2. GAIA KALKULU BEKTORIALA

FISIKA

Ariketak: 27. orr

①

$\vec{A}(6,-3,0)$ $\vec{B}(-3,4,0)$ $\vec{C}(2,5,0)$

a) $\vec{D} = \vec{A} + \vec{B} + \vec{C}$

$\vec{D} = (5, 6, 0)$

b) $|\vec{D}|$ + kosinu zuzentzaileak

$|\vec{D}| = \sqrt{5^2+6^2} = \sqrt{61}$

$\cos\alpha = \frac{5}{\sqrt{61}}$ $\cos\beta = \frac{6}{\sqrt{61}}$ $\cos\gamma = \frac{0}{\sqrt{61}} = 0$

②

$\vec{A}$? XY planoan $\measuredangle(\vec{A}, 0X) = 30°$ $|\vec{A}| = 10$

$\hat{k} = 0$

$\cos\alpha = \frac{A_x}{|A|}$ $A_x = \cos\alpha \cdot |A| = \cos 30 \cdot 10 = 8'66$

$|A| = \sqrt{A_x^2 + A_y^2 + \cancel{A_z^2}^{\,0}}$

$|A|^2 = A_x^2 + A_y^2 \longrightarrow A_y = \sqrt{|A|^2 - A_x^2} = 5$

$\boxed{\vec{A}(8'66, 5, 0)}$

③

$\vec{A}$? XY planoan $\measuredangle(\vec{A}, 0X) = 37°$ $|\vec{A}| = 8$ $\hat{k} = 0$

$\vec{B}(3,-5,0)$

$\vec{C}(-6,3,0)$

$A_x = \cos|A| = \cos 37 \cdot 8 = 6'39$

$A_y = \sqrt{|A|^2 - A_x^2} = 4'81$

$\vec{A}(6'4, 4'8, 0)$

a) $\vec{A} + \vec{C} = (0'4, 7'8, 0)$

$\vec{B} - \vec{A} = (-3'4, -9'8, 0)$

$\vec{A} - 2\vec{B} + 3\vec{C} = (0'4, 14'8, 0) + (-18, 9, 0) = (-17'6, 13'8, 0)$

b) $\vec{G}$? $\vec{G} - \vec{B} = \vec{A} + 2\vec{C} + 3\vec{G}$

$\vec{G}(1-3) = \vec{A} + 2\vec{C} + \vec{B}$

$$\vec{G} = \frac{\vec{A} + 2\vec{C} + \vec{B}}{-2} = (1'3, -2'9, 0)$$

$\vec{A} + 2\vec{C} + \vec{B} = (-2'6, 5'8, 0)$

c) $\vec{A} \cdot \vec{C} = (6'4, 4'8, 0) \cdot (-6, 3, 0) = -38'4 + 14'4 + 0 = -24$

$$\vec{A} \times \vec{C} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 6'4 & 4'8 & 0 \\ -6 & 3 & 0 \end{vmatrix} = 19'2\,\hat{k} + 28'8\,\hat{k} = 48\,\hat{k} = (0, 0, 48)$$

$(\vec{A} \times \vec{C}) \cdot \vec{B} = (0, 0, 48) \cdot (3, -5, 0) = 0$

$$(\vec{A} \times \vec{C}) \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & 48 \\ 3 & -5 & 0 \end{vmatrix} = 144\,\hat{j} + 240\,\hat{i} = (240, 144, 0)$$

5

$\left.\begin{array}{l} \vec{A}\,(3,2,4) \\ \vec{B}\,(0,0,4) \end{array}\right\}$ $\vec{C} \longrightarrow \perp$ / unitario

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 2 & 4 \\ 0 & 0 & 4 \end{vmatrix} = 8\hat{i} - 12\hat{j} = (8, -12, 0)$$

$|\vec{A} \times \vec{B}| = \sqrt{8^2 + 12^2} = 14'4 = 4\sqrt{13}$

$$\hat{c} = \frac{\vec{A} \times \vec{B}}{|\vec{A} \times \vec{B}|}(\hat{i}, \hat{j}, \hat{k}) = 0'55\,\hat{i} - 0'83\,\hat{j} + 0\,\hat{k}$$

$\boxed{\hat{C}\,(0'55, -0'83, 0)}$

$\boxed{\hat{c}\left(\frac{2}{\sqrt{13}}, -\frac{3}{\sqrt{13}}, 0\right)}$

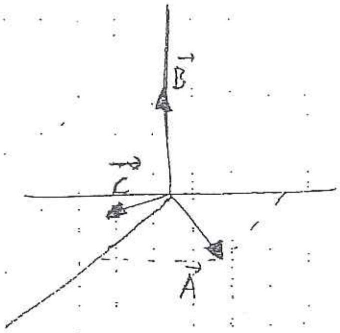
2. GAIA: KALKULU BEKTORIALA — Fisika

⑥

$\vec{A}\,(3,2,0)$

$\vec{B}\,(0,0,4)$

$\vec{C}\left(\frac{2}{\sqrt{13}}, \frac{-3}{\sqrt{13}}, 0\right)$

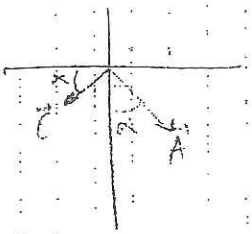


$|A| = \sqrt{3^2+2^2} = \sqrt{13}$

$|B| = 4$

$\cos\alpha = \frac{A_x}{|A|} = \frac{3}{\sqrt{13}}$

$\underline{\alpha = 33'7^\circ}$



$|\vec{C}| = 4\sqrt{13}$

$\cos\alpha = \frac{A_x}{|C|} = \frac{12}{4\sqrt{13}} = \frac{3}{\sqrt{13}}$

$\underline{\alpha = 33'7^\circ}$

⑦

$\phi = 3x^2z + y \qquad d\phi?$

$\partial\phi = \frac{\partial\phi}{\partial x}dx + \frac{\partial\phi}{\partial y}dy + \frac{\partial\phi}{\partial z}dz = \boxed{6zx\,dx + dy + 3x^2dz}$

⑧

$\vec{V} = 2xy\,\hat{i} + z^2\,\hat{j} \qquad d\vec{V}?$

$\partial\vec{V} = \frac{\partial\vec{V}}{\partial x}dx + \frac{\partial\vec{V}}{\partial y}dy + \frac{\partial\vec{V}}{\partial z}dz = \boxed{2y\hat{i}\,dx + 2x\hat{i}\,dy + 2z\hat{j}\,dz}$

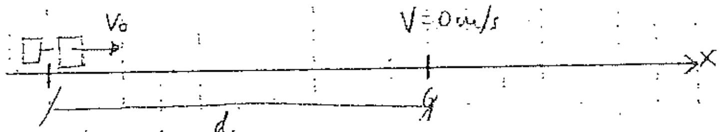
3. GAIA PARTIKULAREN ZINEMATIKA

Ariketak

45. orr

(1)

$t_0 = 0\ s$



Trena: abiadura konst. ($V = V_0 \longrightarrow$ HZU)

$$\left| \; x = x_0 + V_{0x}(t - \cancel{t_0})^0 \qquad x = x_0 + V_{0x} t \right.$$

Bagoia: HZUA

$$V_x = V_{0y} + a(t - \cancel{t_0})^0 \qquad V_x = V_{0x} + at \xrightarrow{t \text{ isolatu}} t = \frac{V_x - V_{0x}}{a}$$

$$x = x_0 + V_{0x}(t - \cancel{t_0})^0 + \tfrac{1}{2} a (t - \cancel{t_0})^2 \quad x = x_0 + V_{0x} t + 0{,}5 a t^2$$

ordezkatu

$$t = \frac{V_x - V_{0x}}{a}$$

$$x = x_0 + V_{0x} \frac{V_x - V_{0x}}{a} + \tfrac{1}{2} a \left(\frac{V_x - V_{0x}}{a} \right)^2$$

$$V_x^2 - V_{0x}^2 = 2a(x - x_0) \longrightarrow V_x^2 - V_{0x}^2 = 2a(x - x_0)$$

$$0^2 - V_{0x}^2 = 2a\,d$$

$$a = \frac{-V_0^2}{2d} \qquad \rightarrow \quad t = \frac{\cancel{V_x}^0 - V_{0x}}{a} = \frac{-\cancel{V_0} \cdot 2d}{-V_0^{\cancel{2}}} = \boxed{\frac{2d}{V_0}}$$

Bagoiak geltokia iritsi eta gelditu

$$\Delta t = \frac{V_x - V_{0x}}{a} \rightarrow t = \frac{0 - V_{0x} \cdot 2d}{V_{0x}^2} = \frac{2d}{V_{0x}} \longrightarrow$$

arte behar duen denbora

Trena:

$$x = \cancel{x_0}^0 + V_0 t = V_0 \cdot \frac{2d}{V_0} = \boxed{2d}$$

Trenak ibilitako distantzia bagoia askatu duenetik

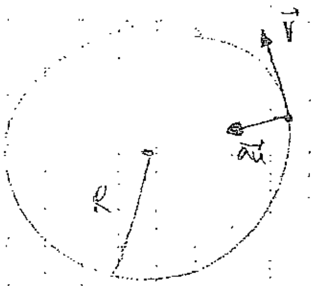
Trenetik geltokira: $2d - d = d$

Trena geltokitik d distantziara egongo da, bagoia geldi dagoenean

(4)

$\omega = 15000 \frac{bira}{min} \cdot \frac{2\pi\, rad}{1\, bira} \cdot \frac{1\, min}{60\, s} = 1571\ rad/s = \omega$

$R = 15\ cm = 0{,}15 m$



a) $|\vec{v}|\ |\vec{a_n}|$?

$V = \omega \cdot R = 1571\ rad/s \cdot 0'15\ m = \boxed{236\ m/s}$

$a_n = \frac{V^2}{R} = \omega^2 \cdot R = 1571^2\ rad/s^2 \cdot 0'15\ m = \boxed{370\,000\ m/s^2}$

b)

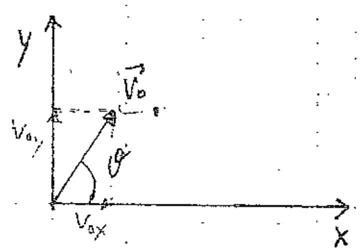
t = 90 s-tan azeleratzen $\longrightarrow \omega = 1571$ rad/s-ra arte. a_T?

$\omega = \cancel{\omega_0}^{0} + \alpha(t - \cancel{t_0})^{0}$

$\omega = \alpha t$; $\alpha = \frac{\omega}{t} = 17'46\ rad/s^2$

$a_T = \alpha \cdot R = 17'46\ rad/s^2 \cdot 0'15\ m = \boxed{2'62\ m/s}$

(5)



$\vec{a} = -g\hat{j}$ X ardatza → HZU y ardatza → HZUA

$\vec{V_0} = \begin{cases} V_{0x} = V_0 \cos\theta + \cancel{a_x}^{0} t \\ V_{0y} = V_0 \sin\theta + a_y t \ldots, -gt \end{cases}$

$V_{0y} = V_0 \sin\theta - gt$

a) $\vec{v}(t)$?

$\vec{v} = \vec{v_0} + \vec{a}\,t = V_0 \cos\theta\ \hat{i} + V_0 \sin\theta\ \hat{j} - gt\,\hat{j}$

$\boxed{\vec{v}(t) = V_0 \cos\theta\,\hat{i} + (V_0 \sin\theta - gt)\,\hat{j}\ (m/s)}$

b) X → HZU Y → HZUA

$\vec{r} \begin{cases} x = x_0 + V_{0x}(t - t_0) + \frac{1}{2}\cancel{a_x}^{0}(t - t_0)^2 = \overbrace{V_0 \cos\theta}^{V_{0x}}\, t \\ y = \cancel{y_0}^{0} + V_{0y}(t - t_0) + \frac{1}{2} a_y (t - t_0)^2 = V_0 \sin\theta \cdot t - \frac{1}{2} g t^2 \end{cases}$

$\boxed{\vec{r}(t) = V_0 \cos\theta \cdot t\,\hat{i} + (V_0 \sin\theta \cdot t - \frac{1}{2} g t^2)\,\hat{j}\ (m)}$

②

XY planoan

$\vec{a}(4,3)\ (m/s^2) \longrightarrow \vec{a} = 4\hat{i} + 3\hat{j}\ m/s^2 \leftarrow$ kte

$t = 0\ s \rightarrow \begin{cases} \vec{r}(4,3)\ (m) \longrightarrow \vec{r} = 4\hat{i} + 3\hat{j}\ m \\ \vec{v}(2,-9)\ (m/s) \longrightarrow \vec{v} = 2\hat{i} - 9\hat{j}\ m/s \end{cases}$ HZUA

$x = x_0 + v_{0x} t + \frac{1}{2} a t^2$

$v_x = v_{0x} + a t$

$v_x^2 - v_{0x}^2 = 2 a t$

a) $\vec{v}(t = 2\ s)$?

$\vec{v} = \vec{v_0} + \vec{a} t$

$\vec{v} = 2\hat{i} - 9\hat{j} + (4\hat{i} + 3\hat{j}) \cdot 2\ \ (m/s)$

$\vec{v} = 2\hat{i} + 8\hat{i} - 9\hat{j} + 6\hat{j}\ \ (m/s)$

$\vec{v} = 10\hat{i} - 3\hat{j}\ \ (m/s)$

$\boxed{\vec{v}(t = 2s) = (10, -3)\ (m/s)}$

b) $\vec{r}(t = 4\ s)$

$\vec{r} = \vec{r_0} + \vec{v_0} t + \frac{1}{2} \vec{a} t^2$

$\vec{r} = 4\hat{i} + 3\hat{j} + 4(2\hat{i} - 9\hat{j}) + 8(4\hat{i} + 3\hat{j})$

$\boxed{\vec{r} = 44\hat{i} - 9\hat{j}\ (m)}$ $\boxed{|\vec{r}| = 44'9\ m}$

$\cos\alpha = \frac{r_x}{|r|}\ \frac{44}{44'9}\ \alpha = \arccos\frac{r_x}{|r|} = \boxed{11'55°}$ $\boxed{\gamma = 90°}$ ↳ z ardatzarekin

$\cos\beta = \frac{r_y}{|r|}\ \frac{-9}{44'9}\ \beta = \arccos\frac{r_y}{|v|} = \boxed{101,66°}$

③ $\vec{r} = (3t^2 - 10)\hat{i} + 2t^2\hat{j}\ (m)$

a) $\vec{v}, \vec{a}$?

$\vec{v} = \frac{d\vec{r}(t)}{dt} = 6t\hat{i} + 4t\hat{j}\ (m/s)$ // $\vec{a} = \frac{d\vec{v}(t)}{dt} = 6\hat{i} + 4\hat{j}\ (m/s^2)$

b) ibilbidearen ekuazioa?

$\begin{cases} x = 3t^2 - 10 + 6t^2 + \frac{6t^2}{2} = 12t^2 - 10 \longrightarrow x = 12 \cdot \frac{y}{8} - 10 \\ y = 2t^2 + 4t^2 + 2t^2 = 8t^2 \rightarrow t^2 = y/8 \end{cases}$

$\boxed{x = 1'5y - 10}$

$$\vec{r}: \begin{cases} x = V_0 \cos\theta \cdot t & \longrightarrow t = \dfrac{x}{V_0 \cos\theta} \\ y = V_0 \sin\theta \cdot t - \frac{1}{2} g t^2 \end{cases}$$

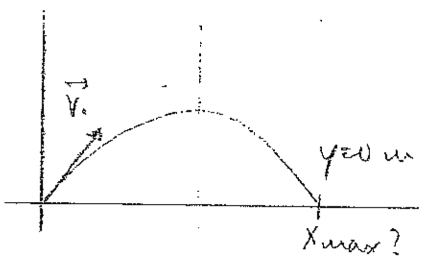
$$y = \cancel{V_0} \sin\theta \cdot \frac{x}{\cancel{V_0}\cos\theta} - \frac{1}{2} g \left(\frac{x}{V_0 \cos\theta}\right)^2$$

$$\boxed{y = x \cdot \operatorname{tg}\theta - \frac{g}{2 V_0 \cos\theta} \cdot x^2}$$

hemen, parabola bat dela ikusten da.

Beraz, ibilbide parabolikoa du.

c) OX-eko irismen maximoa?



Y → HZUA

$$y = V_0 \sin\theta \cdot t - \frac{1}{2} g t^2$$

$$0 = (V_0 \sin\theta - \frac{1}{2} g t)\, t$$

- $t = 0$ s
- $V_0 \sin\theta - \frac{1}{2} g t = 0$

$$t = \frac{2 V_0 \cdot \sin\theta}{g}$$

puntu altuenera iristeko $t/2$ behar du

X → HZU $\quad x = x_0 + V_0 t$

$$x = V_0 \cos\theta \cdot t$$

$$x = \underbrace{V_0 \cdot \cos\theta}_{V_{0x}} \cdot \overbrace{\frac{2 V_0 \sin\theta}{g}}^{t} = \frac{2 V_0^2 \sin\theta \cdot \cos\theta}{g}$$

$$\sin(\theta + \theta) = \sin\theta \cdot \cos\theta + \sin\theta \cdot \cos\theta$$

$$\boxed{X_{max} = \frac{V_0^2 \sin 2\theta}{g}}$$

d) puntorik altuena? t_{ymax}?

$Y_{max} \longrightarrow V_y = 0$ m/s

$$V_y = V_0 \sin\theta - g t$$

$$0 = V_0 \sin\theta - g t$$

$$\boxed{t = \frac{V_0 \sin\theta}{g}} \quad \text{igotzeko}$$

$$t = \frac{2 V_0 \sin\theta}{g} \quad \text{igotzeko eta jaisteko}$$

3. GAIA: PARTIKULAREN ZINEMATIKA

FISIKA

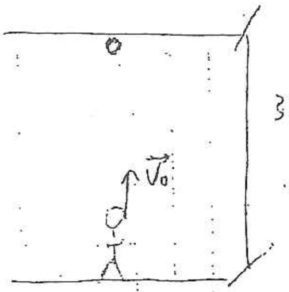
HZUA ekuazioak

$$Y = Y_0 + V_{0y} t + 0{,}5 a t^2$$
$$V = V_{0y} + at$$
$$V^2 - V_0^2 = 2a(Y - Y_0)$$

(6)

Bola bakoitzeko: 0'3 s

h = 3 m



Bolen mugidura: HZUA

$$\begin{cases} Y = Y_0 + V_{0y} t - \frac{1}{2} g t^2 \\ V_y = V_{0y} - g t \\ V_y^2 - V_{0y}^2 = -2g(Y - \cancel{Y_0}^{\,0}) \end{cases}$$

$$\cancel{V}^{0} - V_0^2 = -2gY$$

$$\boxed{V_0 = \sqrt{2gY}}$$

$$\cancel{V}^{0} = V_0 - gt \longrightarrow 0 = \sqrt{2gY} - gt \qquad 2\cancel{g}Y = \cancel{g}^{}t^2$$

$$t = \sqrt{\frac{2Y}{g}} = 0'78\ s$$

Bola batek igotzeko behar duen denbora.

igo eta jaitsi: $t = 0'78 \cdot 2\ s = 1'56\ s$

Bola kopurua:

$$\left.\begin{array}{l} 0'3\ s \longrightarrow 1\ \text{bola} \\ 1'56 \longrightarrow ?\ \text{bola} \end{array}\right\} \longrightarrow 5'2\ \text{bola}$$

$$\boxed{\text{Beraz, 5ekin bai, 6rekin ez}}$$

(7)

$\vec{r}(t) = \left(V_0 t \cdot \cos\theta , \quad V_0 t \sin\theta - \frac{1}{2} g t^2 \right)$

a) $\vec{V}(t)$? $\vec{a}(t)$?

$\vec{V}(t) = \frac{d\vec{r}(t)}{dt} = V_0 \cos\theta \,\hat{i} + (\overbrace{V_0 \sin\theta}^{V_{0y}} - gt)\,\hat{j}$ m/s

$\vec{a}(t) = \frac{d\vec{v}(t)}{dt} = -g\,\hat{j}$ m/s²

$$\boxed{\begin{array}{l} \vec{v}(t) = (V_0 \cos\theta , \quad V_0 \sin\theta - gt) \\ \vec{a}(t) = (0 , \quad -g) \end{array}}$$

V_0 = hasierako abi. modulua

θ = $\vec{V}_0$-ren θ ardatzekiko

g = azelerazioaren modulua

b) X → IRU Y → IRUA

$$\vec{r} \begin{cases} x = V_0 t \cos\theta \longrightarrow t = \frac{x}{V_0 \cos\theta} \\ y = V_0 t \sin\theta - \frac{1}{2} g t^2 \end{cases}$$

$$y = -\frac{1}{2} g \left(\frac{x}{V_0 \cos\theta} \right)^2 + \cancel{V_0} \sin\theta \cdot \frac{x}{\cancel{V_0} \cos\theta}$$

$$\boxed{y = -\frac{g x^2}{2 V_0^2 \cos^2\theta} + x \operatorname{tg}\theta}$$

$\theta = 0° \longrightarrow$ higikariaren ekuazioa: $y = -\frac{g}{2V_0^2} x^2$

$\theta = 90° \dashrightarrow$ higikariaren ekuazioa $y = \emptyset$

→ jaurtiketa horizontala

→ jaurtiketa bertikala

→ ibilbide zuzena

c) y_{max}?

$\cancel{V_y^2} = V_{0y}^2 - 2g(y_{max} - \cancel{y_0})$

$$\boxed{y_{max} = \frac{V_0^2 \sin^2\theta}{2g}}$$

$\cancel{y} = y_0 + \cancel{V_{0y}} t - \frac{1}{2} g t^2$

$\frac{+ V_0^2 \sin^2\theta}{\cancel{2}g} = +\cancel{\frac{1}{2}} g t^2$

$\frac{V_0^2 \sin^2\theta}{g^2} = t^2$

$t = \frac{V_0 \sin\theta}{g}$ goitik behera

$t = \frac{2V_0 \sin\theta}{g}$ hasieratik behera

$V_y^2 - V_{0y}^2 = -2g(Y_{max} - Y_0)$

$$\boxed{Y_{max} = \frac{V_0^2 \sin^2\theta}{2g}}$$ c, b)

3. GAIA : PARTIKULAREN ZINEMATIKA — Fisika

X ardatzean ⟶ HZU

$x = x_0 + V_{0x}(t - t_0) \longrightarrow x = \quad + V_0 \cos\theta \, t$

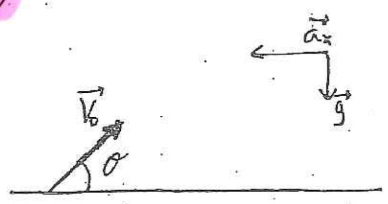
$t = \dfrac{2 V_0 \sin\theta}{g} \longrightarrow x = V_0 \cos\theta \cdot \dfrac{2 V_0 \sin\theta}{g}$

$$\boxed{x_{max} = \frac{V_0^2 \sin 2\theta}{g}}$$

$V_0 = 20 \ m/s \quad \theta = 30° \quad g = 10 \ m/s^2$

$\boxed{y_{max} = 5 \ m} \quad \boxed{t = 2 \ s} \quad x = 35 \ m$

*(8)



$a = 5'7 \ m/s^2$

$V_0 = 180 \ km/h$

$\hookrightarrow \begin{cases} V_{0x} = V_0 \cos\theta \\ V_{0y} = V_0 \sin\theta \end{cases}$

X ardatzean HZUA
Y ardatzean HZUA

$180 \ km/h \cdot \dfrac{1 h}{3600 s} \cdot \dfrac{1000 m}{1 km} = \underline{50 \ m/s}$

a) θ? Baloia hasierako puntura itzul dadin.

$\cancel{x}^0 = \cancel{x_0}^0 + V_{0x} t - \frac{1}{2} a t^2$

$V_{0x} t = \frac{1}{2} a t^2 \longrightarrow V_0 \cos\theta = \frac{1}{2} t \cdot a$

$\cancel{V_0} \cos\theta = \dfrac{\cancel{2} V_0 \sin\theta \cdot a}{\cancel{2} g}$

$\cancel{V_y}^0 = V_0 \sin\theta - \frac{1}{2} g t$

$\dfrac{g}{a} = \tan\theta$

$\theta = \arctan\left(\dfrac{g}{a}\right) = 59'8°$

$t = \dfrac{2 V_0 \sin\theta}{g} \longrightarrow$ igotzeko

$\boxed{\theta = 59'8°}$

b) x_{max}?

$V_x = 0 \ m/s$

$\cancel{V_x^2}^0 = V_{0x}^2 - 2a(x - \cancel{x_0})$

$V_{0x}^2 = 2ax \longrightarrow x = \dfrac{V_0^2 \cos^2\theta}{2a} = \boxed{55'48 \ m}$

$x = 55'48$ m

$x = \cancel{x_0}^0 + V_{0x} t - \frac{1}{2} a t^2$

$t (V_{0x} - \frac{1}{2} a t) = 55'48$

$\rightarrow \cancel{t = 55'48}$ s $\longrightarrow$ inkoherentea

$\rightarrow V_{0x} - \frac{1}{2} a t = 55'48$

$$t = \frac{(55'48 - V_0 \cos \theta) 2}{-a} = 10'64 \text{ s}$$

(X ardatzean $V_x = 0$ izatetik behar dira daborra)

$y = \cancel{y_0}^0 + V_{0y} t - \frac{1}{2} g t^2$

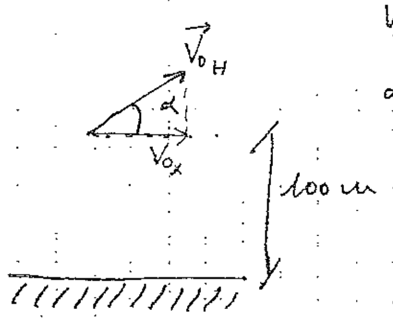
$y = V_0 \sin \theta t - \frac{1}{2} g t^2 = 459'79 - 554'72$

$y = 95'3 \hat{j}$

$\vec{r} = 55'46 \hat{i} + 95'3 \hat{j}$ (m) $\quad |\vec{V}| = 0$

47. orr

(9)



$V_{0H} = 200$ m/s

$\alpha = 60°$

Fardelaren abiadura: $V_0 = 200$ m/s

$V_{0y} = V_0 \sin\alpha = 173{,}20$ m/s

$V_{0x} = V_0 \cos\alpha = 100$ m/s etc

Beraz, Fardela $\longrightarrow$ $\begin{cases} x \longrightarrow HZU \longrightarrow & x = x_0 + V_{0x} t \\ y \longrightarrow HZUA \longrightarrow & y = y_0 + V_{0y} t - \frac{1}{2} g t^2 \\ & V_y = V_{0y} - g t \end{cases}$

a) Non erori da? Askapen puntuarekiko?

$\overset{0}{\cancel{y}} = \overset{100}{y_0} + V_{0y} t - \frac{1}{2} g t^2$

$\frac{g}{2} t^2 - \overset{V_{0y}}{V_0 \sin\alpha}\, t - 100 = 0 \qquad t = \frac{V_0 \sin\alpha \pm \sqrt{(V_{0y})^2 - 4(\frac{g}{2})(-100)}}{2(\frac{g}{2})} =$

$= \frac{173'20 \pm \sqrt{31960}}{g} \quad \begin{cases} t = 35'92 \text{ s} \\ t = -0'56 \text{ s} \end{cases}$

$x = \cancel{x_0} + V_{0x} t$

$x = V_0 \cdot \cos\alpha \cdot t = 100 \text{ m/s} \cdot 35'91 \text{ s} = \boxed{3590 \text{ m}}$

Askapen puntutik 3590 m-tara erori da fardela

b) denbora?

(kalkuluak aurreko galderan) $\boxed{t = 35'92 \text{ s}}$

c) $\vec{v}$? $|\vec{v}|$?

$V_y = V_{0y} - g t = -179$ m/s (negatiboa aurkako norantza dauka)

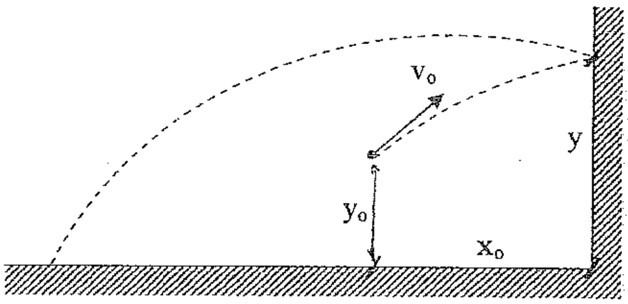
$\boxed{\vec{v} = 100\,\hat{i} - 179\,\hat{j} \text{ m/s}}$

$\boxed{|\vec{v}| = 204{,}87 \text{ m/s}}$

48. orr

10

Pilota jaurtitzen denean, ibilbide gorria deskribatzen du. Hormaren kontra errebote egin ostean, ibilbide urdina. Bi ibilbideak *parabolak* dira.



$x_o = 4$ m
$y_o = 2$ m.
$v_o = 10\ \hat{i} + 10\ \hat{j}$ (m/s)
$g = -10$ m/s^2 $\hat{j}$

Kalkula dezagun zenbat *denbora* pasatuko den pilotak horma jo arte:

$x = x_o + v_{ox} \cdot t \quad \Rightarrow \quad t = \frac{x - x_o}{v_{ox}} = \frac{4\ m}{10\ m/s} = \boxed{0.4\ s}$

Eta zein altuera daukan jotze-puntuak:

$y = y_o + v_{oy} \cdot t - \frac{1}{2} g \cdot t^2 \quad = 2 + 10 \cdot 0.4 - \frac{1}{2} 10 \cdot 0.4^2 = \boxed{5.2\ m.}$

ABIADURA TALKAREN ALDIUNEAN

$v_x = v_{ox} = \boxed{10\ m/s}$
$v_y = v_{oy} - g \cdot t \ = \ 10 - 10 \cdot 0.4 = \boxed{6\ m/s}$

ABIADURA JUSTU TALKAREN OSTEAN:
$v = \mathbf{-10}\ \hat{i} + \mathbf{6}\ \hat{j}$ (m/s) (x osagaia alderantzikatuta)

ONDOREN, noiz joko du pilotak lurra (y=0) $\quad y = y_o + v_{oy} \cdot t - \frac{1}{2} g \cdot t^2$

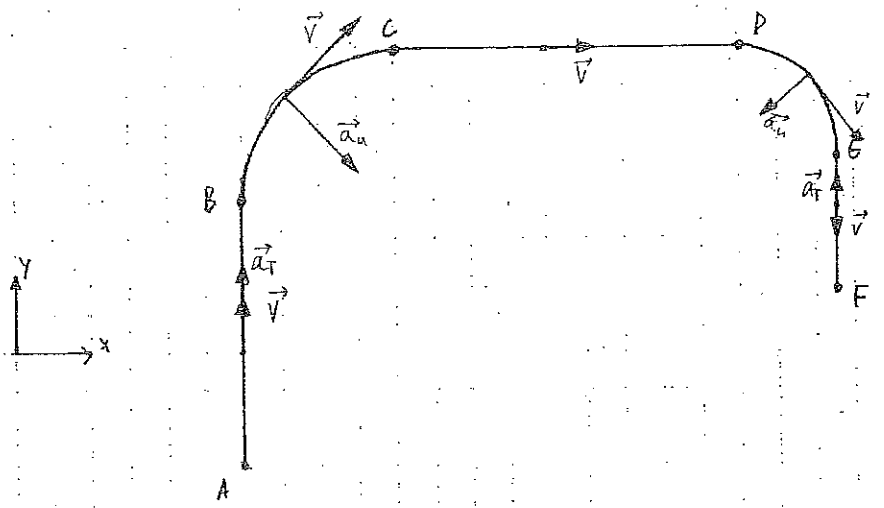
$\Rightarrow \ 0 = 5.2 + 6 \cdot t - \frac{1}{2} 10 \cdot t^2$ bi emaitza $\begin{cases} t = -0.58\ s \\ \boxed{t = +\ 1.78\ s} \end{cases}$ emaitza positiboa aukeratu

Eta non joko du lurra?

$x = x_o + v_{ox} \cdot t = 0 + 10\ m/s \cdot 1.78\ s = \boxed{17.8\ m}$

3. GAIA: PARTIKULAREN ZINEMATIKA — Fisika

*(11)



c) Arkuetan dun azelerazioa azelerazio normala da.

Azelerazio normala, abiadura zenbat eta handiagoa izan, edo kurbadura erradioa zenbat eta txikiagoa izan, handiagoa izango da.

$BC \longrightarrow R$ $\qquad DE \longrightarrow \frac{R}{2}$ $\qquad V_{BC} = V_{DE} = V = kte$

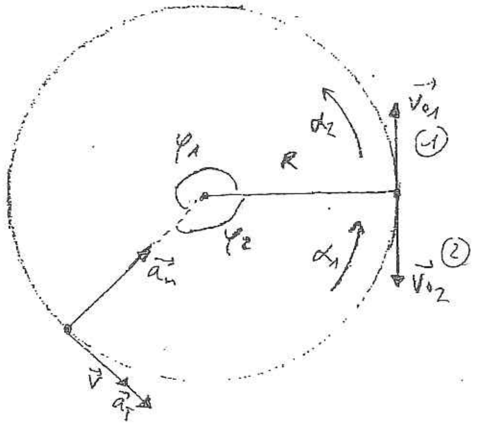
$$a_n = \frac{V^2}{R} \left\{ \begin{array}{l} BC: \quad a_n = \frac{V^2}{R_{BC}} = \frac{V^2}{R} \\ DE: \quad a_n = \frac{V^2}{R_{DE}} = \frac{V^2}{\cancel{R/2}} = \frac{2V^2}{R} \end{array} \right.$$

$$\boxed{2\, a_{n_{BC}} = a_{n_{DE}}}$$

DE-an azelerazioa handiagoa da, R txikiagoa delako.

(12) $\rightarrow$ liburuan eginda

*(13)



$V_{0_1} = V_{0_2} = V_0$

$\alpha_1 = \alpha_2 = \alpha$

$V = \omega \cdot R$

(1), (2) $\rightarrow$ HZr UA $\left\{ \begin{array}{l} \varphi = \varphi_0 + \omega_0 t + \frac{1}{2}\alpha t^2 \\ \omega = \omega_0 + \alpha t \end{array} \right.$

$a_T = \alpha \cdot R$

$a_n = \frac{V^2}{R} = \omega^2 \cdot R$

a) a_T?

$V_0 = \omega_0 \cdot R \longrightarrow \omega_0 = \frac{V_0}{R}$

(2) $\rightarrow \cancel{\varphi}^0 = \omega_0 - \alpha t$

$\frac{V_0}{R} = \alpha t \quad \alpha = \frac{V_0}{Rt} \longrightarrow a_T = \alpha \cdot R = \frac{V_0}{t} = \frac{V_0}{\frac{R\pi}{V}} = \boxed{\frac{V_0^2}{R\pi}}$

Guntzatzen diren aldiuan $\varphi_1 + \varphi_2 = 2\pi$

$\left. \begin{array}{l} \varphi_1 = \cancel{\varphi_0}^0 + \omega_0 t + \frac{1}{2}\alpha t^2 \\ \varphi_2 = \varphi_0 + \omega_0 t - \frac{1}{2}\alpha t^2 \end{array} \right| \rightarrow \omega_0 t + \cancel{\frac{1}{2}\alpha t^2} + \omega_0 t - \cancel{\frac{1}{2}\alpha t^2} = 2\pi$

$\cancel{2}\omega_0 t = \cancel{2}\pi \; ; \quad t = \frac{\pi}{\omega_0} = \frac{R\pi}{V_0}$

b) $\alpha_1 = \alpha_2 = \alpha \xRightarrow{R \text{ kte}} a_{T_1} = a_{T_2} = a_T = \frac{V_0^2}{R\pi}$

$a_{T_1} = a_{T_2} = \frac{V_0^2}{R\pi}$

$a_{n_1} = \frac{V_1^2}{R} = \omega_1^2 \cdot R = \qquad \omega = \omega_0 + \alpha t = \omega_0 + \frac{V_0^2}{\pi} \cdot \frac{R\pi}{V_0} = 2\omega_0$

$= 4\omega_0^2 R = \frac{4V_0^2 R}{R^2} = \frac{4V_0^2}{R}$

$a_{n_2} = \frac{V_2^{2^{\,0}}}{R} = 0$

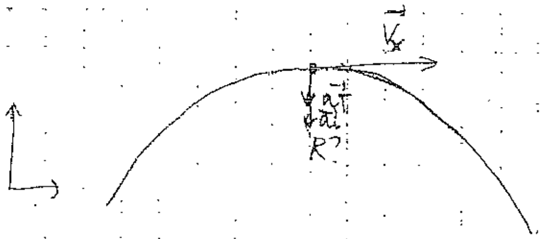
(1) $a_T = \frac{V_0^2}{R\pi}$ $a_n = \frac{4V_0^2}{R}$

(2) $a_T = \frac{V_0^2}{R\pi}$ $a_n = 0$

(14)

tiro parabolikoa ⟶ puntrik altuenetako kurbadura erradioa

V_0, g, θ



X: HZU | $X = X_0 + V_{0x} t$

Y: HZUA: $y = y_0 + v_0 t - 1/2\, g\, t^2$

$v_y = V_{0y} - g \cdot t$

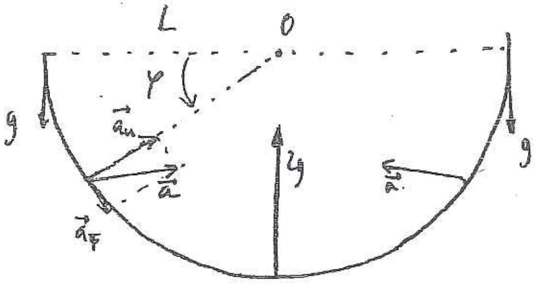
$V^2 - V_0^2 = 2a(\Delta y)$

Puntrik altuenean: $\vec{a} = -g\hat{j}$; $a = g$

$a^2 = a_T^2 + a_n^2$

$\rho = \frac{v^2}{a_n}$ $\qquad v^2 = V_0^2 \cos^2\theta = V_{0x}^2$

1b



$\dot{\varphi}^2 = w^2 = \left(\frac{2g}{L}\right)\sin\varphi$

$w = \sqrt{\frac{2g}{L}\sin\varphi}$

a) a_T? a_n? a_{max}? a_{min}?

$$a_T = \frac{dv}{dt} = \frac{d(w\cdot L)}{dt} = L\,\frac{dw}{dt} = L\cdot\frac{dw}{d\varphi}\cdot\underbrace{\frac{d\varphi}{dt}}_{w} =$$

$$= L\cdot\left[\frac{1}{2}\cdot\left(\frac{2g}{L}\sin\varphi\right)^{-1/2}\cdot\frac{2g}{L}\cos\varphi\right]\cdot\sqrt{\frac{2g}{L}\sin\varphi} =$$

$$= \cancel{L}\,\frac{\frac{\cancel{2}g\cos\varphi}{\cancel{L}}}{\cancel{2}\sqrt{\frac{2g\sin\varphi}{L}}}\cdot\sqrt{\frac{2g}{L}\sin\varphi} = g\cos\varphi = a_T$$

$$a_n = \frac{v^2}{L} = w^2\cdot L = \left(\sqrt{\frac{2g}{\cancel{L}}\sin\varphi}\right)^2\cdot\cancel{L} = 2g\sin\varphi = a_n$$

$$|\vec{a}| = \sqrt{a_T^2 + a_n^2} = \sqrt{g^2\cos^2\varphi + 4g^2\sin^2\varphi} = g\sqrt{\cos^2\varphi + 4\sin^2\varphi}$$

$\cos^2\varphi < 4\sin^2\varphi$

⟶ mínima $\cos^2\varphi = 1$, $4\sin^2\varphi = 0$ ($\varphi = 0$)

$a = g$

⟶ máxima $\cos^2\varphi = 0$, $4\sin^2\varphi = 1$ ($\varphi = \pi/2$)

$a = 2g$

b)

$a_{Ty} = a_T\sin\varphi = g\cdot\cos\varphi\cdot\sin\varphi$ $a_{Ty} = a_{ny}$?

$a_{ny} = a_n\sin\varphi = 2g\sin^2\varphi$

$\cancel{g}\cos\varphi\,\cancel{\sin\varphi} = 2\cancel{g}\sin^{\cancel{2}}\varphi$

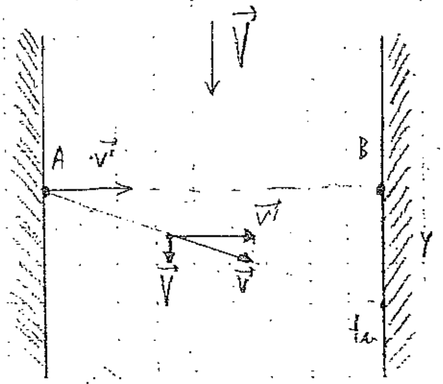
$\frac{1}{2} = \frac{\sin\varphi}{\cos\varphi}$ $\tan\varphi = 1/2$

$\varphi = \arctan 1/2 =$

$\varphi = \arctan\frac{1}{2} = \begin{cases} 26'56° \\ 153'43° \end{cases}$

(18)

1. era



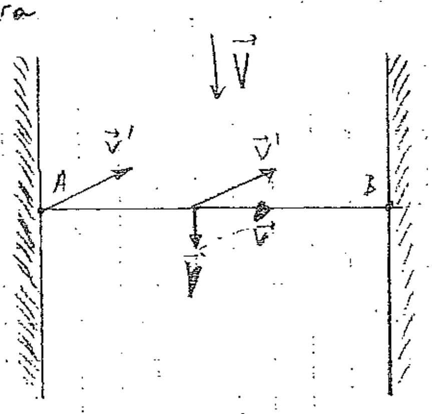
$t_u = \frac{L}{v'} = \frac{2\ km}{5\ km/h} = 0'4\ h = 24\ min$

$y = V \cdot t_u = 4\ km/h \cdot 0'4\ h = 1'6\ km$

$t_k = \frac{y}{V_k} = \frac{1'6\ km}{8\ km/h} = 0'2\ h = 12\ min$

$t = t_u + t_k = 12 + 24 = \boxed{36\ min}$

2. era



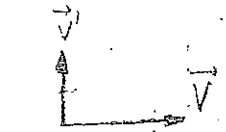
$V^2 + v^2 = v'^2$ (Pitag)

$v = \sqrt{v'^2 - V^2} = \sqrt{5^2 - 4^2} = 3\ km/h$

$t = \frac{L}{v} = \frac{2\ km}{3\ km/h} = 0'66\ h = \boxed{40\ min}$

1. era hartzea komeni zaio

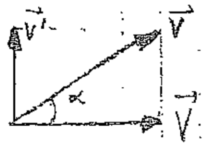
(20)



$v' = 34\ m/s$

$V = 72\ km/h \cdot \frac{1\ h}{3600\ s} \cdot \frac{1000\ m}{1\ km} = 20\ m/s$

$v = 39'44\ m/s$



α:

$V = v \cos\alpha$

$\alpha = \arccos \frac{V}{v}$

$\alpha = 59'5°$

a) $\cancel{v_y}^0 = v_{0y} - g t \longrightarrow t = \frac{v'}{g} = 3'46\ s$ igotzen

$t = 6'94\ s$ igotzen eta jaisten

$x = \cancel{x_0}^0 + V_x t \longrightarrow x = V t = 138,7\ m$ mugitu da

kamioiaren gainean, 138,7 m aurrerago eroriko da

b) $v = 39'44\ m/s$

c) $x = \cancel{x_0}^0 + V t \longrightarrow x = V t \quad t = \frac{X}{V} = 5\ s$ 100 m-ak egiteko

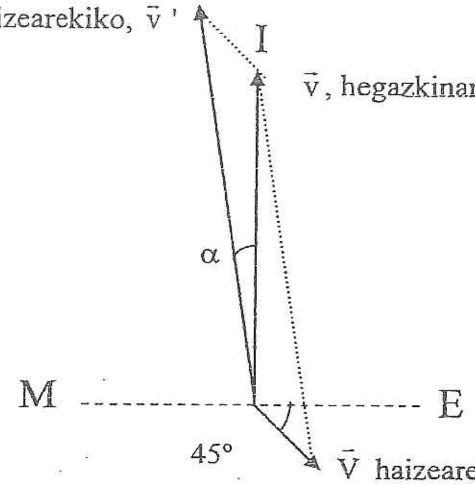
$v_y = v' - g t = -15\ m/s$

$\vec{v} = 20\ \hat{i} - 15\ \hat{j}\ m/s$ $\quad \boxed{|\vec{v}| = 2[illegible]}$

Liburuko ariketak

* 19 ariketa (52 or.)

hegazkinaren abiadura haizearekiko, $\vec{v}\,'$



$\vec{v}$, hegazkinaren abiadura lurrarekiko

$\vec{v} = \vec{V} + \vec{v}\,'$

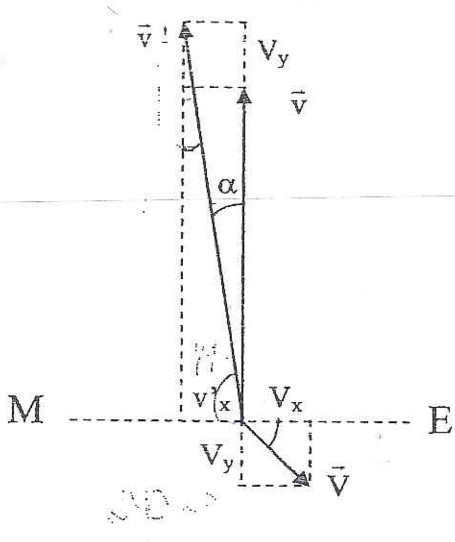
$\vec{V}$ haizearen abiadura lurrarekiko

Ezagunak: v' (modulua, 240 km/h), V (modulua, 50 km/h eta norabidea, 45°) eta v (norabidea, iparralderantz).
Ezezagunak: v modulua eta v' norabidea (α).

