4.3. HIGIDURA KANTITATEA: EULERREN EKUAZIOA

1. HIGIDURA KANTITATEAREN T^MA^: EULERREN EKUAZIOA
2. JARIAKIN FLUXUAK SOLIDO BATEN GAINEAN ERAGITEN DUEN INDARRA.
   1. KONDUKZIO HODIETAN.
   2. ZORROTADA BATEK OZTOPO BATEN GAINEAN ERAGITEN DUEN INDARRA.
   3. PROPULTSIO SISTEMAK.
      1. HELIZEAK.
      2. AEROGENERADOREAK.
      3. TURBOERREAKTOREAK.
      4. KOHETEAK.

1. HIGIDURA KANTITATEAREN T^{MA}: EULERREN EKUAZIOA
    1. ADIERAZPEN OROKORRA.
        1. REYNOLDSEN GARRAIO TMA.
        2. NEWTONEN 2. LEGEA.
    2. APLIKAZIOA BALDINTZAK.
    3. β : HIGIDURA KANTITATEAREN ZUZENKETA FAKTOREA.
    4. EULERREN 1. EKUAZIOAREN ADIERAZPENA.
2. JARIAKIN FLUXUAK SOLIDO BATEN GAINEAN ERAGITEN DUEN INDARRA.
    1. KONDUKZIO HODIETAN.
        1. UKONDOAK.
    2. ZORROTADA BATEK OZTOPO BATEN GAINEAN ERAGITEN DUEN INDARRA.
        1. FINKOAK.
        2. MUGIKORRAK
        3. ALABEEN SEGIDA.
    3. PROPULTSIO SISTEMAK
        1. HELIZEAK.
        2. AEROGENERADOREAK.
        3. TURBOERREAKTOREAK.
        4. KOHETEAK.

1. HIGIDURA KANTITATEAREN T^{MA}: EULERREN EKUAZIOA

1. ADIERAZPEN OROKORRA.

1. REYNOLDSEN GARRAIO TMA.

$$\frac{dB^S}{dt} = \frac{\partial}{dt}\iiint_{KB} b\rho dV + \iint_{KG} b\rho \vec{U}_r d\vec{A}_{KG}$$

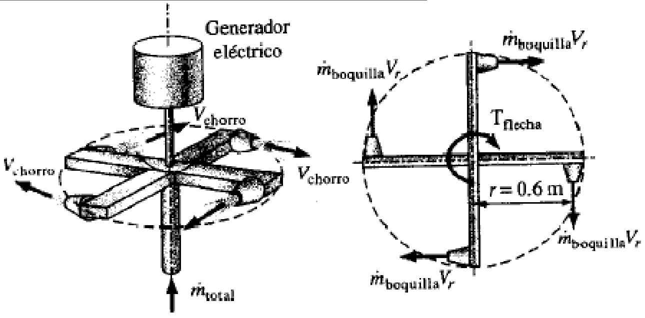
2. NEWTONEN 2. LEGEA.

$$\sum \vec{F}_{ext} = \vec{P} + \vec{G} = \frac{d\vec{M}^S}{dt} = \frac{d\left(m\vec{U}\right)^S}{dt}$$

3. REYNOLDSEN GARRAIO TMA HIGIDURA KANTITATEARI APLIKATUTA

$$b = \vec{U} = masa\ unitateko\ izango\ den\ higidura\ kantitatea.$$

$$\sum \vec{F}_{ext} = \frac{d\left(m\vec{U}\right)^S}{dt} = \frac{\partial}{dt}\iiint_{KB} \vec{U}\rho dV + \iint_{KG} \vec{U}\rho \vec{U}_r d\vec{A}_{KG}$$



2. APLIKAZIO BALDINTZAK.

1. FLUXU IRAUNKORRA.

$$\frac{\partial}{dt}\iiint_{KB}\vec{U}\rho dV=0$$

2. FLUXU UNIDIREKZIONALA.
3. KG FLUXUAREKIKO ELKARTZUTA.

$$\iint_{KG}\vec{U}\rho\vec{U}_R d\vec{A}_{KG}=\iint_{A_1}\vec{U}_1\rho\vec{U}_{R_1}d\vec{A}_1+\iint_{A_2}\vec{U}_2\rho\vec{U}_{R_1}d\vec{A}_2+\iint_{A_L}\vec{U}_L\rho\vec{U}_{R_L}d\vec{A}_L$$
$$=\iint_{A_1}\vec{U}_1\rho\vec{U}_{R_1}d\vec{A}_1+\iint_{A_2}\vec{U}_2\rho\vec{U}_{R_2}d\vec{A}_2=\iint_{A_2}\vec{U}_2\rho U_{R_2}dA_2-\iint_{A_1}\vec{U}_1\rho U_{R1}dA_1$$

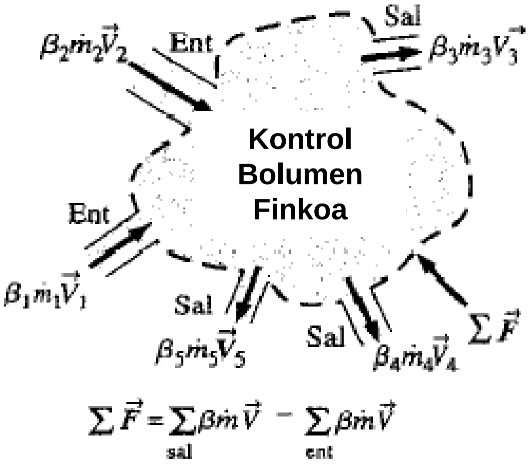
3. β : HIGIDURA KANTITATEAREN ZUZENKETA FAKTOREA.

$$\beta=\frac{1}{A}\int\left(\frac{U}{U_m}\right)^2 dA\Rightarrow\begin{cases}LAMINARRA\Rightarrow\beta=4/3\\TURBULENTOA\Rightarrow\beta=1\end{cases}$$

4. EULERREN 1. EKUAZIOAREN ADIERAZPENA.

$$\sum \vec{F}_{ext} = \frac{d\left(m\vec{U}\right)^S}{dt} = \iint_{A_2} \vec{U}_2 \rho U_2 dA_2 - \iint_{A_1} \vec{U}_1 \rho U_1 dA_1 = \vec{U}_2 \iint_{A_2} \rho U_2 dA_2 - \vec{U}_1 \iint_{A_1} \rho U_1 dA_1 =$$

$$= q_{m_2} \vec{U}_2 - q_{m_1} \vec{U}_1 = \vec{M}_2 - \vec{M}_1$$



Azterketa ardatzetan eginez :

$$x \Rightarrow \sum \vec{F}_{ext\,x} = q_{m_2} \vec{U}_{2_x} - q_{m_1} \vec{U}_{1_x} = q_m \left(\vec{U}_{2_x} - \vec{U}_{1_x}\right)$$

$$y \Rightarrow \sum \vec{F}_{ext\,y} = q_{m_2} \vec{U}_{2_y} - q_{m_1} \vec{U}_{1_y} = q_m \left(\vec{U}_{2_y} - \vec{U}_{1_y}\right)$$

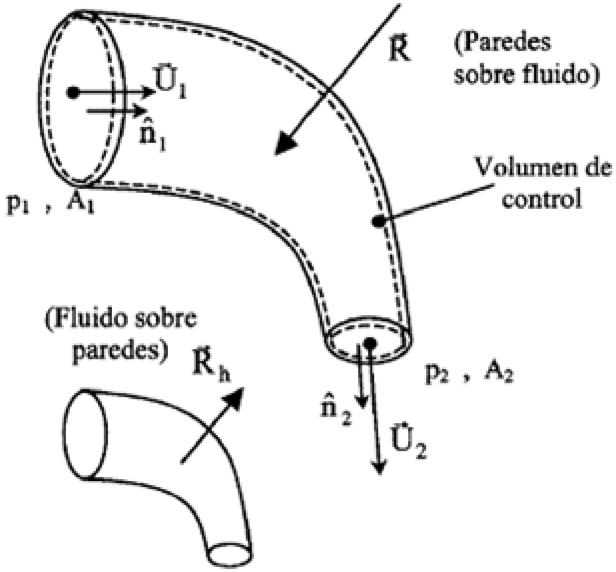
$$z \Rightarrow \sum \vec{F}_{ext\,z} = q_{m_2} \vec{U}_{2_z} - q_{m_1} \vec{U}_{1_z} = q_m \left(\vec{U}_{2_z} - \vec{U}_{1_z}\right)$$

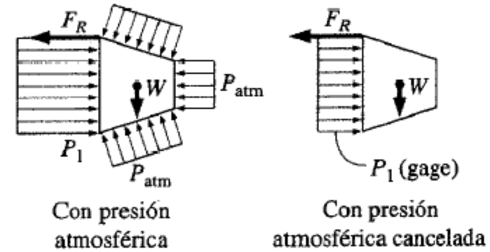
2. JARIAKIN FLUXUAK SOLIDO BATEN GAINEAN ERAGITEN DUEN INDARRA.

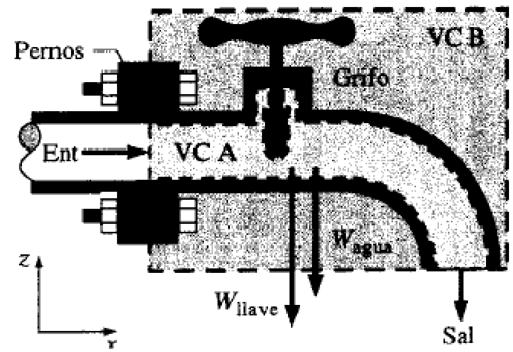
1. KONDUKZIO HODIETAN.
    1. UKONDOAK.
    2. BAT BATEKO ZABALGUNEA/ ESTUGUNEA.
2. GORAPEN HIDRAULIKOA.
3. ZORROTADA BATEK OZTOPO BATEN GAINEAN ERAGITEN DUEN INDARRA.
    1. FINKOAK.
        1. PARETA BERTIKALA.
        2. OZTOPO KONIKO SIMETRIKOA.
        3. ALABEA (PLAKA INKLINATUA).
    2. MUGIKORRAK.
        1. PARETA BERTIKALA.
        2. OZTOPO KONIKO SIMETRIKOA.
        3. ALABEA
    3. ALABEEN SEGIDA.
        1. GURPIL BATETAN KOKATUTAKO PLAKA BERTIKALEN SEGIDA.
        2. GURPIL BATETAN KOKATUTAKO PLAKA INKLINATUEN SEGIDA.
        3. PELTON TURBINA.

