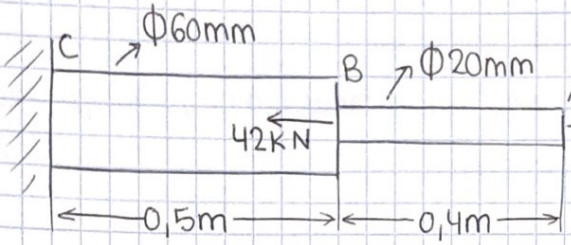
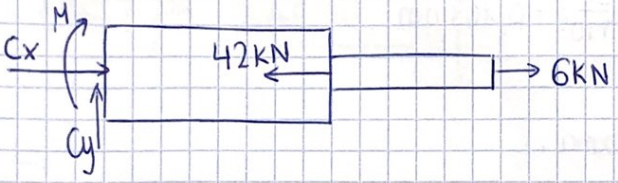
A, B, C barra aluminiozkoa da. Kalkulatu A eta B puntuen desplazamendua. E=70GPa



Hainbat zati badaude, aztertu zatika

$$\delta = \Sigma \frac{N_i \cdot L_i}{E_i \cdot A_i}$$

1) Barrak askatu → erreakzioak



$\Sigma Fx = 0$

$Cx - 42 + 6 = 0$

$Cx = 36KN$

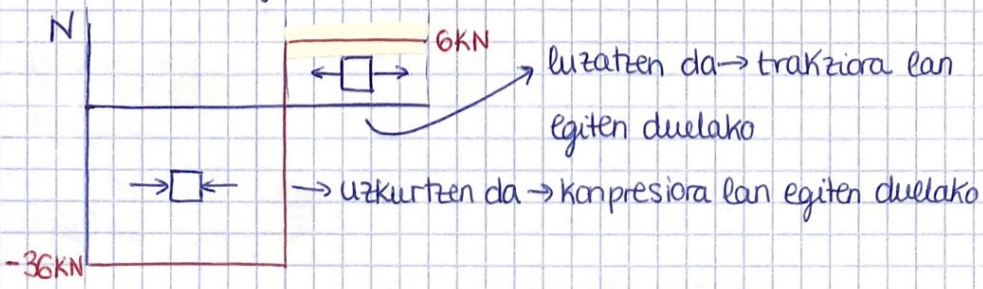
$\Sigma Fy = 0$ $\quad \Sigma M_C = 0 \rightarrow \overset{\curvearrowleft}{M_C} = 0$

Solido askearen diagrama:

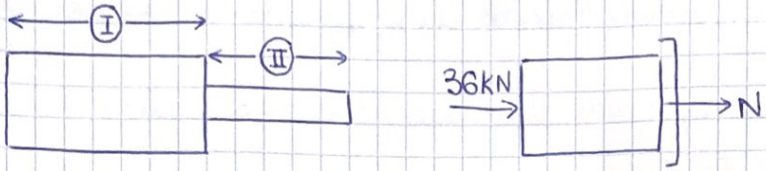
sartu (konpresioa) (-)

atera (trakzio) (+)

Barne indarren diagrama edo N diagrama (mutur bakoitza sekzioa da):



Ez da areka, barne indarrak



$\Sigma F_X = 0 \rightarrow N = -36KN$ (KIE beti)

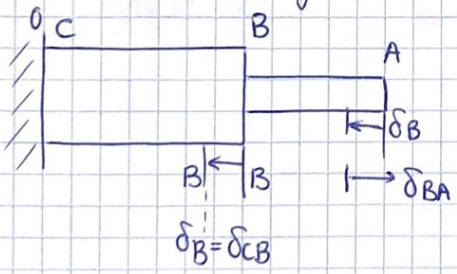
Karga maximoa $\left(\sigma = \frac{42000}{A}\right)$ isurpen tentsioarekin (σ_y) konparatu segurtasun koefizientea ateratzeko