Batetik, batuketa soil bat da:

$$\vec{v} = \vec{V} + \vec{v}\,' \begin{cases} v_x = V_x + v'_x = \mathbf{0} \text{ (iruditik); beraz, } v'_x = -V_x = -V\cos 45 = -50\dfrac{\sqrt{2}}{2} = \mathbf{-35.4\ km/h} \\ v_y = V_y + v'_y \text{ (bi ezezagun: } v_y \text{ eta } v'_y \text{)} \end{cases}$$

Ondoko irudian hobeto ikus ditzakegu osagai cartesiarrok...



Bestalde, $v'^2_x + v'^2_y = v'^2$, beraz,

$$v'_y = \sqrt{v'^2 - v'^2_x} = \sqrt{240^2 - \frac{50^2}{2}} = \mathbf{237.4\ km/h}$$

v'-ren norabidea: $\alpha = \text{arctg}\,\dfrac{v'_x}{v'_y} =$

$= \text{arctg}\,\dfrac{35.4}{237.4} = \mathbf{8.5°}$ (iparraldearekiko).

v-ren modulua (v_y): $v_y = V_y + v'_y = -50\,\dfrac{\sqrt{2}}{2} + 237.4 = \mathbf{202\ km/h}$

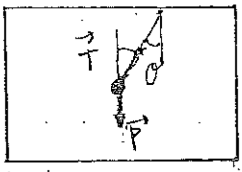
Eta azkenik, iraupena: $t = \dfrac{d}{v} = \dfrac{520\ km}{202\ km/h} = 2.57\ h = \mathbf{2\ h\ eta\ 34\ min}$

3. GAIA: PARTIKULAREN ZINEMATIKA

Fisika

21

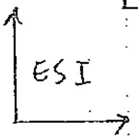
2.- 60. orr

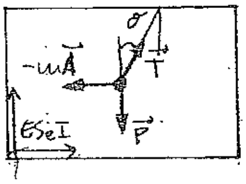


$\vec{a} = \vec{A}$

$$Y \left| \; T \cdot \cos\theta - mg = 0 \right.$$
$$X \left| \; T \cdot \sin\theta = m \cdot a = m \cdot A \right.$$

(abiadura $T\sin\theta$ ematen dio)



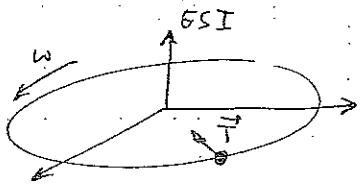


$\vec{A}$

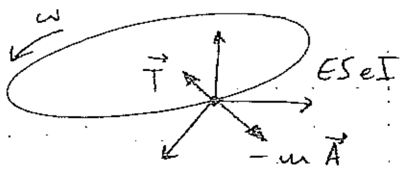
$$Y \left| \; T\cos\theta - mg = 0 \right.$$
$$X \left| \; T\sin\theta - mA = 0 \right.$$

Barnekoentzako biak ez dira ikusten

3.- 60.- 61. orr



$$\left| \begin{array}{l} N - mg = 0 \\ T = m \cdot a_n = m\frac{V^2}{R} = m\omega^2 R \end{array} \right.$$



$$\left| \begin{array}{l} N - mg = 0 \\ T - mA = 0 \end{array} \right.$$

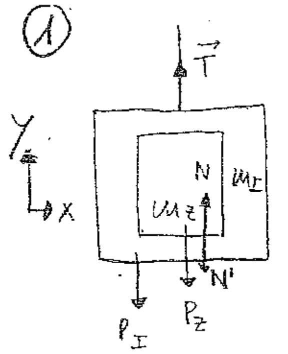
$a' = 0$

Formula berdinak baina kontzeptu desberdina

ARIKETAK

61. orr

(1)



$V = 4\ m/s$

$N = N'$

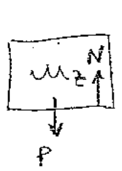
$m_Z = 600\ kg$ $g = 10\ m/s^2$

$m_I = 1200\ kg$

$a = 4\ m/s^2$

$a = \frac{\Delta V}{\Delta t} = \frac{4 m/s}{1 s} = 4 m/s^2$

a) N'? (azeleratzean)



$a = 4 m/s^2$, V

$\hat{j}$ ardatzean : $N - P = m \cdot a$

$N = N'$; $N = m(a + g)$

$N = 600\ (4 + 10)\ (N)$

$N = 8400\ N$

$\boxed{N' = 8400\ N}$

b) N'? (V = kte joaterakoan)

$V = 4 m/s^2$

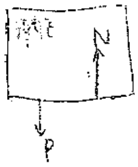
$a = 0\ m/s^2$

$\hat{j}$ ardatzean : $N - P = m \cdot \cancel{a}^{0}$

$N = N'$; $N = P = m_Z \cdot g = 6000\ N$

$\boxed{N' = 6000\ N}$

c) N'? (frenatzerakoan)



V

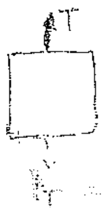
$a = 4 m/s^2$

$\hat{j}$ ardatza : $P - N = m \cdot a$

$N = N'$ $N = P - m \cdot a = 3600\ N$

$\boxed{N' = 3600\ N}$

d) T? (azeleratzean)

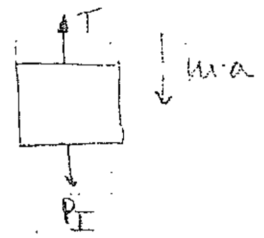


a $T - P_I = m a$

$T = m a + g(m_Z + m_I) = 25200\ N$

$\boxed{T = 25200\ N}$

d) T? (beherantz, lutrile)



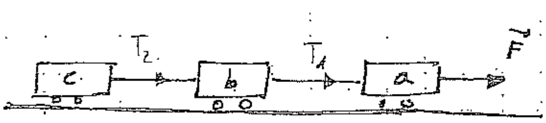
$P_I - T = m \cdot a$

$T = P_I - m \cdot a = m_I g - m_I a = 7200$ N

mg

$\boxed{T = 7200 \text{ N}}$

(2)

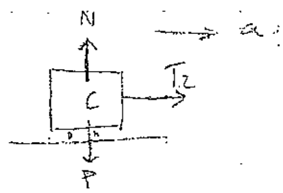


$m_a = 10$ kg $F = 50$ N

$m_b = 15$ kg

$m_c = 20$ kg

a) a_c, a_b, a_a, Tentsioak



$\hat{j}$ | $N - P = 0$

$\hat{i}$ | $T_2 = m_c \cdot a_c$

$\boxed{a_c = a_b = a_a \longrightarrow \text{lotuta daudelako}}$

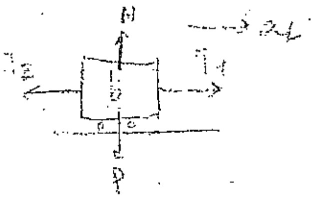
$T_2 = m_c \cdot a$

$T_1 = m_b \cdot a + m_c \cdot a$

$F = m_a \cdot a + m_b \cdot a + m_c \cdot a$

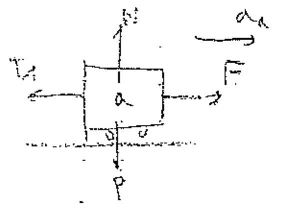
$F = a(m_a + m_b + m_c)$

$\boxed{a = 1,11 \text{ m/s}^2}$



$\hat{i}$ | $N - P = 0$

$\hat{j}$ | $T_1 - T_2 = m_B \cdot a_b$

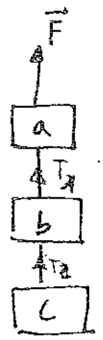


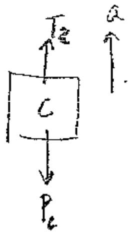
$\hat{i}$ | $N - P = 0$

$\hat{j}$ | $F - T_1 = m_A \cdot a_a$

$\boxed{\begin{array}{l} T_2 = 22'22 \text{ N} \\ T_1 = 38'89 \text{ N} \end{array}}$

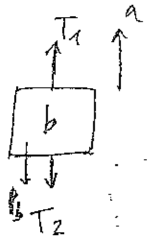
b) Bertikalki



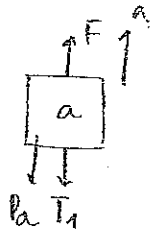


$\hat{j}: \left| \; T_2 - P_c = m_c \cdot a \right.$

$a_c = a_b = a_a$ 3-ak soka berdinari lotuta baitaude, eta indar berdinak bultzatzen baitu



$\hat{j}: \left| \; T_1 - T_2 - P_b = m_b \cdot a \right.$



$\hat{j}: \left| \; F - T_1 - P_a = m_a \cdot a \right.$

$$\left| \begin{array}{l} T_2 = m_c (a+g) \\ T_1 = m_b (a+g) + m_c (a+g) \\ F = m_a (a+g) + m_b (a+g) + m_c (a+g) \end{array} \right.$$

$m_{tot} = m_a + m_b + m_c$ $\qquad F = (a+g)(m_a + m_b + m_c)$

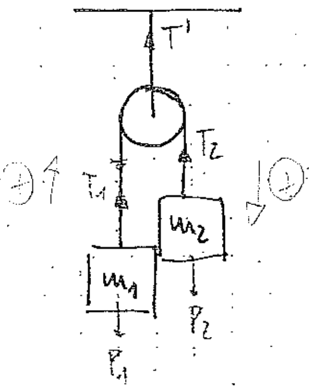
$$a = \frac{F}{m_{tot}} - g = -8'89 \ m/s^2$$

F-ren aurka doala esan nahi du, beherantz

$$\boxed{\begin{array}{l} a = 8'89 \ m/s^2 \ \text{(F-ren aurka)} \\ T_1 = 38'85 \ N \\ T_2 = 22'2 \ N \end{array}}$$

(3)

a) T?



$$\begin{array}{ll}(1) & T_1 - P_1 = m_1 \cdot a_1 \\ (2) & T_2 - P_2 = m \cdot a_2\end{array} \longrightarrow \begin{cases} m_1 g - T = m_1 \cdot a \rightarrow a = \dfrac{m_1 g - T}{m_1} \\ T - m_2 g = m_2 a \end{cases}$$

(a₁ gorantz badoa, a₂ beherantz, edo alderantziz)

$a_2 = -a_1 = a$

$T_1 = T_2$

(soka berdina delako)

$$T - m_2 g = m_2 \cdot \frac{m_1 g - T}{m_1}$$

$$m_1 T - m_2 m_1 g = m_2 m_1 g - m_2 T$$

$$(m_1 + m_2) T = m_2 m_1 g + m_2 m_1 g$$

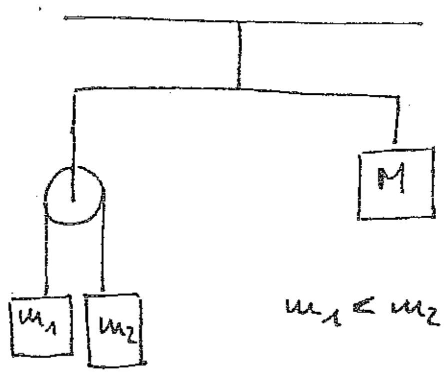
$$\boxed{T = \frac{2 m_2 m_1 g}{m_1 + m_2}}$$

$$\begin{array}{ll}(1) & T = m_1 (g - a) \\ (2) & T = m_2 (g + a)\end{array}$$

$$m_1 g - m_1 a = m_2 g + m_2 a$$

$$\boxed{a = \frac{(m_1 - m_2) g}{m_2 + m_1}}$$

④

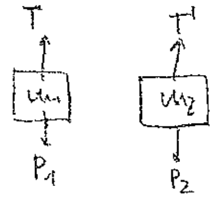


Polea geldi ⟶ orekan

Frenoa askatu ta gero, norantza zein?

$m_1 < m_2$

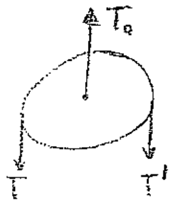
Polea geldi: (balantza orekan)

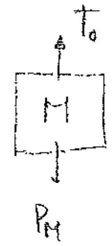


$$\begin{cases} T - P_1 = 0 \\ T' - P_2 = 0 \end{cases}$$

$T_0 = T + T'$

$T_0 = m_1 g + m_2 g$



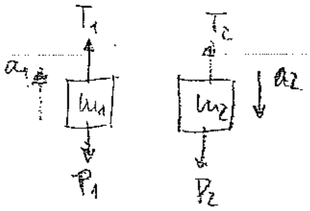


$$\begin{cases} T_0 - P_M = 0 \\ P_M = T_0 \end{cases}$$

$P_M = m_1 g + m_2 g$

$M = m_1 + m_2$

Polea ez-geldi:



$T = T_1 = T_2 \quad a_1 = -a_2 = a$

$$\begin{cases} T - P_1 = m_1 a_1 \\ T - P_2 = m_2 a_2 \end{cases} \longrightarrow \begin{cases} T - P_1 = m_1 a \rightarrow T = m_1(a-g) \\ P_2 - T = m_2 a \rightarrow T = m_2(g-a) \end{cases}$$

$\dfrac{T - m_1 g}{m} = a$

$m_1(a+g) = m_2(g-a)$ $\quad \dfrac{mg - T}{m_2} = a$

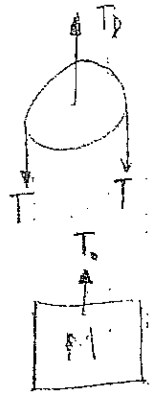
$m_1 a + m_1 g = m_2 g - m_2 a \longrightarrow$

$$a = \frac{(m_2 - m_1)g}{m_1 + m_2}$$

$m_2 T - m_2 m_1 g = m_2 m_1 g - m_1 T$

$$T = \frac{2 m_2 m_1 g}{m_2 + m_1}$$

Balantza:



$$T_D = \frac{4m_1m_2g}{m_1+m_2} \quad (2T)$$

$$T_0 \overset{?}{\gtrless} T_D$$

$$T_0 = Mg$$

$$T_0 = (m_1+m_2)g$$

$$\frac{4m_1m_2\cancel{g}}{m_1+m_2} > (m_1+m_2)\cancel{g}$$

$$4m_1m_2 > (m_1+m_2)^2$$

$$4m_1m_2 > m_1^2+m_2^2+2m_1m_2$$

$$2m_1m_2 > m_1^2+m_2^2$$

$$0 > m_1^2+m_2^2-2m_1m_2$$

$$0 > (m_1-m_2)^2 \longrightarrow \text{ez da posible}$$

gaitzi dago ikurra.

$$\frac{4m_1m_2g}{m_1+m_2} = m_1+m_2 a$$

$$4m_1m_2 = (m_1+m_2)\cdot(m_1+m_2)$$

$$4m_1m_2 = m_1^2+m_2^2+2m_1m_2$$

$$2m_1m_2 = m_1^2+m_2^2$$

$$0 = m_1^2+m_2^2-2m_1m_2$$

$$0 = (m_1-m_2)^2$$

Ez da posible!

$$\cancel{T_D > T_0}$$

$T_D < T_0$ ✓ $\longrightarrow$ Ezkumarantz mugituko da.

b) Zein kendu beharko litzateke masa balantza orekatzeko?

$$T_D = \frac{4m_1m_2}{m_1+m_2}g \qquad T_D = T_0$$

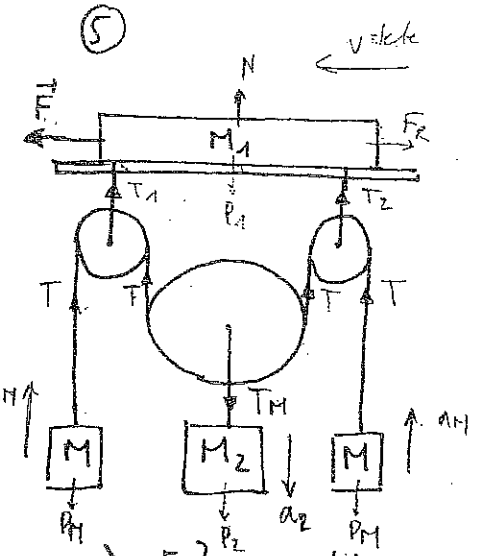
$$T_0 = M_Z\, g \qquad \frac{4m_1m_2}{m_1+m_2}\cancel{g} = M_Z\cancel{g}$$

$$(m_1+m_2) = M \qquad M - M_Z = \text{zenbat kendu} =$$

$$= m_1+m_2 - \frac{4m_1m_2}{m_1+m_2} = \frac{(m_1+m_2)^2-4m_1m_2}{m_1+m_2} =$$

$$= \frac{m_1^2+m_2^2-2m_1m_2}{m_1+m_2} = \boxed{\frac{(m_1-m_2)^2}{m_1+m_2}}$$

5

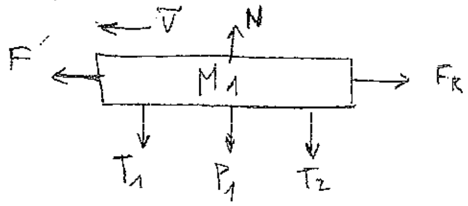


$M_1 = 1000$ kg $M = 100$ kg

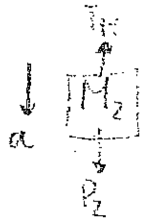
$\mu_d = 0'05$ $M_2 = 2000$ kg

$v = 1$ m/s $g = 10$ m/s²

a) F? v = kte izateko

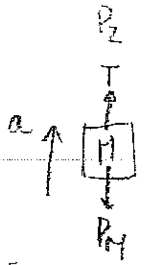


$$\begin{cases} \vec{i} & F - F_R = 0 \\ \vec{j} & N - P_1 - T_1 - T_2 = 0 \end{cases} \longrightarrow \begin{cases} \mu_d N - F = 0 \\ N = P_1 + T_1 + T_2 \end{cases}$$



$$\left\{ P_2 - T_M = M_2 \cdot a \right. \; ; \quad T_M = M_2(g-a)$$

$$T_M = 2T$$



$$\left\{ T - P_M = M \cdot a \right.$$

$$T = M(a+g)$$

$$T_M = 2M(a+g)$$

$$2M(a+g) = M_2(g-a)$$

$$2Ma + 2Mg = M_2 g - M_2 a$$

$$M_2 a + 2Ma + 2Mg - M_2 g = 0$$

$$a = \frac{(M_2 - 2M)g}{M_2 + 2M} = \boxed{8'02 \text{ m/s}^2}$$

$$T_1 = T_2 = 2T = 2M(a+g)$$

$$N = M_1 g + T_1 + T_2 = M_1 g + 4M(a+g) = 17272'7 \text{ N}$$

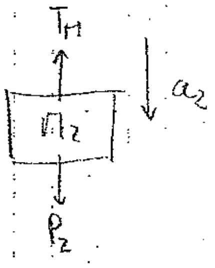
· F_R-k eragiten dion deselerazioa desegiteko $F = F_R$ izan behar du [illegible]; $F_1 - F = M_1 a_1$ ekuazioan [illegible] [illegible], $a_1 = 0$ m/s² [illegible] baita.

Beraz, F indarra kalkulatzeko:

$$\mu_d N - F = 0 \longrightarrow \mu_d N = \boxed{F = 863'6\ N}$$

v=1 m/s konstantez higi dadin

b) t? m_2-k 20 m jaisteko



$a_2 = 8'02\ m/s^2$

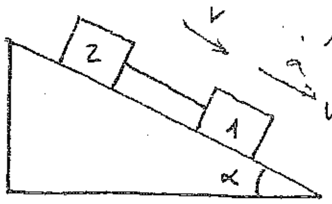
HZUA

$$y = \cancel{y_0}^{0} + \cancel{v_{0y}}^{0} t + \frac{1}{2} a t^2$$

$$20 = \frac{1}{2} a t^2$$

$$\boxed{t = \sqrt{\frac{40}{a}} = 2'2\ s}$$

⑥

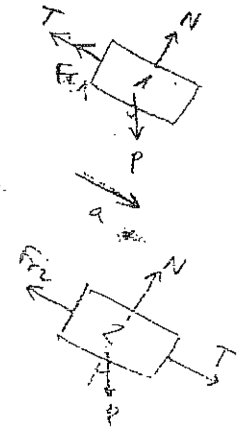


$\mu_1 < \mu_2$

$m_1 = m_2$ (a bietan berdina da, soka berdinera "konektatuta" daudelako)

a) T? b) a?

(m-ak berdinak direlako, N-ak ere berdinak dira)



$$\hat{j}: \quad N - P\cos\alpha = 0$$
$$\hat{i}: \quad P\sin\alpha - T - F_{r_1} = m \cdot a$$

$$N = P\cos\alpha$$
$$F_{r_1} = \mu_1 P = \mu_1 m g \cos\alpha$$
$$F_{r_2} = \mu_2 P = \mu_2 m g \cos\alpha$$

$$\hat{j}: \quad N - P\cos\alpha = 0$$
$$\hat{i}: \quad T + P\sin\alpha - F_{r_2} = m \cdot a$$

$$2\cancel{m}g\sin\alpha \cancel{+T}\cancel{-T} - F_{r_1} - F_{r_2} = 2\cancel{m}a$$
$$2g\sin\alpha - \mu_1 g\cos\alpha - \mu_2 g\cos\alpha = 2a$$
$$2g\sin\alpha - g\cos\alpha\,(\mu_1+\mu_2) = 2a$$
$$\boxed{a = g\left(\sin\alpha - \frac{\cos\alpha\cdot(\mu_1+\mu_2)}{2}\right)}$$

$$\cancel{P\sin\alpha} - \cancel{P\sin\alpha} - T - T - F_{r_1} + F_{r_2} = \cancel{ma} - \cancel{ma}$$
$$+2T = -F_{r_1} + F_{r_2}$$
$$\boxed{T = \frac{(\mu_2-\mu_1)\,m g\cos\alpha}{2}}$$

c) α? $v = kte$

$a = 0 \ m/s^2$

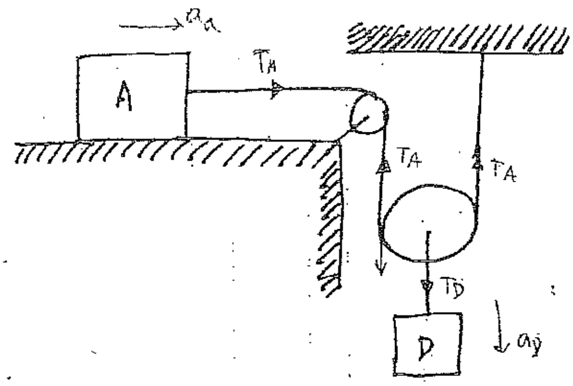
$$a = g\left(\sin\alpha - \frac{\cos\alpha\cdot(\mu_1+\mu_2)}{2}\right) = 0 \quad \begin{cases} g = 0 \ \times \\ (\ ) = 0 \end{cases}$$

$$\sin\alpha = \frac{\cos\alpha\cdot(\mu_1+\mu_2)}{2}$$

$$\tan\alpha = \left(\frac{\mu_1+\mu_2}{2}\right) \qquad \boxed{\alpha = \arctan\left(\frac{\mu_1+\mu_2}{2}\right)}$$

4. GAIA: MEKANIKA KLASIKOAREN OINARRIAK — Fisika

(7)



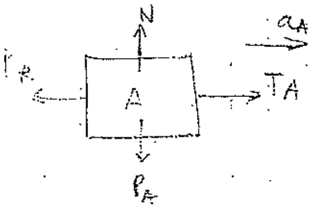
$m_A = 1\,kg$

$m_D = 2\,kg$ $\longrightarrow 2m_A = m_D$

(A) $\mu_d = 0'5$

$T_D = 2T_A$

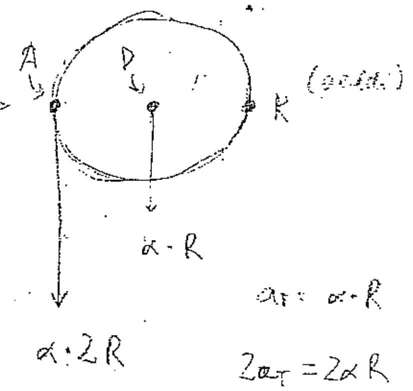
a) a_A? a_D?



$\vec{i}: \; T_A - F_R = m_A \cdot a_A$

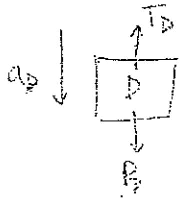
$\vec{j}: \; N - P_A = 0 \longrightarrow N = P_A$

$a_A = 2a_D \longrightarrow$



$a_T = \alpha \cdot R$

$2a_T = 2\alpha R$



$\vec{i}: \; \varnothing$

$\vec{j}: \; P_D - T_D = m_D \cdot a_D$

$$\begin{cases} P_D - T_D = m_D \cdot a_D \\ T_A - F_R = m_A \cdot a_A \end{cases} \rightarrow \quad T_D = 2T_A \quad a_A = 2a_D$$

$$\begin{cases} P_D - 2T_A = m_D \cdot \frac{a_A}{2} \\ T_A - \mu_d \cdot P_A = m_A \cdot a_A \end{cases} \longrightarrow T_A = m_A(a_A + \mu_d g) = \frac{m_D}{2}(a_A + \mu_d g)$$

$$g\,\cancel{m_D} - 2\frac{\cancel{m_D}(a_A + \mu_d g)}{2} = \cancel{m_D} \cdot a_A/2$$

$$g - a_A - \mu_d g = a_A/2 \qquad \frac{3}{2}a_A = g(1-\mu_d)$$

$$a_A = \frac{2g(1-\mu_d)}{3} = 3'27\ m/s^2$$

$a_D = 1'63\ m/s^2$

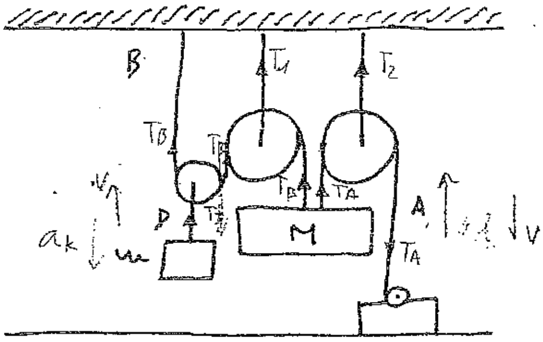
$T_A = 8'17\ N$

$T_D = 16'34\ N$

$a_A = 3'27\ m/s^2$

$a_D = 1'63\ m/s^2$

⑧



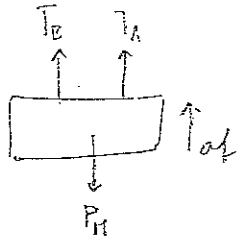
$M = 250$ kg

$m = 50$ kg

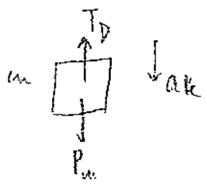
$a_f = 1\ m/s^2$

$T_2 = 2T_A$

$T_1 = 2T_B$

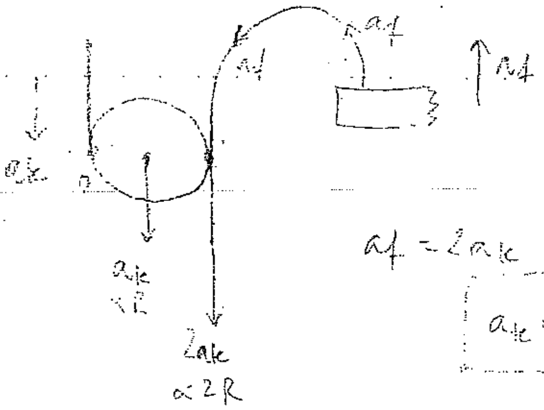


$\uparrow: \quad T_B + T_A - P_M = M \cdot a_f$



$\downarrow: \quad P_m - T_D = m \cdot a_k \quad \rightarrow \quad mg - 2T_B = m \cdot a_k$

$T_D = 2T_B$



$a_f = 2a_k$

$$\boxed{a_k = \frac{a_f}{2} = 0'5\ m/s^2}$$

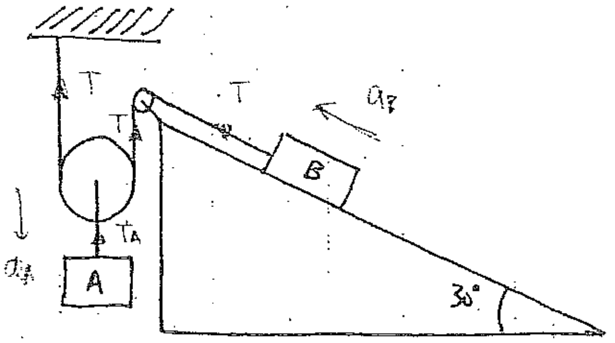
$$\begin{cases} T_B + T_A - Mg = Ma_f \\ mg - 2T_B = m \cdot a_k \end{cases}$$

$T_B = \dfrac{m(g - a_k)}{2} = 237'5$ N

$T_A = M(a_f + g) - T_B = 2513$ N

$$\boxed{\begin{aligned} T_B &= 237'5\ N \\ T_D &= 475\ N \\ T_A &= 2513\ N \end{aligned}}$$

(9)

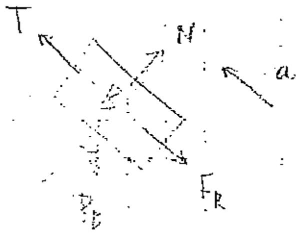


$m_A = 200$ kg

$m_B = 100$ kg

$\mu_B = 0'25$

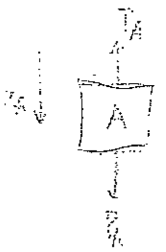
$m_A = 2 m_B$



$N - P_B \cos 30° = 0$ $\quad N = P_B \cos 30°$

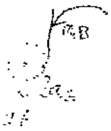
$T - P_B \sin 30° - F_R = m_B a_B$ $\quad T = m_B(2a_A + g \sin 30°) + \mu N$

$N = P_B \cos 30° = 849$ N



$P_A - T_A = m_A a_A \longrightarrow P_A - 2T = m_A a_A$

$2T = T_A$



$2a_A = a_B$

$T = 2m_B a_A + m_B g \sin 30° + \mu N$

$2T = m_A g - m_A a_A \longrightarrow 2T = m_A (g - a_A)$

$T = m_B (g - a_A)$

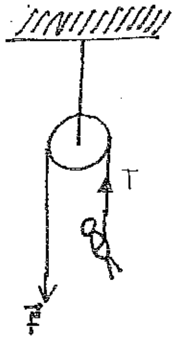
$2 m_B a_A + m_B g \ldots \frac{\sqrt{3} m_B g \cos 30°}{2} = m_B (g - a_A)$

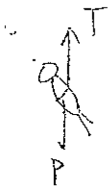
$- g \ldots - \frac{\sqrt{3} \mu g}{2} + g = + 3a_A$

$a_A = \frac{g(\ldots - \frac{\sqrt{3}\mu}{2})}{3} = 0'93 \text{ m/s}^2 \iff a_B = 1'85 \text{ m/s}^2$

$$\boxed{\begin{aligned} a_A &= 0'93 \text{ m/s}^2 \\ a_B &= 1'85 \text{ m/s}^2 \end{aligned}}$$

(10) F? v = kte-z igotzeko.

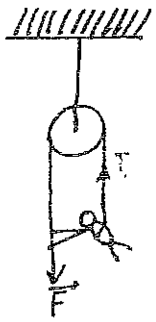




$F = T$

$T - P = m \cdot \cancel{a}^0 = 0$ $\quad T = P$; $F = P = mg$

$\boxed{F = mg}$

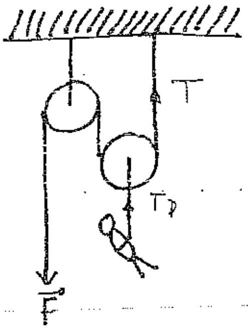


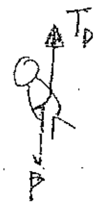
$F = T$

$2T - P = m\cancel{a}^0 = 0$

$\boxed{T = \frac{mg}{2}}$

Soka berdina denez, berak egingo duen indarra, sokaren tentsioa izango da



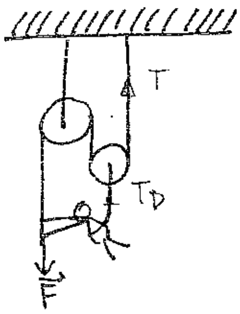


$T_D = 2T$

$2T - P = m\cancel{a}^0 = 0$

$2T = mg$

$\boxed{T = \frac{mg}{2}}$



$F = T \quad T_D = 2T$

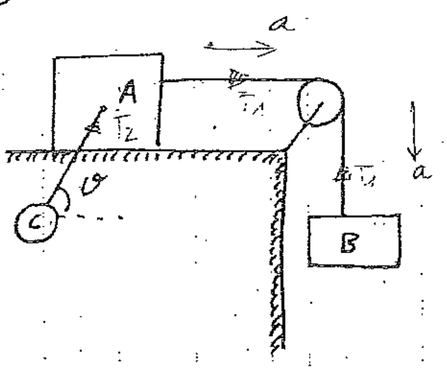
$F + T_D - P = m \cdot \cancel{a}^0 = 0$

$T + 2T - P = 0$

$\boxed{T = \frac{mg}{3}}$

(akzio-erreakzio legea aplikatzen da 2, 4 kasuetan)

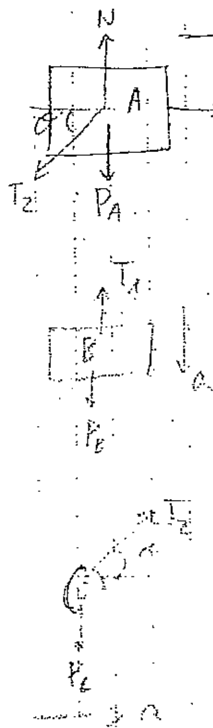
(11)



$m_A = 3$ kg F_R ∅

$m_B = 20$ kg

$m_C = 5$ kg



$$\begin{cases} N - P_A - T_2 \sin\theta = 0 \\ T_1 - T_2 \cos\theta = m_A \cdot a \end{cases}$$

$$P_B - T_1 = m_B\, a$$

$$\begin{cases} T_2 \sin\theta = P_C \;\; \longrightarrow \;\; T_2 = \dfrac{P_C}{\sin\theta} \\ T_2 \cos\theta = m_C \cdot a \end{cases}$$

$$\begin{cases} T_1 - P_C \operatorname{cotg}\theta = m_A\, a \\ P_C \operatorname{cotg}\theta = m_C\, a \\ P_B - T_1 = m_B \cdot a \end{cases} \quad \longrightarrow T_1 = a(m_A + m_C)$$

$$m_B g - a(m_A + m_C) = m_B \cdot a$$

$$m_B g = a(m_A + m_C + m_B)$$

$$a = \frac{m_B g}{m_A + m_C + m_B} = \boxed{7 \text{ m/s}^2}$$

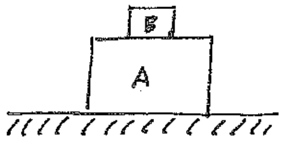
$T_1 = a(m_A + m_C) = 56$ N

$P_C \operatorname{cotg}\theta = m_C \cdot a$; $\theta = \operatorname{arc\,tg} \dfrac{P_C}{m_C \cdot a} = \operatorname{arctg} \dfrac{g}{a} = 54'46°$

$T_2 = \dfrac{P_C}{\sin\theta} = 60'21$ N

$$\boxed{\begin{array}{ll} T_1 = 56 \text{ N} & \theta = 54'46° \\ T_2 = 60'21 \text{ N} & \end{array}}$$

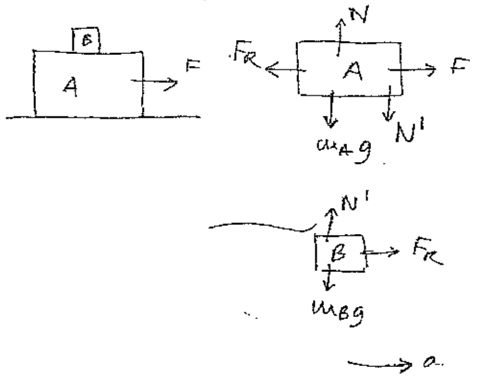
(12)



$m_B = 3\ kg \longrightarrow \mu_s = 0'2 \qquad g = 10\ m/s^2$

$m_A = 5\ kg \qquad \mu_d = 0'1$

a) Bi blokeak elkarrekin higitzeko F_{max}? b) a?



