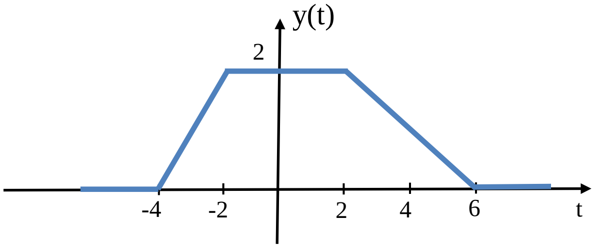
SEINALEEN PROZESATZEA. Lehen partziala

Azterketak 3 ariketa ditu. Ariketa bakoitzak 10 puntu balio ditu, eta 1. ariketako galdera guztiek pisu berdina dute. Bi ordu dituzue.

1. ARIKETA (10 puntu, 40 minutu)

1. Irudiko y(t) seinalea x(t) seinaletik sortu da honela eginaz: $y(t) = x\left(\frac{-t+6}{2}\right)$



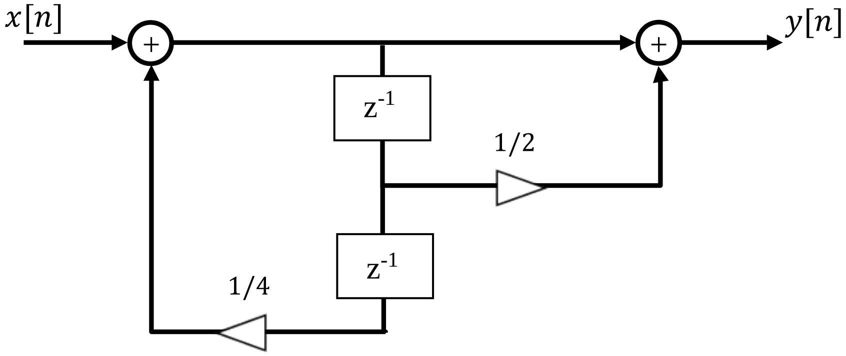
   a. Adierazi zein izan den aldagai askearen aldaketen sekuentzia (tolestea, desplazatzea, eskalatzea, …) x(t) seinaletik abiatuta y(t) seinalea lortzeko.
   b. Aurreko aldaketen sekuentzia jarraituz eta x(t) seinaletik abiatuz irudikatu aldagai aldaketa bakoitzaren emaitza y(t) seinalera iritsi arte.

2. Izan bedi honako irteera-sarrera erlazioa duen LTI sistema: $y[n] = \sum_{k=4}^{\infty} x\,[n-k]a^{k-4}$
   a. Kalkulatu sistemaren pultsu-erantzuna, h[n], eta unitate-maila sekuentziaren menpe adierazi.
   b. Aztertu sistemaren kausalitatea eta egonkortasuna h[n] kontutan hartuz eta $0 < a < 1$ betetzen dela jakinda.
   c. Lortu sistemaren irteera-seinalea, y[n], honako sarrera-seinalearentzat: $x[n] = a^n u[n]$.

3. Izan bedi honako pultsu-erantzuna duen LTI sistema, $h[n] = \begin{cases} 1/4, & 0 \leq n < 4 \\ 0, & \text{bestea} \end{cases}$. Kalkulatu eta irudikatu sistemaren erantzuna honako sarrera-seinalearentzat: $x[n] = \sum_{k=0}^{\infty} \delta\,[n-2k]$.

2. ARIKETA (10 puntu, 40 minutu)

Irudiko sisteman, hasierako baldintzak nuluak direla kontutan hartuta, honakoak eskatzen dira:



a. Lortu sistemaren diferentzia-ekuazioa era arrazoituan. **(3 p)**

b. Arrazoitu sistemaren mota eta ordena. **(1 p)**

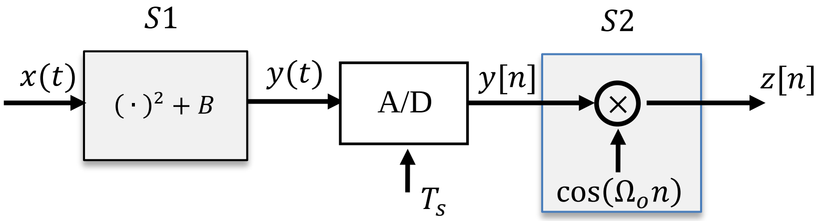
c. Lortu eta irudikatu sistemaren pultsu-erantzuna, h[n], eta unitate-mailaren menpe adierazi. **(2 p)**

Hartu sarrera-seinalea x[n] = {$\underline{1}$, 1, 1} eta lortu y[n] irteera-seinalearen lehenengo hiru laginak, bi eratara:

d. Iragaziz. **(2 p)**

e. Konboluzionatuz. **(2 p)**

3. ARIKETA (10 puntu, 40 minutu)



Irudiko eskema kontutan hartuta, honakoak eskatzen dira:

a. Frogatu $x(t) = A\cos(\omega_0 t)$ seinalea periodikoa dela, T_0 periododuna. Erlazionatu T_0 eta ω_0 maiztasuna angeluarra. Kalkulatu seinalearen batezbesteko potentzia, P_x. **(3 p)**

b. Irudiko S1 sistema honako sarrera-irteera erlazioak definitzen du: $y(t) = x(t)^2 + B$. Aztertu lineala, denboran aldaezina, kausala edota egonkorra den. *B* balio konstantea da. **(2 p)**

c. S1 sistemaren erantzuna, y(t), a) ataleko x(t) seinaleari, lagindu egiten T_s (s) laginketa periodoarekin y[n] seinalea lortzeko. Adierazi era analitikoan y[n] seinalea, eta aztertu T_s eta T_0 balioen arteko erlazioa y[n] seinalea periodikoa izan dadin. **(3 p)**

d. Aztertu irudiko S2 sistema lineala, denboran aldaezina, kausala edota egonkorra den. Ω_0 balio konstantea da. **(2 p)**

TRATAMIENTO DE SEÑALES: PRIMER PARCIAL

La puntuación total del examen es de 30 puntos divididos en:

Problema 1: 10 puntos. Todas las cuestiones tienen el mismo peso.

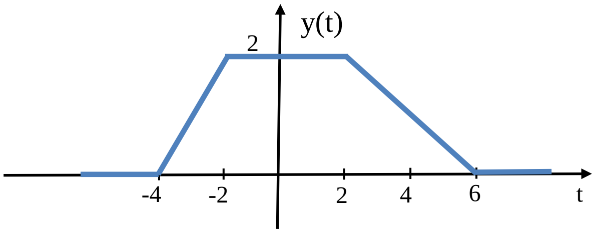
Problema 2: 10 puntos.

Problema 3: 10 puntos.

El tiempo estimado para resolver el examen es de dos horas.

PROBLEMA 1 (10 puntos, 40 minutos)

1. La señal de la figura y(t) de la figura se ha obtenido a partir de x(t) mediante la siguiente expresión $y(t) = x\left(\frac{-t+6}{2}\right)$.



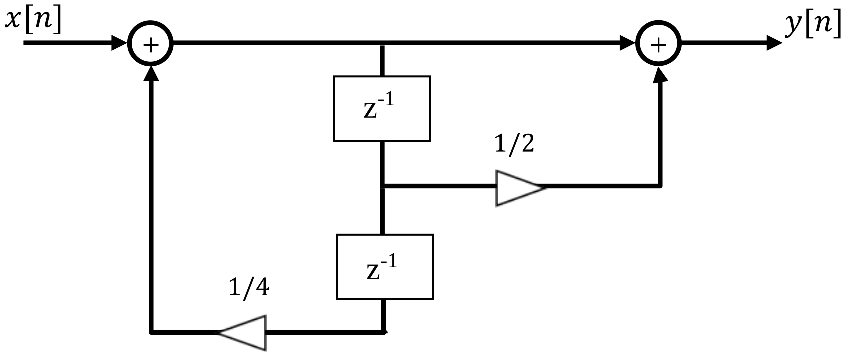
   a. Indicar una secuencia de transformaciones básicas de la variable independiente (inversión, desplazamiento, escalado) sobre x(t) para llegar a obtener y(t).
   b. Según la secuencia planteada en el apartado anterior dibujar cada una de las transformaciones hasta llegar a la y(t) dada y obtener la x(t) de partida.

2. Sea el sistema LTI con la característica entrada-salida $y[n] = \sum_{k=4}^{\infty} x\,[n-k]a^{k-4}$.
   a. Calcular la respuesta impulsional h[n] expresándola en función del escalón unidad.
   b. Analizar la causalidad y estabilidad del sistema a partir de h[n] si $0 < a < 1$.
   c. Obtener la respuesta del sistema y[n] para una entrada $x[n] = a^n u[n]$.

3. Sea el sistema LTI cuya respuesta impusional es $h[n] = \begin{cases} 1/4, & 0 \le n < 4 \\ 0, & \text{resto} \end{cases}$. Calcular y dibujar la respuesta del sistema para una entrada $x[n] = \sum_{k=0}^{\infty} \delta\,[n-2k]$.

PROBLEMA 2 (10 puntos, 40 minutos)

Sea el sistema descrito por el esquema de implementación de la figura:



Se pide:

a. Obtener de forma razonada la ecuación en diferencias del sistema. **(3 p)**

b. Determinar de forma razonada el tipo y orden del sistema. **(1 p)**

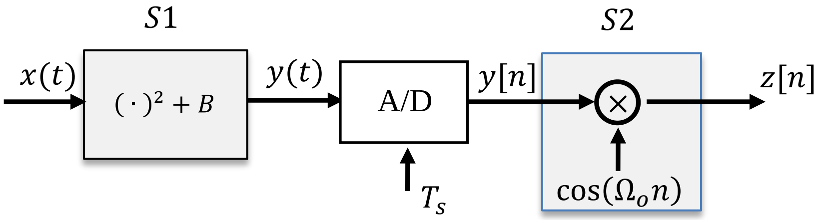
c. Obtener y dibujar la respuesta impulsional, expresando h[n] en función del escalón unidad. **(2 p)**

Obtener las tres primeras muestras de la respuesta del sistema y[n] a la entrada x[n] = {$\underline{1}$, 1, 1}:

d. Mediante filtrado. **(2 p)**

e. Mediante la suma de convolución. **(2 p)**

PROBLEMA 3 (10 puntos, 40 minutos)



Considerando el esquema de la figura, se pide:

a. Demostrar que la señal $x(t) = A\cos(\omega_0 t)$ es una señal periódica de periodo T_0. Relacionar T_0 con la frecuencia angular ω_0. Calcular la potencia media de la señal, P_x. **(3 p)**

b. Analizar si el sistema S1 de la figura, descrito por $y(t) = x(t)^2 + B$, es lineal, invariante en tiempo, causal y/o estable. Considerar que *B* es un valor constante **(2 p)**

c. La respuesta y(t) del sistema S1 a la señal x(t) del apartado a) se muestrea con un periodo de muestreo T_s (s) para obtener la señal y[n]. Expresar analíticamente la señal así obtenida y analizar la relación entre T_s y T_0 para que y[n] sea una señal periódica. **(3 p)**

d. Analizar si el sistema S2 de la figura es lineal, invariante en tiempo, causal y/o estable. Considerar que Ω_0 es un valor constante. **(2 p)**

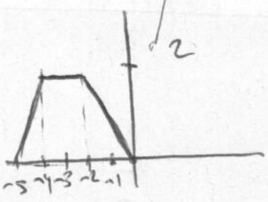
$\boxed{\text{PROBLEMA 1}}$

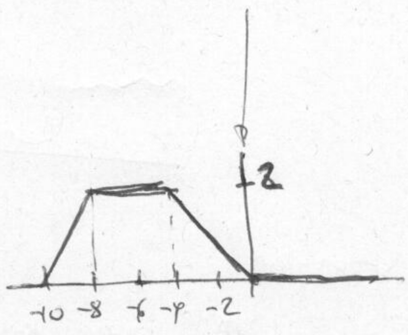
①. $y(t) = x\left(\frac{-t+6}{2}\right)$

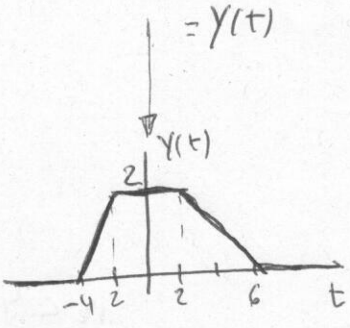
a) $x(t) \xrightarrow{t\to -t} x(-t) \xrightarrow{t\to \frac{t}{2}} x\left(\frac{-t}{2}\right) \xrightarrow{t\to t-6} x\left(\frac{-(t-6)}{2}\right) = x\left(\frac{-t+6}{2}\right)$

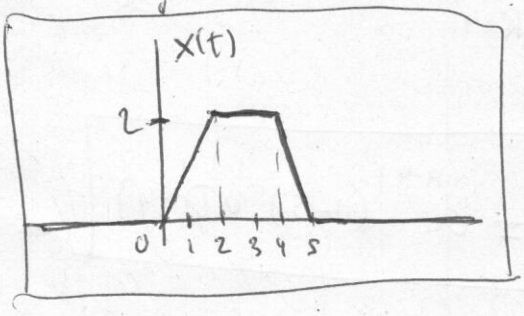
$= y(t)$

b)









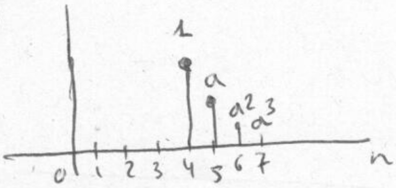
②. a) $y[n] = \sum_{k=4}^{\infty} x[n-k]\, a^{k-4} = \sum_{k=4}^{\infty} x[n-k]\, a^{k-4}\, u[k-4]$

$$= \sum_{k=-\infty}^{\infty} x[n-k]\, a^{k-4}\, u[k-4] = \sum_{k=-\infty} x[n-k]\, h[k]$$

$$h[k] = a^{k-4}\, u[k-4]$$

$$\boxed{h[n] = a^{n-4}\, u[n-4]}$$

Otra forma: $h[n] = \sum_{k=4}^{\infty} \delta[n-k]\, a^{k-4}$ —



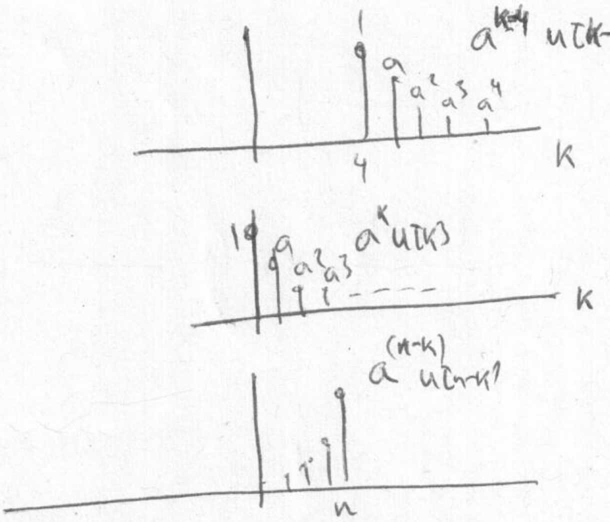
$$\boxed{h[n] = a^{n-4} \cdot u[n-4]}$$

b) $\boxed{\text{Es causal}}$ porque $h[n] = 0$ $\forall n<0$

Estabilidad $\sum_{n=-\infty}^{\infty} |h[n]| < \infty = 1 + a + a^2 + a^3 + --- a^{\infty}$ Serie geométrica de razón $r = a < 1$

Es convergente $\boxed{\text{Es estable}}$

c) $y[n] = x[n] * h[n] = a^n u[n] * a^{n-4} u[n-4]$



$n < 4 \quad y[n] = 0$

$$n \geq 4 \quad y[n] = \sum_{k=4}^{n} a^{n-k} \cdot a^{k-4} = \sum_{k=4}^{n} a^{n-4} = a^{n-4} \cdot \sum_{k=4}^{n} 1$$

