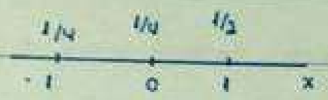
1. Zorizko aldagai diskretu baten probabilitate - banaketa honako hau da:

balioak	$x = -1$	0	1
probabilitateak	$1/4$	$1/4$	$1/2$

a) Kalkulatu x-en lehenengo bi momentuak, funtzio karakteristikoaren metodoaren bitartez.

b) Kalkulatu 3x+2ren funtzio karakteristikoa.

a)



α_1, α_2 funtzio karakteristikoa

$$\Psi(u) = \sum_{\forall x} e^{iux} P(x)$$

$$\Psi(u) = e^{-iu} \cdot \frac{1}{4} + e^{iu0} \cdot \frac{1}{4} + e^{iu} \cdot \frac{1}{2} = \frac{1}{4} e^{-iu} + \frac{1}{4} + \frac{1}{2} e^{iu}$$

5 propietatea kontutan izanik

$$\alpha_K : \underbrace{\alpha^{K)}(0)}_{} \rightarrow \boxed{\Psi^{K)}(0) = i^K \alpha_K}$$

funtzio sortzailea $\rightarrow$ liburuan dagoena

$$\alpha(u) = E(e^{ux})$$
$$\Psi(u) = E(e^{iux})$$

$$\frac{d(\Psi)}{du} = -i \frac{1}{4} e^{-iu} + \frac{1}{2} i e^{iu}$$

$$\frac{d(\Psi')}{du} = +i^2 \frac{1}{4} e^{-iu} + \frac{1}{2} i^2 e^{iu}$$

$$\alpha_1 = \frac{\Psi'(0)}{i^1} = \frac{-\cancel{i}\, 1/4\, \cancel{e^{-i \cdot 0}}^{\,1} + 1/2\, \cancel{i} \cdot \cancel{e^{i \cdot 0}}^{\,1}}{\cancel{i}} = -\frac{1}{4} + \frac{1}{2} = \boxed{\frac{1}{4}}$$

$$\alpha_2 = \frac{\Psi''(0)}{i^2} = \frac{\cancel{i^2} \cdot 1/4\, \cancel{e^{-i0}}^{\,1} + 1/2\, \cancel{i^2}\, \cancel{e^{i0}}^{\,1}}{\cancel{i^2}} = \frac{1}{4} + \frac{1}{2} = \boxed{3/4}$$

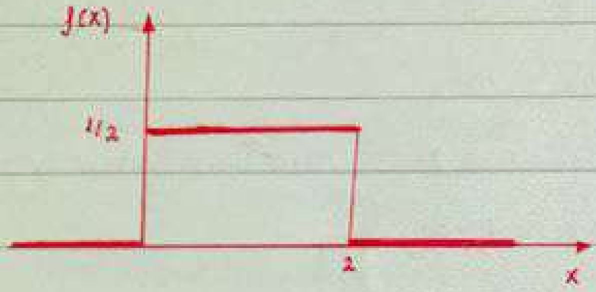
b) $y = 3x + 2 \longrightarrow \Psi_y(u) = \,?$ (3. propietatea)

$$\Psi_y(u) = E(e^{iuy}) = E(e^{iu(3x+2)}) = E(e^{iu \cdot 3x} \cdot \underbrace{e^{2iu}}_{Kte}) = e^{2iu} E(e^{iu3x}) =$$

$$= e^{2iu} \Psi_x(3u) = \boxed{e^{2iu} \left(\frac{1}{4} e^{-3iu} + \frac{1}{4} + \frac{1}{2} e^{3iu} \right)}$$

$$\Psi_x = \frac{1}{4} e^{-iu} + \frac{1}{4} + \frac{1}{2} e^{iu}$$

2. X zorizko aldagai baten dentsitate - funtzioa honako hau da:



a) Zoriz hiru balio dituen laginketa arrunt bat egiten da. Kalkula ezazu 3 balio horietatik bi 0'5 - 1'7 tartean egoteko eta hirugarrena tarte horretatik kanpo egoteko probabilitatea.