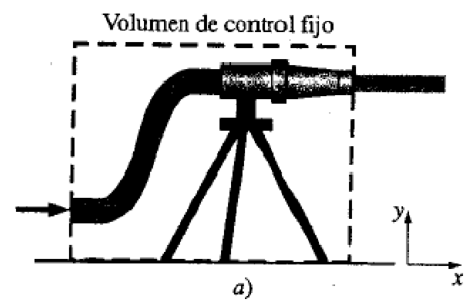
1. KONDUKZIO HODIETAN.

1. UKONDOAK.



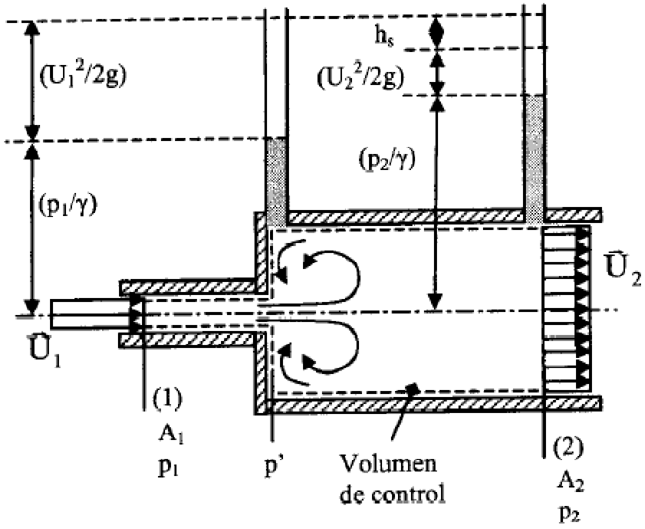






$$\sum \vec{F}_{ext} = \vec{R} + \vec{P}_1\vec{A}_1 + \vec{P}_2\vec{A}_2 = \vec{M}_2 - \vec{M}_1 = q_{m_2}\vec{U}_2 - q_{m_1}\vec{U}_1$$

2. BAT BATEKO ZABALGUNEA/ ESTUGUNEA



$$\sum \vec{F}_{ext} = \vec{P}_1\vec{A}_1 + \vec{P}_2\vec{A}_2 + \vec{P}'\vec{A}' = \vec{M}_2 - \vec{M}_1 = q_{m_2}\vec{U}_2 - q_{m_1}\vec{U}_1 = \rho Q(\vec{U}_2 - \vec{U}_1)$$

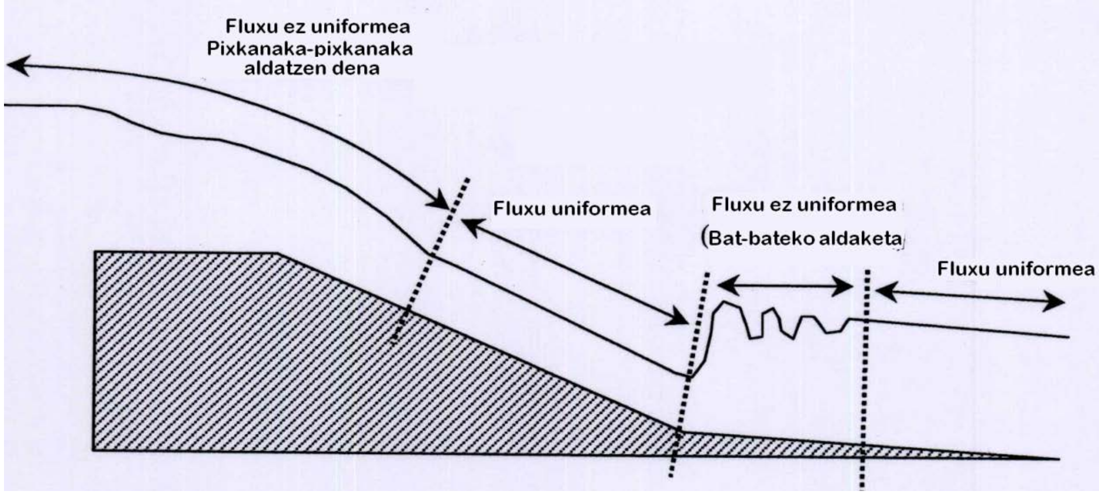
$$P_1A_1 + P'A' - P_2A_2 = \rho Q(U_2 - U_1)$$

$$P_1A_1 + P'\left(A_2 - A_1\right) - P_2A_2 = \rho Q(U_2 - U_1)$$

$$P_1A_1 + P_1\left(A_2 - A_1\right) - P_2A_2 = \rho Q(U_2 - U_1)$$

$$P_1A_2 - P_2A_2 = \left(P_1 - P_2\right)A_2 = \rho A_2 U_2(U_2 - U_1) \Rightarrow \left(P_1 - P_2\right) = \rho U_2^{\,2}(1 - \frac{A_2}{A_1})$$

3. GORAPEN HIDRAULIKOA.



$$\sum \vec{F}_{ext} = \vec{P}_1\vec{A}_1 + \vec{P}_2\vec{A}_2 = \vec{M}_2 - \vec{M}_1 = q_{m_2}\vec{U}_2 - q_{m_1}\vec{U}_1 = \rho Q(\vec{U}_2 - \vec{U}_1)$$

$$\vec{P}_1\vec{A}_1 = \gamma h_{G_1} A_1 \hat{i} = \gamma \frac{h_1}{2} h_1 B \hat{i} = \gamma \frac{h_1^{\,2}}{2} B \hat{i}$$

$$\vec{P}_2\vec{A}_2 = \gamma h_{G_2} A_2 - \hat{i} = \gamma \frac{h_2}{2} h_2 B - \hat{i} = \gamma \frac{h_2^{\,2}}{2} B - \hat{i}$$

$$P_1 A_1 - P_2 A_2 = \rho Q(U_2 - U_1)$$

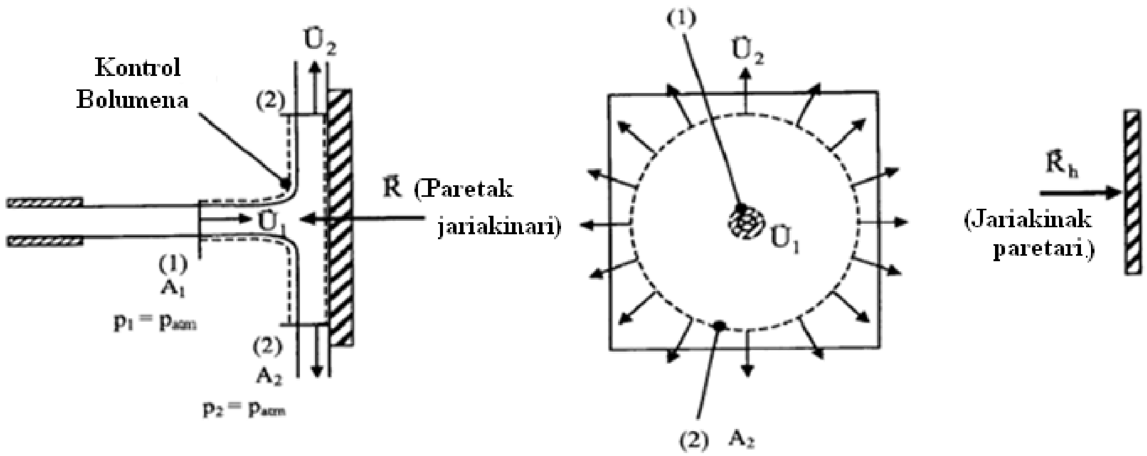
$$\left.\begin{matrix} Q_1 = A_1 U_1 = h_1 B U_1 \\ Q_2 = A_2 U_2 = h_2 B U_2 \end{matrix}\right\} \Rightarrow Q_1 = Q_2 \Rightarrow h_1 B U_1 = h_2 B U_2 \Rightarrow U_2 = \frac{h_1}{h_2} U_1$$

$$\gamma \frac{h_1^{\,2}}{2} B - \gamma \frac{h_2^{\,2}}{2} B = \rho h_1 B U_1 (\frac{h_1}{h_2} U_1 - U_1) \Rightarrow h_2 = \frac{-h_1 \pm \sqrt{h_1^{\,2} + 8 g h_1 U_1^{\,2}}}{2}$$

3. ZORROTADA BATEK OZTOPO BATENGAN ERAGITEN DUEN INDARRA.

1. FINKOAK.

1. PARETA BERTIKALA



$$\sum \vec{F}_{ext} = \vec{R} = \vec{M}_2 - \vec{M}_1$$

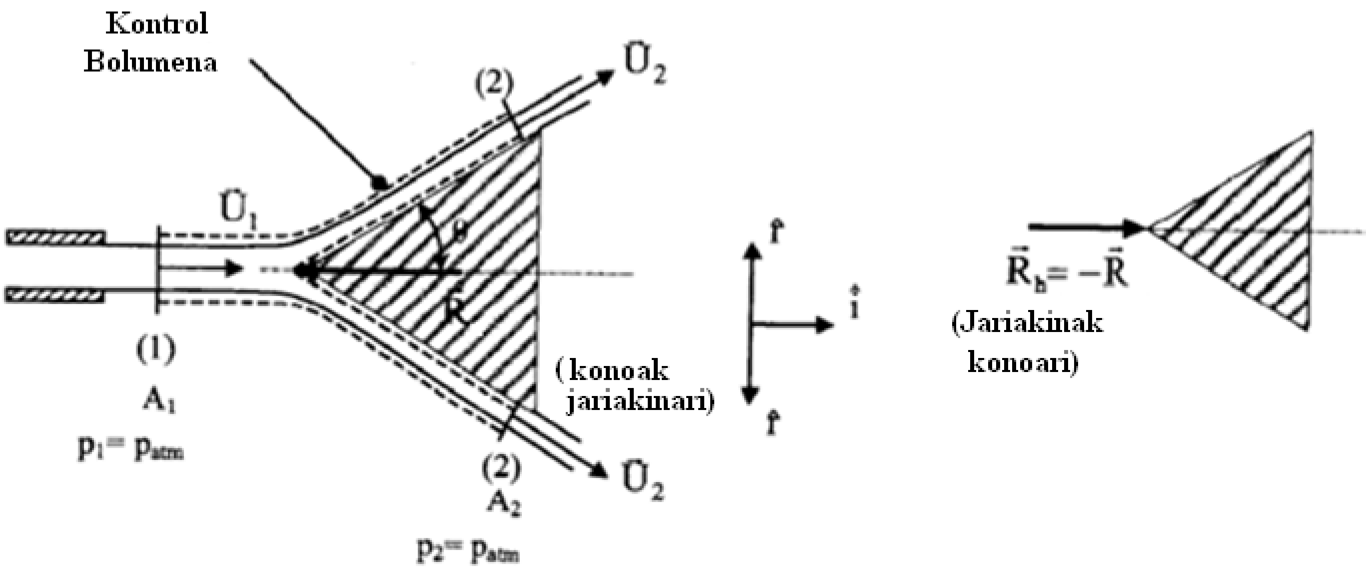
$$\vec{M}_2 = \iint_{A_2} \vec{U}_2 \rho U_2 dA_2 = 0$$