◉ Desplazamenduak: ⚠ Newtonetan, ez KN

$$\delta_{CB} = \frac{N_{AC} \cdot L_{CB}}{E \cdot A_{CB}} = \frac{-36000 \cdot 500}{70 \cdot 10^3 \cdot \pi \cdot 30^2} = -0,091mm$$

$$\delta_{BA} = \frac{N_{BA} \cdot L_{BA}}{E \cdot A_{BA}} = \frac{6000 \cdot 400}{70 \cdot 10^3 \cdot \pi \cdot 10^2} = 0,109mm$$

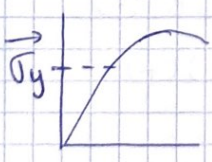
✳ Desplazamendu diagrama:



$\delta_A = \delta_B + \delta_{BA}$

$\delta_B = \delta_{CB} = -0,091mm \leftarrow$

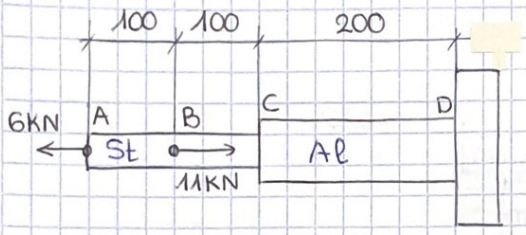
$\delta_A = \overleftarrow{\delta_B} + \overrightarrow{\delta_{BA}} = -0,091 + 0,109 = 0,018mm \rightarrow$



Irudiko euskarri zuzenak bi karga axial jasaten ditu, eta D muturrean lanlatua dago. Euskarria material ezberdineko bi barrez osatua dago: ABC zatia altzairuzkoa da eta $50mm^2$-ko sekzioa dauka; CD barra, berriz, aluminiozkoa da eta bere sekzioa $70mm^2$-koa da. Kalkulatu

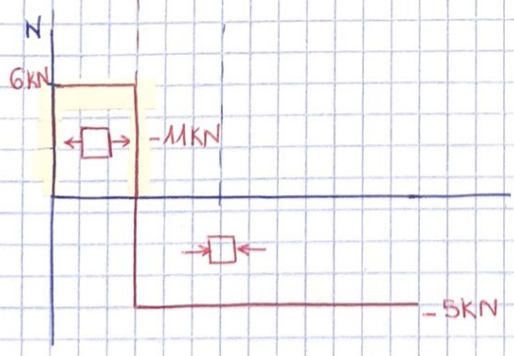
a) A eta B puntuek jasango duten desplazamendu (Datuak: $E_{St}=210GPa$; $E_{Al}=70GPa$).

b) Ze tartetan dago tentsio normal maximoa? Ze balio dauka?



a) 1) Egitura askatu:

(Solido askearen diagrama)



Deformazioak:

AB zatia

$$\delta_{AB}=\frac{N_{AB}\cdot L_{AB}}{E_{AB}\cdot A_{AB}}=\frac{6000\cdot 100}{210\cdot 10^3\cdot 50}=0,057mm$$

↳ mm-tan egiteko MPa jarri behar da.

BC zatia

$$\delta_{BC} = \frac{-5 \cdot 10^3 \cdot 100}{210 \cdot 10^3 \cdot 50} = -0{,}048\text{mm}$$

CD zatia

$$\delta_{CD} = \frac{-5 \cdot 10^3 \cdot 200}{70 \cdot 10^3 \cdot 70} = -0{,}204\text{mm}$$

Desplazamenduak:

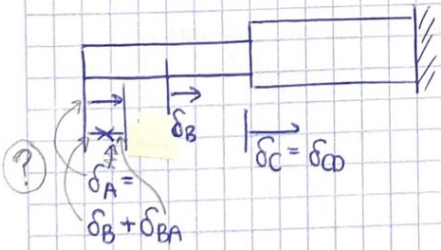
D puntua oinarria

$\delta_C = \delta_{CD} = \overrightarrow{0{,}204\text{mm}}$

$\delta_B = \delta_C + \delta_{BC} = 0{,}204 + \overleftarrow{0{,}048} = \boxed{\overrightarrow{0{,}252\text{mm}}}$

