3. GAIA

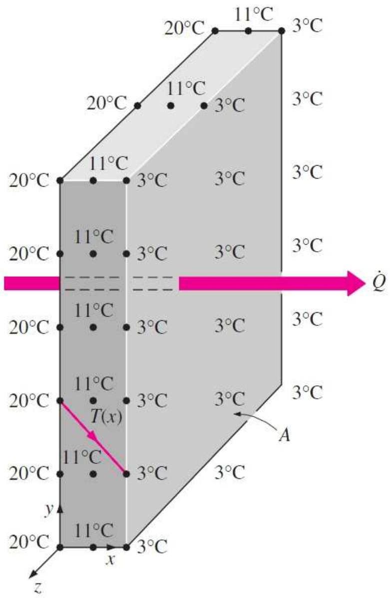
BERO-EROAPEN GELDIKORRA

3.0 - HELBURUAK

- **Erresistentzia termikoaren** kontzeptua eta haren mugak ulertu, eta erresistentzia termikoen sareak garatu bero-eroapeneko problema praktikoetarako.
- Geruza anitzeko geometria angeluzuzen, zilindriko edo esferikoen **eroapen geldikorreko** problemak ebatzi.
- **Kontaktu-erresistentzia** termikoa eta hori zer egoeratan izan daitekeen garrantzitsua era intuitiboan ulertzeko gaitasuna landu.
- Isolatuz gero bero-transferentzia handitu dezaketen aplikazioak identifikatu (**erradio kritikoa**).
- **Gainazal hegaldunak** aztertu, eta kalkulatu zenbateko errendimendua eta eraginkortasuna duen hegalak bero-transferentzia handitzeko.

3.1 – BERO-EROAPENA HORMA LAUETAN

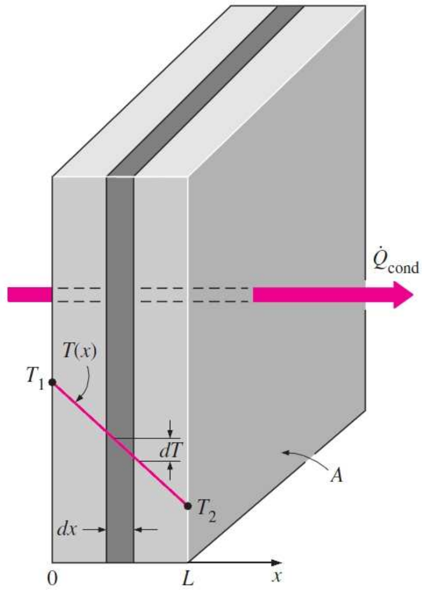
- Etxe baten horma neguan



$$\dot{Q}_{in} - \dot{Q}_{out} = \frac{dE_{wall}}{dt} = 0$$

$$\dot{Q}_{cond,wall} = -kA\frac{dT}{dx}$$

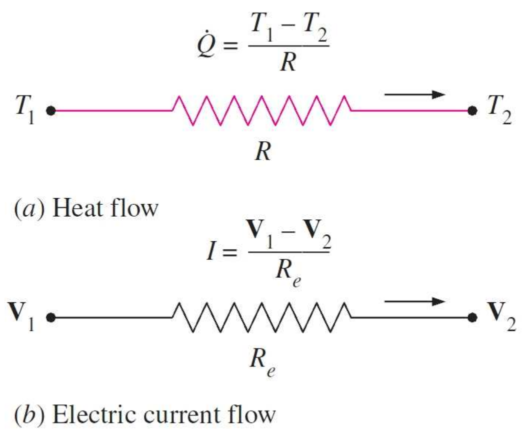
$$\dot{Q}_{cond,wall} = kA\frac{T_1 - T_2}{L}$$



3.1 – BERO-EROAPENA HORMA LAUETAN

ERRESISTENTZIA TERMIKOAREN KONTZEPTUA

Analogia termoelektrikoa



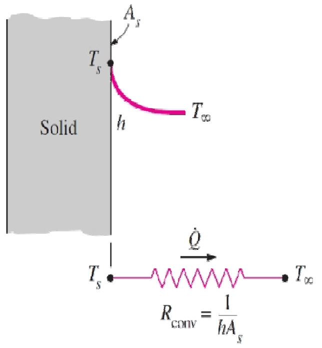
Baldintzak:
- Egoera egonkorra
- Bero-sorrera gabe

