ADVANCED NUMERICAL METHODS

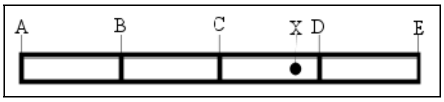
Degree in Industrial Technology Engineering

MAY 29, 2017

TIME: 3 hours **29 points total**

N.B. Exercises needing a calculator should be solved with rounding to 6 significant digits.

1.- A body moving along a 10 m-long underground pipe emits a signal whose intensity could be measured at 5 equally-spaced points according to the scheme below; the intensities obtained are the ones shown in the table.



A	B	C	D	E
1.61534	2.18174	2.25203	1.98124	1.56423

One wants to estimate the intensity of the signal at point X, located at a distance of 2 m from the pipe's midpoint, via interpolation.

a) From the estimation of truncation errors, decide whether the quadratic or the cubic interpolation is more advisable. (3 points)

b) Using a numerically optimal method, and according to the answer to the previous section, estimate the intensity of the signal at point X. (2 points)

2.- a) We know the cubic spline with boundary conditions determined by the nodes $x_0 < x_1 < \ldots < x_n$ and the values $f(x_0), f(x_1), \ldots, f(x_n), f'(x_0), f'(x_n)$ is optimal. Explain in what sense, and state precisely the corresponding theoretical result. (1.5 points)

b) One wants to build a quadratic spline of class C^1 with the nodes $x_0 < x_1 < \ldots < x_n$ and the corresponding ordinates $y_0, y_1, \ldots, y_n$. Can it be done ensuring that $f'(x_0) = f'(x_n) = 0$? Justify the answer. (1.5 points)

3.- Calculate $\int_0^{\pi} e^{\cos x} dx$ with 0.5% 'precision' using Gauss quadrature. (3.5 points)

4.- Calculate $\int_1^3 \left(\int_2^4 (7y^3 + y^2 x) dy \right) dx$ exactly with Newton-Cotes formulas and the least possible computational cost. Justify the choice of the formulas used. (3 points)

5.- The position in space of a moving body is described by the following system of differential equations:

$$\begin{cases} x'(t) = x(t) - y(t) + t\, z(t) \\ y'(t) = -x(t) - y(t) + z(t) \\ z'(t) = t\, x(t) + y(t) - z(t) \end{cases}$$

At the initial instant the body is at point $(1,0,-1)$. Use the **Enhanced Euler (Heun) method** to estimate its position at instants 0.1 and 0.2 (step size $h=0.1$). (5.5 points)

6.- Find a and b for the method $y_n = y_{n-2} + h\,[a\,f_n + b\,f_{n-3}]$ to be convergent with the maximum possible order. Is the method obtained explicit or implicit? Of how many steps? Justify the answers. (3 points)

7.- a) Obtain a numerical differentiation formula, as well as its error term in its simplest form, to estimate $f'(z)$ from the values of f at the nodes x_0, $x_1 = x_0 + h$, $x_2 = x_1 + 2h$, with $z = x_0 + 0.5h$. Do it using Taylor series. (4 points)

b) Justify if the formula obtained in section a) would give the exact value of $f'(x_0 + 0.5h)$ for this function:

$$f(x) = \begin{cases} q(x) & x \le x_1 \\ r(x) & x > x_1 \end{cases}$$

where q and r are polynomials of degrees 2 and 3, respectively. (0.5 points)

c) For $f(x) = \mathrm{Ln}(3x)$, $x_0 = 2$, and precision $\varepsilon = 10^{-4}$, calculate the optimal step size h_{opt} to apply the formula obtained in section a). (1.5 points)