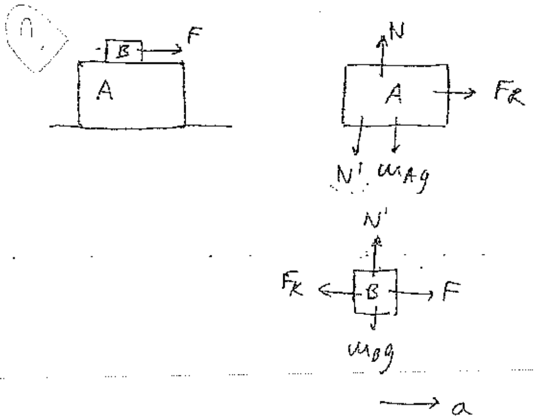
$$\begin{cases} N - m_A g - N' = 0 \\ F - F_R = m_A a \longrightarrow F = m_A a + \mu_s m_B g = \mu_s g (m_A + m_B) \end{cases}$$

$$\begin{cases} N' - m_B g = 0 \qquad N' = m_B g \\ F_R = m_B \cdot a \longrightarrow a = \frac{\mu_s \cancel{m_B} g}{\cancel{m_B}} = \mu_s g \end{cases}$$

$F_R = \mu_s m_B g$

$\boxed{F = 16\ N}$ $\boxed{a = 2\ m/s^2}$

b)



$$\begin{cases} N - m_A g - N' = 0 \\ F_R = m_A a \quad ; \quad a = \frac{\mu_s m_B g}{m_A} \end{cases}$$

$$\begin{cases} N' - m_B g = 0 \longrightarrow N' = m_B g \\ F - F_R = m_B a \longrightarrow F = m_B^2 \frac{\mu_s g}{m_A} + \mu_s m_B g \end{cases}$$

$$F = \mu_s m_B g \left(\frac{m_B}{m_A} + 1 \right) = \boxed{9'6\ N}$$

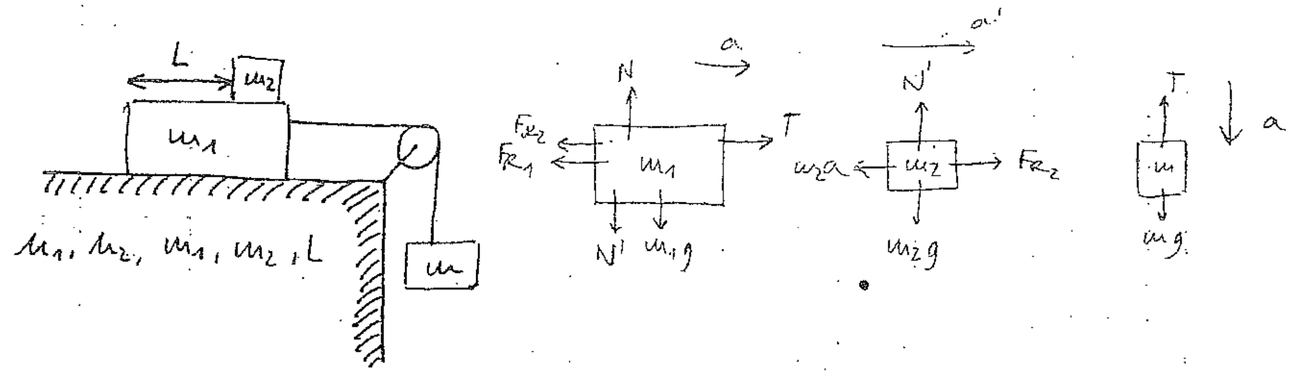
$\boxed{a = 1'2\ m/s^2}$

c)

F_{max} baino indar handiagoarekin bultzatzerakoan, bi blokeak, haietako azeleraziorik egongo dira, eta higitzen egongo dira. Ez dira geldi egongo eta beraz marruskadura koefiziente dinamikoa hartu behar da kontuan:

- A-ri indarra, B blokeak $a_B = \mu_d g = \boxed{1\ m/s^2}$
- B-ri indarra, A blokeak $a_A = \mu_d g \frac{m_B}{m_A} = \boxed{0'6\ m/s^2}$

(13)



$\mu_1, \mu_2, m_1, m_2, L$

$m_1: \begin{cases} N - N' - m_1 g = 0 \\ T - F_{R_1} - F_{R_2} = m_1 \cdot a \end{cases}$ $\quad m_2: \begin{cases} N' - m_2 g = 0 \\ F_{R_2} - m_2 a = m_2 a' \end{cases}$ $\quad m: \begin{cases} mg - T = m \cdot a \\ T = m(g-a) \end{cases}$

$T - \mu_1 N - \mu_2 m_2 g = m_1 a \qquad N' = m_2 g$

$m(g-a) - \mu_1 g(m_1+m_2) - \mu_2 m_2 g = m_1 a$

$$\frac{(m - \mu_1(m_1+m_2) - \mu_2 m_2)g}{m_1+m} = a$$

$F_{R_2} - m_2 a = m_2 a' \longrightarrow \mu_2 \cancel{m_2} g - \cancel{m_2} a = \cancel{m_2} a'$

$$\mu_2 g - \frac{(m - \mu_1(m_1+m_2) - \mu_2 m_2)g}{m_1+m} = a' ;$$

$$a' = \frac{\mu_2 g(m_1+m) - mg + \mu_1 g(m_1+m_2) + \mu_2 g m_2}{m_1+m} =$$

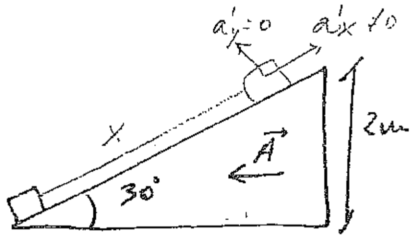
$$a' = \frac{(-m + \mu_1(m_1+m_2) + \mu_2(m+m_1+m_2))g}{m_1+m}$$

HZOA:

$L = -\frac{1}{2} a' t^2$

$$\boxed{t = \sqrt{\frac{2L}{g} \cdot \frac{m_1+m}{-\mu_1(m_1+m_2) - \mu_2(m+m_1+m_2) + m}}}$$

(14)

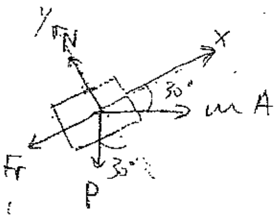


$\mu_d = 0'2$

$A = 12\ m/s^2$

$x \cdot \sin 30° = 2$

$x = \frac{2}{\sin 30°} = 4\ m$



$$\begin{aligned} \hat{i} &: \left| \; mA\cos 30° - F_r - mg\sin 30° = m \cdot a'_x \right. \\ \hat{j} &: \left| \; N - mg\cos 30° - mA\sin 30° = 0 \right. \end{aligned}$$

$$N = m(A\sin 30° + g\cos 30°)$$

$$\cancel{m}A\cos 30° - \mu_d \cdot \cancel{m}(A\sin 30° + g\cos 30°) - \cancel{m}g\sin 30° = \cancel{m} \cdot a'_x$$

$$\frac{\sqrt{3}A}{2} - \frac{\mu_d A}{2} - \frac{\sqrt{3}\mu_d g}{2} - \frac{g}{2} = a'_x = 2'59\ m/s^2$$

$$\boxed{a'_x = 2'59\ m/s^2}$$

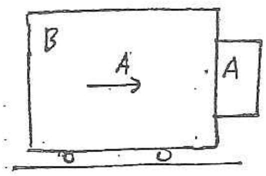
HZUA

$$x = \cancel{x_0}^{\,0} + \cancel{v_0}^{\,0} t + \frac{1}{2} a t^2$$

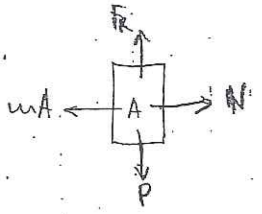
$$t = \sqrt{\frac{2x}{a}} = 1'76\ s$$

$$\boxed{t = 1'76\ s}$$

15



μ_s marruskadura koefiziente estatikoa



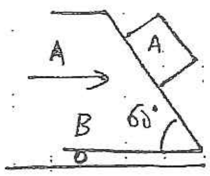
ESeI

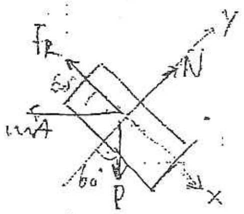
$$\hat{i}: \; N - mA = 0 \longrightarrow N = mA$$
$$\hat{j}: \; F_R - P = 0$$

$$\mu N = mg$$

$$\mu \cancel{m} A = \cancel{m} g \longrightarrow A = \frac{g}{\mu}$$

$$\boxed{A_{min} \geq g/\mu_s}$$





ESeII

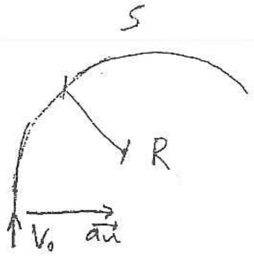
$$\hat{i}: \; P\sin 60° - F_R - mA\cos 60° = 0$$
$$\hat{j}: \; N - P\cos 60° - mA\sin 60° = 0 \longrightarrow N = m(g\cos 60° + A\sin 60°)$$

$$\cancel{m}g\sin 60° - \mu_s \cancel{m}(g\cos 60° + A\sin 60°) - \cancel{m}A\cos 60° = 0$$

$$\frac{\frac{\sqrt{3}g}{2} - \frac{\mu g}{2}}{\frac{\sqrt{3}\mu}{2} + \frac{1}{2}} = A = \frac{\sqrt{3}g - \mu g}{\sqrt{3}\mu + 1} = \frac{(\sqrt{3} - \mu)g}{\sqrt{3}\mu + 1}$$

$$\boxed{A_{min} \geq \frac{(\sqrt{3} - \mu)g}{\sqrt{3}\mu + 1}}$$

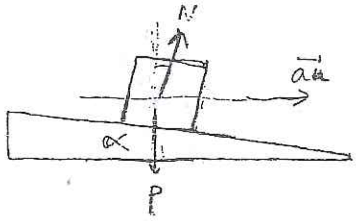
(16)



$V_0 = 90\ km/h = 25\ m/s \qquad g = 10\ m/s^2$

$R = 150\ m$

$m = 750\ kg$



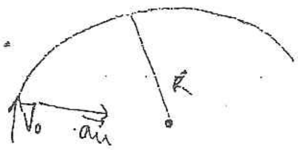
$$\begin{cases} N \sin\alpha = m \cdot a_n \\ N\cos\alpha - P = 0 \longrightarrow N = \dfrac{mg}{\cos\alpha} \end{cases}$$

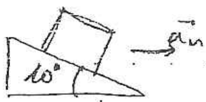
$$\frac{\cancel{m}g}{\cos\alpha} \cdot \sin\alpha = \cancel{m} \cdot \frac{V^2}{R}$$

$$tg\alpha = \frac{V^2}{Rg} \; ; \; \alpha = arctg\, \frac{V^2}{Rg} = 22'6°$$

$$\boxed{\alpha = 22'6°}$$

(17)



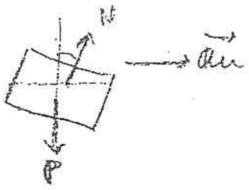


$V_0 = 85\ km/h = 23'61\ m/s$

$m = 800\ kg \qquad g = 10\ m/s^2$

$R = 150\ m$

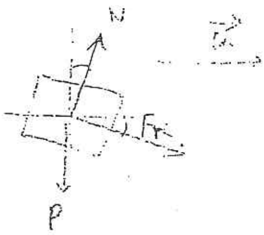
a) N?



$$\begin{cases} N \sin 10° = m \cdot a_n \\ N\cos 10° - P = 0 \longrightarrow N = \dfrac{mg}{\cos 10} = 8123'4\ N = \end{cases}$$

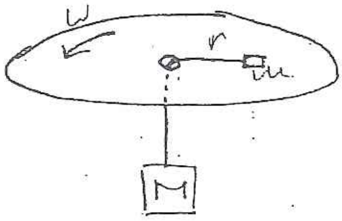
$$\boxed{N = 8'1\ kN}$$

b) F_R? μ? min



$$\begin{cases} N\cos\alpha - mg - F_R \sin\alpha = 0 \\ F_R \cos\alpha + N\sin\alpha = m\, a_n \end{cases}$$

18

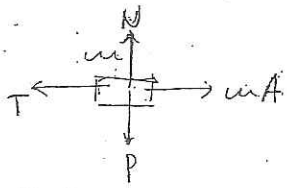


$m = m$ kg

$M = M$ kg

$w = w$ rad/s

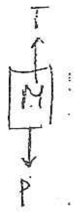
w, m labain ez dadin?



$$\begin{cases} \hat{i} & N - P = 0 \\ \hat{j} & T - mA = 0 \end{cases} \longrightarrow T = m \cdot a_n$$

EseI

$$Mg = m \cdot w^2 r$$



$$\begin{cases} \hat{i} & \emptyset \\ \hat{j} & T - P = 0 \end{cases} \longrightarrow T = Mg$$

$$w = \sqrt{\frac{Mg}{mr}}$$

ESI

$$\boxed{w \geq \sqrt{\frac{Mg}{mr}}}$$

5. GAIA PARTIKULAREN DINAMIKA OROKORRA — FISIKA

83. orr

(1) $\vec{r} = (6t^2 - 6t)\,\hat{i} - 4t^3\,\hat{j} + (3t+2)\,\hat{k}$ (m) $\qquad m = 6$ kg

a) F?

$$\vec{v} = \frac{d\vec{r}}{dt} = (12t - 6)\,\hat{i} - 12t^2\,\hat{j} + 3\,\hat{k} \quad (m/s)$$

$$\vec{a} = \frac{d^2\vec{r}}{dt^2} = 12\,\hat{i} - 24t\,\hat{j} \quad (m/s^2)$$

$$\boxed{\vec{F} = m\cdot\vec{a} = 72\,(\hat{i} - 2t\,\hat{j})\ N}$$

b) M_o?

$$\vec{M}_o = \vec{r}\times\vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 6(t^2-t) & -4t^3 & 3t+2 \\ 72 & -72\cdot 2t & 0 \end{vmatrix} = 72(3t+2)\,\hat{j} - 864\,(t^3 - t^2)\,\hat{k} - 288t^3\,\hat{k} + 144(3t+2)\,\hat{i} =$$

$$= 72\left[(3t+2)\,\hat{j} - 12(t^3 - t^2)\,\hat{k} - 4t^3\,\hat{k} + 2(3t+2)\,\hat{i}\right]\ kg\,\frac{m^2}{s^2}$$

$$\boxed{\vec{M}_o = 72\left[(6t+4)\,\hat{i} + (3t+2)\,\hat{j} + (-16t^3 + 12t^2)\,\hat{k}\right]\ kg\,\frac{m^2}{s^2}}$$

c) $\vec{L}_o$? $\vec{p}$?

$$\vec{p} = m\cdot\vec{v} = 6\left[(12t - 6)\,\hat{i} - 12t^2\,\hat{j} + 3\,\hat{k}\right]\ kg\,\frac{m}{s}$$

$$\vec{L}_o = \vec{r}\times\vec{p} = 6\cdot\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 6(t^2-t) & -4t^3 & 3t+2 \\ 6(2t-1) & -12t^2 & 3 \end{vmatrix}$$

$$\boxed{\vec{L}_o = 36\left[(4t^3 + 4t^2)\,\hat{i} + (3t^2 + 4t - 2)\,\hat{j} + (-4t^4 + 8t^3)\,\hat{k}\right]\ kg\,\frac{m^2}{s}}$$

d) $\vec{F} = \dfrac{d\vec{p}}{dt}$

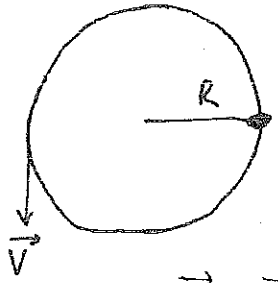
$$\vec{F} = m\cdot\vec{a} = m\cdot\frac{d\vec{v}}{dt} = \frac{d(m\vec{v})}{dt} = \frac{d\vec{p}}{dt} = m\cdot\frac{d^2\vec{r}}{dt}$$

$$\vec{M}_o = \frac{d\vec{L}_o}{dt}$$

$$\frac{d\vec{L}_o}{dt} = \frac{d\vec{r}}{dt}\times\vec{p} + \vec{r}\times\frac{d\vec{p}}{dt} = \vec{r}\times\vec{F} = \vec{M}_o$$

②

$m = 0'1$ kg $\quad t = 5s \quad R = 0'5$ m $\quad M = 0'5$ Nm



a) $\vec{M_0} = \vec{r} \times \vec{F}$

$M_0 = r \cdot F \cdot \underbrace{\sin\theta}_{1} \longrightarrow F = \frac{M_0}{r} = 1$ N

$F = m \cdot a \longrightarrow a = \frac{F}{m} = 10$ m/s^2 $= \alpha \cdot R$

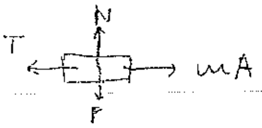
HZrUA $\quad t = 5s \qquad \alpha = 20$ rad/s^2

$\varphi = \cancel{\varphi_0}^{0} + \cancel{\omega_0}^{0} t + \frac{1}{2}\alpha t^2 \qquad \varphi = 250 \text{ rad} \cdot \frac{1 \text{ bira}}{2\pi \text{ rad}} = \boxed{39'8 \text{ bira}}$

$\omega = \cancel{\omega_0} + \alpha t \qquad \omega = 100$ rad/s

$$\boxed{v = \omega \cdot R = 50 \text{ m/s}}$$

ESeI



$\{ T - mA = 0 \longrightarrow T = mA = m\frac{v^2}{R} = 500$ N $\qquad \boxed{T = 500 \text{ N}}$

③

8s $\longrightarrow V_0 = 0$ km/h $\to v = 100$ km/h $= 27'78$ m/s $\qquad m = 800$ kg

a) P?

750 W $\to 1$ ZP

$dE_z = P\,dt$

$\int_0^{E_z} dE_z = \int_0^t P\,dt \longrightarrow E_z = Pt \longrightarrow \frac{1}{2}mv^2 = Pt \qquad \boxed{P = \frac{mv^2}{2t} = 38'6 \text{ kW}}$

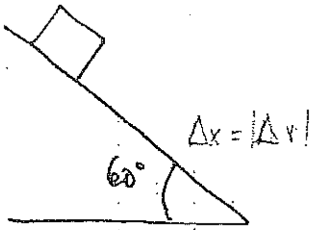
$$\boxed{P = 51'44 \text{ Z.P.}}$$

b) $v(t=4)$?

$P = \frac{E_z}{t} = \frac{mv^2}{2t} \qquad v = \sqrt{\frac{2Pt}{m}} = 19'6$ m/s $= 70'71$ km/h

$$\boxed{v(t=4s) = 70'71 \text{ km/h}}$$

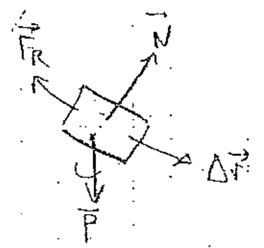
(4)



$m = 6\ kg$ $\Delta r = 2\ m$

$\mu_d = 0'1$

$$W = \int F \cdot dx = F \int dx = F\Delta x$$



$$W = \int \vec{F} \cdot d\vec{r} = \vec{F} \cdot \Delta\vec{r} = |F| \cdot |\Delta r| \cdot \cos\alpha$$

$$\begin{cases} W_R = F_R \cdot \Delta r \cdot \cos\alpha = \mu N \cdot \Delta r \cos\alpha \\ W_N = N \cdot \Delta r \cdot \cos\alpha \\ W_P = mg\ \Delta r \cos\alpha \end{cases}$$

$W_R = \mu_d \cdot mg \cdot \Delta r \cdot \cos 180 = -6\ J$

$W_N = mg \sin 60° \cdot \Delta r \cdot \cos 90° = 0\ J$

$W_P = mg \cdot \Delta r \cdot \cos 30° = 101{,}84\ J$

b) W_{tot}?

$W_{tot} = W_R + W_N + W_P = 95'8\ J$

c)

$W_{tot} = \Delta E_z = E_z - E_{z_0} = \frac{1}{2} m v^2 - \frac{1}{2} m v_0^2$

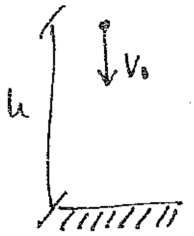
$$v = \sqrt{\frac{2W_{tot}}{m}} = 5'65\ m/s$$

d) $v_0 = 2\ m/s$?

$W_{tot} = \Delta E_z = E_z - E_{z_0} = \frac{1}{2} m v^2 - \frac{1}{2} m v_0^2$

$$v = \sqrt{\frac{(W_{tot} + \frac{1}{2} m v_0^2) \cdot 2}{m}} = 6'00\ m/s$$

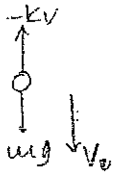
(5)



Marruskadura indarra: $\vec{F} = -k\vec{v}$

m masa

a) $v(t)$?



$F = m\cdot a$; $mg - kv = m\frac{dv}{dt}$; $\frac{dt}{m} = \frac{dv}{mg-kv}$

$$\int_0^t \frac{dt}{m} = \int_{V_0}^{V} \frac{dv}{mg-kv} \; ; \quad \frac{(t-0)}{m} = -\frac{1}{k}\ln(mg-kv)\Big]_{V_0}^{V} =$$

$u = mg\ kv$

$du = -k\,dv \dashrightarrow dv = \frac{-du}{k}$

$$= -\frac{1}{k}\left[\ln(mg-kv) - \ln(mg-kV_0)\right]$$

$$\int \frac{\frac{-du}{k}}{u} = -\frac{1}{k}\int \frac{du}{u}$$

$$\frac{-kt}{m} = \ln\left(\frac{mg-kV}{mg-kV_0}\right)$$

$$\frac{mg-kv}{mg-kV_0} = e^{-\frac{kt}{m}}$$

$$+kv = mg - (mg-kV_0)\cdot e^{-\frac{kt}{m}}$$

$$\boxed{v = \frac{mg}{k} - \left(\frac{mg}{k} - V_0\right)\cdot e^{-\frac{kt}{m}}}$$

b) muga-abiadura $t \to \infty$

$$\boxed{v = \frac{mg}{k}} \longrightarrow \boxed{V_1 = \frac{mg}{k}}$$

c) $v = \frac{dz}{dt} \longrightarrow \int_h^z dz = \int_0^t -V(t)\,dt$; $(z-h) = -\int_0^t \left(\frac{mg}{k} + \left(V_0 - \frac{mg}{k}\right)e^{-\frac{kt}{m}}\right)dt$

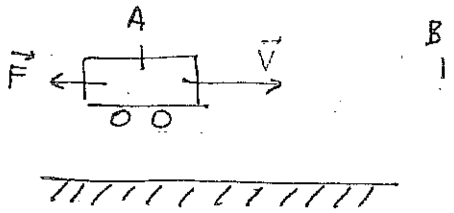
$$z - h = -\frac{mg}{k}t - \left(V_0 - \frac{mg}{k}\right)\int_0^t e^{-\frac{kt}{m}}\,dt = -\frac{mg}{k}t - \frac{m}{k}\left(V_0 - \frac{mg}{k}\right)\left(1 - e^{-\frac{kt}{m}}\right)$$

$$\int_0^t e^{-\frac{kt}{m}}\cdot dt = -\frac{m}{k}\int_0^t e^{-\frac{kt}{m}}\cdot -\frac{k}{m}\cdot dt = -\frac{m}{k}\left(e^{-\frac{kt}{m}} - 1\right)$$

$$z(t) = h - V_1 t + \frac{m}{k}(V_1 - V_0)\left(1 - e^{-\frac{kt}{m}}\right)$$

(6)

$m = 10^4\ kg \qquad \vec{V} = 100\,\hat{i}\ m/s \qquad \vec{F} = -kv^2\,\hat{i}\ N \quad k = 10\ kg/m$



$V_A = 100\ m/s$

$V_B = 50\ m/s$

a) $V_A \rightarrow V_B$ distantzia?

$F = m \cdot a \longrightarrow -kv^2 = m\frac{dv}{dt} \longrightarrow \int -\frac{k}{m}dt = \int \frac{dv}{v^2}$

$$\int_0^t \frac{dt}{m} = \int_{V_0}^{V} \frac{dv}{-kv^2}\ ; \qquad \frac{t}{m} = \frac{-1}{k}\int_{V_0}^{V}\frac{dv}{v^2}$$

$$\frac{t}{m} = -\frac{1}{k}\cdot\left[\frac{-1}{V}\right]_{V_0}^{V} = \frac{1}{k}\left(\frac{1}{V} - \frac{1}{V_0}\right) \Bigg\} \quad -\frac{k}{m}t = \frac{1}{V_0} - \frac{1}{V}$$

$$\frac{t}{m} = \frac{1}{k}\left(\frac{V_0 - V}{V \cdot V_0}\right)\ ; \qquad \frac{t}{m} = \frac{V_0 - V}{kV\cdot V_0}$$

$$t = \frac{m(V_0 - V)}{k\cdot V\cdot V_0} = 10\ s$$

(1)

$$\frac{dx}{dt}(-kv^2) = m\frac{dv}{dt}\cdot\frac{dx}{dt} \quad v$$

$$\int -\frac{k}{m}\,dx = \int_{V_0}^{V} \frac{dv \cdot v}{v^2}$$

$\boxed{t = 10\ s \text{ igaro dira tarte horretan}}$

⑦

$\vec{F}_m = -kv^2 \hat{i}$

$m = 1000$ kg

$t = 20$ s v erdira gutxitzeko.

$v_0 = 120$ km/h $= 33'\widehat{33}$ m/s

$v_1 = 60$ km/h $= 16'67$ m/s

a) k ?

$F = m \cdot a \qquad -kv^2 = m \cdot a \quad ; \quad -kv^2 = m \cdot \frac{dv}{dt}$

$$\int_0^t \frac{dt}{m} = \int_{v_0}^{v} \frac{dv}{-kv^2} \quad ; \quad \frac{t}{m} = \frac{1}{-k} \cdot \left[-\frac{1}{v}\right]_{v_0}^{v}$$

$$\frac{t}{m} = \frac{-1}{k} \cdot \left(\frac{1}{v_0} - \frac{1}{v}\right)$$

$$k = \frac{m}{t}\left(\frac{1}{v} - \frac{1}{v_0}\right)$$

$$\boxed{k = 1'5 \ \text{kg/m}}$$

b) autoaren abiadura, erditik laurdenera t?

$v_2 = 8'33$ m/s

$$t = \frac{m}{k}\left(\frac{1}{v_2} - \frac{1}{v_1}\right) = 40 \text{ s} \qquad \boxed{t = 40 \text{ s}}$$

c) ~~[illegible]~~ Potentzia

$v = 90$ km/h $= 25$ m/s $W \cdot \Delta r = 10000$ m

$v = 120$ km/h

$P = F \cdot v = kv^2 \cdot v = kv^3$

$P(90 \text{ km/h}) = 23'6$ kW

$P(120 \text{ km/h}) = 55'6$ kW

$$W = \int \vec{F}(\vec{r})\,d\vec{r} = \int -kv^2 \cdot dr \cdot \cos 180^\circ = \int kv^2\,dr = kv^2 r =$$

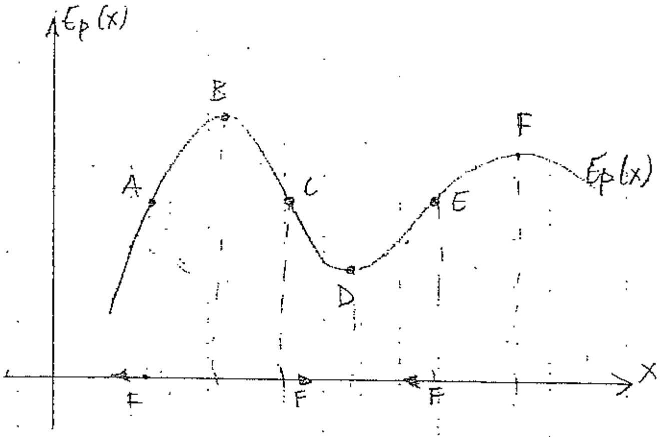
$W(90 \text{ km/h}) = 9375$ kJ

$W(120 \text{ km/h}) = 16600$ kJ

$E_k(90 \text{ km/h}) = \frac{1}{2} m v^2 = 312$ kJ

5. GAIA: PARTIKULAREN DINAMIKA OROKORRA — FISIKA

(8)



a) A: $E_p'(x) > 0$ delako $F < 0$

F; B: $F = 0$ → oreka puntua (ez-egonkorra)

C: $F > 0$; $E_p'(x) < 0$

D: $F = 0$ → oreka puntua (egonkorra)

E: $F < 0$, $E_p'(x) > 0$

$-dE_p = F(x)\cdot dx$ ⟶ $E_p'(x) = -F(x)\,dx$

$E_{p_0} = 0$ suposatu

$E_p = \int -F(x)\,dx$

b) $F = max$ ⟶ $E_p'(x) = max$ den tokian izango da, hau da $E_p(x)$ funtzioak malda handiena duen puntuan, kasu honetan, A puntuan.

c) Oreka posizioak: B, D, F.

$F = 0$-ko puntuak dira, non E_p max edo min den.

E_p → max bada ⟶ ez egonkorra $\frac{d^2E_p}{dx} < 0$

E_p → min bada ⟶ egonkorra $\frac{d^2E_p}{dx} > 0$

(9) $m = 1\,kg$ $\quad E_p(x) = \frac{x^4}{4} - 2x^2 + 4$

a) $\vec{F}$?

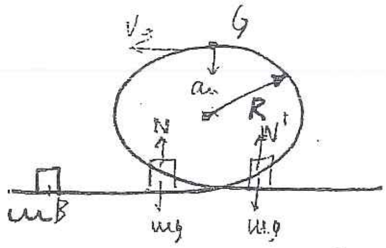
$-dE_p(x) = \vec{F}(x)\cdot dx$ $\qquad \vec{F}(x) = (-x^3 + 4x)\,\hat{i}$ (N)

b) Ø c)

$F(x) = -x^3 + 4x = 0$ ⟶ $x = 0$, $x = -2$, $x = 2$

d) $E_p(x) = \frac{x^4}{4} - 2x^2 + 4 = 2$ eta x-aren balioak atera

10



$\mu = 0 \quad F_R = \emptyset$

a) V_{min} bueltaa emateko?

$N + mg = m \cdot a_n = m \frac{V^2}{R}$

$\boxed{\text{Muga} \quad N = 0}$

Ez erortzeko.

$N = \cancel{m}\frac{V_g^2}{2} - \cancel{m}g = 0 \qquad \frac{V_g^2}{2} = g \qquad \boxed{V_g \geq \sqrt{Rg}}$

ENERGIAREN KONTSERBAZIO PRINTZIPIOA (Marruskadurarik ez)

$E_{mB} = E_{mG}$

$\cancel{E_{PB}}^{0} + E_{ZB} = E_{ZG} + E_{PG}$

$\frac{1}{2}\cancel{m}V_B^2 = \frac{1}{2}\cancel{m}V_g^2 + \cancel{m}g\,2R \; ; \quad V_B = \sqrt{(\frac{1}{2}Rg + 2gR)\,2} = \sqrt{Rg + 4Rg} = \sqrt{5Rg}$

$\boxed{V_B \geq \sqrt{5Rg}}$

b)

$V_B' = 2\sqrt{5Rg}$

$E_{mB} = E_{mg}$

$\frac{1}{2}\cancel{m}V_B^2 = \frac{1}{2}\cancel{m}V_g^2 + \cancel{m}g\,2R \qquad V_g^2 = V_B^2 - 4gR = \sqrt{4(5Rg) - 4Rg} = \sqrt{16Rg}$

$V_g' = \sqrt{16Rg} = 4\sqrt{Rg}$

$N + mg = m\frac{V_g'^2}{R} \; ; N = m\frac{V_g'^2}{R} - mg$

$N = m\frac{16gR}{R} - mg = 15mg \qquad \boxed{N = 15mg}$

c)

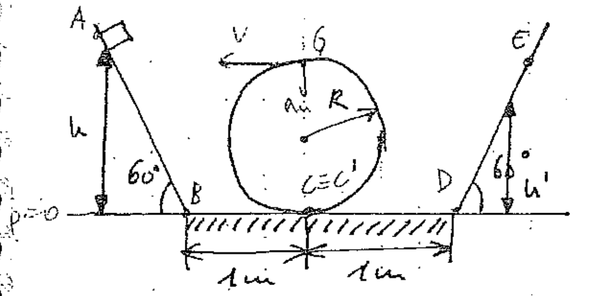
Lehen: $N - mg = 0 \qquad \boxed{N = mg}$

Gero: $N - mg = m \cdot a_n \longrightarrow N' = mg + m\frac{V_B'^2}{R} = mg + m\frac{4 \cdot 5gR}{R} = 21mg$

$\boxed{N' = 21mg}$

5 GAIA: PA... ...IKA OROKORRA Fisika.

(11)



$R = 1\,m$ $g = 10\,m/s^2$

$m = 300\,g$

$\mu_d = 0'1$

a) ~~...~~? h? $N = \frac{mg}{5}$

$N + mg = m \cdot a_n = m \frac{V_g^2}{R}$ $\frac{1}{5} mg + mg = m \cdot \frac{V_g^2}{R}$

ENERGIAREN KONTSERBAZIO PRINTZIPIOA $V_g \geq \sqrt{\frac{6gR}{5}} = 3'46\,m/s$

- $E_{mg} = E_{mc}$

$\frac{1}{2} m V_g^2 + mg2R = \frac{1}{2} m V_c^2$; $V_c = \sqrt{\frac{6gR}{5} + 4gR} = \sqrt{\frac{26gR}{5}} = 7'21\,m/s$

- $E_{mC} = E_{mB}$

$\frac{1}{2} m V_C^2 = \frac{1}{2} m V_B^2 - W_{BC}$

$\frac{1}{2} m V_B^2 = \frac{1}{2} m V_c^2 - \mu_d \cdot mg \cdot \Delta r \cdot \cos 180°$ $V_B = \sqrt{\frac{26gR}{5} + 2\mu_d g \Delta r} = 7'35\,m/s$

- $E_{mA} = E_{mB}$ ($V_A = 0\,m/s$)

$\cancel{\frac{1}{2} m V_A^2}^{0} + mgh = \frac{1}{2} m V_B^2$ $h = \frac{\frac{1}{2} V_B^2}{g} = 2'7\,m$ $\boxed{h = 2'7\,m}$

b)

Marruskadurarik ez dagoenez, $E_{mC} = E_{mC'}$, eta beraz, $V_C = V_C'$

$N - mg = m \cdot a_n = m \frac{V_c'^2}{R}$

$N = m\left(\frac{V_c'^2}{R} + g\right) = 18'6\,N$ $\boxed{N_C = 18'6\,N}$

c) $E_{mC'} = E_{mD} - W_R$

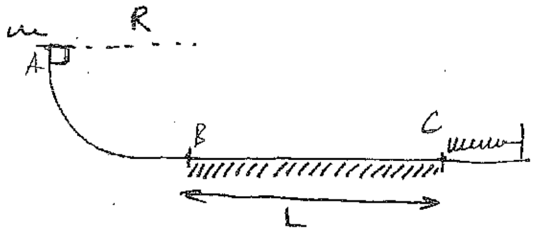
$\frac{1}{2} m V_c'^2 = \frac{1}{2} m V_D^2 - \mu_d mg \Delta r \cos 180°$ $V_D = \sqrt{V_c'^2 - 2\mu_d \Delta r g} = 7'07\,m/s$

$V_D = 7'07\,m/s$

$E_{mD} = E_{mE}$

$\frac{1}{2} m V_D^2 = \frac{1}{2} m \cancel{V_E^2}^{0} + mgh'$ $\boxed{h' = 2'5\,m}$

(12)



$m = 2\,kg \qquad L = 9\,m \qquad k = 7200\,kg/m$

$R = 5\,m \qquad \mu_d = 0'2$

a) V_B?

$E_{mA} = E_{mB}$

$\cancel{1/2\, m\, V_A^2}^{\,0} + \cancel{m} g h_A = 1/2 \cancel{m} V_B^2 \qquad V_B = \sqrt{2gh_A} = \sqrt{2gR} = 10\,m/s$

$\boxed{V_B = 10\,m/s}$

b) V_c?

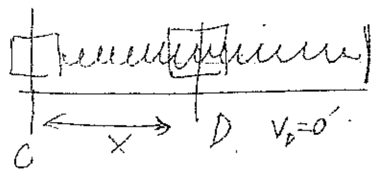
$E_{mB} = E_{mc} - W_R$

$1/2\,\cancel{m} V_B^2 = 1/2\,\cancel{m} v_c^2 - \mu_d \cancel{m} g\, L \cos 180°$

$1/2\, V_B^2 - \mu_d g L = 1/2\, m v_c^2$

$V_C = \sqrt{V_B^2 - 2\mu_d g L} = 8\,m/s \qquad \boxed{V_c = 8\,m/s}$

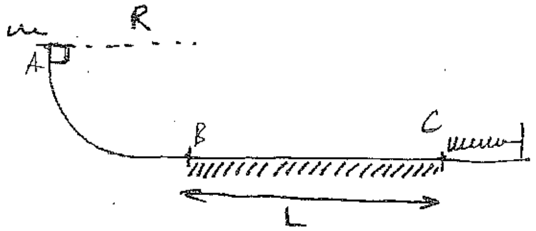
c) x?



$E_{mc} = E_{mD}$

$\cancel{1/2}\, m v_c^2 = 1/2\, k x^2 \qquad x = \sqrt{\frac{m V_c^2}{k}} = 0'13\,m \qquad \boxed{x = 0'13\,m}$

(12)



$m = 2\,kg$ $\quad L = 9\,m$ $\quad k = 7200\ kg/m$

$R = 5\,m$ $\quad \mu_d = 0'2$

a) V_B?

$Em_A = Em_B$

$\cancel{\tfrac{1}{2} m v_A^2}^{\,\to 0} + \cancel{m} g h_A = \tfrac{1}{2} \cancel{m} v_B^2$ $\qquad V_B = \sqrt{2gh_A} = \sqrt{2gR} = 10\ m/s$

$\boxed{V_B = 10\ m/s}$

b) V_c?

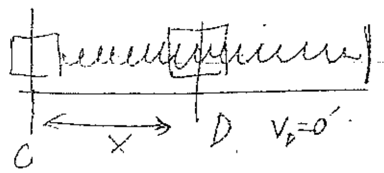
$Em_B = Em_C - W_R$

$\tfrac{1}{2} \cancel{m} v_B^2 = \tfrac{1}{2} \cancel{m} v_c^2 - \mu_d \cancel{m} g\, L \cos 180^\circ$

$\tfrac{1}{2} V_B^2 - \mu_d g L = \tfrac{1}{2} m v_c^2$

$V_C = \sqrt{V_B^2 - 2\mu_d g L} = 8\ m/s$ $\qquad \boxed{V_c = 8\ m/s}$

c) x?

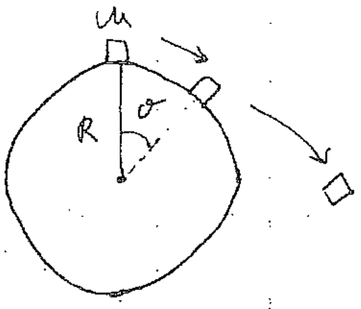


$Em_C = Em_D$

$\tfrac{1}{2} m v_C^2 = \tfrac{1}{2} k x^2$ $\qquad x = \sqrt{\dfrac{m V_C^2}{k}} = 0'13\ m$ $\qquad \boxed{x = 0'13\ m}$

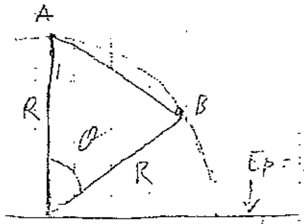
5. GAIA: ~~[illegible]~~ PARTIKULAREN DINAMIKA OROKORRA — FISIKA

(13)



$F_R = \emptyset$

a) $E_Z(\theta)$?



ENERGIAREN KONTSERBAZIO PRINTZIPIOA

$$E_{mA} = E_{mB}$$

$$\tfrac{1}{2}\cancel{m}\overset{0}{V_A^2} + \cancel{m}gR = \tfrac{1}{2}\cancel{m}V_B^2 + R\cos\theta\,\cancel{m}g$$

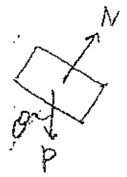
$$V_B = \sqrt{2gR - 2R\cos\theta\, g} = \sqrt{2Rg\,(1-\cos\theta)}$$

$$E_Z = \tfrac{1}{2}mV_B^2 = \tfrac{1}{\cancel{2}}m\,\cancel{2}Rg\,(1-\cos\theta) = Rgm\,(1-\cos\theta)$$

$$\boxed{E_Z = Rgm\,(1-\cos\theta)}$$

b) N?

(B)



$$\begin{cases} P\cos\theta - N = m\cdot a_n \\ P\sin\theta = m\cdot a_t \end{cases}$$

$$N = mg\cos\theta - m\cdot\frac{V_B^2}{R}$$

$$N = m\left(g\cos\theta - \frac{2\cancel{R}g\,(1-\cos\theta)}{\cancel{R}}\right)$$

$$N = mg\,(\cos\theta - 2(1-\cos\theta))$$

$$N = mg\,(\cos\theta - 2 + 2\cos\theta) = \boxed{mg\,(3\cos\theta - 2)}$$

c) N=0?

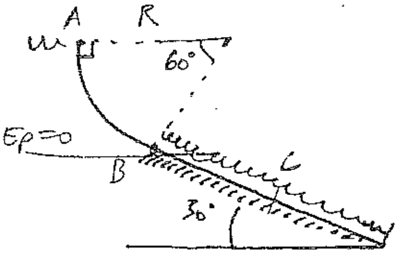
$$N = mg\,(3\cos\theta - 2) = 0$$

$$3\cos\theta - 2 = 0$$

$$\cos\theta = \frac{2}{3} \longrightarrow \theta = \arccos\frac{2}{3} = 48'19°$$

$$\boxed{\theta = 48'19°}$$

(14)



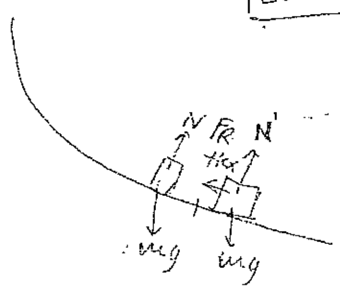
$R = 30\ m$ $\qquad \mu_d = 0'2$

$m = 70\ kg$ $\qquad k = 27\ N/m$

$g = 10\ m/s^2$

a) N_B?

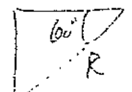
Lehen



$$\begin{cases} N - mg\cos 30^\circ = m a_n = m\dfrac{V_B^2}{R} \\ \cancel{m}g \sin 30 = \cancel{m}\, a_T \end{cases}$$

ENERGIAREN KONTSERBAZIO PRINTZIPIOA

$E_{mA} = E_{mB}$



$\cancel{m}gR\sin 60^\circ = \frac{1}{2}\cancel{m}v_B^2 \qquad V_B = \sqrt{2Rg\sin 60^\circ} = 22'79\ m/s$

$$N = m\left(\frac{V_B^2}{R} + g\cos 30^\circ\right) = m\left(\frac{2\cancel{R}g\sin 60^\circ}{\cancel{R}} + g\cos 30^\circ\right)$$

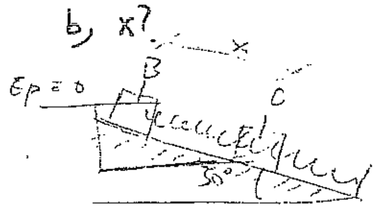
$\boxed{N = 1820\ N}$

Gero

$$\begin{cases} N - mg\cos 30^\circ = 0 \longrightarrow N = mg\cos 30^\circ \\ +kx + \mu_d N - mg\sin 30^\circ = m \cdot a_T \end{cases}$$

$\boxed{N' = 606\ N}$

b) x?



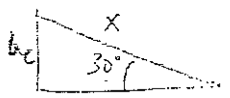
$E_{mB} = E_{mC}$

$\frac{1}{2}mV_B^2 = mgh_C + \frac{1}{2}kx^2 - \mu_d mg\cos 30^\circ x \cdot \cos 180$

$\frac{1}{2}mV_B^2 = -mgx\sin 30^\circ + \frac{1}{2}kx^2 + \mu_d mg\cos 30^\circ\ x$

$\frac{1}{2}kx^2 + mg(\mu_d\cos 30^\circ - \sin 30^\circ)x - \frac{1}{2}mV_B^2 = 0$

$13'5x^2 - 229x - 18180 = 0$



$\sin 30 = \frac{h_C}{x}$

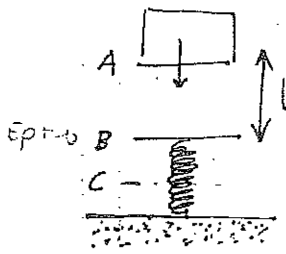
$-x \cdot \sin 30 = h_C$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \begin{cases} 46'14\ m \\ \cancel{-29'18}\ m \end{cases}$$

$\boxed{x = 46'14\ m}$

5. GAIA: PARTIKULAREN DINAMIKA OROKORRA — FISIKA

(15)



$m = 2\,kg$ $\quad F = 1000\,N$ $\quad \Delta t = 10^{-2}\,s$ $\quad g = 10\,m/s^2$

$h = 10\,m$ $\quad k = 2000\,N/m$

a) V_B?

$F = \frac{dp}{dt}$ $\quad \Delta p = \int F\,dt = F\int dt = F\cdot\Delta t = 1000\,N \cdot 10^{-2}\,s = 10\,kg\,m/s$

$\Delta p = p - p_0 = m(v - \cancel{v_0}^{\,0}) \longrightarrow \Delta p = mv \quad V_A = \frac{\Delta p}{m} = 5\,m/s$

$mv - 0 = F\cdot\Delta t = I \quad V_A = \frac{F\cdot\Delta t}{m} = 5\,m/s$

ENERGIAREN KONTSERBAZIO PRINTZIPIOA

$E_{mA} = E_{mB}$

$\frac{1}{2}\cancel{m}V_A^2 + \cancel{m}gh = \frac{1}{2}\cancel{m}V_B^2 \qquad V_B = \sqrt{V_A^2 + 2gh} = 15\,m/s$

$\boxed{V_B = 15\,m/s}$

b) x?

$E_{mB} = E_{mC}$

$\frac{1}{2}mv_B^2 = \frac{1}{2}kx^2 + mgh_C$

$\frac{1}{2}mv_B^2 = \frac{1}{2}kx^2 - mgx \quad ; \quad 1000\,x^2 - 20x - 225 = 0$

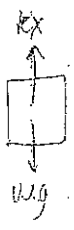
$x = \frac{20 \pm \sqrt{400 - 4(1000)(-225)}}{2000} =$

$= \frac{20 \pm 948'894}{2000} = \begin{cases} 0'48\,m \\ -0'46\,m \end{cases}$

$\boxed{x = 0'48\,m}$

c) $kx = 2000 \cdot 0'48 = \boxed{969\,N}$

d)



$mg - kx = m\cancel{a}^{\,0} = 0$

$mg = kx$

$x = \frac{mg}{k} = 0'01\,m \qquad \boxed{x = 0'01\,m}$

[illegible]

5. GAIA: PARTIKULAREN DINAMIKA OROKORRA — Fisika

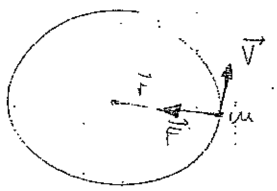
(17)

$R_L = 1{,}5 \cdot 10^8$ km $= 1'5 \cdot 10^{11}$ m $R_I = 385000$ km $= 385 \cdot 10^6$ m

$M_E = 2 \cdot 10^{30}$ kg $M_L = 6 \cdot 10^{24}$ kg

a) T? Vorb?

