[illegible]

2020ko ekainaren 30a

1.

$$z = e^{(x^2+y^2+2x-2y)}$$

Puntu kritikoak kalkulatzeko, lehenengo x eta y-rekiko deribatu partzialak kalkulatu behar ditugu.

$$\frac{\partial z}{\partial x} = (2x+2)e^{(x^2+y^2+2x-2y)} = 0 \longrightarrow 2x+2=0,\quad \boxed{x=-1}$$

$$\frac{\partial z}{\partial y} = (2y-2)e^{(x^2+y^2+2x-2y)} = 0 \longrightarrow 2y-2=0,\quad 2y=2,\quad \boxed{y=1}$$

$(-1,1)$

Puntu kritikoa zein motatakoa den jakiteko hessianoa kalkulatuko dugu.

$$|H| = \begin{vmatrix} \frac{\partial^2 z}{\partial x^2} & \frac{\partial^2 z}{\partial x \partial y} \\ \frac{\partial^2 z}{\partial y \partial x} & \frac{\partial^2 z}{\partial y^2} \end{vmatrix} = \begin{vmatrix} 2e^{-2} & 0 \\ 0 & 2e^{-2} \end{vmatrix} = 4e^{-4} > 0$$

$2e^{-2} > 0$

Minimo erlatiboa dugu $(-1,1)$ puntuan.

$$\frac{\partial^2 z}{\partial x^2} = 2e^{(x^2+y^2+2x-2y)} + (2x+2)(2x+2)e^{(x^2+y^2+2x-2y)} =$$

$$= e^{(x^2+y^2+2x-2y)}(2+4x^2+4x+4x+4) =$$

$$= e^{(x^2+y^2+2x-2y)}(6+4x^2+8x) \rightarrow \text{puntua ordezkatuz.} \rightarrow$$

$$\rightarrow e^{((-1)^2+1^2+(-2)-2)}(6+4(-1)^2+8(-1)) = 2e^{-2}$$

$$\frac{\partial^2 z}{\partial x \partial y} = (2x+2)(2y-2)e^{(x^2+y^2+2x-2y)} \rightarrow \text{puntua ordezkatuz} \rightarrow$$

$$\rightarrow (2\cdot(-1)+2)(2\cdot 1-2)e^{((-1)^2+1^2+2\cdot(-1)-2\cdot 1)} = 0$$

$$\frac{\partial^2 z}{\partial y \partial x} = (2y-2)(2x+2)e^{(x^2+y^2+2x-2y)} \rightarrow \text{puntua ordezkatuz} \rightarrow$$

$$\rightarrow = 0$$

$$\frac{\partial^2 z}{\partial y^2} = 2e^{(x^2+y^2+2x-2y)} + (2y-2)(2y-2)e^{(x^2+y^2+2x-2y)} =$$

$$= e^{(x^2+y^2+2x-2y)}(2+4y^2-4y-4y+4) =$$

$$= e^{(x^2+y^2+2x-2y)}(6+4y^2-8y) \rightarrow \text{puntuak ordezkatuz} \rightarrow$$

$$\rightarrow 2e^{-2}$$

(-1, 1) puntuan ebaluatuz, taylorren polinomioa lortuko dugu:

$$z(-1,1) = e^{-2} \qquad \frac{\partial^2 z}{\partial x \partial y} = 0$$

$$\frac{\partial z}{\partial x}(-1,1) = 0 \qquad \frac{\partial^2 z}{\partial x^2} = 2e^{-2}$$

$$\frac{\partial z}{\partial y}(-1,1) = 0 \qquad \frac{\partial^2 z}{\partial y^2} = 2e^{-2}$$

taylor-en garapena:

$$P(x,y) \equiv f(a,b) + \frac{1}{1!}\left(\frac{\partial f}{\partial x}(a,b)(x-a) + \frac{\partial f}{\partial y}(a,b)(y-b) \right) +$$
$$\frac{1}{2!}\left(\frac{\partial^2 f}{\partial x^2}(a,b)(x-a)^2 + \frac{\partial^2 f}{\partial y^2}(a,b)(y-b)^2 + 2\frac{\partial^2 f}{\partial x \partial y}(a,b)(x-a)(y-b)\right)$$