$$\vec{M}_1 = \iint_{A_1} \vec{U}_1 \rho U_1 dA_1 = q_{m_1} \vec{U}_1$$

$$\sum \vec{F}_{ext} = \vec{R} = 0 - \vec{M}_1 = -q_{m_1} \vec{U}_1$$

$$\begin{cases} x \Rightarrow -R_X = -q_{m_1} U_1 \Rightarrow R_X = q_{m_1} U_1 = \rho A_1 U_1^{\,2} \\ \qquad \vec{R}_X = \rho A_1 U_1^{\,2} (-\hat{i}) \Rightarrow \vec{R}_{H_X} = \rho A_1 U_1^{\,2} \, \hat{i} \\ y \Rightarrow R = 0 \Rightarrow \vec{R}_{H_Y} = 0 \end{cases}$$

2. OZTOPO KONIKO SIMETRIKOA.



$$\sum \vec{F}_{ext} = \vec{R} = \vec{M}_2 - \vec{M}_1$$

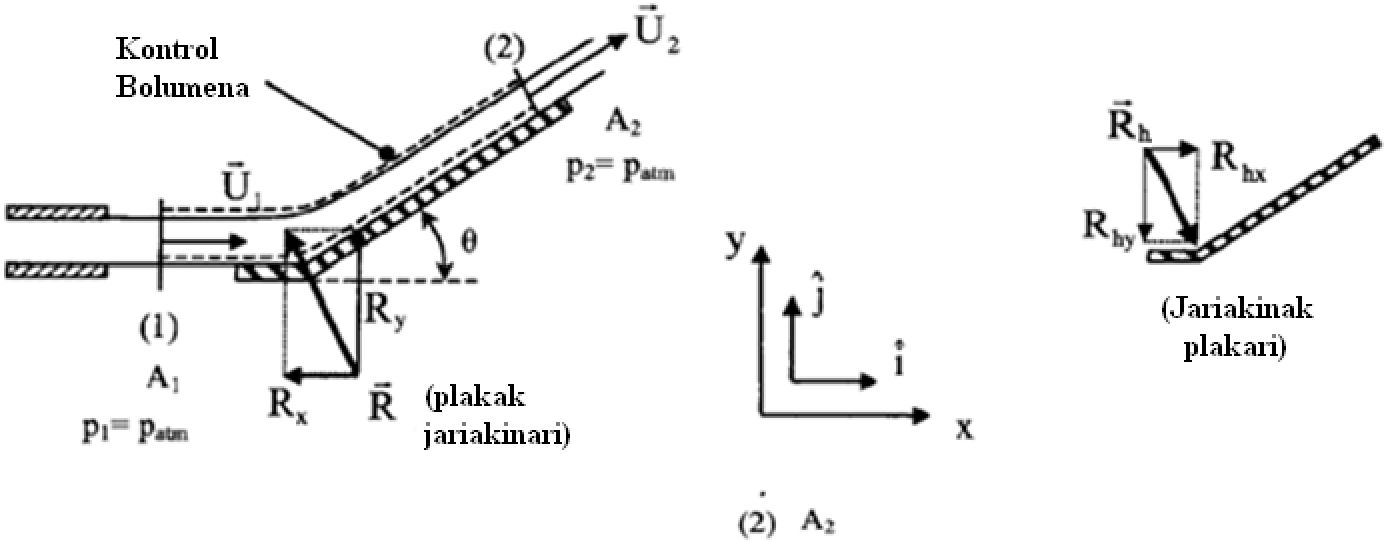
$$\vec{M}_2 = \iint_{A_2} \vec{U}_2 \rho U_2 dA_2 = \iint_{A_2} \left(\vec{U}_{2_x} + \vec{U}_{2_y}\right) \rho U_2 dA_2 = \iint_{A_2} \vec{U}_{2_x} \rho U_2 dA_2 + \iint_{A_2} \vec{U}_{2_y} \rho U_2 dA_2 = \vec{M}_{2_x} + 0 =$$

$$\vec{M}_1 = \iint_{A_1} \vec{U}_1 \rho U_1 dA_1 = q_{m_1} \vec{U}_1$$

$$\sum \vec{F}_{ext} = \vec{R} = \vec{M}_2 - \vec{M}_1 = \vec{M}_{2_x} - q_{m_1} \vec{U}_1$$

$$\begin{cases} x \rightarrow -R_X = q_{m_2} U_{2_X} - q_{m_1} U_{1_X} = q_{m_1} (U_2 \cos\theta - U_1) = q_{m_1} U_1 (\cos\theta - 1) \\ \qquad R_X = q_{m_1} U_1 (1 - \cos\theta) \Rightarrow \vec{R}_X = q_{m_1} U_1 (1 - \cos\theta)(-\hat{i}) \Rightarrow \vec{R}_{H_X} = \rho A_1 U_1^2 (1 - \cos\theta)\hat{i} \\ y \rightarrow R_Y = 0 \Rightarrow R_{H_Y} = 0 \end{cases}$$

3. ALABEA (PLAKA INKLINATUA)



$$\sum \vec{F}_{ext} = \vec{R} = \vec{M}_2 - \vec{M}_1 = q_{m2}\vec{U}_2 - q_{m1}\vec{U}_1 = q_{m2}\vec{U}_{2y} - q_{m1}\vec{U}_{1x}$$

$$x \Rightarrow -R_X = (q_{m2}U_2\cos\theta - q_{m1}U_1) \Rightarrow R_X = q_{m1}U_1(1-\cos\theta) = \rho A_1 U_1^2(1-\cos\theta)$$

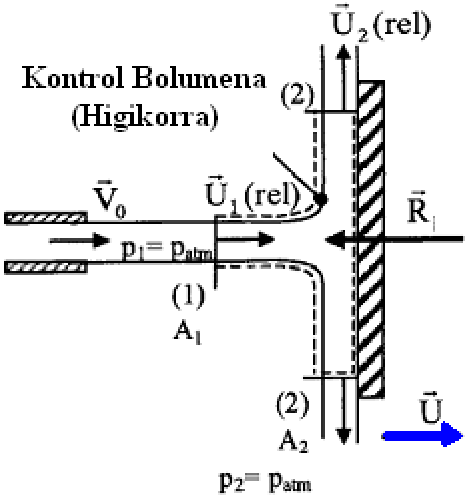
$$\vec{R}_X = \rho A_1 U_1^2(1-\cos\theta)\left(-\hat{i}\right) \Rightarrow \vec{R}_{H_X} = \rho A_1 U_1^2(1-\cos\theta)\hat{i}$$

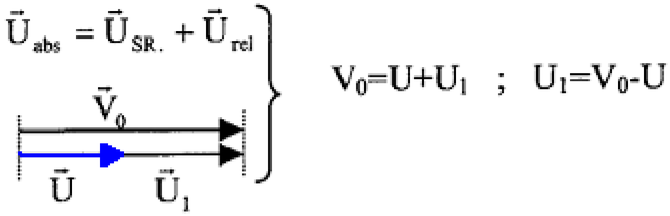
$$y \Rightarrow R_Y = q_{m2}U_2 sen\theta \Rightarrow R_Y = q_{m1}U_1 sen\theta = \rho A_1 U_1^2 sen\theta$$

$$\vec{R}_Y = \rho A_1 U_1^2 sen\theta\,\hat{j} \Rightarrow \vec{R}_{H_Y} = \rho A_1 U_1^2 sen\theta(-\hat{j})$$

2. MUGIKORRAK

1. PARETA BERTIKALA.





$$\sum \vec{F}_{ext} = \vec{R} = \vec{M}_2 - \vec{M}_1$$

$$\vec{M}_2 = \iint_{A_2} \vec{U}_{R_2} \rho U_{R_2} dA_2 = 0$$

$$\vec{M}_1 = \iint_{A_1} \vec{U}_{R_1} \rho \vec{U}_{R_1} dA_1 = \vec{U}_{R_1} \iint_{A_1} \rho \vec{U}_{R_1} dA_1 = \vec{U}_{R_1} q_{m R_1} = q_{m R_1} \vec{U}_{R_1}$$

$$\vec{U}_{R_1} = (\vec{V}_0 - \vec{U}) = (V_0 - U)\hat{i}$$

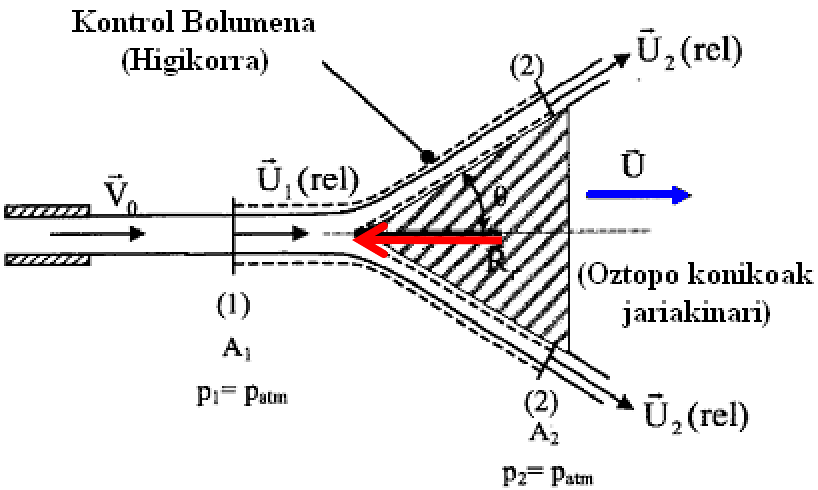
$$\sum \vec{F}_{ext} = \vec{R} = \vec{M}_2 - \vec{M}_1 = 0 - q_{m R_1} \vec{U}_{R_1}$$

$$x \Rightarrow -R_X = -q_{m R_1} \vec{U}_{R_1} \Rightarrow R_X = \rho A_1 U_{R_1} U_{R_1} = \rho A_1 U_{R_1}^2 = \rho A_1 (V_0 - U)^2$$

$$\vec{R}_X = \rho A_1 (V_0 - U)^2 \left(-\hat{i}\right) \Rightarrow \vec{R}_{H_X} = \rho A_1 (V_0 - U)^2 \hat{i}$$

$$y \Rightarrow R_Y = 0 \Rightarrow R_{H_Y} = 0$$

2. OZTOPO KONIKOA



$$\sum \vec{F}_{ext} = \vec{R} = \vec{M}_2 - \vec{M}_1 = q_{m_{R_2}} \vec{U}_{R_{2x}} - q_{m_{R_1}} \vec{U}_{R_{1x}}$$

$$\vec{U}_{R_1} = (V_0 - U)\hat{i}$$

$$\sum \vec{F}_{ext} = \vec{R} = \vec{M}_2 - \vec{M}_1 = q_{m_{R_2}} \vec{U}_{R_{2x}} - q_{m_{R_1}} \vec{U}_{R_{1x}}$$