$F = m \cdot a_n \longrightarrow G\frac{M\cancel{m}}{r^{\cancel{2}}} = \cancel{m}\frac{V^2}{\cancel{r}}$



$V = \sqrt{\frac{GM}{r}}$

Luna eguzkiaren inguruan | Ilargia lurraren inguruan

$V_L = \sqrt{G\frac{M_E}{R_L}} = 29821'69$ m/s $= \boxed{29'82 \text{ km/s}}$

$V_I = \sqrt{G\frac{M_L}{R_I}} = 1019{,}5$ m/s $= \boxed{1 \text{ km/s}}$

$T_L = \frac{2\pi R_L}{V_L} = 31603768{,}8$ s $= \boxed{365'78 \text{ egun}}$

$T_I = \frac{2\pi R_I}{V_I} = 2206356{,}39$ s $= \boxed{25'5 \text{ egun}} \simeq 28$ egun

b) orbita geoest? h? ~~R~~? v? $R_L = 6400$ km $= 6400000$ m

[1] ok geoest. $T = 24$ h

$T = \frac{2\pi R}{V} = \frac{2\pi(R_L+h)}{V} = 24h = 86400$ s

$F = m \cdot a_n$; $G\frac{M\cancel{m}}{(R_L+h)^{\cancel{2}}} = \cancel{m}\frac{V^2}{\cancel{(R_L+h)}}$ $R_L + h = \frac{GM}{V^2}$

$\frac{2\pi\frac{GM}{V^2}}{V} = \frac{2\pi GM}{V^3} = 86400$ s $V = \sqrt[3]{\frac{2\pi GM}{86400}} = 3075'9$ m/s $= \boxed{3' \text{ km/s}}$

$h = \frac{GM}{V^2} - R_L = 35899249{,}3$ m

$\boxed{h = 3'59 \cdot 10^7 \text{ m}}$

[2] $h_s = \frac{h}{10} = 3'59 \cdot 10^6$ m , $m_s = 100$ kg

$T = \frac{2\pi(R_L+h_s)}{V} = 9917'2$ s $\rightarrow \boxed{= 2'75 \text{ h}}$

$V = \sqrt{\frac{GM}{(R_L+h_s)}} = 6329'3$ m/s $\rightarrow \boxed{= 6'33 \text{ km/s}}$

$E_1 = \frac{1}{2}mv_1^2 - \frac{GM_L m}{(R_L+h)} = \boxed{-4'73 \cdot 10^5 \text{ J}}$

$E_2 = \frac{1}{2}mv_2^2 - G\frac{M_L m}{(R_L+h_s)} = \boxed{-3 \cdot 10^9 \text{ J}}$

(18)

$M_L = 6 \cdot 10^{24}$ kg $\quad R_L = 6400000$ m

$M_M = 6{,}6 \cdot 10^{23}$ kg $\quad R_M = 3400000$ m

a) g_L? g_M?

$$F = m \cdot a_n = m \cdot g \xrightarrow{h=0\,m} g = \frac{G\frac{M\cancel{m}}{R^2}}{\cancel{m}} = G\frac{M}{R^2}$$

Lluna: $g_L = G\frac{M_L}{R_L^2} = \boxed{9'77\ m/s^2}$

Marte: $g_M = G\frac{M_M}{R_M^2} = \boxed{3'81\ m/s^2}$

b) h? $g_L/2$?

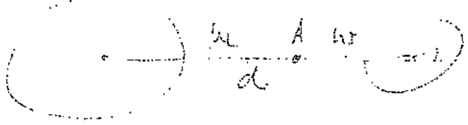
$$F = m\, a_n = m\, g_L \longrightarrow F = m\frac{g_L}{2} \longrightarrow G\frac{M_L\cancel{m}}{(R_L+h)^2} = \cancel{m}\frac{g_L}{2}$$

$$h = \sqrt{\frac{2GM_L}{g_L}} - R_L = \boxed{2651\ Km}$$

c) $g_L = 0$?

d = 385000 km

$M_I = 7'4 \cdot 10^{22}$ kg



$h_I = d - h_L$

$$g_L = G\frac{M_L}{h_L^2} \qquad g_I = G\frac{M_I}{(d-h_L)^2} \qquad g_I = g_L?$$

$$\cancel{G}\frac{M_L}{h_L^2} = \cancel{G}\frac{M_I}{(d-h_L)^2}$$

$$M_L(d-h_L)^2 = M_I h_L^2$$

$$M_L d^2 + M_L h_L^2 - 2M_L d h_L = M_I h_L^2$$

$$(M_L - M_I)h_L^2 - 2M_L d\, h_L + M_L d^2 = 0$$

$a = 5'926 \cdot 10^{24}$

$b = -4'62 \cdot 10^{33}$

$c = 8'89 \cdot 10^{41}$

$$h_L = \frac{-b \pm \sqrt{b^2 - 4(a\,c)}}{2a} = \frac{4'62 \cdot 10^{33} \pm 5'21 \cdot 10^{32}}{2 \cdot 5'926 \cdot 10^{24}} =$$

$h_{L1} = \cancel{433774792'4}$ m

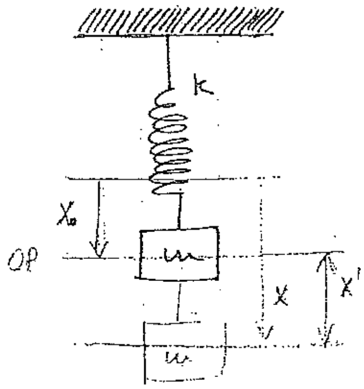
$h_{L2} = 345848801{,}9$ m

$\boxed{h_L = 346000\ km}$

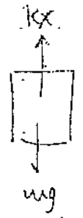
6. GAIA   HIGIDURA OSZILAKORRA   Fisika

Ariketak

(1)



a) Frogatu HHS dela $\ddot{x} + Cx = 0$



$F = m \cdot a$

$mg - kx = ma$

$kx_0 - kx = m\ddot{x}$

$k(x_0 - x) = m\ddot{x}$

$-k(x - x_0) = m\ddot{x}$

$-kx' = m\ddot{x}$

oreka posizioa:

$kx = mg$

$kx_0 = mg$

$x_0 = \frac{mg}{k}$

$x' = x - x_0$ (kte)

$\ddot{x}' = \ddot{x}$

$$\boxed{\ddot{x}' + \frac{k}{m}x' = 0} \longrightarrow \boxed{x'(t) = A\sin(\omega_0 t + \delta)}$$

$\omega^2 = \frac{k}{m}$

b) $E_p = \frac{1}{2}kx^2$

$x = x' + x_0 \qquad x_0 = \frac{mg}{k} \qquad x' = x - x_0$

$E_p = E_{p_e} + E_{p_g}$

$\frac{1}{2}kx'^2 = \frac{1}{2}kx^2 + mgx' =$

$$\frac{1}{2}kx^2 + mgx' = \frac{1}{2}k(x' + x_0)^2 + mgx' = \frac{1}{2}k(x'^2 + x_0^2 + 2x'x_0) + mgx' =$$

$$= \frac{1}{2}kx'^2 + \frac{1}{2}kx_0^2 + \cancel{\tfrac{1}{2}}kx'x_0\cancel{\cdot 2} + mgx' = \frac{1}{2}kx'^2 + \frac{1}{2}k\left(\frac{mg}{k}\right)^2 + kx'\left(\frac{mg}{k}\right) +$$

$$+ mgx' = \frac{1}{2}kx'^2 + \frac{1}{2}\frac{(mg)^2}{k} + mgx' + mgx' = \frac{1}{2}kx'^2 + \frac{(mg)^2}{2k} + 2mgx' =$$

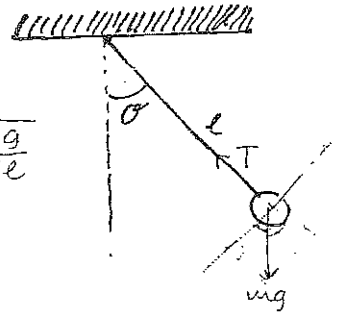
$$= \frac{1}{2}kx'^2 + \frac{m^2g^2}{2k} + 2mg(x - x_0) = \frac{1}{2}kx'^2 + \frac{m^2g^2}{2k} + 2mgx - 2mgx_0 =$$

$$= \frac{1}{2}kx'^2 + \frac{m^2g^2}{2k} + 2mgx - 2mg\frac{mg}{k} = \frac{1}{2}kx'^2 + 2mgx + \left(\frac{1}{2} - 2\right)\frac{m^2g^2}{k} =$$

$$= \frac{1}{2}kx'^2 + 2mg(x' + x_0) - \frac{3}{2}\frac{m^2g^2}{k} = \frac{1}{2}kx'^2 + 2mgx' + \left(2 - \frac{3}{2}\right)\frac{m^2g^2}{k} =$$

$$= \frac{1}{2}kx'^2 + 2mgx' + \frac{m^2g^2}{4k}$$

(2)



$w_0 = \sqrt{\frac{g}{\ell}}$

$\frac{v^2}{R} = w^2 R$

a) $\ddot{\theta} + C\theta = 0$ ($C \to \omega_0^2$)

$$\begin{cases} T - mg\cos\theta = m \cdot a_n = m \cdot w^2 \ell = m\dot{\theta}^2 \ell \\ -mg\sin\theta = m \cdot a_T = m\ddot{\theta}\ell \end{cases}$$

$\ddot{\theta}\ell \to \alpha \ell$

$\alpha = \frac{dw}{dt}$; $w = \frac{d\theta}{dt}$

2. eke hartuta: $\cancel{m}\ddot{\theta}\ell + \cancel{m}g\sin\theta = 0$

$\ddot{\theta} + \frac{g}{\ell}\sin\theta = 0$ $\quad$ ($\sin\theta \simeq \theta$ (oszilazio txikiak direnean))

$\boxed{\ddot{\theta} + \frac{g}{\ell}\theta = 0}$ $\to$ $w^2 = \frac{g}{\ell}$ $\quad w = \sqrt{\frac{g}{\ell}}$ $\quad \tau = \frac{2\pi}{w} = 2\pi\sqrt{\frac{\ell}{g}}$

b) $t=0 \to \begin{cases} \theta = \theta_0 \\ V_0 = 0 \end{cases}$ $\quad \ddot{\theta} + \frac{g}{\ell}\theta = 0 \longrightarrow \begin{cases} \theta(t) = A\sin(\omega_0 t + \delta) \\ \dot{\theta}(t) = A\omega_0\cos(\omega_0 t + \delta) \end{cases}$

$\dot{\theta}(0) = A\omega_0\cos\delta = 0 \longrightarrow \cos\delta = 0$

$\theta(t) = A\sin(\omega_0 t + \pi/2)$ $\quad\quad \delta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{n\pi}{2}$ $\quad n = 1,2,3\ldots$

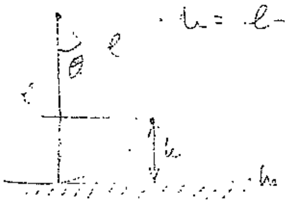
$\theta(0) = A\sin\pi/2 = A = \theta_0$

$\boxed{\theta(t) = \theta_0\sin(\omega_0 t + \pi/2)}$

c) $E_p(t)$?

$\cos\theta \simeq 1 - \frac{\theta^2}{2}$

$E_p(t) = mgh(t) = mg\ell(1-\cos\theta) = mg\ell \cdot \frac{\theta^2}{2} = \boxed{\frac{mg\ell\theta_0^2}{2}\sin^2(\omega_0 t + \pi/2)}$



$h = \ell - \ell\cos\theta = \ell(1-\cos\theta)$

d) $E_z(t)$?

$E_z(t) = 1/2\, m v(t)^2 = 1/2\, m(\dot{\theta}\ell)^2 = \boxed{\frac{1}{2} m\ell^2\theta_0^2\omega_0^2\cos^2(\omega_0 t + \pi/2)}$

e) E = kte?

$E = E_p(t) + E_z(t) = \frac{mg\ell\theta_0^2}{2}\sin^2(\omega_0 t + \pi/2) + \frac{1}{2}m\ell^2\theta_0^2\omega_0^2\cos^2(\omega_0 t + \pi/2) =$

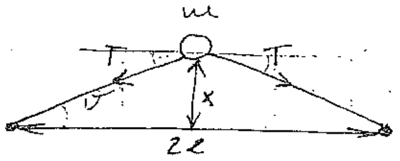
$= \frac{mg\ell\theta_0^2}{2} \cdot \frac{m\ell^2\theta_0^2}{2} \cdot \frac{g}{\ell} = \frac{1}{2}mg\ell\theta_0^2$

$\boxed{E = 1/2\, mg\ell\theta_0^2 \neq E(t)}$ $\quad$ [illegible]

6. GAIA: HIGIDURA OSZILAKORRA

FISIKA

③



$\ddot{\theta} + c\theta = 0$

Tentsioa oso handia dela suposatuz, pisua arbuiatu egin daiteke.

$$\begin{cases} T\cos\theta - T\cos\theta = 0 \\ -T\sin\theta - T\sin\theta = m a = m\cdot\ddot{x} \end{cases}$$

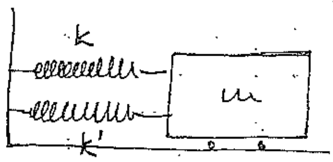
$$l\cdot\sin\theta = x \longrightarrow \sin\theta = \frac{x}{l}$$

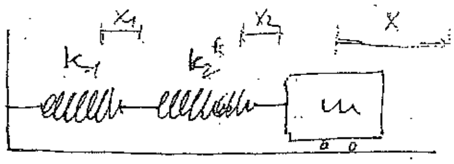
$$-2T\sin\theta = m\cdot\ddot{x} \longrightarrow -2T\cdot\frac{x}{l} = m\ddot{x}$$

$$\boxed{\ddot{x} + \frac{2T}{ml}x = 0}$$

$$\rightarrow \omega_0^2 = \frac{2T}{ml} \qquad \boxed{\omega_0 = \sqrt{\frac{2T}{ml}}}$$

④





$$-kx - k'x = ma = m\ddot{x}$$

$$\frac{-x(k+k')}{m} = \ddot{x}$$

$$\ddot{x} + \frac{k+k'}{m}x = 0$$

$$\omega_0^2 = \frac{k+k'}{m}\ ; \quad \boxed{\omega_0 = \sqrt{\frac{k+k'}{m}}}$$

masatik tiratzen dugula suposatuz, x_0,

$$x = x_1 + x_2$$

$$x = \frac{F}{k} \qquad x_1 = \frac{F_1}{k_1} \qquad x_2 = \frac{F_2}{k_2}$$

$$\frac{F}{k} = \frac{F_1}{k_1} + \frac{F_2}{k_2}$$

Baina $F = F_1 = F_2$ denez,

$$\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2} \qquad k = \frac{k_1 k_2}{k_1 + k_2}$$

$$-kx = ma = m\ddot{x}$$

$$\ddot{x} + \frac{k}{m}x = 0 \quad ; \quad \ddot{x} + \frac{k_1 k_2}{m(k_1+k_2)}x = 0$$

$$\omega_0^2 = \frac{k_1 k_2}{m(k_1+k_2)} \quad ; \quad \boxed{\omega_0 = \sqrt{\frac{k_1\cdot k_2}{(k_1+k_2)m}}}$$

(5)

$x_1 = 6\ cm \rightarrow 2\ cm/s = V_1$ $\quad x(t) = A \sin(\omega_0 t + \delta) \longrightarrow \sin^2(\omega_0 t + \delta) = \left(\frac{x(t)}{A}\right)^2$

$x_2 = 8\ cm \rightarrow 1'5\ cm/s = V_2$ $\quad v(t) = A\omega_0 \cos(\omega_0 t + \delta) \longrightarrow \cos^2(\omega_0 t + \delta) = \left(\frac{v(t)}{A\omega_0}\right)^2$

$$a(t) = -A\omega_0^2 \sin(\omega_0 t + \delta)$$

$\sin^2(\ ) + \cos^2(\ ) = 1$

$$\frac{x^2(t)}{A^2} + \frac{v^2(t)}{A^2\omega_0^2} = 1$$

$$\frac{x^2(t)\,\omega_0^2 + v^2(t)}{A^2\omega_0^2} = 1 \quad ; \quad x^2(t)\,\omega_0^2 + v^2(t) = A^2\omega_0^2$$

$$\omega_0^2\left(x^2(t) - A^2\right) + v^2(t) = 1$$

$$\omega_0^2 = \frac{-v^2(t)}{x^2(t) - A^2} = \frac{v^2(t)}{A^2 - x^2(t)}$$

Bi datu hauekin ω_0 berdina dela,

$$\omega_{01}^2 = \frac{V_1^2}{A^2 - X_1^2} = \omega_{02}^2 = \frac{V_2^2}{A^2 - X_2^2} \qquad \frac{V_1^2}{A^2 - X_1^2} = \frac{V_2^2}{A^2 - X_2^2}$$

$$(A^2 - X_2^2)V_1^2 = V_2^2(A^2 - X_1^2)$$

$$A^2(V_1^2 - V_2^2) = X_2^2 V_1^2 - X_1^2 V_2^2 \qquad A^2 = \frac{X_2^2 V_1^2 - X_1^2 V_2^2}{V_1^2 - V_2^2} = 100\ cm$$

$$\boxed{A = 10\ cm}$$

$$\omega_0^2 = \frac{V_1^2}{A^2 - X_1^2} = \frac{V_2^2}{A^2 - X_2^2} = 0'0625 \qquad \boxed{\omega_0 = 0'25\ rad/s}$$

$a(t) = -A\omega_0^2 \sin(\omega_0 t + \delta)$ $\quad a_{max} \cdots \sin(\ ) = 1$

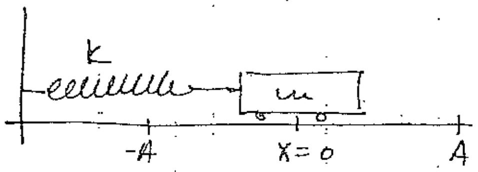
$$|a_{max}| = |-A\omega_0^2| = A\omega_0^2 = 0'625\ cm/s^2$$

$$\boxed{a_{max} = 0'625\ cm/s^2}$$

6. GAIA : HIGIDURA OSZILAKORRA   FISIKA.

(6)

$m = 0'5$ kg $\tau = 0'1$ s $A = 10$ cm



		O.P. (x=0)	A	x = 5 cm = 0'05 m
(m/s^2)	a	0	394'76	±197'4
(N)	F	0	197'4	98'7
(J)	E_p	0	9'87	2'46
(J)	E_z	9'87	0	7'40

$a(t) = -A\omega_0^2 \sin(\omega_0 t + \delta)$

$F = -kx$

$E_p = \frac{1}{2} k x^2$

$E_z = \frac{1}{2} m v^2$

$x(t) = A \sin(\omega_0 t + \delta)$

$v(t) = A\omega_0 \cos(\omega_0 t + \delta)$

$\omega_0 = \sqrt{\frac{k}{m}} = \frac{2\pi}{\tau} = 62'83$ rad/s

$\frac{k}{m} = \frac{4\pi^2}{0'1^2}$; $k = \frac{4\pi^2 m}{0'01} = 1974$ N/kg

$E_{p\,max} = \frac{1}{2} k A^2 = 9'87$ J

$E_p(x = 0'05\,m) = \frac{1}{2} k x^2 = 2'46$ J

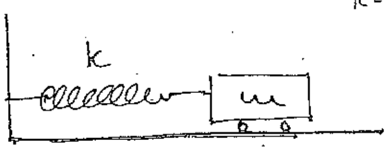
$|F_{max}| = |-kA| = 197'4$ N

$|a_{max}| = |-A\omega_0^2| = A\omega_0^2 = 394'76$ m/s^2

$a(t) = -\omega_0^2 x \longrightarrow a(x = 0'05\,m) = \pm 197'4$ m/s^2

(7)

$k = 12$ N/m $m = 3$ kg



a) $x(t)$?

$\begin{cases} N - P = 0 \\ -kx = ma = m \ddot{x} \end{cases} \longrightarrow -kx = m\ddot{x}$

$\omega_0^2 = \frac{k}{m} = 4$

$-\frac{k}{m} x = \ddot{x} \longrightarrow \ddot{x} + \frac{k}{m} x = 0$

$x(t) = A \sin(\omega_0 t + \delta)$

$x(t) = A \sin(2t + \delta)$

b) A? δ? E?

$V_0 = 0$

$X_0 = -3$ m

$v(0) = A\omega_0 \cos(\omega_0 t + \delta) = 0$; $\cos(0 + \delta) = 0 \longrightarrow \delta = \frac{\pi}{2}, \frac{3\pi}{2}, -\frac{\pi}{2}$

$v_0 = 0$, $x_0 = -3$ m $\delta = \pi/2, 3\pi/2, n\pi/2 \ldots$ A? E?

$x(0) = A \sin(0 + \delta) = A = -3$ m

$\pi/2 \longrightarrow \sin(\,) > 0$
$3\pi/2 \longrightarrow \sin(\,) < 0$ $\longrightarrow x_0 = -3$ — $\boxed{\delta = \frac{3\pi}{2} \quad A = \pm 3 \text{ m}}$

$E = \frac{1}{2} k A^2 = 54$ J $\boxed{E = 54 \text{ J}}$

b) A? δ? E?

$V_0 = -1$ m/s $X_0 = 1$ m

$V(0) = A w_0 \cos(\delta) = -1 = 2A \cos\delta$

$x(0) = A \sin(0 + \delta) = 1 = A \sin \delta$

$-1 = 2 \operatorname{cotg} \delta$

$\operatorname{cotg}\delta = -1/2 \longrightarrow \operatorname{tg}\delta = -2$

$A \cdot \sin\delta = 1 > 0 \longrightarrow A \cdot \sin 116'5 = 1$ $\delta = \operatorname{arctg}(-2) = -63'43° + n 180°$
$n = 1, 2, 3$

$\sin -63'43° < 0$ $A = \dfrac{1}{\sin 116'5°}$

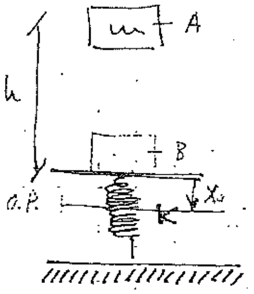
$\sin 116'5° > 0$

$\boxed{\delta = 116'5° \quad A = 1'12 \text{ m}}$

$E = \frac{1}{2} k A^2 = 7'5$ J $\boxed{E = 7'5 \text{ J}}$

6. GAIA: HIGIDURA OSTILAKORRA   Fisika

(8)



$g = 10\ m/s^2$ $h = 0'0125\ m = 1'25\ cm$

$k = 200\ N/m$ $m = 0'5\ kg$

ERORKETA

$E_{mA} = E_{mB}$

$\cancel{m}gh = 1/2 \cdot \cancel{m} v_B^2$ $V_B = \sqrt{2gh} = 0'5\ m/s$

OSTILAZIOAK

$x(t) = A \sin(\omega_0 t + \delta)$

$v(t) = A\omega_0 \cos(\omega_0 t + \delta)$

$\omega_0 = \sqrt{\frac{k}{m}} = 20\ rad/s$

OREKA POSIZIOA

$mg = kx_0$

$x_0 = \frac{mg}{k} = 0'025\ m = 2'5\ cm$

$$\begin{cases} x(0) = A \cdot \sin(\cancel{\omega_0 t}^{0} + \delta) = 0'025\ m \longrightarrow A \sin\delta = 0'025 \\ v(0) = A\omega_0 \cos(\cancel{\omega_0 t}^{0} + \delta) = 0'5\ m/s \longrightarrow A\omega_0 \cos\delta = 0'5 \end{cases}$$

$\frac{tg\,\delta}{\omega_0} = 0'05$

$tg\,\delta = 0'05 \cdot 20 = 1$

$\delta = arctg\ 1 = 45° = \frac{\pi}{4}, \frac{n\pi}{4}\quad n = 1, 2, 3 \ldots$

$A = \frac{x_0}{\sin\delta} = \frac{0'025}{\sin 45°} = 0'035\ m$

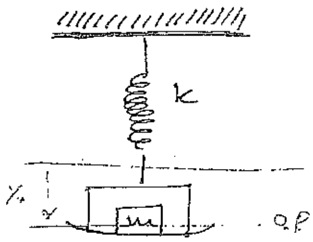
$\boxed{A = 0'035\ m}$

$F = -kx = -k A \sin(\omega_0 t + \delta) =$

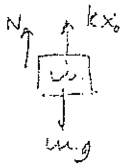
$= -7 \sin(20t + \pi/4)$

$\boxed{F = -7 \sin(20t + \pi/4)}$

⑨



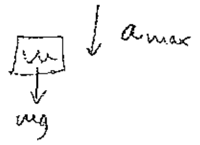
OREKA POSITIBOA



$N + kx_0 = mg$

A-max:

$a(t) = -A\omega_0^2 \sin(\omega_0 t - \delta)$ $\qquad \omega_0^2 = \sqrt{\frac{k}{m}}$



$|a(t)| = |-\omega_0^2 x| = |-\omega_0^2 A|$

$\cancel{m}g = m \cdot a = \cancel{m} \cdot (\omega_0^2 A)$

$A = \frac{g}{\omega_0^2} = \frac{g}{\frac{k}{m}} = \frac{mg}{k}$

$\boxed{A_{max} = \frac{mg}{k}}$ → kontaktuan ez galtzeko, beraz,

$\boxed{A = \frac{mg}{k}}$

⑩

$A \longrightarrow \frac{A}{2}$ $t = 15\,s \longrightarrow 117$ oszil. Ω? γ? $\qquad \Omega = \sqrt{\omega_0^2 + \gamma^2}$

$15\,s \longrightarrow 117$ osz. $\qquad \frac{15\ seg}{117\ osz} = \tau$

x — 1 osz.

$x(t) = A_0 e^{-\gamma t} \sin(\omega_0 t + \delta)$

$A(t) = A_0 e^{-\gamma t}$

$\frac{\cancel{A_0}}{2} = \cancel{A_0} e^{-\gamma t}$

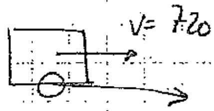
$\ln \frac{1}{2} = -\gamma t$ $\qquad \gamma = -\frac{\ln 1/2}{t} = \frac{-\ln 1/2}{15} = 0'046\ s^{-1}$

$\boxed{\gamma = 0'046\ s^{-1}}$

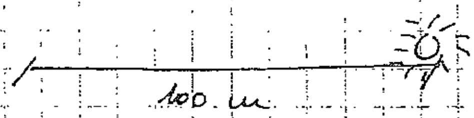
$\Omega = \sqrt{\omega_0^2 + \gamma^2} = \frac{2\pi}{\tau} = \frac{2\pi \cdot 117}{15} = 49$ rad/s $\qquad \boxed{\Omega = 49\ \text{rad/s}}$

7. GAIA: PARTIKULA MULTZOEN DINAMIKA

②

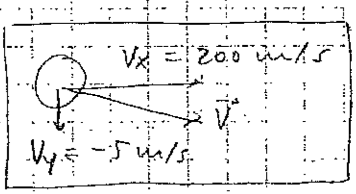


$V = 720\ km/h = 200\ m/s$ (unif) $\qquad V_0 = 200\ m/s \qquad g = 10\ m/s^2$



a) Granada: $\begin{cases} x: x = v_{0x} t \longrightarrow t = \dfrac{x}{v_{0x}} = \dfrac{100\ m}{200\ m/s} = 0'5\ s \\ y: y = v_{0y} t - \frac{1}{2} g t^2 \end{cases}$

edo $V_y = \cancel{V_{0y}}^{0} - gt = -gt \qquad V_y = -10\ m/s^2 \cdot 0'5\ s = -5\ m/s$

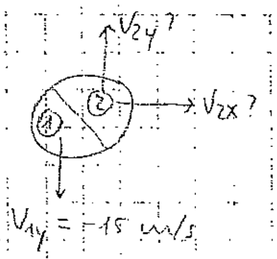


lehen

b)

masa bereko bi zatitan $\longrightarrow \frac{m}{2} = m_1 = m_2$

$\vec{V}_1 = -15\ m/s\ \hat{j} \longrightarrow V_{1y} = -15\ m/s$



$\dfrac{d\vec{p}}{dt} = \vec{F}^{kanp}$

$d\vec{p} = \underset{0}{\vec{F}^{kanp}} \cdot \underset{0}{dt} \simeq 0 \longrightarrow \vec{p}$ kte.

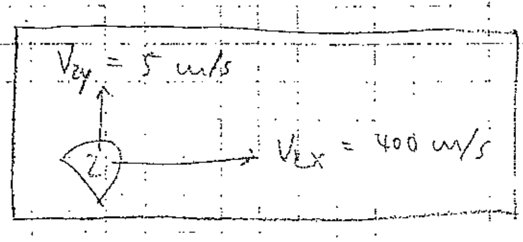
ondoren

$\vec{P}_{lehen} = \vec{P}_{ondoren}$

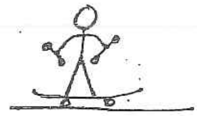
$\begin{cases} P_{x\,lehen} = P_{x\,ondoren} : \cancel{m} V_x = \frac{\cancel{m}}{2} \cancel{V_{1x}}^{0} + \frac{\cancel{m}}{2} V_{2x} \\ P_{y\,lehen} = P_{y\,ondoren} : \cancel{m} V_y = \cancel{m}/2\, V_{1y} + \cancel{m}/2\, V_{2y} \end{cases}$

$V_x = \frac{1}{2} V_{2x} \longrightarrow V_{2x} = 2V_x = 400\ m/s$

$V_y = \frac{1}{2} V_{1y} + \frac{1}{2} V_{2y} \qquad V_{2y} = (V_y - \frac{1}{2} V_{1y})\,2 = 5\ m/s$



③



$m_s = 40$ kg $\quad m_z = 5$ kg

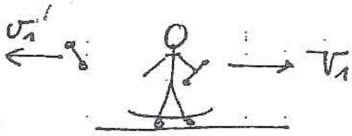
$m_p = 3$ kg

$v_0 = 0$ m/s

$F^{kanp} = mg - N = 0$

$F^{kanp} = \frac{dp}{dt} = 0 \longrightarrow \vec{p}$ kte

a) $v_1' = 7$ m/s



$\vec{P}_{lehen} = \vec{P}_{ondoren}$ (x ardatzean dena)

$$\begin{cases} P_{lehen} = mv_0 = 0 \\ P_{ondoren} = \underbrace{(m_s + m_p + m_z)}_{M} V_1 + m_z v_1 = \end{cases}$$

$$v_1 = V_1 + \underbrace{v_1'}_{7 \text{ m/s}}$$

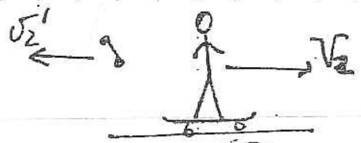
$$= M V_1 + m_z(V_1 + v_1') = V_1(M + m_z) + m_z v_1'$$

$$V_1(M + m_z) + m_z v_1' = 0$$

$$V_1 = \frac{-m_z v_1'}{M + m_z} = \frac{-m_z v_1'}{(m_s + m_p + m_z) + m_z} = -0'66 \text{ m/s}$$

$$\boxed{V_1 = -0'66 \text{ m/s}}$$

b) $v_2' = 7$ m/s



$\vec{P}_{lehen} = \vec{P}_{ondoren}$

$$\begin{cases} P_{lehen} = (m_s + m_p + m_z) V_1 \\ P_{ondoren} = (m_s + m_p) V_2 + m_z v_2 = (m_s + m_p) V_2 + m_z (V_2 + v_2') \end{cases}$$

$$v_2 = V_2 + \underbrace{v_2'}_{7}$$

$$(m_s + m_p + m_z) V_1 = (m_s + m_p) V_2 + m_z v_2' + m_z V_2$$

$$V_2 = \frac{(m_s + m_p + m_z) V_1 - m_z v_2'}{(m_s + m_p + m_z)} = -1'39 \text{ m/s}$$

$$\boxed{V_2 = -1'39 \text{ m/s}}$$

c) bi zamak batera?

(a) ataleko egoera berdina denez (baina masak aldatuta), bertako ebazpena erabil daiteke)

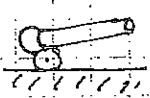
$$V' = \frac{-(2m_z) v'}{m_s + m_p + 2m_z} = -1'32 \text{ m/s}$$

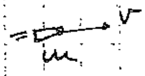
$v' = 7$ m/s

$$\boxed{V' = -1'32 \text{ m/s}}$$

(4)

M





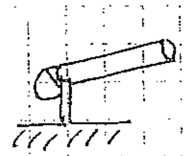
m v

$W^{Bar} = 5 \cdot 10^7\ J \longrightarrow W^{Bar} = \Delta E_z$

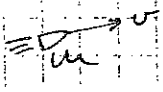
$m = 100\ kg$

$W^{Bar} + \cancel{W^{Kan}}^{0} = \Delta E_z$

a)



Lurrean finko



m v

$W^{Bar} = \Delta E_z = E_z - E_{z_0} =$

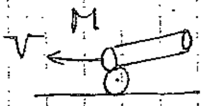
$= \frac{1}{2} m v^2 - \cancel{\frac{1}{2} m v_0^2}^{0} = \frac{1}{2} m v^2$

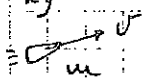
$v = \sqrt{\frac{W^{Bar} \cdot 2}{m}} = 1000\ m/s$

$\boxed{v = 1000\ m/s}$

b) $M = 1000\ kg$

M V





m v

$W^{Bar} = \Delta E_z = E_z - \cancel{E_{z_0}}^{0}$ ($v_0 = 0$ eta $V_0 = 0$ direlako)

$W^{Bar} = \Delta E_z$

$\downarrow$

$W^{Kan} = 0$

$\downarrow$

$\vec{F}^{kan} = \frac{d\vec{p}}{dt} = 0$

p kte

$$\left.\begin{aligned} W^{Bar} &= \tfrac{1}{2} m v^2 + \tfrac{1}{2} M V^2 \\ 0 &= m v + M V \end{aligned}\right\}$$

$V = \frac{-m v}{M}$ $\downarrow$

$W^{Bar} = \frac{1}{2} m v^2 + \frac{1}{2} M \left(\frac{-m v}{M}\right)^2$

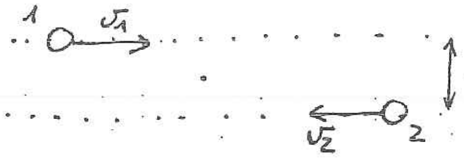
$W^{Bar} = \frac{1}{2} m v^2 + \frac{1}{2} \frac{m^2 v^2}{M}$

$v = \sqrt{\frac{W^{Bar} \cdot 2}{m\left(1 + \frac{m}{M}\right)}} = 953'5\ m/s$

$\boxed{\begin{aligned} v &= 953'5\ m/s \\ V &= -95'3\ m/s \end{aligned}}$

$V_{atzerakada} = 95'3\ m/s$

5



$m_1 = m_2 = m = 70$ kg

$v_1 = v_2 = v = 4$ m/s

a) Bakoitzarena ez.

7. GAIA: PARTIKULA MULTZOEN DINAMIKA

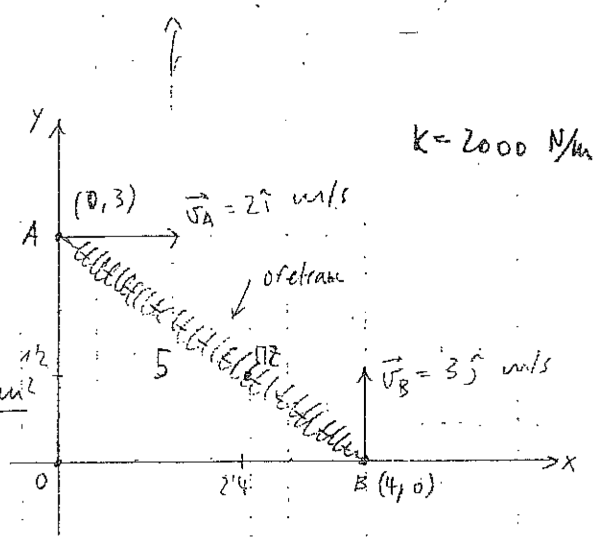
Fisika

⑥

$m_A = 4$ kg A(0,3) $\vec{v}_A = 2\hat{i}$ m/s

$m_B = 6$ kg B(4,0) $\vec{v}_B = 3\hat{j}$ m/s

K = 2000 N/m



a) $\vec{L}_{tot_o}$?

$\vec{L}_{tot_o} = \vec{L}_{B_o} + \vec{L}_{A_o} = \vec{r}_B \times m\vec{v}_B + \vec{r}_A \times m\vec{v}_A$

$\vec{L}_{B_o} = \vec{r}_B \times m\vec{v}_B = 6 \cdot \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 0 & 0 \\ 0 & 3 & 0 \end{vmatrix} = 6 \cdot 12\hat{k} = 72\hat{k}\ \frac{kg\, m^2}{s}$

$\vec{L}_{A_o} = \vec{r}_A \times m\vec{v}_A = 4 \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 3 & 0 \\ 2 & 0 & 0 \end{vmatrix} = 4(-6)\hat{k} = -24\ \frac{kg\, m^2}{s}$

$\vec{L}_{tot_o} = 72\hat{k} - 24\hat{k} = 48\hat{k}\ \frac{kg\, m^2}{s}$

$\boxed{\vec{L}_{tot_o} = 48\hat{k}\ \frac{kg\, m^2}{s}}$

b) E_{Ztot}?

$E_{Ztot} = E_{Z_1} + E_{Z_2} = \frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B v_B^2 = \frac{1}{2}(m_A v_A^2 + m_B v_B^2) = 35$ J

$\boxed{E_{Ztot} = 35\ J}$

c) Ez, barne indarra delako. Malgukiak eragindako indarra berdina (baina aurkakoa) izango da lerro zuzenean, eta akzio-erreakzio indarrak dienez, elkar anulatu egingo dute, eta beraz, ez dute masa-zentroaren higiduran eraginik sortuko

d) v-ak konstanteak badira, Ez konstantea izango da, Energia potentzial elastikoa konstantea izango da, indar elastikoa kontserbakorra delako. Beraz, barne lana konstantea da (v-ak konstanteak direla suposatuz)

e) luzapena = 5 cm

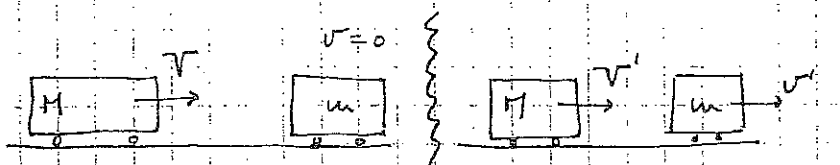
$\vec{R}_{MZ} = \frac{\sum_{k=1}^{N} m_k \cdot \vec{r}_k}{M} = \frac{12\hat{j} + 24\hat{i}}{10} = 1'2\hat{j} + 2'4\hat{i}$ cm

7. GAIA: PARTIKULA MULTZOEN DINAMIKA

Fisika

(7)

ezagunak: M, m, V



p kte ← Talka erabat elastikoa eta frontala

a) P_T kte.

$$p = p' \rightarrow \begin{cases} \text{lehen;} & p = MV + m\cancel{v}^{0} = MV \\ \text{ondoren;} & p' = MV' + mv' \end{cases} \quad MV = MV' + mv'$$

talka erabat elastikoa $\longrightarrow (v' - V') = \underset{\downarrow 1}{e}(V - \cancel{v}^{0}) \longrightarrow v' - V' = V$

$$v' = V + V'$$

$$MV = MV' + m(V + V')$$

$$MV = MV' + mV + mV' \longrightarrow \boxed{V' = \frac{(M-m)V}{M+m}}$$

$$v' = V + \frac{(M-m)V}{M+m} = \frac{\cancel{mV} + MV + MV - \cancel{mV}}{M+m} = \frac{2M}{M+m}V \qquad \boxed{v' = \frac{2M}{M+m}V}$$

b)

$\boxed{M = m}$: $V' = 0$, $v' = V$

$\boxed{M < m}$: $V' < 0$, $v' > 0$ eta aurkako noranzkoan

$\boxed{M > m}$: $V' < V$, $v' > V$ | $V' > 0$, $v' > 0$ eta biak noranzko berean

c)

$\boxed{M <<< m}$

$$\lim_{M \to 0} V' = \lim_{M \to 0} \frac{(M-m)V}{M+m} = -V$$

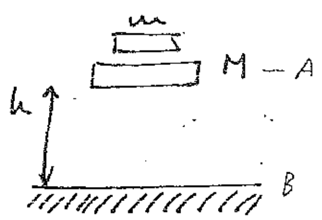
$$\lim_{M \to 0} v' = \lim_{M \to 0} \frac{2M}{M+m} V = 0$$

$\boxed{M >>> m}$

$$\lim_{m \to 0} V' = \lim_{m \to 0} \frac{(M-m)V}{M+m} = V$$

$$\lim_{m \to 0} v' = \lim_{m \to 0} \frac{2M}{M+m} V = 2V$$

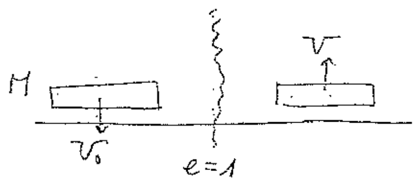
(8)



$E_{mA} = E_{mB}$

$\cancel{M} g h = \frac{1}{2} \cancel{M} v_B^2 \qquad v_B = \sqrt{2gh} = V_0 = v_0 = v$

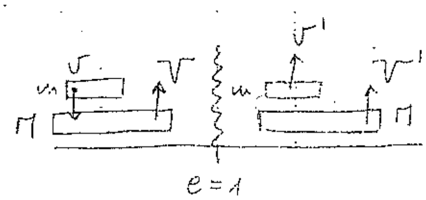
ⓐ M-k lurraren kontra errebotean (elastikoki errebotatu ⟹ e = 1)



p kte

$p_0 = p \longrightarrow M V_0 = M V \longrightarrow V_0 = V = \sqrt{2gh}$

ⓑ m M-ren aurkako talka (talka elastikoa ⟶ e = 1)



V', v'

p kte

$p = p' \begin{cases} \text{lehen: } p = mv + MV \\ \text{ondoren } p' = mv' + MV' \end{cases}$

$V' - v' = \underset{\uparrow}{e}(v - V)$

$V' - v' = v - V$

$V = -v$

$\begin{cases} (1) & mv + MV = mv' + MV' \\ (2) & V' - v' = v - V \longrightarrow V' = v - V + v' \end{cases}$

(1) $mv + MV = mv' + Mv - MV + Mv'$

$mv - Mv - Mv - Mv = (m + M) v'$

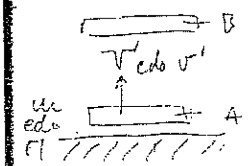
$v' = \dfrac{(m - 3M) v}{m + M} = \dfrac{(m - 3M)\sqrt{2gh}}{m + M}$

(2): $V' = v - V + v' =$

$= 2v + \dfrac{(m - 3M) v}{m + M} = \dfrac{2mv + 2Mv + mv - 3Mv}{m + M} = \dfrac{(3m - M) v}{m + M}$

$V' = \dfrac{(3m - M) v}{m + M} = \dfrac{(3m - M)}{m + M}$

ⓒ igoerak

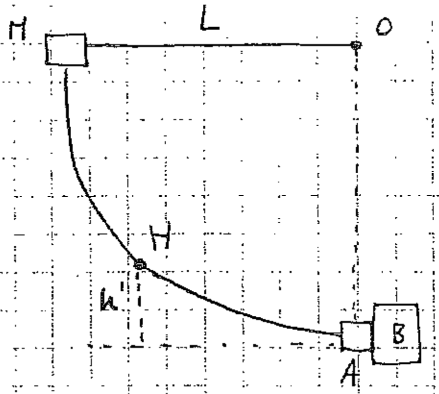


$E_{mA} = E_{mB}$

$\frac{1}{2} \cancel{m} v^2 = \cancel{m} g h_{m/M} \begin{cases} \boxed{h_m = \dfrac{v'^2}{2g} = \dfrac{(m - 3M)^2 \cdot h}{(m + M)^2}} \\ \boxed{h_M = \dfrac{V'^2}{2g} = \dfrac{(3m - M)^2 \cdot h}{(m + M)^2}} \end{cases}$

$\boxed{\begin{array}{l} M = 3m \text{ bada;} \\ h_m = 4 \text{ m} \\ h_M = 0 \text{ m} \end{array}}$

⑨



$M = 20$ kg $L = 1'5$ m $g = 10$ m/s²

$m_B = 25$ kg

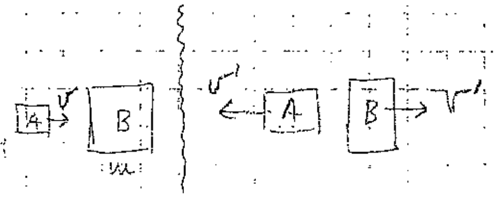
TALKA ELASTIKOA

Energiaren kontserbazio printzipioa

$E_{m_O} = E_{m_A}$

$\cancel{M} g L = \frac{1}{2}\cancel{M} v_A^2$; $v_A = \sqrt{2gL} = v$

TALKAREN EGOERA:



$e = 1$

$V' - v' = v - \cancel{V}^{=0}$

$V' - v' = v \longrightarrow V' = v + v'$

$p = p'$ $\begin{cases} \text{lehen: } p = Mv \\ \text{ondoren: } p' = Mv' + m_B V' \end{cases}$

$Mv = Mv' + m_B V'$

$Mv = Mv' + m_B v + m_B v'$

$v' = \dfrac{(M - m_B)v}{M + m_B} = \dfrac{(M - m_B)\sqrt{2gL}}{M + m_B}$

$MV - m_B V = MV' + m_B V'$

$(M - m_B)V = (M + m_B)V'$

$V' = \dfrac{(M - m_B)V}{M + m_B}$

H-ra igoera

$E_{m_A} = E_{m_H}$

$\frac{1}{2}\cancel{M} v'^2 = \cancel{M} g h' \longrightarrow h' = \dfrac{v'^2}{2g} = \dfrac{(M - m_B)^2 L}{(M + m_B)^2}$

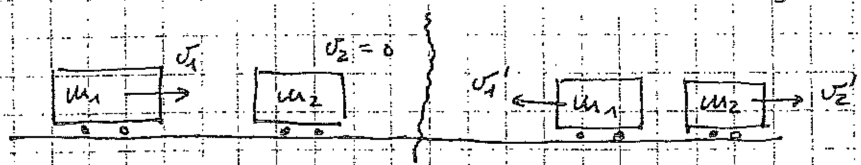
$\boxed{v_A = 5'47 \text{ m/s}}$ $\boxed{h' = 0'0185 \text{ m} = 1'85 \text{ cm}}$ $\boxed{\Delta E_z = -296'3 \text{ J}}$

$\Delta E_z = E_z' - E_z = \frac{1}{2}Mv'^2 - \frac{1}{2}Mv^2 = -296'3$ J

(10)

$m_1 = 2\ kg \quad m_2 = 4\ kg \quad v_1 = 6\ m/s$

$v_1' = -1\ m/s$



a) v_2' ?