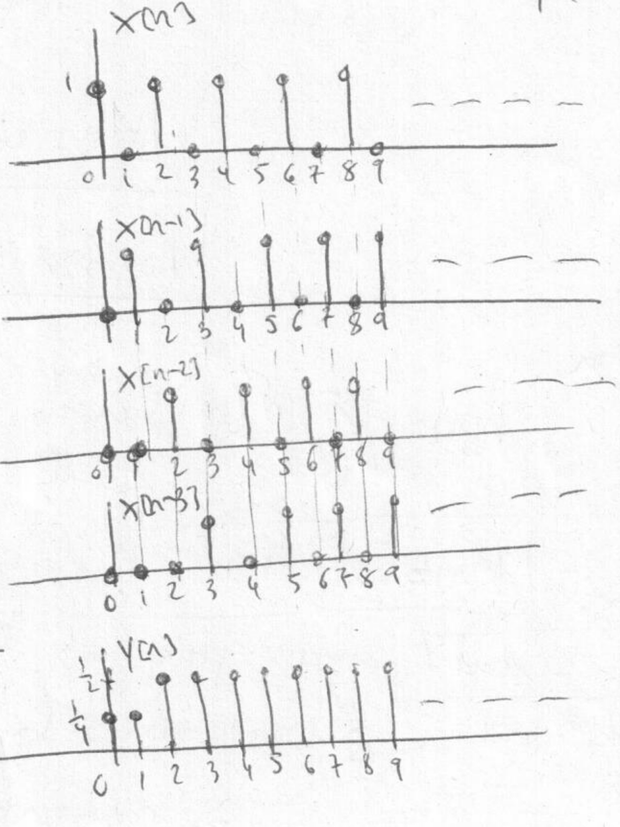
$$= a^{n-4} \cdot (n-3)$$

$$\boxed{y[n] = a^{(n-4)} \cdot (n-3)\, u[n-4]}$$

③ $h[n] = \begin{cases} 1/4 & 0 \leq n < 4 \\ 0 & \text{resto} \end{cases}$

FIR de ecuación en diferencias $y[n] = \frac{1}{4}\left(x[n] + x[n-1] + x[n-2] + x[n-3]\right)$

$x[n] = \sum_{k=0}^{\infty} \delta[n-2k]$



$$\boxed{y[n] = \frac{1}{4} u[n] + \frac{1}{4} u[n-2]}$$

$$\boxed{y[n] = \frac{1}{4}\left(\delta[n] + \delta[n-1]\right) + \frac{1}{2} u[n-2]}$$

PROBLEMA 2

a. Teoría Tema 2, apartado 3.2.2 $\Rightarrow y[n] = x[n] + \frac{1}{2}x[n-1] + \frac{1}{4}y[n-2]$
(Sistema en Forma Directa II)

b. Teoría Tema 2, apartado 3.2, 3.2.1 $\Rightarrow$ Sistema IIR, orden 2

c. $x[n] = \delta[n] \Rightarrow y[n] = h[n] \Rightarrow h[n] = \delta[n] + \frac{1}{2}\delta[n-1] + \frac{1}{4}h[n-2]$

Sistema causal $h[n] = 0 \quad \forall\, n < 0$

$n=0 \quad h[0] = \delta[0] + \frac{1}{2}\cancelto{0}{\delta[-1]} + \frac{1}{4}\cancelto{0}{h[-2]} = 1$

$n=1 \quad h[1] = \cancelto{0}{\delta[1]} + \frac{1}{2}\delta[0] + \frac{1}{4}\cancelto{0}{h[-1]} = \frac{1}{2}$

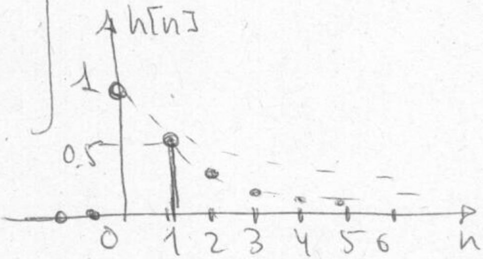
$n=2 \quad h[2] = \cancelto{0}{\delta[2]} + \frac{1}{2}\cancelto{0}{\delta[1]} + \frac{1}{4}h[0] = \frac{1}{4}$

$n=3 \quad h[3] = \delta[3] + \frac{1}{2}\delta[2] + \frac{1}{4}h[1] = \frac{1}{8}$

$n=4 \quad h[4] = \frac{1}{16}$

$\vdots \qquad \vdots \qquad \vdots$

$h[n] = \frac{1}{2^n} \cdot u[n]$



d. $x[n] = \{\underline{1}, 1, 1\}$ Filtrar = aplicar la ecuación en diferencia

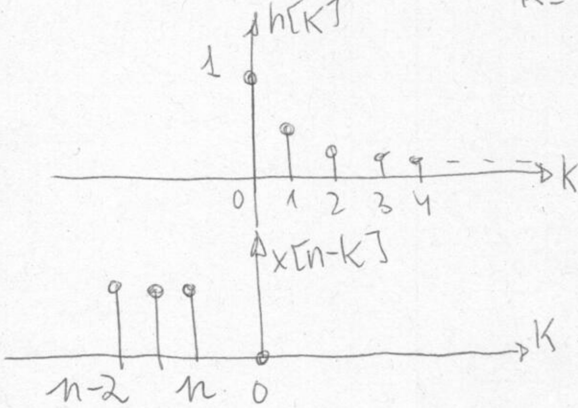
$y[n] = 0 \quad \forall\, n < 0$

$y[0] = \cancelto{1}{x[0]} + \frac{1}{2}\cancelto{0}{x[-1]} + \frac{1}{4}\cancelto{0}{y[-2]} = 1$ //

$y[1] = x[1] + \frac{1}{2}x[0] + \frac{1}{4}\cancelto{0}{y[-1]} = 1 + \frac{1}{2} = \frac{3}{2}$ //

$y[2] = x[2] + \frac{1}{2}x[1] + \frac{1}{4}y[0] = 1 + \frac{1}{2} + \frac{1}{4} = \frac{7}{4}$ //

e. $y[n] = x[n] * h[n] = \sum_{K=-\infty}^{\infty} x[K]\, h[n-K]$



$n < 0 \quad y[n] = 0$

$0 \leq n \leq 2 \quad y[n] = \sum_{0}^{n} \frac{1}{2^K}$ ⊛

$n > 2 \quad y[n] = \sum_{n-2}^{n} \frac{1}{2^K}$

⊛ $n=0 \quad y[0] = 1$ //

$n=1 \quad y[1] = 1 + \frac{1}{2} = \frac{3}{2}$ //

$n=2 \quad y[2] = 1 + \frac{1}{2} + \frac{1}{4} = \frac{7}{4}$ //

a) Periódica si : $x(t) = x(t + kT_0) \quad \forall t$

$$A \cos(w_0 t) = A \cos(w_0(t + kT_0))$$

con $k = 1 \rightarrow w_0 k T_0 = w_0 T_0 = p \cdot 2\pi$ para que se cumpla

$$w_0 = \frac{2\pi p}{T_0} \rightarrow \boxed{T_0 = \frac{w_0}{2\pi}} = \text{periodo fund.}$$

para $p = 1$

señal periódica, T_0

$$P_x = \frac{1}{T_0}\int_{T_0} A^2 \cos^2 w_0 t \, dt = \frac{1}{T_0}\int_{T_0} \frac{A^2}{2} dt + \frac{1}{T_0}\int_{T_0} A^2 \frac{\cos 2w_0 t}{2} dt$$

$$P_x = \frac{A^2}{2T_0}\int_0^{T_0} dt + \frac{A^2}{2T_0}\cancelto{0}{\int_0^{T_0} \cos 2w_0 t \, dt} = \boxed{\frac{A^2}{2}}$$

b) $y(t) = x^2(t) + B$

Lineal → No $\quad S\{a x_1(t) + b x_2(t)\} \neq a S\{x_1(t)\} + b S\{x_2(t)\}$

$$[a x_1(t) + b x_2(t)]^2 + B \neq a \cdot [x_1^2(t) + B] + b [x_2^2(t) + B]$$

T. Invariante → Sí $\quad S\{x(t - t_0)\} = y(t - t_0)$

$$x^2(t - t_0) + B = x^2(t - t_0) + B$$

Causal → sí $\quad y(t)$ depende de valores de la entrada $x(t)$ en el mismo instante t, no de valores futuros.

Estable → sí $\quad$ Para $|x(t)| < A$, la respuesta $|y(t)| \leq A^2 + B$

c) $y(t) = A^2 \cos^2(w_0 t) + B = \frac{A^2}{2} + \frac{A^2}{2} \cos 2w_0 t + B$

$$y[n] = y(t = nT_s) = \frac{A^2}{2} + \frac{A^2}{2} \cos 2 w_0 n T_s + B = \frac{A^2}{2} + \frac{A^2}{2} \cos 2 \cdot \frac{2\pi}{T_0} n T_s + B =$$

$$\boxed{y[n] = \frac{A^2}{2} + \frac{A^2}{2} \cos \Omega_q n + B} \quad \text{con } \Omega_q = 2 \cdot \frac{2\pi}{T_0} T_s$$

Para que $y[n]$ sea periódica: $\Omega_q = \frac{2\pi k}{N} \rightarrow \boxed{\frac{2 \cdot T_s}{T_0} = \frac{k}{N} = \text{nº racional}}$

d) $y[n]$

$z[n] = y[n] \cdot \cos \Omega_0 n$

Lineal → Sí $\quad S\{a y_1[n] + b y_2[n]\} = a S\{y_1[n]\} + b S\{y_2[n]\}$

$$(a y_1[n] + b y_2[n]) \cos \Omega_0 n = a \cdot y_1[n] \cos \Omega_0 n + b y_2[n] \cos \Omega_0 n$$

T. invariante → No $\quad S\{y[n - n_0]\} \neq z[n - n_0]$

$$y[n - n_0] \cdot \cos \Omega_0 n \neq y[n - n_0] \cos \Omega_0 (n - n_0)$$

Causal → Sí $\quad z[n]$ depende de los valores de la entrada en el instante n.

Estable → Sí $\quad$ si $|y[n]| \leq A \rightarrow |z[n]| \leq A$ p.q. el $\cos \Omega_0 n$ está acotado ± 1

TRATAMIENTO DE SEÑALES
Convocatoria ordinaria. Primer parcial

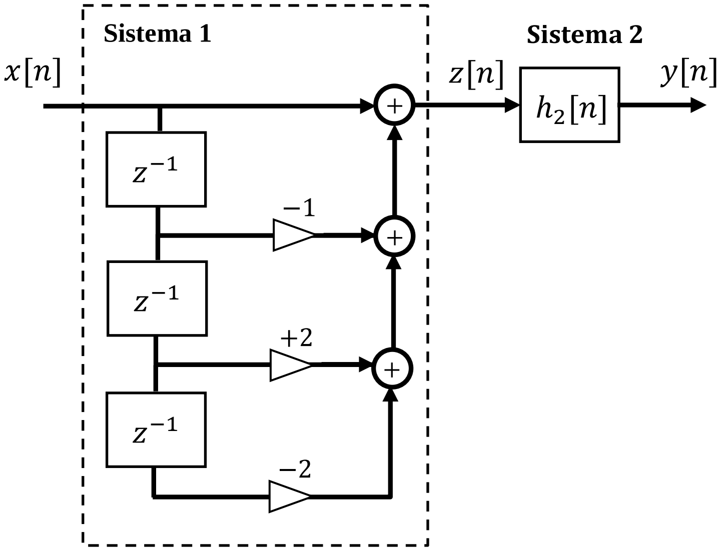
La puntuación total del examen es de 30 puntos divididos en:

- Problema 1: 10 puntos.
- Problema 2: 10 puntos. Todas las cuestiones tienen el mismo peso.
- Problema 3: 10 puntos.

El tiempo estimado para resolver el examen es de **una hora y media**.

PROBLEMA 1 (10 puntos, 30 minutos)

Sea la asociación de sistemas SLI de la figura, donde $h_2[n] = u[n-3]$.



Se pide:

a. Obtener y dibujar la salida $y[n]$ cuando la entrada al sistema 2 es $z[n] = 2u[n]$. **(3p)**

b. Obtener la respuesta impulsional del sistema 1, $h_1[n]$. **(1p)**

c. Obtener mediante filtrado la salida del sistema 1, $z[n]$, para una entrada $x[n] = \{\underline{1}, 1,1,1\}$ **(1p)**

d. Obtener la respuesta impulsional del sistema completo, $h[n]$, en función de δ[n]. **(2p)**

e. Determinar razonadamente el tipo (FIR o IIR) y orden (cuando sea posible) del sistema 1, del sistema 2 y del sistema completo. **(1p)**

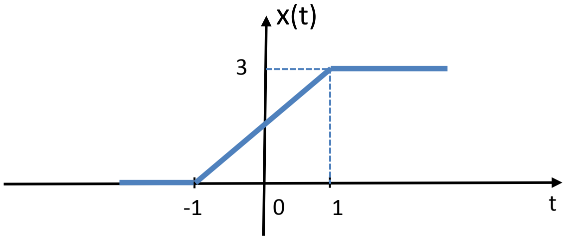
f. Obtener y dibujar la salida $y[n]$ para una entrada $x[n] = a^n u[n]$, con $0 < a < 1$. **(2p)**

PROBLEMA 2 (10 puntos, 30 minutos)

1. Sea x(t) la señal de la figura:

   Representar gráficamente la señal

   $y(t) = v\left(\frac{t}{2} + 4\right)$ para $v(t) = x\left(\frac{-t+3}{3}\right)$.



   Indicar razonadamente las transformaciones de la variable independiente que se realizan y sus representaciones gráficas.

2. Sea el sistema LTI cuya relación entrada-salida viene dada por: $y(t) = \int_2^{\infty} x(t-\tau)e^{-\tau+2}d\tau$.
   a. Obtener la respuesta impulsional $h(t)$.
   b. Analizar la causalidad y estabilidad del sistema.
   c. Obtener la respuesta para una entrada $x(t) = e^{-t}u(t)$.

3. Sea el sistema cuya ecuación en diferencias es: $y[n] = \mathrm{x}[\mathrm{n}-2] + \alpha \mathrm{y}[\mathrm{n}-2]$

   Indicar para qué valores de α el sistema es estable.

PROBLEMA 3 (10 puntos, 30 minutos)

Sea un sistema LTI del que se conoce que para una entrada $x_1(t) = \prod\left(\frac{t}{2}\right)$ la salida es $y_1(t) = \wedge\left(\frac{t-2}{2}\right)$. Sea la señal $x_2(t) = 2\prod\left(\frac{t+1}{2}\right) + 3\prod\left(\frac{t-4}{4}\right) - 5\prod\left(\frac{t-8}{2}\right)$

Se pide:

a. Dibujar $x_2(t)$ y expresarla en función de $x_1(t)$, utilizando solo desplazamientos temporales. **(3p)**

b. Aplicando las propiedades de linealidad e invarianza temporal del sistema, obtener y dibujar la salida $y_2(t)$ cuando la entrada es $x_2(t)$. **(3p)**

c. Obtener la respuesta impulsional del sistema, $h(t)$, sabiendo que $\prod\left(\frac{t}{T}\right) * \prod\left(\frac{t}{T}\right) = T\wedge\left(\frac{t}{T}\right)$. **(2p)**

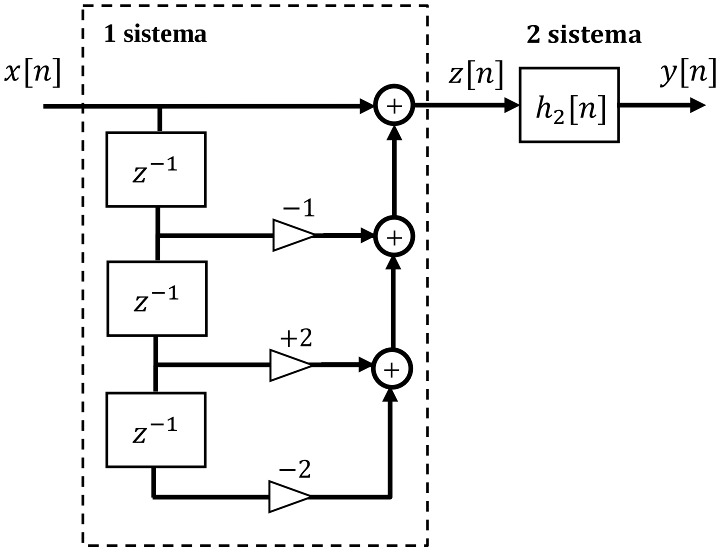
d. Obtener y dibujar la salida $y(t)$ cuando la entrada es $x(t) = \sum_{k=-\infty}^{\infty}\delta(t-k3)$. **(2p)**

SEINALEEN PROZESATZEA: Ohiko deialdia
Lehen partziala

Azterketak 3 ariketa ditu. Ariketa bakoitzak 10 puntu balio ditu, eta 2. ariketako galdera guztiek pisu berdina dute. **Ordu t'erdi duzue**.