$$P(x,y) = e^{-2} + \frac{1}{\cancel{2}!}\left(\cancel{2}e^{-2}(x-a)^2 + \cancel{2}e^{-2}(y-b)^2 \right)$$

$$P(z,(-1,1)) = e^{-2} + e^{-2}\left((x+1)^2 + (y-1)^2\right)$$

$$\boxed{P(z,(-1,1)) = e^{-2}\left[1 + \left((x+1)^2 + (y-1)^2\right)\right]}$$

Gainazalarekiko plano ukitzaileak kalkulatzeko:

$$\underbrace{\frac{\partial z}{\partial x}}_{0}(x_0,y_0)(x-x_0) + \underbrace{\frac{\partial z}{\partial y}}_{0}(x_0,y_0)(y-y_0)$$

Bai x eta bai y-rekiko deribatua zero da, ondorioz eremu erreal ($\mathbb{R}^2$) guztia da plano ukitzailea.

• (0,0) puntuan

$$\frac{\partial z}{\partial x}(0,0) = 2$$

$$\frac{\partial z}{\partial y}(0,0) = -2$$

Planoaren ekuazio inplizitua ondorengo hau izango da:

$$2x + 2y = 0 \longrightarrow \boxed{x + y = 0}$$

2.) $f(x,y) = 7 - 2x^2 - 3y^2$

$P(1,1)$

Deribatuen balioak → (-4 eta -6)

$$\vec{\nabla}f = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right) = \frac{\partial f}{\partial x}\vec{u_x} + \frac{\partial f}{\partial y}\vec{u_y}$$

$$\left.\begin{array}{l} \dfrac{\partial f}{\partial x} = -4x \\ \\ \dfrac{\partial f}{\partial y} = -6y \end{array}\right\} \vec{\nabla}f\Big|_{(1,1)} = \left(\frac{\partial f}{\partial x}\Big|_{(1,1)}, \frac{\partial f}{\partial y}\Big|_{(1,1)}\right) = (-4,-6)$$

$$* \frac{\partial f}{\partial v}\Big|_{(-1,2)} = \vec{\nabla}f\Big|_{(1,1)} \frac{\vec{v}}{|\vec{v}|} = (-4,-6)\cdot\frac{\vec{v}}{|\vec{v}|} = \begin{cases} -4 \\ -6 \end{cases}$$

Bilatu nahi ditugun norabideak, $\vec{u}$ eta $\vec{v}$ bektoreen bidez definituak egongo dira.

$$(-4,-6)\cdot\vec{u} = -4 \longrightarrow \boxed{\vec{u}(1,0)}$$

$$(-4,-6)\vec{v} = -6 \longrightarrow \boxed{\vec{v}(0,1)}$$

Konprobatzen badugu goiko formularekin* ikusten dugu, zuzena dela norabidea. Koordenatuek deribatuaren balioa dutenez, soilik y edo x norabidean doa kasu bakoitzean.

Escaneado con CamScanner

8.

$x^2+y^2+(z-3)^2=4$ (goitik)

$x^2+y^2=4$

$z=0$ (behetik)

Ebakidura:

$$\begin{cases} x^2+y^2+(z-3)^2=4 \\ x^2+y^2=4 \end{cases} \rightarrow \begin{cases} \cancel{x^2}+\cancel{y^2}+(z-3)^2-\cancel{4}=\cancel{x^2}+\cancel{y^2}-\cancel{4} \\ (z-3)^2=0 \\ \boxed{z=3} \end{cases}$$

Koordenatu zilindrikoak: $\begin{cases} x=\rho\cos\theta \\ y=\rho\sin\theta \end{cases}$