- Eroapena:

$$\dot{Q}_{cond,wall} = kA\frac{T_1 - T_2}{L} = \frac{T_1 - T_2}{R_{wall}} \Rightarrow \boxed{R_{wall} = \frac{L}{kA}}$$

ERRESISTENTZIA TERMIKOAREN KONTZEPTUA

• <u>Konbekzioa</u>:

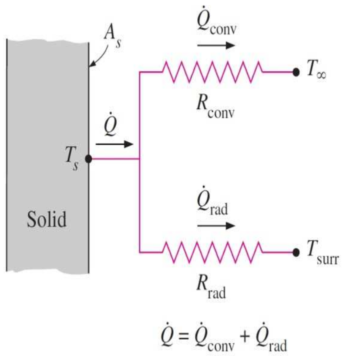


$$\dot{Q}_{conv} = hA\left(T_s - T_\infty\right) \Rightarrow$$

$$\Rightarrow \dot{Q}_{conv} = \frac{T_s - T_\infty}{R_{conv}}$$

$$\boxed{R_{conv} = \frac{1}{hA_s}}$$

• <u>Erradiazioa</u>:

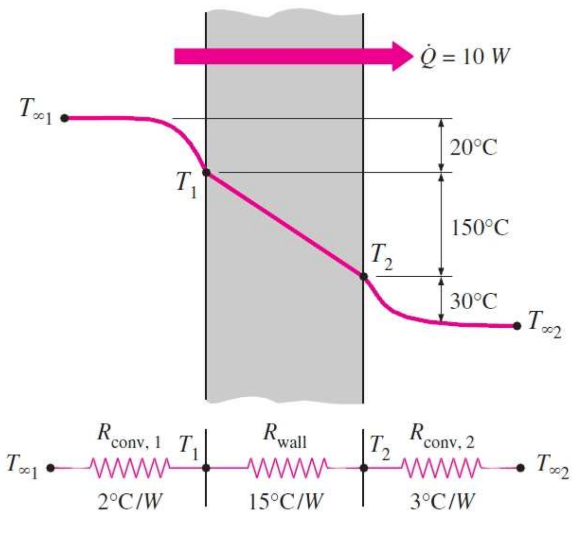


$$\dot{Q}_{rad} = \varepsilon \cdot \sigma \cdot A_s\left(T_s^4 - T_{surr}^4\right) = h_{rad} \cdot A_s\left(T_s - T_{surr}\right) =$$

$$= \frac{T_s - T_{surr}}{R_{rad}}$$

$$\boxed{R_{rad} = \frac{1}{h_{rad} A_s}}$$

ERRESISTENTZIA TERMIKOEN SAREA



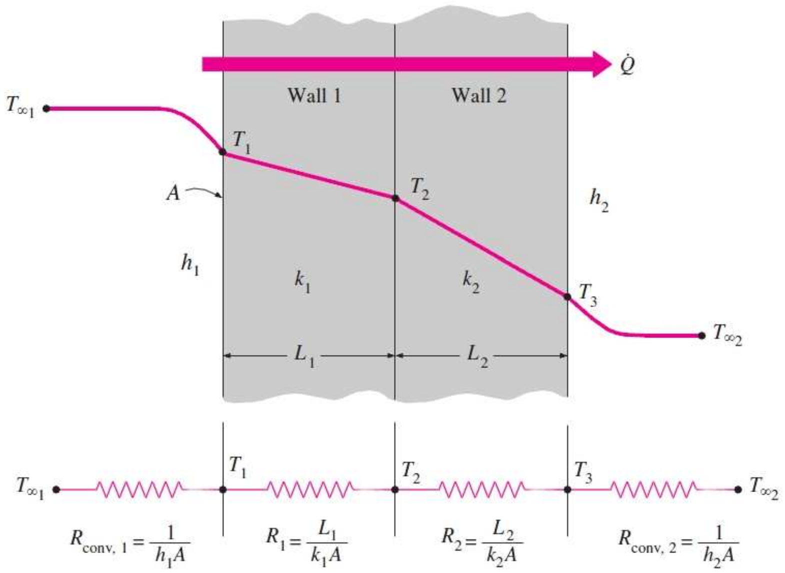
$$\dot{Q} = \frac{\Delta T}{R} = UA\Delta T$$

