Universidad del País Vasco
Euskal Herriko Unibertsitatea

Departamento de Ingeniería Mecánica
Ingeniaritza Mekanikoa Saila

4. Gaia I

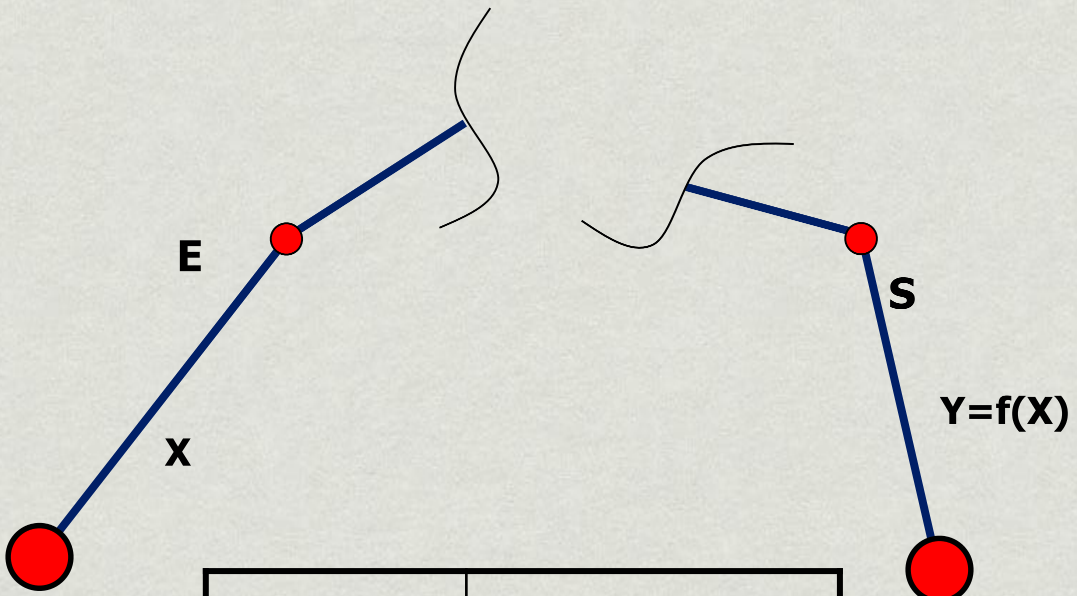
Mekanismo lauen sintesi dimentsionala

Aurkibidea

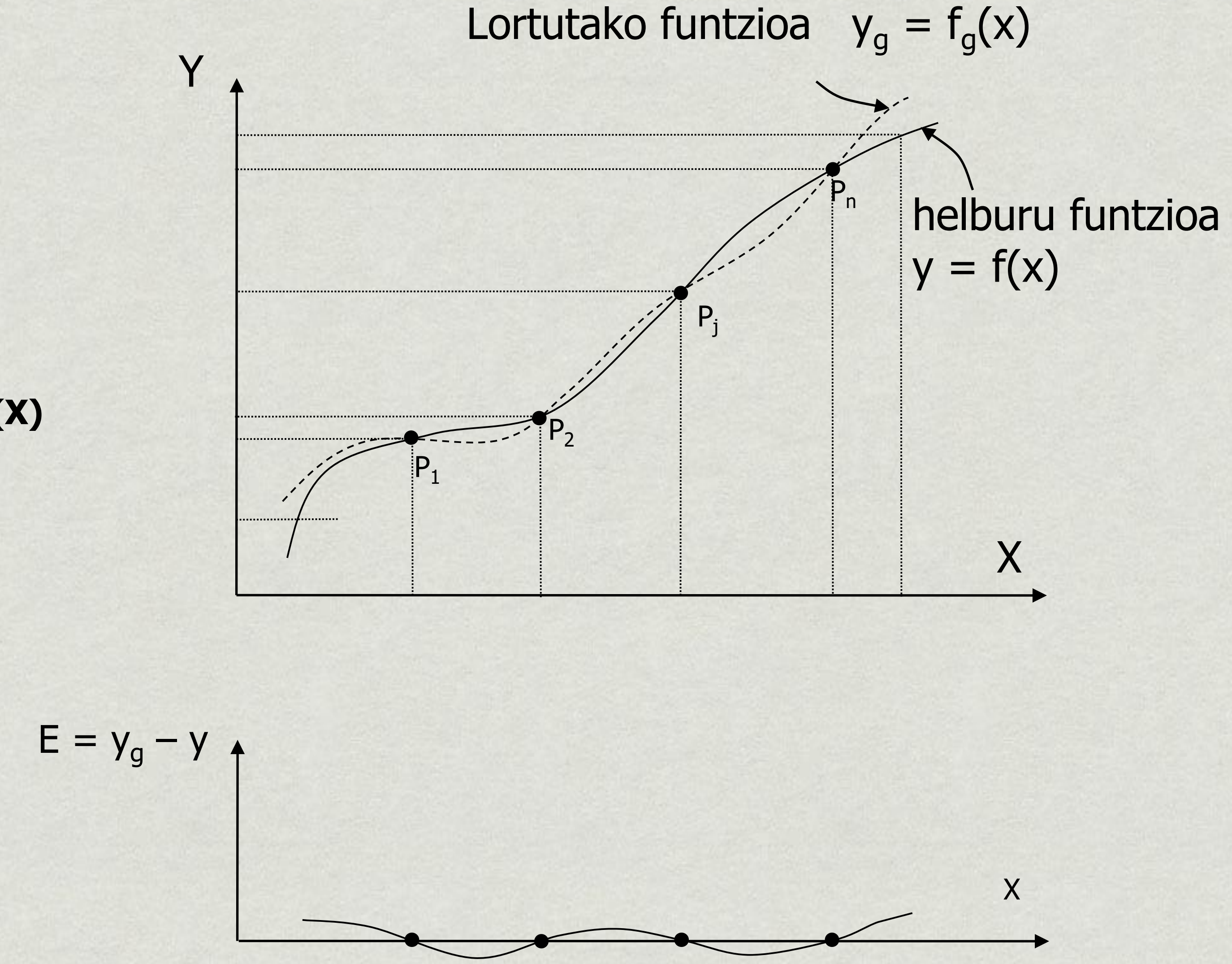
1. Sintesi dimentsional motak.
2. Funtzio sorkuntza sintesia. Freudenstein-en ekuazioa.
3. Ibilbide sorkuntza sintesia
    1. Kurbatura egonkorreko kubika, **kek**.
    2. kubatura zentro egonkorreko kubika, **kzek.**
4. Ball-en puntua eta kokapen zikloidala.

