Universidad del País Vasco
Euskal Herriko Unibertsitatea

Departamento de Ingeniería Mecánica
Ingeniaritza Mekanikoa Saila

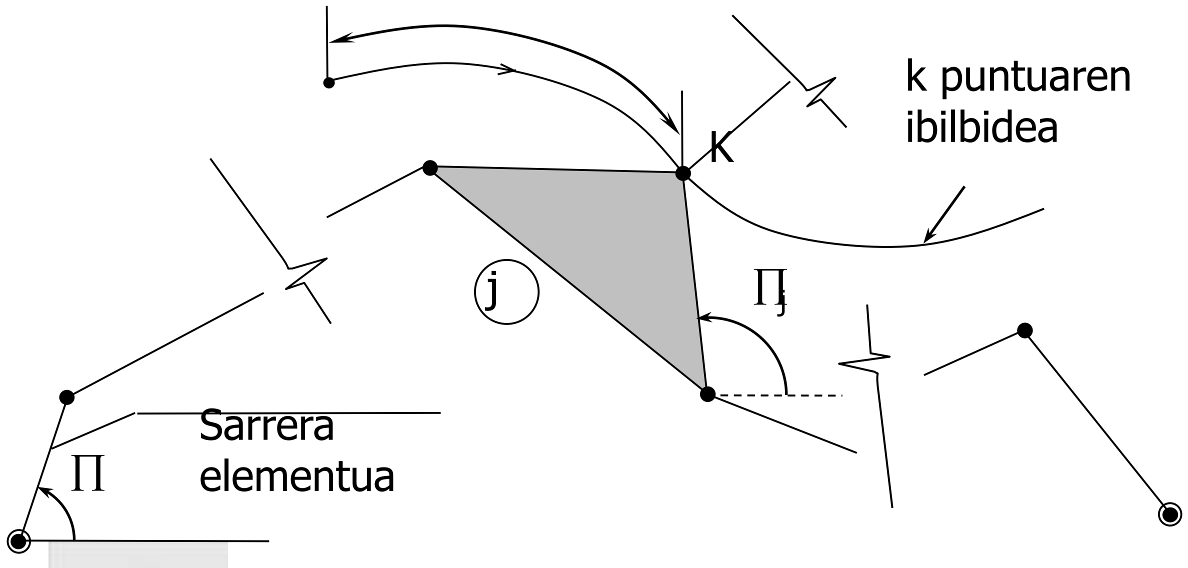
3. Gaia II

Lazo bakarreko mekanismoen analisi zinematikoa

Aurkibidea

1. Eragin koefizienteak.
2. Kokapen arazoaren planteametu analitikoa.
3. Abiadura eta azelerazioen kalkulua.
4. Interes puntuen parametro zinematikoen kalkulua.
5. kokapen singularren detekzioa.

Eragin koefizienteak



1 ag-ko
mekanismoak

$$\varphi_j = f_j(\varphi)$$
$$S_K = \bar{f}_K(\varphi)$$

$$\frac{d\varphi_j}{dt} = \frac{df_j}{d\varphi}\frac{d\varphi}{dt} \longrightarrow \omega_j = g_j\,\omega \longrightarrow g_j(\varphi) = \frac{df_j}{d\varphi}$$

$$\frac{dS_K}{dt} = \frac{d\bar{f}_K}{d\varphi}\frac{d\varphi}{dt} \longrightarrow v_K = \bar{g}_K\,\omega \longrightarrow \bar{g}_K(\varphi) = \frac{d\bar{f}_K}{d\varphi}$$

Abiadura eragin
koefizienteak

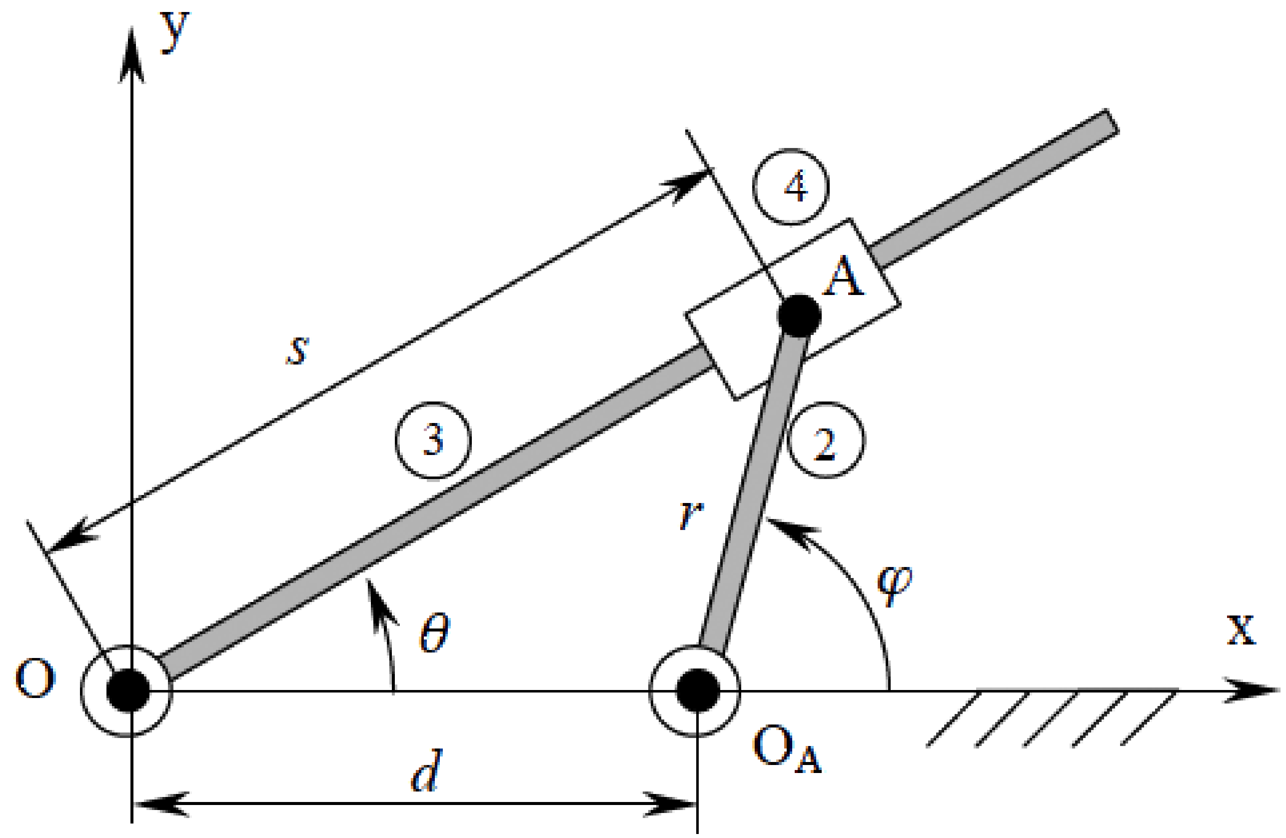
$$\alpha_j = \frac{dg_j}{d\varphi}\omega^2 + g_j\,\alpha \longrightarrow \alpha_j = h_j\,\omega^2 + g_j\,\alpha \longrightarrow h_j = \frac{dg_j}{d\varphi}$$

$$a_K^t = \frac{d\bar{g}_K}{d\varphi}\omega^2 + \bar{g}_K\,\alpha \longrightarrow a_K^t = \bar{h}_K\,\omega^2 + \bar{g}_K\,\alpha \longrightarrow \bar{h}_K = \frac{d\bar{g}_K}{d\varphi}$$

Azelerazio eragin
koefizienteak

Kokapen arazoaren planteamendu analitikoa

- Sarrera elementuak -> **koordenatu orokortuak**
- Aldagai edo **koordenatu sekundarioak**:
  - Transmisio elementuak -> aldagai pasiboak.
  - Irteera elementuak -> irteerako aldagaiak.