TALKA $\longrightarrow$ $p = kte$ $\longrightarrow$ $p = p'$

$p = p'$ lehe: $\left| \begin{array}{l} p = m_1 v_1 + \cancel{m_2 v_2}^{\,0} \\ p' = m_1 v_1' + m_2 v_2' \end{array} \right| \longrightarrow m_1 v_1 = m_1 v_1' + m_2 v_2'$

$$v_2' = \frac{m_1 (v_1 - v_1')}{m_2} \qquad \boxed{v_2' = 3'5\ m/s}$$

b) ΔE_z ?

$$\Delta E_z = E_z' - E_z = E_{z1}' + E_{z2}' - E_{z1} - E_{z2} = \frac{1}{2}\left(m_1 v_1'^2 + m_2 v_2'^2 - m_1 v_1^2 - \cancel{m_2 v_2^2}^{\,0}\right) =$$

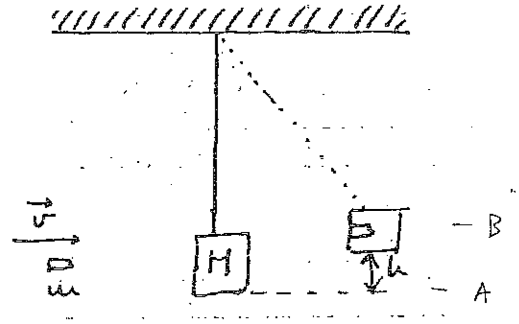
$$= -10'5\ J$$

$\boxed{10'5\ J \text{ galdu dira talkan}}$

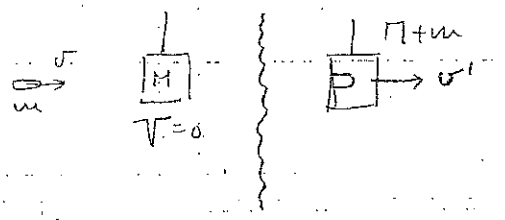
c) e?

$$v_2' - v_1' = e\,(v_1 - \cancel{v_2}^{\,0}) \longrightarrow e = \frac{v_2' - v_1'}{v_1} \qquad \boxed{e = 0'75}$$

11



a)



$$p = p' \left| \begin{array}{l} \text{lehen } p = mv + \cancel{MV}^{0} \\ \text{ondoren } p' = (M+m)v' \end{array} \right.$$

$$mv = (M+m)v'$$

p kte

$$v = \frac{M+m}{m} v' = \frac{M+m}{m}\sqrt{2gh}$$

$$\boxed{v = \frac{M+m}{m}\sqrt{2gh}}$$

$$\rightarrow v' = \frac{m}{M+m} v$$

Energiaren kontserbazio printzipioa

$$E_{m_A} = E_{m_B}$$

$$\frac{1}{2}\cancel{(M+m)}v'^2 = \cancel{(M+m)}gh \longrightarrow v' = \sqrt{2gh}$$

b)

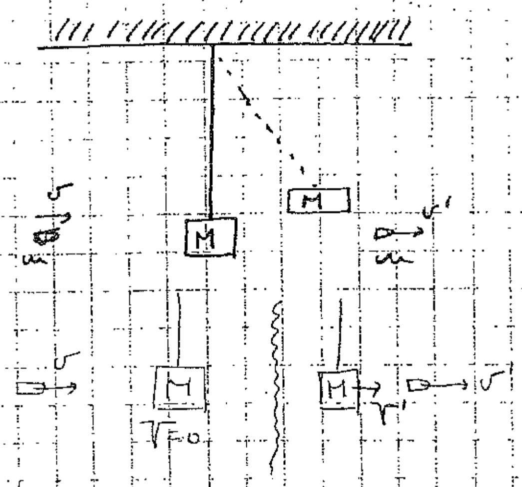
$$E_{z\,lehen} = \frac{1}{2}mv^2$$

$$E_{z\,gero} = \frac{1}{2}(M+m)v'^2 = \frac{1}{2}(M+m)\left(\frac{m}{M+m}v\right)^2 = \frac{1}{2}\frac{m^2}{M+m}v^2 =$$

$$= \frac{1}{2}mv^2 \cdot \frac{m}{M+m} = E_{z\,lehen}\,\frac{m}{M+m}$$

$$\boxed{E_{z\,gero} = E_{z\,lehen} \cdot \frac{m}{M+m}}$$

(12)



$$v' = \frac{v}{2}$$

$$p = p' \begin{cases} \text{lehen} & p = mv + \cancel{MV}^{\,0} \\ \text{ondoren} & p' = mv' + MV' \end{cases}$$

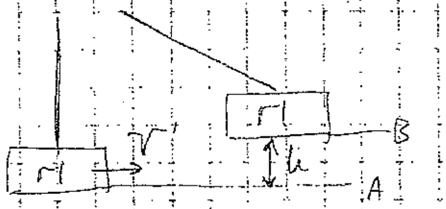
$$mv = mv' + MV'$$

$$mv = m\frac{v}{2} + MV'$$

p kte

Energiaren kontserbazio printzipioa

$$V' = \frac{mv}{2M}$$



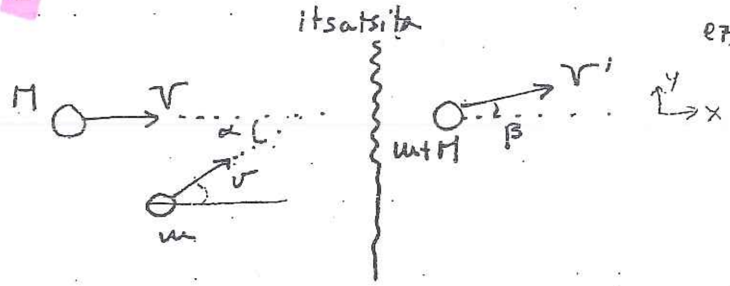
$$E_{m_A} = E_{m_B}$$

$$\frac{1}{2}\cancel{M}V'^2 = \cancel{M}gh$$

$$\frac{1}{2}\cdot\frac{m^2v^2}{4M^2} = gh$$

$$\boxed{h = \frac{m^2v^2}{8M^2g}}$$

13



ezagunak: M, V, m, v

$\alpha, \beta < \pi/2$

$\vec{p}$ kte $\longrightarrow$ $\vec{p} = \vec{p}\,'$ $\begin{cases} \text{lehen} \quad \vec{p} = M\vec{V} + m\vec{v} & \begin{cases} p_x = MV + mv\cos\alpha \\ p_y = mv\sin\alpha \end{cases} \\ \text{ondoren} \quad \vec{p}\,' = (M+m)\vec{V}' & \begin{cases} p'_x = (M+m)V'\cos\beta \\ p'_y = (M+m)V'\sin\beta \end{cases} \end{cases}$

$\vec{p} = \vec{p}\,'$

$\begin{cases} p_x = p'_x \ ; \\ p_y = p'_y \ ; \end{cases} \begin{cases} MV + mv\cos\alpha = (M+m)V'\cos\beta \longrightarrow \cos\beta = \dfrac{MV + mv\cos\alpha}{(M+m)V'} \\ mv\sin\alpha = (M+m)V'\sin\beta \longrightarrow \sin\beta = \dfrac{mv\sin\alpha}{(M+m)V'} \end{cases}$

$$\frac{MV + mv\cos\alpha}{mv\sin\alpha} = \operatorname{cotg}\beta \qquad \operatorname{tg}\beta = \frac{mv\sin\alpha}{MV + mv\cos\alpha}$$

$$\boxed{\beta = \operatorname{arctg}\left(\frac{mv\sin\alpha}{MV + mv\cos\alpha}\right)}$$

$$\cos^2\beta + \sin^2\beta = 1 \longrightarrow \left(\frac{MV + mv\cos\alpha}{(M+m)V'}\right)^2 + \left(\frac{mv\sin\alpha}{(M+m)V'}\right)^2 = 1$$

$$\frac{(MV + mv\cos\alpha)^2 + (mv\sin\alpha)^2}{(M+m)^2 V'^2} = 1$$

$$\frac{M^2V^2 + m^2v^2\cos^2\alpha + 2MV\,mv\cos\alpha + m^2v^2\sin^2\alpha}{(M+m)^2 V'} = 1$$

$$\boxed{\frac{\sqrt{M^2V^2 + m^2v^2 + 2MVmv\cos\alpha}}{M+m} = V'}$$

7. GAIA: PARTIKULAK MULTZOEN DINAMIKA

FISIKA

(14)

$m_1 = 1\ kg \qquad P_1 = (1, 0, -1) \qquad \vec{F}_1 (0, 1, t)$

$m_2 = 2\ kg \qquad P_2 = (0, 1, 1) \qquad \vec{F}_2 (1, t, 2t)$

a) R_{MZ}? $t = 0\ s$

$$\vec{R}_{MZ} = \frac{1(\hat{i} - \hat{k}) + 2(\hat{j} + \hat{k})}{3} = \frac{\hat{i} + 2\hat{j} + \hat{k}}{3} \qquad \boxed{\vec{R}_{MZ} = (1/3,\ 2/3,\ 1/3)\ m}$$

b) R_{MZ}? $t = 2\ s$

$$\vec{F}_{MZ} = \vec{F}_1 + \vec{F}_2 = (1, t, 3t) = m\,\vec{A}_{MZ} = m\frac{d\vec{V}_{MZ}}{dt} = m\frac{d^2\vec{R}_{MZ}}{dt}$$

$$\int_{\vec{V}_0 = 0}^{V} d\vec{V}_{MZ} = \int_{t_0 = 0}^{t} \frac{(\hat{i} + t\hat{j} + 3t\hat{k})}{m}\, dt$$

$$\vec{V}_{MZ} = \frac{\left(t, \frac{t^2}{2}, \frac{3t^2}{2}\right)}{m} \longrightarrow \int_{R_0}^{R} d\vec{R}_{MZ} = \frac{1}{m}\int_{t_0=0}^{t} \left(t, \frac{t^2}{2}, \frac{3t^2}{2}\right) dt$$

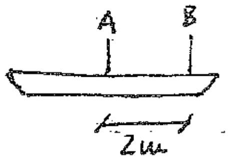
$$\vec{R}_{MZ} - \vec{R}_0 = \frac{\left(\frac{t^2}{2}, \frac{t^3}{6}, \frac{3t^3}{6}\right)}{m}$$

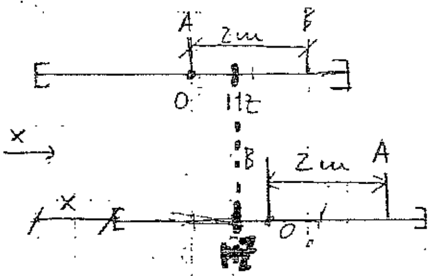
$$\vec{R}_{MZ} = \left(\frac{t^2}{6}, \frac{t^3}{18}, \frac{t^3}{9}\right) + \left(1/3, 2/3, 1/3\right) = \vec{R}_{MZ}(t)$$

$$\vec{R}_{MZ}(t = 2s) = \left(\frac{1}{3} + \frac{1}{2}, \frac{27}{18} + \frac{2}{3}, \frac{8}{9} + 1/3\right)$$

(15)

$m_A = 40 \text{ kg} \quad m_B = 60 \text{ kg} \quad M = 30 \text{ kg}$





$$R_{MZ} = \frac{M\cdot 0 + m_A \cdot 0 + m_B \cdot 2}{130} = 0'923 \text{ m}$$

$$R_{MZ}' = \frac{Mx + m_B x + m_A(2+x)}{130} =$$

$\boxed{R_{MZ} = R_{MZ}'}$

$$\frac{Mx + m_B x + m_A(2+x)}{130} = 0'923$$

$$x(M + m_B + m_A) + 2m_A = 0'923 \cdot 130$$

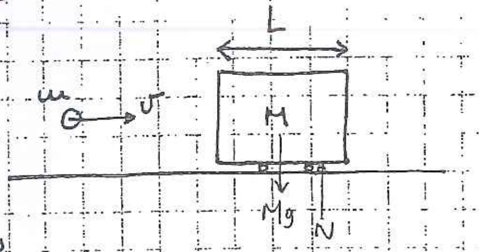
$$x = \frac{0'923 \cdot 130 - 2m_A}{M + m_B + m_A} = 0'31 \text{ m}$$

$\boxed{x = 0'31 \text{ m}}$

P = AkIP

7. GAIA: PARTIKULA MULTZOEN DINAMIKA — Fisika

16



$m = 20\ g = 0'02\ kg$ $\quad L = 20\ cm = 0'2\ m$

$v = 300\ m/s$ $\quad F_R = 578\ N$

$M = 400\ g = 0'4\ kg$

a)

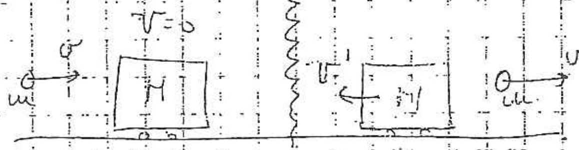
$Mg - N = 0 \longrightarrow$ Bertikalean geldi baitago

$\vec{F}^{kan} = 0 = \dfrac{d\vec{p}}{dt} \longrightarrow \vec{p}$ kte

$\Delta E_z = W^{Bar} + \cancel{W^{kan}} < 0$ $\quad \Delta E_z \neq 0 \longrightarrow E_z \neq$ kte $\quad$ Ez elastikoa

m-ren deskaldura

b)



$p \rightarrow$ kte $\quad p = p' \longrightarrow \; mv = mv' + MV'$

$\Delta E_z = W^{Bar} + \cancel{W^{kan}}^{0} = W_R = -F_R \cdot L$

$E_z' - E_z \qquad \frac{1}{2}mv'^2 + \frac{1}{2}MV'^2 - \frac{1}{2}mv^2 = -F_R \cdot L$

$$\begin{cases} m(v' - v) = -MV' \\ \frac{1}{2}m(v'^2 - v^2) = -F^{Bar} \cdot L - \frac{1}{2}MV'^2 \end{cases}$$

$$\frac{\frac{1}{2}\cancel{m}(v'^2 - v^2)}{\cancel{m}(v' - v)} = \frac{-F^{Bar} \cdot L - \frac{1}{2}MV'^2}{-MV'} \; ; \; \frac{\cancel{(v' - v)}(v' + v)}{2\cancel{(v' - v)}} = \frac{-F^{Bar} \cdot L - \frac{1}{2}MV'^2}{-MV'}$$

$$v' + v = \frac{+2F^{Bar} \cdot L}{+MV'} + \frac{\cancel{M}V'^{\cancel{2}}}{\cancel{M}\cancel{V'}} = \frac{2F^{Bar} \cdot L}{MV'} + V'$$

$$\begin{cases} v' + v = \dfrac{2F^{Bar} \cdot L}{MV'} + V' \\ mv = mv' + MV' \end{cases}$$

$$\begin{cases} mv = mv' + MV' \longrightarrow V' = \dfrac{m(v-v')}{M} \\ v'+v = \dfrac{2F^{Bar}\cdot L}{MV'} + V' \end{cases}$$

$$v'+v = \frac{2F^{Bar}\cdot L}{\cancel{M}\cdot \frac{m(v-v')}{\cancel{M}}} + \frac{m(v-v')}{M} \; ; \; v'+v = \frac{2F^{Bar}\cdot L\cdot M + m^2(v-v')^2}{mM(v-v')}$$

$$Mm(v'+v)(v-v') = 2F^{Bar}LM + m^2(v^2+v'^2-2vv')$$

$$-Mm(v^2-v'^2) = 2F^{Bar}LM + m^2(v^2+v'^2-2vv')$$

$$-Mmv^2 + Mmv'^2 - m^2v^2 - m^2v'^2 + \boxed{m^2 2vv'} = 2F^{Bar}\cdot L\cdot M$$

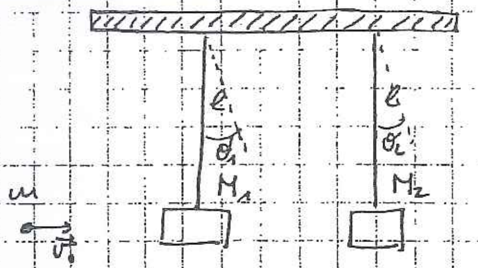
$$(Mm-m^2)v'^2 + 2m^2v\cdot v' - Mmv^2 - 2F^{Bar}\cdot L\cdot M = 0$$

GARAPENA TXARTO DAGO; Baina:

$$v' = \frac{300 \pm \sqrt{300^2 - 1'87}}{2\cdot 1\cdot 10^{-3}} = 280 \ m/s \qquad \boxed{v' = 280 \ m/s}$$

$$\boxed{V' = 1 \ m/s}$$

17



$l = 2\ m$ $\quad \theta_1 = 5'73°$

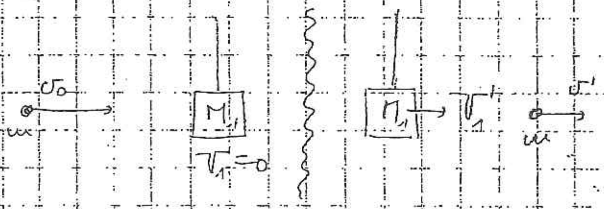
$M_1 = M_2 = M = 10\ kg$ $\quad \theta_2 = 2'87°$

$m = 10\ g = 0'01\ kg$

a) v_0 ?

$F^{kan} = 0 = \frac{dp}{dt} \longrightarrow p$ kte $\longrightarrow p = p' = p''$

$\boxed{p = p'}$



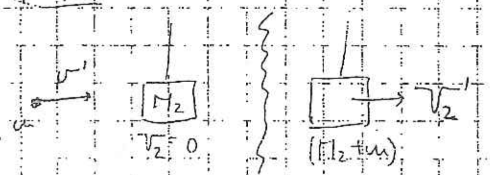
$p = m v_0 + \cancel{M_1 V_1}^{0}$

$p' = M_1 V_1' + m v'$

$m v_0 = M_1 V_1' + m v'$

$v_0 = \frac{M_1 V_1' + m v'}{m}$

$\boxed{p = p''}$



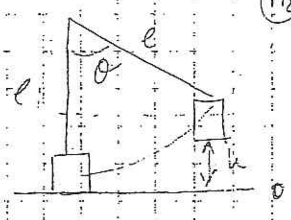
$p = m v' + \cancel{M_2 V_2}^{0}$

$p'' = (M_2 + m) V_2'$

$m v' = (M_2 + m) V_2'$

$v' = \frac{(M_2 + m) V_2'}{m}$

Energiaren kontserbazio printzipioa



$l \cos\theta = l - h$

$h = l - l\cos\theta$

M_2 $\quad E_{mA} = E_{mB}$; $\quad \frac{1}{2}\cancel{(M_2 + m)} V_2'^2 = \cancel{(M_2 + m)} g h$

$\frac{1}{2} V_2'^2 = g(l - l\cos\theta_2)$

$V_2' = \sqrt{2g(l - l\cos\theta_2)} = 0'22\ m/s$

$v' = \frac{(M_2 + m) V_2'}{m} = 225\ m/s$

M_1 $\quad E_{mA} = E_{mB}$ $\quad \frac{1}{2}\cancel{M_1} V_1'^2 = \cancel{M_1} g h$

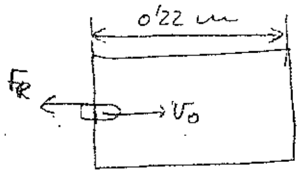
$\frac{1}{2} V_1'^2 = g(l - l\cos\theta_1)$

$V_1' = \sqrt{2g(l + l\cos\theta_1)} = 0'45\ m/s$

$v_0 = \frac{M_1 V_1' + m v'}{m} = 670\ m/s$

$\boxed{v_0 = 670\ m/s}$

b) t? M_1, lodiera 22 cm



$F = m \cdot a$

$F_R = m \cdot a$

sartu $\longrightarrow v_0$

irten $\longrightarrow v'$ $\Big\}$ $\longrightarrow$ dezelerazio konstantea $\longrightarrow$

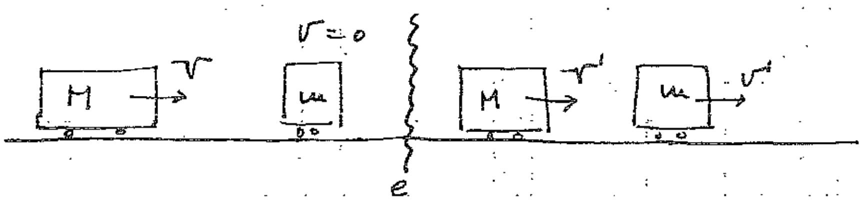
$$\left| \begin{array}{l} v' = v_0 - a t \\ x = v_0 t - \frac{1}{2} a t^2 \\ v'^2 = v_0^2 + 2ax \longrightarrow a = \dfrac{v'^2 - v_0^2}{2x} \end{array} \right.$$

$$t = \frac{v' - v_0}{-a} = 0'0005 \ s$$

$$\boxed{t = 0'5 \ ms}$$

$$a = -905170 \ m/s^2$$

(18)



$F^{kan} = 0 = \frac{dp}{dt} \longrightarrow p$ kte $\qquad V', v'$?

$$p = p' \longrightarrow \left| \begin{array}{l} p = MV + \cancel{m v}^{\,0} \\ p' = MV' + m v' \end{array} \right. \qquad MV = MV' + mv'$$

$$(v' - V') = e(V - \cancel{v}^{\,0}) \longrightarrow v' - V' = eV \longrightarrow \left| \begin{array}{l} v' = eV + V' \\ V' = v' - eV \end{array} \right.$$

$$MV = M(v' - eV) + mv' \longrightarrow MV = Mv' - MeV + mv'$$

$$\boxed{v' = \frac{M(1+e)V}{M+m}}$$

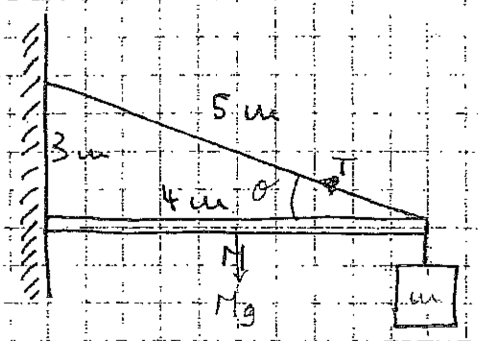
$$MV = MV' + m(eV + V') \longrightarrow MV = MV' + meV + mV'$$

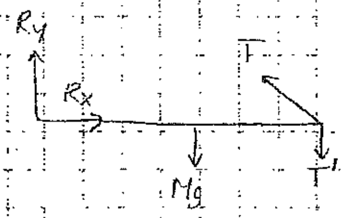
$$\boxed{V' = \frac{(M - me)V}{M+m}}$$

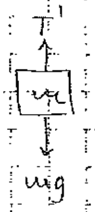
8. GAIA: SOLIDO ZURRUNAREN HIGIDURA — FISIKA

Ariketak

①

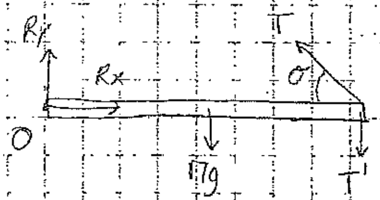






$\vec{F} = \vec{0} \longrightarrow T' - mg = 0 \longrightarrow mg = T'$

$\sum \vec{F} = \vec{0}$



$$\begin{cases} R_y + T\sin\theta - Mg - T' = 0 \\ R_x - T\cos\theta = 0 \end{cases}$$

$\sum \vec{M} = \vec{0}$

$$\cancel{R_y} \cdot 0 + \cancel{R_x} \cdot 0 + Mg \cdot 2 - T\,4\sin\theta + T' \cdot 4 = 0$$

$$2Mg - 4T\sin\theta + 4T' = 0$$

$$2Mg - 4T\sin\theta + 4mg = 0$$

$\sin\theta = \frac{3}{5}$ $\quad \cos\theta = \frac{4}{5}$

$$T = \frac{2Mg + 4mg}{4\sin\theta} = \frac{(M+2m)g}{2\sin\theta}$$

$$\boxed{T = \frac{5}{6}(M+2m)g}$$

- $R_y = T' + Mg - \frac{3T}{5}$

$= mg + Mg - \frac{3}{\cancel{5}} \cdot \frac{\cancel{5}}{6}(M+2m)g =$

$= (m+M)g - \frac{1}{2}(M+2m)g =$

$= \cancel{mg} - \cancel{mg} + Mg - \frac{1}{2}Mg = \frac{1}{2}Mg$

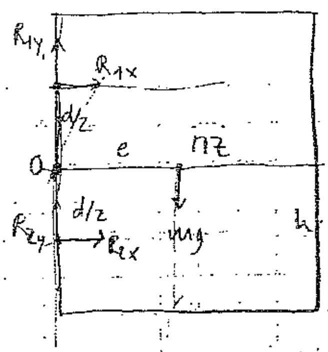
$$\boxed{R_y = \frac{1}{2}Mg}$$

- $R_x = \frac{4}{5}T = \frac{4}{\cancel{5}} \cdot \frac{\cancel{5}}{6}(M+2m)g =$

$= \frac{2}{3}(M+2m)g$

$$\boxed{R_x = \frac{2}{3}(M+2m)g}$$

②



$\Sigma \vec{F} = \vec{0}$

$$\begin{cases} R_{1y} + R_{2y} - mg = 0 \\ R_{1x} + R_{2x} = 0 \longrightarrow R_{1x} = -R_{2x} \end{cases}$$

$\Sigma \vec{M} = 0$

$$R_{1x} \cdot d/2 - R_{2x}\, d/2 + mge = 0$$

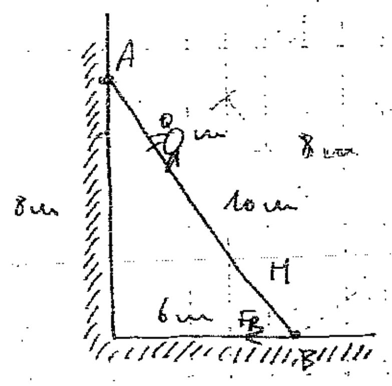
$$-R_{2x}\, d/2 - R_{2x}\, d/2 + mge = 0$$

$$-R_{2x}\, d + mge = 0$$

$$\boxed{R_{2x} = \frac{mge}{d}}$$

(Bertikalak ezin daitezke atera, erlazioa bakarrik)

③

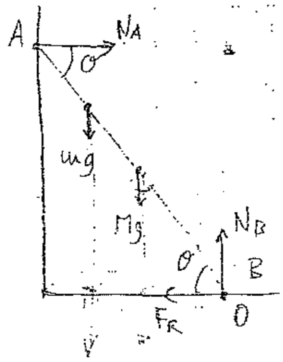


$M = 40$ kg $\qquad g = 10\ m/s^2$

$m = 60$ kg $\rightarrow$ 8 m igo daiteke

$\Sigma \vec{F} = 0 \quad \begin{cases} N_A - F_R = 0 \longrightarrow N_A = F_R \\ N_B - mg - Mg = 0 \end{cases}$

$\Sigma \vec{M} = 0 \quad \{ N_A \cdot 10 \cdot \sin\theta - Mg5\cos\theta - mg8\cos\theta = 0$



$F_R = \mu N_B$

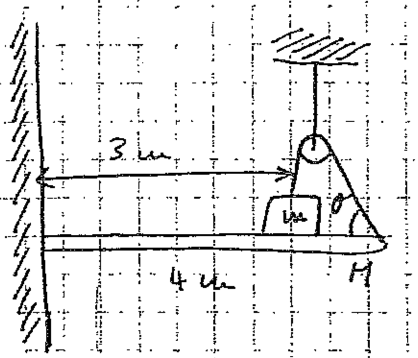
$$N_A = \frac{Mg5\frac{6}{10} + mg8\cdot\frac{6}{10}}{\cancel{10}\cdot\frac{8}{\cancel{10}}} = 510\ N$$

$N_B = mg + Mg = 1000\ N$

$\mu = \frac{F_R}{N_B} = \frac{N_A}{N_B} = 0'51$

$$\boxed{\begin{aligned} N_A &= 510\ N \\ N_B &= 1000\ N \\ \mu &= 0'51 \end{aligned}}$$

④

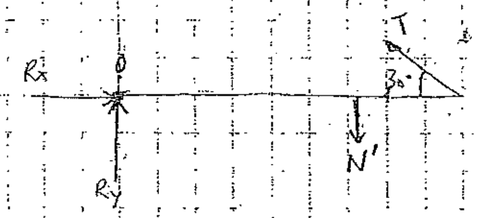


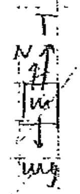
$\theta = 30^\circ$ $M = 0\ kg$

$m = 50\ kg$

$g = 10\ m/s^2$

a)





b)

$$\Sigma \vec{F} = \vec{0} \quad \begin{cases} R_x - T\cos\theta = 0 \\ R_y + T\sin\theta - N = 0 \end{cases}$$

$$\Sigma \vec{F} = 0 \quad \begin{cases} T + N - mg = 0 \\ N = mg - T \end{cases}$$

$$\Sigma \vec{M} = \vec{0} \quad 4T\sin 30^\circ - 3N = 0$$

$$4T\sin 30^\circ = 3N$$

$$4T\sin 30^\circ = 3(mg - T)$$

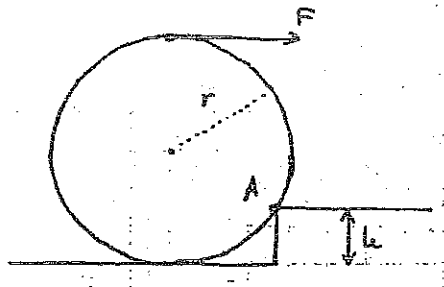
$$4T\sin 30^\circ + 3T = 3mg \qquad T = \frac{3mg}{4\sin 30^\circ + 3} = \boxed{300\ N}$$

$$\boxed{N = 200\ N}$$

$$R_x = T\cos 30^\circ = \boxed{100\sqrt{3}\ N}$$

$$R_y = N - T\sin 30^\circ = 200 - 300 \cdot \tfrac{1}{2} = \boxed{50\ N}$$

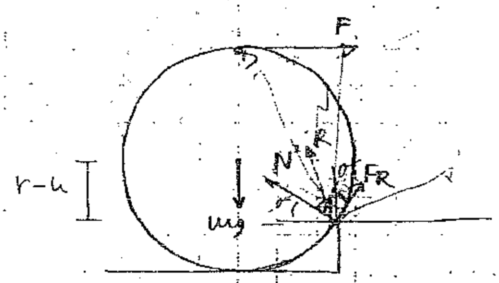
⑤



ez du A-n irrist egiten

$m = 10\ kg \qquad g = 10\ m/s^2$

$h = \frac{r}{2}$

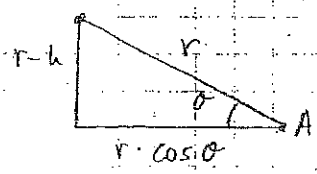


$\sum \vec{F} = \vec{0}$

$$\begin{cases} (1) & N'\cos\theta - F - F_r\sin\theta = 0 \\ (2) & N'\sin\theta + F_r\cos\theta - mg = 0 \end{cases}$$

$\sum \vec{M} = \vec{0}$

$$-mg\,r\cos\theta + F[r + (r-h)] = 0$$



$\sin\theta = \frac{r-h}{r} = \frac{r/2}{r} = 1/2$

$\downarrow$

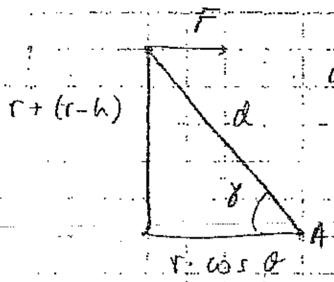
$r - h = r\cdot\sin\theta$

$$-mg\,r\cos\theta + F(r + r\sin\theta) = 0$$

$$-mg\,r\,\frac{\sqrt{3}}{2} + F\,\frac{3}{2} = 0$$

$$F = \frac{mg\sqrt{3}}{\cancel{2}}\cdot\frac{\cancel{2}}{3} = \frac{mg}{\sqrt{3}}$$

$$\boxed{F = \frac{100}{\sqrt{3}}\ N}$$



$d\cdot\sin\gamma = r + (r-h)$

$$\cos\theta = \sqrt{1-\sin^2\theta} = \sqrt{1-\frac{1}{4}} = \frac{\sqrt{3}}{2}$$

$$(2) \longrightarrow F_r = \frac{mg - N'\sin\theta}{\cos\theta} = \frac{100 - 100\cdot\frac{1}{2}}{\sqrt{3}/2} = \frac{100}{\sqrt{3}}$$

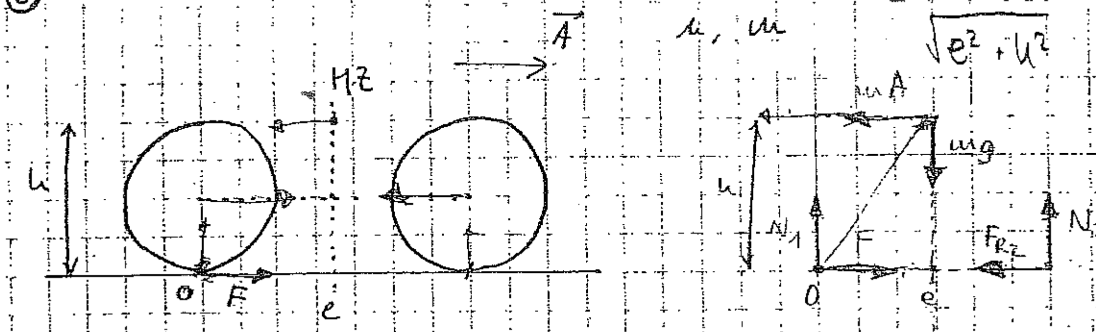
$$\longrightarrow (1) \Rightarrow \quad N'\cos\theta - F - \left(\frac{mg - N'\sin\theta}{\cos\theta}\right)\sin\theta = 0$$

$$N'\cos\theta + N'\frac{\sin^2\theta}{\cos\theta} - F - \frac{mg\sin\theta}{\cos\theta} = 0$$

$$N' = \frac{\frac{mg\sin\theta}{\cos\theta} + F}{\cos\theta + \frac{\sin^2\theta}{\cos\theta}} = \frac{\frac{mg}{\sqrt{3}} + \frac{100}{\sqrt{3}}}{\frac{\sqrt{3}}{2} + \frac{1}{2\sqrt{3}}} = \frac{\frac{200}{\cancel{\sqrt{3}}}}{\frac{4}{2\cancel{\sqrt{3}}}} = 100\ N$$

$$R = \sqrt{N'^2 + F_r^2} = \sqrt{\left(\frac{100}{\sqrt{3}}\right)^2 + 100^2} = \frac{200}{\sqrt{3}}\,N \qquad \boxed{R = \frac{200}{\sqrt{3}}\ N}$$

⑥



$\sum \vec{F} = \vec{0}$ F $\sum F = mA$ (kasu honetan $F_{R_1} = F$ higidura beragatik delako posible, marruskaduragatik.)

(1) $F - F_{R_2} = mA$ ~~$=0$~~

(2) $N_1 + N_2 - mg = 0$

$\sum \vec{M}_0 = \vec{0}$

(3) $mAh + N_2 e - mg\frac{e}{2} = 0$

(3) $N_2 = \dfrac{mg\frac{e}{2} - mAh}{e}$

(2) $N_1 = mg - N_2 = \dfrac{mg\frac{e}{2} + mAh}{e}$

a) ez irristatzeko? $N_2 \neq 0$ $\xrightarrow[\text{Baina}]{}$ $F_{R_2} \ll F$ $F < \mu N_1$

$F - \cancel{F_{R_2}}^{0} = mA$

$\mu N_1 = mA$

$\mu\left(\dfrac{\cancel{m}g\frac{e}{2} + \cancel{m}Ah}{e}\right) = \cancel{m}A$; $\dfrac{\mu g}{2} + \dfrac{\mu Ah}{e} = A$; $\dfrac{\mu g}{2} = A\left(1 - \dfrac{\mu h}{e}\right)$

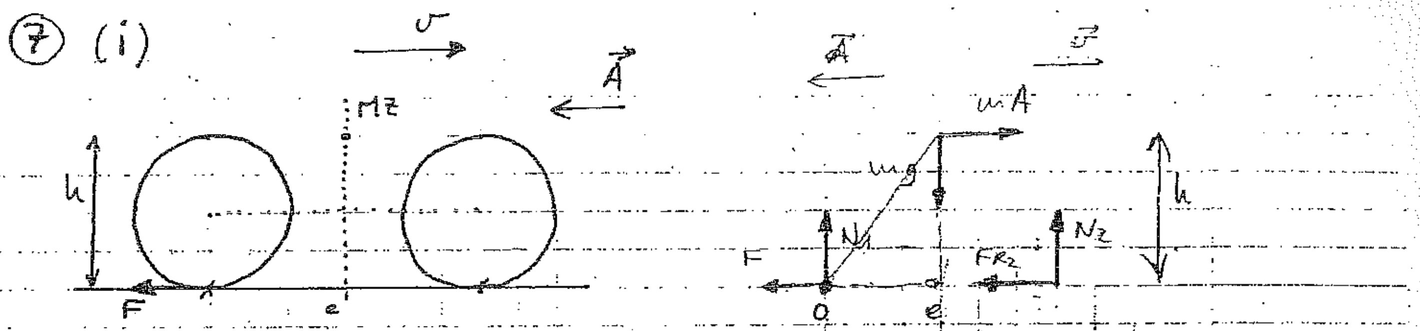
$A = \dfrac{\frac{\mu g}{2}}{\left(1 - \frac{\mu h}{e}\right)} = \dfrac{\mu g e}{2(e - \mu h)} \longrightarrow \boxed{A < \dfrac{\mu g e}{2(e - \mu h)}}$

b) irauli ez dadin? $N_2 = 0$

(3) $mAh + \cancel{N_2 e}^{0} - mg\frac{e}{2} = 0$

$\cancel{m}Ah - \cancel{m}g\frac{e}{2} = 0 \longrightarrow A = \dfrac{eg}{2h} \longrightarrow \boxed{A < \dfrac{eg}{2h}}$

⑦ (i)



$\sum\vec{F}=\vec{0}$

$$\begin{cases} F+F_{R_2}-mA=0 \\ N_1+N_2-mg=0 \end{cases}$$

$$N_2=\frac{mg\frac{e}{2}+mAh}{e}$$

$$N_1=mg-N_1=\frac{mg\frac{e}{2}-mAh}{e}$$

$\sum\vec{M}_0=\vec{0}$

$$\Big\{ -mg\frac{e}{2}-mAh+N_2e=0$$

a) ez irristatzeko $N_2\neq0 \longrightarrow F_{R_2}\ll F$, $F<\mu N_1$

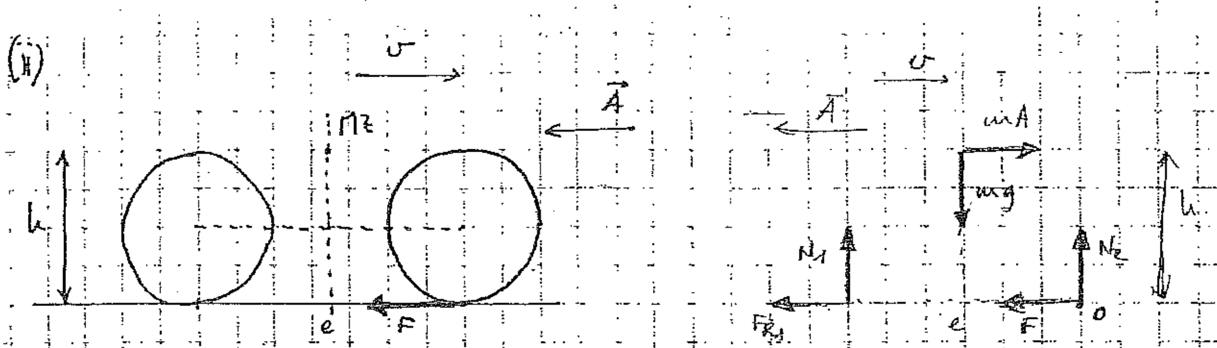
$F+\cancel{F_{R_2}}^{0}=mA$

$\mu N_1=mA \longrightarrow \quad \mu\frac{\cancel{m}g}{2}-\mu\frac{\cancel{m}Ah}{e}=\cancel{m}A; \quad \mu g=\left(1+\frac{\mu h}{e}\right)A$

$$A=\frac{\mu g}{2\left(1+\frac{\mu h}{e}\right)}=\frac{\mu g e}{2(e+\mu h)} \longrightarrow \boxed{A<\frac{\mu g e}{2(e+\mu h)}}$$

b) ez iraultzeko $N_1=0$

(1)



$\sum \vec{F} = \vec{0}$

$$\begin{cases} F_{R_1} + F - mA = 0 \\ N_1 + N_2 - mg = 0 \end{cases}$$

$\sum \vec{\Pi}_o = \vec{0}$

$$N_1 e + mAh - mg\frac{e}{2} = 0$$

$$N_1 = \frac{mg\frac{e}{2} - mAh}{e}$$

$$N_2 = mg - N_1 = \frac{mg\frac{e}{2} + mAh}{e}$$

a) ez irristatzeko $N_1 \neq 0 \longrightarrow F_{R_1} \ll F \qquad F < \mu N_2$

$\cancel{F_R}^{0} + F - mA = 0$

$\mu N_2 = mA \;;\; \mu \frac{\cancel{m}g}{2} + \mu \frac{\cancel{m}Ah}{e} = \cancel{m}A \;;\; \frac{\mu g}{2} = \left(1 - \frac{\mu h}{e}\right) A$

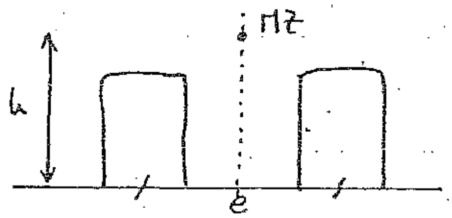
$$A = \frac{\mu g}{2\left(1 - \frac{\mu h}{e}\right)} = \frac{\mu g e}{2(e - \mu h)} \longrightarrow \boxed{A < \frac{\mu g e}{2(e - \mu h)}}$$

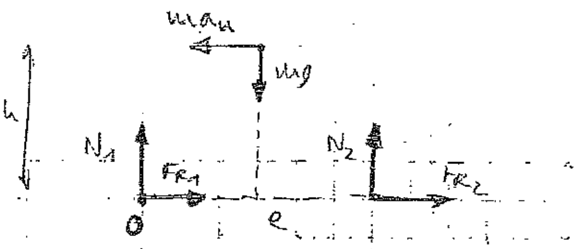
b) ez iraultzeko $N_1 = 0$

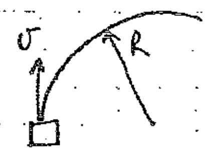
$\cancel{N_1 e}^{0} + mAh - mg\, e/2 = 0 \;;\; \cancel{m}Ah = \cancel{m}g\, e/2$

$$A = \frac{eg}{2h} \longrightarrow \boxed{A < \frac{eg}{2h}}$$

⑧







$\Sigma \vec{F} = \vec{0}$

$$\begin{cases} F_{R_1} + F_{R_2} - ma_n = \vec{0} \\ N_1 + N_2 - mg = 0 \end{cases}$$

$$N_2 = \frac{mg\,e/2 - mah}{e}$$

$$N_1 = mg - N_2 = \frac{mg\,e/2 + mah}{e}$$

$\Sigma \vec{M}_0 = \vec{0}$

$$\Big\} \; mah - mg\,e/2 + N_2 e = 0$$

a) ez labaintzeko $\quad F_{R_1} < \mu N_1 \quad F_{R_2} < \mu N_2$

$$\mu N_1 + \mu N_2 = ma_n$$

$$\mu \frac{\cancel{m} g}{2} + \mu \cancel{\frac{mah}{e}} + \mu \frac{\cancel{m} g}{2} - \mu \cancel{\frac{mah}{e}} = \cancel{m} a_n \; ; \quad \frac{\mu g}{2} + \frac{\mu g}{2} = a_n = \frac{v^2}{R}$$

$$v = \sqrt{\mu g R} \longrightarrow \boxed{v < \sqrt{\mu g R}}$$

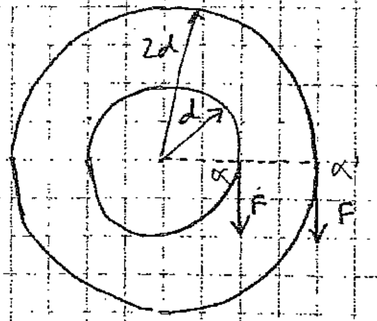
b) ez irauliteko $\quad N_2 = 0$

$$\cancel{m} a_n h - \cancel{m} g\,e/2 + \cancel{N_2}^{\,0} e = 0$$

$$\frac{v^2}{R} h = g\,e/2 \; ; \quad v^2 = \frac{Rge}{2h}$$

$$v = \sqrt{\frac{geR}{2h}} \longrightarrow \boxed{v < \sqrt{\frac{geR}{2h}}}$$

(9)



Diskoa $\longrightarrow$ $I = \frac{1}{2} MR^2$

$d \cdot F \longrightarrow \alpha$

$\Sigma \vec{M} = I_{0z} \cdot \alpha$

$F \cdot d = \frac{1}{2} MR \cdot \alpha \longrightarrow F = \dfrac{MR^2 \alpha}{2d}$

$F \cdot 2d = \frac{1}{2} MR\, \alpha'$

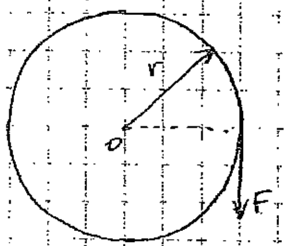
$\dfrac{\cancel{MR^2}\alpha}{\cancel{2d}} \cdot \cancel{2d} = \frac{1}{2} \cancel{MR^2} \alpha'$

$\boxed{\alpha' = 2\alpha}$

(10)

$m = 2'5$ kg $r = 11$ cm $= 0'11$ m

$F = 17$ N Zilindroa: $I_{0z} = \frac{1}{2} m r^2$



a) $\vec{M}_0$?

