

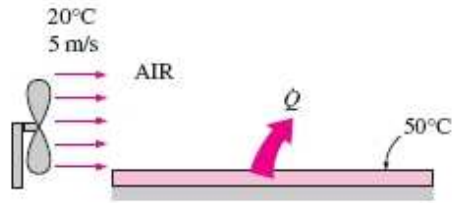
6. GAIA

KONBEKZIOAREN OINARRIAK

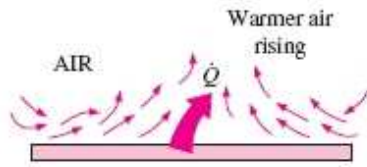
6.0 - HELBURUAK

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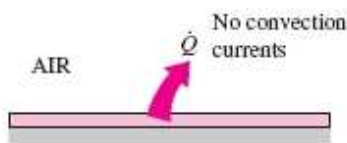
- Konbekzioaren mekanismo fisikoak eta sailkapena ulertu.
- Gainazalen gaineko fluxuen **abiaduraren mugalde-geruza** eta **mugalde-geruza termikoa** irudikatu.
- Reynoldsen, Prandtlen eta Nusselten **zenbaki dimentsiogabeen** ezagutza praktikoa izan.
- Fluxu **laminarrak** eta **turbulentuak** bereizi, eta fluxu turbulentuen momentu- eta bero-transferentziako mekanismoak ulertu
- Konbekzioa deskribatzen duten **ekuazio diferentzialak garatu**, masa-, momentu- eta energia-balantzeetan oinarrituta
- Konbekzio-ekuazioak **dimentsiogabetu**, eta marruskaduraren eta bero-transferentziaren koefizienteen forma **funtzionalak** lortu.
- Momentu- eta bero-transferentziaren arteko **analogiak** erabili, eta bero-transferentziaren koefizientea kalkulatu, marruskadura-koefizientetik abiatuta



(a) Forced convection



(b) Free convection



(c) Conduction

Konbekzio bero-transferentzia hurrengo parametroen arabaerakoa da:

Biskositate dinamikoa μ_{fluid}

Eroankortasun termikoa k_{fluid}

Dentsitatea ρ_{fluid}

Bero espezifikoa $C_{p,\text{fluid}}$

Jariakinaren abiadura V_{fluid}

Konfigurazio geometrikoa

Solidoaren zimurtasuna

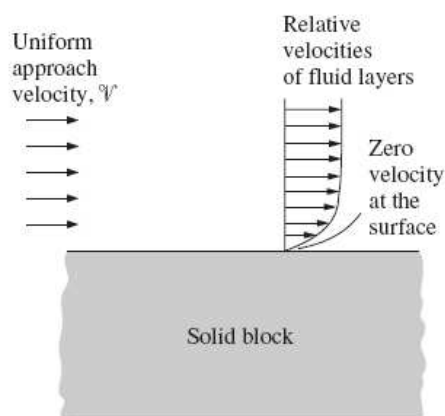
Jariakinaren fluxu mota (laminarra edo turbulentua)

Newtonen hozte legea

$$\dot{Q} = h \cdot A \cdot (T_s - T_\infty) \quad [\text{W}]$$

TERMOTEKNIA

6.1 – KONBEKZIAOREN MEKANISMO FISIKOA



Irristadurarik ezaren baldintza

Mugalde geruza

$$\dot{q}_{\text{cond}} = -k_{\text{fluid}} \cdot \left. \frac{\partial T}{\partial y} \right|_{y=0} \quad [\text{W/m}^2]$$