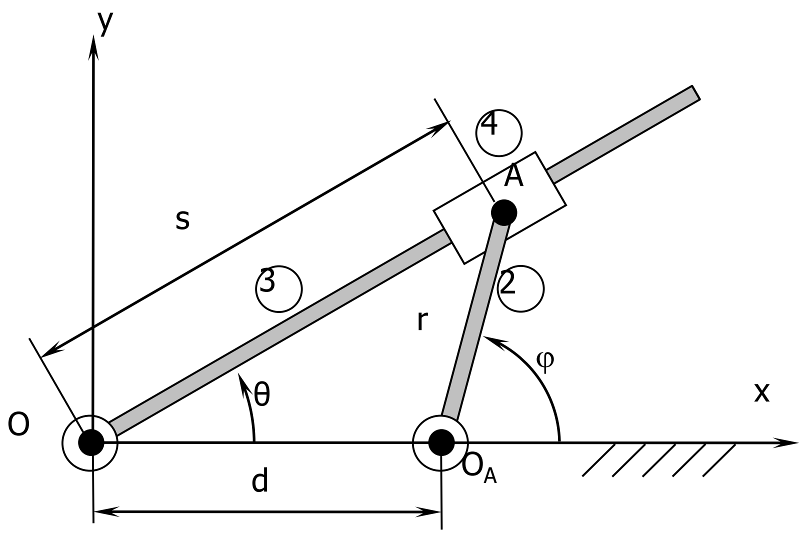


Koordenatu orokortua φ

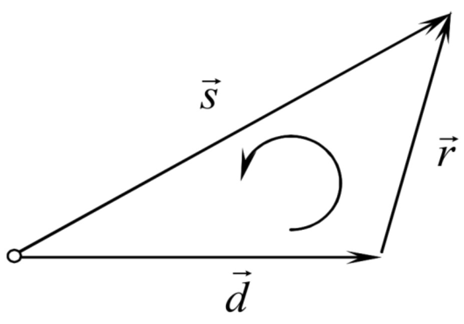
Aldagai sekundarioak

- Irteerakoa θ
- Pasiboa s

Kokapen arazoa



θ, s



$$\vec{d} + \vec{r} - \vec{s} = \vec{0}$$

$$\begin{cases} d + r\cos\varphi - s\cos\theta = 0 \\ r\,\mathrm{sen}\,\varphi - s\,\mathrm{sen}\,\theta = 0 \end{cases}$$

$$\mathrm{sen}\,\theta = \frac{r}{s}\mathrm{sen}\,\varphi$$

$$\cos\theta = \frac{d + r\cos\varphi}{s}$$

$$\theta = \mathrm{Atan2}(r\,\mathrm{sen}\varphi,\ d + r\cos\varphi)$$

s

$$s^2 = d^2 + r^2 + 2dr\cos\varphi$$

Abiaduren kalkulua

$$\begin{bmatrix} -\cos\theta & s\,\mathrm{sen}\theta & -r\,\mathrm{sen}\varphi \\ -\mathrm{sen}\theta & -s\cos\theta & r\cos\varphi \end{bmatrix} \begin{Bmatrix} \dot{s} \\ \dot{\theta} \\ \dot{\varphi} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

$$\begin{bmatrix} -\cos\theta & s\,\mathrm{sen}\theta \\ -\mathrm{sen}\theta & -s\cos\theta \end{bmatrix} \begin{Bmatrix} \dot{s} \\ \dot{\theta} \end{Bmatrix} = \dot{\varphi} \begin{Bmatrix} r\,\mathrm{sen}\varphi \\ -r\cos\varphi \end{Bmatrix}$$

$$\dot{s} = \dot{\varphi}\; r sen(\theta - \varphi)$$

$$\dot{\theta} = \frac{\dot{\varphi}\, r}{s}\cos(\theta - \varphi)$$

$$\frac{\dot{s}}{\dot{\varphi}} = \frac{ds}{d\varphi} = r\,\mathrm{sen}(\theta - \varphi) = \overline{g}_s(\varphi)$$

$$\frac{\dot{\theta}}{\dot{\varphi}} = \frac{d\theta}{d\varphi} = \frac{r}{s}\cos(\theta - \varphi) = g_\theta(\varphi)$$

Azelerazioen kalkulua

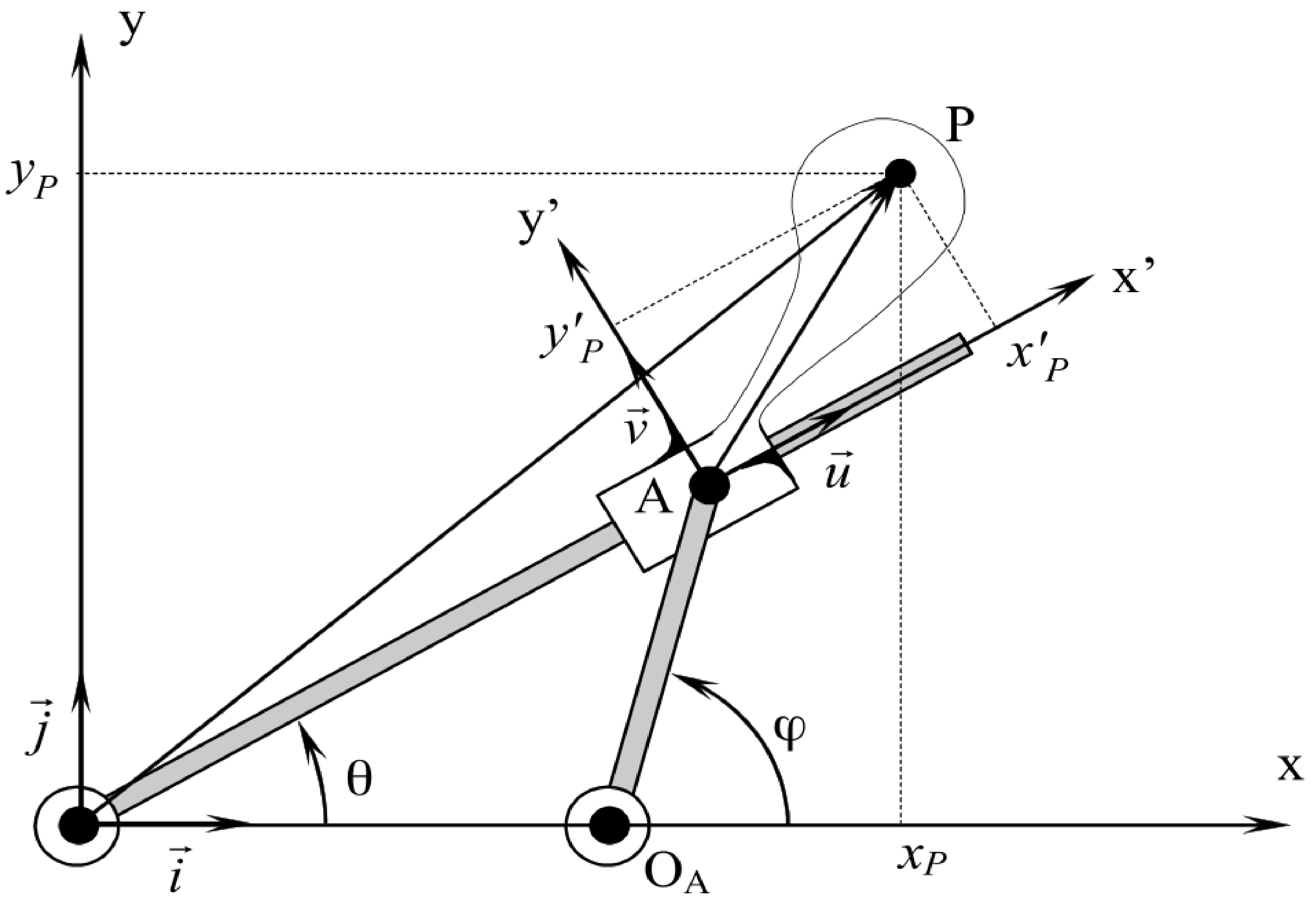
$$\begin{bmatrix} -\cos\theta & s\,\mathrm{sen}\theta \\ -\mathrm{sen}\theta & -s\cos\theta \end{bmatrix} \begin{Bmatrix} \ddot{s} \\ \ddot{\theta} \end{Bmatrix} = \begin{Bmatrix} r\ddot{\varphi}\,\mathrm{sen}\varphi + r\dot{\varphi}^2\cos\varphi - 2\dot{s}\dot{\theta}\,\mathrm{sen}\theta - s\dot{\theta}^2\cos\theta \\ -r\ddot{\varphi}\cos\varphi + r\dot{\varphi}^2\mathrm{sen}\varphi + 2\dot{s}\dot{\theta}\cos\theta - s\dot{\theta}^2\mathrm{sen}\theta \end{Bmatrix}$$

