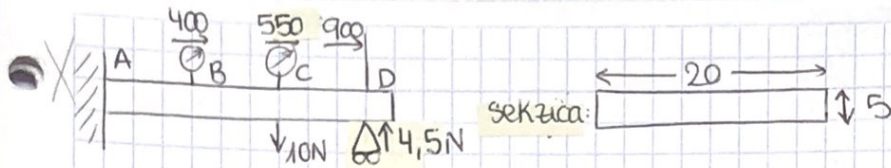
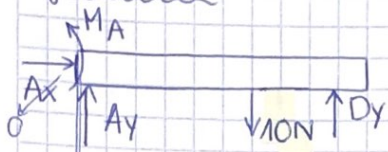


6. GAIA: Makurdura

RESIS



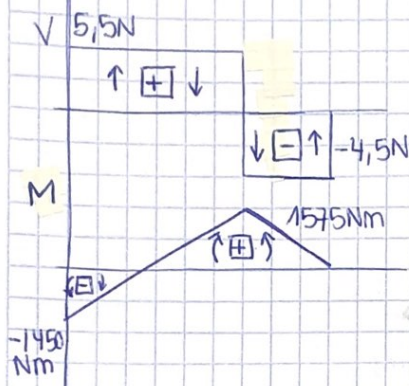
Egitura askatu (solido askearen diagrama)



$$\sum F_y = 0 \rightarrow A_y - 10 + 4,5 = 0 \rightarrow A_y = 5,5 \text{ N}$$

$$\sum M_A = 0 \rightarrow M_A - 10 \cdot 550 + 4,5 \cdot 900 = 0 \rightarrow$$

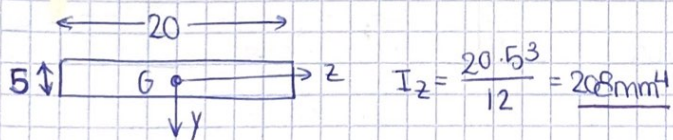
$$\hookrightarrow M_A = 1450 \text{ Nmm}$$



C → sekzio kritikoa

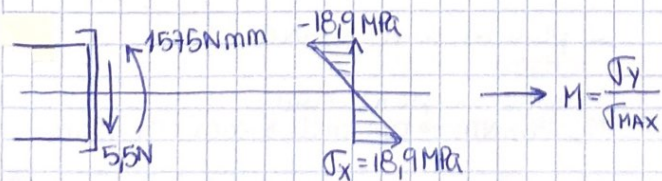
* Ezkerrean → $M_c = 1575 \text{ Nmm} \rightarrow \sigma_x$

↳ $V_c = 5,5 \text{ N} \rightarrow \tau_{xy}$

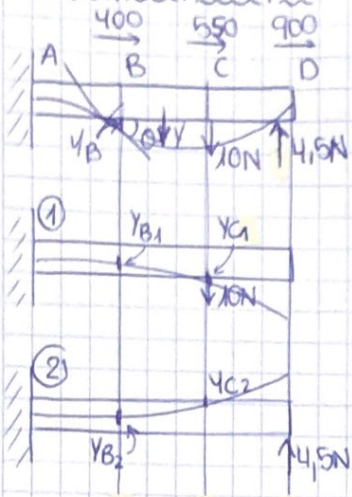


Tentsioak

$$M_z \rightarrow \sigma_x = \frac{M_z y}{I_z} = \frac{1575 \text{ Nmm} \cdot 2,5 \text{ mm}}{208 \text{ mm}^4} = 18,9 \text{ N/mm}^2 = 18,9 \text{ MPa}$$