1. ARIKETA (10 puntu, 30 minutu)

Demagun irudiko LTI sistemen konexioa, non $h_2[n] = u[n-3]$ den.



a. Lortu eta irudikatu $y[n]$, 2. sistemaren sarrera-seinalea $z[n] = 2u[n]$ bada. **(3p)**

b. Lortu 1. sistemaren pultsu-erantzuna, $h_1[n]$. **(1p)**

c. Iragazi $x[n] = \{\underline{1}, 1, 1, 1\}$ seinalea 1. sistema erabiliz. **(1p)**

d. Lortu sistema osoaren $h[n]$ pultsu-erantzuna eta adierazi δ[n] menpe. **(2p)**

e. Adierazi modu arrazoituan sistema mota (FIR/IIR) eta sistemaren ordena (posible denean), irudiko sistema guztientzako, 1. sistema, 2. sistema eta sistema osoa. **(1p)**

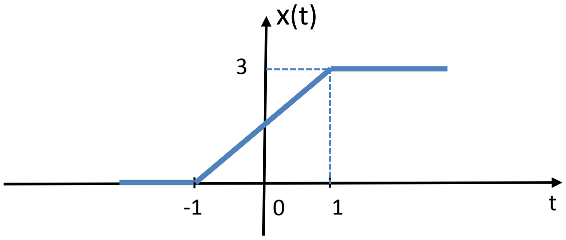
f. Lortu eta irudikatu irteera $y[n]$, sarrera $x[n] = a^n u[n]$ denean, non $0 < a < 1$. **(2p)**

2. ARIKETA (10 puntu, 30 minutu)

1. Demagun irudiko x(t) seinalea:

   Irudikatu seinalea:

   $y(t) = v\left(\frac{t}{2} + 4\right)$ non $v(t) = x\left(\frac{-t+3}{3}\right)$.



   Adierazi modu arrazoituan aldagai askearen transformazio sekuentzia, eta irudikatu itzazu.

2. Demagun LTI sistema ondoko irteera-sarrera erlazioarekin: $y(t) = \int_{2}^{\infty} x(t-\tau)e^{-\tau+2}d\tau$.
   a. Lortu pultsu-erantzuna, $h(t)$.
   b. Aztertu sistemaren kausalitatea eta egonkortasuna.
   c. Lortu sistemaren irteera-seinalea sarrera-seinalea $x(t) = e^{-t}u(t)$ denean.

3. Demagun hurrengo diferentzia ekuazioa duen sistema: $y[n] = \mathrm{x}[\mathrm{n}-2] + \alpha \mathrm{y}[\mathrm{n}-2]$

   Lortu α-ren balioak sistema egonkorra izan dadin.

3. ARIKETA (10 puntu, 30 minutu)

Demagun LTI sistema bat non $x_1(t) = \prod\left(\frac{t}{2}\right)$ sarrerak $y_1(t) = \wedge\left(\frac{t-2}{2}\right)$ irteera eragiten duen. Demagun hurrengo sarrera-seinalea dugula: $x_2(t) = 2\prod\left(\frac{t+1}{2}\right) + 3\prod\left(\frac{t-4}{4}\right) - 5\prod\left(\frac{t-8}{2}\right)$.

a. Irudikatu $x_2(t)$ eta adierazi $x_1(t)$-ren menpe, denbora desplazamenduak erabiliz soilik. **(3p)**

b. Linearitate eta denboran aldaezintasun propietateak erabilita, lortu eta irudikatu $y_2(t)$ irteera-seinalea sarrera-seinalea $x_2(t)$ denean. **(3p)**

c. Lortu sistemaren $h(t)$ pultsu-erantzuna, erabili: $\prod\left(\frac{t}{T}\right) * \prod\left(\frac{t}{T}\right) = T\wedge\left(\frac{t}{T}\right)$. **(2p)**

d. Lortu eta irudikatu $y(t)$ sarrera-seinalea $x(t) = \sum_{k=-\infty}^{\infty}\delta(t-k3)$ denean. **(2p)**

SIGNAL PROCESSING
Final exam. First mid-term

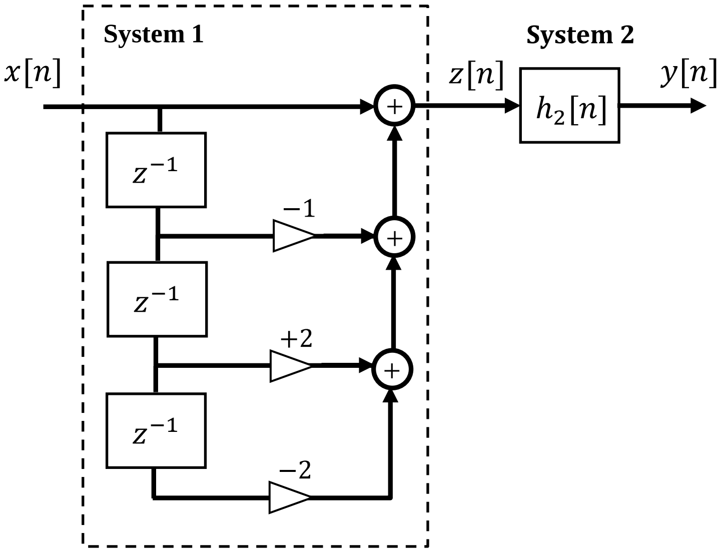
The exam scores a total of 30 points divided as follows:

- Problem 1: 10 points.
- Problem 2: 10 points. All questions have equal weight.
- Problem 3: 10 points.

The estimated time to complete the exam is **one hour thirty minutes**.

PROBLEM 1 (10 points, 30 minutes)

Consider the connection of LTI systems in the figure, where $h_2[n] = u[n-3]$.



a. Obtain and sketch the output $y[n]$ when the input to the system 2 is $z[n] = 2u[n]$. **(3p)**

b. Obtain the impulse response of system 1, $h_1[n]$. **(1p)**

c. Filter the signal $x[n] = \{\underline{1},1,1,1\}$ through system 1. **(1p)**

d. Obtain the impulse response of the whole system, $h[n]$, as a function of δ[n]. **(2p)**

e. Indicate and justify the type of system (FIR or IIR) and the order (when possible) of system 1, system 2, and the whole system. **(1p)**

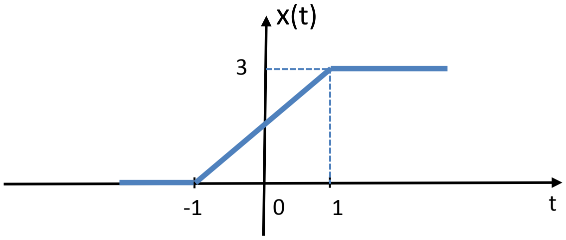
f. Obtain and sketch the output $y[n]$ for an input $x[n] = a^n u[n]$, with $0 < a < 1$. **(2p)**

PROBLEM 2 (10 points, 30 minutes)

1. Consider the signal x(t) of the figure:

   Sketch the signal:

   $y(t) = v\left(\frac{t}{2} + 4\right)$ with $v(t) = x\left(\frac{-t+3}{3}\right)$.



   Indicate and justify the sequence of transformations of the independent variable and sketch them.

2. Consider the LTI system with the input-output relationship: $y(t) = \int_2^{\infty} x(t-\tau)e^{-\tau+2}d\tau$.
   a. Obtain the impulse response, $h(t)$.
   b. Analyze the causality and stability of the system.
   c. Obtain the output of the system for an input $x(t) = e^{-t}u(t)$.

3. Consider the system with the difference equation: $y[n] = \mathrm{x}[\mathrm{n}-2] + \alpha\mathrm{y}[\mathrm{n}-2]$

   Indicate the values of α for which the system is stable.

PROBLEM 3 (10 points, 30 minutes)

Consider a LTI system in which the input $x_1(t) = \prod\left(\frac{t}{2}\right)$ produces an output $y_1(t) = \wedge\left(\frac{t-2}{2}\right)$. Consider the signal $x_2(t) = 2\prod\left(\frac{t+1}{2}\right) + 3\prod\left(\frac{t-4}{4}\right) - 5\prod\left(\frac{t-8}{2}\right)$.

a. Sketch $x_2(t)$ and express it in terms of $x_1(t)$, using only time shifts. **(3p)**

b. Using the properties of linearity and time-invariance of the system, obtain and sketch the output $y_2(t)$ when the input is $x_2(t)$. **(3p)**

c. Obtain the impulse response of the system, $h(t)$, using $\prod\left(\frac{t}{T}\right) * \prod\left(\frac{t}{T}\right) = T\wedge\left(\frac{t}{T}\right)$. **(2p)**

d. Obtain and sketch the output $y(t)$ when the input is $x(t) = \sum_{k=-\infty}^{\infty}\delta(t-k3)$. **(2p)**

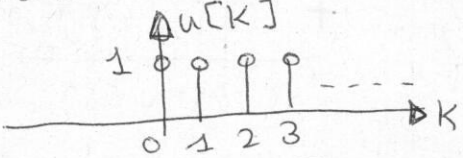
2018/2019. Convocatoria Ordinaria. Prime parcial

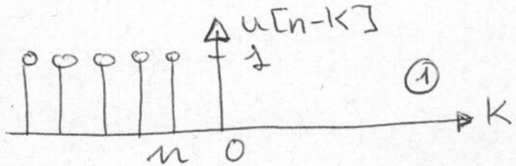
PROBLEMA 1

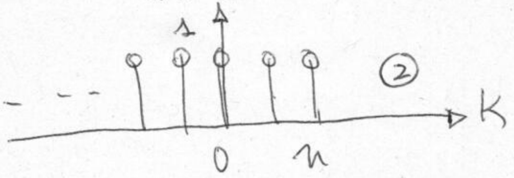
a.) $z[n]=2u[n]$, $h_2[n]=u[n-3]$ } $y[n]=z[n]*h_2[n]=2u[n]*u[n-3]$

Se trabaja con la señal auxiliar: $u[n]*u[n]=w[n]$, de forma que $y[n]=2w[n-3]$

Obtenemos $w[n]=u[n]*u[n]=\sum_{K=-\infty}^{\infty}u[K]u[n-K]$







① $n<0$ $w[n]=0$

② $n\geq 0$ $w[n]=\sum_{0}^{n}1=(n+1)$

Por lo tanto: $w[n]=(n+1)\cdot u[n]$

Finalmente:

$$\underset{\substack{\uparrow \\ n\rightarrow n-3}}{y[n]=}2(n+1-3)\cdot u[n-3]=2(n-2)u[n-3]$$

$$\boxed{y[n]=2(n-2)\cdot u[n-3]}$$

⊛ Dibujo, al final

b.) $z[n]=x[n]-x[n-1]+2x[n-2]-2x[n-3]$

$x[n]=\delta[n]$
$z[n]=h_1[n]$

$\left[h_1[n]=\delta[n]-\delta[n-1]+2\delta[n-2]-2\delta[n-3]\right]$

sistema FIR, orden 3

↓ z[n] solo depende de x[n], x[n-1], etc.. (el sistema no tiene parte recurrente.)

c.) $x[n]=\{\overset{0}{\underline{1}},\overset{1}{1},\overset{2}{1},\overset{3}{1}\}$

$z[0]=x[0]=1$; $z[1]=x[1]-x[0]=0$

$z[2]=x[2]-x[1]+2x[0]=2$

$z[3]=x[3]-x[2]+2x[1]-2x[0]=1-1+2-2=0$

} $z[n]=\{\underline{1},0,2,0\}$

d.) $\left[h[n]=h_1[n]*h_2[n]=(\delta[n]-\delta[n-1]+2\delta[n-2]-2\delta[n-3])*u[n-3]=\right.$

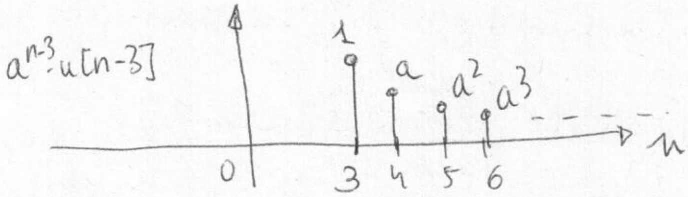
$$=\underbrace{u[n-3]-u[n-4]}_{\delta[n-3]}+\underbrace{2u[n-5]-2u[n-6]}_{2\delta[n-5]}=\underline{\underline{\delta[n-3]+2\delta[n-5]}}$$

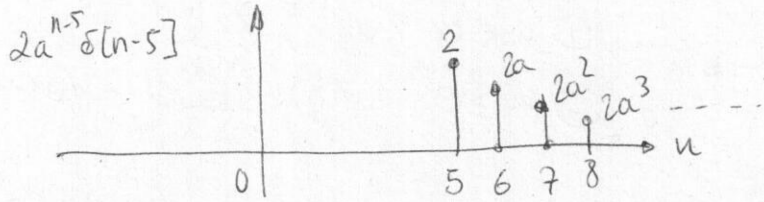
e.) Sistema 1: FIR, orden 3 (ver b.))
Sistema 2: IIR, porque $h_2[n]$ es de longitud ∞. El orden no se puede determinar en este caso.

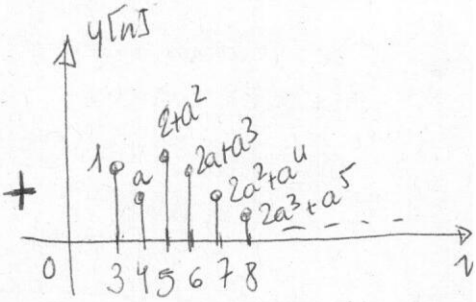
Sistema completo: FIR, orden 5

f.) $x[n] = a^n \cdot u[n]$, con $0 < a < 1$

$$\left[y[n] = a^n \cdot u[n] * (\delta[n-3] + 2\delta[n-5]) = \right.$$
$$\left. = a^{n-3} u[n-3] + 2a^{n-5} \delta[n-5] \right]$$

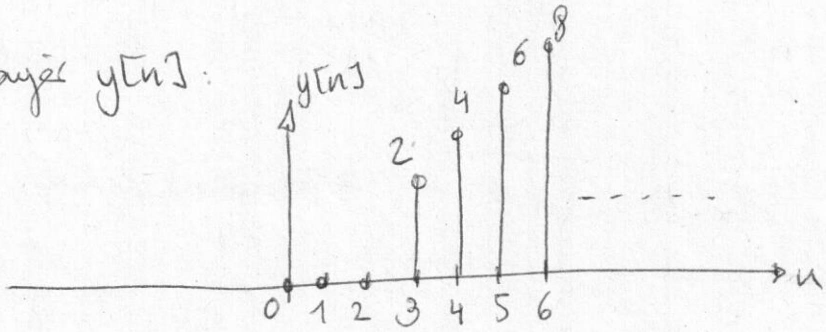




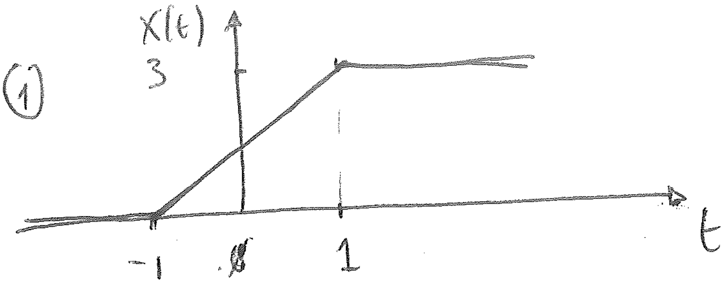


Otra expresión para y[n]: $y[n] = \delta[n-3] + a\,\delta[n-4] + (a^2+2)a^{n-5}\,\delta[n-5]$

a) ⊛ Dibujar y[n]:



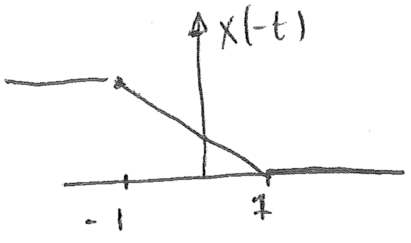
② ①



$$v(t) = x\left(\frac{-t+3}{3}\right) = x\left(\frac{-t}{3}+1\right)$$

$$y(t) = v\left(\frac{t}{2}+4\right) = v\left(\frac{t+8}{2}\right)$$

$$x(t) \rightarrow x(-t) \rightarrow x\left(\frac{-t}{3}\right) \rightarrow x\left(\frac{-t+3}{3}\right)$$



