)

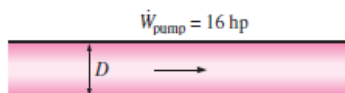
$$\Delta P = \rho g h$$

Karga Galera

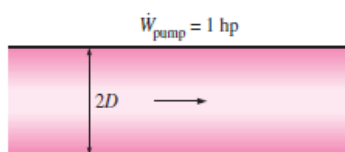
$$h_L = \frac{\Delta P_L}{\rho g} = f \frac{L}{D} \frac{\rho V_{prom}^2}{2g}$$



PONPATZE-POTENTZIA



$$W_{bomba,L} = \dot{V} \Delta P_L = \dot{V} \rho g h_L = \dot{m} g h_L$$

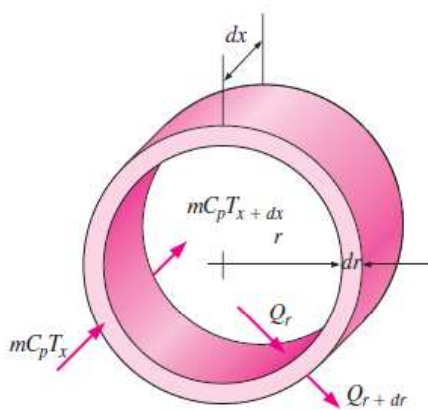


$$V_{prom} = \frac{(P_1 - P_2) R^2}{8\mu L} = \frac{\Delta P D^2}{32\mu L}$$

Poiseville-ren
Legea

$$\dot{V} = V_{prom} A_c = \frac{(P_1 - P_2) R^2}{8\mu L} \pi R^2 = \frac{\Delta P \pi D^4}{128\mu L}$$

TENPERATURA-PROFILA ETA NUSSELTEN ZENBAKIA

Fluxu Laminarra, geldikorra, $u = u(r)$

Energia Balantzea

$$\dot{m}c_p T_x - \dot{m}c_p T_{x+dx} + \dot{Q}_r - \dot{Q}_{r+dr} = 0$$

$$u \frac{\partial T}{\partial x} = - \frac{1}{2\rho c_p \pi dx} \frac{\partial \dot{Q}}{\partial r}$$

$$u \frac{\partial T}{\partial x} = \frac{\alpha}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right)$$

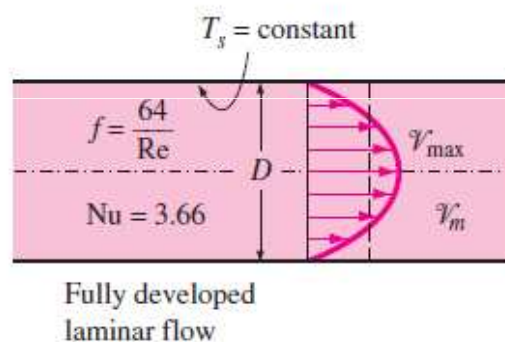
8.5 – FLUXU LAMINARRA HODIETAN

GAINAZALEKO BERO-FLUXU KONSTANTEA ($q_s = \text{cte}$)

$$Nu = \frac{hD}{k} = 4,36$$


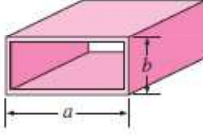
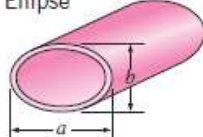
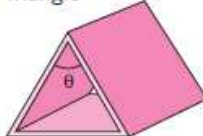
GAINAZAL-TENPERATURA KONSTANTEA ($T_s = \text{cte}$)

$$Nu = \frac{hD}{k} = 3,66$$



FLUXU LAMINARRA HODI EZ-ZIRKULARRETAN

$$Nu = \frac{hD}{k} \Rightarrow$$

| Tube Geometry | a/b or θ° | Nusselt Number | | Friction Factor f |
|--|----------------------------|-----------------------|-----------------------------|------------------------|
| | | $T_s = \text{Const.}$ | $\dot{q}_s = \text{Const.}$ | |
| Circle  | — | 3.66 | 4.36 | 64.00/Re |
| Rectangle  | a/b | | | |
| | 1 | 2.98 | 3.61 | 56.92/Re |
| | 2 | 3.39 | 4.12 | 62.20/Re |
| | 3 | 3.96 | 4.79 | 68.36/Re |
| | 4 | 4.44 | 5.33 | 72.92/Re |
| | 6 | 5.14 | 6.05 | 78.80/Re |
| | 8 | 5.60 | 6.49 | 82.32/Re |
| | ∞ | 7.54 | 8.24 | 96.00/Re |
| Ellipse  | a/b | | | |
| | 1 | 3.66 | 4.36 | 64.00/Re |
| | 2 | 3.74 | 4.56 | 67.28/Re |
| | 4 | 3.79 | 4.88 | 72.96/Re |
| | 8 | 3.72 | 5.09 | 76.60/Re |
| | 16 | 3.65 | 5.18 | 78.16/Re |
| Triangle  | θ | | | |
| | 10° | 1.61 | 2.45 | 50.80/Re |
| | 30° | 2.26 | 2.91 | 52.28/Re |
| | 60° | 2.47 | 3.11 | 53.32/Re |
| | 90° | 2.34 | 2.98 | 52.60/Re |
| | 120° | 2.00 | 2.68 | 50.96/Re |