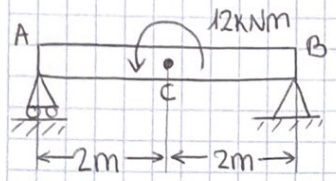
Gaujarpen printzipiala



$$y_B = \delta_B = y_{B1} - y_{B2}$$

$$y_C = y_{C1} - y_{C2}$$

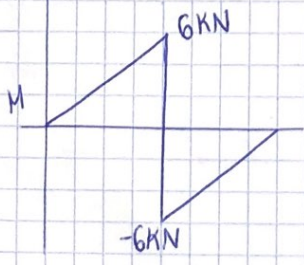
o) $\tau_y = 1100 \text{ MPa}; n = 2$



1) Egitura askatu



2) Solido askearen diagrama



Tentsioak:

Sezio kritikoa \rightarrow zati okerrena aukeratu

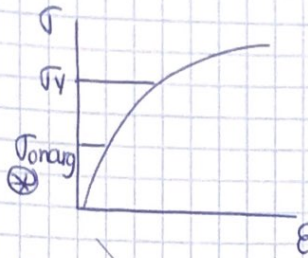
C puntuaren sezio kritikoa:

\rightarrow momentua puntu kritikoa
 $M_z = 6 \text{ kNm} \rightarrow$ berdindu \oplus edo \ominus simetrikoa

delako (es como doblar hacia arriba o hacia abajo, se va a romper igual, lo que cambia es el norabide)

$$\sigma_x = \frac{Mz y}{I_z}$$

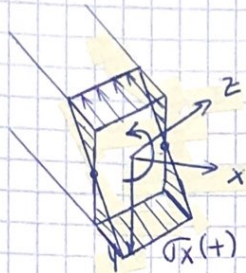
$$\tau_{onarg} = \frac{\tau_y}{2}$$



$$\text{Inertia-momentua: } I_z = \frac{bh^3}{12} = \frac{a^4}{12}$$

$$\tau_{onarg} = \frac{\tau_y}{n} = \frac{1100 \text{ MPa}}{2} = \underline{550 \text{ MPa}}$$

$$550 \text{ MPa} = \frac{6 \cdot 10^6 \cdot a/2}{a^4/12}$$



$$\sum F_x = 0$$

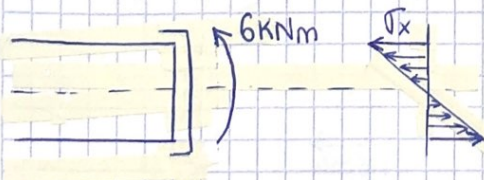
$$\sum M = Mz \rightarrow \text{momentu bat sortzen du}$$

$$N = 0$$

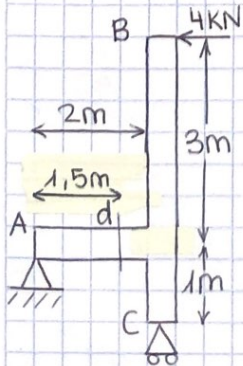
Albo batetik ihusita.



$$a^3 = \frac{12 \cdot 6 \cdot 10^6}{2 \cdot 550} \rightarrow a = 40,3 \text{ mm}$$

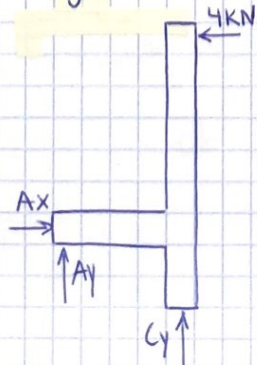


o



$$\tau_y = 1200 \text{ MPa}$$

1) Egitura askatu



$$\sum F_x = 0 \rightarrow Ax - 4 = 0 \rightarrow \underline{Ax = 4 \text{ kN}}$$

$$\sum F_y = 0 \rightarrow Ay + Cy = 0$$

$$\sum M_A = 0 \rightarrow Cy \cdot 2 + 4 \cdot 3 = 0 \rightarrow$$

$$\hookrightarrow \underline{Cy = 6 \text{ kN}}$$

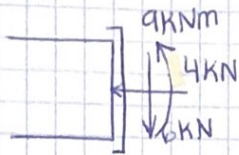
$$\underline{Ay = -6 \text{ kN}}$$

Barne-indarrak:

$$\sum F_x = 0 \rightarrow 4 + N = 0 \rightarrow \underline{N = -4 \text{ kN}}$$

$$\sum F_y = 0 \rightarrow 6 - V = 0 \rightarrow \underline{V = 6 \text{ kN}} \rightarrow z_{xy}$$

$$\sum M = 0 \rightarrow -6 \cdot 1,5 + M = 0 \rightarrow \underline{M = 9 \text{ kNm}}$$