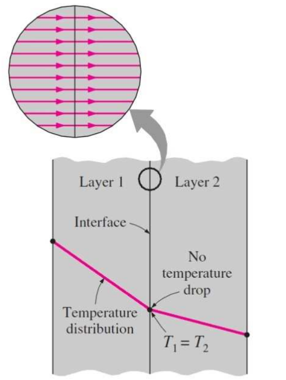
$$\boxed{UA = \frac{1}{R_{total}}}$$

$$\dot{Q} = \frac{T_{\infty 1} - T_1}{R_{conv,1}} = \frac{T_1 - T_2}{R_{cond}} = \frac{T_2 - T_{\infty 2}}{R_{conv,2}} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}}$$

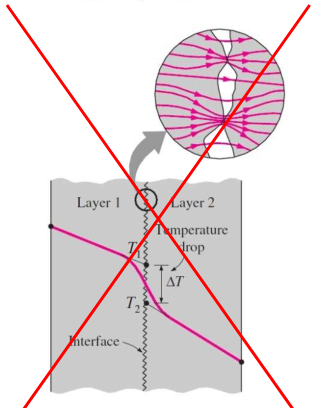
3.1 – BERO-EROAPENA HORMA LAUETAN

GERUZA ANITZEKO HORMA LAUAK



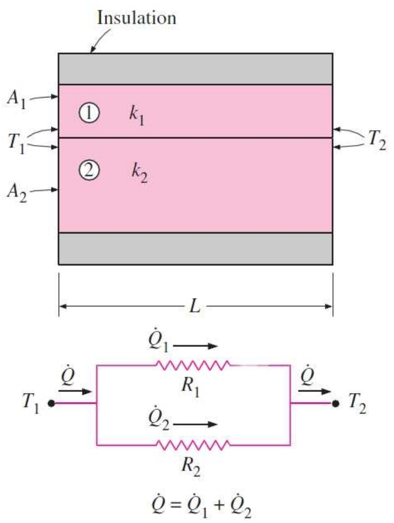


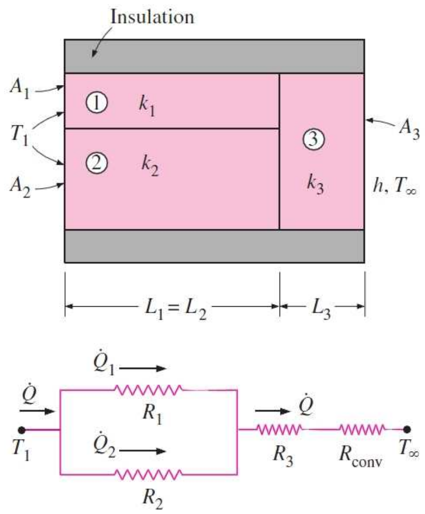
(a) Ideal (perfect) thermal contact

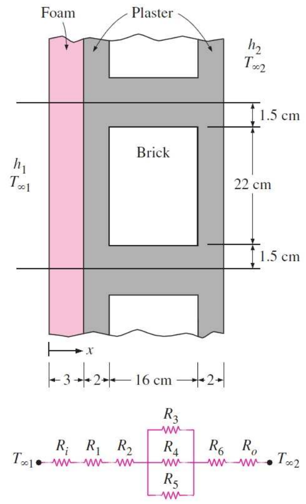


(b) Actual (imperfect) thermal contact

3.3 – ERRESISTENTZIA TERMIKOEN SARE OROKORTUAK

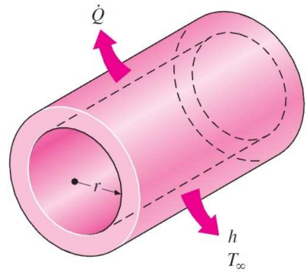




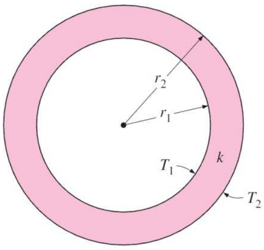


Helburua

$$\dot{Q}_{cond} = -kA\frac{dT}{dr} \Rightarrow \dot{Q}_{cond} = \frac{\Delta T}{R}$$



• Zilindroak:

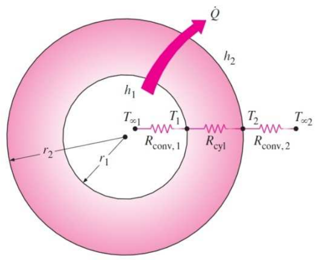


$$\int_{r_1}^{r_2} \frac{\dot{Q}_{cond}}{A} dr = -\int_{T_1}^{T_2} kdT \Rightarrow \dot{Q}_{cond} = 2\pi Lk\frac{T_1 - T_2}{\ln\left(r_2 / r_1 \right)}$$

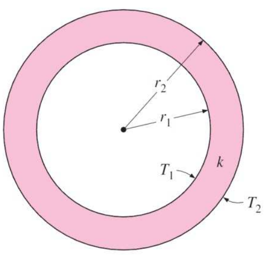
$$\boxed{R_{cil} = \frac{\ln\left(r_2 / r_1 \right)}{2\pi Lk}}$$

Helburua

$$\dot{Q}_{cond} = -kA\frac{dT}{dr} \Rightarrow \dot{Q}_{cond} = \frac{\Delta T}{R}$$



• Esferak:



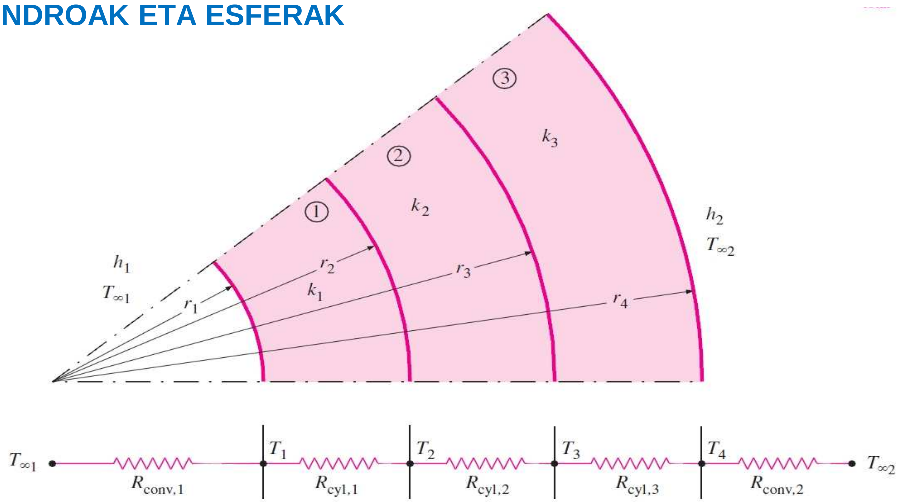
$$\int_{r_1}^{r_2} \frac{\dot{Q}_{cond}}{A} dr = -\int_{T_1}^{T_2} kdT \Rightarrow \dot{Q}_{cond} = 4\pi \cdot r_1 r_2 k\frac{T_1 - T_2}{r_2 - r_1}$$

$$\boxed{R_{esf} = \frac{r_2 - r_1}{4\pi \cdot r_1 r_2 k}}$$

3.4 – BERO-EROAPENA ZILINDROETAN ETA ESFERETAN

GERUZA ANITZEKO ZILINDROAK ETA ESFERAK

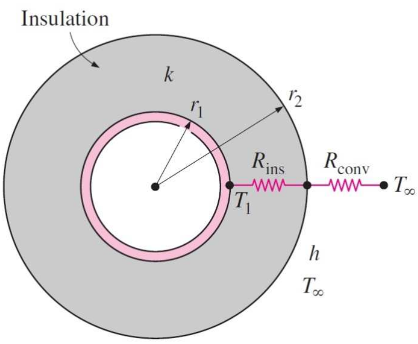
$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}}$$



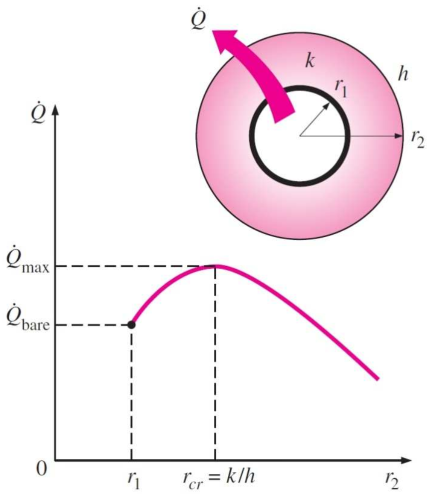
$$R_{total} = R_{conv,1} + R_{cyl,1} + R_{cyl,2} + R_{cyl,3} + R_{conv,2} \Rightarrow$$

$$\Rightarrow R_{total} = \frac{1}{h_1 A_1} + \frac{\ln\left(r_2/r_1\right)}{2\pi L k_1} + \frac{\ln\left(r_3/r_2\right)}{2\pi L k_2} + \frac{\ln\left(r_4/r_3\right)}{2\pi L k_3} + \frac{1}{h_2 A_4}$$