$\rho^2=x^2+y^2$

$\boxed{\rho=2}$

$\underbrace{x^2+y^2}_{\rho^2}+(z-3)^2=4 \rightarrow (z-3)^2=4-\rho^2$

$z=\sqrt{4-\rho^2}+3$

$$\iiint_V dV=\int_0^{2\pi}\int_0^2\int_0^{\sqrt{4-\rho^2}+3}\rho\,dz\,d\rho\,d\theta=\int_0^{2\pi}\int_0^2 \rho\, z\Big|_0^{\sqrt{4-\rho^2}+3}d\rho\,d\theta=$$

$$=\int_0^{2\pi}\int_0^2\rho\left(\sqrt{4-\rho^2}+3\right)d\rho\,d\theta=\int_0^{2\pi}d\theta\int_0^2\left(\rho\sqrt{4-\rho^2}+3\rho\right)d\rho=$$

$$=\theta\Big|_0^{2\pi}\left[-\frac{1}{2}\frac{(4-\rho^2)^{3/2}}{3/2}+\frac{3\rho^2}{2}\right]_0^2=2\pi\cdot\left[-\frac{1}{2}\frac{\cancel{0}}{3/2}+\frac{3\cdot 2^2}{\cancel{2}}+\frac{1}{\cancel{2}}\frac{(4)^{3/2}}{\cancel{3/2}}\right]$$

$$=2\pi\left(6+\frac{(4)^{3/2}}{3}\right)=2\pi\left(\frac{18+8}{3}\right)=\boxed{\frac{52}{3}\pi}$$

$$X_G=\frac{\int_0^{2\pi}\int_0^2\int_0^{\sqrt{4-\rho^2}+3}\cancel{k}\rho^2\cos\theta\,dz\,d\rho\,d\theta}{\int_0^{2\pi}\int_0^2\int_0^{\sqrt{4-\rho^2}+3}\cancel{k}\rho\,dz\,d\rho\,d\theta}=\frac{3}{52\pi}\int_0^{2\pi}\int_0^2 z\Big|_0^{\sqrt{4-\rho^2}+3}\rho^2\cos\theta\,d\rho\,d\theta=$$

$$=\frac{3}{52\pi}\int_0^{2\pi}\underbrace{\cancel{\cos\theta\,d\theta}}_{0}\int_0^2\rho^2\left(\sqrt{4-\rho^2}+3\right)d\rho=\boxed{0}$$

cos-ren deribatua sinua da eta sin 0 eta $2\pi=0$ da, orduan $X_G=0$.

$$Y_G=\frac{\int_0^{2\pi}\int_0^2\int_0^{\sqrt{4-\rho^2}+3}\cancel{k}\rho^2\sin\theta\,dz\,d\rho\,d\theta}{\int_0^{2\pi}\int_0^2\int_0^{\sqrt{4-\rho^2}+3}\cancel{k}\rho\,dz\,d\rho\,d\theta}=\frac{3}{52\pi}\int_0^{2\pi}\sin\theta\,d\theta\int_0^2\left(\rho^2\sqrt{4-\rho^2}+3\rho^2\right)d\rho=$$

$$=\frac{3}{52\pi}\underbrace{\cancel{(-\cos\theta)\Big|_0^{2\pi}}}_{(-1+1)=0}\int_0^2\left(\rho^2\sqrt{4-\rho^2}+3\rho^2\right)d\rho=\boxed{0}$$

Aurrekoan bezala $Y_G=0$ da!

Escaneado con CamScanner

$$z_G = \frac{\int_0^{2\pi}\int_0^{2}\int_0^{\sqrt{4-\rho^2}+3} z\,\cancel{k}\,\rho\,dz\,d\rho\,d\alpha}{\int_0^{2\pi}\int_0^{2}\int_0^{\sqrt{4-\rho^2}+3} \cancel{k}\,\rho\,dz\,d\rho\,d\alpha} = \frac{3\pi}{52}\int_0^{2\pi}\int_0^{2} \rho\,\frac{z^2}{2}\Big|_0^{\sqrt{4-\rho^2}+3} d\rho\,d\alpha =$$

$$= \frac{3}{52\pi}\int_0^{2\pi} d\alpha \int_0^{2} \rho\left(\frac{(\sqrt{4-\rho^2}+3)^2}{2}\right) d\rho = \frac{3\pi}{\cancel{52}_{26}}\cdot\cancel{2}\pi\int_0^{2} \rho\,\frac{(4-\rho^2+6\sqrt{4-\rho^2}+9)}{2}\,d\rho$$