$M_0 = r \cdot F \cdot \cancel{\sin 90°} = 0'11 \text{ m} \cdot 17 \text{ N} = 1'87$ Nm $\boxed{M_0 = 1'87 \text{ Nm}}$

b) α?

$\Sigma \vec{M} = I_{0z} \cdot \alpha$

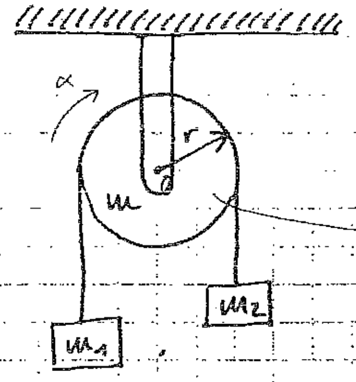
$F r = \frac{1}{2} m r^2 \alpha \longrightarrow \alpha = \dfrac{2F\cancel{r}}{m r^{\cancel{2}}} = \dfrac{2F}{mr}$ $\boxed{\alpha = 124 \text{ rad/s}^2}$

c) $\omega(t = 5s)$?

HZKUA $\longrightarrow$ $\omega = \cancel{\omega_0}^{0} + \alpha(t - \cancel{t_0}^{0})$

$\omega = \alpha t \longrightarrow \boxed{\omega(t=5s) = 618 \text{ rad/s}}$

(11)

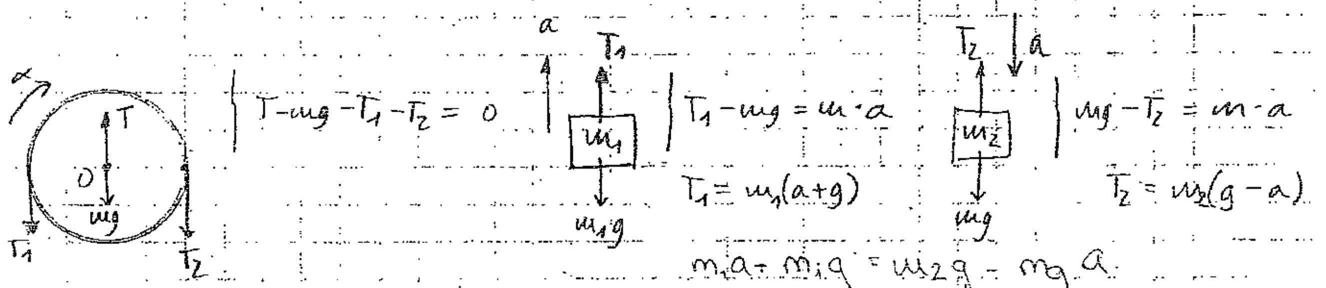


$m = 50\,g = 0'05\,kg \quad r = 4\,cm = 0'04\,m$

$m_1 = 500\,g = 0'5\,kg \qquad g = 9'8\,m/s^2$

$m_2 = 510\,g = 0'51\,kg$

Diskoa $\rightarrow I_{0z} = \frac{1}{2} m r^2$



$T - mg - T_1 - T_2 = 0$

$T_1 - mg = m \cdot a$

$T_1 = m_1(a+g)$

$mg - T_2 = m \cdot a$

$T_2 = m_2(g-a)$

$m_1 a + m_1 g = m_2 g - m_2 a$

~~$m_1 a + m_1$~~ $a = \frac{(m_2 - m_1)g}{(m_1 + m_2)}$

$\sum \vec{M}_0 = I_{0z} \cdot \alpha$

$-T_1 \cancel{r} + T_2 \cancel{r} = I_{0z} \cdot \alpha = \frac{1}{2} m r^{\cancel{2}} \cdot \frac{a}{\cancel{r}} = \frac{1}{2} m a$

$-m_1(a+g) + m_2(g-a) = \frac{1}{2} m a \longrightarrow -m_1 a - m_1 g + m_2 g - m_2 a = \frac{1}{2} m a$

$(m_2 - m_1) g = (\frac{1}{2} m + m_1 + m_2) a \qquad a = \frac{(m_2 - m_1) g}{\frac{1}{2} m + m_1 + m_2}$

$\boxed{a = 0'0947\ m/s^2}$

b)

$T_1 = m_1(a+g) = 4'9473\,N$

$T_2 = m_2(g-a) = 4'9497\,N$

c) $m \simeq 0$

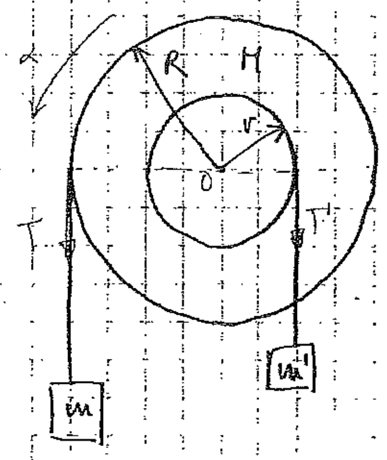
$a = \frac{(m_2 - m_1) g}{\cancel{\frac{1}{2} m} + m_1 + m_2} = \frac{(m_2 - m_1) g}{m_1 + m_2} = 0'097\ m/s^2$

$T_1 = m_1 \left(\frac{(m_2 - m_1) g}{m_1 + m_2} + g \right) = m_1 \frac{-\cancel{m_1 g} + \cancel{m_1 g} + m_2 g + m_2 g}{m_1 + m_2} = \frac{2 m_2 m_1 g}{m_1 + m_2} = \boxed{4'95\ N}$

$T_2 = m_2 \left(g - \frac{(m_2 - m_1) g}{m_1 + m_2} \right) = m_2 \frac{m_1 g + \cancel{m_2 g} - \cancel{m_2 g} + m_1 g}{m_1 + m_2} = \frac{2 m_2 m_1 g}{m_1 + m_2} = \boxed{4'95\ N}$

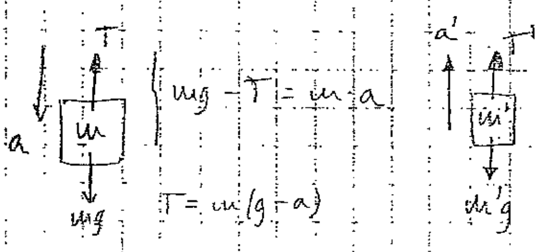
$\boxed{T_1 = T_2}$

(12)



$I = \frac{MR^2}{2}$ $M = 10\ kg$ $m = 5\ kg$ $m' = 3\ kg$

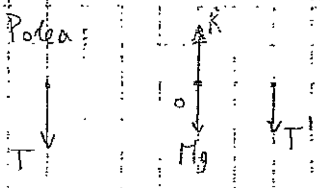
$R = 0'40\ m$ $r = 10\ cm = 0'10\ m$



$mg - T = m\,a$ $\quad T' - m'g = m'a'$

$T = m(g-a)$ $\quad T' = (a'+g)\,m'$

Polea:



$R - T - Mg - T' = 0$

$a = \alpha \cdot R$

$a' = \alpha \cdot r$

$\sum \vec{M}_0 = I_{MZ} \cdot \alpha \quad T \cdot R - T' r = \frac{1}{2} M R^2 \alpha$

$m(g-a)R - (a'+g)m'r = \frac{1}{2} M R^2 \alpha$

$m(g - \alpha R)R - (\alpha r + g)m' r = \frac{1}{2} M R^2 \alpha \ ; \quad (mR - m'r)g = \left(\frac{1}{2} M R^2 + m R^2 + m' r^2\right) \alpha$

$\alpha = \frac{(mR - m'r)\,g}{\frac{1}{2} M R^2 + m R^2 + m' r^2}$ $\quad \alpha = 10'42\ rad/s^2$ $\quad \boxed{a = 4'17\ m/s^2 \quad a' = 1'04\ m/s^2}$

b)

$T = m(g-a)$ $\quad \boxed{T = 29'15\ N}$

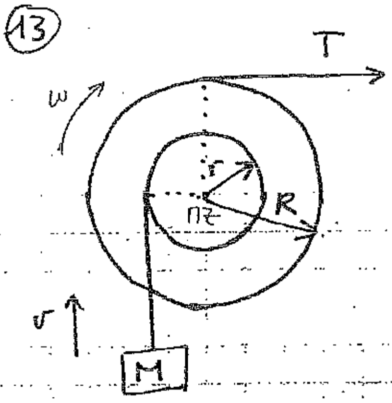
$T' = m'(a'+g)$ $\quad \boxed{T' = 33'12\ N}$

c) HZUA $(t = 5\ s)$ v?

$m:$ $v = \cancel{v_0}^{0} + a\,t$ $\quad \boxed{v = 20'85\ m/s}$

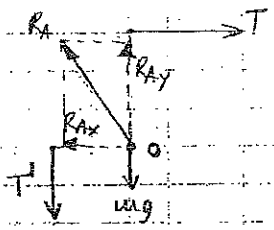
$m':$ $v' = v_0^{?} + a' t$ $\quad \boxed{v' = 5'2\ m/s}$

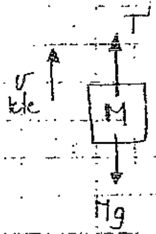
(13)



$M = 100\ kg \quad m = 10\ kg \quad I = \frac{mR^2}{2} \quad g = 10\ m/s^2$

$R = 0'5\ m \quad r = 0'2\ m \quad v = 1'5\ m/s$





$T' - Mg = 0$

$T' = Mg \quad T' = 1000\ N.$

$\Sigma \vec{F} = \vec{0} \qquad\qquad \Sigma \vec{M}_o = \vec{0}$

$\begin{cases} T - R_{Ax} = 0 \\ R_{Ay} - mg - T' = 0 \end{cases} \qquad \{ T'r - TR = 0$

a) T? R_A?

$T = \frac{T'r}{R} = \frac{1000\ N \cdot 0'2\ \cancel{m}}{0'5\ \cancel{m}} = 400\ N \qquad \boxed{T = 400\ N}$

$R_A = \sqrt{R_{Ax}^2 + R_{Ay}^2} \quad \begin{cases} \boxed{R_{Ax} = T} \\ R_{Ay} = T' + mg = Mg + mg = \boxed{(M+m)g} \end{cases}$

b) $P = \frac{dW}{dt} = \frac{d(Fr)}{dt} = F\frac{dr}{dt} = F \cdot v$

T': $P = T' \cdot v = 1000\ N \cdot 1'5\ m/s = 1500$ Watt

$v = \omega \cdot r \quad ; \quad v' = \omega \cdot R$

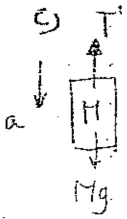
$\omega = \frac{v}{r} \longrightarrow v' = \frac{vR}{r}$

T: $P' = T \cdot v' = T \cdot \frac{vR}{r} = 400\ N \cdot \frac{1'5\ m/s \cdot 0'5\ \cancel{m}}{0'2\ \cancel{m}} =$

$= 1500$ Watt

$\boxed{P = P' = \text{Potencia} = 1500 \text{ watt}}$

c)



$Mg - T' = M \cdot a \qquad\qquad \Sigma \vec{M}_o = I_m \cdot \alpha$

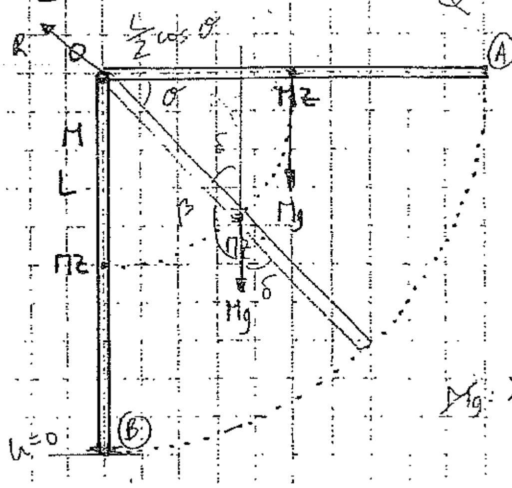
$T' = M(g - a) \qquad\qquad \{ T'r = \frac{mR^2}{2} \cdot \alpha = \frac{mR^2}{2} \cdot \frac{a}{r}$

$M(g-a) \cdot r = \frac{mR^2 a}{2r} \ ; \quad Mgr^2 - Mar^2 = \frac{mR^2}{2} a$

$Mgr^2 = \left(\frac{mR^2}{2} + Mr^2\right) a \qquad a = \frac{Mgr^2}{\frac{mR^2}{2} + Mr^2} = 7'62\ m/s^2$

$\boxed{a = 7'62\ m/s^2}$

(14)



$L = 1m \quad M = 1kg \quad g = 10\ m/s^2$

$I_{Mz} = \frac{1}{12} ML^2$

→ Steiner; $I_0 = I_{m} + Md^2$

$\sum \vec{M}_0 = I_0 \cdot \vec{\alpha}$

$Mg\frac{L}{2} = (I_{Mz} + Md^2)\alpha = \left(\frac{1}{12}ML^2 + M\frac{L^2}{4}\right)\alpha$

$Mg\frac{L}{2} = \frac{L}{2}\left(\frac{1}{6}ML + ML\frac{1}{2}\right)\alpha, \quad g = \frac{2L}{3}\alpha$

a) α?

$\alpha = \frac{3g}{2L} = 15\ rad/s^2$ $\quad \boxed{\alpha = 15\ rad/s^2}$

$v = \omega r$

b) v?

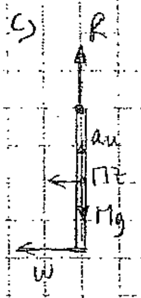
$E_{mA} = E_{mB} \longrightarrow mgL = mg\frac{L}{2} + \frac{1}{2}I_{Mz}\omega^2 + \frac{1}{2}m_{Mz}v_{Mz}^2$ $\quad (I_{Mz} = \frac{1}{12}ML^2)$

$v = \omega\cdot\frac{L}{2}$ $\quad gL = g\frac{L}{2} + \frac{L}{24}L^2\omega^2 + \frac{1}{2}\left(\omega\cdot\frac{L}{2}\right)^2; \quad g/2 = \left(\frac{1}{24}L + \frac{1}{8}L\right)\omega^2$

$\frac{g}{2} = \left(\frac{1}{24} + \frac{1}{8}\right)L\omega^2 \qquad \omega = \sqrt{\frac{6g}{2L}} = \sqrt{\frac{3g}{L}}$ $\quad$ Mutur askea: $v = \omega \cdot L$

$\omega = \omega_0 + \quad a = r\alpha$ $\qquad \boxed{v = \sqrt{3gL} = 5'4\ m/s}$

c)



$R - Mg = M a_n = M\omega^2\frac{L}{2}$

$R = M\left(\omega^2\frac{L}{2} + g\right) = M\left(\frac{3g}{L}\frac{L}{2} + g\right) = \frac{5gM}{2} = \boxed{25\ N}$

a) Bis

$M_0 = I_0\,\alpha \longrightarrow mg\frac{L}{2}\sin\beta = I_0\,\alpha \longrightarrow mg\frac{L}{2}\cos\theta = I_0\,\alpha$ $\quad (\sin\beta = \cos\theta)$

$\alpha = \frac{mg\frac{L}{2}\cos\theta}{\frac{1}{3}mL^2} = \frac{3g}{2L}\cos\theta$

(azelerazioa gutxitzen doa eta behean $\alpha = 0$ da)

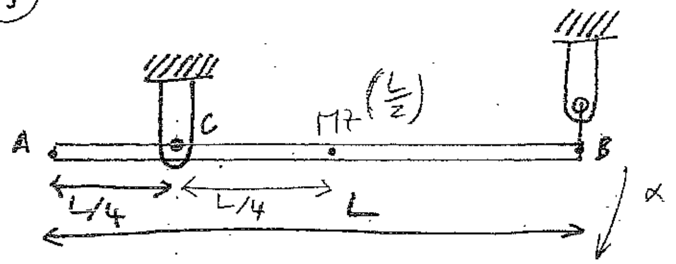
b) Bis

$E_{mA} = E_{mB}$

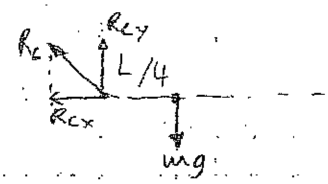
$mgL = mg\frac{L}{2} + \frac{1}{2}m v_{Mz}^2 + \frac{1}{2}I_{Mz}\omega^2$

$mgL = mg\frac{L}{2} + \underbrace{\frac{1}{2}m\left(\omega\cdot\frac{L}{2}\right)^2 + \frac{1}{2}\cdot\frac{1}{12}mL^2\omega^2}_{\frac{1}{2}\cdot\frac{1}{3}mL^2\omega^2}$ ← Hemen Steiner ez

15



L, m,



$\sum \vec{M}_C = I_C \cdot \alpha$

$\frac{L}{4} \cancel{m} g = \left(\frac{1}{12} \cancel{m} L^2 + \cancel{m}\left(\frac{L}{4}\right)^2\right) \cdot \alpha$

$\frac{\cancel{L}}{4} g = \frac{\cancel{L}}{4}\left(\frac{L}{3} + \frac{L}{4}\right) \alpha$

$g = \frac{7L}{12} \alpha$

$\alpha = \frac{12g}{7L} = \frac{a}{L/4}$

$a_{MZ} = \frac{\cancel{L}\, 12g}{7 \cdot 4 \cancel{L}} = \frac{3 \cancel{L} g}{7 \cancel{L}}$

$a_{MZ} = \frac{3g}{7} \downarrow$

MZ: $\begin{cases} mg - R_{cy} = m \cdot a \\ R_{cx} = 0 \end{cases}$

$mg - R_{cy} = ma$

$R_{cy} = (g - a)\, m$

$R_{cy} = \left(g - \frac{3g}{7}\right) m = \left(\frac{4g}{7}\right) m$

$R = \sqrt{R_{cy}^2 + R_{cx}^2} = \frac{4}{7} mg$

$\boxed{R = \frac{4}{7} mg}$

C-tik distantzia berdinera daudenez

$a_A = a_{MZ}$

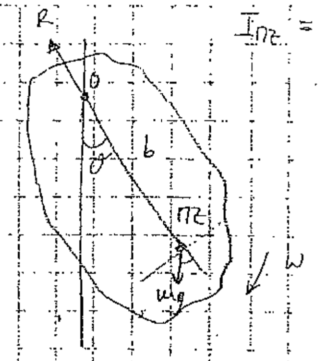
$\boxed{a_A = \frac{3g}{7}}$

8. GAIA: SOLIDO ZURRUNAREN HIGIDURA

Fisika

(16)

a)



$I_{PZ} = I$ m

$\sum \vec{M}_0 = I_0 \alpha$

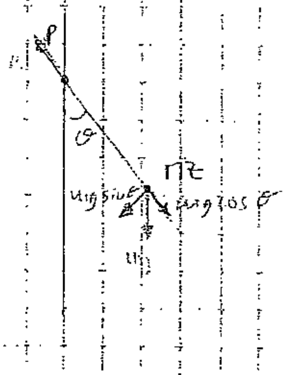
$-mgb \underset{\theta}{\underset{\approx}{\sin\theta}} = I_0 \alpha$

(Angelu txikia dela suposatuz)

$-mgb\,\theta = I_0 \ddot{\theta}$

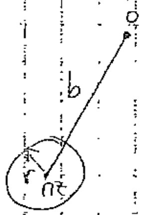
$\ddot{\theta} + \frac{mgb}{I_0}\theta = 0$

$\omega^2 = \sqrt{\frac{mgb}{I_0}}$



$$\tau = \frac{2\pi}{\omega} = \boxed{2\pi\sqrt{\frac{I_0}{mgb}}}$$

b) Diskoa

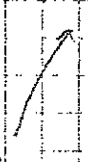


$I_0 = I_{PZ} + mb^2$ (STEINER) $I_{PZ} = \frac{1}{2}mr^2$

$$\tau = 2\pi\sqrt{\frac{1/2\,\cancel{m}r^2 + \cancel{m}b^2}{\cancel{m}gb}} = 2\pi\sqrt{\frac{r^2+2b^2}{2gb}}$$

Pendulu sinplea

$\tau_0 = 2\pi\sqrt{\frac{b}{g}}$



Pendulu sinplearen periodoa:

$\tau = 2\pi\sqrt{\frac{L}{g}}$

$$\frac{\tau}{\tau_0} = \frac{\cancel{2\pi}\sqrt{\frac{r^2+2b^2}{2gb}}}{\cancel{2\pi}\sqrt{\frac{b}{g}}} = \sqrt{\frac{(r^2+2b^2)\cancel{g}}{2\cancel{g}b\cdot b}} =$$

$$= \sqrt{\frac{r^2+2b^2}{2b^2}} = \sqrt{1+\frac{r^2}{2b^2}}$$

$$\boxed{\frac{\tau}{\tau_0} = \sqrt{1+\frac{r^2}{2b^2}}}$$

(17)

$= 6400.000\ m = 6'4 \cdot 10^{6}\ m$

$M_L = 6'0 \cdot 10^{24}\ kg$ $\quad R = 6400\ km$ $\quad R_{orbita} = 1'5 \cdot 10^{11}\ m$ $\quad M_E = 2 \cdot 10^{30}\ kg$

$$\tau = \frac{2\pi}{\omega} \longrightarrow \omega = \frac{2\pi}{\tau} \longrightarrow v = \frac{2\pi R_{orb}}{\tau}$$

$$\omega = \frac{v}{R}$$

$\tau = 365$ egun $= 3153600\ s$

$v = 29885'77\ m/s$

Translaziotik $E_T = \frac{1}{2} M_L v^2 = \boxed{2'68 \cdot 10^{33}\ J}$

$$\tau = \frac{2\pi}{\omega} \longrightarrow \omega = \frac{2\pi}{\tau}$$

$\tau = 1$ egun $= 86400\ s$

$\omega = 7'2722 \cdot 10^{-5}\ rad/s$

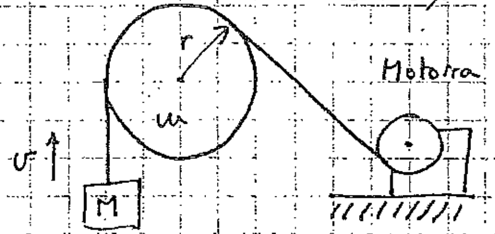
Errotaziotik $E_Z = \frac{1}{2} I_{rz} \omega^2 = \frac{1}{5} M_L R^2 \omega^2 = \boxed{2'6 \cdot 10^{29}\ J}$

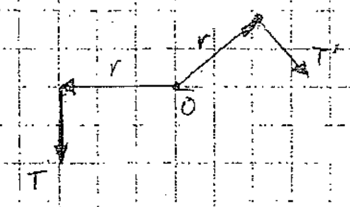
Esfera $I_{rz} = \frac{2}{5} M R^2$

(18)

$r = 30$ cm $= 0'3$ m $M = 2000$ kg $v = 8$ cm/s $= 0'08$ m/s

$m = \emptyset$ $g = 10$ m/s²

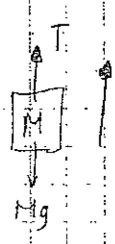




$$\sum \vec{M}_O = \cancel{I_{zz}}^{\,0} \cdot \alpha = 0$$

$$T'r - Tr = 0$$

$$T' = T$$



v kte $\Big\}$ $T - Mg = 0$

$T = Mg$

a)

$T' = T \longrightarrow$ kablearen tentsioa

$T' = T = Mg = 20000$ N $\boxed{T = 20000 \text{ N}}$

b)

T-ren momentua $M = r \cdot T \cdot \sin 90° = r\,T = 6000$ Nm

$\boxed{M = 6000 \text{ Nm}}$

c)

$v = 0'08$ m/s $\longrightarrow$ $\omega \cdot r = v$

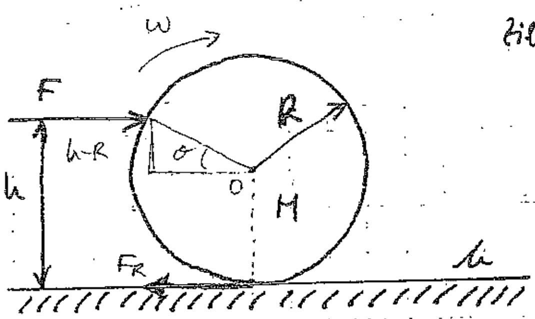
$\omega = \frac{v}{r} = 0'27$ rad/s $\boxed{\omega = 0'27 \text{ rad/s}}$

d)

$P = \frac{dW}{dt} = \frac{d(F \cdot r)}{dt} = F \cdot \frac{dr}{dt} = F \cdot v$ non $F = T = T'$

$P = T \cdot v = 1600$ Watt $\boxed{P = 1600 \text{ Watt}}$

(19)



Zilindroa : $I_{Oz} = \frac{1}{2} M R^2$

$I_O = I_{Oz}$

$\Sigma \vec{F} = m \cdot \vec{a}$

$$\begin{cases} F - F_R = M \cdot a \\ N - Mg = 0 \end{cases}$$

$$a = \frac{F - F_R}{M}$$

$\Sigma \vec{M}_O = I_O \cdot \alpha$

$$F(h-R) + F_R \cdot R = \frac{1}{2} M R^2 \cdot \alpha$$

$$F(h-R) + F_R \cdot R = \frac{1}{2} \cancel{M} R^{\cancel{2}} \frac{\left(\frac{F - F_R}{\cancel{M}}\right)}{\cancel{R}}$$

$$F(h-R) + F_R \cdot R = \frac{1}{2} R (F - F_R)$$

$$2Fh - 2FR + 2F_R R = FR - F_R \cdot R$$

$$(2R + R) F_R = (R + 2R - 2h) F$$

$$3R F_R = (3R - 2h) F$$

$$F_R = \left(1 - \frac{2h}{3R}\right) F$$

a) $h < \frac{3}{2} R$

$$F_R < \left(1 - \frac{\cancel{2} \frac{3}{\cancel{2}} R}{\cancel{3R}}\right) F \qquad F_R < 0$$

b) $h > \frac{3}{2} R$

$$F_R > \left(1 - \frac{\cancel{2} \frac{3}{\cancel{2}} R}{\cancel{3R}}\right) F \qquad F_R > 0$$

c) $h = \frac{3}{2} R$

$$F_R = \left(1 - \frac{2 \cdot \frac{3}{2} R}{3R}\right) F \qquad F_R = 0$$

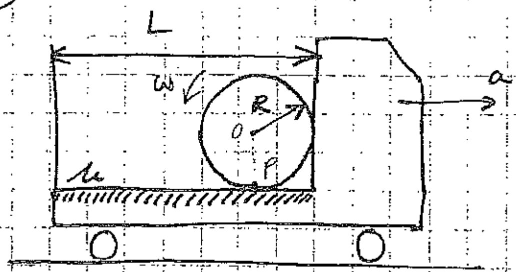
Berdin hiru kasuetan

$$\boxed{F_R = \left(1 - \frac{2h}{3R}\right) F}$$

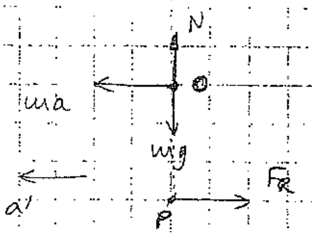
8. GAIA: SOLIDO ZURRUNAREN HIGIDURA

FISIKA

(30)



$I_{MZ} = \frac{1}{2} MR^2$



$\sum \vec{F} = m \cdot \vec{a}$

$\begin{cases} ma - F_R = ma' \\ N - mg = 0 \end{cases}$

$a' = \alpha \cdot R \qquad N = mg$

$\sum \vec{M}_p = I_p \cdot \vec{\alpha}$

$\{ \; ma \cdot R = (I_{MZ} + mR^2)\,\alpha$

$\cancel{m} a \cdot \cancel{R} = \frac{3}{2} \cancel{m} R^{\cancel{2}} \alpha$

$a = \frac{3}{2} R \alpha$

$a = \frac{3}{2} \cancel{R} \frac{a'}{\cancel{R}}$

$a = \frac{3}{2} a'$

a) ez irristatuz

$a' = \frac{2}{3} a \qquad$ HZUA $\longrightarrow \quad L = R + \frac{1}{2} a' t^2 \,, \quad L - R = \frac{1}{3} a t^2$

$t = \sqrt{\frac{3(L-R)}{a}}$

$X = \frac{1}{2} a t^2 = \frac{1}{2} \cancel{a} \frac{3(L-R)}{\cancel{a}} = \frac{3(L-R)}{2}$

$\boxed{X = \frac{3(L-R)}{2}}$

b) irristatuz

Ezin daiteke P puntua erreferentziabat hartu, baina $F_R = \mu N$

$\rightarrow ma - \mu N = m \cdot a' \,, \quad \cancel{m} a - \mu \cancel{m} g = \cancel{m} a'$

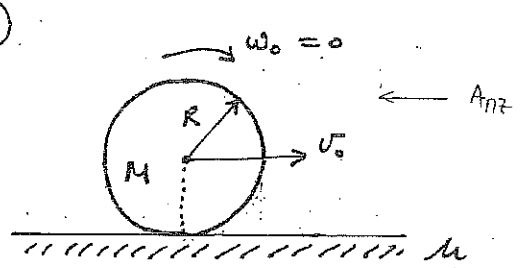
$a' = a - \mu g$

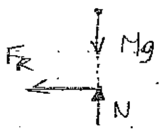
$L = R + \frac{1}{2} a' t^2 \longrightarrow t = \sqrt{\frac{2(L-R)}{a - \mu g}}$

$X' = \frac{1}{2} a t^2 = \frac{1}{2} a \left(\frac{2(L-R)}{a - \mu g} \right)$

$\boxed{X' = \frac{a(L-R)}{a - \mu g}}$

(21)





a) $\Sigma \vec{F} = M \cdot \vec{A_{nz}}$

$$\begin{cases} N - mg = 0 \qquad N = mg \\ F_R = M \cdot A_{nz} \rightarrow \mu N = M \cdot A_{nz} \longrightarrow \mu \cancel{M} g = \cancel{M} A_{nz} \end{cases}$$

$$A_{nz} = \mu g$$

$\Sigma \vec{M}_{nz} = I_{nz} \cdot \vec{\alpha}$

$$\left\{ F_R \cdot R = I_{nz} \cdot \alpha \right.$$

$$\mu \cancel{M} g \cancel{R} = 1/2 \cancel{M} R^{\cancel{2}} \alpha \longrightarrow \alpha = \frac{2\mu g}{R}$$

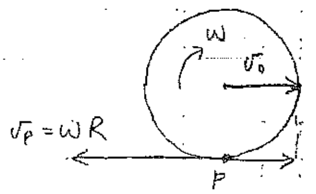
IRRISTATZEN

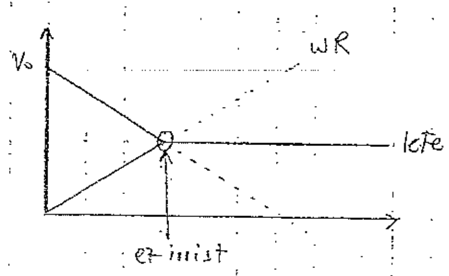
$F_R = \mu N = \mu M g$

$$V_{nz} = V_0 - \cancel{\mu g} \frac{v_0}{3\cancel{\mu g}} = \frac{2 v_0}{3}$$

$$\text{HZUA:} \begin{cases} V_{nz} = V_{0nz} - A_{nz} t \\ \omega = \cancel{\omega_0} + \alpha t \end{cases} \rightarrow \begin{cases} V_{nz} = V_0 - \mu g t \\ \omega = \frac{2\mu g}{R} t \longrightarrow \omega = \frac{2\cancel{\mu g}}{R} \cdot \frac{v_0}{3\cancel{\mu g}} = \frac{2 v_0}{3 R} \end{cases}$$

$v_p = V_{nz} - \omega R$





$V_p = 0 \longrightarrow V_{nz} = \omega R$

$$V_0 - \mu g t = \frac{2\mu g t}{\cancel{R}} \cdot \cancel{R} \longrightarrow v_0 = 3\mu g t \qquad t = \frac{v_0}{3\mu g}$$

$$x = \cancel{x_0}^{0} + v_0 t - \frac{1}{2} A_{nz} t^2 \longrightarrow x = v_0 \frac{v_0}{3\mu g} - 1/2 \mu g \left(\frac{v_0}{3\mu g}\right)^2$$

$$\longrightarrow x = \frac{v_0^2 \, 2 \cdot 3 \cancel{\mu g} - \cancel{\mu g} v_0^2}{(3\mu g)^2 \cdot 2} = \frac{v_0^2 (2 \cdot 3 - 1)}{18 \mu g} = \frac{5 v_0^2}{18 \mu g}$$

$$\boxed{X = \frac{5 v_0^2}{18 \mu g}}$$

b) $W_R = \Delta E_z = \frac{1}{2} I_{nz} \omega^2 + 1/2 m v_{nz}^2 - 1/2 m v_0^2 =$

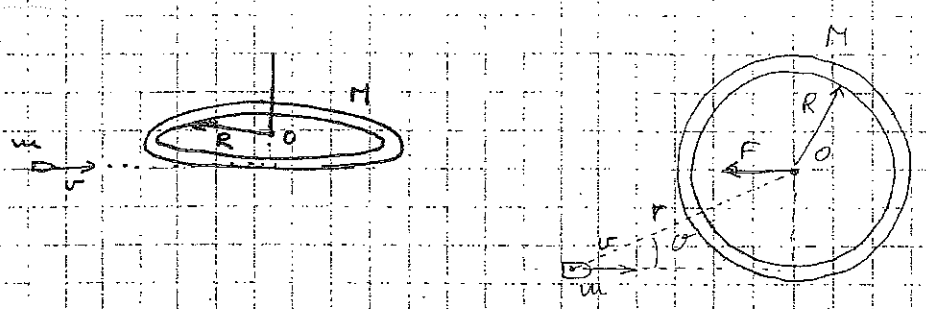
$$= \frac{1}{2} M \left(\frac{2 v_0}{3}\right)^2 + \frac{1}{\cancel{2}} M \cancel{R^2} \frac{1}{\cancel{2}} \left(\frac{2 v_0}{3 \cancel{R}}\right)^2 - 1/2 M v_0^2 = 1/2 M \left(\frac{4 v_0^2}{9} + \frac{2 v_0^2}{9} - v_0^2\right) =$$

$$= \boxed{-1/6 M v_0^2}$$

8. GAIA: SOLIDO ZURRUNAREN HIGIDURA

FISIKA

(22)



F: balarari eusteko ardatzak egiten duen indarra

$V_0 = 0$

$\frac{d\vec{p}}{dt} = \vec{F} \neq 0$ Ardatz bati lotuta badago, ez da kontserbatuko, balarari eusteko, ardatzak indar bat egingo baitu.

$\Delta E_z \neq 0$

$\frac{d\vec{L_0}}{dt} = \vec{M_0} = 0$ Indarra egon badago, baina lerrea zuzenarena O puntuaretik zuzena da.

L_0 kte

partikula (bala) $\vec{L_0} = \vec{r} \times \vec{p}$; $L_0 = r \cdot mv \cdot \sin\theta = R\,mv$

solidoa (eraztuna) $L_0 = I_0 \cdot \omega$

$L_{lehen} = R\,mv$

$L_{ondoren} = I \cdot \omega = (I_b + I_G)\,\omega = (mR^2 + MR^2)\,\omega$

$I_b = \int \rho^2 dm = \rho^2 m = R^2 m$

ρ kte

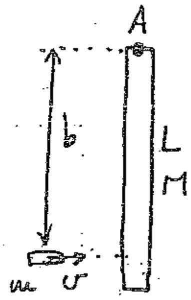
$L_{lehen} = L_{ondoren}$

$Rmv = (mR^2 + MR^2)\,\omega$

$\omega = \frac{mv}{mR + MR} = 1'74$ rad/s

$\boxed{\omega = 1'74 \text{ rad/s}}$

23

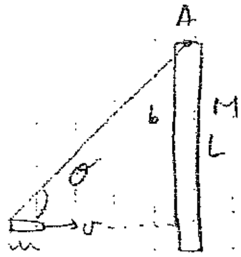


$\frac{d\vec{p}}{dt} = \vec{F} \neq 0$ Pip-teak, balatoari eusteko indar bat aplikatu beharko duelako

$\Delta E_T \neq 0$

$\frac{d\vec{L}_A}{dt} = \vec{M}_A = 0$ Indarra egon arren, A-puntuarekiko, bere momentua nulua da

a)



$L_A(bala) = r \cdot mv \cdot \sin\theta = bmv$

$L_A(solidoa) = I\omega$

$I_b = \frac{1}{12}ML^2$

$$\begin{cases} L_{lehen} = mvb \qquad v' = \omega b \\ L_{ondoren} = mv'b + \left[I_{Mz} + M\left(\frac{L}{2}\right)^2\right]\omega = m\omega b^2 + \left[\frac{1}{12} + \frac{1}{4}\right]ML^2\omega = \end{cases}$$

$$= \left(mb^2 + \frac{1}{3}ML^2\right)\omega$$

b) $L = kte, \longrightarrow L_{lehen} = L_{ondoren}$

$$mvb = \left(mb^2 + \frac{1}{3}ML^2\right)\omega, \quad \boxed{\omega = \frac{mvb}{mb^2 + \frac{1}{3}ML^2}}$$

c) p, p'?

$$\begin{cases} p = mv \\ p' = m\omega b + M\omega \cdot \frac{L}{2} = \left(mb + M\cdot\frac{L}{2}\right)\omega \end{cases}$$

d) b?