3.5 – ISOLAMENDU-ERRADIO KRITIKOA



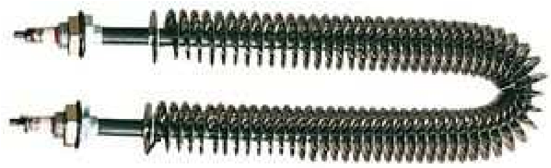
$$\dot{Q} = \frac{T_1 - T_\infty}{R_{ins} + R_{conv}} = \frac{T_1 - T_\infty}{\dfrac{\ln\left(r_2/r_1\right)}{2\pi\, Lk} + \dfrac{1}{h 2\pi\, r_2 L}}$$

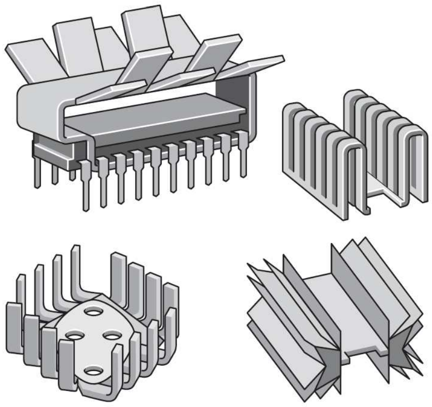


Baldin $r_2 \uparrow$ → $R_{ins} \uparrow$, $R_{conv} \downarrow$ ⇒

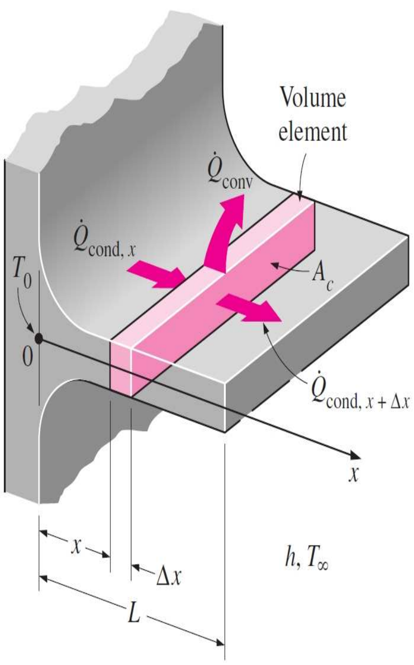
$$\boxed{\frac{d\dot{Q}}{dr_2} = 0 \Rightarrow \begin{cases} r_{cr,cil} = k/h \\ r_{cr,esf} = 2k/h \end{cases}}$$

$$\dot{Q}_{conv} = hA_s\left(T_s - T_\infty\right)$$





HEGAL-EKUAZIOA



$$\left.\begin{aligned} &\dot{Q}_{cond,x} = \dot{Q}_{cond,x+\Delta x} + \dot{Q}_{conv} \\ &\dot{Q}_{conv} = h\left(p\Delta x\right)\cdot\left(T - T_\infty\right) \end{aligned}\right\} \Rightarrow$$

$$\left.\begin{aligned} &\Rightarrow \frac{\dot{Q}_{cond,x+\Delta x} - \dot{Q}_{cond,x}}{\Delta x} + hp\left(T - T_\infty\right) = 0 \xrightarrow{x\to 0} \frac{d\dot{Q}_{cond,x}}{dx} + hp\left(T - T_\infty\right) = 0 \\ &\dot{Q}_{cond,x} = -kA_c\frac{dT}{dx} \end{aligned}\right\} \Rightarrow$$

$$\Rightarrow \frac{d}{dx}\left(kA_c\frac{dT}{dx}\right) - hp\left(T - T_\infty\right) = 0$$