$$\ddot{s} = \frac{d\overline{g}_s}{dt}\dot{\varphi} + \overline{g}_s\ddot{\varphi} = \frac{d\overline{g}_s}{d\varphi}\dot{\varphi}^2 + \overline{g}_s\ddot{\varphi} = \overline{h}_s\dot{\varphi}^2 + \overline{g}_s\ddot{\varphi}$$

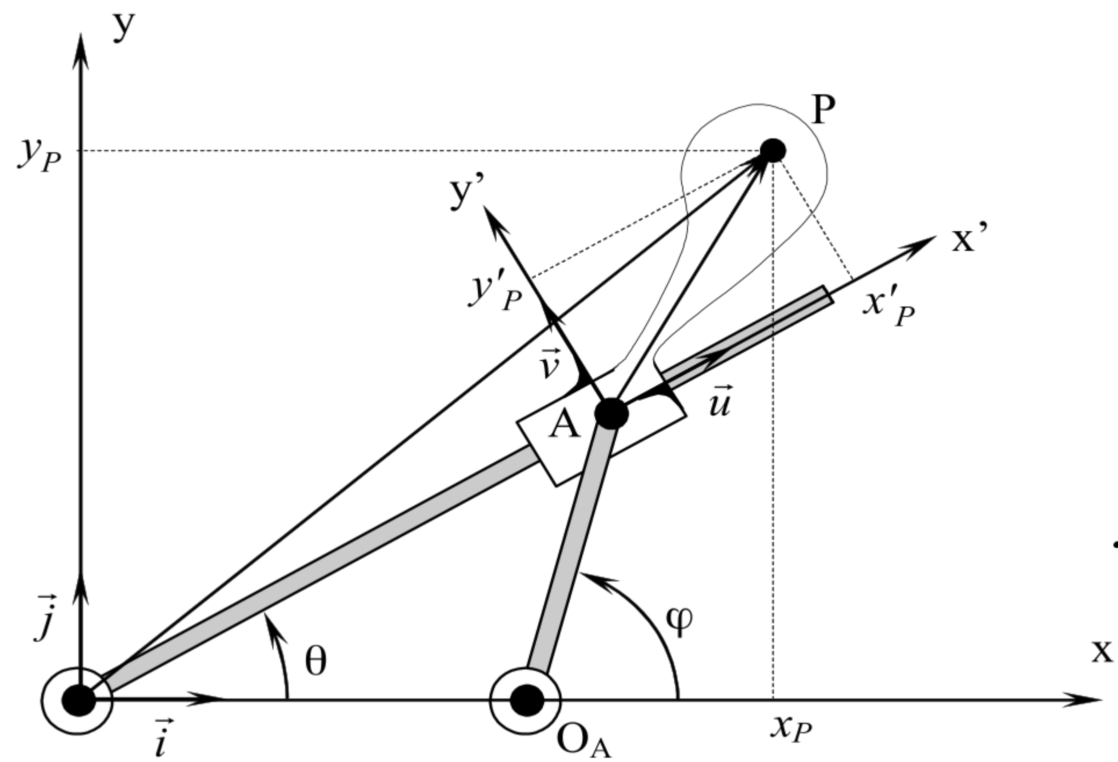
$$\ddot{\theta} = \frac{dg_\theta}{dt}\dot{\varphi} + g_\theta\ddot{\varphi} = \frac{dg_\theta}{d\varphi}\dot{\varphi}^2 + g_\theta\ddot{\varphi} = h_\theta\dot{\varphi}^2 + g_\theta\ddot{\varphi}$$

$$\begin{cases} \overline{h}_s(\varphi) = \dfrac{d\overline{g}_s}{d\varphi} = r\cos(\theta - \varphi)\times\left[\dfrac{d\theta}{d\varphi} - 1\right] = g_\theta\,(g_\theta - 1)s \\[2ex] h_\theta(\varphi) = \dfrac{dg_\theta}{d\varphi} = \dfrac{-r\,\mathrm{sen}(\theta - \varphi)\times\left[\dfrac{d\theta}{d\varphi} - 1\right]s - \dfrac{ds}{d\theta}r\cos(\theta - \varphi)}{s^2} = \dfrac{\overline{g}_s - 2g_\theta\overline{g}_s}{s} \end{cases}$$

Interes puntuen parametro zinematikoen kalkulua



Interes puntuen parametro zinematikoen kalkulua



$$\overrightarrow{OP_4} = \overrightarrow{OA} + \overrightarrow{AP_4}$$

$$x_P = s\cos\theta + x'_P\cos\theta - y'_P\,\mathrm{sen}\theta$$

$$y_P = s\,\mathrm{sen}\theta + x'_P\,\mathrm{sen}\theta + y'_P\cos\theta$$

$$v_{P_4}^x = \dot{s}\cos\theta - s\dot{\theta}\,\mathrm{sen}\theta - x'_P\dot{\theta}\,\mathrm{sen}\theta - y'_P\dot{\theta}\cos\theta$$

$$v_{P_4}^y = \dot{s}\,\mathrm{sen}\theta + s\dot{\theta}\cos\theta + x'_p\dot{\theta}\cos\theta - y'_p\dot{\theta}\,\mathrm{sen}\theta$$

