ADVANCED NUMERICAL METHODS

3rd year Bachelor Degree in Industrial Technology Engineering, 5/23/2013

PART 1

1.- a) Knowing that the following data come from a polynomial of degree 3, correct the ones that are wrong.

x_i	-8	-6	-4	-2	0	2	4	6	8	10	12	14	16
$f(x_i)$	-178	-77	-3	49	84	108	122	137	154	179	217	273	352

(3.5p)

b) Obtain the value of the leading coefficient of that polynomial from the data in the table without calculating the polynomial, *stating* the result used to obtain it. (1.5p)

c) Obtain the polynomial using the last data in the table, and evaluate it optimally at $x = 5$. (2p)

2.- Explain how to carry out the optimal storage in memory of the table of divided differences so as to calculate the interpolation polynomial p(x) of a function f(x) at the nodes $x_0, x_1, \ldots, x_n$. Write the algorithm that implements the process. (3.5p)

3.- Given the function

$$S(x) = \begin{cases} 2x^3 + x^2 - 22x + 26 & 0 \le x \le 1 \\ 7x^2 - 28x + 28 & 1 \le x \le 3 \\ -3x^3 + 34x^2 - 109x + 109 & 3 \le x \le 4 \end{cases}$$

study if it is or it is not the natural cubic spline corresponding to the following data table

x_i	0	1	3	4
$f(x_i)$	26	7	7	25

If it is not, say what properties are not satisfied. (4.5p)

4.- a) From the respective simple formulas, obtain the N-time compound trapezoidal and midpoint rules (without the error terms). (3p)

b) Use the compound formulas above with 3 subintervals of the midpoint rule in the variable x and 2 subintervals of the trapezoidal rule in the variable y for the integral $\int_0^3 \left(\int_1^4 f(x,y)\,dy \right) dx$. (2p)

5.- Calculate the *exact* value of the integral $\int_{-2}^{2} \frac{x^3 + 2x + 1}{\sqrt{4 - x^2}}\,dx$ by means of the optimal numerical method. (3p)

TIME: 1 hour and 45 minutes

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PART 2

1.- Given the differential equation

$$y'' - 0.05\cdot y' + 0.15\cdot y = e^{t}$$

subject to the initial conditions $y(0)=0,\ \ y'(0)=1$, take one step of the Runge-Kutta method of order 4 to obtain an approximation of y(0.1) and y'(0.1). Operate with rounding to 6 significant digits. (6p)

2.- Write the Matlab loop that allows to program the Runge-Kutta method of order 4 for a system $\begin{cases} \underset{\sim}{y}' = \underset{\sim}{f}(t,\underset{\sim}{y}) \\ \underset{\sim}{y}(a) = \underset{\sim}{\alpha} \end{cases}$ $t\in[a,b]$. Specify which is the vector containing the points of the discretization of the interval [a,b], as well as the size of the matrix containing the solution to the system. (2p)

3.- a) Define the concept of local truncation error for a single-step explicit method.

b) Define the concept of a consistent explicit single-step method.

c) Under the conditions of the previous section, when is a method said to be of order p?

d) In what sense does a method of order 3 converge faster than a method of order 2? (4p)

4.- a) *Prove* that the formulas of numerical differentiation are unstable. (2p)

b) Given the formula of numerical differentiation

$$f''(z) = \frac{-f(z-2h)+16\cdot f(z-h)-3\cdot f(z)+16\cdot f(z+h)-f(z+2h)}{12h^2} + \frac{h^4}{90}\cdot f^{vi)}(\xi)$$

for some $\xi \in (z-2h, z+2h)$

calculate the optimal step size h for the function

$$f(x) = e^{x} + \cos(x) \ \text{ with } \ x\in[0,2]$$

if the function data are used with imprecisions not greater than 5×10^{-6}. (3p)

TIME: 1 hour and 15 minutes