Sintesi dimentsional motak

- Funtzio sorkuntza sintesia

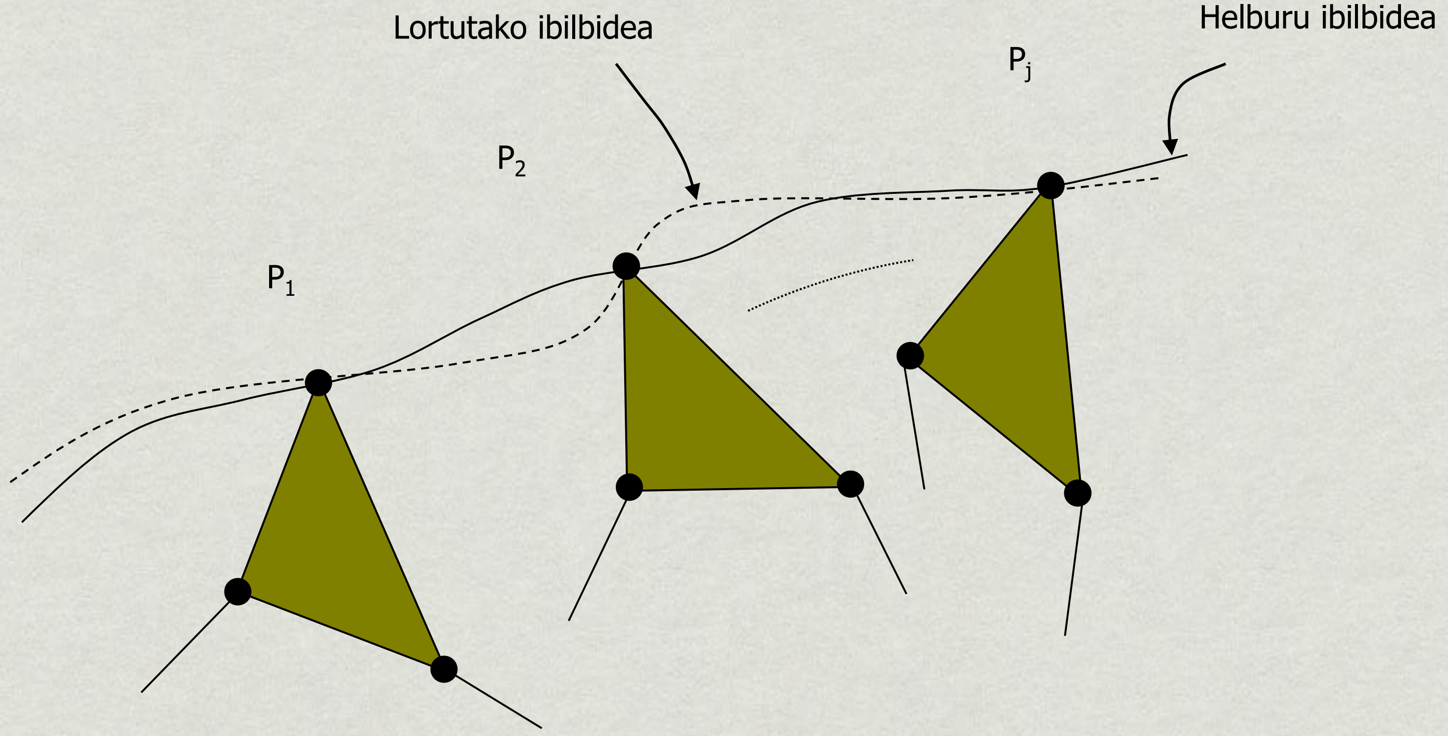


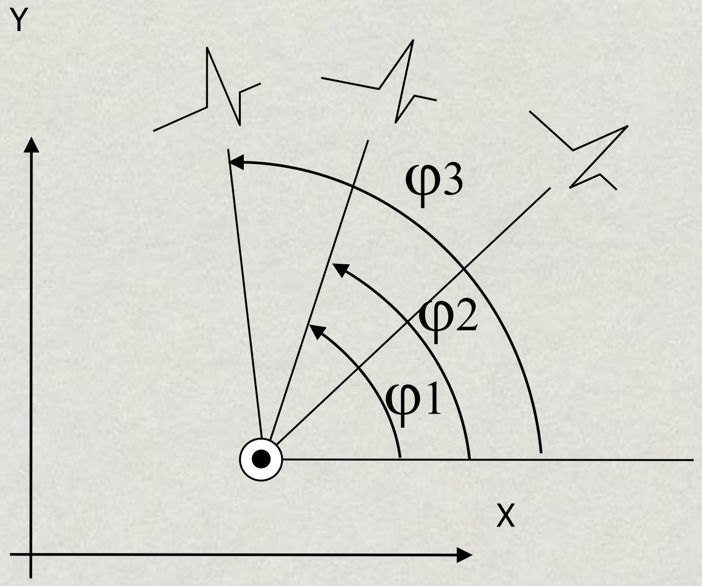
X	Y=f(X)
X_1	Y_1
X_2	Y_2
……	……



Sintesi dimentsional motak

- Ibilbide sorkuntza sintesia

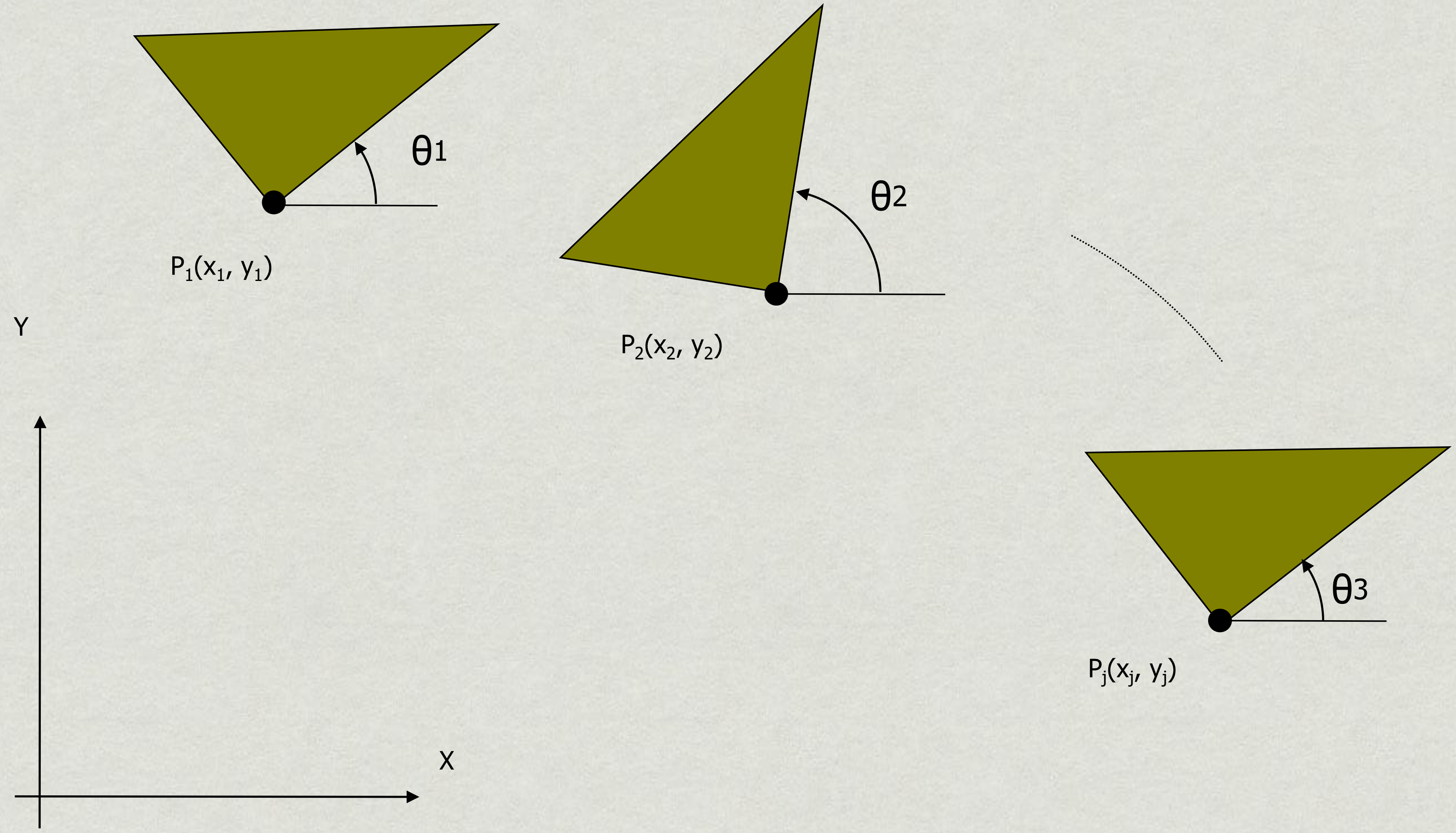




Sarrera parametroaren murrispenarekin

Sintesi dimentsional motak

- Solido zurrunaren gidaketa sintesia



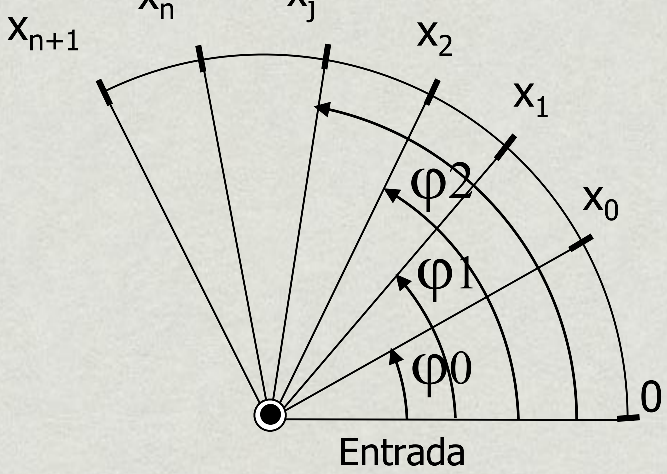
Funtzio sorkuntza sintesia. Freudenstein-en ekuazioa.