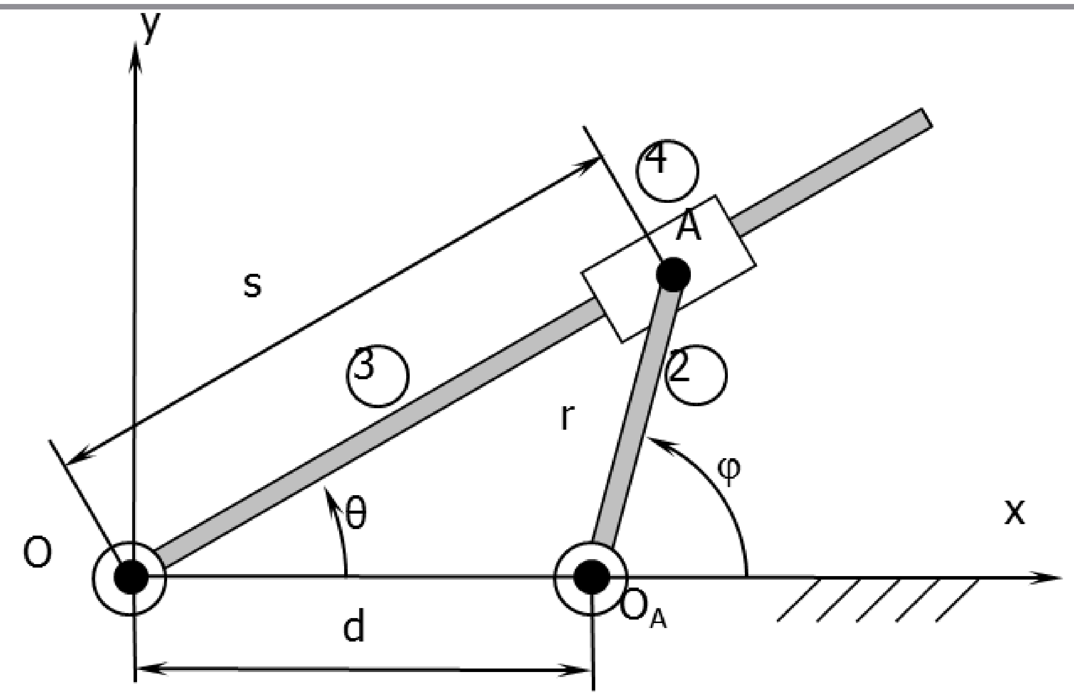
$$\overline{g}_{P_4}^x = \overline{g}_s\cos\theta - s g_\theta\,\mathrm{sen}\theta - x'_P g_\theta\,\mathrm{sen}\theta - y'_P g_\theta\cos\theta$$

$$\overline{g}_{P_4}^y = \overline{g}_s\,\mathrm{sen}\theta + s g_\theta\cos\theta + x'_P g_\theta\cos\theta - y'_P g_\theta\,\mathrm{sen}\theta$$

$$\overrightarrow{a_{P_4}} = \left(\ddot{\varphi}\,\overline{g}_{P_4}^x + \dot{\varphi}^2\overline{h}_{P_4}^x\right)\times\vec{i} + \left(\ddot{\varphi}\,\overline{g}_{P_4}^y + \dot{\varphi}^2\overline{h}_{P_4}^y\right)\times\vec{j}$$

$$\overline{h}_{P_4}^x = \frac{d\overline{g}_{P_4}^x}{d\varphi} = \overline{h}_s\cos\theta + \overline{g}_s g_\theta\,\mathrm{sen}\theta - \overline{g}_s g_\theta\,\mathrm{sen}\theta - s h_\theta\,\mathrm{sen}\theta - s g_\theta^2\cos\theta - x'_P h_\theta\,\mathrm{sen}\theta$$
$$- x'_P g_\theta^2\cos\theta - y'_P h_\theta\cos\theta + y'_P g_\theta^2\,\mathrm{sen}\theta$$

$$\overline{h}_{P_4}^y = \frac{d\overline{g}_{P_4}^y}{d\varphi} = \overline{h}_s\,\mathrm{sen}\theta + \overline{g}_s g_\theta\cos\theta + \overline{g}_s g_\theta\cos\theta + s h_\theta\cos\theta - s g_\theta^2\,\mathrm{sen}\theta + x'_P h_\theta\cos\theta$$
$$- x'_P g_\theta^2\,\mathrm{sen}\theta - y'_P h_\theta\,\mathrm{sen}\theta - y'_P g_\theta\cos\theta$$



Matrize Jakobiarra: **J**

$$\begin{bmatrix} -\cos\theta & s\,\text{sen}\theta & -r\,\text{sen}\varphi \\ -\text{sen}\theta & -s\cos\theta & r\cos\varphi \end{bmatrix} \begin{Bmatrix} \dot{s} \\ \dot{\theta} \\ \dot{\varphi} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

Koordenatu orokortuaren bektorea: **Je**

$$\begin{bmatrix} -\cos\theta & s\,\text{sen}\theta \\ -\text{sen}\theta & -s\cos\theta \end{bmatrix} \begin{Bmatrix} \dot{s} \\ \dot{\theta} \end{Bmatrix} = \dot{\varphi} \begin{Bmatrix} r\,\text{sen}\varphi \\ -r\cos\varphi \end{Bmatrix}$$

Koordenatu sekundarioen matrizea: **Js**

Kokapen singularren detekzioa

- Singularitatea: mekanismo baten askatasun graduen aldaketa.
  - **Mekanismoaren blokeoa** edo **indeterminazioa** (kontrol galketa) sortu dezakete.
  - Singularitatea sortzen deneko kokapena: Kokapen singularra.

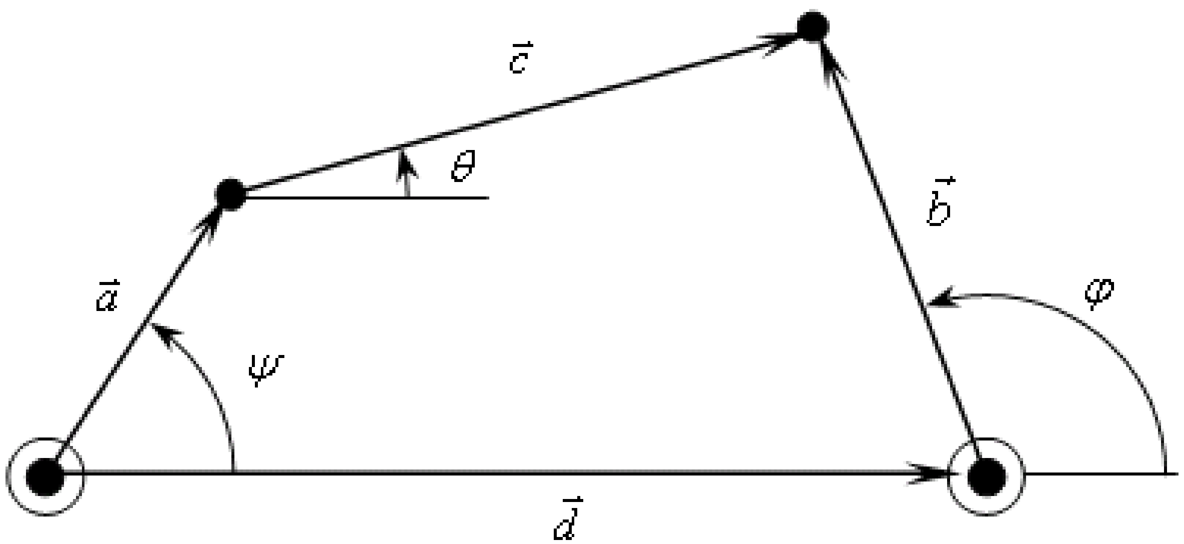
$$\boldsymbol{J} = \begin{bmatrix} \frac{\partial f_1}{\partial s} & \frac{\partial f_1}{\partial \theta} & \frac{\partial f_1}{\partial \varphi} \\ \frac{\partial f_2}{\partial s} & \frac{\partial f_2}{\partial \theta} & \frac{\partial f_2}{\partial \varphi} \end{bmatrix} = \begin{bmatrix} -\cos\theta & s\,\mathrm{sen}\theta & -r\,\mathrm{sen}\varphi \\ -\mathrm{sen}\theta & -s\cos\theta & r\cos\varphi \end{bmatrix}$$