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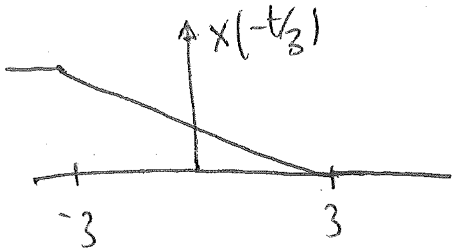
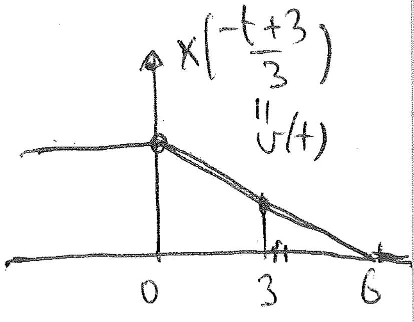
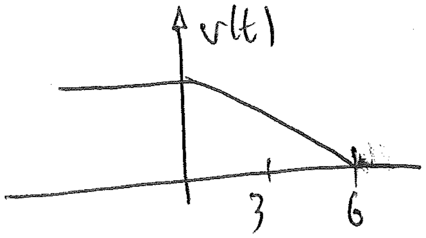
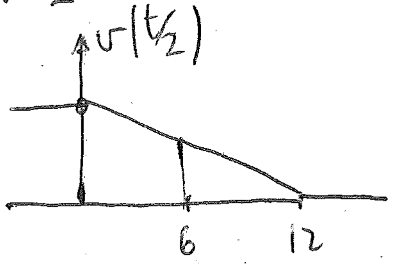
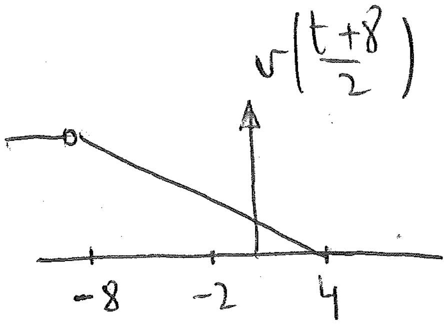




$$v(t) \rightarrow v\left(\frac{t}{2}\right) \rightarrow v\left(\frac{t+8}{2}\right)$$





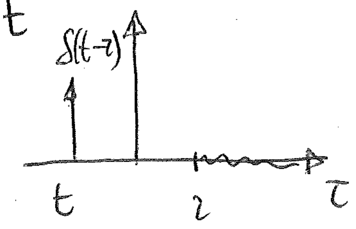


② $y(t) = \int_2^{\infty} x(t-\tau) e^{-\tau+2} \, d\tau$

ⓐ $x(t) = \delta(t) \rightarrow h(t) = \int_2^{\infty} \underline{\delta(t-\tau)} e^{-\tau+2} \, d\tau = \int_2^{\infty} \delta(t-\tau) e^{-t+2} \, d\tau$

$\tau = +t$

$$= e^{-t+2} \int_2^{\infty} \delta(t-\tau) \, d\tau$$

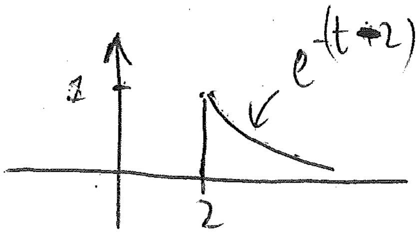


$t<2 \rightarrow h(t) = 0$

$t \geq 2 \rightarrow h(t) = e^{-t+2}$

$h(t) = e^{-(t+2)} \; u(t-2)$

$= \boxed{e^{-(t-2)} u(t-2) = h(t)}$

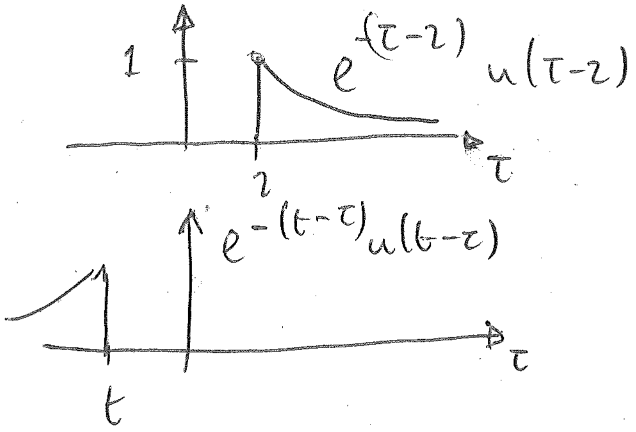


(a) Causal $h(t) = 0 \quad t<0$

(b) Stable $\int_{-\infty}^{\infty} |h(t)| dt = \int_2^{\infty} e^{-(t-2)} dt =$

$$= \int_0^\infty e^{t} dt = \frac{e^{-t}}{-1}\Big]_0^\infty = 1 < \infty.$$

ⓒ $y(t) = x(t) * h(t) = e^{-(t-2)} u(t-2) * e^{-t} u(t)$



$t<2 \quad y(t) = 0$ no overlap

$$t \geq 2 \quad y(t) = \int_2^t e^{-(\tau-2)} e^{-(t-\tau)} d\tau$$

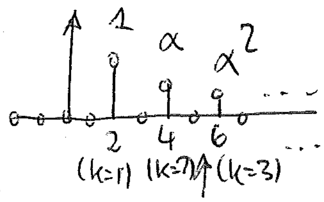
$$= e^{-t} \cdot e^{2} \int_2^t d\tau = e^{-(t-2)} (t-2)$$

$$\boxed{y(t) = (t-2) e^{-(t-2)} u(t-2)}$$

③ $y[n] = x[n-2] + \alpha y[n-2]$

Compute $h[n]$ $\quad$ ($h[n] = 0 \quad n<0$, causal system)

$x[n] = \delta[n]$



$h[0] = \delta[0-2] + \alpha h[-2] = 0$

$h[1] = \delta[-1] + \alpha h[-1] = 0$

$h[2] = \delta[0] + \alpha h[0] = 1$

$h[3] = \alpha h[1] = 0$

$h[4] = \alpha h[2] = \alpha$

$h[5] = \alpha h[3] = 0$

$h[6] = \alpha h[4] = \alpha^2$

$\vdots$

$$h[n] = \begin{cases} 0 & n \leq 0 \\ 0 & n \text{ odd} \\ \alpha^{\left(\frac{n-2}{2}\right)} & n \text{ even.} \end{cases} = \sum_{k=1}^{\infty} \alpha^{k-1} \delta[n-2k]$$

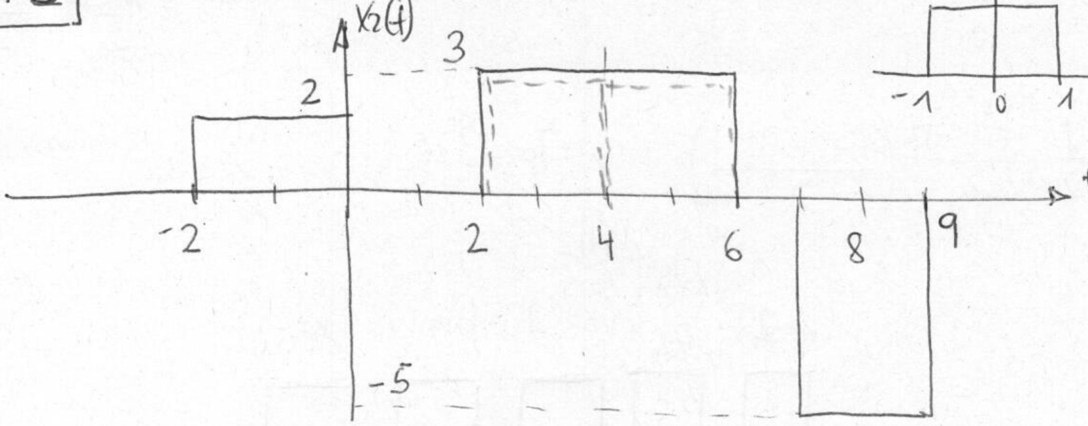
Stable

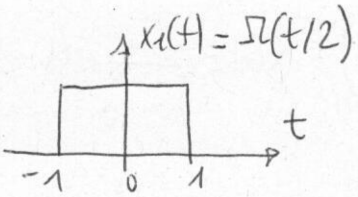
$$\sum_{n=-\infty}^{\infty} |h[n]| = \sum_{n=0,\, n \text{ even}}^{\infty} \alpha^{\left(\frac{n-2}{2}\right)} \underset{\uparrow \text{ geom ser}}{=} \overbrace{1 + \alpha + \alpha^2 + \dots}^{|\alpha|<1}$$

$$\boxed{|\alpha|<1}$$

PROBLEMA 3

a)





$$x_2(t) = 2x_1(t+1) + \underbrace{3x_1(t-3) + 3x_1(t-5)} - 5x_1(t-8) = \text{ ; Solo desplazamiento}$$

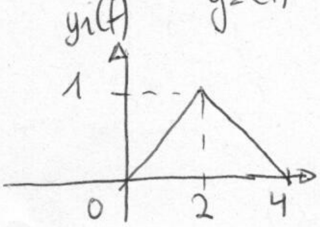
$$= 2\Pi\left(\frac{t+1}{2}\right) + 3\Pi\left(\frac{t-3}{2}\right) + 3\Pi\left(\frac{t-5}{2}\right) - 5\Pi\left(\frac{t-8}{2}\right)$$

b) $\| a x_1(t-t_0) + b x_2(t-t_1) \xrightarrow{LTI} a y_1(t-t_0) + b y_2(t-t_1) \|$ Para cualesquiera $x_1(t), x_2(t), t_0, t_1$

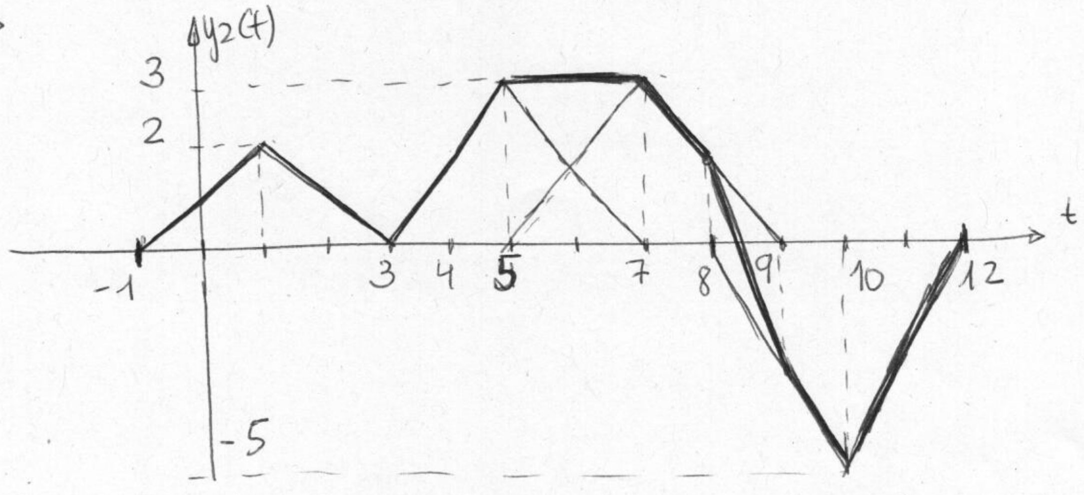
Por lo tanto:

$$y_2(t) = 2y_1(t+1) + 3y_1(t-3) + 3y_1(t-5) - 5y_1(t-8) =$$

$$= 2\Lambda\left(\frac{t-1}{2}\right) + 3\Lambda\left(\frac{t-5}{2}\right) + 3\Lambda\left(\frac{t-7}{2}\right) - 5\Lambda\left(\frac{t-10}{2}\right)$$



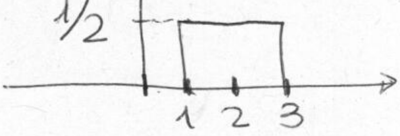
$y_1(t) = \Lambda\left(\frac{t-2}{2}\right)$



c) $y_1(t) = x_1(t) * h(t) \Rightarrow \Lambda\left(\frac{t-2}{2}\right) = \Pi\left(\frac{t}{2}\right) * h(t)$

Aplicando: $\Pi\left(\frac{t}{T}\right) * \Pi\left(\frac{t}{T}\right) = T\Lambda\left(\frac{t}{T}\right)$ y prop. desplazamiento:

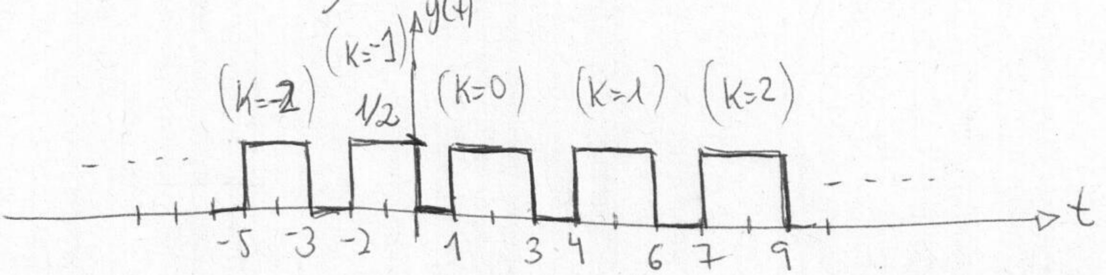
$$\left[\; h(t) = \frac{1}{2}\Pi\left(\frac{t-2}{2}\right) \;\right]$$



|| Podemos comprobar ahora con el apartado b que se han hecho las cosas bien ||

d) $x(t) = \sum_{K=-\infty}^{\infty} \delta(t-3K) \Rightarrow y(t) = x(t) * h(t) = \sum_{K=-\infty}^{\infty} \delta(t-3K) * \frac{1}{2}\Pi\left(\frac{t-2}{2}\right) =$

$$y(t) = \frac{1}{2}\sum_{K=-\infty}^{\infty} \Pi\left(\frac{t-2-3K}{2}\right) \quad ; \; T_0 = 3s$$



TRATAMIENTO DE SEÑALES
Convocatoria ordinaria. Segundo parcial

La puntuación total del examen es de 30 puntos divididos en:

Problema 1: 10 puntos. Todas las cuestiones tienen el mismo peso.

Problema 2: 10 puntos.

Problema 3: 10 puntos.

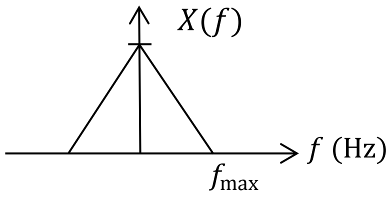
El tiempo estimado para resolver el examen es de **dos horas**.

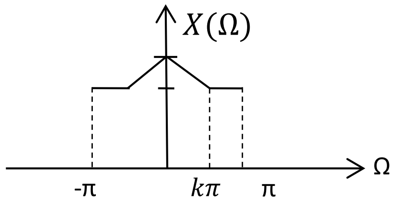
PROBLEMA 1 (10 puntos, 30 minutos)

1. Sea la señal $x(t) = 5\sin(2\pi 100t) + 4\sin(2\pi 200t)$ la señal de entrada a un conversor analógico digital ideal sin filtro antialiasing y $f_{s1} = 300$Hz. La señal digital resultante se pasa por un conversor digital analógico ideal, pero con f_{s2} distinta. La señal resultante es $y(t) = A\sin(2\pi 200t)$.

   Calcula el valor de f_{s2} y A.

2. En la figura se muestra el espectro de una señal analógica, y el espectro de la señal digital resultante de muestrear la señal analógica con frecuencia de muestreo f_s. Se pide hallar la frecuencia de muestreo f_s en función de los valores de k y $f_{\max}$, siendo $0 < k < 1$.





3. Sea un sistema LTI discreto con respuesta impulsional $h[n]$:

$$h[n] = \begin{cases} 1 & |n| \leq N \\ 0 & |n| > N \end{cases}$$

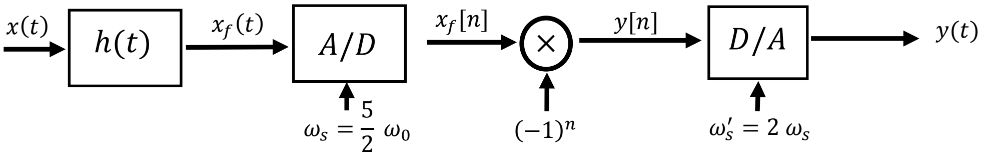
   Se pide:

   a. Dibuja el módulo de la respuesta frecuencial, $|H(\Omega)|$.

   b. Para una señal de entrada. $x[n] = A\cos\left(\frac{2\pi}{9}n\right)$ la salida es $y[n] = 0$. Calcula el mínimo valor de N $(N \neq 0)$ para el que esto es así.