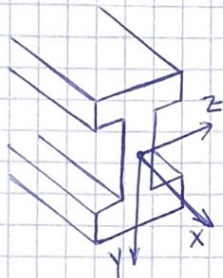


Tentsioak:

$$A = (80 \cdot 8) \cdot 2 + 64 \cdot 8 = \underline{1792 \text{ mm}^2}$$

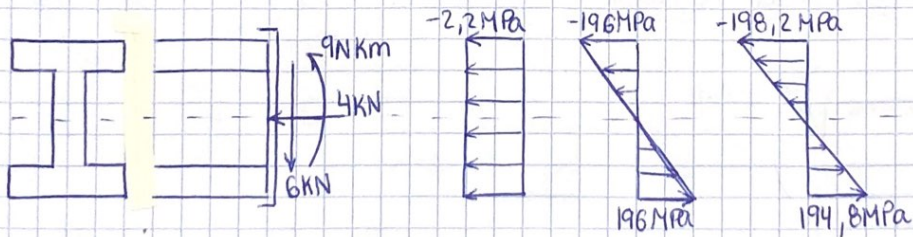
$$\sigma_x = \frac{N}{A} = \frac{-4000}{1792} = \underline{-2,2 \text{ MPa}}$$

$$M \rightarrow \sigma_x = \frac{Mz}{I_z} = \frac{9 \cdot 10^6 \cdot 40}{1,84 \cdot 10^6} = \underline{196 \text{ MPa}}$$



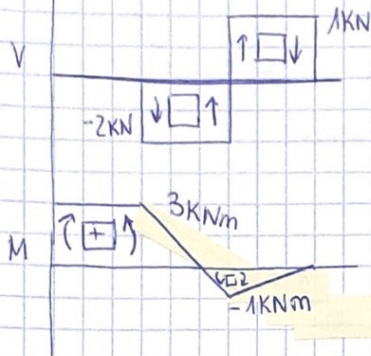
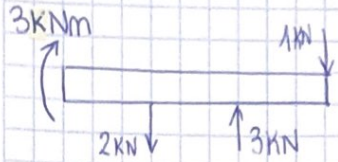
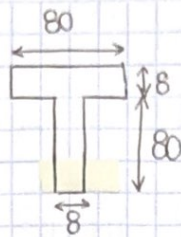
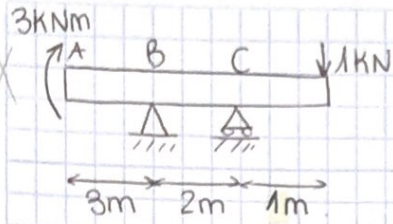
Inertia-momentua: I_z

$$I_z = \frac{80 \cdot 80^3}{12} - 2 \cdot \left(\frac{36 \cdot 64^3}{12} \right) = 1,84 \cdot 10^6 \text{ mm}^4$$

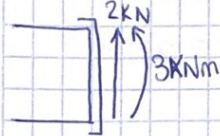


$$\sigma_x = \frac{\sigma_y}{n} \rightarrow n = \frac{1200 \text{ MPa}}{196 \text{ MPa}} = 6,12 \quad (?)$$

z_{xy} (Momentu estatikoak / tentsio eba tentsioak) beti kalkulatu. σ puntuan, maximoa delako eta sekuzaren zabalera (b) alda eta BORTITZA dagoen.