$[J]_h = [J_s]_h =$ koord. sekundarioen kopuru

$$\boldsymbol{J}_S = \begin{bmatrix} \frac{\partial f_1}{\partial s} & \frac{\partial f_1}{\partial \theta} \\ \frac{\partial f_2}{\partial s} & \frac{\partial f_2}{\partial \theta} \end{bmatrix} = \begin{bmatrix} -\cos\theta & s\,\mathrm{sen}\theta \\ -\mathrm{sen}\theta & -s\cos\theta \end{bmatrix}$$

Bi matrizeen heina=2 edozein aldiuneetarako

Mekanismoak ez dauka singularitaterik

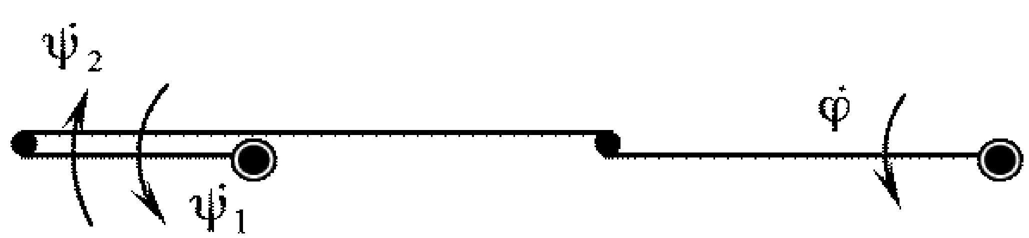


Posición:

$$a\cos\psi + c\cos\theta - b\cos\varphi - d = 0$$
$$a\,sen\,\psi + c\,sen\,\theta - b\,sen\,\varphi = 0$$

Velocidad:

$$\begin{bmatrix} a\,\text{sen}\,\psi & c\,\text{sen}\,\theta & -b\,\text{sen}\,\varphi \\ a\cos\psi & c\cos\theta & -b\cos\varphi \end{bmatrix} \begin{Bmatrix} \dot\psi \\ \dot\theta \\ \dot\varphi \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

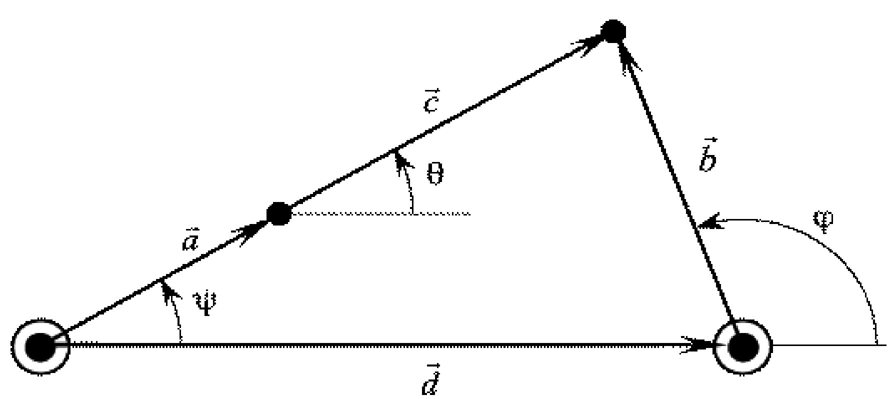


$$\varphi = \psi = 180^o \text{ y } \theta = 0^o$$
$$-a + c + b - d = 0$$
$$-a\dot\psi + c\dot\theta = -b\dot\varphi$$

Heina [J] = Heina [Js] = 1

Ag kop.= 2

Indeterminazio kokapena



$$\psi = \theta$$

$$\begin{bmatrix} a\,\text{sen}\psi & c\,\text{sen}\theta \\ a\cos\psi & c\cos\theta \end{bmatrix} \begin{Bmatrix} \dot\psi \\ \dot\theta \end{Bmatrix} = \dot\varphi\, b \begin{Bmatrix} \text{sen}\varphi \\ \cos\varphi \end{Bmatrix}$$

Heina [J] = 2 Heina [Js] = 1

Blokeo kokapena

$$\dot\psi = -\frac{c}{a}\dot\theta$$

* Laburbilduz:

Kokapen arazoa

$$\begin{cases} f_1(s_1, s_2, \varphi) = 0 \\ f_2(s_1, s_2, \varphi) = 0 \end{cases}$$

Abiadura arazoa

$$\boldsymbol{J}_S\,\dot{\boldsymbol{s}} = -\boldsymbol{J}_E\,\dot{\varphi}$$

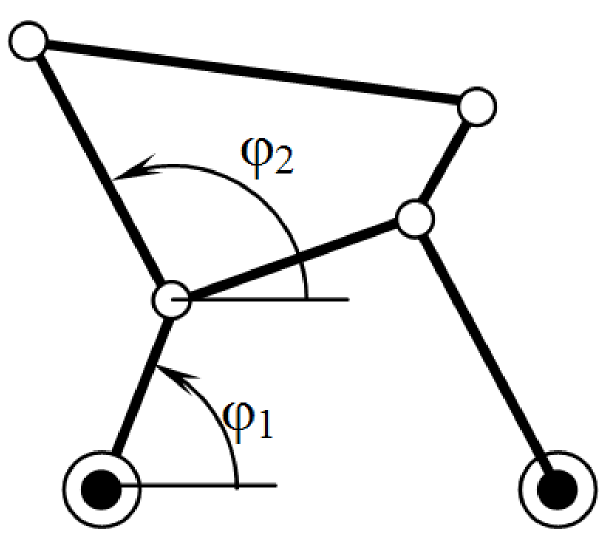
$$\boldsymbol{J}_S\,\frac{\dot{\boldsymbol{s}}}{\dot{\varphi}} = -\boldsymbol{J}_E \qquad \boldsymbol{J}_S\,\boldsymbol{g}_s(\varphi) = -\boldsymbol{J}_E$$