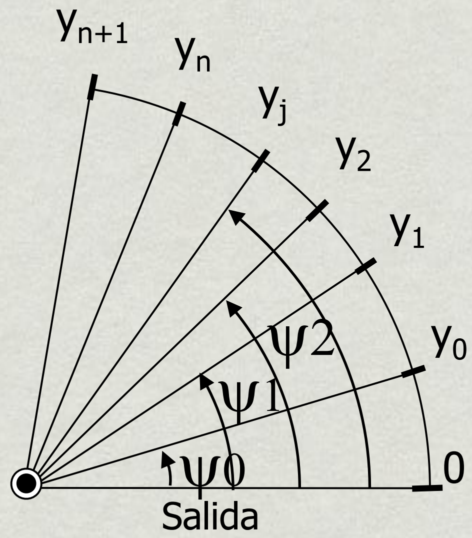
- Erlazio funtzioalak:

X	Y=f(X)
X_1	Y_1
X_2	Y_2
......	......

→

φ	ψ=f(φ)
φ_1	ψ_1
φ_2	ψ_2
......	......



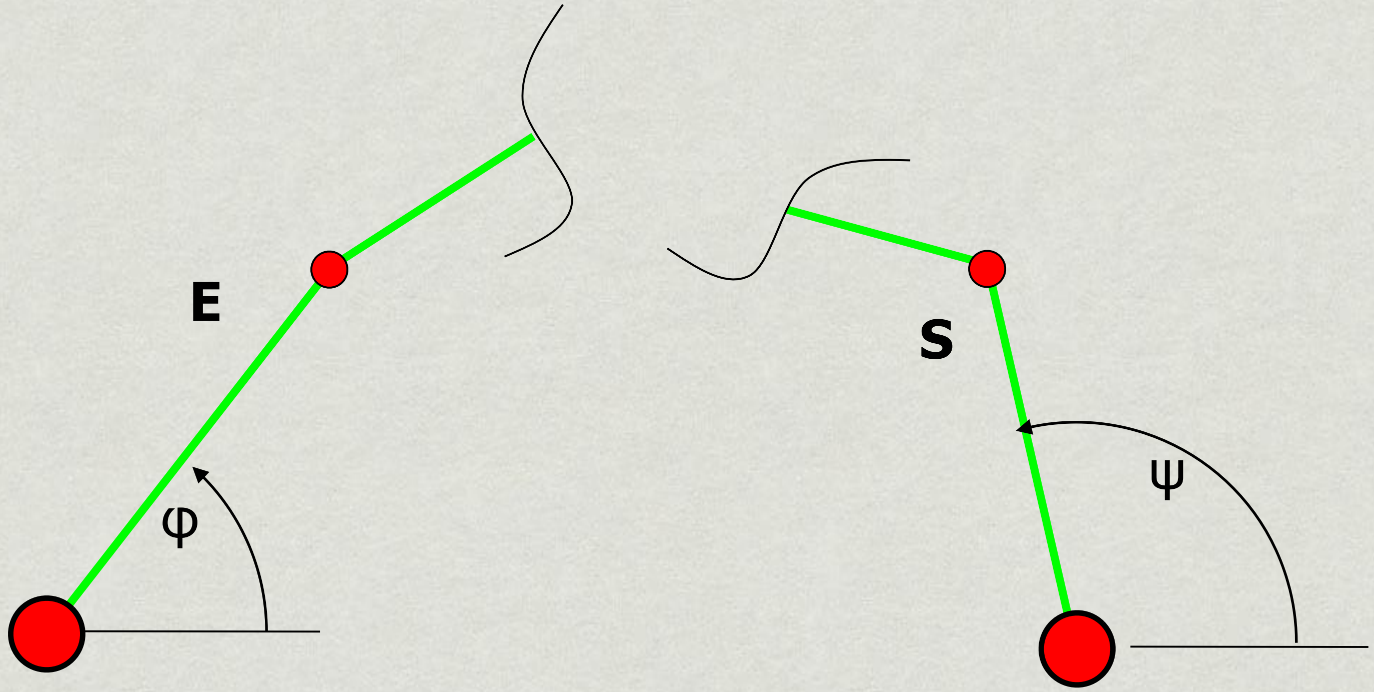


$$\frac{\varphi_j - \varphi_0}{x_j - x_0} = \frac{\Delta\varphi}{\Delta x} \qquad \frac{\psi_j - \psi_0}{y_j - y_0} = \frac{\Delta\psi}{\Delta y}$$

$$\varphi_j = \frac{\Delta\varphi}{\Delta x}(x_j - x_0) + \varphi_0 \qquad \psi_j = \frac{\Delta\psi}{\Delta y}(y_j - y_0) + \psi_0$$

Funtzio sorkuntza sintesia. Freudenstein-en ekuazioa.

A.g. bakarreko mekanismo orokorra



$$f\left(a_1, a_2, a_3 \ldots a_n, \varphi, \psi\right) = 0$$

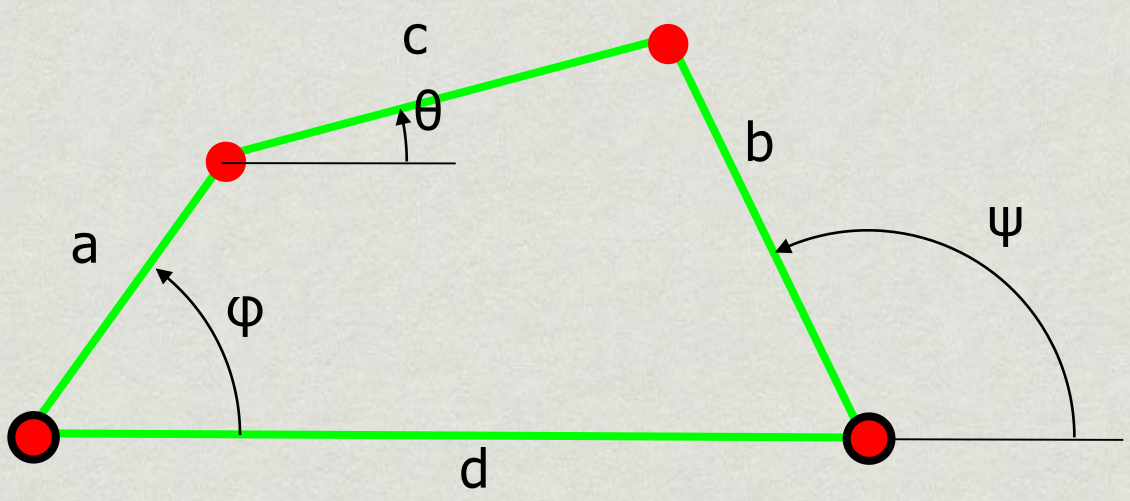
φ	ψ=f(φ)
φ_1	ψ_1
φ_2	ψ_2
……	……

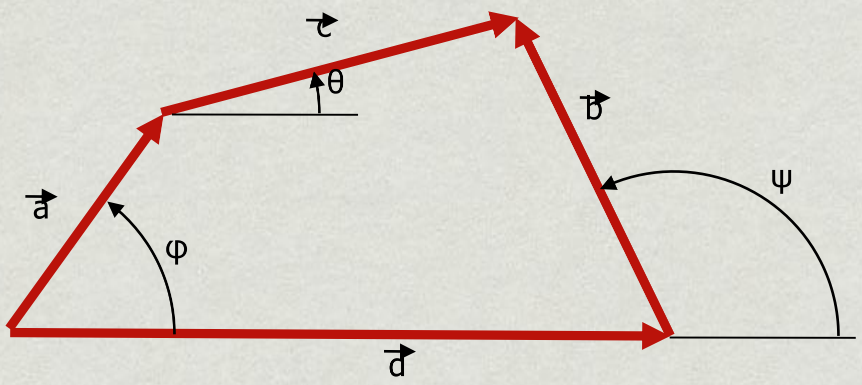
$$f\left(a_1, a_2, a_3 \ldots a_n, \varphi_1, \psi_1\right) = 0$$
$$f\left(a_1, a_2, a_3 \ldots a_n, \varphi_2, \psi_2\right) = 0$$
$$f\left(a_1, a_2, a_3 \ldots a_n, \varphi_3, \psi_3\right) = 0$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$
$$f\left(a_1, a_2, a_3 \ldots a_n, \varphi_n, \psi_n\right) = 0$$

$$\downarrow$$

$$a_1, a_2, a_3 \ldots a_n$$

Funtzio sorkuntza sintesia. Freudenstein-en ekuazioa.