PROBLEMA 2 (10 puntos, 45 minutos)

En el esquema de la figura se utiliza como entrada una señal x(t) que es **real y periódica** de pulsación ω_0. De la señal x(t) se conocen los siguientes coeficientes del desarrollo en serie de Fourier: a_1=5j, a_2=-2j, a_3=1+j, a_K=0 donde K>0 y distinto de 1, 2 y 3. Por otro lado, la respuesta impulsional es $h(t) = 2\frac{\sin\left(\frac{5}{2}\omega_0\, t\right)}{\pi t}$

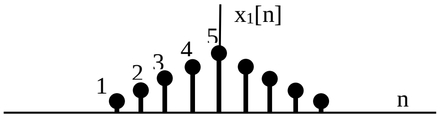


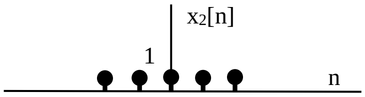
Se pide

a. Expresar x(t) como suma de cosenos. **(0,5 pto)**
b. Expresar analíticamente y representar gráficamente X(ω). **(1 pto)**
c. Obtener $x_f(t)$ como suma de cosenos y representar gráficamente $X_f(\omega)$. **(1,5 ptos)**
d. Obtener la secuencia $x_f[n]$ como suma de cosenos y representar gráficamente $X_f(\Omega)$. **(1,5 ptos)**
e. Analizar si la señal $x_f[n]$ es periódica. En caso afirmativo calcular los coeficientes del desarrollo en serie de Fourier. **(1,5 ptos)**
f. Obtener la secuencia y[n] como suma de cosenos y representar gráficamente Y(Ω).**(2 ptos)**
g. Obtener la señal de salida y(t) como suma de cosenos. Analizar si es periódica. En caso afirmativo calcular los coeficientes del desarrollo en serie de Fourier. **(2 ptos)**

PROBLEMA 3 (10 puntos, 45 minutos)

Sean las señales $x_1[n]$ y $x_2[n]$ de la figura:

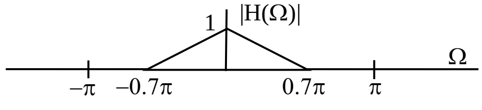


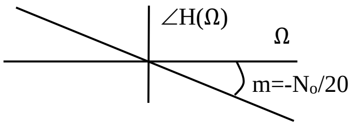


a. Calcular y representar gráficamente la transformada de Fourier de la secuencia $x_1[n]$ a partir de la relación que existe entre $x_1[n]$ y $x_2[n]$. **(2 ptos)**
b. A partir del resultado anterior, hallar los coeficientes del desarrollo en serie de Fourier de **(3 ptos)**

$$x[n] = \sum_{k=-\infty}^{\infty} x_1[n - kN_0] \qquad \text{con} \qquad N_0 = 10$$

c. Obtener y representar gráficamente X(Ω). **(2 ptos)**
d. $x[n]$ es la señal de entrada a un sistema con la respuesta frecuencial $H(\Omega)$ de la figura. Obtener la señal de salida $y[n]$ expresada en forma de suma de cosenos. **(3 ptos)**





SEINALEEN PROZESATZEA: Ohiko deialdia
Bigarren partziala

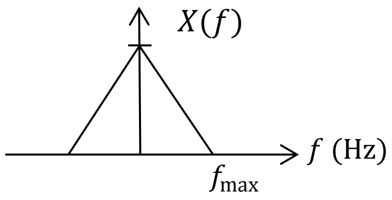
Azterketak 3 ariketa ditu. Ariketa bakoitzak 10 puntu balio ditu, eta 1. ariketako galdera guztiek pisu berdina dute. **Bi ordu dituzue**.

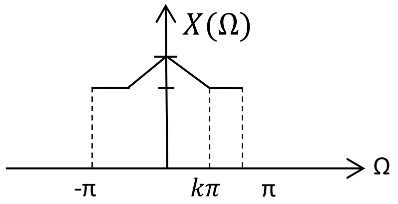
1. ARIKETA (10 puntu, 30 minutu)

1. Izan bedi $x(t) = 5\sin(2\pi 100t) + 4\sin(2\pi 200t)$, antialiasing iragazkirik gabeko analogiko-digital bihurgailu baten sarrera seinalea, laginketa maiztasuna $f_{s1} = 300\text{Hz}$ delarik. Lortutako seinalea digital-analogiko bihurgailu ideal batetatik pasa da baina beste laginketa maiztasun bat erabiliz, f_{s2}. Lortutako seinalea $y(t) = A\sin(2\pi 200t)$ da.

   Kalkulatu f_{s2} eta A balioak.

2. Irudian seinale analogiko eta digital baten espektroak irudikatu dira, seinale digitala seinale analogikoa f_s laginketa maiztasunarekin lagintzetik lortu delarik. Kalkulatu laginketa maiztasuna f_s , irudiko k eta $f_{\max}$ balioen menpe, non $0 < k < 1$.





3. Demagun ondoko $h[n]$ pultsu-erantzuna duen LTI sistema:

$$h[n] = \begin{cases} 1 & |n| \leq N \\ 0 & |n| > N \end{cases}$$

   Erantzun:

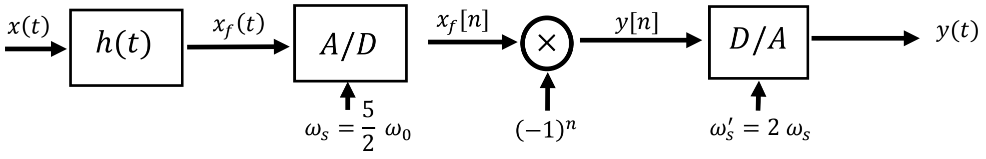
   a. Irudikatu maiztasun-erantzunaren modulua: $|H(\Omega)|$.

   b. Sarrera-seinalea $x[n] = A\cos\left(\frac{2\pi}{9}n\right)$ bada, irteera-seinalea $y[n] = 0$ da. Kalkulatu N-ren $(N \neq 0)$ balio minimoa horrela izan dadin.

2. ARIKETA (10 puntu, 45 minutu)

Irudiko diagraman x(t) sarrera-seinalea **erreala eta periodikoa** da, eta ω_0 oinarrizko maiztasuna du. x(t)-ren hurrengo Fourierren koefizienteak ezagutzen ditugu: a_1=5j, a_2=-2j, a_3=1+j, a_K=0 beste edozein k>0. LTI sistemaren pultsu-erantzuna hau da:

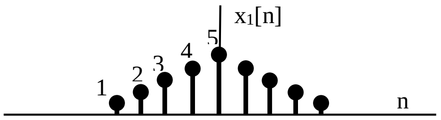
$$h(t) = 2\frac{\sin\left(\frac{5}{2}\,\omega_0\,t\right)}{\pi t}$$

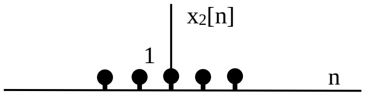


a. Lortu x(t) kosinuen batuketa bezala. **(0,5 pts)**
b. Lortu eta irudikatu X(ω) adierazpen analitikoa. **(1 pts)**
c. Lortu x_f(t) kosinuen batuketa bezala, eta irudikatu $X_f(\omega)$. **(1,5 pts)**
d. Lortu x_f[n] kosinuen batuketa bezala, eta irudikatu $X_f(\Omega)$. **(1,5 pts)**
e. Periodikoa al da x_f[n]? Horrela bada, lortu bere Fourier-ren koefizienteak. **(1,5 pts)**
f. Lortu y[n] kosinuen batuketa bezala, eta irudikatu Y(Ω). **(2 pts)**
g. Lortu y(t) kosinuen batuketa bezala. Periodikoa al da? Horrela bada, lortu bere Fourier-ren koefizienteak. **(2 pts)**

3. ARIKETA (10 puntu, 45 minutu)

Izan bitez irudiko $x_1[n]$ eta $x_2[n]$ seinaleak:

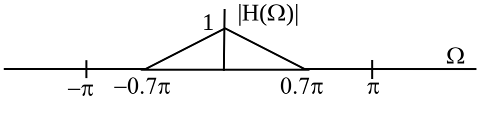


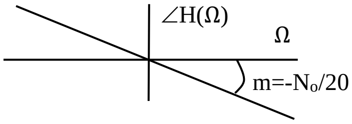


a. Kalkulatu $x_1[n]$ seinalearen Fourierren transformatua, abiatu $x_1[n]$ eta $x_2[n]$ seinaleen arteko erlaziotik. **(2 pts)**
b. Aurreko emaitzatik abiatuta, lortu ondoko seinalearen Fourierren koefizienteak **(3 pts)**

$$x[n] = \sum_{k=-\infty}^{\infty} x_1[n-kN_0] \qquad \text{non} \qquad N_0 = 10$$

c. Lortu eta irudikatu X(Ω). **(2 pts)**
d. Irudiko $H(\Omega)$ maiztasun erantzuna duen sistemaren sarrera-seinalea $x[n]$ da. Lortu irteera-seinalea $y[n]$ kosinuen batuketa bezala. **(3 pts)**





SIGNAL PROCESSING: Final exam
Second mid-term

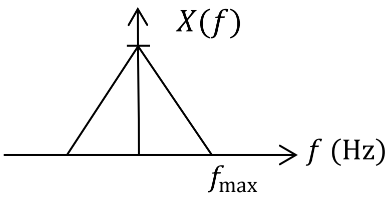
The estimated time to solve the exam is **two hours**.
The 3 short questions in problem 1 have all the same value.

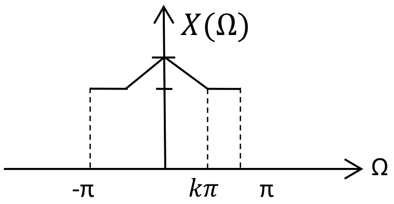
PROBLEM 1 (10 points, 30 minutes)

1. Consider the signal $x(t) = 5\sin(2\pi 100t) + 4\sin(2\pi 200t)$, which is the input to an analog to digital converter without antialiasing filter and $f_{s1} = 300$Hz. The digital signal is then passed through an ideal digital to analog converter but with different sampling frequency, f_{s2}. The resulting signal is $y(t) = A\sin(2\pi 200t)$.

   Compute the value of f_{s2} and A.

2. The figure shows the spectrum of an analog signal, and the spectrum of a digital signal obtained by sampling the analog signal with sampling frequency f_s. Obtain the sampling frequency f_s in terms of the values k and $f_{\max}$, with $0 < k < 1$.





3. Consider a LTI system with the following impulse response $h[n]$:

$$h[n] = \begin{cases} 1 & |n| \leq N \\ 0 & |n| > N \end{cases}$$

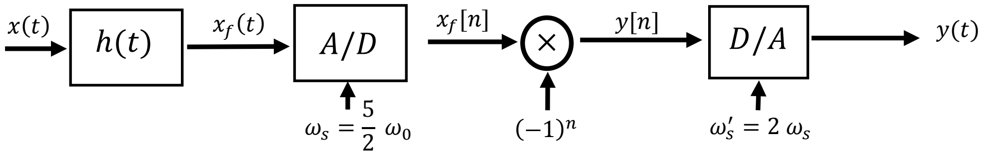
   Answer the following questions:

   a. Sketch the modulus of the frequency response: $|H(\Omega)|$.
   b. For an input signal $x[n] = A\cos\left(\frac{2\pi}{9}n\right)$, the output is $y[n] = 0$. Compute the minimum value of N $(N \neq 0)$ so this holds.

PROBLEM 2 (10 points, 45 minutes)

In the diagram of the figure the input signal x(t) is **real and periodic** with angular frequency ω_0. We know the following Fourier coefficients of x(t): $a_1=5j$, $a_2=-2j$, $a_3=1+j$, $a_K=0$ for other k>0. The impulse response of the LTI system is

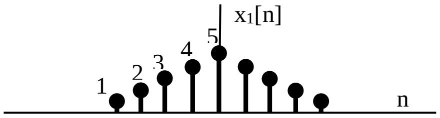
$$h(t) = 2\frac{\sin\left(\frac{5}{2}\,\omega_0\,t\right)}{\pi t}$$

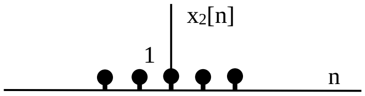


a. Express x(t) as a sum of cosines. **(0,5 pts)**
b. Obtain the analytical expression of X(ω), and plot it. **(1 pts)**
c. Obtain $x_f(t)$ as a sum of cosines, and plot $X_f(\omega)$. **(1,5 pts)**
d. Obtain $x_f[n]$ as a sum of cosines and plot $X_f(\Omega)$. **(1,5 pts)**
e. Is $x_f[n]$ periodic? If so, obtain its Fourier coefficients. **(1,5 pts)**
f. Obtain y[n] as sum of cosines and sketch Y(Ω). **(2 pts)**
g. Obtain y(t) as sum of cosines. Is it periodic? If so, obtain its Fourier coefficients. **(2 pts)**

PROBLEM 3 (10 points, 45 minutes)

Consider the signals $x_1[n]$ and $x_2[n]$ of the figure:

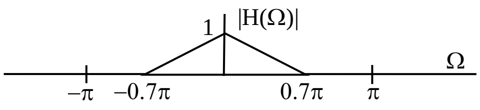


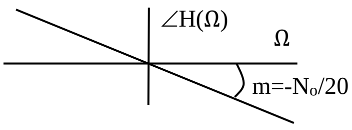


a. Compute and represent the Fourier transform of $x_1[n]$ from the relation between $x_1[n]$ and $x_2[n]$. **(2 pts)**
b. From the previous results, obtain the Fourier coefficients of **(3 pts)**

$$x[n] = \sum_{k=-\infty}^{\infty} x_1[n - kN_0] \qquad \text{with} \qquad N_0 = 10$$

c. Obtain and sketch X(Ω). **(2 pts)**
d. $x[n]$ is the input signal to a system with the frequency response $H(\Omega)$ of the figure. Obtain the output signal $y[n]$ as a sum of cosines. **(3 pts)**





① ① $x(t) = 5\sin(2\pi\cdot 100t) + 4\cdot\sin(2\pi\cdot 200t)$

$f_{s_1} = 300\,Hz \qquad t = n/300$

$$x[n] = 5\sin(2\pi \underbrace{\tfrac{100}{300}}\, n) + 4\sin(2\pi \underbrace{\tfrac{200}{300}}\, n)$$

$\Omega_1 = \frac{2\pi}{3} \qquad \Omega_2 = \frac{4\pi}{3} \equiv \frac{4\pi}{3} - 2\pi = -\frac{2\pi}{3}$

$|\Omega_1| < \pi \qquad |\Omega_2| > \pi$

$\sin(-x) = -\sin(x)$

$$x[n] = 5\sin(2\pi \tfrac{100}{300} n) + 4\sin(-2\pi\tfrac{100}{300} n) =$$

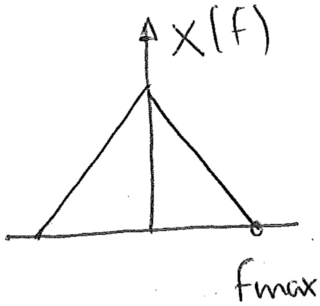
$$= 5\sin(2\pi\tfrac{100}{300}n) - 4\sin(2\pi\tfrac{100}{300}n) = 1\cdot\sin(2\pi\tfrac{100}{300}n)$$

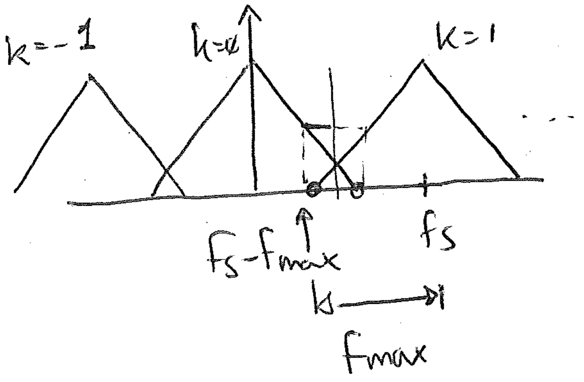
f_{s_2}? $\qquad y(t) = A\sin(2\pi\cdot 200t) = 1\cdot\sin(2\pi \underbrace{\tfrac{100}{300}\cdot f_{s_2}}_{= 200Hz}\cdot t)$

$t = \frac{n}{f_{s_2}} \rightarrow n = t\cdot f_{s_2}$

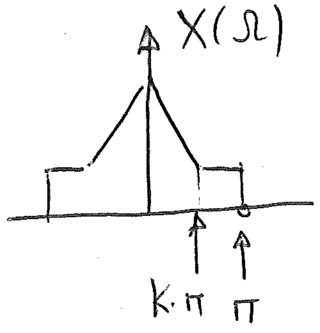
$\frac{100}{300}\cdot f_{s_2} = 200 \rightarrow \boxed{\begin{array}{l} f_{s_2} = 600Hz \\ A = 1 \end{array}}$