Azelerazio arazoa

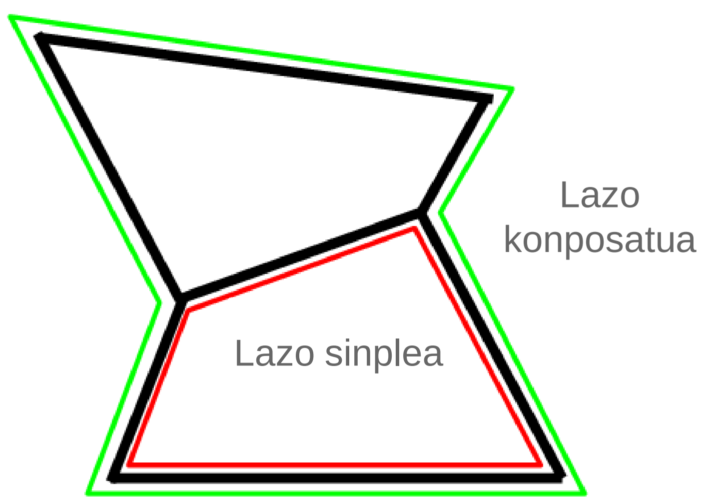
$$\frac{d\boldsymbol{J}_S}{dt}\dot{\boldsymbol{s}} + \boldsymbol{J}_S\,\ddot{\boldsymbol{s}} = -\boldsymbol{J}_E\,\ddot{\varphi} - \frac{d\boldsymbol{J}_E}{dt}\dot{\varphi} \quad \boldsymbol{J}_S\,\ddot{\boldsymbol{s}} = -\frac{d\boldsymbol{J}_S}{d\varphi}\boldsymbol{g}_s\dot{\varphi}^2 - \boldsymbol{J}_E\ddot{\varphi} - \frac{d\boldsymbol{J}_E}{d\varphi}\dot{\varphi}^2$$

$$\dot{\boldsymbol{s}} = \boldsymbol{g}_s \cdot \dot{\varphi} \quad \Longrightarrow \quad \ddot{\boldsymbol{s}} = \boldsymbol{g}_s \cdot \ddot{\varphi} + \frac{d\boldsymbol{g}_s}{d\varphi} \cdot \dot{\varphi}^2 = \boldsymbol{g}_s \cdot \ddot{\varphi} + \boldsymbol{h}_s \cdot \dot{\varphi}^2$$