$$\vec{a} + \vec{c} = \vec{b} + \vec{d}$$

$$a\cos\varphi + c\cos\theta = d + b\cos\psi$$

$$a\sin\varphi + c\sin\theta = b\sin\psi$$

θ kenduz:

$$\frac{d}{a}\cos\psi - \frac{d}{b}\cos\varphi + \frac{a^2 + b^2 + d^2 - c^2}{2ab} = \cos(\psi - \varphi)$$

Funtzio sorkuntza sintesia. Freudenstein-en ekuazioa.

$$\frac{d}{a}\cos\psi - \frac{d}{b}\cos\varphi + \frac{a^2 + b^2 + d^2 - c^2}{2ab} = \cos(\psi - \varphi)$$

Suposatuz: $K_1 = \frac{d}{a}$ $K_2 = \frac{d}{b}$ $K_3 = \frac{a^2 + b^2 + d^2 - c^2}{2ab}$

Emaitza: $K_1 \cos\psi - K_2 \cos\varphi + K_3 = \cos(\psi - \varphi)$

Funtzio sorkuntza sintesia. Freudenstein-en ekuazioa.

$$K_1 \cos\psi - K_2 \cos\varphi + K_3 = \cos(\psi - \varphi)$$

φ	ψ=f(φ)
φ_1	ψ_1
φ_2	ψ_2
φ_3	ψ_3

$$K_1 \cos\psi_1 - K_2 \cos\varphi_1 + K_3 = \cos(\psi_1 - \varphi_1)$$
$$K_1 \cos\psi_2 - K_2 \cos\varphi_2 + K_3 = \cos(\psi_2 - \varphi_2)$$
$$K_1 \cos\psi_3 - K_2 \cos\varphi_3 + K_3 = \cos(\psi_3 - \varphi_3)$$

$$\begin{bmatrix} \cos\psi_1 & -\cos\varphi_1 & 1 \\ \cos\psi_2 & -\cos\varphi_2 & 1 \\ \cos\psi_3 & -\cos\varphi_3 & 1 \end{bmatrix} \begin{Bmatrix} K_1 \\ K_2 \\ K_3 \end{Bmatrix} = \begin{Bmatrix} \cos(\psi_1 - \varphi_1) \\ \cos(\psi_2 - \varphi_2) \\ \cos(\psi_3 - \varphi_3) \end{Bmatrix}$$

K_1, K_2, K_3

↓ ← d finkatuz

a, b, c

- Diseinatu y=√x funtzioa jarraitzen duen lauki artikulatua

$$x_0 = 0; \quad x_4 = 2; \quad \Delta x = 2 \qquad y_0 = 0; \quad y_4 = 1.4142; \quad \Delta y = 1.4142$$

Chebyshev-en tartekatze optimoa:

$$x_i = \frac{1}{2}(x_0 + x_{n+1}) - \frac{1}{2}(x_{n+1} - x_0)\cos\frac{\pi(2i-1)}{2n} \qquad i = 1, 2, \ldots, n$$

$$x_1 = \frac{1}{2}(0+2) - \frac{1}{2}(2-0)\cos\frac{\eth(2\times 1 - 1)}{2\times 3} = 0.13397$$

$$x_2 = \frac{1}{2}(0+2) - \frac{1}{2}(2-0)\cos\frac{\eth(2\times 2 - 1)}{2\times 3} = 1$$

$$x_3 = \frac{1}{2}(0+2) - \frac{1}{2}(2-0)\cos\frac{\eth(2\times 3 - 1)}{2\times 3} = 1.86603$$

Doitasun puntuak

x	y=f(x)
0.1339	**0.3660**
1	**1**
1.8660	**1.3660**

$$\varphi_0 = 45^o; \quad \Delta\varphi = 60^o; \quad \psi_0 = 0^o; \quad \Delta\psi = 60^o$$

$$\varphi_j = \varphi_0 + \frac{\Delta\varphi}{\Delta x}(x_j - x_0)$$

$$\psi_j = \psi_0 + \frac{\Delta\psi}{\Delta y}(y_j - y_0)$$

$$\varphi_1 = \frac{60^o}{2}(0.13397 - 0) + 45^o = 49.0191^o$$

$$\varphi_2 = \frac{60^o}{2}(1 - 0) + 45^o = 75^o$$

$$\varphi_3 = \frac{60^o}{2}(1.86603 - 0) + 45^o = 100.9809^o$$

$$\psi_1 = \frac{60^o}{1.41421}(0.36602 - 0) = 15.52895^o$$

$$\psi_2 = \frac{60^o}{1.41421}(1 - 0) = 42.42651^o$$

$$\psi_3 = \frac{60^o}{1.41421}(1.36603 - 0) + 45^o = 57.95559^o$$

x	y=f(x)
0.1339	0.3660
1	1
1.8660	1.3660

→

φ	ψ=f(φ)
49.019	15.5289
75	42.4265
100.981	57.9556

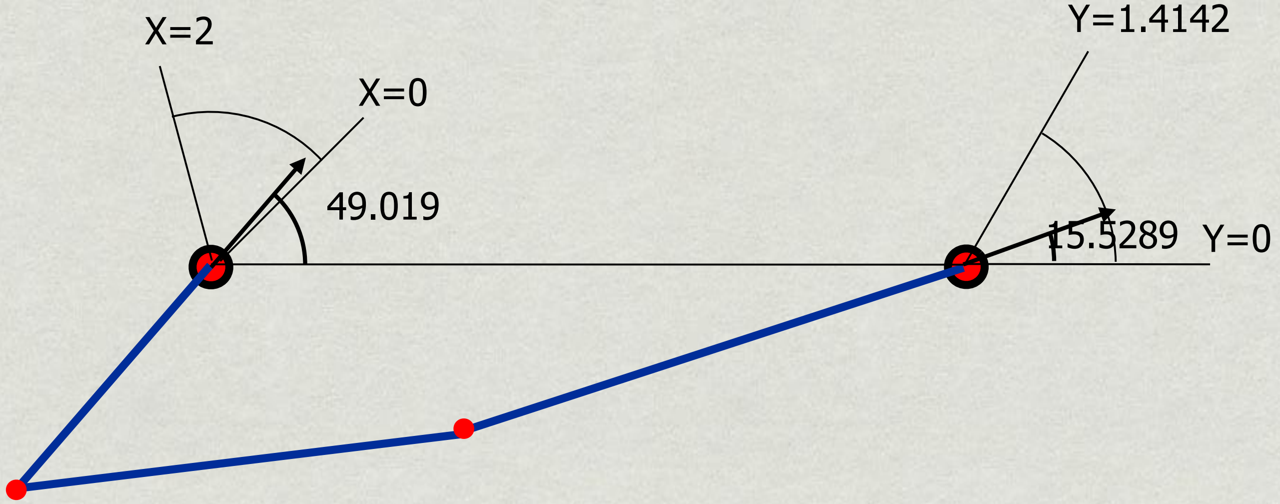
$$\begin{Bmatrix} K_1 \\ K_2 \\ K_3 \end{Bmatrix} = \begin{bmatrix} \cos\psi_1 & -\cos\varphi_1 & 1 \\ \cos\psi_2 & -\cos\varphi_2 & 1 \\ \cos\psi_3 & -\cos\varphi_3 & 1 \end{bmatrix}^{-1} \begin{Bmatrix} \cos(\psi_1 - \varphi_1) \\ \cos(\psi_2 - \varphi_2) \\ \cos(\psi_3 - \varphi_3) \end{Bmatrix} = \begin{Bmatrix} -2.55857 \\ -1.43046 \\ 2.36105 \end{Bmatrix}$$

Aukeratuz $d = 10$

$$a = -3.9084$$

$$b = -6.99076$$

$$c = 5.92456$$



Kurbatura egonkorreko kubika

- Puntu baten ibilbidearen eta bere zirkunferentzia oskulatzailearen artean bigarren mailako kontaktua dago.
- Orden handiagoko kontaktua egoteko hurrengo adierazpena balioztatu behar da:

$$\frac{d\rho}{dt} = 0 \qquad \frac{d\rho}{d\varphi_e} = 0$$

- Adierazpen hori balioztatzen duten puntuen leku geometrikoa kurbatura egonkorreko kubika da.