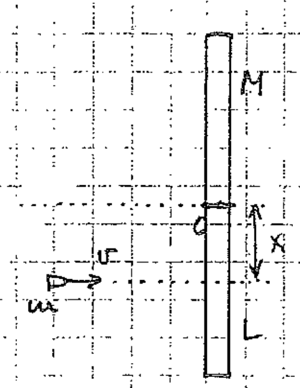
$$p = p' \; ; \quad \cancel{mv} = \left(mb + M\frac{L}{2}\right)\frac{\cancel{mv}b}{mb^2 + \frac{1}{3}ML^2} \; ; \quad \cancel{mb^2} + \frac{1}{3}\cancel{M}L^{\cancel{2}} = \cancel{mb^2} + \cancel{M}\frac{\cancel{L}}{2}b$$

$$\frac{1}{3}L = \frac{b}{2} \longrightarrow \boxed{b = \frac{2L}{3}}$$

(24)

$I_{m} = \frac{1}{12} M L^2$

Talka ondoren $m <<< M$



a)

Ardatzik ez dagoenez, $\frac{dp}{dt} = F = 0$; eta beraz, $p = p'$

$p = mv$

$p' = (\cancel{m}^0 + M) v'$

$mv = Mv' \longrightarrow \boxed{v' = \frac{mv}{M}}$

b)

$L_{aurretik} = mvx$

$L_{ondoren} = I_{m}\omega = \frac{1}{12} M L^2 \omega$

$p = ktk$ denez, $L = ktk$

$L_{aurretik} = L_{ondoren}$

$mvx = \frac{1}{12} M L^2 \omega$

$\boxed{\omega = \frac{12 mvx}{ML^2}}$

c) $x = 0$ $x = \frac{L}{6}$ $x = \frac{L}{2}$

$\boxed{x = 0}$

$\omega = 0$ rad/s $\longrightarrow$ ez du abiadura angeluarrik edukiko

$\boxed{x = \frac{L}{6}}$

$\omega = \frac{12 mv \frac{L}{6}}{ML^2} = \frac{2mv}{ML}$

$\omega = 0$ duen puntutik $x = \frac{L}{6}$-ra

$\omega \cdot R = v'$

$\frac{2mv}{ML} \cdot R = \frac{mv}{M}$ $\boxed{R = \frac{L}{2}}$ -ra ez edukiko du $\omega = 0$

$\boxed{x = \frac{L}{2}}$

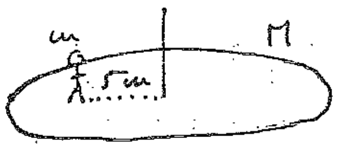
$\omega = \frac{12 mv \frac{L}{2}}{ML^2} = \frac{6mv}{ML}$

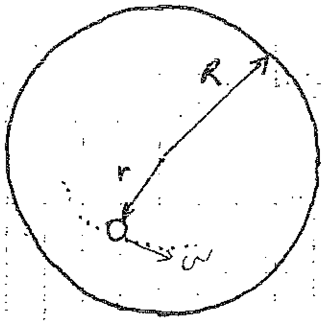
$\omega \cdot R = v'$

$\frac{6mv}{ML} R = \frac{mv}{M}$ $\boxed{R = \frac{L}{6}}$ -ra edukiko du $\omega = 0$

25

$M = 100\ kg \quad m = 60\ kg \quad R = 10\ m \quad r = 5\ m$





$I = 1/2\, MR^2$

a) $v' = 1\ m/s$

$v = v' + V$

$v = v' + \omega r$

$L_{lehen} = 0$

$L_{ondoren} = rmv + I_{Mz}\omega$

L kte

$rmv + I_{Mz}\omega = 0$

$rm(v' + \omega r) + I_{Mz}\omega = 0$

$rmv' + r^2 m\omega + I_{Mz}\omega = 0$

$\omega = -\dfrac{rmv'}{r^2 m + 1/2 MR^2} = -0'046\ rad/s$ (kontrako dala...)

$\boxed{\omega = 0'046\ rad/s}$

b) $\omega_0 = 1{,}047\ rad/s = \pi/3\ rad/s$

(i) $r' = 2\ m$

$L_{lehen} = r_0 m v_0 + I_{Mz}\omega_0$

$L_{ondoren} = r' m v + I_{Mz}\omega_0'$

L kte

$r_0 m v_0 + I_{Mz}\omega_0 = r' m v + I_{Mz}\omega_0'$

$r_0 m \omega_0 r_0 + I_{Mz}\omega_0 = r' m \omega_0' r' + I_{Mz}\omega_0'$

$\omega_0' = \dfrac{(r_0^2 m + \frac{1}{2} MR^2)\,\omega_0}{r'^2 m + 1/2\, MR^2} = \boxed{1'30\ rad/s}$

(ii)

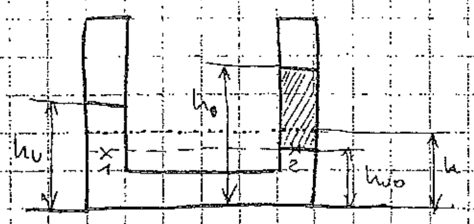
10. GAIA: FLUIDOAK

FISIKA

Ariketak

(1)

$\rho_0 = 0'78\ g/cm^3 \qquad h_u = 34\ cm \qquad h = 28\ cm$



$$P_1 = P_2$$

$$\cancel{P_{atm}} + \rho \cancel{g}(h_u - h_{uo}) = \cancel{P_{atm}} + \rho_0 \cancel{g}(h_o - h_{uo})$$

$$\rho(h_u - h_{uo}) = \rho_0(h_o - h_{uo})$$

$2h = h_u + h_{uo}$ — Berdinak

$h - h_{uo} = h_u - h$

$h_{uo} = 2h - h_u$

$$\rho(h_u - 2h + h_u) = \rho_0(h_o - 2h + h_u)$$

$$2\rho(h_u - h) = \rho_0(h_o - 2h + h_u)$$

$$h_o = \frac{2\rho(h_u - h)}{\rho_0} + 2h - h_u \qquad \boxed{h_o = 37'38\ cm}$$

eta beraz, $\boxed{h_{uo} = 22\ cm}$

(2)

$r_1 = 2\ cm \qquad r_2 = 20\ cm$

$M = 1500\ kg$

$$\Delta P_1 = \frac{F_1}{S_1} = \frac{F_2}{S_2} = \Delta P_2$$

$S_1 = \pi r_1^2$

$S_2 = \pi r_2^2$

$$\frac{F_1}{S_1} = \frac{F_2}{S_2} \longrightarrow F_1 = \frac{S_1}{S_2} Mg = \frac{\cancel{\pi} r_1^2}{\cancel{\pi} r_2^2} Mg = \left(\frac{r_1}{r_2}\right)^2 Mg$$

$$\boxed{F_1 = 147\ N}$$

Autoa h igotzeko W?

$$W_1 = F_1 \cdot h' = \left(\frac{r_1}{r_2}\right)^2 Mg \left(\frac{r_2}{r_1}\right)^2 h = Mgh$$

$$W_2 = F_2 \cdot h = Mgh$$

$$\boxed{W_1 = W_2}$$

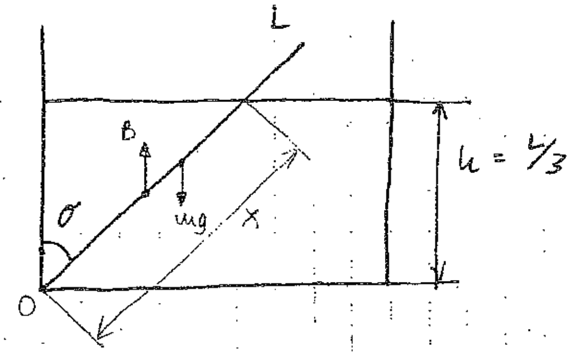
$\Delta V: \quad \Delta P = \Delta P$

$h S_2 = h' \cdot S_1$

$$h' = \frac{S_2}{S_1} \cdot h = \frac{\cancel{\pi} r_2^2}{\cancel{\pi} r_1^2} h$$

$$h' = \left(\frac{r_2}{r_1}\right)^2 h$$

③



$\rho' = 0'45\ g/cm^3$

Hagatxoaren azalera = S

$B = \rho V \cdot g = \rho S \cdot x \cdot g$

$mg = \rho' S \cdot L g$

$\vec{M} = \vec{0}$

$B \cdot \frac{x}{\cancel{2}} \cdot \cancel{\sin\theta} - mg \frac{L}{\cancel{2}} \cancel{\sin\theta} = 0$

$\cos\theta = \frac{L/3}{x} = \frac{L}{3x}$

$Bx - mgL = 0$

$x = \frac{L}{3\cos\theta}$

$\rho \cancel{S} x^2 \cancel{g} - \rho' \cancel{S} L^2 \cancel{g} = 0$

$\rho x^2 - \rho' L^2 = 0$

$\rho \left(\frac{\cancel{L}}{3\cos\theta}\right)^2 - \rho' \cancel{L^2} = 0 \longrightarrow \frac{\rho}{9\cos^2\theta} = \rho'$

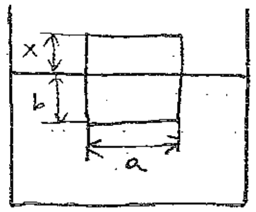
$\cos^2\theta = \frac{\rho}{9\rho'} \longrightarrow \cos\theta = \sqrt{\frac{\rho}{9\rho'}}$

$\theta = \arccos\sqrt{\frac{\rho}{9\rho'}} = 60'20^\circ$ $\boxed{\theta = 60'20^\circ}$

4

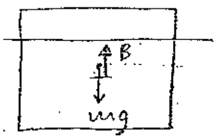
(5)

kubo bat $a = 20\ cm = 0'2\ m$ // $\rho' = 0'65 \cdot 10^3\ kg/m^3$



$x + b = a$ $V = a^3$ $g = 10\ m/s^2$

a) zenbat dago uretatik kanpo?



$\Sigma \vec{F} = \vec{0} \longrightarrow B - mg = 0$

$\rho a^2 b \cancel{g} = \rho' V \cancel{g}$

$\rho a^2 b = \rho' V$

$B = \rho V' = \rho a^2 \cdot b g$

$mg = \rho' V g$

$b = \frac{\rho' V}{a^2 \rho} = \frac{\rho' a^{\cancel{3}}}{\cancel{a^2} \rho} = \frac{\rho'}{\rho} a$ $b = 0'13\ m$

$x = 0'07\ m$

$\boxed{x = 7\ cm}$

b) $b = a$ izateko?

$x = 0$, $b = a$ izateko? $\longrightarrow B = \rho V' = \rho a^3 g$

$B - (m + m')g = 0$

$\rho a^3 \cancel{g} = (m + m')\cancel{g} \longrightarrow m' = \rho a^3 - m = \rho a^3 - \rho' a^3 = a^3(\rho - \rho')$

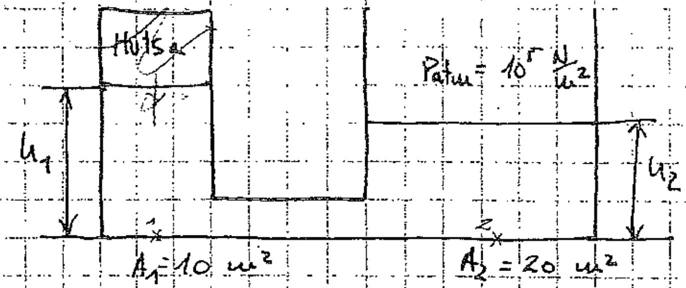
$\rho' = \frac{m}{V} = \frac{m}{a^3}$

P' (erantsi beharreko pisua) $= a^3(\rho - \rho')g$

$m = \rho' a^3$

$\boxed{P' = 28\ N}$

(6)



$h_1 = 15\ m$ $g = 10\ m/s^2$

$h_2 = 10\ m$

Presioak: $P_1 = P_2$ (beti orekan egonda)

$\rho g h_1' = P_{atm} + \rho g h_2' \longrightarrow 10^4 h_1' - 10^4 h_2' = 10^5$

Bolumenak: $V_{lehen} = V_{orain}$ (konstante dirau)

$A_1 h_1 + A_2 h_2 = A_1 h_1' + A_2 h_2' \longrightarrow 10 h_1' + 20 h_2' = 350$

Beraz, daukagun sistema hurrengoa da:

$$\begin{cases} h_1' - h_2' = 10 \\ h_1' + 2h_2' = 35 \end{cases} \qquad \left(\begin{array}{cc|c} 1 & -1 & 10 \\ 1 & 2 & 35 \end{array}\right) \xrightarrow[E_2 - E_1]{} \left(\begin{array}{cc|c} 1 & -1 & 10 \\ 0 & 3 & 25 \end{array}\right)$$

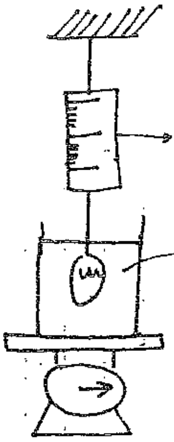
$h_1' - h_2' = 10$ $h_2' = \frac{25}{3}$

$h_1' = 10 + \frac{25}{3} = \frac{55}{3}$ $h_1' = \frac{55}{3}$

Beraz, $\Delta V_1 = A_1 (h_1' - h_1) = 33'3\ m^3$

$\boxed{V = 33'3\ m^3}$ ur pasatu da alde batetik bestera oreka lortzeko.

7

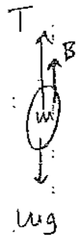


$m = 1\ kg$

outzitaren masa: 50 g.

→ 8'53 N markatzen du. ⟶ $T = 8'53\ N$ (B-k mg-n dauka eragina)

→ $0'35\ dm^3$ ur



$\Sigma \vec{F} = \vec{0}$

$T + B - mg = 0$

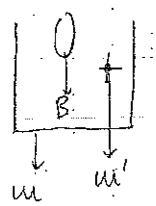
non $B = \rho V g$

$T + \rho V g - mg = 0$

$V = \dfrac{mg - T}{\rho g}$ $\qquad V = 1'29 \cdot 10^{-4}\ m^3 = 129\ cm^3$

$\boxed{V = 129\ cm^3}$

b) balantzak markatzen duen pisua



$mg + B + m'g = (m + \rho V + m')g = (0'05\ kg + 10^3 \frac{kg}{m^3} \cdot 1'29 \cdot 10^{-4}\ m^3 + 0'35\ kg) \cdot 9'8\ m/s^2$

$= 5'18\ N$

$m' \longrightarrow 0'35\ dm^3 \cdot \dfrac{1\ m^3}{10^3\ dm^3} \cdot \dfrac{1000\ kg}{1\ m^3} \longrightarrow 0'35\ kg$

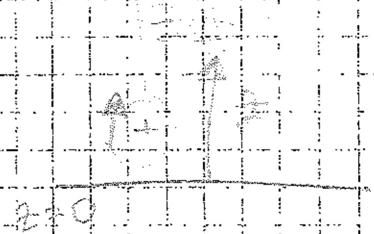
Balantzak markatzen duen pisua: $\boxed{P = 5'18\ N}$

8

$$\int_{P_0}^{P} dP = \int_{z_0}^{z} -\rho g \, dz \qquad \text{non} \quad \rho = \frac{\rho_0}{P_0} P$$

a) $P(z)$? Integratzio mugak $d\rho$? $\longrightarrow$ $P = P_0 \, e^{-\frac{\rho_0}{P_0} g z}$

$$\frac{dP}{dz} = -\rho g \qquad \rho = \frac{\rho_0}{P_0} P \qquad \frac{dP}{dz} = -\frac{\rho_0 g}{P_0} P$$



$$\frac{dP}{P} = -\frac{\rho_0 g}{P_0} dz$$

$$\ln\left(\frac{P(z)}{P_0}\right) = -\frac{\rho_0 g}{P_0} z$$

$$P(z) = P_0 \, e^{-\frac{\rho_0 g}{P_0} z}$$

b) $P_0 = 1{,}013 \cdot 10^5 \ Pa$ $\quad \rho_0 = 1'3 \ kg/m^3$ $\quad z = 8848 \ m$ $\quad P$?

$$P = P_0 \, e^{-\frac{\rho_0}{P_0} g z} \longrightarrow P = 1{,}013 \cdot 10^5 \frac{N}{m^2} \cdot e^{-\frac{1'3 \ kg/m^3}{1'013 \cdot 10^5 \ N/m^2} \cdot 9'8 \ m/s^2 \cdot 8848 \ m} = 33292'0 \ Pa \left(\frac{N}{m^2}\right)$$

$$\boxed{P = 33292 \ Pa = 0'33 \cdot 10^5 \ Pa}$$

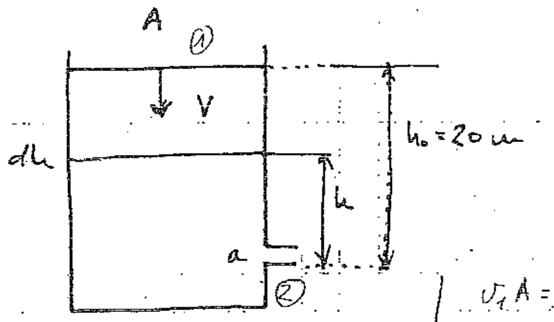
c) Noiz da $P = \frac{P_0}{2}$

$$P = \cancel{P_0} \, e^{-\frac{\rho_0}{P_0} g z} = \frac{\cancel{P_0}}{2} \longrightarrow \ln e^{-\frac{\rho_0}{P_0} g z} = \ln \frac{1}{2}$$

$$-\frac{\rho_0}{P_0} g z = \ln \frac{1}{2}$$

$$z = \frac{\ln \frac{1}{2} \cdot P_0}{-\rho_0 g} \longrightarrow \boxed{z = 5511'44 \ m}$$

(9)



$A = 10\ m^2$

$a = 1\ dm^2 = 100\ cm^2 = 0'01\ m^2$

Badakigu:

$$\begin{cases} v_1 A = v_2 a \longrightarrow v_1 = \dfrac{a}{A} v_2 \\ \cancel{\rho} g h + \frac{1}{2}\cancel{\rho} v_1^2 = \frac{1}{2}\cancel{\rho} v_2^2 \longrightarrow 2gh + v_1^2 = v_2^2 \end{cases}$$

$$2gh + \left(\frac{a}{A} v_2\right)^2 = v_2^2 \longrightarrow 2gh = \left[1 - \left(\frac{a}{A}\right)^2\right] v_2^2$$

$$v_1 = \frac{a}{A}\sqrt{\frac{2gh}{1-\left(\frac{a}{A}\right)^2}} \qquad v_2 = \sqrt{\frac{2gh}{1-\left(\frac{a}{A}\right)^2}}$$

Depositutik irteten den ura: $a v_2$ denbora unitateko, $a v_2 dt$.

Hortaz, depositutko ura h jaitsi egingo da:

$$-A\,dh = a v_2\,dt \longrightarrow -A\,dh = a \cdot \sqrt{\frac{2gh}{1-\left(\frac{a}{A}\right)^2}} \cdot dt$$

$$-\frac{\cancel{A}\,dh}{\sqrt{h}} = \cancel{A} a \sqrt{\frac{2g}{A^2 - a^2}}\,dt \longrightarrow -\frac{dh}{\sqrt{h}} = a\sqrt{\frac{2g}{A^2-a^2}}\,dt$$

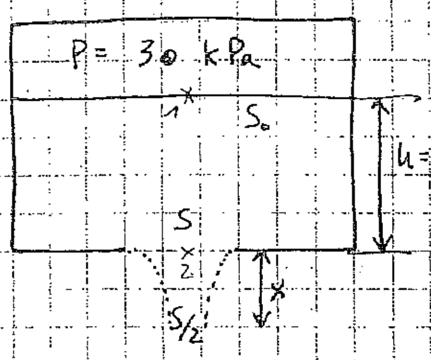
$$-\int_{h_0}^{h} \frac{dh}{\sqrt{h}} = a\sqrt{\frac{2g}{A^2-a^2}} \int_0^t dt \longrightarrow -\left[2\sqrt{h}\right]_{h_0}^{h} = a\sqrt{\frac{2g}{A^2-a^2}}\; t$$

$2\sqrt{h_0} - 2\sqrt{h} = a\sqrt{\dfrac{2g}{A^2-a^2}}\; t$, baina $h=0$ jartzen badugu, hau da, noiz arte husten bada,

$$2\sqrt{h_0} = \sqrt{\frac{2g}{\frac{A^2}{a^2}-1}}\; t \longrightarrow t = 2\sqrt{h_0}\cdot\sqrt{\frac{\frac{A^2}{a^2}-1}{2g}} = \sqrt{\left(\frac{A^2}{a^2}-1\right)\frac{2h_0}{g}}$$

Eta beraz, datuak ordezkatuz: $\boxed{t = 2020'3\ s}$

(10)



$S_0 = 10\ m^2$ $S = 10\ cm^2 = 1 \cdot 10^{-3}\ m^2$

$P = P_m = 30\ kPa = 3 \cdot 10^4\ Pa$

$\rho = 10^3\ \frac{kg}{m^3}$

$P_1 = P_m + P_{atm}$

$P_2 = P_{atm}$

a)

$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h = P_2 + \frac{1}{2}\rho v_2^2 + \cancel{\rho g h}^{\,0}$ // $v_1 S_0 = v_2 S$

$P_m + \cancel{P_{atm}} + \frac{1}{2}\rho v_1^2 + \rho g h = \cancel{P_{atm}} + \frac{1}{2}\rho v_2^2$ $v_1 = \frac{S}{S_0} v_2$

$\frac{1}{2}\rho v_2^2 - \frac{1}{2}\rho v_1^2 = P_m + \rho g h$

$\frac{1}{2}\rho v_2^2 - \frac{1}{2}\rho\left(\frac{S}{S_0}v_2\right)^2 = P_m + \rho g h$

$\frac{1}{2}\rho v_2^2 \left(1 - \left(\frac{S}{S_0}\right)^2\right) = P_m + \rho g h \longrightarrow v_2 = \sqrt{\frac{2(P_m + \rho g h)}{\rho\left[1 - \left(\frac{S}{S_0}\right)^2\right]}} = 16\ m/s$

$\boxed{v_2 = 16\ m/s}$

b)

$\cancel{P_{atm}} + \frac{1}{2}\cancel{\rho} v_2^2 = \cancel{P_{atm}} + \frac{1}{2}\cancel{\rho} v_x^2 + \cancel{\rho} g x$

$\frac{1}{2}v_2^2 = \frac{1}{2}4v_2^2 + gx$ $v_2 \cancel{S} = v_x \cancel{S}/2 \rightarrow v_x = 2v_2$

$\frac{1}{2}v_2^2 = 2v_2^2 + gx$

$\left(\frac{1}{2} - 2\right)v_2^2 = gx \longrightarrow x = \frac{\left(\frac{1}{2} - 2\right)v_2^2}{g} = -39'2\ m$ (minusa hartutako erreferentzi sistemaren alde negatiborantz delako da)

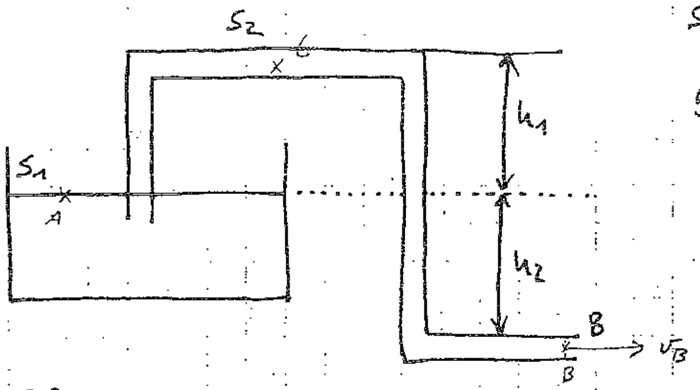
$\boxed{x = 39'2\ m}$-ra iristen da S/2

c)

$v_x = 2v_2 = 32\ m/s$ -ko abiadura edukiko du posizio horretan

$\boxed{v_x = 32\ m/s}$

(11)



$S_1 >>> S_2$

$g = 10\ m/s^2$

a) v_B?

$$v_A S_1 = v_B S_2 \longrightarrow S_1 >>> S_2 \longrightarrow v_A \approx 0$$

$$\cancel{P_A} + \cancel{\tfrac{1}{2}\rho v_A^2}^{\,0} + \rho g h_2 = \cancel{P_B} + \tfrac{1}{2}\rho v_B^2 + \cancel{\rho g h_B}^{\,0}$$ non $P_A = P_{atm}$, $P_B = P_{atm}$

$$\cancel{\rho} g h_2 = \tfrac{1}{2}\cancel{\rho} v_B^2$$

$$g h_2 = \tfrac{1}{2} v_B^2 \longrightarrow \boxed{v_B = \sqrt{2 g h_2}}$$

b) $h_2 = 3\ m$

$$h_2 = 3\ m \longrightarrow v_B = \sqrt{2 g h_2} \longrightarrow \boxed{v_B = 7'74\ m/s}$$

c) $h_2 = 3\ m$ badu $\longrightarrow$ h_1 max?

$$v_B S_2 = v_C S_2 \longrightarrow v_B = v_C$$

$$P_B + \tfrac{1}{2}\cancel{\rho} v_B^2 + \cancel{\rho g h_B}^{\,0} = P_C + \tfrac{1}{2}\cancel{\rho} v_C^2 + \rho g (h_1 + h_2) \qquad P_C \geq 0$$

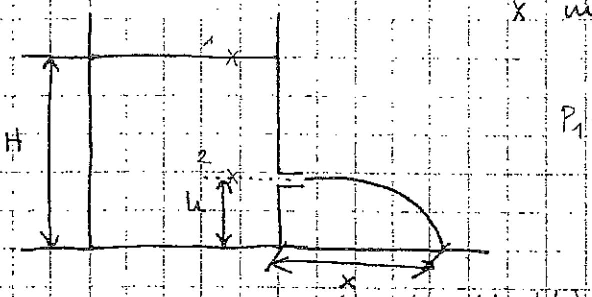
$P_B = P_C + \rho g (h_1 + h_2)$ // $P_B = P_{atm}$, $P_C = 0$ (muga antzekoa)

$$P_{atm} = \rho g (h_1 + h_2)$$

$$h_1 = \frac{P_{atm}}{\rho g} - h_2 = \frac{10^5\ Pa\left(\frac{N}{m^2}\right)}{10^3\ \frac{kg}{m^3} \cdot 10\ m/s^2} - 3\ m = 7\ m$$

$$\boxed{h_1 = 7\ m}$$

(12)



x maximoa $h = H/2$ denean

$P_1 = P_2 = P_{atm}$

Maximoa: $\frac{dx}{dh} = 0$

$$\cancel{P_1} + \cancel{\tfrac{1}{2}\rho v_1^2}^{0} + \rho g h_1 = \cancel{P_2} + \tfrac{1}{2}\rho v_2^2 + \cancel{\rho g h_2}^{0}$$

$$g h_1 = \tfrac{1}{2} v_2^2 \longrightarrow v_2 = \sqrt{2g(H-h)}$$

Higidura parabolikoa deskribatuko du:

$$v_2 = v_{0x}, \quad v_{0y} = 0 \longrightarrow \begin{cases} x: & x = v_2 t \\ y: & y = h - \tfrac{1}{2} g t^2 \end{cases} \longrightarrow x = \sqrt{2g(H-h)}\sqrt{\frac{2h}{g}}$$

$$0 = h - \tfrac{1}{2} g t^2$$

$$t = \sqrt{\frac{-2h}{-g}} = \sqrt{\frac{2h}{g}}$$

$$x = \sqrt{2\cancel{g}(H-h)\cdot\frac{2h}{\cancel{g}}} = \left[4(H-h)h\right]^{1/2} = \left[4Hh - 4h^2\right]^{1/2}$$

$$\frac{dx}{dh} = 0 \longrightarrow \tfrac{1}{2}\left[4Hh - 4h^2\right]^{-1/2}\cdot(4H - 8h) = 0$$

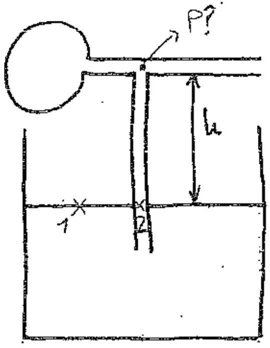
$$\frac{4H - 8h}{2\sqrt{4Hh - 4h^2}} = 0$$

Egia izateko: $4H - 8h = 0$ izan behar da

$$4H = 8h$$

$$\boxed{h = \frac{H}{2}}$$ Frogatuta

⑬



$h = 10\ cm = 0'1\ m$ $\quad \rho_A = 1'3\ kg/m^3$

$\rho = 10^3\ kg/m^3$

$$\left.\begin{array}{l} P_1 = P_{atm} \\ P_2 = P + \rho g h \end{array}\right\} \quad P_1 = P_2$$

$$P_{atm} = P + \rho g h$$

$$P = P_{atm} - \rho g h = 10^5\ Pa - 10^3 \frac{kg}{m^3} \cdot 9'8\ m/s^2 \cdot 0'1\ m =$$

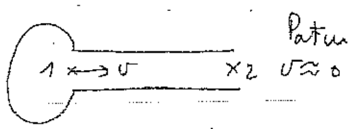
$$= \underbrace{10^5}_{P_{atm}} - 980 = 99020$$

eta $\quad P = P_{atm} + P_m$

$$P_m = P - P_{atm} = 99020 - 10^5 = -980\ Pa$$

$$\boxed{P_m = -980\ Pa}$$

b) v_{min} ?

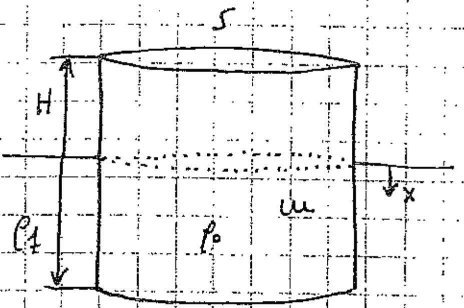


(v_{min}, P-rekin; P-delako Pmin wa gorarte igo dadin)

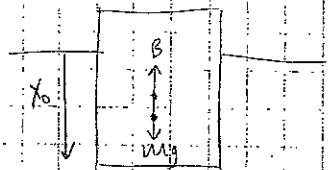
$$P + \tfrac{1}{2}\rho_A v^2 = P_{atm} + \cancel{\tfrac{1}{2}\rho_A v^2}^{\,0}$$

$$v = \sqrt{\frac{(P_{atm} - P)\cdot 2}{\rho_A}} = 38'8\ m/s \qquad \boxed{v_{min} = 38'8\ m/s}$$

(14)



Orekan



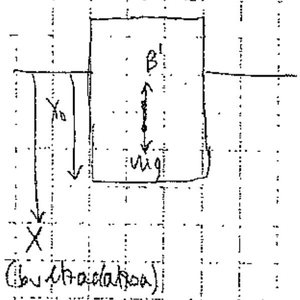
$\sum \vec F = \vec 0$

$B - mg = 0$

$B = mg = \rho_1 x_0 S g$

$B = \rho_1 x_0 \cdot S g$

Orekatik kanpo



$F = m a$

$P - B' = m \cdot a$

$mg - \rho_1 (x_0 + x) S g = m a$

$\underbrace{mg - \rho_1 x_0 S g}_{0} - \rho_1 x S g = m a$

$-\rho_1 x S g = m \ddot x$

$\boxed{\ddot x + \frac{\rho_1 S g}{m} x = 0}$ → Higidura harmonikoa? higitzen da

$\omega^2 = \frac{\rho_1 S g}{m}$ → $\boxed{\omega = \sqrt{\frac{\rho_1 S g}{m}}}$

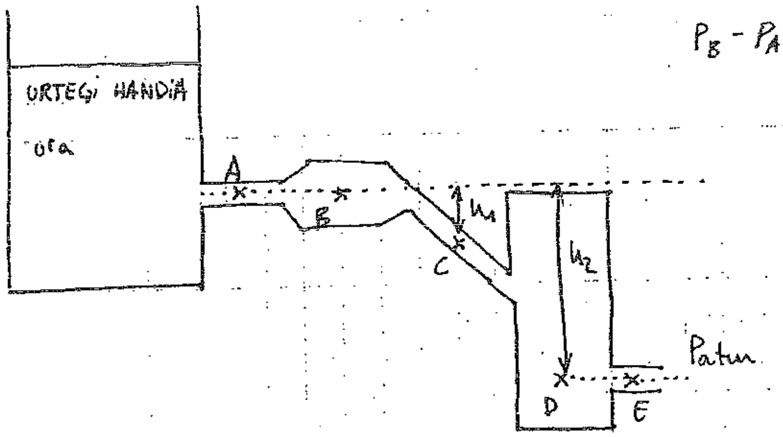
(15)

$S_A = S_C = S_E = 10\ cm^2 = 10^{-3}\ m^2$ $\quad S_B = 20\ cm^2 = 2\cdot 10^{-3}\ m^2$ $\quad S_D = 80\ cm^2 = 8\cdot 10^{-3}\ m^2$

$P_B - P_A = 500\ Pa$

$h_1 = 15\ cm = 0'15\ m$

$h_2 = 30\ cm = 0'3\ m$



$P_A + \frac{1}{2}\rho v_A^2 = P_B + \frac{1}{2}\rho v_B^2$ $\quad // \quad v_A S_A = v_B S_B$

$\frac{1}{2}\rho(v_A^2 - v_B^2) = P_B - P_A$ $\quad v_A = \frac{S_B}{S_A} v_B$

$\frac{1}{2}\rho v_B^2 \left(\frac{S_B^2}{S_A^2} - 1\right) = P_B - P_A$

$v_B = \sqrt{\frac{2(P_B - P_A)}{\left(\frac{S_B^2}{S_A^2} - 1\right)\rho}}$ $\quad \boxed{v_B = 0'57\ m/s}$

$S_A = S_C = S_E \longrightarrow v_A = v_C = v_E = \frac{S_B}{S_A} v_B$ $\quad \boxed{v_A = v_C = v_E = 1'15\ m/s}$

---

eta beraz,

$v_A S_A = v_D S_D \longrightarrow v_D = \frac{S_A}{S_D} v_A$ $\quad \boxed{v_D = 0'14\ m/s}$

- $P_E = P_{atm} = 10^5\ Pa = 100000\ Pa$ $\quad \boxed{P_E = 100000\ Pa}$
- $P_D + \frac{1}{2}\rho v_D^2 = P_E + \frac{1}{2}\rho v_E^2$ $\quad \boxed{P_D = 100656'25\ Pa}$

  $P_D = P_E + \frac{1}{2}\rho(v_E^2 - v_D^2)$
- $P_C + \frac{1}{2}\rho v_C^2 + \rho g(h_2 - h_1) = P_D + \frac{1}{2}\rho v_D^2$ $\quad \boxed{P_C = 98530\ Pa}$

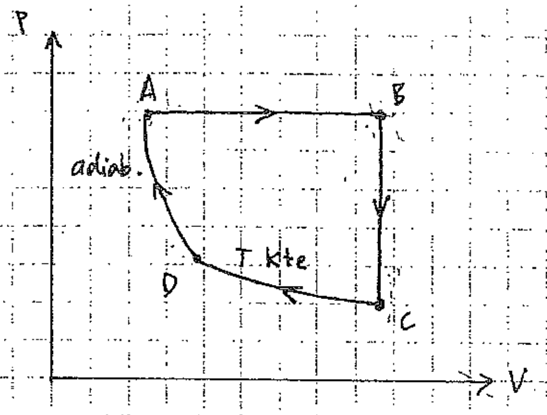
  $P_C = P_D + \frac{1}{2}\rho(v_D^2 - v_C^2) - \rho g(h_2 - h_1)$
- $P_B + \frac{1}{2}\rho v_B^2 + \rho g h_1 = P_C + \frac{1}{2}\rho v_C^2$ $\quad \boxed{P_B = 97560\ Pa}$

  $P_B = P_C + \frac{1}{2}\rho(v_C^2 - v_B^2) - \rho g h_1$
- $P_B - P_A = 500 \longrightarrow P_A = P_B - 500$ $\quad \boxed{P_A = 97060\ Pa}$

11. GAIA TERMODINAMIKA 1. LEHEN PRINTZIPIOA

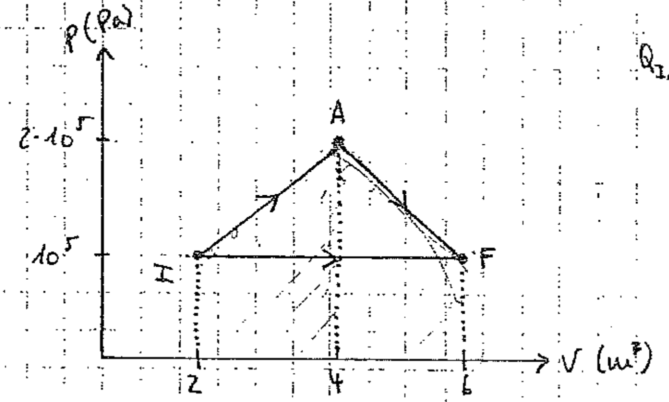
Fisika

①



	Q	W	ΔU	ΔT
AB	+	+	+	+
BC	−	0	−	−
CD	−	−	0	0
DA	0	−	+	+

②



$Q_{IAF} = 8\cdot 10^5$ J

$PV = RnT$

$\frac{PV}{Rn} = T$

a) W_{IAF}

$$W_{IAF} \longrightarrow \text{azalera} \longrightarrow \begin{cases} (6-2)\,m^3 \cdot 10^5\ Pa \\ + \\ \dfrac{(6-2)\,m^3 \cdot (2\cdot 10^5 - 10^5)\,Pa}{2} \end{cases} = 6\cdot 10^5\ J$$

$$\boxed{W_{IAF} = 6\cdot 10^5\ J}$$

b) W_{IF}

$$W_{IF} = \int P\cdot dV = P\int_{V_i}^{V_F} dV = P(V_F - V_I) = 4\cdot 10^5\ J$$

P kte

$$\boxed{W_{IF} = 4\cdot 10^5\ J}$$

c) ΔU

$$Q_{IAF} = 8\cdot 10^5\ J \longrightarrow \Delta U = Q_{IAF} - W_{IAF} = (8-6)\cdot 10^5 = 2\cdot 10^5\ J$$

$$\boxed{\Delta U = 2\cdot 10^5\ J}$$

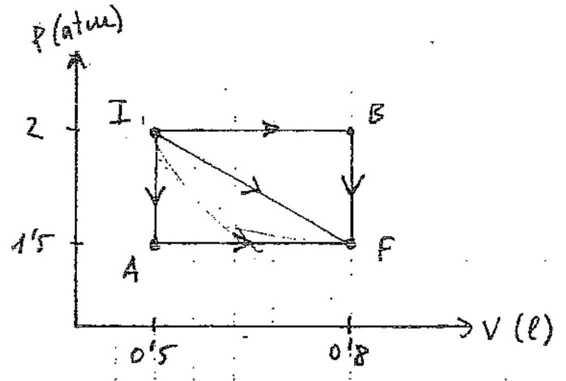
d) Q_{IF}

$$\Delta U = Q_{IF} - W_{IF}$$

$$Q_{IF} = \Delta U + W_{IF} = (2+4)\cdot 10^5\ J$$

$$\boxed{Q_{IF} = 6\cdot 10^5\ J}$$

③



$U_I = 91\ J$

$U_F = 182\ J$ $\Big] \longrightarrow \Delta U = U_F - U_I = 91\ J$

$\Delta U = Q - W$

$2\ atm \cdot \dfrac{1'013 \cdot 10^5\ Pa}{1\ atm} = 2'026 \cdot 10^5\ Pa$

$1'5\ atm \cdot \dfrac{1'013 \cdot 10^5\ Pa}{1\ atm} = 1'52 \cdot 10^5\ Pa$

$0'5\ \ell \cdot \dfrac{1\ m^3}{1000\ \ell} = 5 \cdot 10^{-4}\ m^3$

$0'8\ \ell \cdot \dfrac{1\ m^3}{1000\ \ell} = 8 \cdot 10^{-4}\ m^3$

a) W? 3 bideetatik.

- $W_{IAF} = \cancel{W_{IA}}^{0} + W_{AF} = \int_{V_A}^{V_F} P\,dV = \underset{\substack{\uparrow \\ P\ kte}}{} P_{AF}(V_F - V_A) =$

$= 45'6\ J$

- $W_{IBF} = W_{IB} + \cancel{W_{BF}}^{0} = \int_{V_I}^{V_B} P\,dV = \underset{\substack{\uparrow \\ P\ kte}}{} P_{IB}(V_B - V_I) = 60'78\ J$

- $W_{IF} = (\text{Azalera}) = \underbrace{1'52 \cdot 10^5\ Pa \cdot (8-5) \cdot 10^{-4}\ m^3}_{\text{laukia}} + \underbrace{\dfrac{(2'026 - 1'52) \cdot 10^5\ Pa \cdot (8-5) \cdot 10^{-4}\ m^3}{2}}_{\text{hirukia}} =$

$= 45'6 + 7'59 = 53'19\ J$

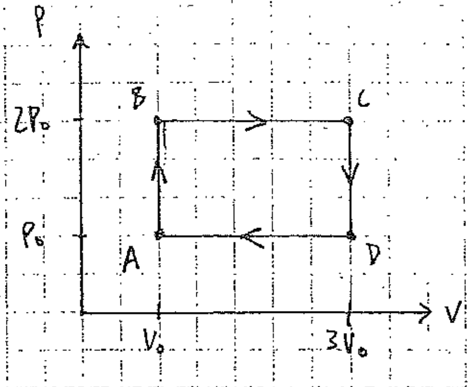
$\boxed{\begin{array}{l} W_{IAF} = 45'6\ J \\ W_{IBF} = 60'78\ J \\ W_{IF} = 53'19\ J \end{array}}$

b) Q? 3 bideetatik

- $Q_{IAF} = \Delta U + W_{IAF} = 136'6\ J$
- $Q_{IBF} = \Delta U + W_{IBF} = 151'78\ J$
- $Q_{IF} = \Delta U + W_{IF} = 144'19\ J$

$\boxed{\begin{array}{l} Q_{IAF} = 136'6\ J \\ Q_{IBF} = 151'78\ J \\ Q_{IF} = 144'19\ J \end{array}}$

(4)



Gas monoatomikoa

$$\begin{cases} c_v = \frac{3}{2} nR \\ c_p = \frac{5}{2} nR \end{cases}$$

$PV = nRT$

a) W?

$W = W_{AB} + W_{BC} + W_{CD} + W_{DA} = 0 + 4P_0V_0 + 0 - 2P_0V_0 = \boxed{2P_0V_0}$

$W_{AB} = 0$

$W_{BC} = \int_{V_B}^{V_C} P\,dV \underset{P\ kte}{=} P(V_C - V_B) = 2P_0(3V_0 - V_0) = 4P_0V_0$

$W_{CD} = 0$

$W_{DA} = \int_{V_D}^{V_A} P\,dV \underset{P\ kte}{=} P(V_A - V_D) = P_0(V_0 - 3V_0) = -2P_0V_0$

b) Q?