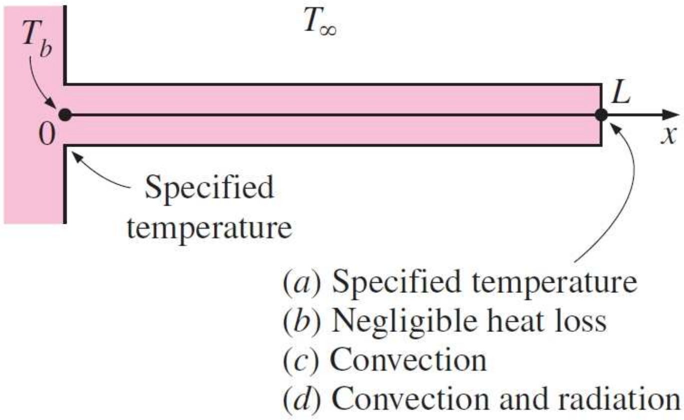
HEGAL-EKUAZIOA

Suposatuz x-ekiko ez direla aldatzen:

- Eroankortasun termikoa
- Zeharkako azalera
- Perimetroa

$$\theta(x) = T(x) - T_\infty$$

$$\boxed{\frac{d^2\theta}{dx^2} - a^2 \theta = 0 \rightarrow a^2 = \frac{hp}{kA_c}}$$



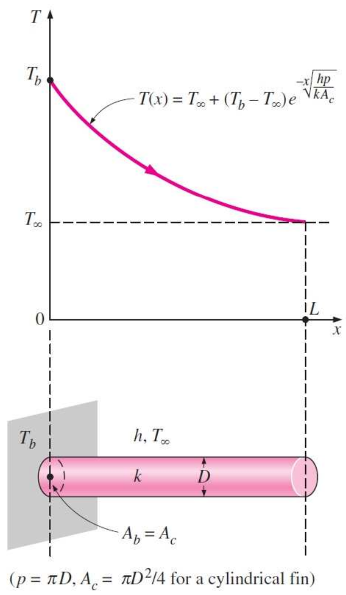
$$\boxed{\theta(x) = C_1 e^{ax} + C_2 e^{-ax}}$$

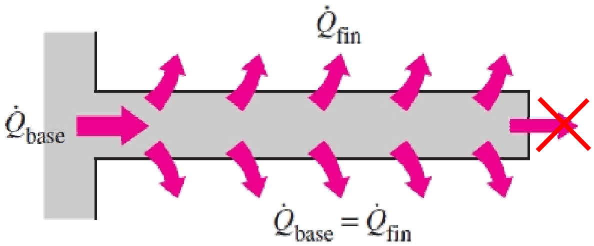
HEGAL-EKUAZIOA

$$\theta(x) = C_1 e^{ax} + C_2 e^{-ax}$$

1. KASUA: Hegal luzera infinitukoa

$$\frac{T(x) - T_\infty}{T_b - T_\infty} = e^{-ax} = e^{-x\sqrt{hp/kA_c}}$$





$$\dot{Q}_{long\ fin} = -kA_c \left.\frac{dT}{dx}\right|_{x=0} = \sqrt{hpkA_c}\,(T_b - T_\infty)$$

HEGAL-EKUAZIOA

Hegalak ez dira izaten beren muturreko tenperatura inguruneko tenperaturara hurbiltzeko adina luzeak → T(L) = T_∞

2. KASUA: Hegalaren muturreko bero-galera baztergarria

$$\frac{T(x)-T_\infty}{T_b-T_\infty}=\frac{\cosh a(L-x)}{\cosh aL}$$

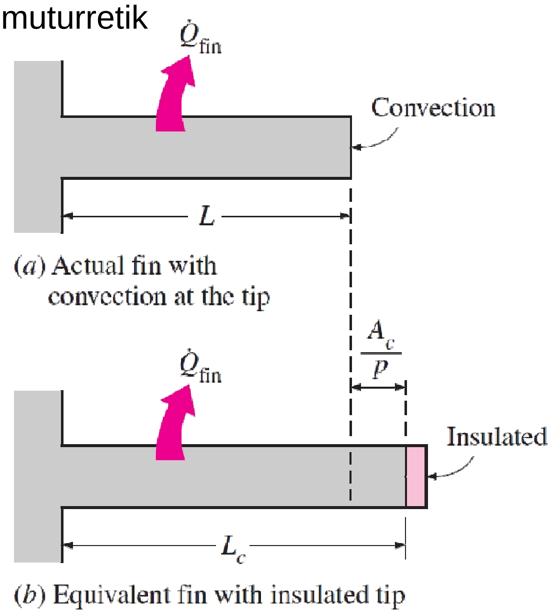
$$T(x)-T_\infty=C_1e^{ax}+C_2e^{-ax}$$

Numerikoki

$$\dot{Q}_{long\ fin}=-kA_c\left.\frac{dT}{dx}\right|_{x=0}=\sqrt{hpkA_c}\,(T_b-T_\infty)\tanh aL$$