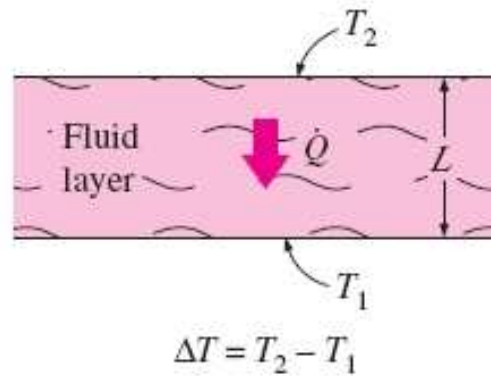
$$\dot{q}_{\text{conv}} = h \cdot (T_s - T_\infty) \quad [\text{W/m}^2]$$

$$h = \frac{-k_{\text{fluid}} \cdot (\partial T / \partial y)_{y=0}}{T_s - T_\infty} \quad [\text{W/m}^2 \cdot ^\circ\text{C}]$$

Bataz besteko konbekzio koefizientea – Konbekzio koefiziente lokala

NUSSELTEN ZENBAKIA

$$Nu = \frac{h \cdot L_c}{k}$$



$$\left. \begin{aligned} \dot{q}_{conv} &= h \cdot \Delta T \\ \dot{q}_{cond} &= k \cdot \frac{\Delta T}{L} \end{aligned} \right\} \Rightarrow \frac{\dot{q}_{conv}}{\dot{q}_{cond}} = \frac{h \cdot \Delta T}{k \cdot \Delta T / L} = \frac{h \cdot L}{k} = Nu$$

6.2 – FLUIDO-FLUXUEN SAILKAPENA

Fluxu likatsua vs ez-likatsua.

Barne-fluxua vs kanpo-fluxua.

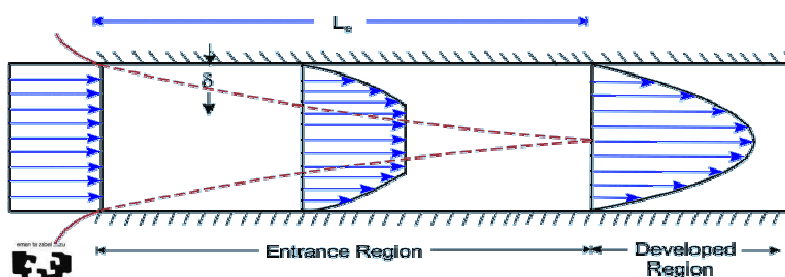
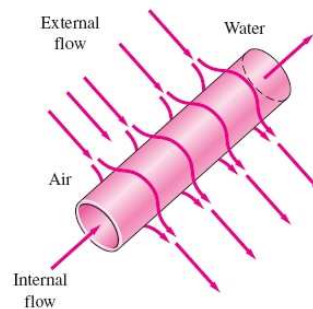
Fluxu konprimagarria vs konprimaezina.

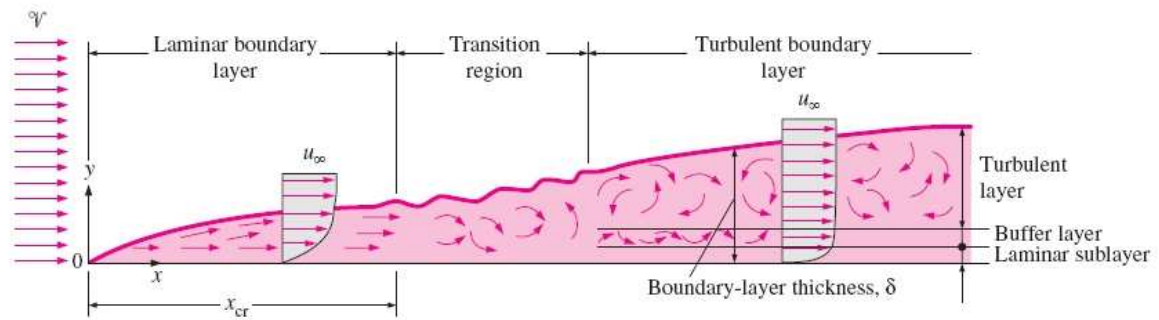
Fluxu laminarra vs turbulentua.

Fluxu naturala vs behartua.