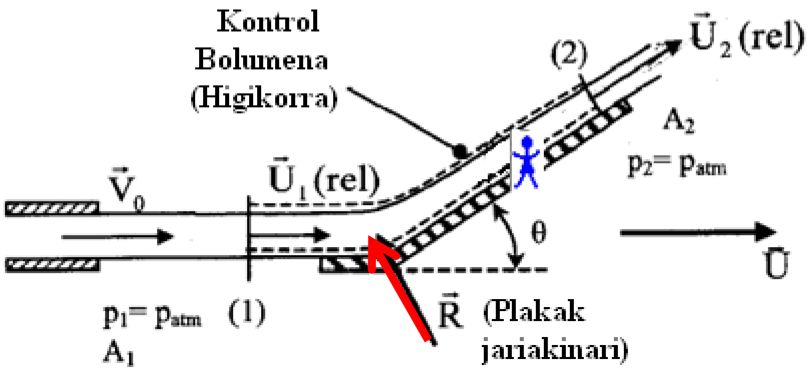
$$x \rightarrow -R_X = q_{m_{R_2}} U_{R_{2x}} - q_{m_{R_1}} U_{R_{1x}} = q_{m_{R_1}} (U_{R_2} \cos\theta - U_{R_1}) = q_{m_{R_1}} U_{R_1} (\cos\theta - 1)$$

$$R_X = \rho A_1 U_{R_1} (1 - \cos\theta) = \rho A_1 (V_0 - U)^2 (1 - \cos\theta)$$

$$\vec{R}_X = \rho A_1 (V_0 - U)^2 (1 - \cos\theta)\left(-\hat{i}\right) \Rightarrow \vec{R}_{H_X} = \rho A_1 (V_0 - U)^2 (1 - \cos\theta)\hat{i}$$

$$y \rightarrow R_Y = 0 \Rightarrow R_{H_Y} = 0$$

3. ALABEA (PLAKA INKLINATUA)



$$\sum \vec{F}_{ext} = \vec{R} = \vec{M}_2 - \vec{M}_1 = q_{m_{R_2}} \vec{U}_{R_2} - q_{m_{R_1}} \vec{U}_{R_1}$$

$$\vec{U}_{R_1} = (V_0 - U)\hat{i}$$

$$x \Rightarrow -R_X = (q_{m_{R_2}} U_{R_2} \cos\theta - q_{m_{R_1}} U_{R_1})$$

$$R_X = q_{m_{R_1}} U_{R_1} (1 - \cos\theta) = \rho A_1 U_{R_1}^2 (1 - \cos\theta)$$

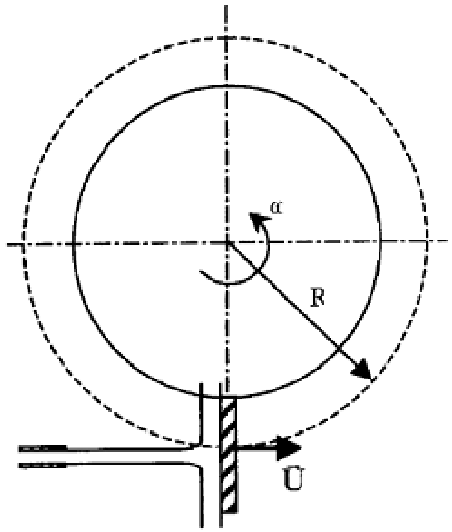
$$\vec{R}_X = \rho A_1 (V_0 - U)^2 (1 - \cos\theta)\left(-\hat{i}\right) \Rightarrow \vec{R}_{H_X} = \rho A_1 (V_0 - U)^2 (1 - \cos\theta)\hat{i}$$

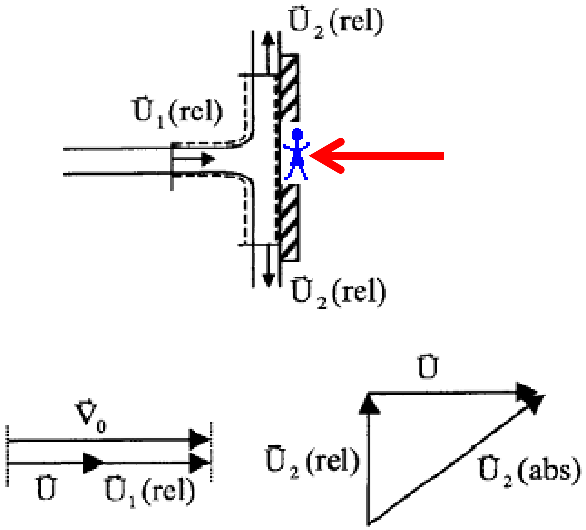
$$y \Rightarrow R_Y = q_{m_{R_2}} U_{R_2} sen\theta \Rightarrow R_Y = q_{m_{R_1}} U_{R_1} sen\theta = \rho A_1 (V_0 - U)^2 sen\theta$$

$$\vec{R}_Y = \rho A_1 (V_0 - U)^2 sen\theta\, \hat{j} \Rightarrow \vec{R}_{H_Y} = \rho A_1 (V_0 - U)^2 sen\theta (-\hat{j})$$

3. ALABEEN SEGIDA:

1. GURPIL BATETAN KOKATUTAKO PLAKA BERTIKALEN SEGIDA:





$$\sum \vec{F}_{ext} = \vec{R} = \vec{M}_2 - \vec{M}_1$$

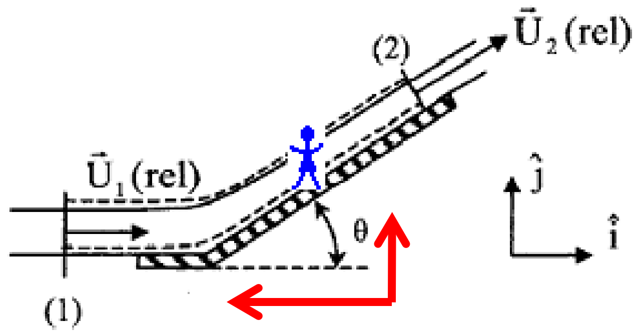
$$\vec{M}_2 = \iint_{A_2} \vec{U}_{2abs} \rho U_{2abs} dA_2 = \iint_{A_2} (\vec{U}_{2R} + \vec{U}) \rho U_{2abs} dA_2 = 0 + \iint_{A_2} \vec{U} \rho U_{2abs} dA_2 = q_{m_{2abs}} \vec{U}$$

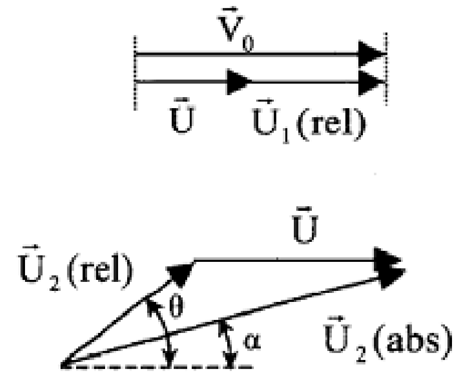
$$\vec{M}_1 = q_{m_{1abs}} \vec{V}_0$$

$$\vec{R} = \vec{M}_2 - \vec{M}_1 = q_{m_0} (\vec{U} - \vec{V}_0)$$

$$x \Rightarrow -R = q_{m_0} (U - V_0) \Rightarrow R = q_{m_0} (V_0 - U) = \underbrace{\rho A_0 V_0}_{q_{m2abs} = q_{m1abs} = q_{m0}} (V_0 - U) \Rightarrow \vec{R}_H = \rho A_0 V_0 (V_0 - U)(-\hat{i}) \Rightarrow \vec{R} = \rho A_0 V_0 (V_0 - U)\hat{i}$$

2. GURPIL BATETAN KOKATUTAKO PLAKA INKLINATUEN SEGIDA.





$$\vec{U}_{2,abs} = (U_2 \cos\theta + U)\hat{i} + U_2 sen\theta \hat{j}$$

$$\sum \vec{F}_{ext} = \vec{R} = \vec{M}_2 - \vec{M}_1 =$$

$$\vec{M}_2 = \iint_{A_2} \vec{U}_{2abs} \rho U_{2abs} dA_2 = \vec{U}_{2abs} \iint_{A_2} \rho U_{2abs} dA_2 = \vec{U}_{2abs} q_{m_{2abs}} = q_{m_0} (\vec{U}_{2R} + \vec{U})$$

$$\vec{M}_1 = \iint_{A_1} \vec{U}_{1abs} \rho U_{1abs} dA_1 = \vec{U}_{1abs} \iint_{A_1} \rho U_{1abs} dA_1 = \vec{U}_{1abs} q_{m_{1abs}} = q_{m_0} \vec{V}_0$$