Lazo anitzkoitzeko mekanismoak



Mekanismoa



Grafoa

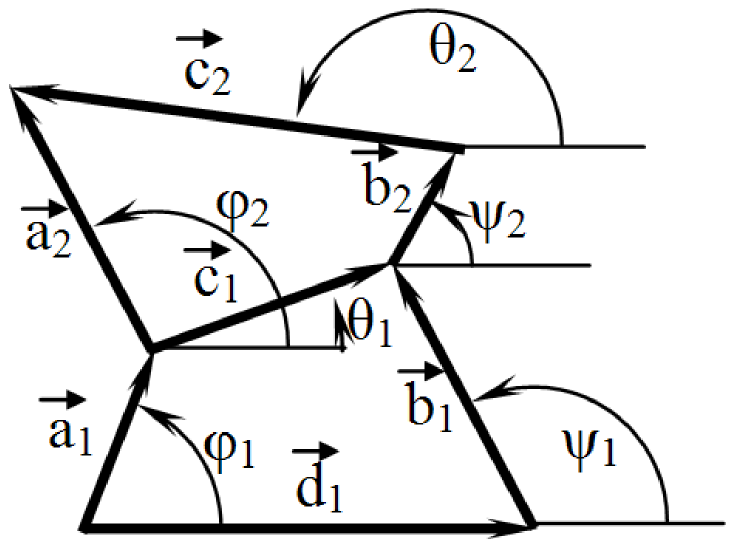


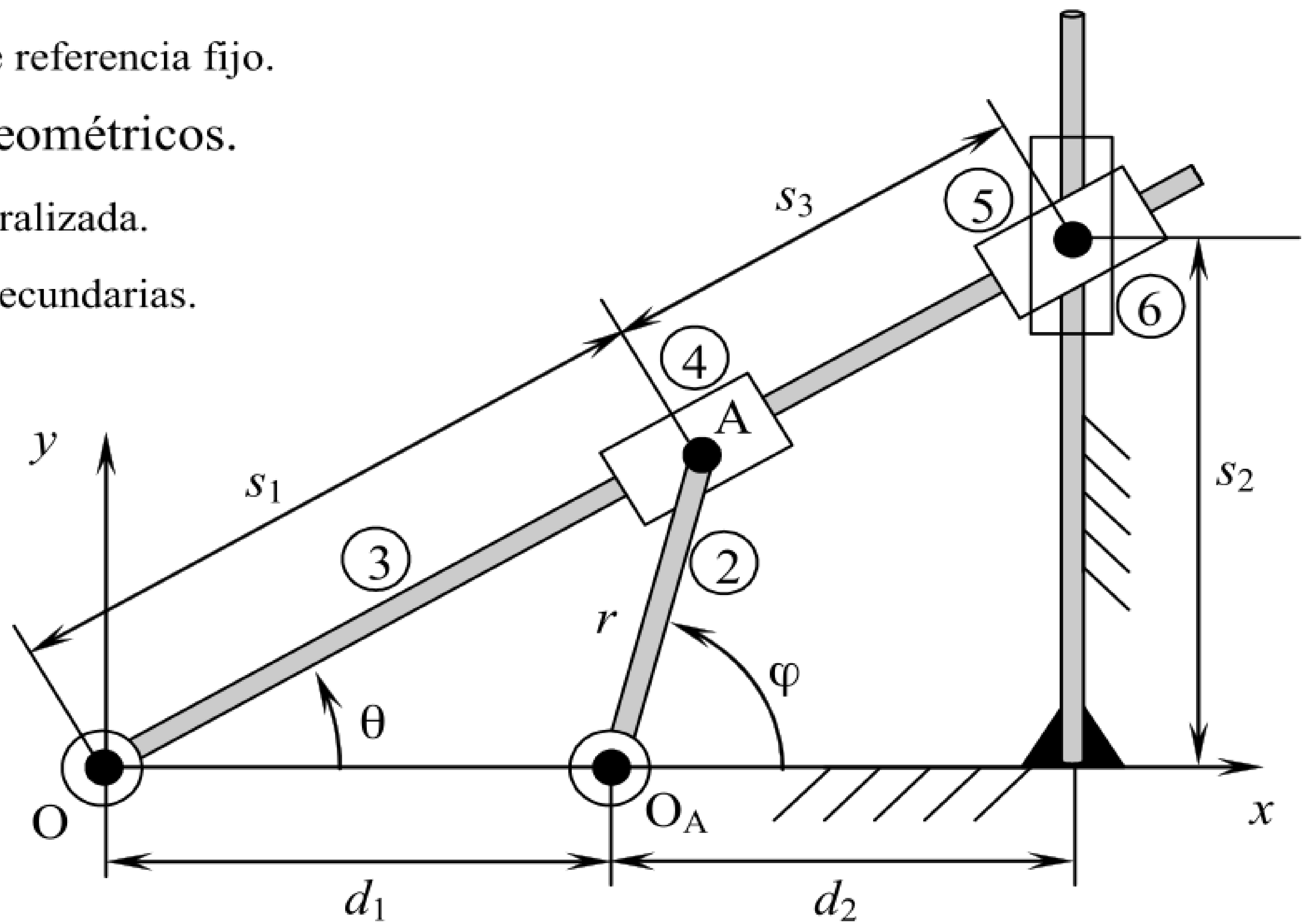
Diagrama bektoriala

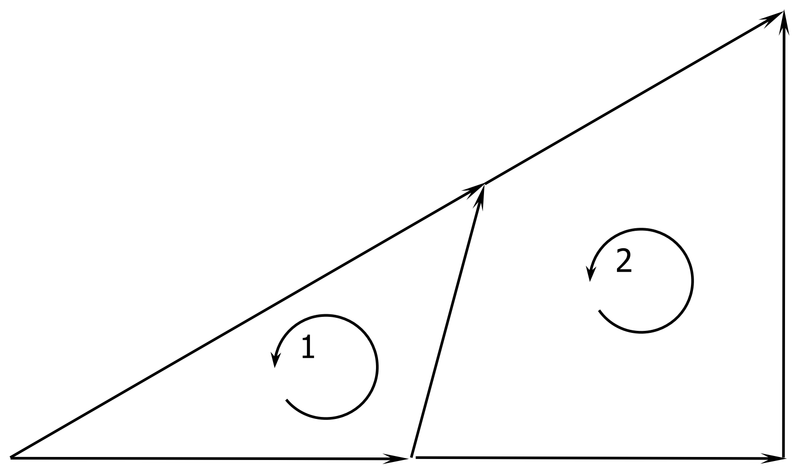
(O, x, y): sistema de referencia fijo.

r, d_1, d_2: datos geométricos.

φ: coordenada generalizada.

s_1, s_2, s_3, θ: coord. secundarias.





$$\vec{d}_1 + \vec{r} - \vec{s}_1 = \vec{0}$$

$$\vec{d}_2 + \vec{s}_2 - \vec{s}_3 - \vec{r} = \vec{0}$$

$$\begin{cases} s_1^2 = d_1^2 + r^2 + 2d_1 r\cos\varphi \\[2ex] \cos\theta = \dfrac{d_1 + r\cos\varphi}{s_1} \qquad \mathrm{sen}\theta = \dfrac{r}{s_1}\mathrm{sen}\varphi \\[2ex] s_2 = \left(\dfrac{d_1 + d_2}{d_1 + r\cos\varphi}\right)\times r\,\mathrm{sen}\varphi \\[2ex] s_3 = s_1 \times \dfrac{d_2 - r\cos\varphi}{d_1 + r\cos\varphi} \end{cases}$$

$$\frac{\partial f_1}{\partial s_1}\dot{s}_1 + \frac{\partial f_1}{\partial s_2}\dot{s}_2 + \frac{\partial f_1}{\partial s_3}\dot{s}_3 + \frac{\partial f_1}{\partial \theta}\dot{\theta} + \frac{\partial f_1}{\partial \varphi}\dot{\varphi} = 0$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

$$\frac{\partial f_4}{\partial s_1}\dot{s}_1 + \frac{\partial f_4}{\partial s_2}\dot{s}_2 + \frac{\partial f_4}{\partial s_3}\dot{s}_3 + \frac{\partial f_4}{\partial \theta}\dot{\theta} + \frac{\partial f_4}{\partial \varphi}\dot{\varphi} = 0$$

$$\Rightarrow \quad \boldsymbol{J}_S\,\dot{\boldsymbol{s}} + \boldsymbol{J}_E\,\dot{\varphi} = \boldsymbol{0}$$

$$\boxed{\boldsymbol{J}_S\,\dot{\boldsymbol{s}} = -\boldsymbol{J}_E\,\dot{\varphi}}$$

$$\boldsymbol{J}_S = \begin{bmatrix} \frac{\partial f_1}{\partial s_1} & \frac{\partial f_1}{\partial s_2} & \frac{\partial f_1}{\partial s_3} & \frac{\partial f_1}{\partial \theta} \\ \frac{\partial f_2}{\partial s_1} & \frac{\partial f_2}{\partial s_2} & \frac{\partial f_2}{\partial s_3} & \frac{\partial f_2}{\partial \theta} \\ \frac{\partial f_3}{\partial s_1} & \frac{\partial f_3}{\partial s_2} & \frac{\partial f_3}{\partial s_3} & \frac{\partial f_3}{\partial \theta} \\ \frac{\partial f_4}{\partial s_1} & \frac{\partial f_4}{\partial s_2} & \frac{\partial f_4}{\partial s_3} & \frac{\partial f_4}{\partial \theta} \end{bmatrix} = \begin{bmatrix} -\cos\theta & 0 & 0 & s_1 \operatorname{sen}\theta \\ -\operatorname{sen}\theta & 0 & 0 & -s_1\cos\theta \\ 0 & 0 & -\cos\theta & s_3\operatorname{sen}\theta \\ 0 & 1 & -\operatorname{sen}\theta & -s_3\cos\theta \end{bmatrix}$$

$$\boldsymbol{J}_E = \frac{\partial \boldsymbol{f}}{\partial \varphi} = \begin{Bmatrix} \frac{\partial f_1}{\partial \varphi} \\ \frac{\partial f_2}{\partial \varphi} \\ \frac{\partial f_3}{\partial \varphi} \\ \frac{\partial f_4}{\partial \varphi} \end{Bmatrix} = \begin{Bmatrix} -r\operatorname{sen}\varphi \\ r\cos\varphi \\ r\operatorname{sen}\varphi \\ -r\cos\varphi \end{Bmatrix} = r\begin{Bmatrix} -\operatorname{sen}\varphi \\ \cos\varphi \\ \operatorname{sen}\varphi \\ -\cos\varphi \end{Bmatrix}$$

Askatasun gradu anitz

Izan bedi $\boldsymbol{\varphi} = [\varphi_1 \quad \varphi_2 \quad \dots \quad \varphi_F]$ Koordenatu orokortuen bektorea

$\boldsymbol{s} = [s_1 \quad s_2 \quad \dots \quad s_n]$ aldagai sekundarioen bektorea

Kokapen arazoa

$$\begin{aligned} f_1(s_1, s_2, \dots s_n, \varphi_1, \varphi_2, \dots \varphi_F) &= 0 \\ f_2(s_1, s_2, \dots s_n, \varphi_1, \varphi_2, \dots \varphi_F) &= 0 \\ &\dots\dots\dots\dots \\ f_n(s_1, s_2, \dots s_n, \varphi_1, \varphi_2, \dots \varphi_F) &= 0 \end{aligned}$$

Abiaduren kalkulua

$$\begin{bmatrix} \frac{\partial f_1}{\partial s_1} & \cdots & \frac{\partial f_1}{\partial s_n} & \frac{\partial f_1}{\partial \varphi_1} & \cdots & \frac{\partial f_1}{\partial \varphi_F} \\ \vdots & & \vdots & \vdots & & \vdots \\ \frac{\partial f_n}{\partial s_1} & \cdots & \frac{\partial f_n}{\partial s_n} & \frac{\partial f_n}{\partial \varphi_1} & \cdots & \frac{\partial f_n}{\partial \varphi_F} \end{bmatrix} \begin{Bmatrix} \dot{s}_1 \\ \vdots \\ \dot{s}_n \\ \dot{\varphi}_1 \\ \vdots \\ \dot{\varphi}_F \end{Bmatrix} = \{0\} \Longrightarrow \boxed{\boldsymbol{J}_S \, \dot{\boldsymbol{s}} = -\boldsymbol{J}_E \, \dot{\boldsymbol{\varphi}}}$$

$$\boldsymbol{J}_S = \begin{bmatrix} \frac{\partial f_1}{\partial s_1} & \cdots & \frac{\partial f_1}{\partial s_n} \\ \vdots & & \vdots \\ \frac{\partial f_n}{\partial s_1} & \cdots & \frac{\partial f_n}{\partial s_n} \end{bmatrix} \qquad \boldsymbol{J}_E = \begin{bmatrix} \frac{\partial f_1}{\partial \varphi_1} & \cdots & \frac{\partial f_1}{\partial \varphi_F} \\ \vdots & & \vdots \\ \frac{\partial f_n}{\partial \varphi_1} & \cdots & \frac{\partial f_n}{\partial \varphi_F} \end{bmatrix}$$