Fluxu geldikorra vs ez-geldikorra (iragankorra).

Dimentsio bakarreko, biko eta hiruko fluxuak.





GAINAZALEKO EBAKIDURA-TENTSIOA $\tau = \mu \frac{\partial u}{\partial y} \Big|_{y=0}$ [N/m²] Jariakin
newtondarrak

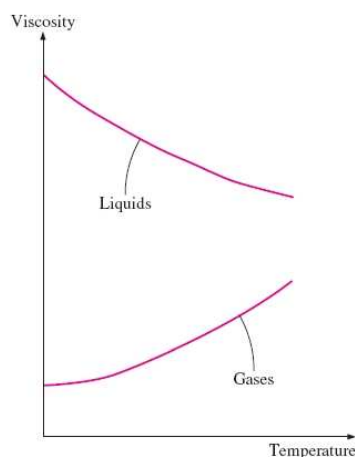
Biskositate dinamikoa: μ [kg/m·s] [poise] 1 poise = 0,1 kg /m·s

Biskositate zinematikoa: $\nu = \frac{\mu}{\rho}$ [m²/s] [stoke] 1 stoke = 10⁻⁴ m²/s

6.3 – ABIADURAREN MUGALDE-GERUZA

Dynamic viscosities of some fluids
at 1 atm and 20°C (unless otherwise
stated)

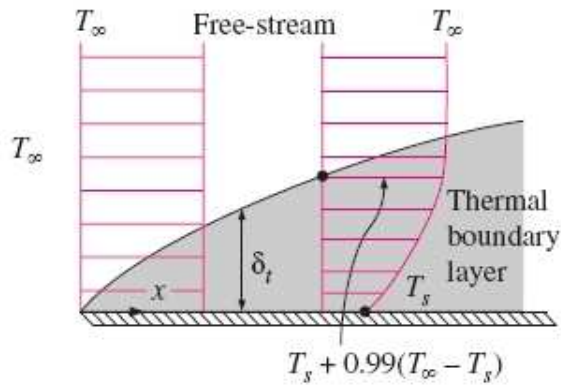
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GAINAZALEKO EBAKIDURA-TENTSIOA

$$\tau_s = C_f \frac{\rho \cdot V^2}{2}$$

↙ Marrusadura-koefizientea

Fluid	Dynamic viscosity μ , kg/m · s
Glycerin:	
−20°C	134.0
0°C	12.1
20°C	1.49
40°C	0.27
Engine oil:	
SAE 10W	0.10
SAE 10W30	0.17
SAE 30	0.29
SAE 50	0.86
Mercury	0.0015
Ethyl alcohol	0.0012
Water:	
0°C	0.0018
20°C	0.0010
100°C (liquid)	0.0003
100°C (vapor)	0.000013
Blood, 37°C	0.0004
Gasoline	0.00029
Ammonia	0.00022
Air	0.000018
Hydrogen, 0°C	0.000009



Typical ranges of Prandtl numbers for common fluids

Fluid	Pr
Liquid metals	0.004–0.030
Gases	0.7–1.0
Water	1.7–13.7
Light organic fluids	5–50
Oils	50–100,000
Glycerin	2000–100,000

PRANDTLEN ZENBAKIA

$$Pr = \frac{\nu}{\alpha} = \frac{\mu \cdot c_p}{k}$$

$$Pr = \frac{\text{Momentuaren difusibitate molekularra}}{\text{Beroaren difusibitate molekularra}}$$