b) Kalkula ezazu banaketaren funtzio karakteristikoa.

a) Zoriz 3 balio laginketa

3 — 2 (0'5 - 1'7) tartean (A)
— 1 (Kanpo) " (B)
Probabilitatea

$f(x) = \frac{1}{2}$

$Pr[X \in (0'5 - 1'7)]$

Balio batentzat → $Pr(X \in (0'5 - 1'7)) = \frac{1}{2}(1'7 - 0'5) = 0'6$

$Pr[A\ A\ B] = \binom{3}{2} 0'6^2 \cdot 0'4^1 = \boxed{0'432}$

b) Kalkulatu funtzio karakteristikoa.

$$\varphi_X(u) = E e^{iuX} = \int_0^2 e^{iux} f(x)\,dx = \int_0^2 e^{iux} \cdot \frac{1}{2}\,dx = \left.\frac{e^{iux}}{2iu}\right]_0^2 = \boxed{\frac{e^{2iu} - 1}{2iu}}$$

3. X eta Y zorizko bi aldagai independente dira. Beren banaketa - funtzioak berdinak dira, eta bakoitzaren funtzio karakteristikoa honako hau da:

$$\varphi(u) = a + bu + cu^2 + \dots$$

a) Kalkulatu a, b eta c aldagai horien batezbestekoa 2 eta desbideratze tipikoa 3 izan dadin.

b) Kalkulatu 3x - ren funtzio karakteristikoa

c) Kalkulatu 2x + y - ren funtzio karakteristikoa

$$\varphi(u) = a + bu + cu^2 + \dots$$

a) $m = 2 \quad \sigma = 3$ bada a, b, c?

6. propietatea $\rightarrow \varphi(u) = 1 + \alpha_1 (iu) + \alpha_2 (iu)^2 + \dots$

$\varphi(u) = a + bu + cu^2 + \dots$ } identifikazioa eginez...

$$a = 1 \qquad \alpha_1 = m$$
$$i\alpha_1 = b \qquad \alpha_2 = \frac{m^2 + \sigma^2}{2}$$

$$\begin{matrix} m = 2 \\ \sigma = 3 \end{matrix} \qquad \begin{matrix} a = 1 \\ b = i \cdot 2 \\ c = i^2 \, \dfrac{2^2 + 3^2}{2} = \dfrac{13}{2} i^2 \end{matrix}$$

b) 3x funtzio karakteristikoa

$$\varphi(u)_{3x} = \varphi(3u)_x = 1 + 2i(3u) = \frac{13}{2}(3u^2) + \dots$$

c) $\varphi(u)_{2x+y} = \varphi(u)_{2x} \cdot \varphi(u)_y = \varphi(2u)_x \cdot \varphi(u)_y =$

$$= \left[1 + 2i(2u) - \frac{13}{2}(2u)^2 + \dots\right] \cdot \left[1 + 2iu - \frac{13}{2}u^2 + \dots\right]$$

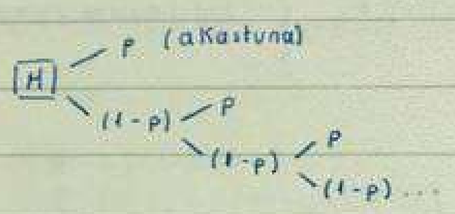
4. Makina batek piezak akastunak egiteko probabilitatea p da (akatsak independienteak dira, eta bata bestearen atzetik gertatzen dira). Makinak lehen pieza akastuna ataldu arte egiten du lan. X zoriko aldagaiak lehen pieza akastuna egin arte ekoitzitako pieza kopurua adierazten du (lehen pieza akastuna barne).