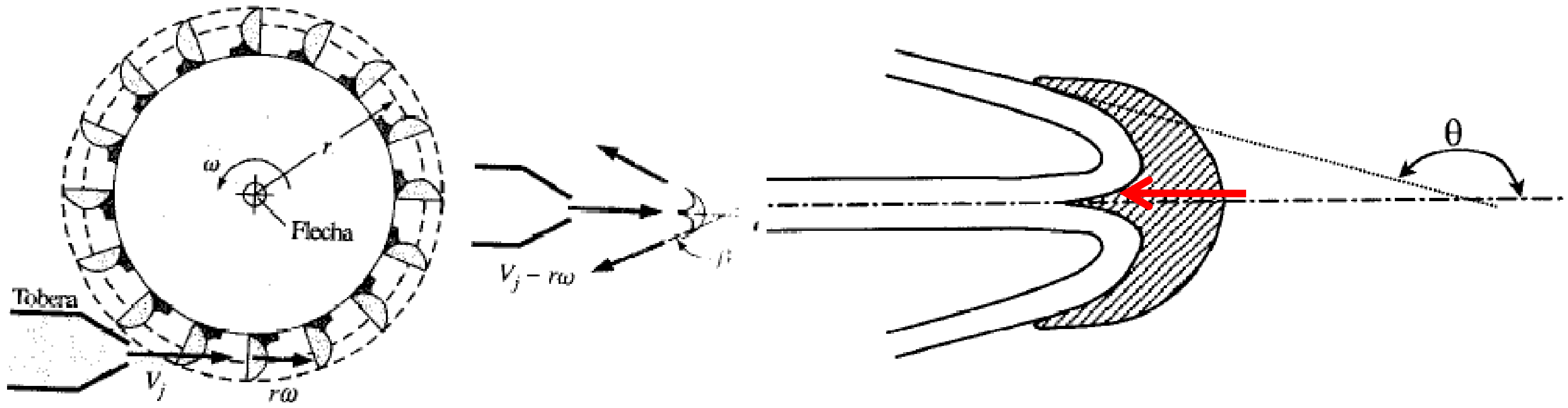
$$x \Rightarrow \; -R_X = q_{m_0} (V_0 - U)(\cos\theta - 1) = \rho A_0 V_0 (V_0 - U)(\cos\theta - 1)$$

$$R_X = \rho A_0 V_0 (V_0 - U)(1 - \cos\theta) \Rightarrow \vec{R}_X = \rho A_0 V_0 (V_0 - U)(1 - \cos\theta)\hat{i} \Rightarrow \vec{R}_{H_X} = \rho A_0 V_0 (V_0 - U)(1 - \cos\theta)(-\hat{i})$$

$$y \Rightarrow R_Y = q_{m0} (V_0 - U) sen\theta$$

$$\vec{R}_Y = \rho A_0 V_0 (V_0 - U) sen\theta \, \hat{j} \Rightarrow \vec{R}_{H_Y} = \rho A_0 V_0 (V_0 - U) sen\theta (-\hat{j})$$

3. PELTON TURBINA:



$$\left.\begin{aligned}\vec{R}_x &= q_{m_0}(V_0-U)(1-\cos\theta)(-\vec{i}) = \rho A_0 V_0 (V_0-U)(1-\cos\theta)(-\vec{i})\\ \vec{R}_y &= 0\end{aligned}\right\}$$

$$\Rightarrow \vec{R}_{H_x} = -\vec{R}_x = q_{m_0}(V_0-U)(1-\cos\theta)\,\vec{i} = \rho A_0 V_0 (V_0-U)(1-\cos\theta)\,\vec{i}$$

$$P_{BALIOGARRIA} = R_H U = \rho A_0 V_0 (V_0-U)(1-\cos\theta)U \Rightarrow \frac{dP}{dU} = 0 \Leftrightarrow U = \frac{V_0}{2}$$

$$\eta = \frac{P_{LORTUTAKO}}{P_{BEHARREZKOA}} = \frac{P_{BALIOGARRIA}}{P_{BEHARREZKOA}} = \frac{P_{BALIOGARRIA}}{P_{ZORROTADA}} = \frac{R_H U}{\frac{1}{2} q_{m_0} V_0^2} = \frac{R_H U}{\frac{1}{2}\rho A_0 V_0^3} = \frac{2(V_0-U)(1-\cos\theta)U}{V_0^2}$$

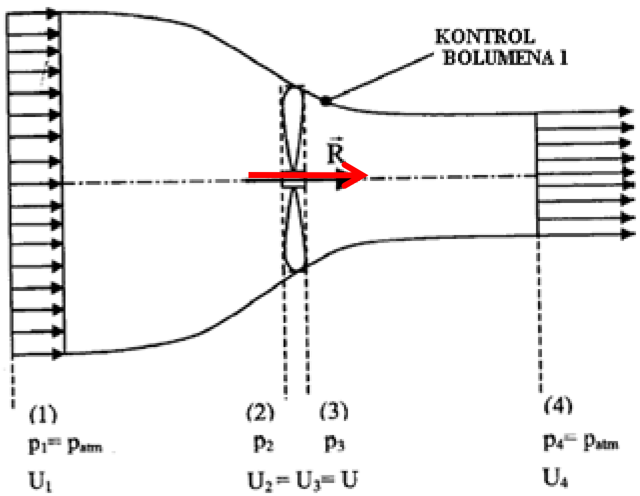
$$\eta = 1 \Leftrightarrow U = \frac{V_0}{2};\ (1-\cos\theta) = 1 \Leftrightarrow \theta = 180^{\circ}$$


WLL

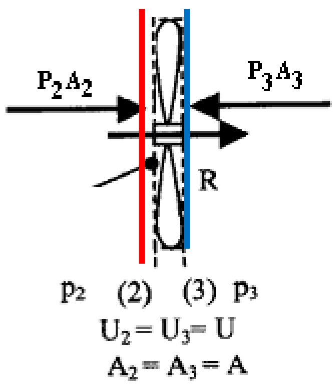
3. PROPULTSIO SISTEMAK

1. HELIZEAK:

1. U_1: sarrerako abiadura; U_4 : irteerako abiadura.
2. Biak abiadura erlatiboak dira (helizearekiko)

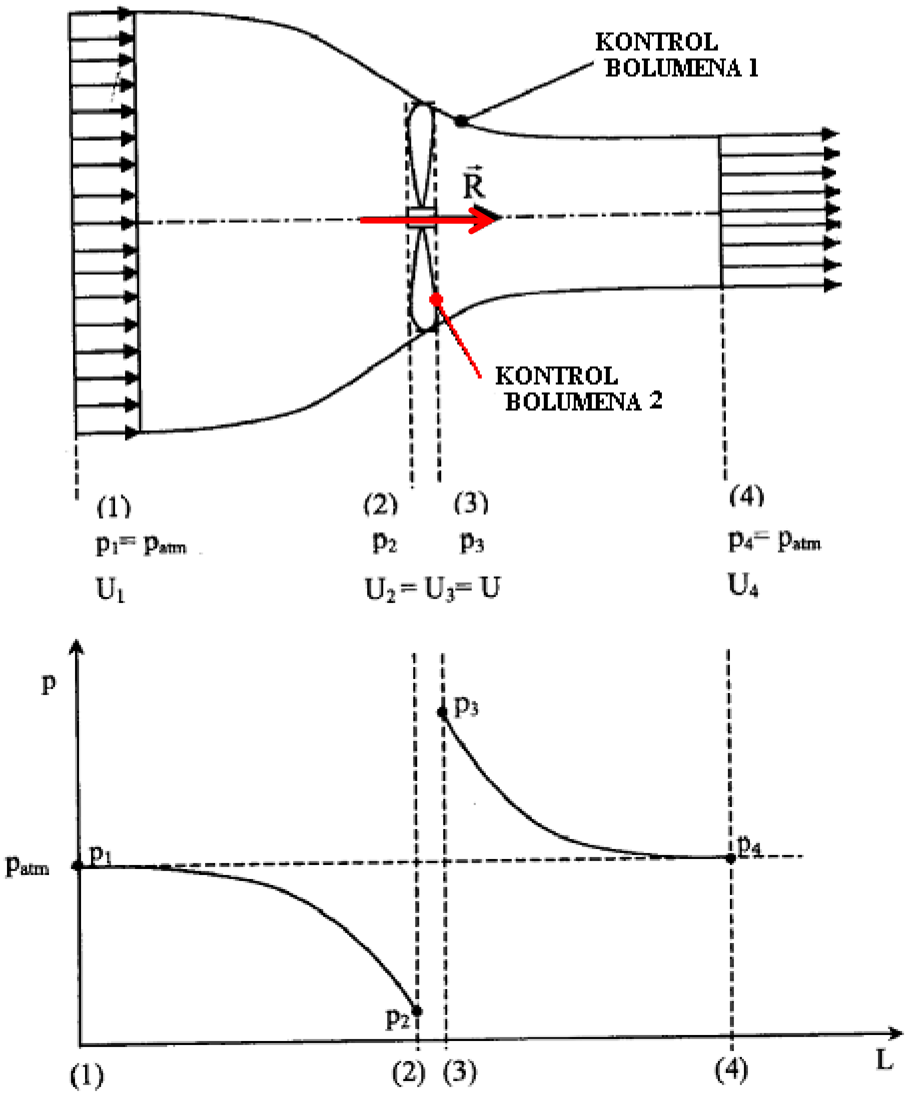


$$HKT^{ma}{}_1^4 \Rightarrow \vec{R} = \vec{M}_4 - \vec{M}_1$$
$$\Rightarrow R = \rho Q_4 U_4 - \rho Q_1 U_1 = \rho Q (U_4 - U_1);\ U_4 > U_1$$
$$HKT^{ma}{}_2^3 \Rightarrow \vec{R} + \vec{P}_2 \vec{A}_2 + \vec{P}_3 \vec{A}_3 = \vec{M}_3 - \vec{M}_2$$
$$\Rightarrow P_2 A_2 - P_3 A_3 + R = \rho Q_3 U_3 - \rho Q_2 U_2 = \rho Q (U_3 - U_2) = 0$$
$$P_3 A_3 - P_2 A_2 = (P_3 - P_2) A = R; \quad p_3 > p_2$$