6.5 – FLUXU LAMINARRAK ETA TURBULENTOAK



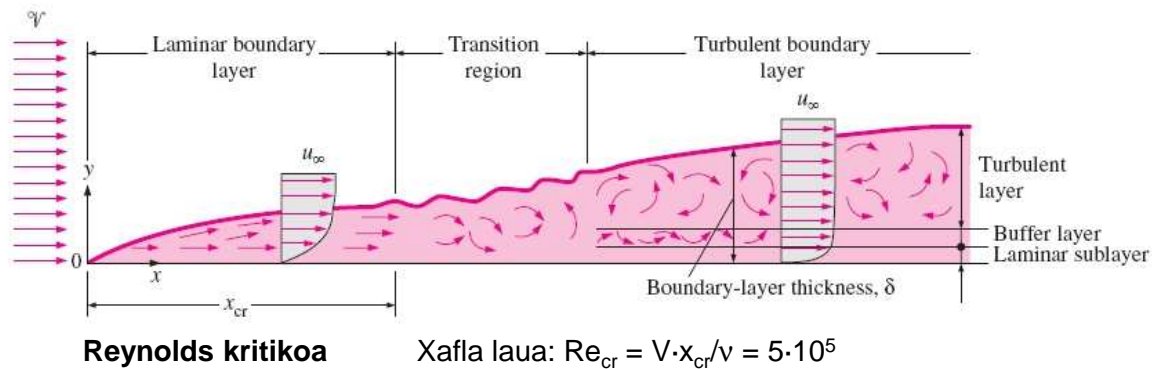
REYNOLDSSEN ZENBAKIA

$$Re = \frac{V \cdot L_c}{\nu} = \frac{\rho \cdot V \cdot L_c}{\mu}$$

$$Re = \frac{\text{Inertzi indarrak}}{\text{Biskositate - indarrak}}$$

Re baxuak → Fluxu laminarra

Re altuak → Fluxu turbulentua

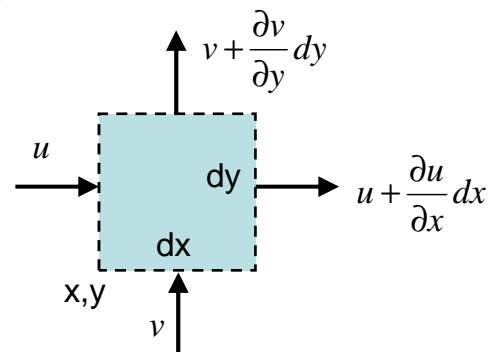
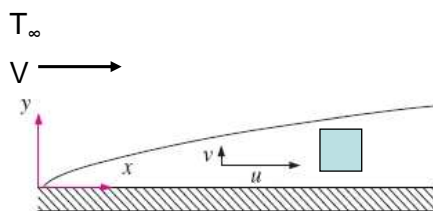


6.7 – KONBEKZIO-EKUAZIO DIFERENTZIALEN GARAPENA

Hipotesiak:

- Fluxu geldikorra bi dimentsiotan
- Jariakin Newtondarra
- Propietateak konstante (ρ, ν, k, \dots)

JARRAITUTASUN-EKUAZIOA

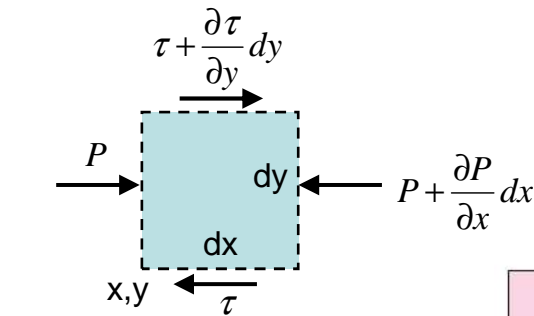


$$\left[\text{Masa-emaria kontrol-bolumenaren barrurantz} \right] = \left[\text{Masa-emaria kontrol-bolumenaren kanporantz} \right]$$

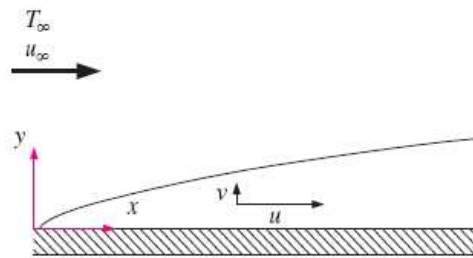
$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