$$= \frac{3}{52\cancel{\pi}}\cancel{\pi}\int_0^{2}(4\rho-\rho^3+6\rho\sqrt{4-\rho^2}+9\rho)\,d\rho =$$

$$= \frac{3}{52}\left(2\rho^2-\frac{\rho^4}{4}+\frac{9\rho^2}{2}-2(4-\rho^2)^{\frac{3}{2}}\right)\Big|_0^{2} =$$

$$= \frac{3}{52}\left(2\cdot 2^2-\frac{2^4}{4}+\frac{9\cdot 2^{\cancel{2}}}{\cancel{2}}-\underbrace{\cancel{2(4-2^2)^{3/2}}}_{0}+2(4)^{3/2}\right) =$$

$$= \frac{3}{52}(8-4+18+16) = \frac{114}{52} = \boxed{\frac{57}{26}}$$

$$\boxed{G\left(0,0,\frac{57}{26}\right)}$$

4.

$$4\frac{d^2y}{dx^2} - y = x e^{-x/2} \quad ; \quad y(0)=1 \quad y'(0)=0$$

$4m^2 - 1 = 0$ Ekuazio karakteristikoa.

Ekuazio karakteristikoaren zeroak kalkulatuko ditugu ekuazio homogeneoaren soluzioa kalkulatzeko.

$$m^2 = \frac{1}{4} \longrightarrow \boxed{m = \pm\frac{1}{2}}$$

$$y_h(x) = C_1 e^{\frac{1}{2}x} + C_2 e^{-\frac{1}{2}x}$$

Behin homogeneoa kalkulatuta, partikularra kalkulatu behar dugu, horretarako ZI mulko bakarra: $Ax^2 + Bx + C$

$$\left.\begin{aligned} y_p(x) &= Ax^2 + Bx + C \\ y'_p(x) &= 2Ax + B \\ y''_p(x) &= 2A \end{aligned}\right\}$$

Hasierako ekuazio diferentzialean ordezkatu behar dira, parametroak kalkulatzeko.

$$4(2A) - (Ax^2 + Bx + C) = x e^{-x/2}$$

$$8A - Ax^2 - Bx - C = x e^{-x/2}$$

$$(8A - B) - Ax^2 - (B + e^{-x/2})x$$

5. $0 \le x \le 1$

$0 \le y \le 1$

$f(x,y) = k(x+y)^4$

$\int_0^1\int_0^1 k(x+y)^4\,dy\,dx = 1$; $\int_0^1 k\,\frac{(x+y)^5}{5}\Big|_0^1 dx =$

$= k\left(\frac{(x+1)^6}{30} - \frac{x^6}{30}\right)\Big|_0^1 = k\left(\frac{2^6}{30} - \frac{1}{30} - \frac{1^6}{30}\right) =$

$= k\left(\frac{62}{30}\right) = 1$

$\boxed{k = \frac{15}{31}}$

$p(x \le \frac{1}{2}) = \frac{15}{31}\int_0^{1/2}\int_0^1 (x+y)^4\,dy\,dx = \frac{15}{31}\int_0^{1/2} \frac{(x+y)^5}{5}\Big|_0^1 dx =$

$= \frac{15}{31}\int_0^{1/2}\left(\frac{(x+1)^5}{5} - \frac{x^5}{5}\right)dx = \frac{15}{31}\left[\frac{(x+1)^6}{5\cdot 6} - \frac{x^6}{6\cdot 5}\right]_0^{1/2} =$

$= \frac{15}{31}\left(\frac{(\frac{1}{2}+1)^6}{30} - \frac{(1/2)^6}{30} - \frac{1^6}{30}\right) = \frac{15}{31}\left(\frac{3^6}{2^6\cdot 30} - \frac{1}{30\cdot 2^6} - \frac{1}{30}\right)$

$= \frac{\cancel{15}}{31\cdot\cancel{30}\,2}\left(\frac{3^6}{2^6} - \frac{1}{2^6} - \frac{2^6}{2^6}\right) = \frac{1}{62}\cdot\left(\frac{664}{64}\right) =$

$= \frac{166}{992} = \boxed{\frac{83}{496}}$