$\delta_A = \delta_B + \delta_{AB} = \overrightarrow{0{,}252} - \overleftarrow{0{,}057} = \boxed{\overrightarrow{0{,}195\text{mm}}}$

Desplazamenduen diagrama:



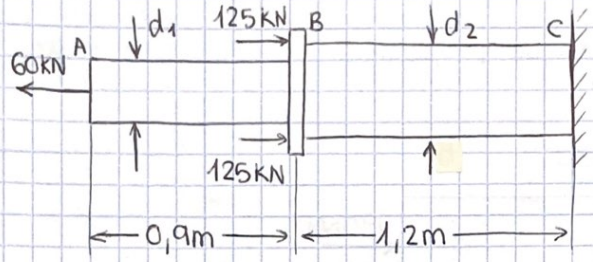
b) $\sigma_{max} = \frac{N_{AB}}{A_{AB}} = \frac{6000}{50} = \boxed{120\text{MPa}}$

2. Irudiko AB eta BC zilindro barne-beteak B puntuan soldatuak daude. Tentsio normalak 150MPa gainditu ezin dituela jakinda, zehaztu:

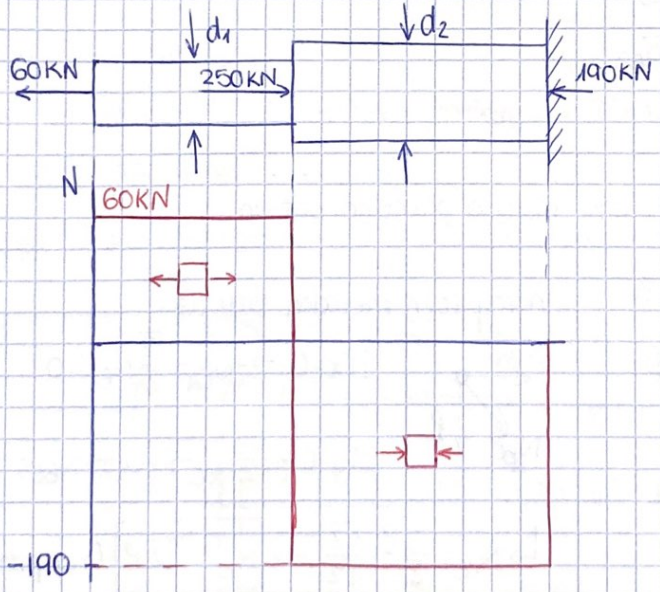
a) d_1 eta d_2 diametro minimo onargarriak.

Diametro horiek erabilita, kalkulatu:

b) A eta B puntuen desplazamendua (200GPa)



a) Egitura askatu: solido askearen diagrama



Diametroak:

AB

$$\sigma_{max_y} = \frac{N_{AB}}{A_{AB}} \rightarrow 150MPa = \frac{60 \cdot 10^3}{\pi r^2} \rightarrow \boxed{d_1 = 22,54mm} \rightarrow A_1 = 127cm^2$$

BC

$$\sigma_{max_y} = \frac{N_{BC}}{A_{BC}} \rightarrow \boxed{d_2 = 40,16mm} \rightarrow ABC = 408mm^2$$

b) Deformazioak:

$$\delta_{AB} = \frac{N_{AB} \cdot L_{AB}}{E_{AB} \cdot A_{AB}} = \frac{60 \cdot 10^3 \cdot 900}{200 \cdot 10^3 \cdot 127} = 0{,}68mm$$

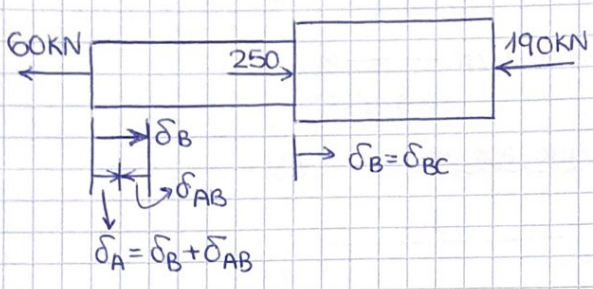
$$\delta_{BC} = \frac{N_{BC} \cdot L_{BC}}{E_{BC} \cdot A_{BC}} = \frac{-190 \cdot 10^3 \cdot 1200}{200 \cdot 10^3 \cdot 403} = -0{,}9mm$$