Kurbatura egonkorreko kubika

- Froga daiteke baldintza hori betezen duten puntek kubika batean kokaturik daudela

$$\frac{1}{r} = \frac{1}{A \times \cos\theta} + \frac{1}{B \times \mathrm{sen}\,\theta}$$

- A eta B kurbaren konstanteak dira

Kurbatura egonkorreko kubika

- Koordenatu kartesiarretara eta parametrikoetara pasatuz:

$$\frac{1}{r^2} = \frac{1}{A \times r \times \cos\theta} + \frac{1}{B \times r \times sen\theta}$$

$$\frac{1}{x^2 + y^2} = \frac{1}{Ax} + \frac{1}{By}$$

$$\left(x^2 + y^2\right)\left(\frac{1}{Ax} + \frac{1}{By}\right) = 1$$

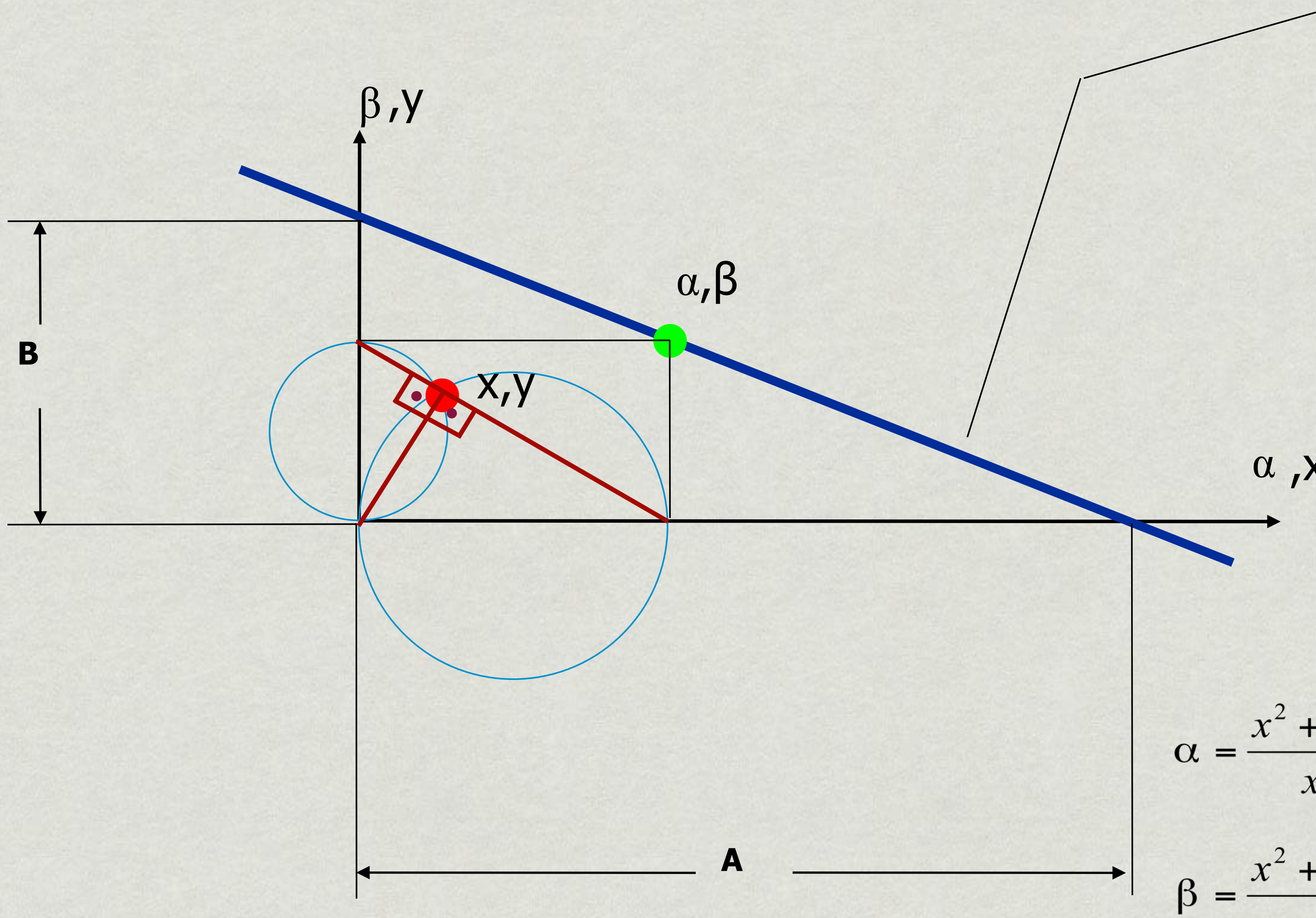
$$\alpha = \frac{x^2 + y^2}{x}; \beta = \frac{x^2 + y^2}{y}$$

Parametroak

$$\frac{\alpha}{A} + \frac{\beta}{B} = 1$$

Kurbatura egonkorreko kubika

$$\frac{\alpha}{A} + \frac{\beta}{B} = 1$$



$$\alpha = \frac{x^2 + y^2}{x} \rightarrow \left(x - \frac{\alpha}{2}\right)^2 + y^2 = \left(\frac{\alpha}{2}\right)^2$$

$$\beta = \frac{x^2 + y^2}{y} \rightarrow x^2 + \left(y - \frac{\beta}{2}\right)^2 = \left(\frac{\beta}{2}\right)^2$$

Kurbatura egonkorreko zentroen kubika

- ¿Non egongo dira kurbatura egonkorreko kubikaren puntuen ibilbideen kurbatura zentroak?

$$\left(\frac{1}{\overline{O_A P^*}} + \frac{1}{\overline{PA^*}}\right) \cdot \text{sen}\,\theta = \frac{1}{\delta^*}$$

$$\overline{PA^*} = r$$

$$\overline{O_A P^*} = \bar{r}$$

$$\frac{1}{\bar{r}} + \frac{1}{r} = \frac{1}{\delta^* \cdot sen\,\theta}$$

$$\frac{1}{r} = \frac{1}{A \cdot \cos\theta} + \frac{1}{B \cdot \text{sen}\,\theta}$$

k.e.k.

$$\frac{1}{\bar{r}} + \frac{1}{A \cdot \cos\theta} + \frac{1}{B \cdot \text{sen}\,\theta} = \frac{1}{\delta^* \cdot \text{sen}\,\theta}$$