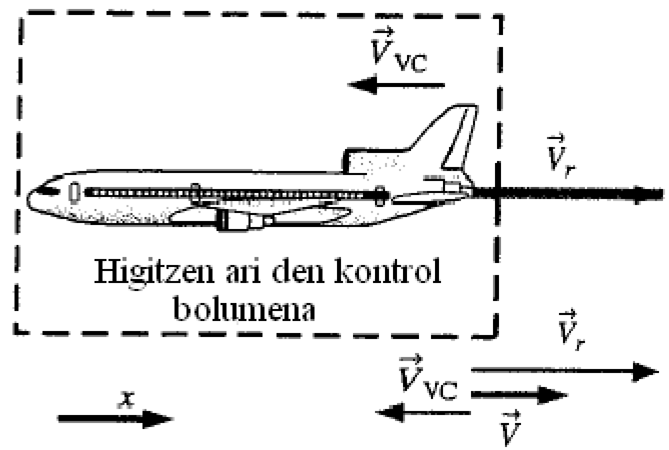


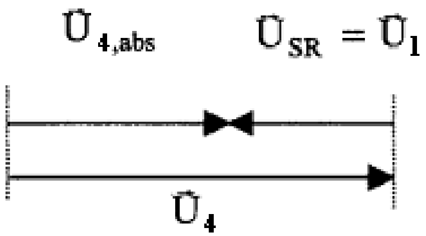
$$B_1^2 \Rightarrow \frac{p_1}{\gamma} + \frac{U_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{U_2^2}{2g} + z_2 \ (3)$$
$$B_3^4 \Rightarrow \frac{p_3}{\gamma} + \frac{U_3^2}{2g} + z_3 = \frac{p_4}{\gamma} + \frac{U_4^2}{2g} + z_4 \ (4)$$
$$\left(\begin{array}{l} p_1 > p_2;\ U_2 > U_1 \\ p_3 > p_4;\ U_4 > U_3 \end{array} \right)$$

$$\xrightarrow{(3)+(4)} \frac{p_3}{\gamma} + \frac{U_1^2}{2g} = \frac{p_2}{\gamma} + \frac{U_4^2}{2g} \Rightarrow \frac{p_3 - p_2}{\gamma} = \frac{U_4^2 - U_1^2}{2g} = \frac{(U_4 - U_1)(U_4 + U_1)}{2g}$$
$$p_3 - p_2 = \rho \frac{(U_4 - U_1)(U_4 + U_1)}{2} = \frac{R}{A} = \frac{\rho Q (U_4 - U_1)}{A}$$
$$\Rightarrow \frac{(U_4 + U_1)}{2} = \frac{Q}{A} = U$$



Presio aldaketa propultsio sistema batetan zehar





$$\bar{U}_{4,abs} = \bar{U}_{SR} + \bar{U}_4$$

$$U_{4,abs} = U_4 - U_1$$

$$\eta = \frac{P_{Baliog}}{P_t} = \frac{R_H U_1}{P_t}$$

$$P_{Baliog} = R_H U_1 = q_m (U_4 - U_1) U_1$$

$$P_t = P_{Baliog} + P_{Galerak} = R_H U_1 + \frac{1}{2} q_m U_4{}^2{}_{abs} = R_H U_1 + \frac{1}{2} q_m (U_4 - U_1)^2 =$$

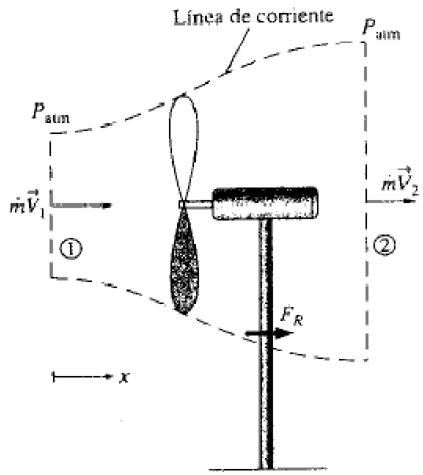
$$= q_m (U_4 - U_1) U_1 + \frac{1}{2} q_m (U_4 - U_1)^2 = q_m (U_4 - U_1) \left[U_1 + \frac{(U_4 - U_1)}{2} \right]$$

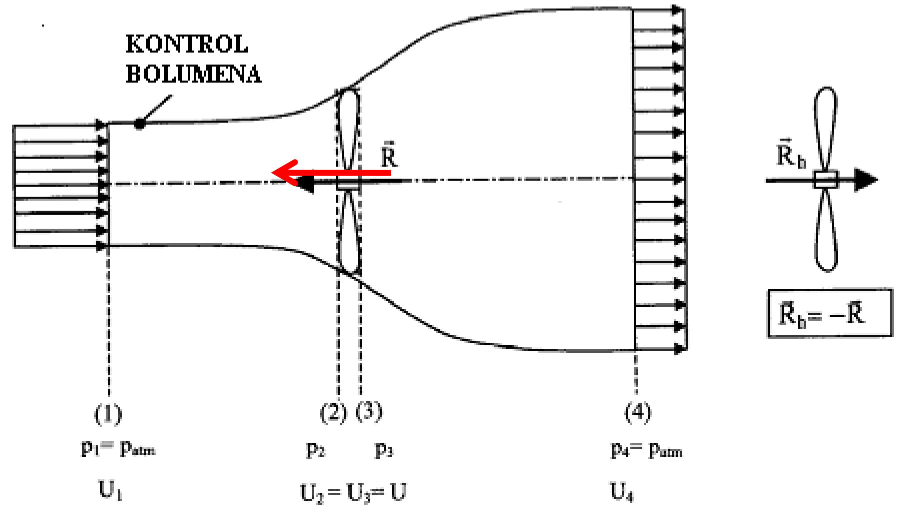
$$= q_m (U_4 - U_1) \frac{(U_4 + U_1)}{2} = R_H U$$

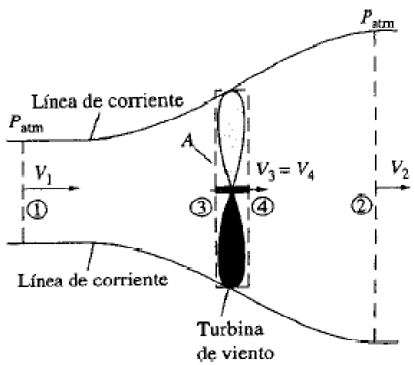
$$\eta = \frac{P_{Baliog}}{P_t} = \frac{R_H U_1}{R_H U} = \frac{U_1}{U} = \frac{U_1}{\frac{U_4 + U_1}{2}} = \frac{U_1}{\frac{U_4 + U_1}{2}} = \frac{2U_1}{U_1 + U_4}$$

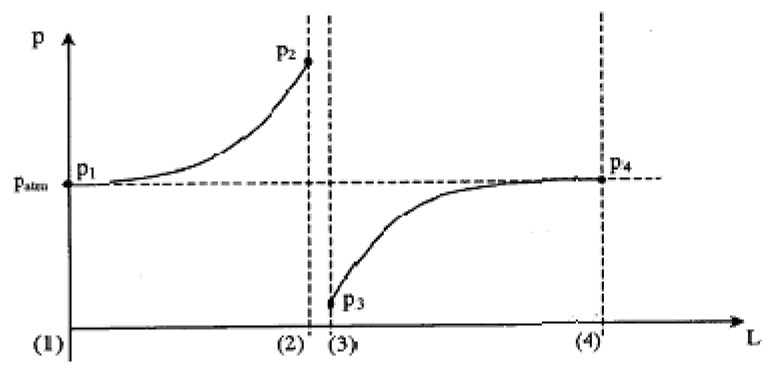
2. AEROSORGAILUA (HAIZE-SORGAILUA):

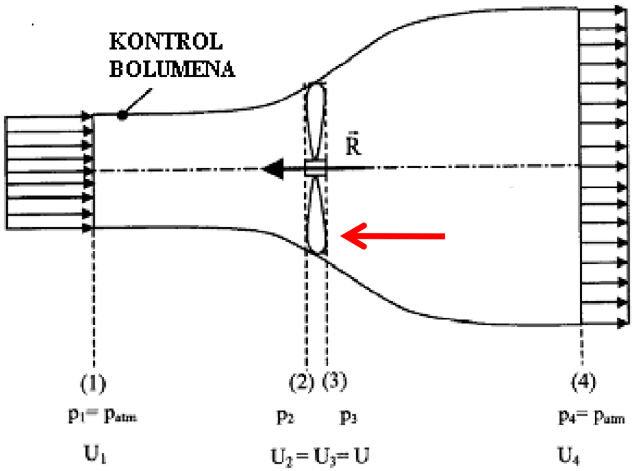
1. U_1: sarrerako abiadura, U_4 : irteerako abiadura.
2. Biak abiadura erlatiboak dira (airesorgailuarekiko).

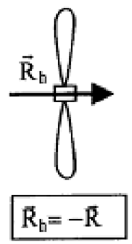




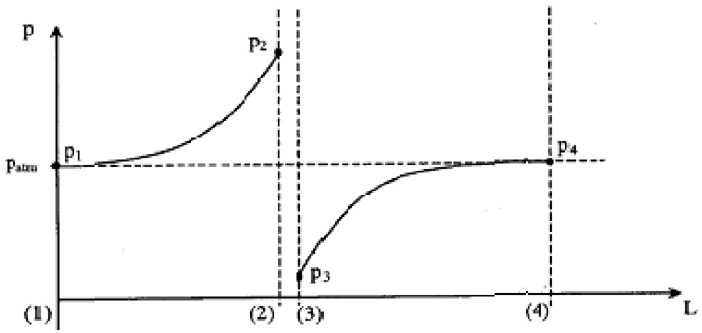








$$HKT^{ma}{}_1^4 \Rightarrow \vec{R}_H = \vec{M}_4 - \vec{M}_1$$
$$\Rightarrow -R_H = \rho Q_4 U_4 - \rho Q_1 U_1 \Rightarrow R_H = \rho Q(U_1 - U_4);\ U_1 > U4$$
$$HKT^{ma}{}_2^3 \Rightarrow \vec{R}_H + \vec{P}_2\vec{A}_2 + \vec{P}_3\vec{A}_3 = \vec{M}_3 - \vec{M}_2$$
$$\Rightarrow P_2A_2 - P_3A_3 - R_H = \rho Q_3 U_3 - \rho Q_2 U_2 = \rho Q(U_3 - U_2) = 0$$
$$P_2A_2 - P_3A_3 = (P_2 - P_3)A = R_H;\quad p_2 > p_3$$



$$B_1^2 \Rightarrow \frac{p_1}{\gamma} + \frac{U_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{U_2^2}{2g} + z_2 \ (3)$$
$$B_3^4 \Rightarrow \frac{p_3}{\gamma} + \frac{U_3^2}{2g} + z_3 = \frac{p_4}{\gamma} + \frac{U_4^2}{2g} + z_4 \ (4)$$
$$\left(\begin{matrix} p_2 > p_1;\ U_1 > U_2 \\ p_4 > p_3;\ U_3 > U_4 \end{matrix} \right)$$