MOMENTU-EKUAZIOAK

$$\left(\text{Masa} \right) \left(\begin{array}{c} \text{Azelerazioa, zehaztutako} \\ \text{noranzkoan} \end{array} \right) = \left(\begin{array}{c} \text{Noranzko horretan eragiten duen indar} \\ \text{garbia (gorputzekoa eta gainazalekoa)} \end{array} \right)$$



$$\rho \left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = \mu \frac{\partial^2 u}{\partial y^2} - \frac{\partial P}{\partial x}$$



- 1) Velocity components:
 $v \ll u$
- 2) Velocity gradients:
 $\frac{\partial v}{\partial x} \approx 0, \frac{\partial v}{\partial y} \approx 0$
 $\frac{\partial u}{\partial x} \ll \frac{\partial u}{\partial y}$
- 3) Temperature gradients:
 $\frac{\partial T}{\partial x} \ll \frac{\partial T}{\partial y}$

$$\frac{\partial P}{\partial y} = 0$$

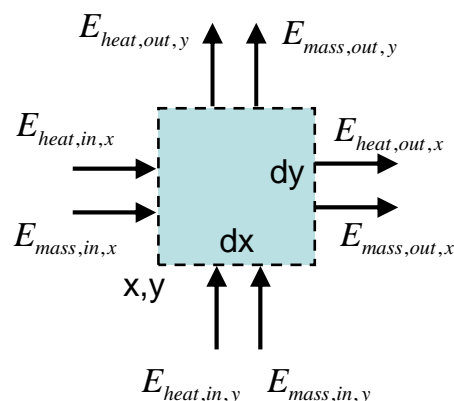
6.7 – KONBEKZIO-EKUAZIO DIFERENTZIALEN GARAPENA

ENERGIA-KONTSERBAZIOAREN EKUAZIOA

$$E_{in} - E_{out} = \Delta E_{sist} \xrightarrow{\text{Egoera egonkorrean:}} \dot{E}_{in} - \dot{E}_{out} = 0$$

$$(\dot{E}_{in} - \dot{E}_{out})_{\text{by heat}} + (\dot{E}_{in} - \dot{E}_{out})_{\text{by work}} + (\dot{E}_{in} - \dot{E}_{out})_{\text{by mass}} = 0$$

$$e_{stream} = h + e_c + e_p = h + \frac{1}{2}V^2 + g \cdot z \approx h = c_p \cdot T$$



ENERGIA-KONTSERBAZIOAREN EKUAZIOA

$$(\dot{E}_{in} - \dot{E}_{out})_{\text{by mass}} = -\rho \cdot c_p \cdot \left(u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) dx \cdot dy$$

$$(\dot{E}_{in} - \dot{E}_{out})_{\text{by heat}} = k \cdot \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) dx \cdot dy$$

$$(\dot{E}_{in} - \dot{E}_{out})_{\text{by work}} = \mu \cdot \Phi \cdot dx \cdot dy = \mu \cdot \left(\frac{\partial u}{\partial y} \right)^2 \cdot dx \cdot dy$$

Abiadura baxuetan mesprezagarria

$$\dot{E}_{in} - \dot{E}_{out} = 0 \quad \rightarrow \quad \boxed{\rho \cdot c_p \cdot \left(u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = k \cdot \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right)}$$

Mugalde geruzaren barnean oso txikia

TERMOTEKNIKIA

6.9 – KONBEKZIO-EKUAZIO DIMENTSIOGABEAK ETA ANTZEKOTASUNA

Aldagai guztiak dimentsiogabetuz:

$$x^* = \frac{x}{L}; \quad y^* = \frac{y}{L}; \quad u^* = \frac{u}{V}; \quad v^* = \frac{v}{V}; \quad P^* = \frac{P}{\rho \cdot V^2}; \quad T^* = \frac{T - T_s}{T_\infty - T_s};$$