na eta Motor
Termikoak Salla
Departamento de Máquinas
y Motores Térmicos

GARAPEN-FASEKO FLUXU LAMINARRA SARRERA-ESKUALDEAN

Hodi zirkularra, L, $T_s = \text{cte}$

$$Nu = 3,66 + \frac{0,0065(D/L) \text{Re} \text{Pr}}{1 + 0,04[(D/L) \text{Re} \text{Pr}]^{2/3}}$$

Hodi zirkularra, L, $T_s \neq T$
fluido

$$Nu = 1,86 + \left(\frac{\text{Re} \text{Pr} D}{L} \right)^{1/3} \left(\frac{\mu_b}{\mu_s} \right)^{0,14}$$

Hodi zirkularra, L, fluxua plaka
paralelo isothermoen artean
 $\text{Re} \leq 2800$

$$Nu = 7,54 + \frac{0,03(D_h/L) \text{Re} \text{Pr}}{1 + 0,016[(D_h/L) \text{Re} \text{Pr}]^{2/3}}$$

HODI LEUNA

Petukhov

$$f = (0.790 \ln Re - 1.64)^{-2} \quad 3000 < Re < 5 \times 10^6$$

Chilton-Colburn

$$\left\{ \begin{array}{l} Nu = 0.125 f Re Pr^{1/3} \\ Nu = 0.023 Re^{0.8} Pr^{1/3} \quad \left(\begin{array}{l} 0.7 \leq Pr \leq 160 \\ Re > 10\,000 \end{array} \right) \\ Nu = 0.023 Re^{0.8} Pr^n \quad \left(\begin{array}{l} n = 0.4 \text{ heating} \\ n = 0.3 \text{ cooling} \end{array} \right) \end{array} \right.$$

Sieder-Tate

$$Nu = 0.027 Re^{0.8} Pr^{1/3} \left(\frac{\mu}{\mu_s} \right)^{0.14} \quad \left(\begin{array}{l} 0.7 \leq Pr \leq 17\,600 \\ Re \geq 10\,000 \end{array} \right)$$

8.6 – FLUXU TURBULENTUA HODIETAN

Re > 10 000 Y HODI LEUNA ETA ZIMURRAK

2ª-Petukhov

$$Nu = \frac{(f/8) Re Pr}{1.07 + 12.7(f/8)^{0.5} (Pr^{2/3} - 1)} \quad \left(\begin{array}{l} 0.5 \leq Pr \leq 2\,000 \\ 10^4 < Re < 5 \times 10^6 \end{array} \right)$$

Gnielinski

$$Nu = \frac{(f/8)(Re - 1000) Pr}{1 + 12.7(f/8)^{0.5} (Pr^{2/3} - 1)} \quad \left(\begin{array}{l} 0.5 \leq Pr \leq 2\,000 \\ 3 \times 10^3 < Re < 5 \times 10^6 \end{array} \right)$$

GAINAZAL ZIMURRAK

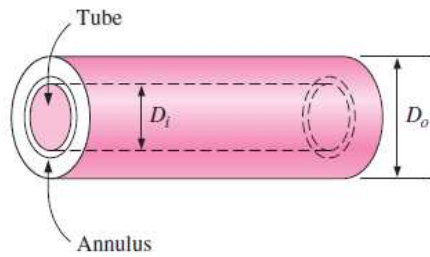
Colebrook

$$\frac{1}{\sqrt{f}} = -2.0 \log \left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{Re \sqrt{f}} \right)$$

Moody

$$\frac{1}{\sqrt{f}} = -1.8 \log \left(\frac{6.9}{Re} + \left(\frac{\varepsilon/D}{3.7} \right)^{1.11} \right)$$

FLUXUA HODI ZENTROKIDEEN ARTEKO ERAZTUN-FORMAKO ESPAZIOAN



$$D_h = \frac{4A_c}{p} = D_o - D_i$$

$$Nu_i = \frac{h_i D_h}{k} \quad Nu_o = \frac{h_o D_h}{k}$$

Erabat garatutako fluxu turbulentuetan

Gnielinski

Gnielinski ekuazioaren obezpena

$$Fi = 0.86 \left(\frac{Di}{Do} \right)^{-0.16} \quad \text{horma adiabatikoa, kanpoko}$$

$$Fo = 0.86 \left(\frac{Di}{Do} \right)^{-0.16} \quad \text{horma adiabatikoa, barnekoa}$$

8.7 – IRAKATSIKO EZ DIREN ATALAK