HEGAL-EKUAZIOA

3. KASUA: Konbekzioa (edo konbekzio eta erradiazio konbinatuak) hegal-muturretik



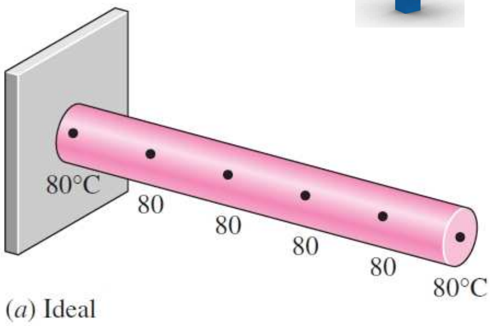
(a) Actual fin with convection at the tip

(b) Equivalent fin with insulated tip

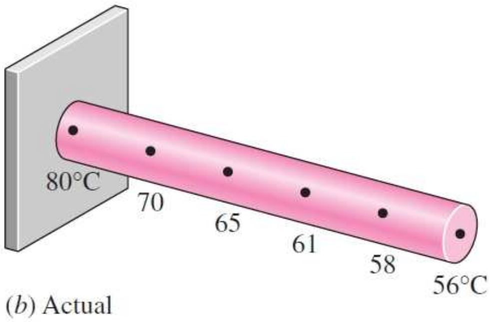
Hegal-mutur adiabatikoaren kasua luzera zuzenduz

$$\boxed{L_c=L+\frac{A_c}{p}}$$

HEGAL ERRENDIMENDUA



(*a*) Ideal



(*b*) Actual

Hegalaren bero-transferentzia maximoa

$$\dot{Q}_{fin,\max} = hA_{fin}\left(T_b - T_\infty\right)$$

Hegalaren errendimendua

$$\boxed{\eta_{fin} = \frac{\dot{Q}_{fin}}{\dot{Q}_{fin,\max}} \Rightarrow \dot{Q}_{fin} = \eta_{fin} \cdot \dot{Q}_{fin,\max}}$$

f(geom,a)

HEGAL ERRENDIMENDUA

<u>1. KASUA</u>: Hegal luzera infinitua

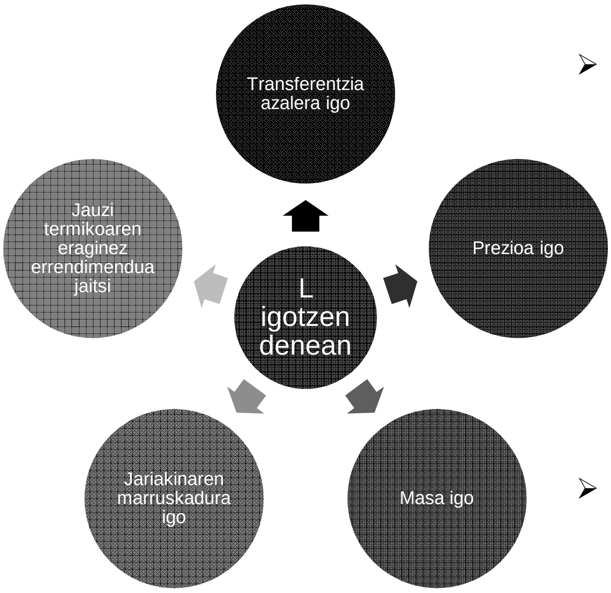
$$\eta_{long\ fin} = \frac{\dot{Q}_{fin}}{\dot{Q}_{fin,\max}} = \frac{\sqrt{hpkA_c}\left(T_b - T_\infty\right)}{hA_{aleta}\left(T_b - T_\infty\right)} = \frac{1}{L}\sqrt{\frac{kA_c}{hp}} = \frac{1}{aL}$$

<u>2. KASUA</u>: Hegalaren muturreko bero-galera baztergarria

$$\eta_{insulated\ tip} = \frac{\dot{Q}_{fin}}{\dot{Q}_{fin,\max}} = \frac{\sqrt{hpkA_c}\left(T_b - T_\infty\right)\tanh aL}{hA_{aleta}\left(T_b - T_\infty\right)} = \frac{\tanh aL}{aL}$$