$Q = Q_{AB} + Q_{BC} + Q_{CD} + Q_{DA} = \left(\frac{3}{2} + 10 - \frac{9}{2} - 5\right)P_0V_0 = \boxed{2P_0V_0}$

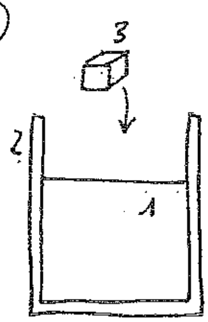
- $Q_{AB} = c_v \Delta T = c_v(T_B - T_A) = \frac{3}{2}\cancel{nR}\left(\frac{2P_0V_0}{\cancel{nR}} - \frac{P_0V_0}{\cancel{nR}}\right) = \frac{3}{2}P_0V_0$
- $Q_{BC} = c_p \Delta T = c_p(T_C - T_B) = \frac{5}{2}\cancel{nR}\left(\frac{2P_0 3V_0}{\cancel{nR}} - \frac{2P_0V_0}{\cancel{nR}}\right) = \frac{5}{2}(4P_0V_0) = 10P_0V_0$
- $Q_{CD} = c_v \Delta T = c_v(T_D - T_C) = \frac{3}{2}\cancel{nR}\left(\frac{P_0 3V_0}{\cancel{nR}} - \frac{2P_0 3V_0}{\cancel{nR}}\right) = \frac{3}{2}(-3P_0V_0) = -\frac{9}{2}P_0V_0$
- $Q_{DA} = c_p \Delta T = c_p(T_A - T_D) = \frac{5}{2}\cancel{nR}\left(\frac{P_0V_0}{\cancel{nR}} - \frac{P_0 3V_0}{\cancel{nR}}\right) = \frac{5}{2}(-2P_0V_0) = -5P_0V_0$

c) $n = 1$ mol $\quad T_A = 0°C$

$A \longrightarrow P_0V_0 = nRT_A = 1\,\cancel{mol} \cdot 8'31 \frac{J}{\cancel{K}\,\cancel{mol}} \cdot 273\,\cancel{K} = 2268'63\ J$

$Q = W = 2P_0V_0 = 2nRT_A = 2 \cdot 2268'63\ J = 4537'26\ J \qquad \boxed{W = 4537'26\ J}$

(5)



<u>Hasierau</u>

1 eta 2 : 17'3 °C

3 : 100 °C

<u>Amaierau:</u>

1, 2 eta 3 : 22'7 °C

$Q = C_P \Delta T = m c_p \Delta T$

$C_{p2} = C_{p3}$

$m_1 = 500$ gr

$m_2 = 200$ gr

$m_3 = 200$ gr

$Q_1 + Q_2 = -Q_3$

$m_1 c_{p1} \Delta T_1 + m_2 c_{p2} \Delta T_2 = -m_3 c_{p3} \Delta T_3$

$m_1 c_{p1} \Delta T_1 + m_2 c_{p3} \Delta T_2 = -m_3 c_{p3} \Delta T_3$

$\Delta T_{C°} = 15 - 5$

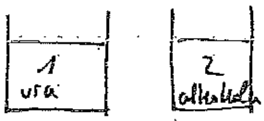
$\Delta T_K = \cancel{273} + 15 - \cancel{273} - 5$

$$c_{p3} = \frac{m_1 c_{p1} \Delta T_1}{-m_3 \Delta T_3 - m_2 \Delta T_2} = \frac{500\,gr \cdot 1 \frac{cal}{gr \cdot °C} \cdot (22'7 - 17'3)°C}{-200\,gr \cdot (22'7 - 100)°C - 200\,gr\,(22'7 - 17'3)°C} = 0'188 \frac{cal}{gr \cdot °C}$$

$Q = m\,c\,\Delta T$

$$\boxed{C_{p2} = C_{p3} = 0'188 \frac{cal}{gr \cdot °C}}$$

(6)



$m_1 = 100$ gr $\quad m_2 = 200$ gr

<u>Hasierau:</u>

1: 16 °C

2: 16 °C

<u>Amaierau:</u>

100 °C (7 min) (irakiten punta)

78 °C (6 min 12 s)

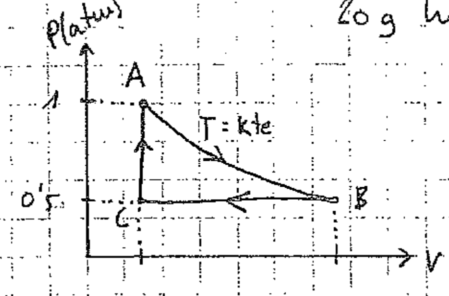
$Q_1 = Q_2$

$$m_1 c_{p1} \Delta T_1 = m_2 c_{p2} \Delta T_2 \longrightarrow c_{p2} = \frac{m_1 c_{p1} \Delta T_1}{m_2 \Delta T_2} = \frac{100\,gr \cdot 1 \frac{cal}{gr \cdot °C} (100°C - 16°C)}{200\,gr\,(78°C - 16°C)} =$$

$$= 0'6\cancel{77} \frac{cal}{gr \cdot °C}$$

$P = \frac{dW}{dt}$

(7)



20 g hidrogeno $\longrightarrow$ diatomikoa $\longrightarrow$ $c_V = \frac{5}{2} nR$ $\quad c_P = \frac{7}{2} nR$

Hasieran: 1 atm, 27°C

$1 \text{ atm} \cdot \frac{1'013 \cdot 10^5 \text{ Pa}}{1 \text{ atm}} = 1'013 \cdot 10^5 \text{ Pa}$

$T_A = T_B = 27°C = 300 \text{ K}$

$0'5 \text{ atm} \cdot \frac{1'013 \cdot 10^5 \text{ Pa}}{1 \text{ atm}} = 0'506 \cdot 10^5 \text{ Pa}$

$20 \text{ g } H_2 \longrightarrow 10 \text{ mol } H_2 = n$

$R = 8'31 \frac{J}{K \cdot mol}$

$W = W_{AB} + W_{BC} + W_{CA}$

$$W_{AB} = \int_{V_A}^{V_B} P\,dV = \int_{V_A}^{V_B} \frac{nRT}{V}\,dV = nRT \ln \frac{V_B}{V_A} = nRT \ln \frac{\frac{\cancel{nRT_B}}{P_B}}{\frac{\cancel{nRT_A}}{P_A}} = nRT \ln \frac{P_A}{P_B} =$$

$$P_B V_B = nRT_B \longrightarrow V_B = \frac{nRT_B}{P_B} \qquad = 17280 \text{ J}$$

$$P_A V_A = nRT_A \longrightarrow V_A = \frac{nRT_A}{P_A}$$

$$W_{BC} = \int_{V_B}^{V_C} P\,dV \underset{P\ kte}{=} P_{BC}(V_C - V_B) = -12480 \text{ J}$$

$$V_C = V_A = \frac{nRT_A}{P_A} = \frac{10 \cancel{mol} \cdot 8'31 \frac{J}{\cancel{K \cdot mol}} \cdot 300 \cancel{K}}{1'013 \cdot 10^5 \text{ Pa}} = 0'246 \text{ m}^3$$

$$V_B = \frac{nRT_B}{P_B} = \frac{10 \cancel{mol} \cdot 8'31 \frac{J}{\cancel{K \cdot mol}} \cdot 300 \cancel{K}}{0'506 \cdot 10^5 \text{ Pa}} = 0'492 \text{ m}^3$$

$W_{CA} = 0 \quad$ (V kte)

$$W = W_{AB} + W_{BC} + \cancel{W_{CA}}^{\,0} = 4800 \text{ J} \qquad \boxed{W = 4800 \text{ J}}$$

$\Delta U = Q - W$

$\Delta U_{AB} = 0 \qquad \boxed{\Delta U = 0 \text{ J}}$

$\Delta U_{BC} = c_V \Delta T = \frac{5}{2} nR (T_C - T_B) = -31162 \text{ J}$

$$T_C = \frac{P_C V_C}{nR} = 150 \text{ K}$$

$\Delta U_{CA} = c_V \Delta T = \frac{5}{2} nR (T_A - T_C) = 31162 \text{ J}$

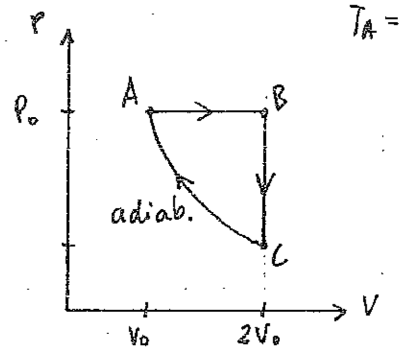
Q

$Q_{AB} = \cancel{\Delta U_{AB}}^{\,0} + W_{AB} = 17280 \text{ J}$

$Q_{BC} = c_P \Delta T = \frac{7}{2} nR (T_C - T_B) = -43627 \text{ J}$

$Q_{CA} = c_V \Delta T = \frac{5}{2} nR (T_A - T_C) = 31162 \text{ J}$

⑧



$T_A = T_0$ Helioa $\longrightarrow$ monoatomikoa $\longrightarrow \gamma = 5/3$

$P_A = P_0$ $\qquad P_A V_A = nRT_A \longrightarrow n = \frac{P_A V_A}{RT_A} = \frac{P_0 V_0}{RT_0}$

$V_A = V_0$

$T_A = T_0$

a)

$P_B = P_0 \qquad V_B = 2V_0 \qquad T_B = \frac{P_B V_B}{nR} = \frac{2P_0 V_0}{nR} = \frac{2\cancel{P_0}\cancel{V_0}}{\frac{\cancel{P_0 V_0}}{RT_0}\cdot\cancel{R}} = 2T_0$

$V_C = 2V_0 \qquad P_A V_A^{\gamma} = P_C V_C^{\gamma} \qquad T_C = \frac{P_C V_C}{nR} = \frac{0'31\,\cancel{P_0}\,2\cancel{V_0}}{\frac{\cancel{P_0 V_0}}{RT_0}\cdot\cancel{R}} = 0'62\,T_0$

$P_0 \cancel{V_0^{5/3}} = P_C (2\cancel{V_0})^{5/3} \longrightarrow P_C = \frac{P_0}{2^{5/3}} = 0'31\,P_0$

b)

$\underline{W} = W_{AB} + W_{BC} + W_{CA}$

$\bullet\ W_{AB} = \int_{V_A}^{V_B} P\,dV \underset{\substack{\uparrow \\ P\ kte}}{=} P_{AB}(V_B - V_A) = P_0 V_0$ $\qquad -\frac{5}{3}+1 = \frac{-2}{3}$

$\bullet\ W_{BC} = 0$

$\bullet\ W_{CA} = \int_{V_C}^{V_A} P\,dV = \int_{V_C}^{V_A} \frac{kte}{V^{\gamma}}\,dV = kte\int_{V_C}^{V_A} V^{-\gamma}\,dV = kte\left[\frac{V^{-\gamma+1}}{-\gamma+1}\right]_{V_C}^{V_A} = kte\left[\frac{V^{-2/3}}{-2/3}\right]_{V_C}^{V_A}$

$P_A V_A^{\gamma} = P\cdot V^{\gamma} = kte \longrightarrow P = \frac{kte}{V^{\gamma}}$

$= kte\left[\frac{V_A^{-2/3}}{-2/3} - \frac{V_C^{-2/3}}{-2/3}\right] = kte\left[-3/2\left(V_0^{-2/3} - (2V_0)^{-2/3}\right)\right] =$

$= P_0 V_0^{\frac{5}{3}}\left[-3/2\cdot V_0^{-2/3}\cdot(1-2^{-2/3})\right] = -0'555\,P_0 V_0^{\frac{5}{3}-\frac{2}{3}} = -0'555\,P_0 V_0$

$\underline{Q}$

$\bullet\ Q_{AB} = C_P\,\Delta T = \frac{5}{2}nR(2T_0 - T_0) = \frac{5}{2}nRT_0 = \frac{5}{2}P_0 V_0 = 2'5\,P_0 V_0$

$\bullet\ Q_{BC} = C_V\,\Delta T = \frac{3}{2}nR(0'62T_0 - 2T_0) = -2'07\,nRT_0 = -2'07\,P_0 V_0$

$\bullet\ Q_{CA} = 0$

$\underline{\Delta U}$

$\bullet\ \Delta U_{AB} = C_V\,\Delta T = \frac{3}{2}nR(T_B - T_A) = \frac{3}{2}nR(2T_0 - T_0) = \frac{3}{2}nRT_0 = 1'5\,P_0 V_0$

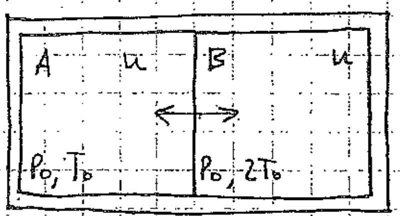
$\bullet\ \Delta U_{BC} = Q_{BC} - \cancel{W_{BC}}^{\,0} = -2'07\,P_0 V_0$

$\bullet\ \Delta U_{CA} = \cancel{Q_{CA}}^{\,0} - W_{CA} = 0'555\,P_0 V_0$

(9)

$u = 1$ mol H_2 $\quad H_2 \rightarrow$ diatomikoa $\rightarrow \begin{cases} C_v = \frac{5}{2}uR \\ C_p = \frac{7}{2}uR \end{cases}$

$PV = uRT$



Amaieran: biek T'

a) V_{sis}?

$V_{sis} = V_A + V_B$

$V_A = \frac{uRT_A}{P_A} = \frac{uRT_0}{P_0} = \frac{RT_0}{P_0}$

$V_B = \frac{uRT_B}{P_B} = \frac{2uRT_0}{P_0} = \frac{2RT_0}{P_0}$

$\boxed{V_{sis} = \frac{3RT_0}{P_0}}$

b)

$\Delta U = \Delta U_A + \Delta U_B$

$\Delta U_A = C_v \cdot \Delta T = \frac{5}{2}uR(T' - T_0)$

$\Delta U_B = C_v \cdot \Delta T = \frac{5}{2}uR(T' - 2T_0)$

$\Delta U = \frac{5}{2}uR(T' - T_0 + T' - 2T_0) = \boxed{\frac{5}{2}uR(2T' - 3T_0)}$

c)

$W_A = -W_B \longrightarrow W_T = 0$

$Q_A = -Q_B \longrightarrow Q_T = 0$

$\longrightarrow \Delta U = \cancel{Q_T}^0 - \cancel{W_T}^0 = 0 = \frac{5}{2}uR(2T' - 3T_0)$

$2T' - 3T_0 = 0$

$T' = \frac{3}{2}T_0 = 1'5\, T_0$ $\quad \boxed{T' = 1'5\, T_0}$

d)

$V_A' = \frac{uRT'}{P_A'}$

$V_B' = \frac{uRT'}{P_B'}$

$P_A' = P_B' = P'$, $V_A' = V_B' = V'$, T'

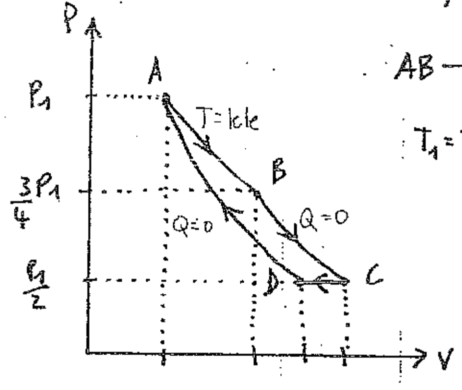
$V_T = V_A + V_B = V_A' + V_B' = \frac{2\cancel{R}T'}{P'} = \frac{3\cancel{R}T_0}{P_0}$

$\frac{2 \cdot \frac{3}{2}\cancel{T_0}}{P'} = \frac{3\cancel{T_0}}{P_0}$ $\quad \boxed{P' = P_0 = P_A' = P_B'}$

$V_A' = V_B' = \frac{uRT'}{P_0} = \frac{R\frac{3}{2}T_0}{P_0} = \frac{3RT_0}{2P_0}$

$\boxed{V_A' = V_B' = \frac{3RT_0}{2P_0}}$

(10)



Gas monoatomikoa, $n=1$ mol $\longrightarrow \gamma = 5/3$

AB $\rightarrow$ isotermoa   BC, DA $\rightarrow$ adiabatikoak

$T_1 = T_A$

	A	B	C	D
P	P_1	$\frac{3}{4}P_1$	$\frac{1}{2}P_1$	$\frac{1}{2}P_1$
T	T_1	T_1	$0'85\,T_1$	$0'76\,T_1$
V	$\frac{RT_1}{P_1}$	$\frac{4}{3}\frac{RT_1}{P_1}$	$1'70\frac{RT_1}{P_1}$	$1'52\frac{RT_1}{P_1}$

a) (A)

$P_A = P_1 \qquad T_A = T_1 \qquad V_A = \frac{nRT_A}{P_A} = \frac{RT_1}{P_1}$

(B)

$P_B = \frac{3}{4}P_1 \qquad T_B = T_1 \qquad V_B = \frac{nRT_B}{P_B} = \frac{4RT_1}{3P_1}$

(C)

$P_C = \frac{1}{2}P_1 \qquad$ adiab $\rightarrow PV^\gamma = kte \longrightarrow V_C^{5/3} = \frac{P_B V_B^{5/3}}{P_C} = \frac{\frac{3}{4}\cancel{P_1}\left(\frac{4}{3}\frac{RT_1}{P_1}\right)^{5/3}}{\frac{1}{2}\cancel{P_1}}$

$V_C = \left(\frac{3}{2}\right)^{3/5} \cdot \frac{4}{3}\frac{RT_1}{P_1} = 1'70\frac{RT_1}{P_1}$

$T_C = \frac{P_C V_C}{nR} = \frac{\frac{1}{2}P_1 \cdot 1'70 \cdot \frac{RT_1}{P_1}}{R} = 0'85\,T_1$

(D)

$P_D = \frac{1}{2}P_1 \qquad$ adiab $\longrightarrow PV^\gamma = kte \longrightarrow V_D^{5/3} = \frac{P_A \cdot V_A^{5/3}}{P_D} = \frac{\cancel{P_1}\cdot\left(\frac{RT_1}{P_1}\right)^{5/3}}{\frac{1}{2}\cancel{P_1}}$

$V_D = 2^{3/5} \cdot \frac{RT_1}{P_1} = 1'52\frac{RT_1}{P_1}$

$T_D = \frac{P_D V_D}{nR} = \frac{\frac{1}{2}\cancel{P_1} \cdot 1'52\frac{\cancel{R}T_1}{\cancel{P_1}}}{\cancel{R}} = 0'76\,T_1$

b) AB $\quad \cancel{\Delta U_{AB}}^{\,0} = Q_{AB} - W_{AB} \longrightarrow Q_{AB} = W_{AB}$

$\Delta U_{AB} = 0$

$W_{AB} = \int_{V_A}^{V_B} P\,dV = \int_{V_A}^{V_B} \frac{nRT}{V}\,dV = nRT_1 \ln\frac{V_B}{V_A} = RT_1 \cdot \ln\frac{\frac{4}{3}\frac{\cancel{RT_1}}{\cancel{P_1}}}{\frac{\cancel{RT_1}}{\cancel{P_1}}} = 0'29\,RT_1$

$Q_A = W_{AB} = 0'29\,RT_1$

BC $\Delta U_{BC} = \cancel{Q_{BC}}^{0} - W_{BC} \longrightarrow \Delta_{BC} = -W_{BC}$

$$W_{BC} = \int_{V_B}^{V_C} PdV = \int_{V_B}^{V_C} \frac{kte}{V^\gamma} dV = kte \int_{V_B}^{V_C} V^{-\gamma} dV = kte \left[\frac{V^{-\gamma+1}}{-\gamma+1} \right]_{V_B}^{V_C} = P_B V_B^{5/3} \left[\frac{-3}{2} \left(V_C^{-2/3} - V_B^{-2/3} \right) \right] =$$

$P_B V_B^\gamma = PV^\gamma = kte = P_C V_C^\gamma$

$$= \frac{3}{4} P_1 \left(\frac{4}{3} \frac{RT_1}{P_1} \right)^{5/3} \left(-\frac{3}{2} \right) \left[\left(1'70 \frac{RT_1}{P_1} \right)^{-2/3} - \left(\frac{4}{3} \frac{RT_1}{P_1} \right)^{-2/3} \right] =$$

$$= -\frac{3}{4} \cdot \frac{3}{2} \left(\frac{4}{3} \right)^{5/3} \cdot P_1 \cdot \left(\frac{RT_1}{P_1} \right)^{5/3 - 2/3} \left(1'70^{-2/3} - \left(\frac{4}{3} \right)^{-2/3} \right) =$$

$$= 0'23 \cancel{P_1} \frac{RT_1}{\cancel{P_1}} = 0'23 RT_1$$

$Q_{BC} = 0$

$\Delta U_{BC} = -0'23 RT_1$

CD $\Delta U_{CD} = Q_{CD} - W_{CD}$

$$W_{CD} = \int_{V_C}^{V_D} PdV \underset{P\ kte}{=} P_{CD} (V_D - V_C) = \frac{1}{2} P_1 \left(1'52 \frac{RT_1}{P_1} - 1'70 \frac{RT_1}{P_1} \right) = -0'09 \cancel{P_1} \frac{RT_1}{\cancel{P_1}} = -0'09 RT_1$$

$$Q_{CD} = C_P \Delta T = \frac{5}{2} nR (T_D - T_C) = \frac{5}{2} R (0'76 - 0'85) T_1 = -0'23 RT_1$$

$$\Delta U_{CD} = Q_{CD} - W_{CD} = -0'135 RT_1$$

DA $\Delta U_{DA} = \cancel{Q_{DA}}^{0} - W_{DA}$

$Q_{DA} = 0$

$$W_{DA} = -\Delta U_{DA} = \int_{V_D}^{V_A} PdV = \dots = P_A V_A^{5/3} \left[\frac{-3}{2} \left(V_A^{-2/3} - V_D^{-2/3} \right) \right] = P_1 \left(\frac{RT_1}{P_1} \right)^{5/3} \left(\frac{-3}{2} \right) \left[\left(\frac{RT_1}{P_1} \right)^{-2/3} - \left(1'52 \frac{RT_1}{P_1} \right)^{-2/3} \right] =$$

$$= \frac{-3}{2} \cdot P_1 \cdot \left(\frac{RT_1}{P_1} \right)^{5/3 - 2/3} \cdot \left(1 - 1'52^{-2/3} \right) =$$

$$= \frac{-3}{2} \cancel{P_1} \frac{RT_1}{\cancel{P_1}} \left(1 - 1'52^{-2/3} \right) = -0'36 RT_1$$

$\Delta U_{DA} = 0'36 RT_1$

d)

$$\Delta U = Q - W$$

$\Delta U = 0 \longrightarrow$ ziklo itzulgarria delako Gainera $\Delta U = \Delta U_{AB} + \Delta U_{BC} + \Delta U_{CD} + \Delta U_{DA} = 0$

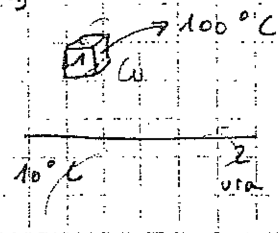
$Q_{torg} = 0'29 R T_1$

$W_T = W_{AB} + W_{BC} + W_{CD} + W_{DA} = Q_T = 0'07 R T_1$

Ariketak: 216. orr

(1)

a)



$m_1 = 400\ g \qquad c_p = 0'0924\ \frac{cal}{g\cdot °C}$

$Q_{Cu} = C_p \Delta T = m_1 \cdot c_p \cdot \Delta T = 400\ \cancel{g} \cdot 0'0924 \frac{cal}{\cancel{g}\cdot \cancel{°C}} \cdot (10-100)\ \cancel{°C} =$

$= -3326'4\ cal$

$\Delta S_{Cu} = \int \frac{\delta Q_{itzul}}{T} = \int \frac{C_p\, dT}{T} = c_p\, m \int \frac{dT}{T} = m \cdot c_p \cdot \ln \frac{T_F}{T_I} =$

$= 400\ g \cdot 0'0924 \frac{cal}{g\cdot K} \cdot \ln \frac{283}{373} = -10'21 \frac{cal}{K}$

$\Delta S_{fok} = \int \frac{\delta Q}{T} = \frac{1}{T} \int \delta Q = \frac{Q}{T} = \frac{-Q_{Cu}}{T} = \frac{3326'4\ cal}{283\ °K} = 11'75 \frac{cal}{°K}$

T ktk

$\Delta S_{unib} = \Delta S_{Cu} + \Delta S_{fok} = 1'55 \frac{cal}{°K}$ $\qquad \boxed{\Delta S_{unib} = 1'55 \frac{cal}{°K}}$

b)

Cu

1 [373 K | 273 °K] 2 ⟶ [T' | T']

$\left.\begin{array}{l} Q_1 = C_p \Delta T = m c_p (T' - T_1) \\ Q_2 = C_p \Delta T = m c_p (T' - T_2) \end{array}\right\} \longrightarrow Q_1 = -Q_2$

$\cancel{m}\cancel{c_p}(T' - T_1) = -\cancel{m}\cancel{c_p}(T' - T_2) \longrightarrow 2T' = T_2 + T_1$

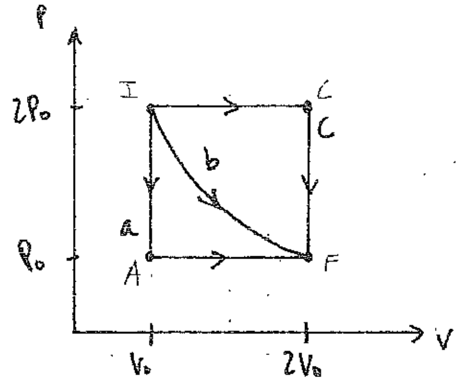
$T' = 323\ °K$

$\Delta S_1 = \int \frac{\delta Q_1}{T} = m c_p \int \frac{dT}{T} = m c_p \ln\left(\frac{T'}{T_1}\right)$

$\Delta S_2 = \int \frac{\delta Q_2}{T} = m c_p \int \frac{dT}{T} = m c_p \ln\left(\frac{T'}{T_2}\right)$

$\Delta S_{unib} = \Delta S_1 + \Delta S_2 = m c_p \ln\left(\frac{T'^2}{T_1 \cdot T_2}\right) = 0'90 \frac{cal}{°K}$ $\qquad \boxed{\Delta S_{unib} = 0'90 \frac{cal}{°K}}$

(2)



$\Delta S_a = \Delta S_b = \Delta S_c$?

$PV = nRT$

b → isoterma (T_0)

$T_A = \frac{P_A V_A}{nR} = \frac{P_0 V_0}{nR} = \frac{1}{2} T_0$ $\quad T_C = \frac{2P_0 2V_0}{nR} = 2T_0$

$T_0 = \frac{2P_0V_0}{nR} = T_I = T_F$

a) $\Delta S_a = \Delta S_{a_1} + \Delta S_{a_2}$

- $\Delta S_{a_1} = \int \frac{\delta Q}{T} = \int \frac{c_V\, dT}{T} = c_V \ln \frac{T_A}{T_I} = c_V \ln \frac{1}{2} = -c_V \ln 2$
- $\Delta S_{a_2} = \int \frac{\delta Q}{T} = \int \frac{c_P\, dT}{T} = c_P \ln \frac{T_F}{T_A} = c_P \ln 2$

$$\Delta S_a = (c_P - c_V)\ln 2 = nR \ln 2 = 2 \cdot \ln 2 \cdot \frac{P_0 V_0}{T_0}$$

$c_P = c_V + nR$

b) $\Delta S_b = \int \frac{\delta Q}{T} \underset{T\ \text{ctte}}{=} \frac{1}{T_0}\int \cancel{dU}^{\,0} + \delta W = \frac{1}{T_0}\int P \cdot dV = \frac{1}{T_0}\int \frac{nRT}{V}\, dV = \frac{nR\cancel{T_0}}{\cancel{T_0}} \ln \frac{V_F}{V_I} =$

$$= nR \ln \frac{2V_0}{V_0} = \ln 2\, nR = 2 \ln 2 \frac{P_0 V_0}{T_0}$$

c) $\Delta S_c = \Delta S_{c_1} + \Delta S_{c_2}$

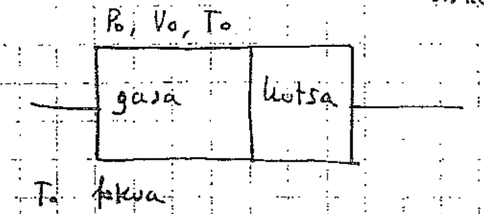
- $\Delta S_{c_1} = \int \frac{\delta Q}{T} = \int \frac{c_P\, dT}{T} = c_P \ln \frac{T_C}{T_I} = c_P \ln 2$
- $\Delta S_{c_2} = \int \frac{\delta Q}{T} = \int \frac{c_V\, dT}{T} = c_V \ln \frac{T_F}{T_C} = c_V \ln \frac{1}{2} = -c_V \ln 2$

$$\Delta S_c = (c_P - c_V)\ln 2 = nR \ln 2 = 2 \ln 2 \frac{P_0 V_0}{T_0}$$

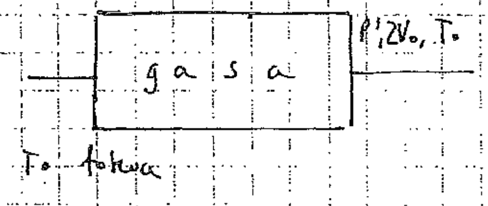
$$\boxed{\Delta S_a = \Delta S_b = \Delta S_c}$$

③

aske hedatzen da $PV = nRT$



$$\left.\begin{array}{l} P_0 V_0 = nRT_0 \\ P' 2V_0 = nRT_0 \end{array}\right\} \longrightarrow P_0 \cancel{V_0} = P' 2 \cancel{V_0} \qquad P' = \frac{P_0}{2}$$



Hasieran	Amaieran
P_0, V_0, T_0	$\frac{P_0}{2}, 2V_0, T_0$

b) Prozesua "askea" denez, $W = 0 = Q = \Delta U = 0$ (T kte), baina entropia kalkulua egiteko, prozesu itzulgarri batean pentsatu beharra dago: (T kte, isotermoa)

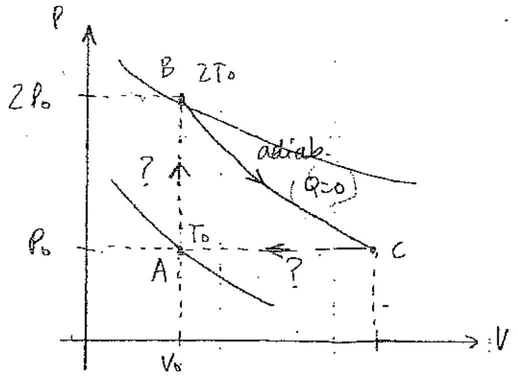
$$\Delta S_{gas} = \int \frac{\delta Q}{T} = \int \frac{\cancelto{0}{dU} + \delta W}{T} = \int \frac{P\,dV}{T} = \int \frac{nR\,dV}{V} = nR \ln \frac{V_F}{V_I} = nR \ln 2 =$$

$$= \frac{P_0 V_0}{T_0} \ln 2$$

$$\Delta S_{foku} = \int \frac{\delta Q}{\underset{\uparrow\, T_0\ kte}{T}} = \frac{1}{T_0} \int \delta Q = \frac{\cancelto{0}{Q}}{T_0} = 0$$

$$\Delta S_{unib} = \Delta S_{gas} + \cancelto{0}{\Delta S_{foku}} = \frac{P_0 V_0}{T_0} \ln 2 \qquad \boxed{\Delta S_{unib} = \frac{P_0 V_0}{T_0} \ln 2}$$

(4) Gas monoatomikoa $\longrightarrow \gamma = 5/3$



A: P_0, V_0, T_0

B: $2P_0, 2T_0, V_0$

C: $P_0, 2^{3/5}T_0, 2^{3/5}V_0$

BC: adiab $\longrightarrow PV^{\gamma} = kte$

$$\frac{2\cancel{P_0} V_0^{5/3}}{\cancel{P_0}} = V_C^{5/3} \longrightarrow V_C = 2^{3/5} V_0$$

$$T_C = \frac{P_0 2^{3/5} V_0}{nR} = 2^{3/5} T_0$$

GAS $\Delta S_{gas} = \Delta S_{AB} + \Delta S_{BC} + \Delta S_{CA}$

- $\Delta S_{AB} = \begin{pmatrix} \text{Prozesu} \\ \text{itzulg.} \\ \text{V kte} \end{pmatrix} = \int_A^B \frac{\delta Q}{T} = \int_A^B \frac{C_V\, dT}{T} = C_V \ln\left(\frac{T_B}{T_A}\right) = \frac{3}{2} nR \ln 2$

- $\Delta S_{BC} = \int \frac{\cancel{\delta Q}^{\,0}}{T} = 0$

- $\Delta S_{CA} = \begin{pmatrix} \text{Prozesu} \\ \text{itzulg.} \\ \text{P kte} \end{pmatrix} = \int_C^A \frac{\delta Q}{T} = \int_C^A \frac{C_P\, dT}{T} = C_P \ln \frac{T_A}{T_C} = \frac{5}{2} nR \ln \frac{1}{2^{3/5}} = -\frac{\cancel{5}}{2}\frac{3}{\cancel{5}} nR \ln 2 =$

$$= -\frac{3}{2} nR \ln 2$$

$$\Delta S_{gas} = 0 + \frac{3}{2} nR \ln 2 - \frac{3}{2} nR \ln 2 = 0 \qquad \boxed{\Delta S_{gas} = 0}$$

FOKUAK $\qquad Q = -Q_{foku}$

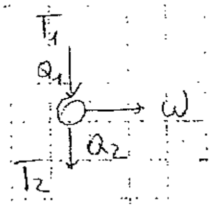
- $\Delta S_{T_0} = \int \frac{-\delta Q}{T} = T\,kte = \frac{1}{T_0}\int -\delta Q = \frac{1}{T_0}\cdot\left[-C_P (T_A - T_C)\right] = \frac{-\frac{5}{2}\cancel{T_0}(1 - 2^{3/5})nR}{\cancel{T_0}} = 1'29\, nR$

- $\Delta S_{2T_0} = \int \frac{\delta Q}{T} = T\,kte = \frac{1}{2T_0}\int \delta Q = \frac{1}{2T_0}\cdot\left[-C_V (T_B - T_A)\right] = \qquad \boxed{\Delta S_{T_0} = 1'29\, nR}$

$$= \frac{-\frac{3}{2} nR \cancel{T_0}}{2\cancel{T_0}} = -\frac{3}{4} nR = -0'75\, nR \qquad \boxed{\Delta S_{2T_0} = -0'75\, nR}$$

$$\Delta S_{unib} = \Delta S_{gas} + \Delta S_{T_0} + \Delta S_{2T_0} = 0'54\, nR \qquad \boxed{\Delta S_{unib} = 0'54\, nR}$$

5 (4)



Carnot-en motorea $\longrightarrow$ $W = 1000\ J$

$T_1 = 400\ K \qquad T_2 = 300\ K$

2) $\eta_{carnot} = 1 - \frac{T_2}{T_1} = 1 - \frac{Q_2}{Q_1} \longrightarrow \frac{T_2}{T_1} = \frac{Q_2}{Q_1} \longrightarrow \boxed{Q_1 = \frac{T_1 Q_2}{T_2} = 4000\ J.}$

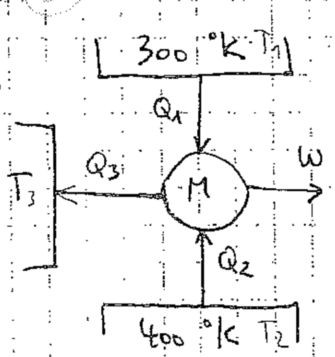
1) Motore batek $\longrightarrow$ $Q_1 = W + Q_2$

$$\frac{T_1 Q_2}{T_2} = W + Q_2 \longrightarrow T_1 Q_2 = T_2 W + T_2 Q_2$$

$$Q_2 (T_1 - T_2) = T_2 W \longrightarrow Q_2 = \frac{T_2 W}{T_1 - T_2} = 3000\ J$$

$Q_2 = 3000\ J$ emango dizkio foku hotzari
$Q_1 = 4000\ J$ hartuko ditu foku berotik

6 (5)



$Q_1 = 300\ J \qquad Q_2 = 200\ J \qquad W = 100\ J$

$$Q_1 + Q_2 + Q_3 = W$$

$$Q_3 = W - Q_2 - Q_1 = -400\ J \qquad \boxed{Q_3 = -400\ J}$$

$$\Delta S_{tot} = \Delta S_{T_1} + \Delta S_{T_2} + \Delta S_{T_3} = \frac{Q_1}{T_1} + \frac{Q_2}{T_2} + \frac{Q_3}{T_3} \geq 0$$

Berdintza gertatuko da (=) prozesu guztiak itzulgarriak badira, eta itzulgarriak direnez, $\boxed{\Delta S_{tot} = 0}$

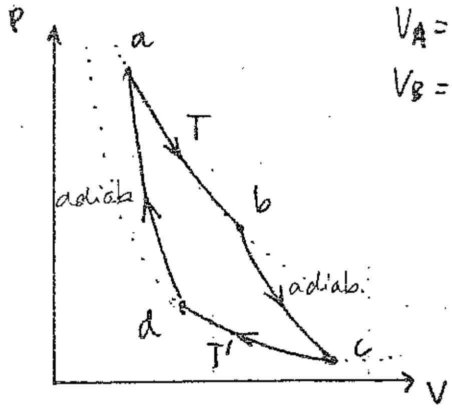
$$\frac{Q_1}{T_1} + \frac{Q_2}{T_2} + \frac{Q_3}{T_3} = 0 \longrightarrow T_3 = \frac{Q_3}{-\frac{Q_1}{T_1} - \frac{Q_2}{T_2}} = 267\ K$$

$$\boxed{T_3 = 267\ °K}$$

6/ 7 Carnot-en motorea $T = 373\ K$ $T' = 273\ K$ $n = 1\ mol$ G. monoatomikoa $\rightarrow \gamma = 5/3$

$V_A = 1\ l = 1 \cdot 10^{-3}\ m^3$ $R = 8'31 \frac{J}{K \cdot mol}$ $P\ (Pa)$

$V_B = 2\ l = 2 \cdot 10^{-3}\ m^3$



a)

(A) $V_A = 1 \cdot 10^{-3}\ m^3$ $T_A = 373\ K$ $P_A = \frac{nRT_A}{V_A} = 3'10 \cdot 10^6\ Pa$

(B) $V_B = 2 \cdot 10^{-3}\ m^3$ $T_B = 373°K$ $P_B = \frac{nRT_B}{V_B} = 1'55 \cdot 10^6\ Pa$

(C) $V_C = 3'19 \cdot 10^{-3}\ m^3$ $T_C = 273°K$ $P_C = 7'10 \cdot 10^5\ Pa$

adiab $\longrightarrow$ $PV^\gamma = kte$ $\longrightarrow$ $P_C = \frac{P_B V_B^{5/3}}{V_C^{5/3}} = \frac{P_B V_B^{5/3}}{\left(\frac{RT_C}{P_C}\right)^{5/3}}$; $P_C^{1-5/3} = P_B \left(\frac{V_B}{RT_C}\right)^{5/3}$

$$P_C^{-2/3} = P_B \left(\frac{V_B}{RT_C}\right)^{5/3} \longrightarrow P_C = \left[P_B\left(\frac{V_B}{RT_C}\right)^{5/3}\right]^{-3/2} = 7'10 \cdot 10^5\ Pa$$

$$V_C = \frac{RT_C}{P_C} = 3'19 \cdot 10^{-3}\ m^3$$

(D) $V_D = 1'6 \cdot 10^{-3}\ m^3$ $T_D = 273°K$ $P_D = 1'42 \cdot 10^6\ Pa$

adiab $\longrightarrow$ $PV^\gamma = kte$ $\longrightarrow [\ \dots\ ] \longrightarrow$ $P_D = \left[P_A\left(\frac{V_A}{RT_D}\right)^{5/3}\right]^{\frac{1}{1-\gamma}} = 1'42 \cdot 10^6\ Pa$

$$V_D = \frac{RT_D}{P_D} = 1'6 \cdot 10^{-3}\ m^3$$

b)

$$W_T = W_{AB} + W_{BC} + W_{CD} + W_{DA}$$

- $W_{AB} = \int P\,dV = \int \frac{nRT\,dV}{V} = nRT \ln \frac{V_B}{V_A} = RT \ln 2 = 2148'5\ J$

- $W_{BC} = \cancel{Q_{BC}}^{\,0} - \Delta U_{BC} = -c_V(T_C - T_B) = -\frac{3}{2} nR(T_C - T_B) = 1246'5\ J$

- $W_{CD} = \int P\,dV = \int \frac{nRT\,dV}{V} = nRT \ln \frac{V_D}{V_C} = -1565'4\ J$

$$\bullet\ W_{DA} = \cancel{Q_{DA}}^{0} - \Delta U_{DA} = -c_v(T_A - T_D) = -\frac{3}{2}nR(T_A - T_D) = -1246'5\ J$$

$$W_T = W_{AB} + W_{BC} + W_{CD} + W_{DA} = 583'1\ J \qquad \boxed{W_T = 583'1\ J}$$

c)

$$Q_{ab} = \cancel{\Delta U_{ab}}^{0} + W_{ab} = 2148'5\ J \qquad Q_{bc} = 0\ J \qquad \boxed{Q_{torg} = 2148'5\ J}$$

gasak kanpotik hartu

$$Q_{cd} = \cancel{\Delta U_{cd}}^{0} + W_{cd} = -1565'4\ J \qquad Q_{da} = 0\ J$$

gasak foku hotzari eman

$$\eta = \frac{W_T}{Q_{torg}} = \frac{583'1\ J}{2148'5\ J} = 0'27$$

$$\eta_{max} = 1 - \frac{T'}{T} = 0'27$$

errendimendua altuena da máximoa da

(8)

5 ZP $\qquad 1\ ZP = 735\ W$

$Q = 4500\ kcal/h$

$T = 1400\ °C$

$$5\ \cancel{ZP} \cdot \frac{735\ W}{1\ \cancel{ZP}} = 3675\ W\ \left(\frac{J}{s}\right)$$

$T' = 300\ °C$

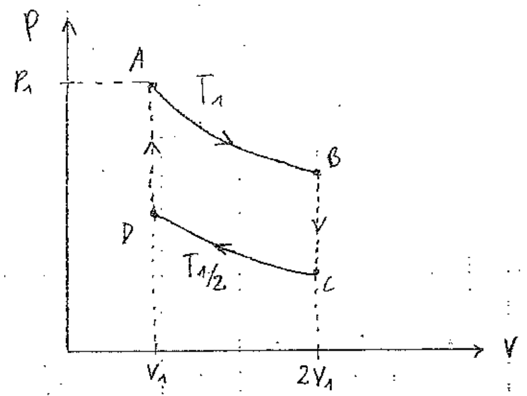
$$\frac{Q}{t} = 4500\ \frac{kcal}{h} \cdot \frac{1\ h}{3600\ s} \cdot \frac{4'18 \cdot 10^3\ J}{1\ kcal} = 5225\ J/s\ (W)$$

$$\eta = \frac{W}{Q_{torg}} = \frac{3675\ W}{5225\ W} = 0'70 \qquad \%70'3$$

$$\eta_{max} = 1 - \frac{T'}{T} = 1 - \frac{300+273}{1400+273} = 0'65 \qquad \%65'7$$

$\eta_{max} < \eta$ baino? $\boxed{\text{Ezinezkoa da!}}$

9 Gas diatomiko bat $\longrightarrow$ $C_V = \frac{5}{2} nR$ $C_P = \frac{7}{2} nR$



$PV = nRT$

$P_1 = \frac{nRT_1}{V_1}$

$P_B = \frac{nRT_1}{2V_1} = \frac{1}{2} P_1$

$P_C = \frac{nR\frac{T_1}{2}}{2V_1} = \frac{1}{4} P_1$

$P_D = \frac{nR\frac{T_1}{2}}{V_1} = \frac{1}{2} P_1$

a) $W_T = W_{AB} + W_{BC} + W_{CD} + W_{DA}$

$\cdot W_{AB} = \int P\,dV = \int \frac{nRT}{V} dV = nRT_1 \ln \frac{V_B}{V_A} = nRT_1 \ln 2 = \ln 2\, P_1 V_1$

$\cdot W_{BC} = 0$

$\cdot W_{CD} = \int P\,dV = \int \frac{nRT}{V} dV = nR\frac{T_1}{2} \ln \frac{V_D}{V_C} = nR\frac{T_1}{2} \ln \frac{1}{2} = \frac{1}{2} \ln \frac{1}{2} \cdot P_1 V_1$

$\cdot W_{DA} = 0$

$W_T = \left(\ln 2 + \frac{1}{2} \ln \frac{1}{2}\right) P_1 V_1 = 0'35\, P_1 V_1$ $\boxed{W_T = 0'35\, P_1 V_1}$

b) T_1-etik xurgatutako beroa: Q_{DA}