②





$\Rightarrow$



$\pi \rightarrow fs/2$

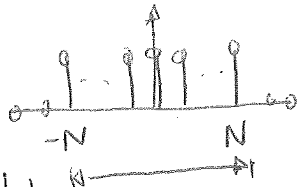
$K\cdot\pi \rightarrow K\cdot\frac{fs}{2}$

$k\cdot\frac{fs}{2} = fs - f_{max}$

$k\cdot f_s = 2f_s - 2f_{max} \qquad 2f_s - kf_s = 2f_{max}$

$\boxed{f_s = \frac{2}{2-k}\cdot f_{max}} \qquad 0<k<1$

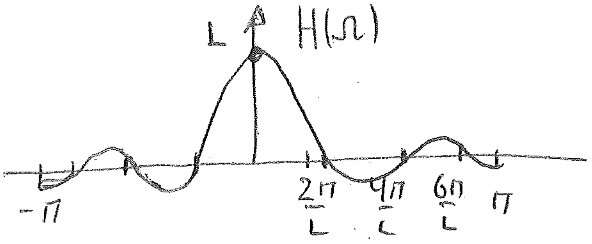
③ $h[n] = \begin{cases} 1 & |n| \leq N \\ 0 & |n| > N \end{cases}$



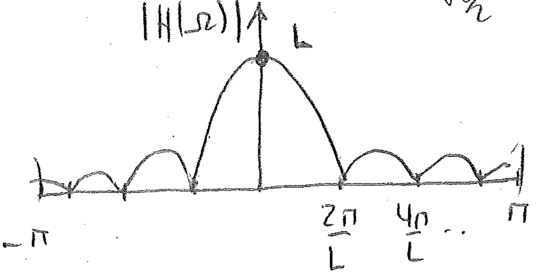
$L = 2N+1$ samples

Basic transform pairs $H(\Omega) = \dfrac{\sin\left(\frac{\Omega L}{2}\right)}{\sin\left(\frac{\Omega}{2}\right)}$

Zeros $\dfrac{\Omega L}{2} = k\pi$ $\quad \Omega = \dfrac{2k\pi}{L}$ $\quad \Omega = 0 \quad H(0) \sim \dfrac{\frac{\Omega L}{2}}{\Omega/2} = L$



$\Rightarrow$



$$x[n] = A \cdot \cos\left(\frac{2\pi}{9} n\right) \xrightarrow{\text{Real system}} y[n] = A \cdot \left|H\left(\frac{2\pi}{9}\right)\right| \cos\left(\frac{2\pi}{9} n + \phi\left(\frac{2\pi}{9}\right)\right) =$$

$\downarrow$

$\Omega_0 = \frac{2\pi}{9}$

$= 0$

$\Rightarrow \left|H\left(\frac{2\pi}{9}\right)\right| = 0$

looking at $|H(\Omega)| \rightarrow \dfrac{\cancel{2\pi}}{9} = \dfrac{\cancel{2\pi}}{L}$ $\quad \boxed{L = 9} \Rightarrow 2N+1 = 9$

(smallest possible) $\quad \boxed{N = 4}$ //

PROBLEMA 2

1.5

a) Señal periódica $x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t} \quad \forall k$

Señal real $a_k = a_{-k}^*$ Conocemos todos a_k $k \geq 0$

En este caso $X(t) = a_0 + \sum_{k=1}^{\infty} 2|a_k| \cos(k\omega_0 t + \measuredangle a_k)$

$$a_1 = 5j = 5 e^{j\pi/2}$$

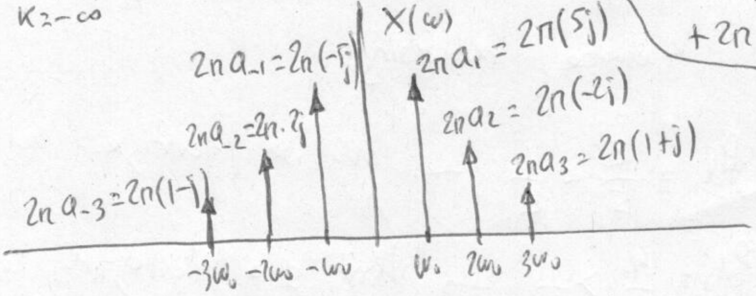
$$a_2 = -2j = 2 e^{-j\pi/2}$$

$$a_3 = 1+j = \sqrt{2} e^{j\pi/4}$$

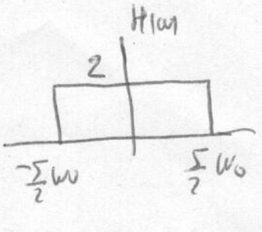
$$\boxed{X(t) = 10 \cos(\omega_0 t + \pi/2) + 4 \cos(2\omega_0 t - \pi/2) + 2\sqrt{2} \cos(3\omega_0 t + \pi/4)}$$

b) Señal periódica de pulsación ω_0

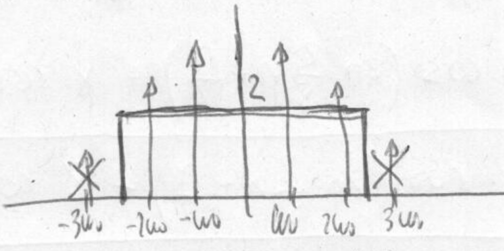
$$X(\omega) = \sum_{k=-\infty}^{\infty} 2\pi a_k \delta(\omega - k\omega_0) = 2\pi(5j)\delta(\omega-\omega_0) + 2\pi(-5j)\delta(\omega+\omega_0) + 2\pi(-2j)\delta(\omega-2\omega_0) + 2\pi(2j)\delta(\omega-2\omega_0) + 2\pi(1+j)\delta(\omega-3\omega_0) + 2\pi(1-j)\delta(\omega+3\omega_0)$$



c) $h(t) = 2 \dfrac{\operatorname{sen}(\frac{5}{2}\omega_0 t)}{\pi t} \xrightarrow{TF} H(\omega) = 2\, \Pi\left(\dfrac{\omega}{5\omega_0}\right)$

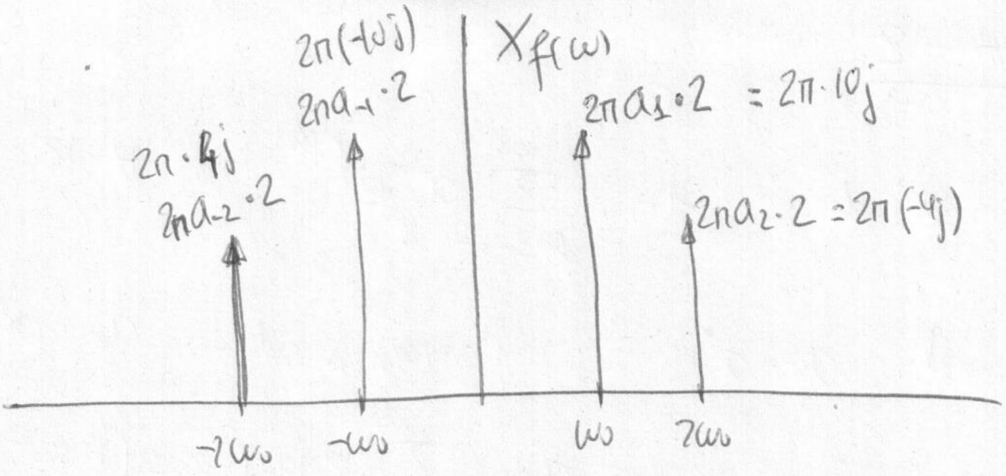


$X_f(\omega) = X(\omega) \cdot H(\omega)$



Se elimina el armónico tercero y se amplifican por 2 la componente fundamental y el segundo armónico.

$$\boxed{X_f(t) = 20 \cos(\omega_0 t + \pi/2) + 8 \cos(2\omega_0 t - \pi/2)}$$



d)

$$X_f[n] = X_f(t)\Big|_{t=nT_s} = X_f(t)\Big|_{t=n\cdot 2\pi\frac{2}{5\omega_0}}$$

$$\omega_s = \frac{2\pi}{T_s}$$

$$T_s = \frac{2\pi}{\omega_s} = \frac{2\pi}{\frac{5}{2}\omega_0} = 2\pi\cdot\frac{2\cancel{\omega}}{5\omega_0}$$

$$= 20\cos\left(\omega_0 t + \pi/2\right) + 8\cos\left(2\omega_0 t - \pi/2\right)\Big|_{t=n\cdot 2\pi\cdot\frac{2}{5\omega_0}}$$

$$X_f[n] = 20\cos\left(\cancel{\omega_0}\, 2\pi\cdot\frac{2}{5\cancel{\omega_0}} n + \pi/2\right) + 8\cos\left(2\cancel{\omega_0}\, 2\pi\frac{2}{5\cancel{\omega_0}} n - \pi/2\right)$$

$$X_f[n] = 20\cos\left(2\pi\cdot\underset{f_{d_1}}{\boxed{\frac{2}{5}}} n + \frac{\pi}{2}\right) + 8\cos\left(2\pi\underset{f_{d_2}}{\boxed{\frac{4}{5}}} n - \pi/2\right)$$

$\frac{5}{2}\omega_0 = \omega_s < 2(2\omega_0)$ Aparece aliasing.

$f_{d_1} = \frac{2}{5} < \frac{1}{2} \rightarrow$ No se corrige.

$f_{d_2} = \frac{4}{5} > \frac{1}{2} \rightarrow$ Hay que corregir

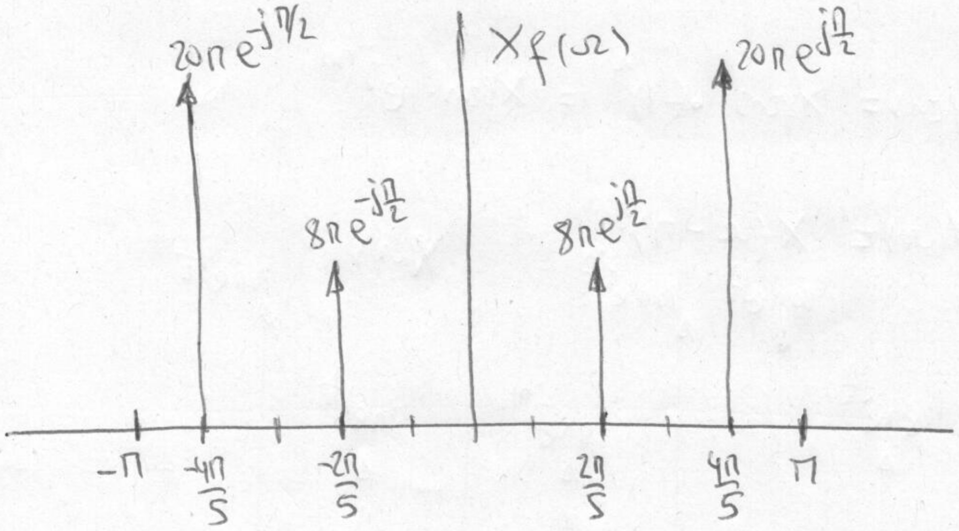
$f_{d_2} = \frac{4}{5} - 1 = \frac{-1}{5}$

$$X_f[n] = 20\cos\left(2\pi\frac{2}{5}n + \frac{\pi}{2}\right) + 8\cos\left(2\pi\left(\frac{-1}{5}\right)n - \frac{\pi}{2}\right)$$

$$\boxed{X_f[n] = 20\cos\left(2\pi\frac{2}{5}n + \frac{\pi}{2}\right) + 8\cos\left(2\pi\frac{1}{5}n + \frac{\pi}{2}\right)}$$

$$X_f(\Omega) = 20\left[\pi e^{j\frac{\pi}{2}}\delta\left(\Omega - \frac{4\pi}{5}\right) + \pi e^{-j\frac{\pi}{2}}\delta\left(\Omega + \frac{4\pi}{5}\right)\right] + 8\left[\pi e^{j\frac{\pi}{2}}\delta\left(\Omega - \frac{2\pi}{5}\right) + \pi e^{-j\pi/2}\delta\left(\Omega + \frac{2\pi}{5}\right)\right]$$

$$|\Omega| \leq \pi$$



e)

$$x_f[n] = 8\cos\left(\underbrace{2\pi \tfrac{1}{5}}_{\Omega_1} n + \tfrac{\pi}{2}\right) + 20\cos\left(\underbrace{2\pi \tfrac{2}{5}}_{\Omega_2} n + \tfrac{\pi}{2}\right)$$

$\Omega_1 = 2\pi \frac{1}{5}$ periódica de periodo $N_1 = 5$

$\Omega_2 = 2\pi \frac{2}{5}$ periódica de periodo $N_2 = 5$

$N_0 = m.c.m(5, 5) = 5$ $\Omega_0 = 2\pi \frac{1}{5}$

$$x_f[n] = a_0 + \sum_{k=1}^{\frac{N_0-1}{2}} 2|a_k| \cos(\cancel{k}\Omega_0 n + \angle a_k) \quad \Rightarrow \underline{N_0 \text{ impar}}$$

$$x_f[n] = a_0 + \sum_{k=1}^{2} 2|a_k| \cos\left(k\, 2\pi \tfrac{1}{5} n + \angle a_k\right)$$

$$\boxed{\begin{aligned} a_0 &= 0 \\ a_1 &= 4e^{j\pi/2} \\ a_2 &= 10\, e^{j\pi/2} \\ a_3 &= 10\, e^{-j\pi/2} \\ a_4 &= 4\, e^{-j\pi/2} \end{aligned}}$$

$2|a_1| = 8$ $|a_1| = 4$ $2|a_2| = 20$ $|a_2| = 10$

$\angle a_1 = \pi/2$ $\angle a_2 = \frac{\pi}{2}$

$a_k = a^*_{N_0-k}$

$a_3 = a^*_{5-3} = a^*_2$

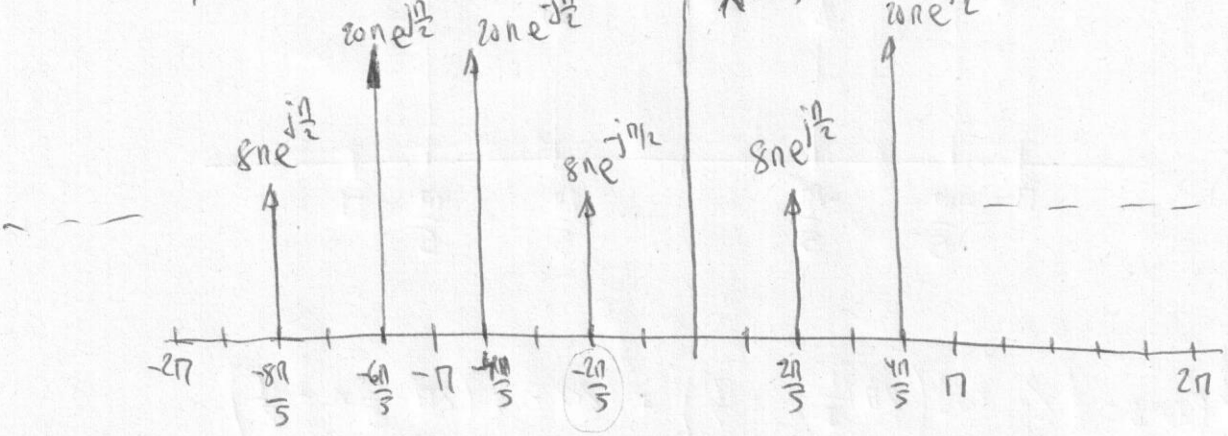
$a_4 = a^*_{5-4} = a^*_1$

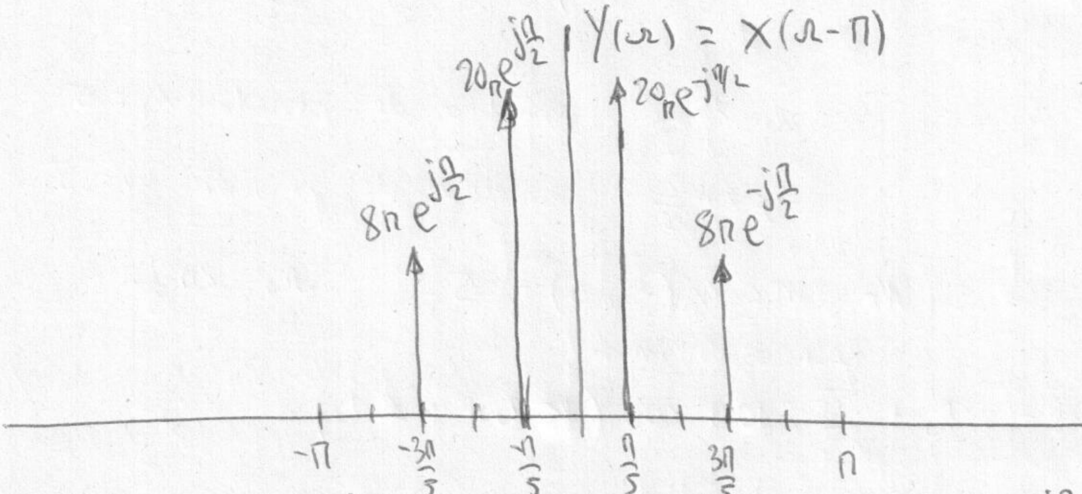
f)