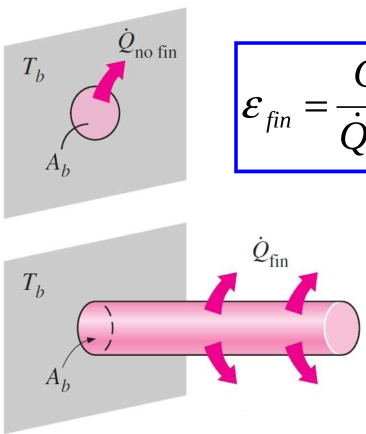
HEGAL ERRENDIMENDUA

Hegalaren luzerari buruzko iruzkinak:



- Hegal-errendimendua % 60tik jaitsarazten duten luzerak ezin dira justifikatu ekonomiaren ikuspegitik, normalean
- Praktikan erabiltzen diren hegal gehienen errendimendua % 90etik gorakoa da.

HEGAL-ERAGINKORTASUNA



$$\boxed{\varepsilon_{fin} = \frac{\dot{Q}_{fin}}{\dot{Q}_{no\ fin}}}$$

ε<1
- Hegalak isolatzaile moduan jokatzen du

ε=1
- Ez du bero-transferentzian eraginik

ε>1
- Bero-transferentzia handitzen da
- Errentagarria da?

$$\varepsilon_{fin} = \frac{\dot{Q}_{fin}}{\dot{Q}_{no\ fin}} = \frac{\eta_{fin} h A_{fin} (T_b - T_\infty)}{h A_b (T_b - T_\infty)} = \frac{A_{fin}}{A_b}\eta_{fin}$$

HEGAL-ERAGINKORTASUNA

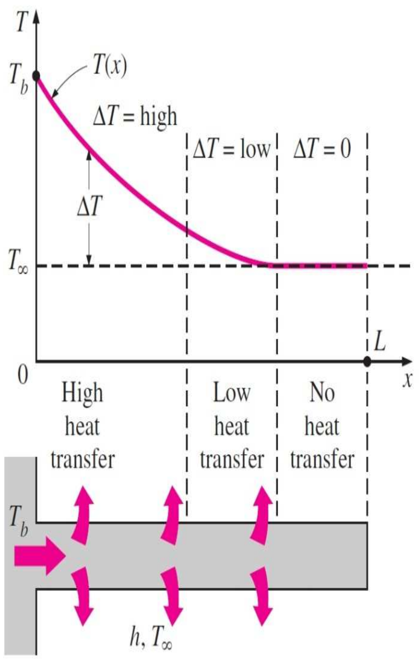
<u>1. KASUA</u>: Hegal luzera infinitua

$$\varepsilon_{long\ fin} = \frac{\dot{Q}_{fin}}{\dot{Q}_{no\ fin}} = \frac{\sqrt{hpkA_c}\,(T_b - T_\infty)}{hA_b\,(T_b - T_\infty)} = \sqrt{\frac{kp}{hA_c}}$$

<u>Ondorioak</u>:

- ✓ h txikia → Eraginkortasun handiagoa konbekzio naturalean eta gasetan
- ✓ k altua (metalak)
- ✓ p/A_c erlazio altua (xafla meheko hegalak)

HEGALAREN LUZERA EGOKIA



$$\frac{\dot{Q}_{fin}}{\dot{Q}_{long\ fin}} = \frac{\sqrt{hpkA_c}\,(T_b - T_\infty)\tanh aL}{\sqrt{hpkA_c}\,(T_b - T_\infty)} = \tanh aL$$

aL	$\dfrac{\dot{Q}_{fin}}{\dot{Q}_{long\ fin}} = \tanh aL$
0.1	0.100
0.2	0.197
0.5	0.462
1.0	0.762
1.5	0.905
2.0	0.964
2.5	0.987
3.0	0.995
4.0	0.999
5.0	1.000

3.7 – IRAKATSIKO EZ DIREN ATALAK

- 3.2 atala: KONTAKTU-ERRESISTENTZIA TERMIKOA
- 3.7 atala: BERO-TRANSFERENTZIA KONFIGURAZIO ARRUNTETAN

Irudi Iturria:
ÇENCEL, Y.A. TRANSFERENCIA DE CALOR Y MASA, Un enfoque práctico. McGraw-Hil.3 Edición. 2007