Desplazamenduak:

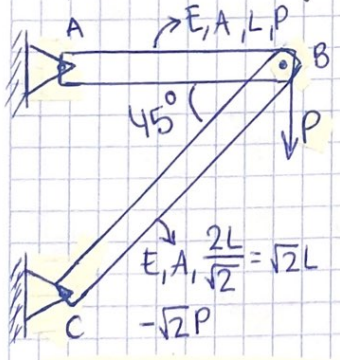
$\delta_B = \delta_{BC} = \boxed{0{,}9 \overrightarrow{mm}}$

$\delta_A = \delta_B + \delta_{AB} = -0{,}\overrightarrow{9} + \overleftarrow{0{,}68} = \boxed{\overrightarrow{0{,}22mm}}$

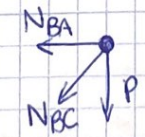
Desplazamendu diagrama:



Desplazamendu diagrama metodo praktiko bat da



Korapiloen metodoa erabiliz:



$\Sigma Fx = 0 \rightarrow -N_{BA} - \frac{\sqrt{2}}{2} N_{BC} = 0$

$\Sigma Fy = 0 \rightarrow \frac{-\sqrt{2}}{2} N_{BC} - P = 0 \rightarrow N_{BC} = \frac{-P2}{\sqrt{2}} \rightarrow$

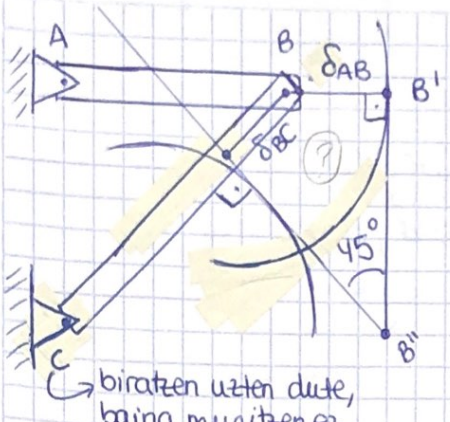
$\hookrightarrow N_{BC} = \frac{-2P}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} \rightarrow \boxed{N_{BC} = -\sqrt{2}P}$ (Konpresioa)

$-N_{BA} - \frac{\sqrt{2}}{2}(-\sqrt{2}P) = 0 \rightarrow \boxed{N_{BA} = P}$ (Traktzioa)

Deformazioak:

AB: $\delta_{AB} = \frac{N_{AB} \cdot L_{AB}}{E \cdot A} = \boxed{\frac{PL}{EA}}$

BC: $\delta_{BC} = \frac{N_{BC} \cdot L_{BC}}{E \cdot A} = \frac{-\sqrt{2}P \cdot \sqrt{2} \cdot L}{EA} = \boxed{\frac{-2PL}{EA}}$



biratzen uzten dute, baina mugitzen ez.

B-n biratu edo desplazatuko dira, bisagra bat delako eta A eta C finkoak daudelako.

SUPOSATU daiteke BIRAKETA ⊥ direla

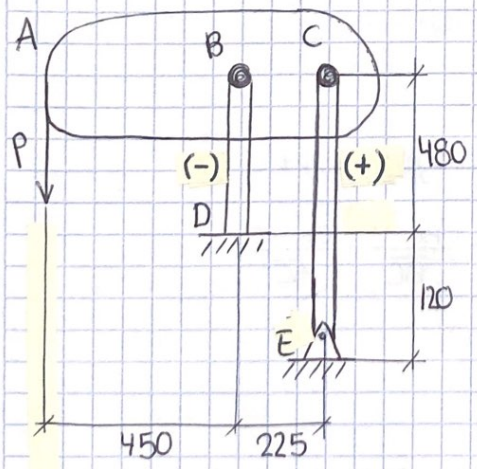
Desplazamenduak:

$$\delta_B^H = \delta_{AB}^{\rightarrow} = \frac{PL}{EA}$$

BC uzkurtu

$$\delta_B^V = \delta_{AB} + \frac{\delta_{BC}}{\sqrt{2}/2} = \frac{PL}{EA} + \frac{2\sqrt{2}PL}{EA} \downarrow$$

X Irudiko egitura ABC habe horizontalak eta BD, CE bertikalek osatzen dute ABC habea zurruntzat hartuz, kalkulatu P karga maximo onargarria A puntuko desplazamendu maximoa 1mm-koa izan dadin.