$$O_A \rightarrow \bar{r}, \bar{\theta}$$

$$\bar{\theta} = \theta + \pi$$

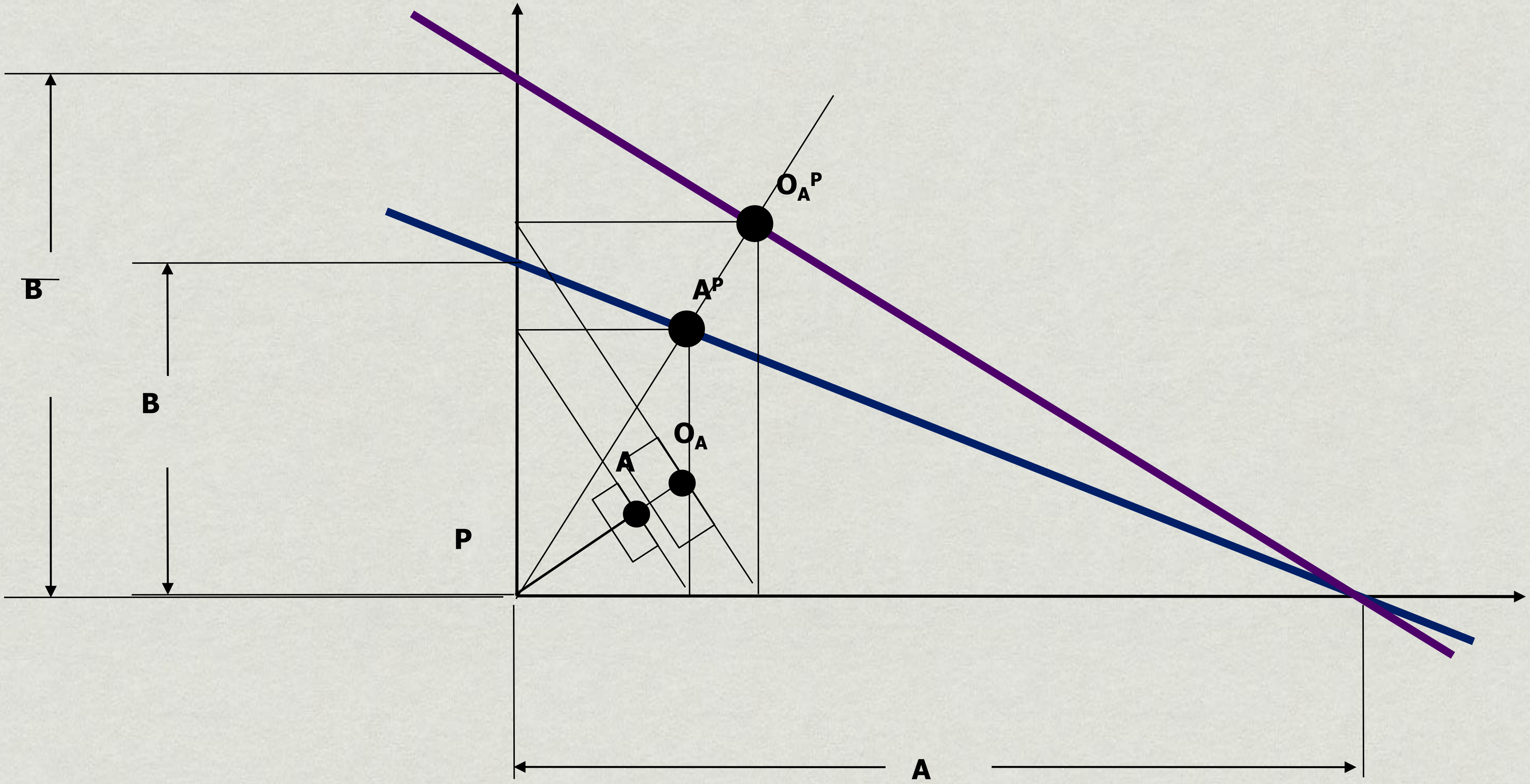
$$\frac{1}{\bar{r}} = \frac{1}{A \cdot \cos\bar{\theta}} + \frac{1}{\bar{B} \cdot \text{sen}\,\bar{\theta}}$$

k.e.z.k.

Non: $\frac{1}{\bar{B}} = \frac{1}{B} - \frac{1}{\delta^*}$

Kurbatura zentro egonkorreko kubika

- Bi kurbak koordenatu parametrikoetan adieraziz



k.e.k. eta k.z.e.k.

- Kubiken propietateak:
    - Poloan puntu bikoitza dute
    - Bi kurbek asintotak dituzte

k.e.k.

$$\frac{1}{r} = \frac{1}{A \times \cos\theta} + \frac{1}{B \times \mathrm{sen}\,\theta} \qquad \tan\theta = -\frac{A}{B}$$

$$r \to \infty$$

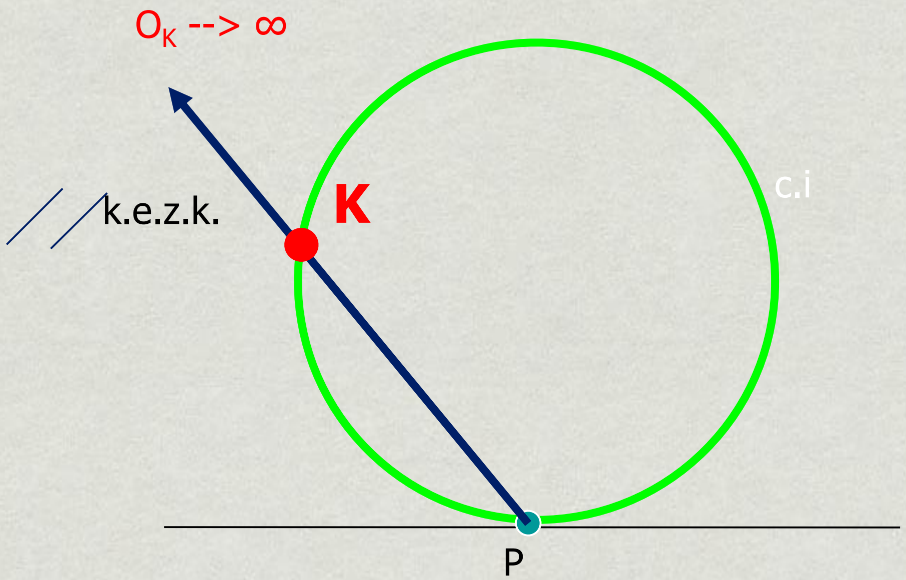
k.e.z.k.

$$\frac{1}{\bar{r}} = \frac{1}{A \times \cos\theta} + \frac{1}{\bar{\bar{B}} \times sen\,\theta} \qquad \tan\bar{\theta} = -\frac{A}{\bar{\bar{B}}}$$