Jarraitutasuna:

$$\frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*} = 0$$

Momentua:

$$u^* \frac{\partial u^*}{\partial x^*} + v^* \frac{\partial u^*}{\partial y^*} = \frac{1}{\text{Re}_L} \frac{\partial^2 u^*}{\partial y^{*2}} - \frac{\partial P^*}{\partial x^*}$$

Energia:

$$u^* \frac{\partial T^*}{\partial x^*} + v^* \frac{\partial T^*}{\partial y^*} = \frac{1}{\text{Re}_L \cdot \text{Pr}} \left(\frac{\partial^2 T^*}{\partial y^{*2}} \right)$$

$$u^*(0, y^*) = 1; \quad u^*(x^*, 0) = 0; \quad u^*(x^*, \infty) = 1; \quad v^*(x^*, 0) = 0;$$

$$T^*(0, y^*) = 1; \quad T^*(x^*, 0) = 0; \quad T^*(x^*, \infty) = 1;$$

“Bi fenomeno fisiko *antzekoak* dira, baldin eta deskribatzen dituzten ekuazio diferentzialen eta mugalde-baldintzen forma dimentsiogabe berak badituzte.”

$$\begin{array}{c} L \\ V \\ T_{\infty} \\ T_s \\ \nu \\ \alpha \end{array} \longrightarrow \begin{array}{c} Re_L \\ Pr \end{array}$$

“Geometria jakin batean, antzekotasun-parametroen balio **bera** duten problemek **ebazpen berdin-berdinak** dituzte.”

6.10 – MARRUSKADURA- ETA KONBEKZIO-KOEFIZIENTEEN FORMA FUNTZIONALAK

$$\left\{ \begin{array}{ll} 3 \text{ ekuazio dimentsiogabeak} & \begin{array}{l} 3 \text{ ezezagun: } u^*, v^* \text{ y } T^* \\ 2 \text{ aldagai independente: } x^*, y^* \\ 2 \text{ parametro: } Re_L \text{ y } Pr \end{array} \end{array} \right.$$

Lehenengo bi ekuazioetatik: $u^* = f_1(x^*, y^*, Re_L)$

Ebakidura-tentsioa: $\tau_s = \mu \left. \frac{\partial u}{\partial y} \right|_{y=0} = \frac{\mu \cdot V}{L} \left. \frac{\partial u^*}{\partial y^*} \right|_{y^*=0} = \frac{\mu \cdot V}{L} f_2(x^*, Re_L)$

Marruskadura-koefizientea:

$$C_{f,x} = \frac{\tau_s}{\rho \cdot V^2 / 2} = \frac{\mu \cdot V / L}{\rho \cdot V^2 / 2} f_2(x^*, Re_L) = f_3(x^*, Re_L)$$

6.10 – MARRUSKADURA- ETA KONBEKZIO-KOEFIZIENTEEN FORMA FUNTZIONALAK

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Azkenengo ekuaziotik.:

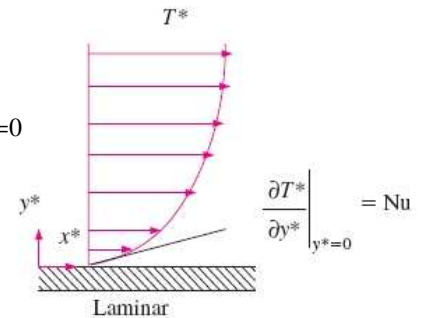
$$T^* = g_1(x^*, y^*, \text{Re}_L, \text{Pr})$$