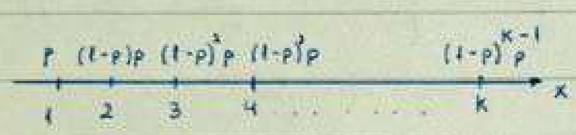
Kalkulatu:

a) Probabilitate - funtzioa.

b) Funtzio karakteristikoa.

c) m eta σ^2, funtzio karakteristikotik abiatuta.





a) $\boxed{P(x) = (1-p)^{x-1} p}$

b) Funtzio karakteristikoa

$$\Psi_x(u) = \sum_{\forall x} e^{iux}(1-p)^{x-1} \cdot p = e^{iu}(1-p)^0 p + e^{2iu}(1-p)\cdot p + e^{3iu}(1-p)^2 \cdot p + \dots$$

$$= p\left[(1-p)^0 e^{iu} + (1-p)e^{2iu} + (1-p)^2 e^{3iu} + \dots\right] = p\cdot e^{iu}\left[1 + (1-p)e^{iu} + (1-p)^2 e^{2iu} + \dots\right]$$

serie geometrikoa $\quad S = \frac{x_0}{1-r}$

$$= \boxed{\frac{p\cdot e^{iu}}{1-(1-p)e^{iu}}}$$

c) m / σ^2

$$\left(\frac{a}{b}\right)' \quad \frac{a'b - ab'}{b^2}$$

$$i\,p\,e^{iu}\left[1-(1-p)e^{iu}\right] - pe^{iu}\left(-(1-p)ie^{iu}\right)$$

$$\Psi'(0) = \frac{ip\left[1-(1-p)\right] + p(1-p)i}{(1-(1-p))^2} = \frac{i(\cancel{p} - \cancel{p} + p^2) + i(p-p^2)}{p^2} = \frac{ip}{p^2} = \frac{i}{p}$$

$$\alpha_1 = m = \frac{\Psi'(0)}{i} = \frac{i/p}{i} = \boxed{1/p}$$

$$\sigma_x^2 = \alpha_2 - \alpha_1^2$$

$$\alpha_2 = \frac{\Psi''(0)}{i^2}$$

$$\frac{d^2\Psi(u)}{du} = \frac{\left[i^2 p e^{iu}(1-(1-p)e^{iu}) + ipe^{iu}\left[-(1-p)ie^{iu}\right] + ipe^{iu}\left[(1-p)ie^{iu}\right] + pe^{iu}(1-p)ie^{iu}\right]}{\left[1-(1-p)e^{iu}\right]^4}$$

$$\cdot \frac{(1-(1-p)e^{iu})^2 - \left[ipe^{iu}\left[1-(1-p)e^{iu}\right] + pe^{iu}\left[(1-p)ie^{iu}\right] \cdot 2\left(-(1-p)ie^{iu}\right)(1-(1-p)e^{iu}\right]}{(1-(1-p)e^{iu})^4}$$

$$\Psi''(0) = \frac{p^3 i^2 - 2(p^2 i)(i - pi)}{p^4} = \frac{p^3 i^2 - 2p^2 i^2 + 2p^3 i^2}{p^4} = \frac{\left[1+(1-p)\right] i^2}{p^2}$$

$$\left[\ \alpha_2 = \frac{\Psi'(0)}{i^2} = \frac{2-p}{p^2}\ \right]$$

$$\left[\ \sigma_x^2 = \alpha_2 - \alpha_1^2 = \frac{2-p}{p^2} - \frac{1}{p^2} = \boxed{\frac{1-p}{p^2}}\right.$$

5. X zorizko aldagaiak banaketa laukizuzena du. Kalkula ezazu bere funtzio karakteristikoa.

funtzio karakteristikoa

$$f(x) = \begin{cases} \frac{1}{b-a} & x \in (a,b) \\ 0 & \text{bestela} \end{cases}$$

$$\Psi(u)_x = Ee^{iux} = \int_a^b e^{iux}\frac{1}{b-a}\,dx = \frac{1}{b-a}\,\frac{1}{iu}\,e^{iux}\Big]_a^b = \boxed{\frac{e^{iub} - e^{iua}}{iu(b-a)}}$$