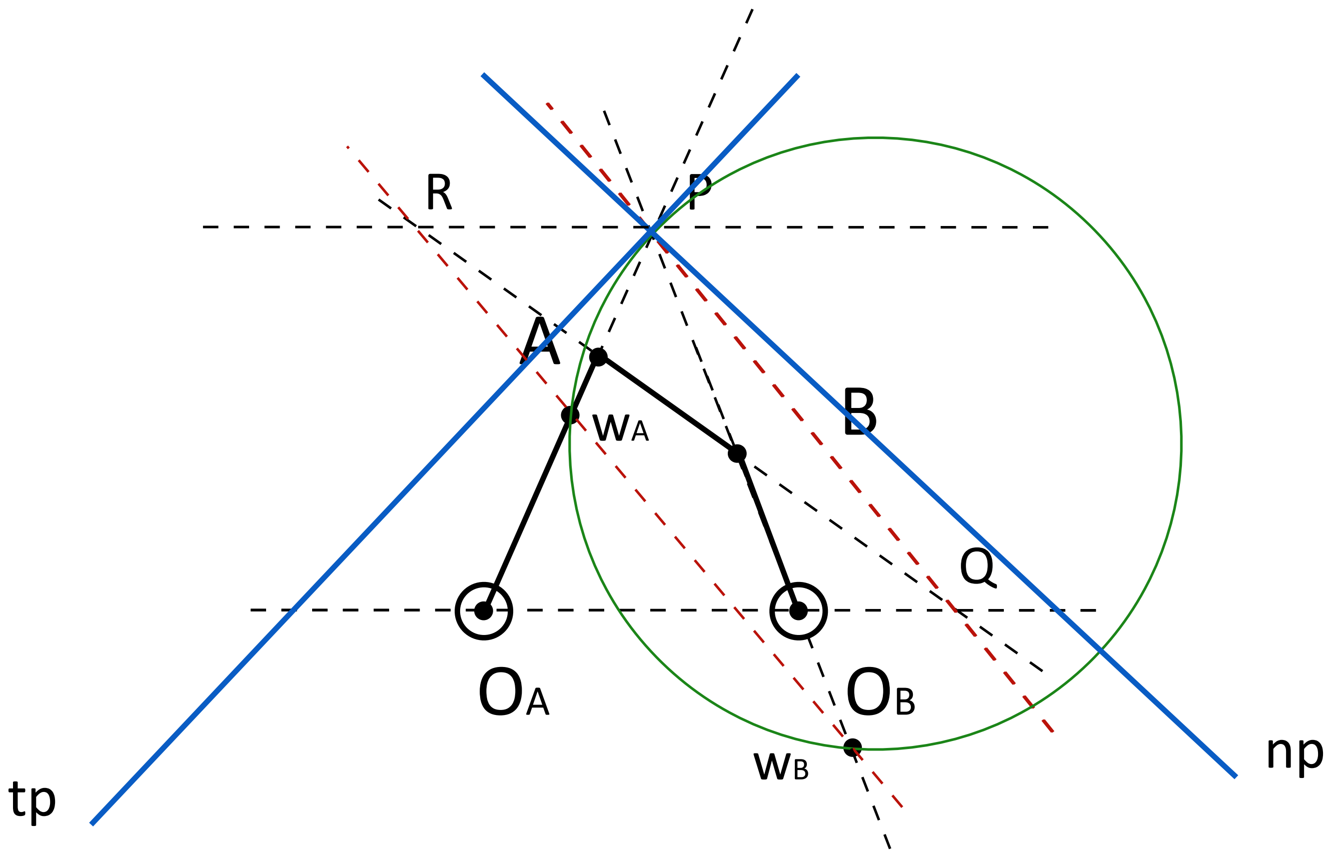
$$\bar{r} \to \infty$$

Punto de Ball

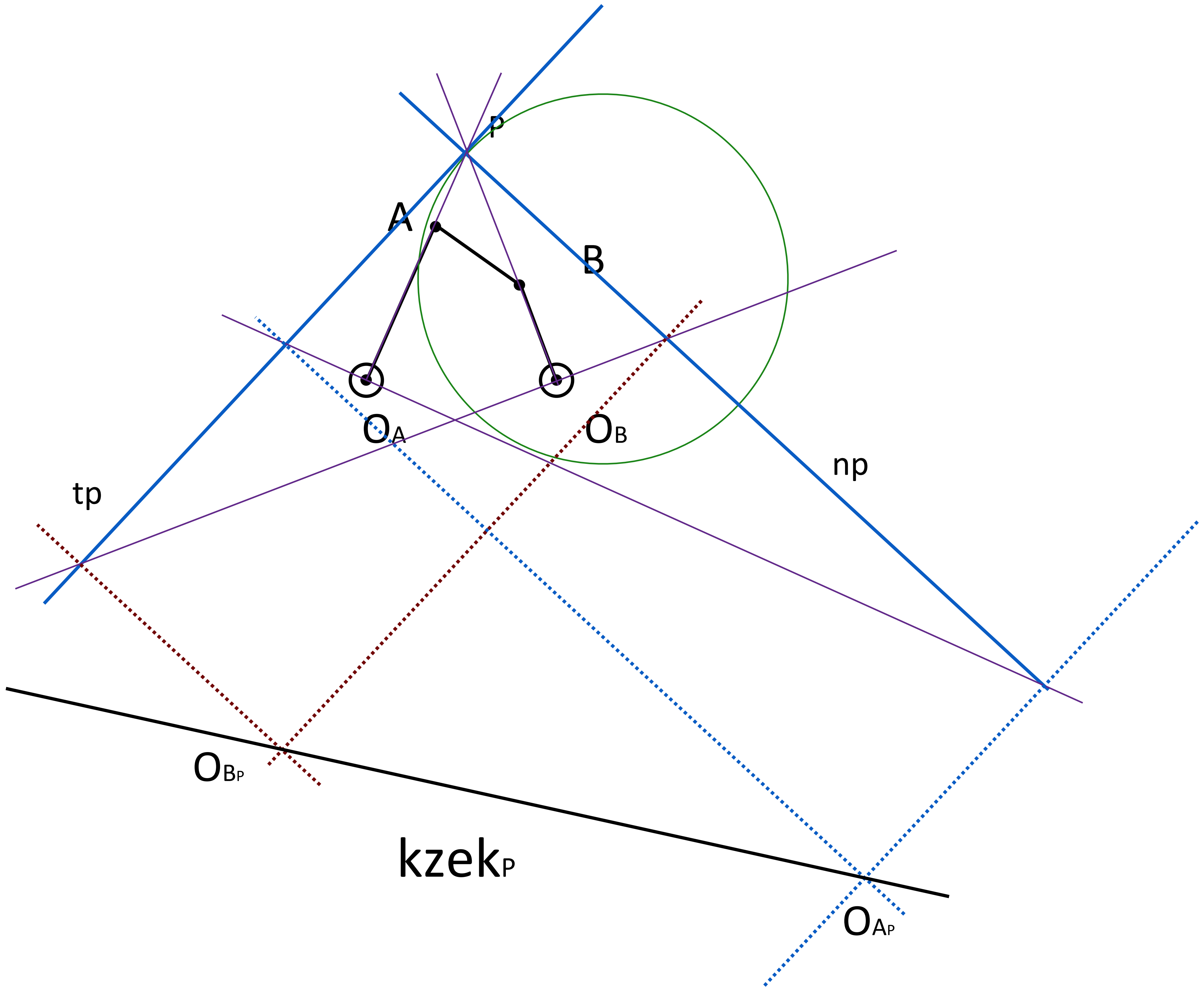
- Ball puntuaren kurbatura zentroa infinituan dago.
- Ball-en puntua k.e.k.-ren barnean dago beraz bere kurbatura zentroa k.z.e.k.-ren barnean egongo da.
- Infinituan eta k.z.e.k.-ren barnean egonda bere asintotaren norabidean egongo da.



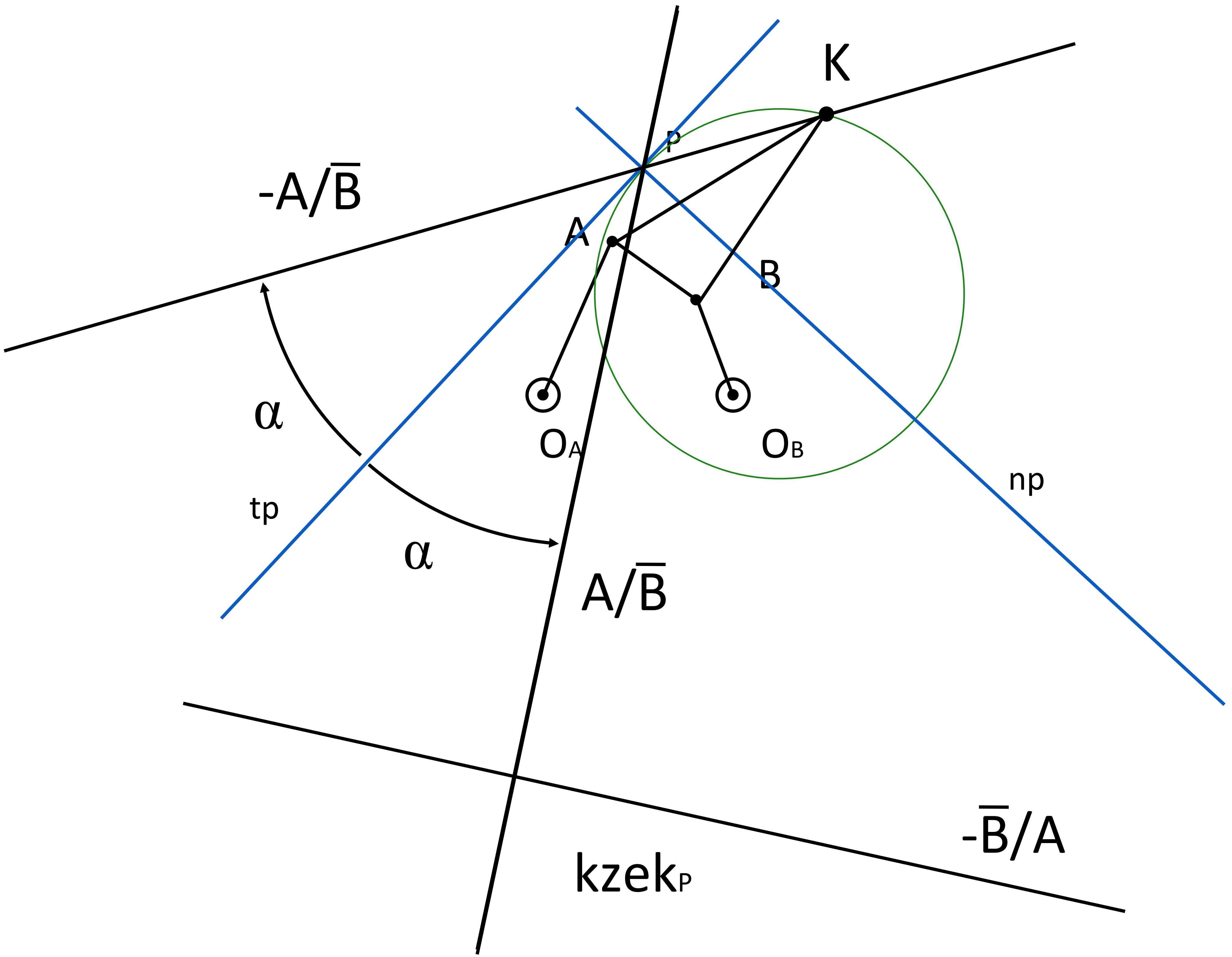
- Ball puntuaren kalkulua lauki artikulatu batean