$$y[n] = x_f[n] \cdot (-1)^n = x_f[n] \cdot e^{j\pi n}$$

TF

$$Y(\Omega) = X(\Omega - \pi)$$





$$-\frac{2\pi}{5} + \pi = \frac{3\pi}{5}$$

$$-\frac{4\pi}{5} + \pi = \frac{\pi}{5}$$

$$-\frac{6\pi}{5} + \pi = -\frac{\pi}{5}$$

$$-\frac{8\pi}{5} + \pi = -\frac{3\pi}{5}$$

$$Y(\Omega) = 20\left[\pi e^{-j\frac{\pi}{2}} \delta(\Omega - \pi/5) + \pi e^{j\pi/2} \delta(\Omega + \pi/5)\right] + 8\left[\pi e^{-j\frac{\pi}{2}} \delta(\Omega - \tfrac{3\pi}{5}) + \pi e^{j\frac{\pi}{2}} \delta(\Omega + \tfrac{3\pi}{5})\right]$$

$$|\Omega| \leq \pi$$

$$\boxed{y[n] = 20 \cos\left(\frac{\pi}{5} n - \pi/2\right) + 8 \cos\left(\frac{3\pi}{5} n - \frac{\pi}{2}\right)}$$

g)

$$y(t) = y[n] \Big|_{n = \frac{t}{T_s'} = t \frac{5W_0}{2\pi}}$$

$$W_s' = 2W_s = 5W_0$$

$$T_s' = \frac{2\pi}{W_s'} = \frac{2\pi}{5W_0}$$

$$y(t) = 20 \cos\left(\frac{\pi}{5} n - \frac{\pi}{2}\right) + 8 \cos\left(\frac{3\pi}{5} n - \frac{\pi}{2}\right) \Bigg|_{n = \frac{5W_0}{2\pi} \cdot t}$$

$$y(t) = 20 \cos\left(\frac{\cancel{\pi}}{\cancel{5}} \cdot \frac{\cancel{5} W_0}{2\cancel{\pi}} t - \frac{\pi}{2}\right) + 8 \cos\left(\frac{3\cancel{\pi}}{\cancel{5}} \cdot \frac{\cancel{5} W_0}{2\cancel{\pi}} t - \frac{\pi}{2}\right)$$

$$\boxed{y(t) = 20 \cos\left(\frac{W_0}{2} t - \frac{\pi}{2}\right) + 8 \cos\left(\frac{3}{2} W_0 t - \frac{\pi}{2}\right)}$$

$w_1 = \frac{w_0}{2}$ $\quad w_2 = \frac{3w_0}{2}$ $\quad w_0' = MCD(w_1, w_2) = \frac{w_0}{2}$

Es periódica de pulsación fundamental $w_0' = \frac{w_0}{2}$

$$y(t) = b_0 + \sum_{k=1}^{\infty} 2|b_k| \cos(k w_0' t + \measuredangle b_k) = 20 \cos(w_0' t - \pi/2) + 8\cos(3w_0' t - \frac{\pi}{2})$$

$$\boxed{\begin{array}{ll} b_1 = 10\, e^{-j\pi/2} & b_{-1} = 10\, e^{j\pi/2} \\ b_3 = 4\, e^{-j\pi/2} & b_{-3} = 4\, e^{j\pi/2} \end{array}}$$

Resto de $\boxed{b_k = 0}$

Ohiko deialdia – 2. partziala. 3. ARIKETA

a) $x_1[n] = x_2[n] * x_2[n] \longrightarrow X_1(\Omega) = X_2(\Omega) \cdot X_2(\Omega) = X_2^2(\Omega)$

$X_2(\Omega) = \dfrac{\operatorname{sen} \frac{\Omega L}{2}}{\operatorname{sen} \Omega/2}$ non $L = 5$

$X_1(\Omega) = \dfrac{\operatorname{sen}^2 2'5\Omega}{\operatorname{sen}^2 0'5\Omega}$

b) $x[n] = \sum\limits_k x_1[n - kN_0]$ $\quad a_k = \dfrac{1}{N_0} X_1(\Omega)\Big|_{\Omega = \frac{2\pi k}{N_0}}$ non $N_0 = 10$

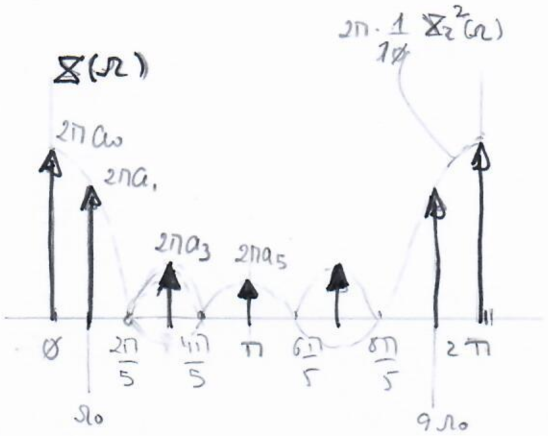
$a_k = \dfrac{1}{10} \dfrac{\operatorname{sen}^2 \pi k/2}{\operatorname{sen}^2 \pi k/10} \rightarrow a_0 = 5/2$

$a_{\pm 1} = 1'04$

$a_{\pm 2} = 0$

$a_{\pm 3} = 0'15$

c) $X(\Omega) = 2\pi \sum\limits_{k=-\infty}^{\infty} a_k \, \delta(\Omega - k\Omega_0)$ non $\Omega_0 = \dfrac{2\pi}{10}$



d) $x[n] = a_0 + \sum\limits_{k=\langle 10 \rangle} 2|a_k| \cos(k\Omega_0 + \varphi_k)$: erreala.

$H(\Omega)$: simetria hermitikoa → $h[n]$ erreala ⇒ $y[n]$ erreala da

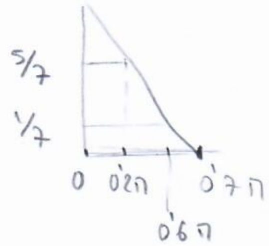
$y[n] = b_0 + \sum\limits_{k=1}^{3} 2|b_k| \cos(k\Omega_0 n + \theta_k)$: $\quad 0'7\pi > 3.\Omega_0$ → 3. harm.

$0'7\pi < 4\Omega_0$

$b_k = a_k \cdot H(k\Omega_0) = a_k |H(k\Omega_0)| \, e^{-j k\Omega_0/2}$

$H(\Omega_0) = H(0'2\pi) = \frac{5}{7} \cdot e^{-j0'1\pi}$ $\quad H(0) = 1.$

$H(3\Omega_0) = H(0'6\pi) = \frac{1}{7} \, e^{-j0'3\pi}$



$a_2 = Ø$

$y[n] = \frac{5}{2} + 2 \cdot |a_1| \cdot \frac{5}{7} \cos(0'2\pi n - 0'1\pi) + 2|a_3| \cdot \frac{1}{7} \cos(0'6\pi n - 0'3\pi)$

SEINALEEN PROZESATZEA: Ezohiko deialdia

Azterketak 3 ariketa ditu. Ariketa bakoitzak 10 puntu balio ditu, eta 1. ariketako galdera guztiek pisu berdina dute. Bi ordu dituzue.

1. ARIKETA (10 puntu, 40 minutu)

1. Izan bedi honako diferentzia-ekuazioak definitzen duen sistema:

$$y[n] = 2\,x[n] - x[n-1] + \frac{1}{2}x[n-2] - \frac{1}{4}x[n-3] + 2y[n-1] - 3y[n-2]$$

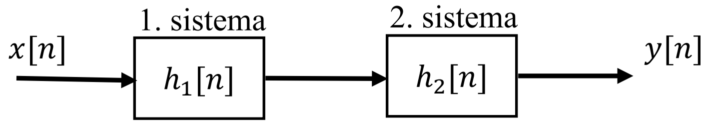
   a. Arrazoitu zein diren sistemaren mota (FIR edo IIR) eta ordena.
   b. Irudikatu sistemaren inplementazioa I. era zuzenean eta II. era zuzenean. Arrazoitu era batetik bestera pasatzeko eman behar diren pausuak.
   c. Kalkulatu y$[n]$ edozein n-rentzat, sarrera-seinalea $x[n] = 8$ bada.

2. Izan bedi $x_a(t)$ seinalea honako espektroa duena: $X_a(\omega) = \wedge\left(\frac{\omega}{2\pi 300}\right)$.
   Aurreko $x_a(t)$ seinalea lagindu egiten da $f_s = 400$Hz laginketa-maiztasunarekin. Irudikatu seinale diskretuaren espektroa, $X_d(\Omega)$, –π eta π tartean, interesgarriak diren pultsazio diskretuak adieraziz honako bi kasutan:
   - Antialiasing iragazkiarekin lagintzen da.
   - Antialiasing iragazkirik gabe lagintzen da.

3. Izan bedi honako sistema jarraitua: $y(t) = \int_{-\infty}^{t+2} x(\tau)d\tau$
   a. Arrazoitu sistema lineala, denboran aldaezina, kausala edota egonkorra den.
   b. Lortu sistemaren pultsu-erantzuna, $h(t)$.
   c. Aurreko a) ataleko emaitzen arabera esan daiteke $y(t) = x(t) * h(t)$ betetzen dela?

2. ARIKETA (10 puntu, 40 minutu)

Izan bedi irudiko sistema, bi sistema konektatuz lortzen dena:



$$h_1[n] = 2\,(-1)^n \, \frac{\sin(\Omega_{c1} n)}{\pi n} \qquad h_2[n] = \frac{\sin\left(\Omega_{c2}(n-4)\right)}{\pi(n-4)}$$

a. Irudikatu sistema osoaren maiztasun-erantzuna (modulua eta fasea) baldin eta, $\Omega_{c1} = {}^{4\pi}/_{5}$ eta $\Omega_{c2} > {}^{\pi}/_{5}$ betetzen badira. Adierazi sistema osoa zer motatako iragazkia den. (3 p)

Izan bedi $x[n] = \sum_{k=-\infty}^{\infty} w[n - kN_0]$ seinale periodikoa, non periodoa $N_0 = 8$ den, eta w[n] seinalearen luzapen periodikoa eginaz sortu den. Azken honen espektroa honako hau da:

$$W(\Omega) = \Lambda\left(\frac{\Omega}{{}^{3\pi}/_{4}}\right) \quad |\Omega| \leq \pi$$

b. Lortu x[n] seinalearen Fourieren koefizienteak, eta adierazi x[n] cosinuen batukari modura. (4 p)

c. Azaldu zein den Ω_{c2} pultsazioaren balio maximoa irteera-sekuentziak honako itxura izan dezan: $y[n] = A\,cos(\Omega_1 n + \theta)$. (1 p)

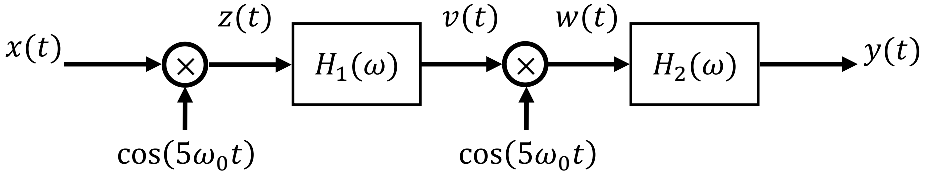
d. Kalkulatu A, Ω_1 eta θ balioak. (2 p)

3. ARIKETA (10 puntu, 40 minutu)

Izan bedi $x(t)$ seinale **erreala** eta periodikoa, oinarrizko pultsazioa ω_0 duena. Seinalea osagai jarraituaz, oinarrizko maiztasunez, eta bigarren eta hirugarren harmonikoez osatuta dago. Honako Fourieren koefizienteak ezagutzen dira: $a_0 = 3,\ a_1 = 5\, e^{j\frac{\pi}{4}},\ a_{-2} = e^{j\frac{\pi}{2}},\ \ a_{-3} = \frac{1}{2}\, e^{-j\frac{\pi}{4}}$

a. Adierazi $x(t)$ seinalea cosinuen batukari modura, eta irudikatu X(ω). (1.5 p)

Aurreko $x(t)$ seinalea honako sistemaren sarrera-seinalea da:



$$H_1(\omega) = \begin{cases} 2\, e^{-j\frac{T_0}{4}\omega} & 4\omega_0 \le |\omega| \le 6\omega_0 \\ 0 & resto \end{cases} \qquad H_2(\omega) = \begin{cases} \frac{1}{2}\, e^{-j\frac{T_0}{4}\omega} & |\omega| \le 2\omega_0 \\ 0 & |\omega| > 2\omega_0 \end{cases}$$

b. Irudikatu $H_1(\omega)$ eta $H_2(\omega)$ (modulua eta fasea), eta adierazi iragazki bakoitzaren mota. (1.5 p)

c. Kalkulatu eta irudikatu Z(ω). (2 p)

d. Kalkulatu eta irudikatu V(ω). (1.5 p)

e. Kalkulatu eta irudikatu W(ω). (2 p)

f. Lortu irteerako-seinalea y(t) eta adierazi cosinuen batukari bezala. (1.5 p)

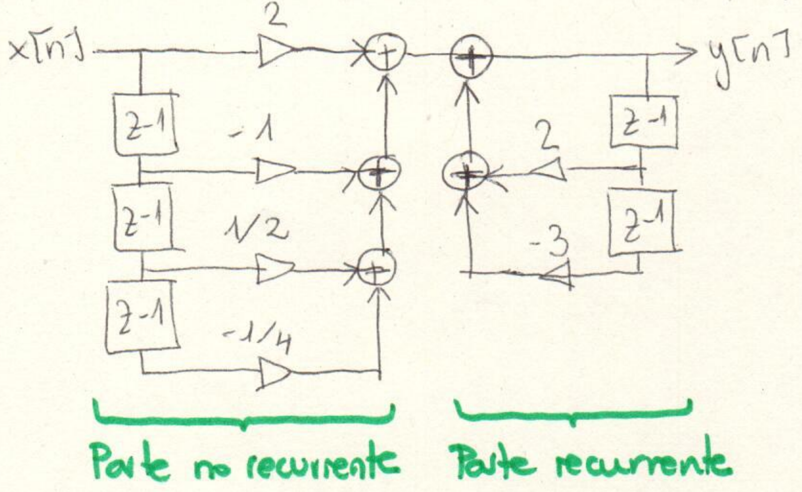
PROBLEMA 1. Cuestión 1 EXTRA. 18/19

$$y[n] = \overbrace{2x[n] - x[n-1] + \frac{1}{2}x[n-2] - \frac{1}{4}x[n-3]}^{\text{Parte no recurrente}} + \underbrace{2y[n-1] - 3y[n-2]}_{\circledast}$$

a. Sistema IIR, parte recurrente $\circledast$

Orden 3, máximo retardo en la ecuación en diferencias: $x[n-3]$

b. Forma Directa I: implementación directa de la ecuación en diferencias, conexión en serie parte no recurrente y parte recurrente:

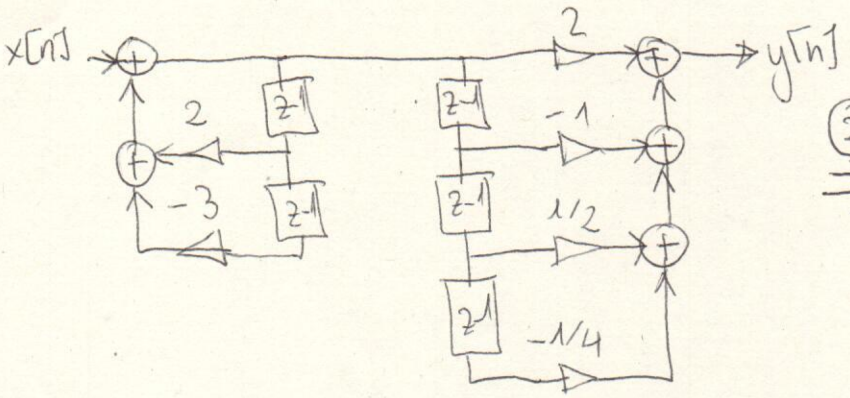


Para pasar a forma directa II:

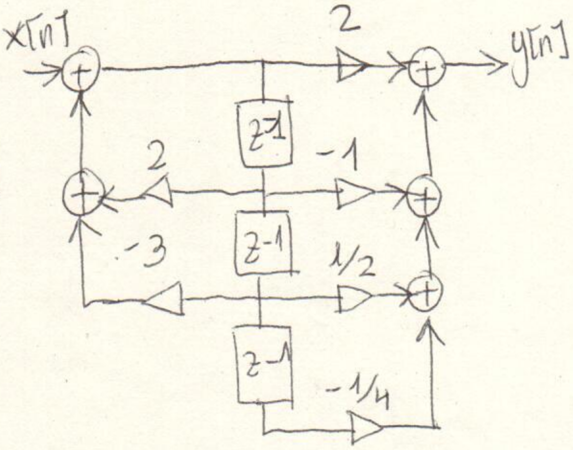
① Intercambiar parte recurrente y no recurrente ⇒ propiedad conmutativa asociación en serie de SLI

② Unificar bloques retardo

①



② ⇒



Forma Directa II

c. $x[n] = 8 \ \forall n \Rightarrow X(\Omega) = 2\pi 8\, \delta(\Omega)$

① Se calcula $H(\Omega)$ a partir de la ecuación en diferencias, aplicando T.F.