$$\overset{(3)+(4)}{\frac{p_3}{\gamma} + \frac{U_1^2}{2g} = \frac{p_2}{\gamma} + \frac{U_4^2}{2g}} \Rightarrow \frac{U_1^2 - U_4^2}{2g} = \frac{(U_1 - U_4)(U_1 + U_4)}{2g} = \frac{p_2 - p_3}{\gamma}$$
$$p_2 - p_3 = \rho\frac{(U_1 - U_4)(U_1 + U_4)}{2g} = \frac{R_H}{A} = \frac{\rho Q(U_1 - U_4)}{A}$$
$$\Rightarrow \frac{(U_1 + U_4)}{2} = \frac{Q}{A} = U$$

$$\eta = \frac{P_{Baliog}}{P_t} =$$

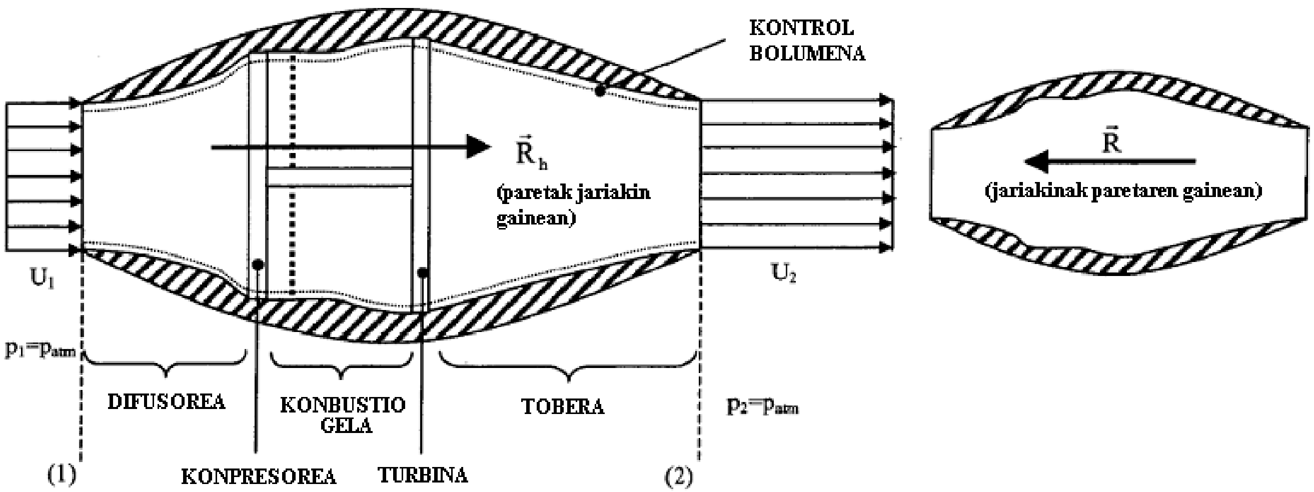
$$P_{Baliog} = R_H U = q_m (U_1 - U_4) \frac{(U_1 + U_4)}{2} = \frac{1}{2} q_m \left(U_1{}^2 - U_4^2 \right) = \frac{1}{2} q_m U_1{}^2 - \frac{1}{2} q_m U_4^2$$

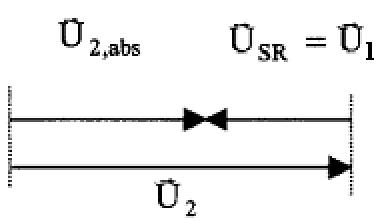
$$P_t = \frac{1}{2} q'_m U_1{}^2 = \frac{1}{2} \rho A U_1 U_1{}^2 = \frac{1}{2} \rho A U_1{}^3$$

$$\eta = \frac{P_{Baliog}}{P_t} = \frac{R_H U}{\frac{1}{2} \rho A U_1{}^3} = \frac{1}{2} \left[1 - \left(\frac{U_4}{U_1} \right)^2 \right] \left[\frac{U_4 + U_1}{U_1} \right] \Rightarrow \frac{\partial \eta}{\partial \left(U_4 / U_1 \right)} = 0 \Leftrightarrow \frac{U_4}{U_1} = \frac{1}{3} \Rightarrow \eta_{\max} \approx 59\%$$

3. TURBOERREAKTOREA

1. M↑↑↑↑↑→ F. konprimagarria





$$\vec{U}_{2,abs} = \vec{U}_{SR} + \vec{U}_2$$

$$U_{2,abs} = U_2 - U_1$$

$$\vec{F} + \vec{G} = \vec{M}_2 - \vec{M}_1$$

$$\vec{R}_H = q_m(\vec{U}_2 - \vec{U}_1)$$

$$\eta = \frac{P_{Baliog}}{P_t} = \frac{R_H U_1}{P_t}$$

$$P_{Baliog} = R_H U_1 = q_m(U_2 - U_1)U_1$$

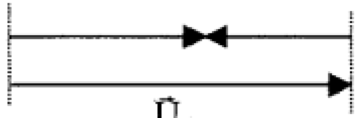
$$P_t = P_{Baliog} + P_{Galerak} = R_H U_1 + \frac{1}{2} q_m U_2{}^2{}_{abs} = R_H U_1 + \frac{1}{2} q_m \left(U_2 - U_1\right)^2 =$$

$$= q_m(U_2 - U_1)U_1 + \frac{1}{2} q_m \left(U_2 - U_1\right)^2 = q_m(U_2 - U_1)\left(U_1 + \frac{(U_2 - U_1)}{2}\right) =$$

$$= q_m(U_2 - U_1)\left(\frac{U_2 + U_1}{2}\right) = R_H U$$

$$\eta = \frac{P_{Baliog}}{P_t} = \frac{R_H U_1}{R_H U} = \frac{U_1}{\frac{U_2 + U_1}{2}} = \frac{U_1}{\frac{U_2 + U_1}{2}}$$

$\vec{U}_{2,abs}$ $\qquad$ $\vec{U}_{SR} = \vec{U}_1$



$\vec{U}_2$

$$\vec{U}_{2,abs} = \vec{U}_{SR} + \vec{U}_2$$

$U_{2,abs} = U_2 - U_1$

$$q_{m_2} \neq q_{m_1}$$

$$q_{m_2} = q_{m_1} + q_c = q_{m_1}(1+k);\ k = erregaiaren\ kontsumo\ espezifikoa = q_c / q_{m_1}$$

$$R_H = q_{m_1}(U_2(1+k) - U_1)$$

$$\eta = \frac{P_{Baliog}}{P_t} = \frac{R_H U_1}{P_t}$$

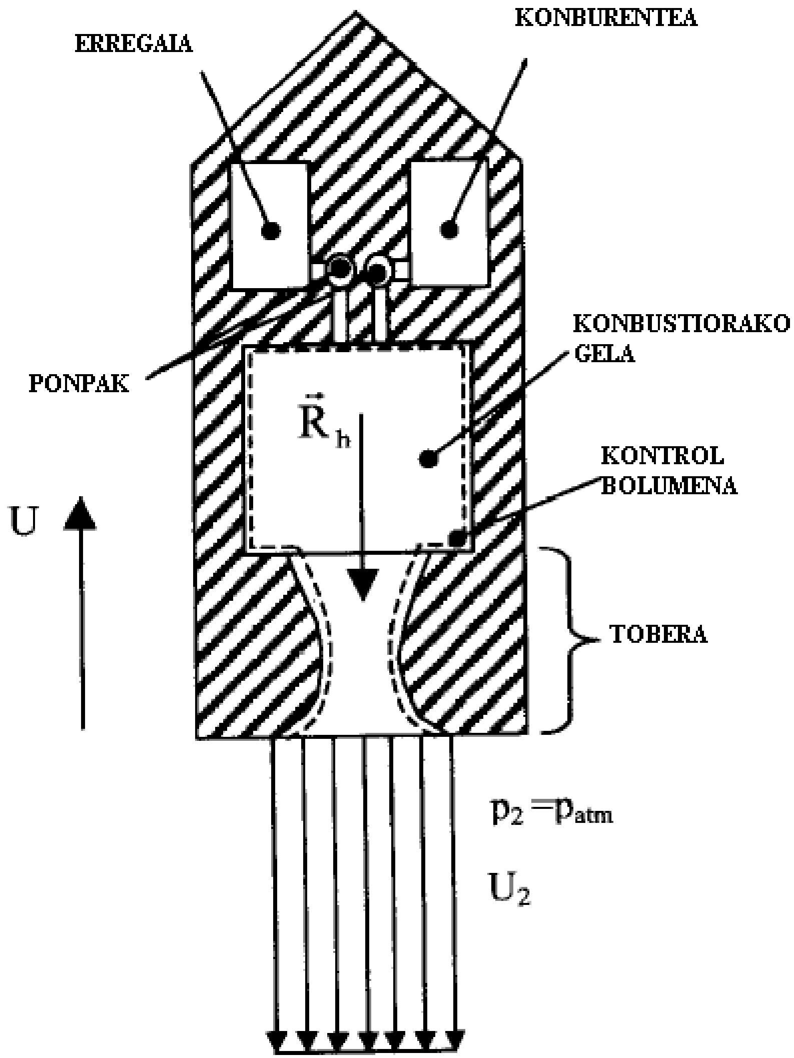
$$P_{Baliog} = R_H U_1 = q_{m_1}(U_2(1+k) - U_1)U_1$$

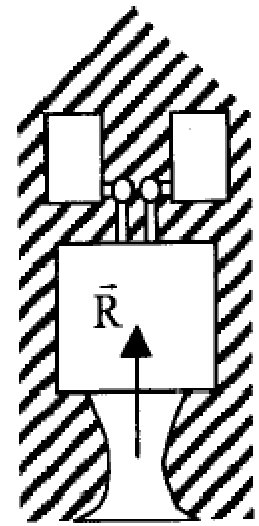
$$P_t = P_{Baliog} + P_{Galerak} = R_H U_1 + \frac{1}{2} q_{m_2} U_{2,abs}{}^2 = R_H U_1 + \frac{1}{2} q_{m_2} \left(U_2 - U_1\right)^2 =$$

$$R_H U_1 + \frac{1}{2} q_{m_1}(1+k)\left(U_2 - U_1\right)^2$$

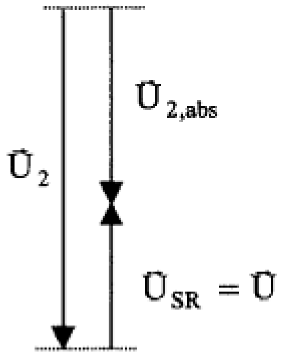
$$\eta = \frac{P_{Baliog}}{P_t} = \frac{1}{1 + \dfrac{(1+k)\left(U_2 - U_1\right)^2}{2\left(U_2(1+K) - U_1\right)U_1}}$$