* Ball puntuaren kalkulua lauki artikulatu batean



- Ball puntuaren kalkulua lauki artikulatu batean



Kokapen zikloidala

$$\frac{d\delta}{dt} = 0 \longrightarrow$$ **Kokapen zikloidala cicloidal**

Kokapen honetan: $\frac{1}{A} = 0$ hurrengoa gertatzen bada $\longrightarrow \delta = CTE$

kokapen zikloidala cicloidal

k.e.k.-ren ekuaziotik 1/A askatuz:

$$\frac{1}{A} = \left(\frac{1}{r} - \frac{1}{B \times sen\theta}\right)\cos\theta = 0$$

$$\cos\theta = 0 \rightarrow Normal \quad Polar$$

$$\left(\frac{1}{r} - \frac{1}{B \times sen\theta}\right) = 0 \rightarrow r = B \times sen\theta$$

En resumen:

k.e.k.

$$r = B \times sen\theta$$

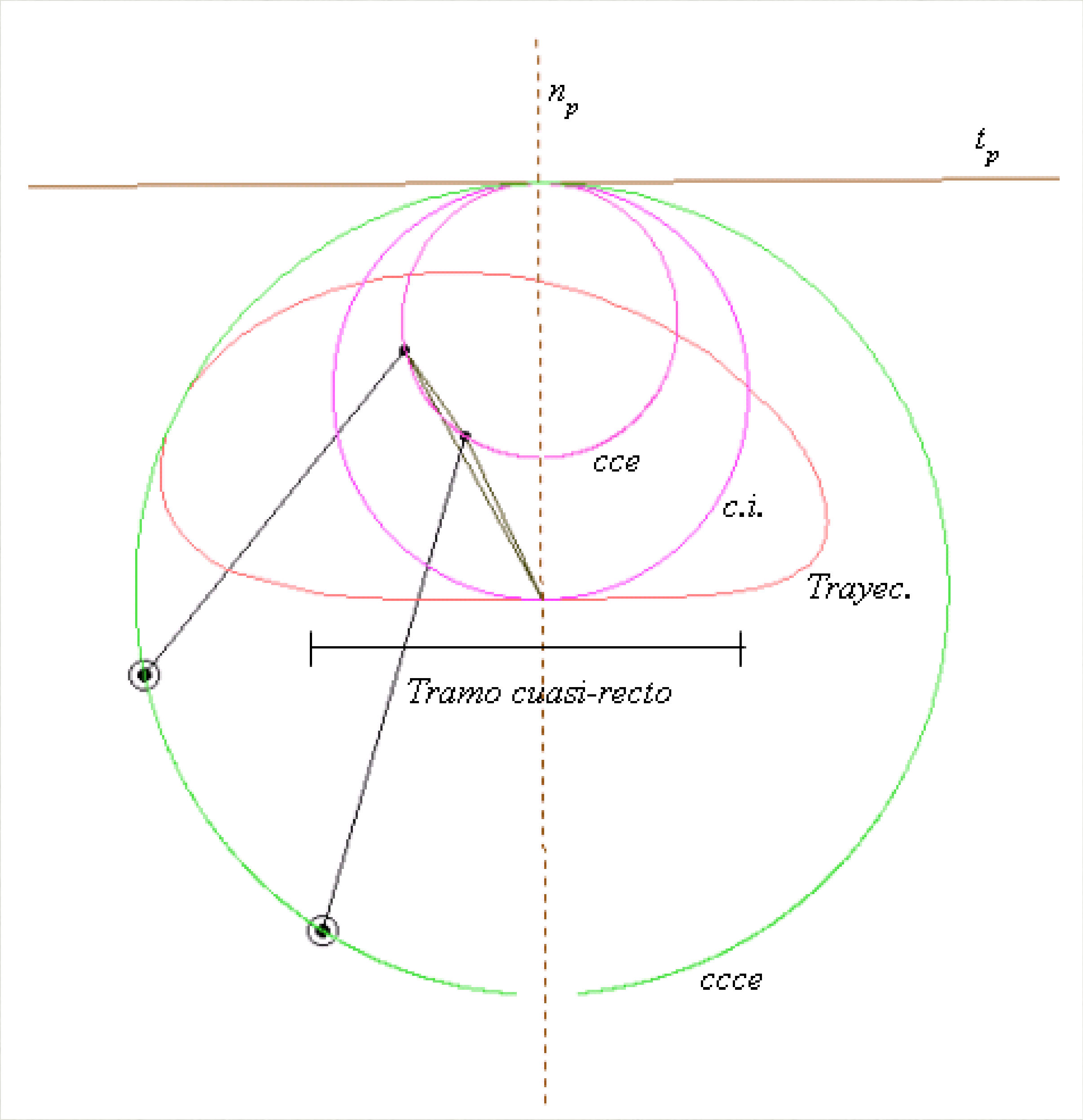
Np

k.z.e.k.

$$\bar{r} = \bar{B} \times sen\bar{\theta}$$

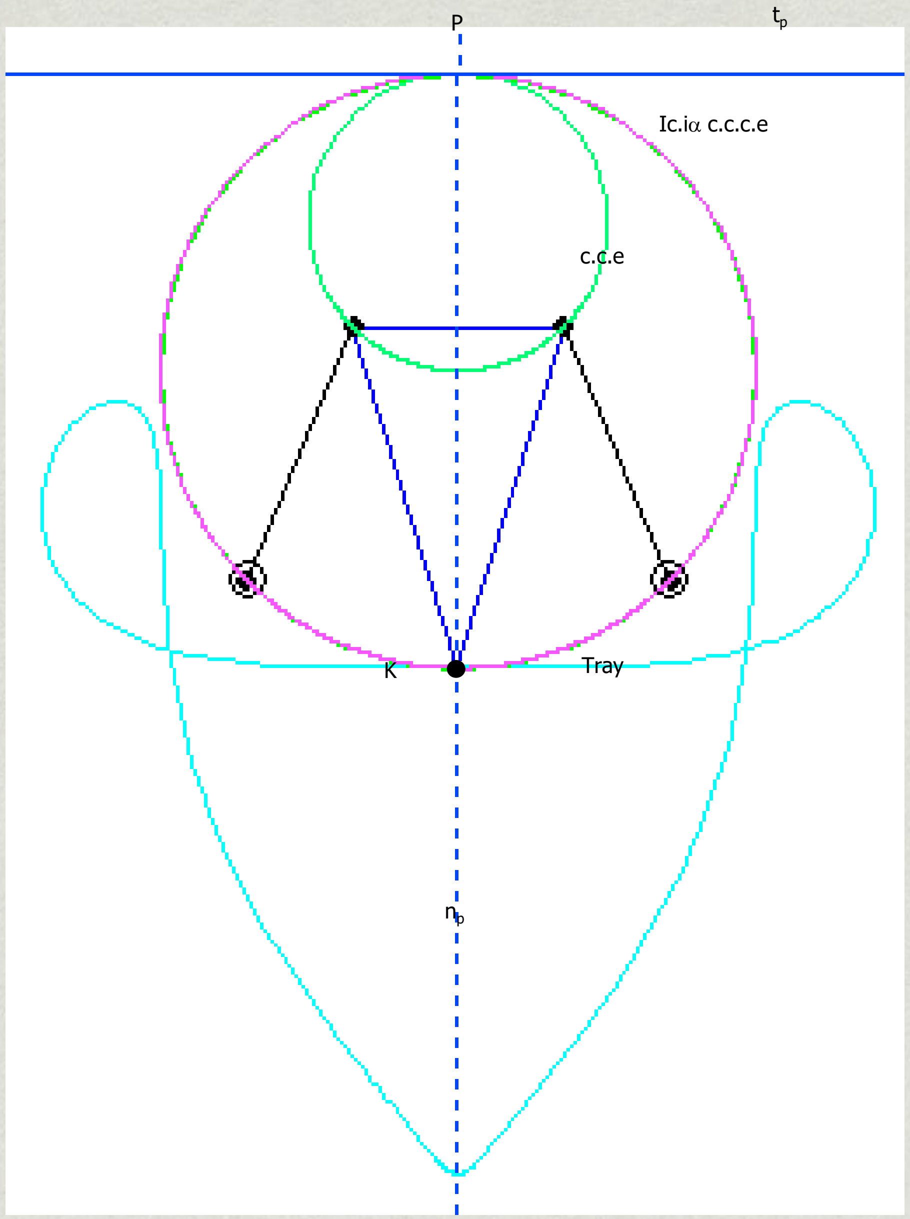
Np

Adibideak



Adibideak

Roberts-en mekanismoa



Chebyshev-en mekanismoa