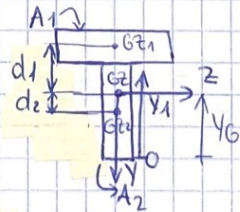


Sehizo kritikoa B ptua (esk)



GZ posizioa:

$$y_0 = \frac{\sum y_i A_i}{\sum A_i} = \frac{80 \cdot 8 \cdot 84 + 80 \cdot 8 \cdot 40}{(80 \cdot 8)^2} = 62 \text{ mm}$$



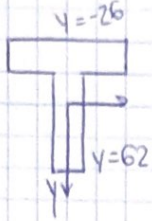
Inertia-momentua: $I_z = I_{z_1} + I_{z_2} = 9,64 \cdot 10^5 \text{ mm}^4$

$$I_{z_1} = \frac{80 \cdot 8^3}{12} + 80 \cdot 8 \cdot (84 + 62)^2 = 3,13 \cdot 10^5$$

$$I_{z_2} = \frac{8 \cdot 80^3}{12} + 80 \cdot 8 \cdot (62 - 40)^2 = 6,51 \cdot 10^5$$

Q_{max} BETI \rightarrow grabitate zentroan

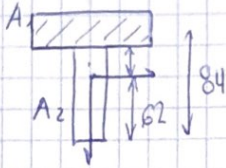
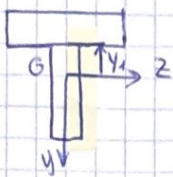
$$M_z = \sigma_x = \frac{M_y}{I_z}$$



$$\sigma_x (y = -26) = \frac{3 \cdot 10^6 (-26)}{9,64 \cdot 10^5} = -53,1 \text{ MPa}$$

$$\sigma_x (y = 62) = \frac{3 \cdot 10^6 \cdot 62}{9,64 \cdot 10^5} = 126,5 \text{ MPa}$$

$$V \rightarrow \tau_{xy} = \frac{VQ}{I_z b}$$



$$y = y_1 \rightarrow Q_1 = A_1 \cdot y_{1G} = 80 \cdot 8 (84 - 62) \rightarrow Q_1 = 14080 \text{ mm}^3$$

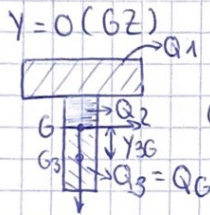
$$\tau_{xy}' = \frac{-2 \cdot 10^3 \cdot 14080}{9,64 \cdot 10^5 \cdot 80} = -0,24 \text{ MPa}$$

$$\tau_{xy}'' = \frac{-2 \cdot 10^3 \cdot 14080}{9,64 \cdot 10^5 \cdot 8} = -2,4 \text{ MPa}$$

b ezberdinak

G plutik A3-ren grabitate zentruarainoko distantzia

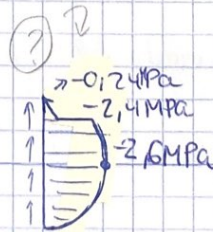
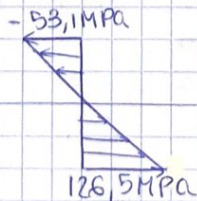
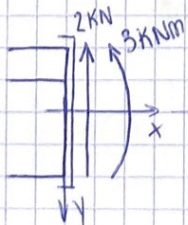
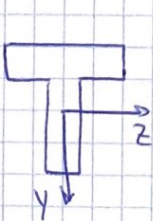
$$Q_G = Q_3 = A_3 \cdot y_{3G} = 8 \cdot 62 \cdot \frac{62}{2} = 15376 \text{ mm}^3$$