CE trakziora egingo du lan, eta BD konpresiora egingo du lan.

$A_{BD} = 1020 mm^2$

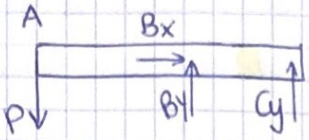
$A_{CE} = 520 mm^2$

$E = 205 GPa$

$\delta_A \rightarrow 1mm \rightarrow P_{max}?$

ABC → zurruna

1) ABC habea askatu



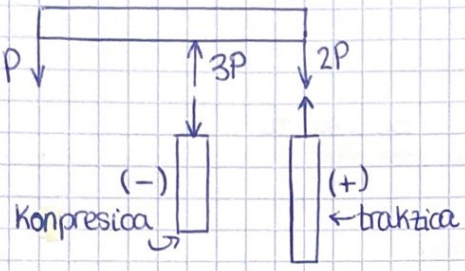
$\Sigma Fx=0 \rightarrow Bx=0$

$\Sigma Fy=0 \rightarrow -P+By+Cy=0$

$\Sigma M_B=0 \rightarrow P\cdot 450+Cy\cdot 225=0 \rightarrow \underline{Cy=-2P}$

↳ $By=-Cy+P=+2P+P \rightarrow \underline{By=3P}$

2) Solido askearen diagrama



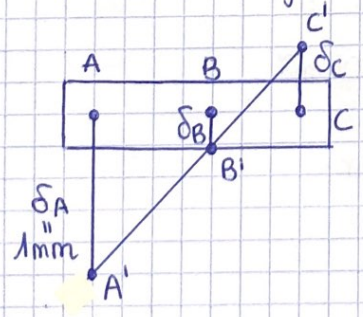
3) Deformazioak

$$\delta_{BD}=\frac{N_{BD}\cdot L_{BC}}{E_{BD}\cdot A_{BD}}=\frac{-3P\cdot 480}{1020\cdot 205\cdot 10^3}=\underline{-6{,}89\cdot 10^{-6}P\ mm}$$

(480 mm; 10^3 → MPa)

$$\delta_{CE}=\frac{N_{CE}\cdot L_{CE}}{E_{CE}\cdot A_{CE}}=\frac{2P\cdot 600}{205\cdot 10^3\cdot 520}=\underline{1{,}12\cdot 10^{-5}P\ mm}$$

4) Desplazamendu diagrama



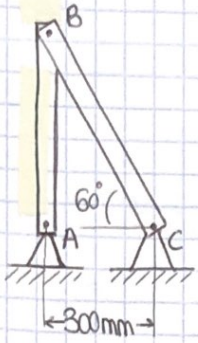
Hiruko enegela erabiliz:

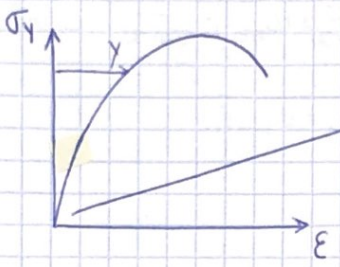
$$\frac{\delta_A-\delta_B}{\overline{AB}}=\frac{\delta_B-\delta_C}{\overline{BC}}=\frac{\delta_A-\delta_C}{\overline{AC}}$$

Bi aukeratuz eta beherantz positibo dela kontuan izanik:

$$\frac{\delta_A-\delta_B}{\overline{AB}}=\frac{\delta_A-\delta_C}{\overline{AC}} \rightarrow \frac{1-6{,}89\cdot 10^{-6}P}{450}=\frac{1-(-1{,}12\cdot 10^{-5}P)}{675} \rightarrow \boxed{P_{max}=23200N}$$