Konbekzio koefizienteak:

$$y = 0 \Rightarrow \dot{q}_{\text{cond}} = \dot{q}_{\text{conv}} \Rightarrow -k_{\text{fluid}} \cdot \frac{\partial T}{\partial y} \Big|_{y=0} = h \cdot (T_s - T_\infty)$$

$$h = - \frac{k \frac{\partial T}{\partial y} \Big|_{y=0}}{T_s - T_\infty} = - \frac{k \frac{T_\infty - T_s}{L} \cdot \frac{\partial T^*}{\partial y^*} \Big|_{y^*=0}}{T_s - T_\infty} = \frac{k}{L} \frac{\partial T^*}{\partial y^*} \Big|_{y^*=0}$$

$$\text{Nu}_x = \frac{h \cdot L}{k} = \frac{\partial T^*}{\partial y^*} \Big|_{y^*=0} = g_2(x^*, \text{Re}_L, \text{Pr})$$



A menudo

$$C_f = f_4(\text{Re}_L) \quad \text{Nu} = g_3(\text{Re}_L, \text{Pr})$$

$$\boxed{\text{Nu} = C \cdot \text{Re}_L^m \cdot \text{Pr}^n}$$

6.11 – MOMENTU- ETA BERO TRANSFERENTZIAREN ARTEKO ANALOGIAK

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Helburua: C_f eta Nu lortu

$$\text{Si } \text{Pr} = 1 \text{ y } \frac{\partial P^*}{\partial x^*} = 0$$

Momentua:

$$u^* \frac{\partial u^*}{\partial x^*} + v^* \frac{\partial u^*}{\partial y^*} = \frac{1}{\text{Re}_L} \frac{\partial^2 u^*}{\partial y^{*2}}$$

Energia:

$$u^* \frac{\partial T^*}{\partial x^*} + v^* \frac{\partial T^*}{\partial y^*} = \frac{1}{\text{Re}_L} \frac{\partial^2 T^*}{\partial y^{*2}}$$

Mugalde baldintzak:

$$u^*(0, y^*) = 1; \quad u^*(x^*, 0) = 0; \quad u^*(x^*, \infty) = 1;$$

$$T^*(0, y^*) = 1; \quad T^*(x^*, 0) = 0; \quad T^*(x^*, \infty) = 1;$$

u^* eta T^* berdinak behar dute izan

$$\frac{\partial u^*}{\partial y^*} \Big|_{y^*=0} = \frac{\partial T^*}{\partial y^*} \Big|_{y^*=0}$$

$$C_{f,x} \frac{Re_L}{2} = Nu_x \quad (Pr = 1)$$

Reynoldsen analogia

$$\frac{C_{f,x}}{2} = St_x \quad (Pr = 1)$$

$$St = \frac{h}{\rho \cdot c_p \cdot V} = \frac{Nu}{Re_L \cdot Pr}$$

Stantonen zenbakia

Prandtl-en zenbakia zuzenduz

Reynoldsen analogia eraldatua edo
Chilton-Colburnen analogia

$$C_{f,x} \frac{Re_L}{2} = Nu_x \cdot Pr^{-1/3}$$

$$\frac{C_{f,x}}{2} = \frac{h_x}{\rho \cdot c_p \cdot V} \cdot Pr^{2/3} \equiv j_H$$

Colburnen j faktorea

Baliogarria: $0,6 < Pr < 60$ tartean

Flujuturbulentua presio-gradienteak badaude ere

Fluxi laminarra baldin eta $\partial P^* / \partial x^* = 0$

6.X – IRAKATSIKO EZ DIREN ATALAK

6.6 atala – Fluxu turbulentuen bero- eta momentu-transferentzia

6.8 atala – Xafla lau baten konbekzio-ekuazioen ebazpenak

Erabilitako bideoen informazio iturria:

Fundamentals of Boundary Layers de The National Committee for Fluid Mechanics Films.

<http://www.youtube.com/watch?v=7SkWxEUXIoM>

<http://www.youtube.com/watch?v=KqqtOb30jWs>

<http://www.youtube.com/watch?v=NplrDarMDF8>

Erabilitako irudien iturria:

ÇENCER, Y.A. TRANSFERENCIA DE CALOR Y MASA, Un enfoque práctico. McGraw-Hil.3 Edición. 2007