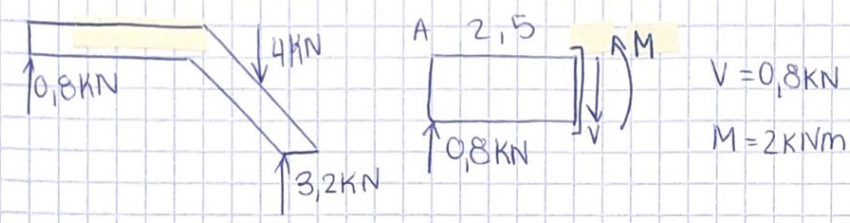
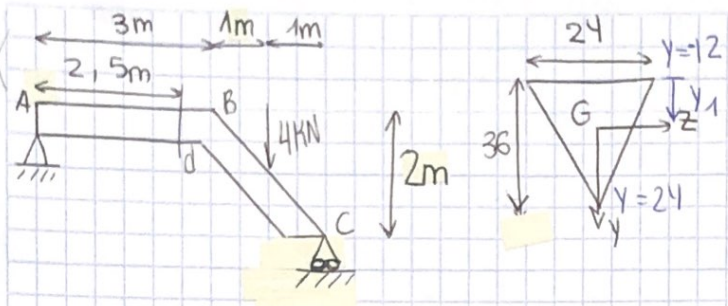


$$Q_G = Q_1 + Q_2$$

$$\tau_{xy}^G = \frac{-2 \cdot 10^3 \cdot 15376}{9,64 \cdot 10^5 \cdot 8} = -2,6 \text{ MPa}$$

Emaitza:





$$V = 0,8 \text{ kN}$$

$$M = 2 \text{ kNm}$$

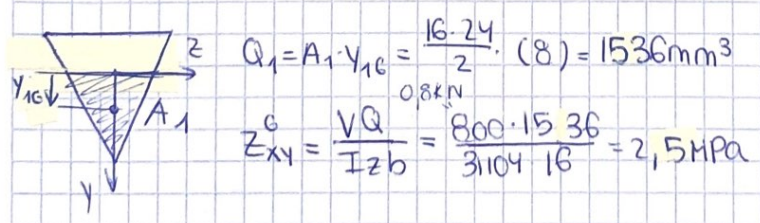
$$y_G = \frac{h}{3} = 12 \text{ mm}$$

$$I_z = \frac{bh^3}{36} = 31104 \text{ mm}^4$$

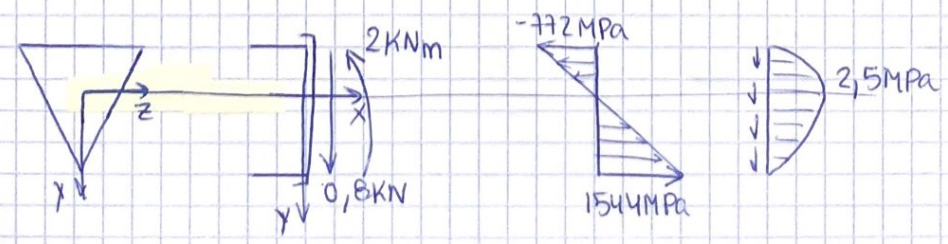
$$M = 2 \text{ kNm} \rightarrow \sigma_x = \frac{My}{I_z}$$

$$\sigma_x (y = -12) = \frac{2 \cdot 10^6 (-12)}{31104} = -772 \text{ MPa}$$

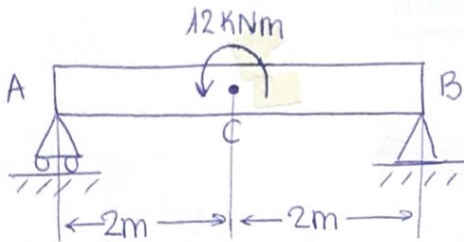
$$\sigma_x (y = 24) = \frac{2 \cdot 10^6 (24)}{31104} = 1544 \text{ MPa}$$



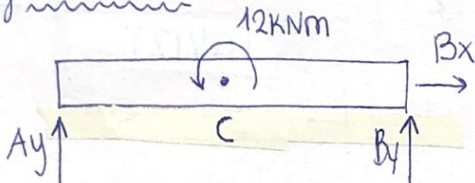
Ematza:



$\sigma_y = 1100 \text{ MPa}; n = 2$



Egitura askatu



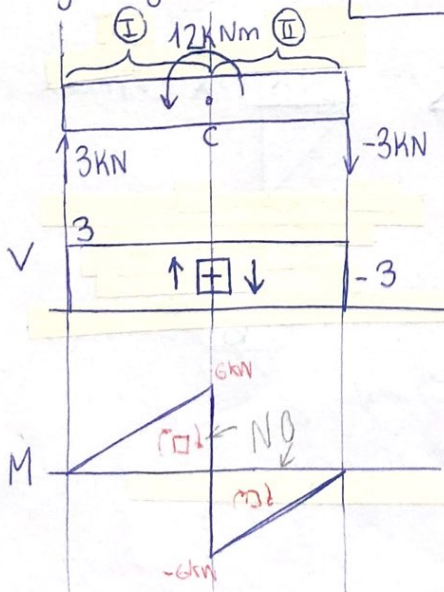
$\Sigma F_x = 0 \rightarrow B_x = 0 \text{ kN} \checkmark$

$\Sigma F_y = 0 \rightarrow A_y + B_y = 0 \rightarrow A_y = -B_y$

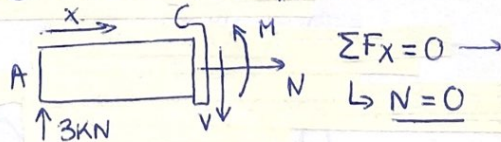
$\Sigma M_C = 0 \rightarrow -2A_y + 12 + 2B_y = 0 \rightarrow$

$\hookrightarrow -2(-B_y) + 12 + 2B_y = 0 \rightarrow 2B_y + 2B_y = -12 \rightarrow 4B_y = -12 \rightarrow B_y = -3 \text{ kN} \checkmark$

$A_y = -B_y = -(-3 \text{ kN}) = 3 \text{ kN} = A_y \checkmark$



◦ I eremua ($x \in (0, 2)$)



$\Sigma F_x = 0 \rightarrow$

$\hookrightarrow N = 0$

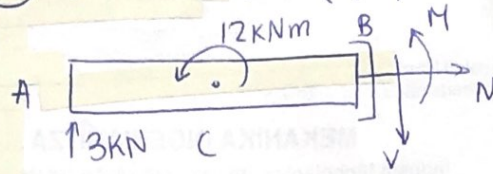
$\Sigma F_y = 0 \rightarrow 3 - V = 0 \rightarrow V = 3 \text{ kN} \checkmark$

$\Sigma M = 0 \rightarrow -3x + M = 0 \rightarrow M = 3x$

$x = 0 \rightarrow M = 0 \text{ kNm}$

$x = 2 \rightarrow M = 6 \text{ kNm} \checkmark$

II) eremua ($x \in (2, 4)$)



$$\sum F_x = 0 \rightarrow N = 0$$

$$\sum F_y = 0 \rightarrow 3 - V = 0 \rightarrow V = 3 \text{ kN} \checkmark$$

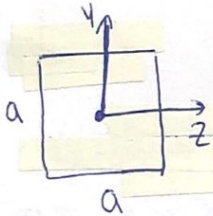
$$\sum M = 0 \rightarrow M + 12 - 3x = 0 \rightarrow M = 3x - 12$$

$$x = 2 \rightarrow M = 3 \cdot 2 - 12 = 6 - 12 = -6 \text{ kNm} \checkmark$$

$$x = 4 \rightarrow M = 3 \cdot 4 - 12 = 12 - 12 = 0 \text{ kNm}$$

C \rightarrow ptu kritikoa $\begin{cases} M = 6 \text{ kNm} \\ V = 3 \text{ kN} \end{cases}$

$$\sigma_{\text{max}} = \frac{\sigma_y}{n} = \frac{1100 \text{ MPa}}{2} = 550 \text{ MPa} \checkmark$$



$$I_z = \frac{b \cdot h^3}{12} = \frac{a \cdot a^3}{12} = \frac{a^4}{12}$$

$$\sigma_{\text{max}} = \frac{M \cdot y_{\text{Gz}}}{I_z}$$

$$550 = \frac{6 \cdot 10^6 \cdot (a/2)}{a^4/12}$$

$$\rightarrow a =$$

(...)