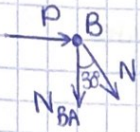
$\sigma = 400MPa$; $n = 4$; $E = 200GPa$; $A = 100mm^2$





$\sigma_{onargarri} = \frac{\sigma_Y}{n} = \frac{400}{4} = \underline{100MPa}$

Korapiloen metodoa, B puntan



$\Sigma F_x = 0 \rightarrow P + N_{BC} \cdot \sin 30° = 0 \rightarrow \underline{N_{BC} = -2P}$ (konpresio)

$\Sigma F_y = 0 \rightarrow -N_{BA} - \frac{\sqrt{3}}{2} N_{BC} = 0 \rightarrow -N_{BA} - \frac{\sqrt{3}}{2}(-2P) = 0 \rightarrow$

$\hookrightarrow \underline{N_{BA} = +\sqrt{3}P}$ (traktzioa)

$\sigma_{onar.} = \frac{N}{A} \rightarrow AB \rightarrow 100 = \frac{\sqrt{3}P}{100} \rightarrow P = \frac{100^2}{\sqrt{3}} = \underline{5773,5027N}$

$BC \rightarrow 100 = \frac{2P}{100} \rightarrow P = \frac{100^2}{2} = \underline{5000N}$ → Balio hau erabiliko dugu, ez 5773,5027N-ekoa → B-ren P onargarria/maximoa 5000N-ekoa denez, indar gehiago jartzen badugu, puskatuko da.

Deformazioak

$N_{AB} = \sqrt{3} \cdot 5000 = 8660$

$L_{AB} = \frac{300}{tg\,30°} = 520mm$

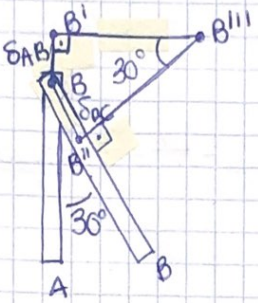
$\delta_{AB} = \frac{N_{AB} \cdot L_{AB}}{EA} = \frac{8660 \cdot 520}{200 \cdot 10^3 \cdot 100} = \underline{0,22mm}$

$N_{BC} = 2 \cdot 5000 = 10000$

$L_{BC} = \frac{300}{\sin 30} = 600mm$

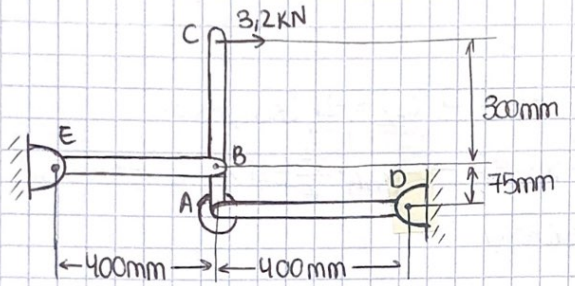
$\delta_{BC} = \frac{N_{BC} \cdot L_{BC}}{EA} = \frac{10000 \cdot 600}{200 \cdot 10^3 \cdot 100} = \underline{0,3mm}$

Desplazamenduu diagrama



$$\delta_B^H = \left(\frac{\delta_{BC}}{\cos 30^\circ} + \delta_{AB}\right) \cdot \frac{1}{\tan 30} = 0{,}98mm \rightarrow$$

$$\delta_B^V = \delta_{AB} = 0{,}22mm \uparrow$$

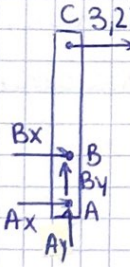


Sekzioa = 6x18mm

Isurpen-tentsioa = 600MPa

E = 200GPa

Habea isolatu



$\Sigma Fx = 0 \rightarrow Bx + 3{,}2 + Ax = 0$

$\Sigma Fy = 0$

$\Sigma M_A = 0 \rightarrow Bx \cdot 75 + 3{,}2 \cdot 375 = 0 \rightarrow \underline{Bx = -16KN}$