Azelerazioen kalkulua

$$\left[\frac{\partial \boldsymbol{J}_S}{\partial s_1}\dot{s}_1 + \cdots + \frac{\partial \boldsymbol{J}_S}{\partial s_n}\dot{s}_n + \frac{\partial \boldsymbol{J}_S}{\partial \varphi_1}\dot{\varphi}_1 + \cdots + \frac{\partial \boldsymbol{J}_S}{\partial \varphi_F}\dot{\varphi}_F \right] \dot{\boldsymbol{s}} + \boldsymbol{J}_S \, \ddot{\boldsymbol{s}} =$$

$$-\left[\frac{\partial \boldsymbol{J}_E}{\partial s_1}\dot{s}_1 + \cdots + \frac{\partial \boldsymbol{J}_E}{\partial s_n}\dot{s}_n + \frac{\partial \boldsymbol{J}_E}{\partial \varphi_1}\dot{\varphi}_1 + \cdots + \frac{\partial \boldsymbol{J}_E}{\partial \varphi_F}\dot{\varphi}_F \right] \dot{\boldsymbol{\varphi}} - \boldsymbol{J}_E \, \ddot{\boldsymbol{\varphi}}$$

$$\Longrightarrow \quad \boldsymbol{J}_S \, \ddot{\boldsymbol{s}} = -\left[\sum_{j=1}^{n} \frac{\partial \boldsymbol{J}_E}{\partial s_j}\dot{s}_j + \sum_{i=1}^{F} \frac{\partial \boldsymbol{J}_E}{\partial \varphi_i}\dot{\varphi}_i \right] \dot{\boldsymbol{\varphi}} - \left[\sum_{j=1}^{n} \frac{\partial \boldsymbol{J}_S}{\partial s_j}\dot{s}_j + \sum_{i=1}^{F} \frac{\partial \boldsymbol{J}_S}{\partial \varphi_i}\dot{\varphi}_i \right] \dot{\boldsymbol{s}} - \boldsymbol{J}_E \, \ddot{\boldsymbol{\varphi}}$$

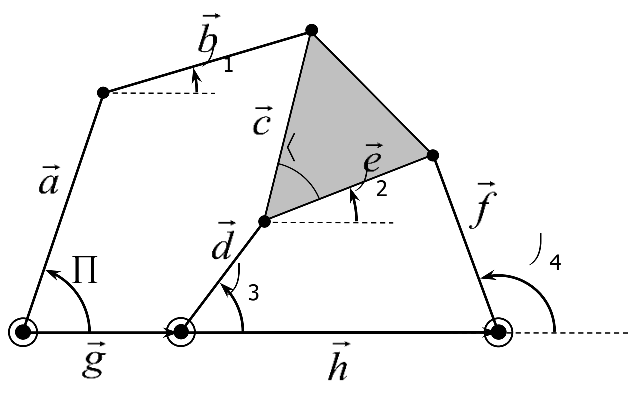
Singulartasunen kalkulua

$$\begin{bmatrix} a\,\text{sen}\varphi & b\,\text{sen}\psi_1 & -c\,\text{sen}(\psi_2+\alpha) & -d\,\text{sen}\psi_3 & 0 \\ -a\cos\varphi & -b\cos\psi_1 & c\cos(\psi_2+\alpha) & d\cos\psi_3 & 0 \\ 0 & 0 & e\,\text{sen}\psi_2 & d\,\text{sen}\psi_3 & -f\,\text{sen}\psi_4 \\ 0 & 0 & -e\cos\psi_2 & -d\cos\psi_3 & f\cos\psi_4 \end{bmatrix} \begin{Bmatrix} \dot\varphi \\ \dot\psi_1 \\ \dot\psi_2 \\ \dot\psi_3 \\ \dot\psi_4 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$

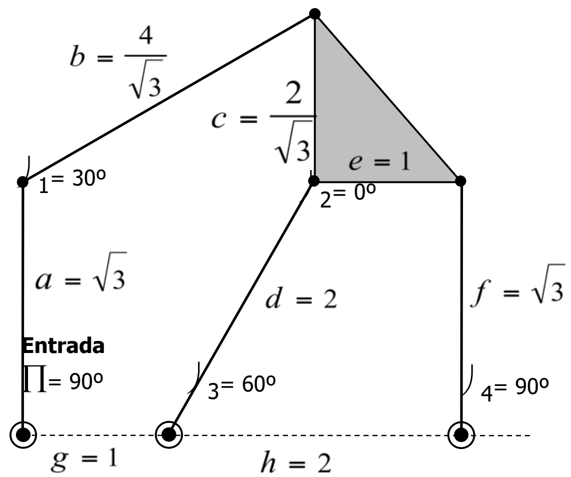
5 coord

Heina **J** =4

A.g. kop=1



a)

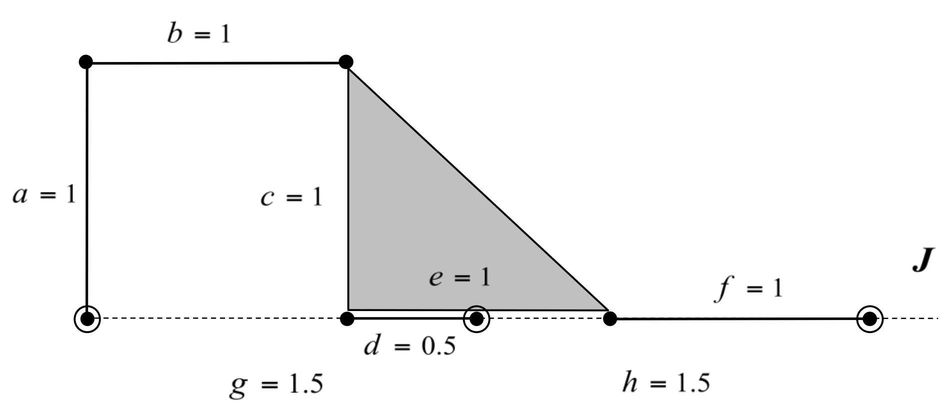


b)

$$Js = \begin{bmatrix} \sqrt{3} & 2/\sqrt{3} & -2/\sqrt{3} & -\sqrt{3} & 0 \\ 0 & -2 & 0 & 1 & 0 \\ 0 & 0 & 0 & \sqrt{3} & -\sqrt{3} \\ 0 & 0 & -1 & -1 & 0 \end{bmatrix}$$

Det $\mathbf{J_s}$ =0

Blokeo kokapena



c)

$$J = \begin{bmatrix} 1 & 0 & -1 & 0 & 0 \\ 0 & -1 & 0 & -1/2 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1/2 & -1 \end{bmatrix}$$

5 coord

Heina **J** =3

A.g. kop=2

Indet. kok.