4. KOHETEAK





“Propultsioa”



$\vec{U}_{2,abs} = \vec{U}_{SR} + \vec{U}_2$

$U_{2,abs}=U_2-U$

$$\vec{F} + \vec{G} = \vec{M}_2 - \vec{M}_1$$

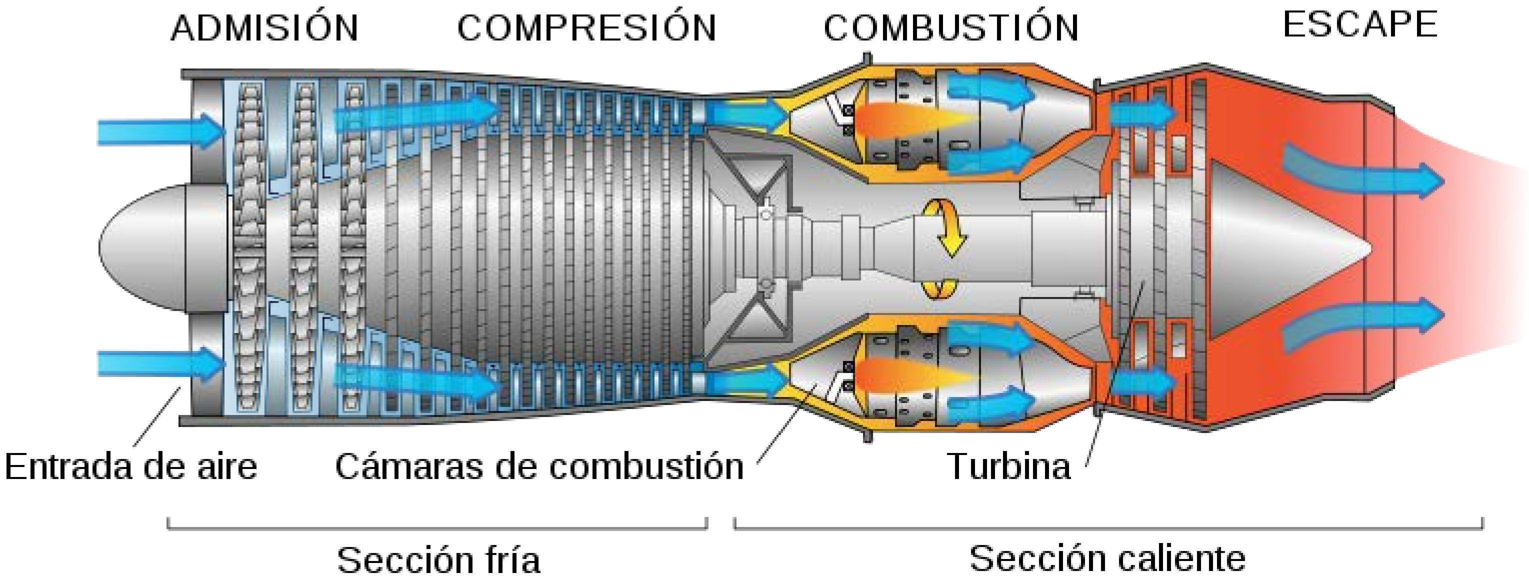
$$x \Rightarrow R_H = q_m(U_2 - U_1); U_1 = 0 \Rightarrow R_H = q_m U_2$$

$$\eta = \frac{P_{Baliog}}{P_t} = \frac{R_H U}{P_t}$$

$$P_{Baliog} = R_H U = q_m U_2 U$$

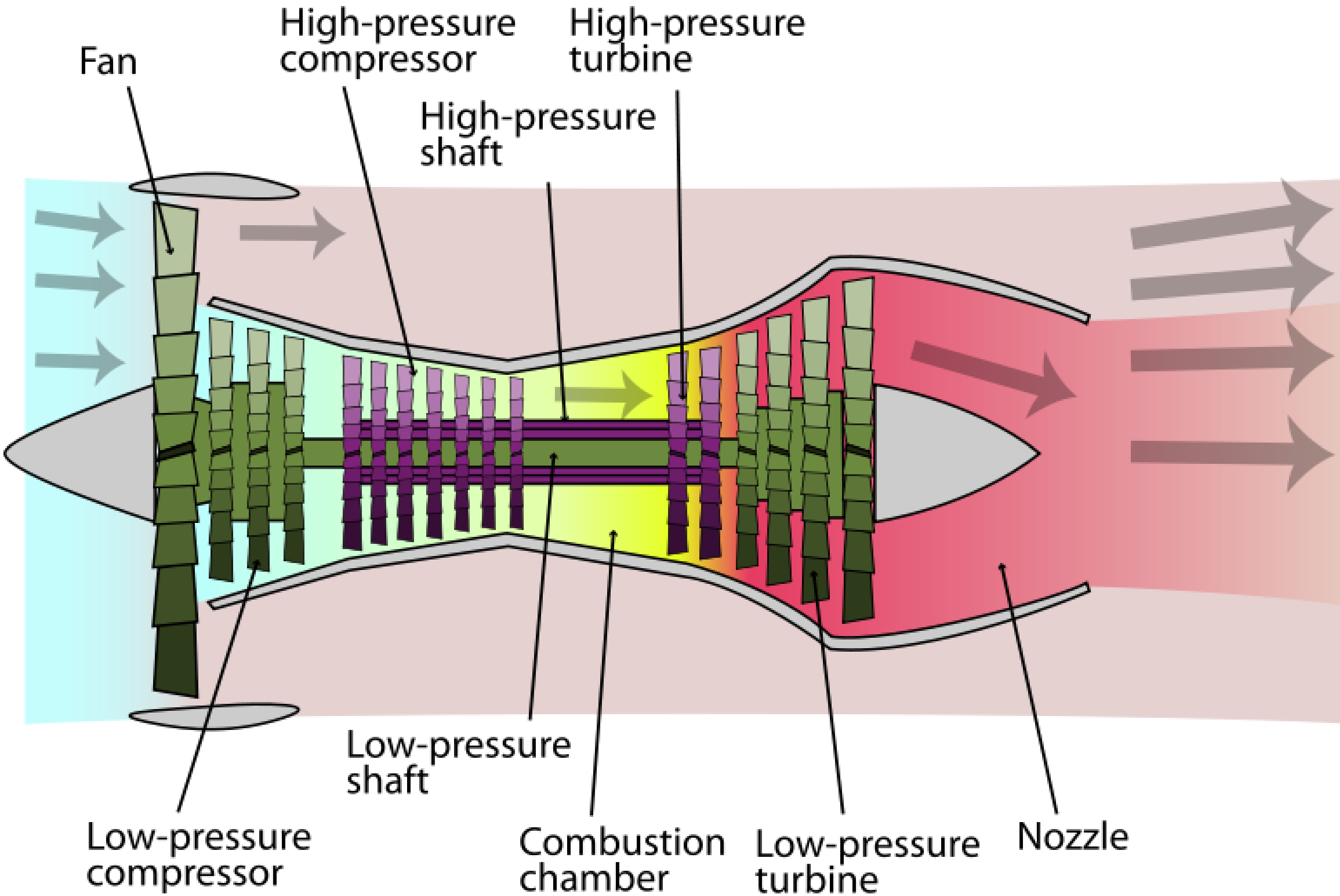
$$P_t = P_{Baliog} + P_{Galerak} = R_H U + \frac{1}{2} q_m U_2{}^2{}_{abs} = R_H U + \frac{1}{2} q_m \left(U_2 - U\right)^2 = q_m U_2 U + \frac{1}{2} q_m \left(U_2 - U\right)^2$$

$$\eta = \frac{P_{Baliog}}{P_t} = \frac{q_m U_2 U}{q_m U_2 U + \frac{1}{2} q_m \left(U_2 - U\right)^2} = \frac{2\left(U_2/U\right)}{1 + \left(U_2/U\right)^2}$$



ADMISIÓN
COMPRESIÓN
COMBUSTIÓN
ESCAPE
Entrada de aire
Cámaras de combustión
Turbina
Sección fría
Sección caliente

2. TURBOFAN


Fan
High-pressure compressor
High-pressure turbine
High-pressure shaft
Low-pressure shaft
Low-pressure compressor
Combustion chamber
Low-pressure turbine
Nozzle

TURBOFAN

1. Fan: situado al frente del motor, Es dónde se inicia la propulsión. Le atraviesa un flujo de aire que se divide en dos corrientes: la primaria y la secundaria o bypass air.

   La corriente primaria entra a través de los compresores a la cámara de combustión.

1. Compresores: el flujo de aire primario pasa a través de diversas etapas de compresores que giran en el mismo sentido del fan. Se suelen utilizar compresores de alta y de baja presión en distintos ejes. La función de estos compresores es aumentar de modo significativo la presión y la temperatura del aire.
2. Cámara de combustión: una vez realizada la etapa de compresión, el aire sale con una presión treinta veces superior de la que tenía en la entrada y a una temperatura próxima a los 600 °C. Se hace pasar este aire a la cámara de combustión, donde se mezcla con el combustible y se quema la mezcla, alcanzándose una temperatura superior a los 1100 °C.
3. Turbinas: el aire caliente que sale de la cámara, pasa a través de los álabes de varias turbinas, haciendo girar diversos ejes. En los motores de bajo bypass el compresor de baja presión y el fan se mueven mediante un mismo eje; mientras que en los de alto bypass se dispone de un eje para cada componente: fan, compresor de baja presión y compresor de alta presión.
4. Escape: una vez el aire caliente ha pasado a través de las turbinas, sale por una tobera por la parte posterior del motor. Las estrechas paredes de la tobera fuerzan al aire a acelerarse. El peso del aire, combinado con esta aceleración produce parte del empuje total. En general, un aumento en el bypass trae como consecuencia una menor participación de la tobera de escape en el empuje total del motor.
5. Conducto del flujo secundario: rodea concéntricamente al núcleo del motor. Sus paredes interna y externa están cuidadosamente perfiladas para minimizar la pérdida de energía del flujo secundario de aire y optimizar su mezcla con el escape del flujo primario.