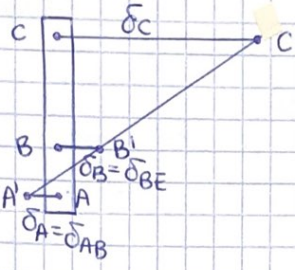
$Ax = -3{,}2 - Bx = \underline{12{,}8KN}$

Deformazioak

$$\delta_{EB} = \frac{N_{BE} \cdot L_{BE}}{EA} = \frac{16000 \cdot 400}{200 \cdot 10^3 \cdot 6 \cdot 18} = \underline{0{,}3mm}$$

$$\delta_{AD} = \frac{N_{AD} \cdot L_{AD}}{EA} = \frac{12800 \cdot 10^3 \cdot 400}{200 \cdot 10^3 \cdot 6 \cdot 18} = \underline{0{,}24mm}$$

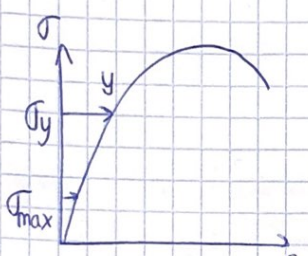
Desplazamendu diagrama



$$\frac{\delta_C - \delta_B}{\overline{CB}} = \frac{\delta_C - \delta_A}{\overline{CA}} = \frac{\delta_B - \delta_A}{\overline{BA}}$$

$$\frac{\delta_C - (-0,24)}{375} = \frac{0,3 - (-0,24)}{75} \rightarrow \frac{\delta_C + 0,24}{375} = \frac{0,54}{75} \rightarrow$$

$$\hookrightarrow \boxed{\delta_C = 2,46 \text{mm} \rightarrow}$$



$$\sigma = \frac{N}{A} \; ; \; N_{max} = N_{EB} = 16000N$$

$$\hookrightarrow \sigma_{max} = \frac{16000}{6 \cdot 18} = 148 MPa$$

$$n = \frac{\sigma_y}{\sigma_{max}} = \frac{\sigma_y}{\sigma_{max}} = \frac{600}{148} = 4$$

Segurtasun-koefizientea handia da (✓)

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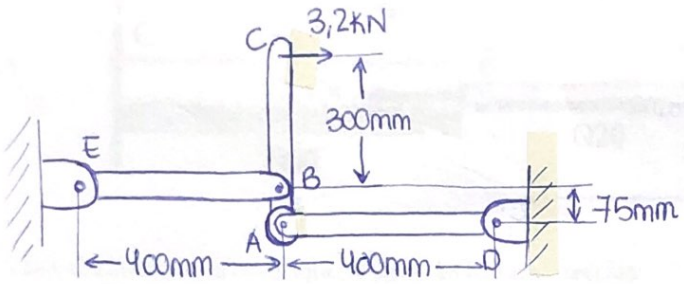
Escuela Universitaria de Ingeniería Técnica Industrial
Eibar

MEKANIKA INGENIARITZA SAILA
Industri Ingeniaritza Teknikorako Unibertsitate Eskola
Eibar

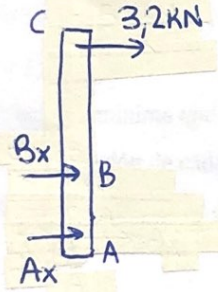


Sekzioa = 6 x 18mm

$\sigma = 600 MPa$

$E = 200 GPa$

1) CBA habea askatu



$\Sigma F_x = 3,2 + Ax + Bx = 0$

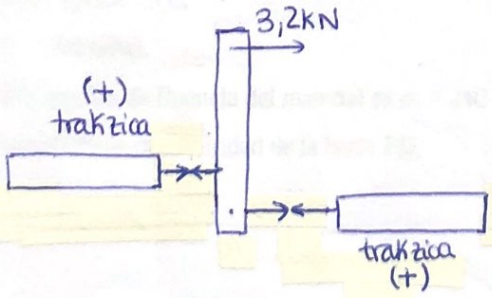
$\Sigma F_y = 0$

$\Sigma M_A = 0 \rightarrow -0,075 \cdot Bx - 3,2 \cdot 0,375 = 0 \rightarrow$

$\hookrightarrow 0,075 Bx = -1,2 \rightarrow \underline{Bx = -16 KN}$

$Ax = -3,2 - (-16) = \underline{12,8 KN}$

2) Solido askearen diagrama

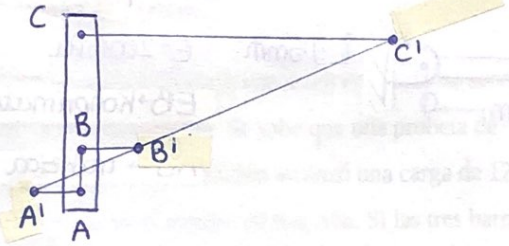


3) Deformazioak

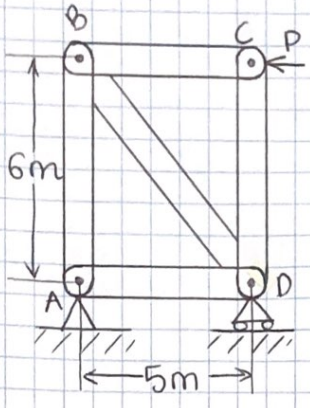
$$\delta_{EB} = \frac{N_{EB} \cdot L_{EB}}{E_{EB} \cdot A_{EB}} = \frac{16000 \cdot 400}{200 \cdot 10^3 \cdot 6 \cdot 18} = 0{,}2963 \text{mm}$$

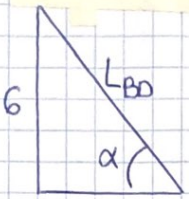
$$\delta_{AD} = \frac{N_{AD} \cdot L_{AD}}{E_{AD} \cdot A_{AD}} = \frac{12800 \cdot 400}{200 \cdot 10^3 \cdot 6 \cdot 18} = 0{,}237 \text{mm}$$

4) Desplazamendu diagrama



3. $A_{BD} = 1920mm^2$; $\delta_{BDmax} = 1,6mm$; $E = 200GPa$; $\sigma_y = 240MPa$
↳ $200 \cdot 10^9 N/m^2 = 200 \cdot 10^3 N/mm^2$



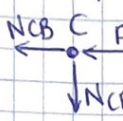


$tg\alpha = \frac{6}{5} \rightarrow \alpha = arctg\left(\frac{6}{5}\right) = \underline{50,19°}$

$\sin\alpha = \frac{6}{L_{BD}} \rightarrow L_{BD} = \frac{6}{\sin\alpha} \rightarrow$

↳ $L_{BD} = 7,81m = \underline{7810,25mm}$

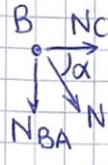
Korapiloen metodoa C puntuan:



$\Sigma F_y = 0 \rightarrow \underline{N_{CD} = 0}$

$\Sigma F_x = 0 \rightarrow \underline{N_{CB} = -P}$

Korapiloen metodoa B puntuan:



$\Sigma F_x = 0 \rightarrow N_{CB} + N_{BD}\cos\alpha = 0 \rightarrow \underline{N_{BD} = \frac{P}{\cos(50,19°)}}$

$\delta_{BDmax} = \frac{N_{BDmax} \cdot L_{BD}}{E \cdot A_{BD}} \rightarrow 1,6mm = \frac{P_{max} \cdot 7810,25mm}{\cos(50,19) \cdot 200 \cdot 10^3 N/mm^2 \cdot 1920mm^2} \rightarrow$

↳ $\boxed{P_{max} = 50360,66N} = P_{onarg}$

$\sigma_{onarg} = E \cdot \frac{\delta_{BDmax}}{L_0} = 200 \cdot 10^3 \cdot \frac{1,6}{7810,25} = \underline{40,97MPa}$

$\sigma_{onarg} = \frac{\sigma_y}{n} \rightarrow n = \frac{240MPa}{40,97MPa} = \boxed{5,86 = n}$