② Se calcula $Y(\Omega) = X(\Omega) \cdot H(\Omega)$

③ Se obtiene $y[n] = TF^{-1}\{Y(\Omega)\}$

① $$H(\Omega) = \frac{2 - e^{-j\Omega} + \frac{1}{2}e^{-j2\Omega} - \frac{1}{4}e^{-j3\Omega}}{1 - 2e^{-j\Omega} + 3e^{-j2\Omega}}$$

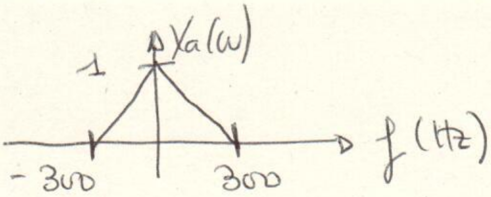
② $Y(\Omega) = X(\Omega) \cdot H(\Omega) = 2\pi 8\, \delta(\Omega) \cdot H(\Omega) = 2\pi 8\, \delta(\Omega) \cdot H(0)$

$$H(0) = \frac{2 - 1 + 1/2 - 1/4}{1 - 2 + 3} = \frac{5}{8} \ ; \quad Y(\Omega) = 2\pi 8 \cdot \frac{5}{8} \cdot \delta(\Omega) = 2\pi 5\, \delta(\Omega)$$

③ $\left[\, y[n] = TF^{-1}\{Y(\Omega)\} = 5 \quad \forall n \,\right]$

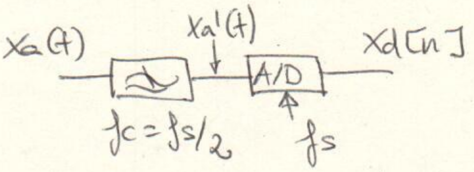
Cuestión 2

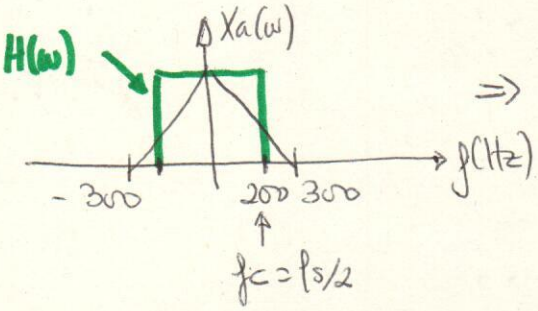
$x_a(t) \Rightarrow X_a(\omega) = \Lambda\left(\frac{\omega}{2\pi 300}\right)$



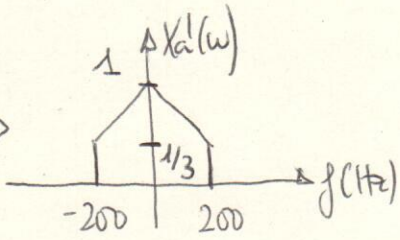
$f_s = 400 Hz$

- Con filtro antialiasing:

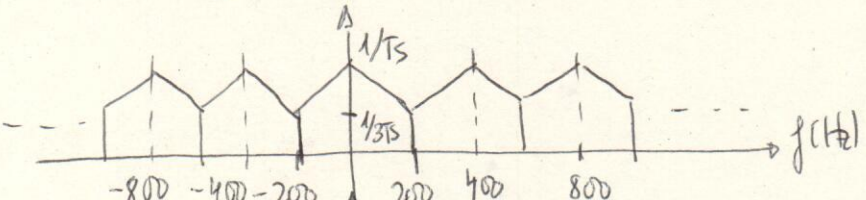


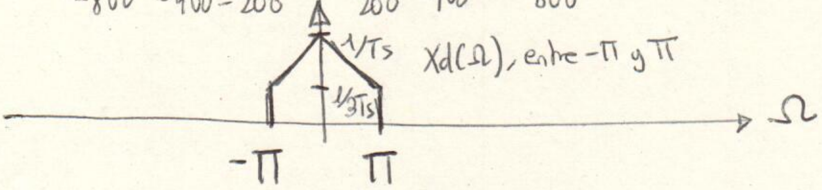


$\Rightarrow$ $X_a'(\omega) = X_a(\omega)\cdot H(\omega)$ $\Rightarrow$



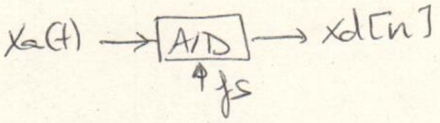
$\Rightarrow$ Muestreo: $X_d(\Omega) = \frac{1}{T_s}\sum_k X_a'(\omega - k\omega_s)$; $\omega_s = \frac{2\pi}{T_s} = 2\pi f_s$; $\Omega = \omega\cdot T_s = \frac{\omega}{f_s} = 2\pi\frac{f}{f_s}$



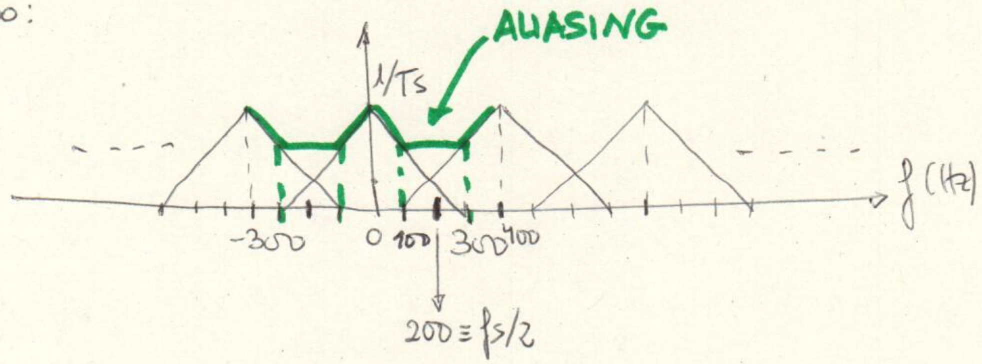


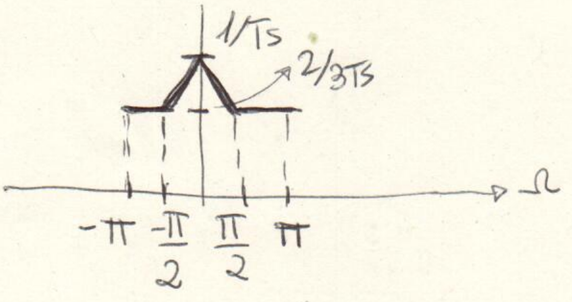
$\left[f_s/2 \Rightarrow \pi \right]$

- Sin filtro antialiasing



Muestreo:





- Hay aliasing porque $f_s \ngeq 2 f_{max}$
- El aliasing aparece en la banda de interés: 100 - 300 Hz

$\boxed{\text{PROBLEMA 1}}$. Cuestión 3 EXTRA. 18/19

$$y(t) = \int_{-\infty}^{t+2} x(\tau)\,d\tau$$

a) $x(t) = ax_1(t) + bx_2(t) \rightarrow y(t) = \int_{-\infty}^{t+2} (ax_1(\tau)+bx_2(\tau))\,d\tau = \int_{-\infty}^{t+2} ax_1(\tau)\,d\tau + \int_{-\infty}^{t+2} bx_2(\tau)\,d\tau =$

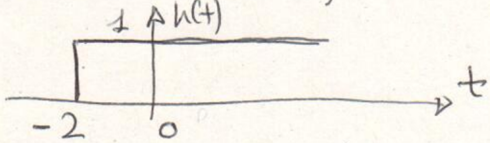
$= ay_1(t) + by_2(t)$ → LINEAL

$x(t-t_0) \rightarrow \int_{-\infty}^{t+2} x(\tau-t_0)\,d\tau \;;\; \tau - t_0 = \tau' \;;\; \int_{-\infty}^{t-t_0+2} x(\tau')\,d\tau' = y(t-t_0)$ → INVARIANTE

NO CAUSAL → para $t=5$ $y(t)$ depende de $x(\tau)$ desde $-\infty$ a 7

INESTABLE → para $x(t) = u(t)$, $y(t)$ crece indefinidamente (para $t \rightarrow \infty$ $y(t) \rightarrow \infty$)

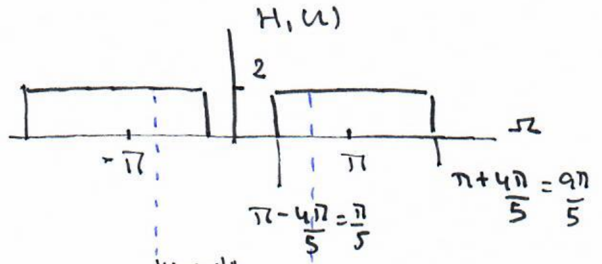
b) $h(t)$? $x(t) = \delta(t) \Rightarrow y(t) = h(t) \Rightarrow h(t) = \int_{-\infty}^{t+2} \delta(\tau)\,d\tau = u(t+2)$



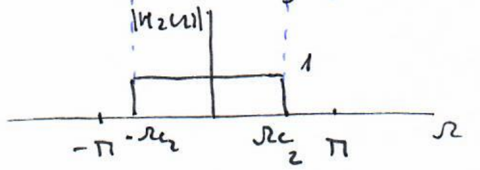
c) Es un SLI, por tanto: $y(t) = x(t) * h(t)$ para cualquier $x(t)$

2018_19 Ezohikoa 2. ARIKETA

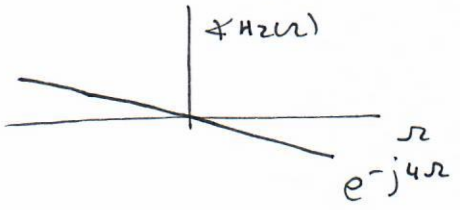
a)

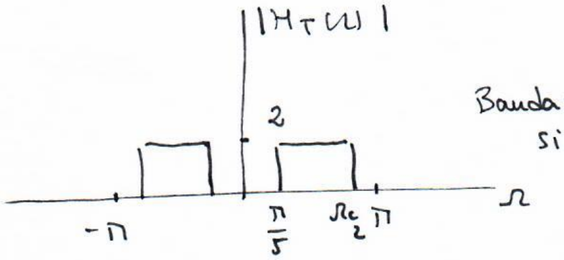


$$H_1(\Omega) = 2\,\Pi\left(\frac{\Omega-\pi}{2\Omega_{c1}}\right)$$

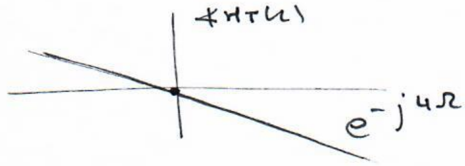


$$H_2(\Omega) = \Pi\left(\frac{\Omega}{2\Omega_{c2}}\right) e^{-j\Omega\cdot 4}$$





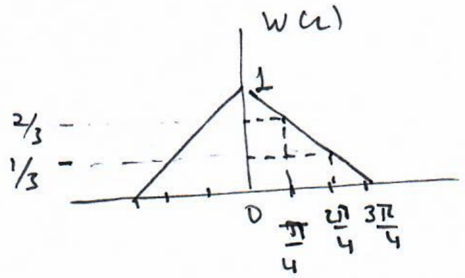
Banda-paseko sistema.



b)

$$a_k = \frac{1}{N_0} X_b(\Omega)\Big|_{\Omega=k\Omega_0} = \frac{1}{N_0} W(\Omega)\Big|_{\Omega=k\Omega_0}$$

$$\Omega_0 = \frac{2\pi}{8} = \frac{\pi}{4}$$



$$a_0 = \frac{1}{8}\cdot 1 = 1/8$$

$$a_{\pm 1} = \frac{1}{8}\cdot\frac{2}{3} = 1/12$$

$$a_{\pm 2} = \frac{1}{8}\ \frac{1}{3} = 1/24$$

$$x[n] = a_0 + 2|a_1|\cos\Omega_0 n + 2|a_2|\cos 2\Omega_0 n$$

$$x[n] = 1/8 + 1/6\cos\frac{\pi}{4}n + 1/12\cos\frac{\pi}{2}n$$

c) $\Omega_{c2} < \frac{\pi}{2}$, 2. harmonikoa ez pasatzeko.

d)

$$y[n] = \frac{1}{6}\underbrace{|H_T(\pi/4)|}_{=2}\cos\Big(\pi/4\, n + \underbrace{\measuredangle H(\pi/4)}_{=-4\cdot\pi/4=-\pi}\Big)$$

$$y[n] = A\cos(\Omega_1 n + \theta) \quad \begin{cases} A = 1/3 \\ \Omega_1 = \pi/4 \\ \theta = -\pi \end{cases}$$

③ x(t) es una señal real con por lo tanto $a_k = a_{-k}^*$

$|a_k| = |a_{-k}|$

$\theta_k = -\theta_{-k}$

ⓐ

$a_0 = 3$

$a_1 = 5e^{j\pi/4}$ $\quad |a_1| = 5 \quad \theta_1 = \frac{\pi}{4}$

$a_{-2} = 1e^{j\pi/2}$ $\quad |a_2| = 1 \quad \theta_2 = -\pi/2$ (real $\theta_K = -\theta_{-K}$)

$a_{-3} = \frac{1}{2}e^{-j\frac{\pi}{4}}$ $\quad |a_3| = \frac{1}{2} \quad \theta_3 = \frac{\pi}{4}$

Para señales reales

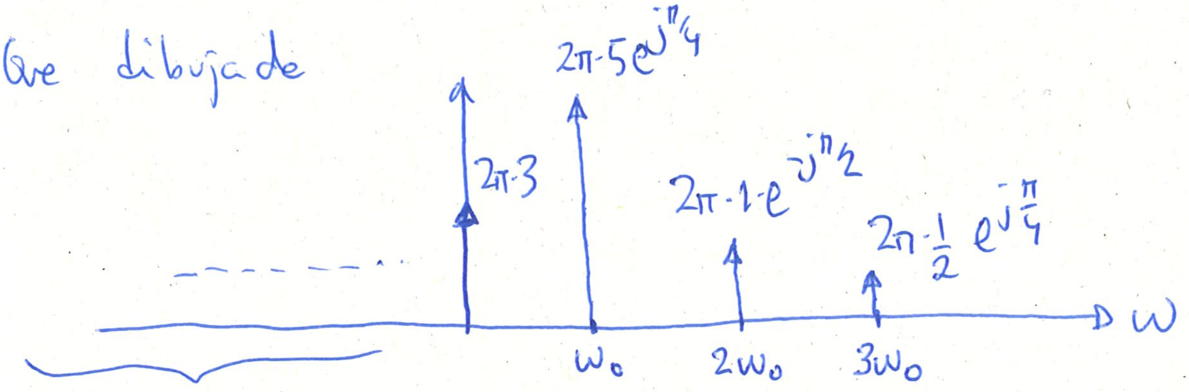
$$x(t) = a_0 + 2 \cdot \sum_{K=1}^{\infty} |a_K| \cos(K \cdot \omega_0 t + \theta_K)$$ es decir:

$$\boxed{x(t) = 3 + 2 \cdot 5 \cos(\omega_0 t + \frac{\pi}{4}) + 2 \cdot 1 \cos(2\omega_0 t - \frac{\pi}{2}) + 2 \cdot \frac{1}{2} \cos(3\omega_0 t + \frac{\pi}{4})}$$

Y sabemos que:

$$X(\omega) = 2\pi \cdot \sum_{K=-\infty}^{\infty} a_K \cdot \delta(\omega - K\omega_0)$$

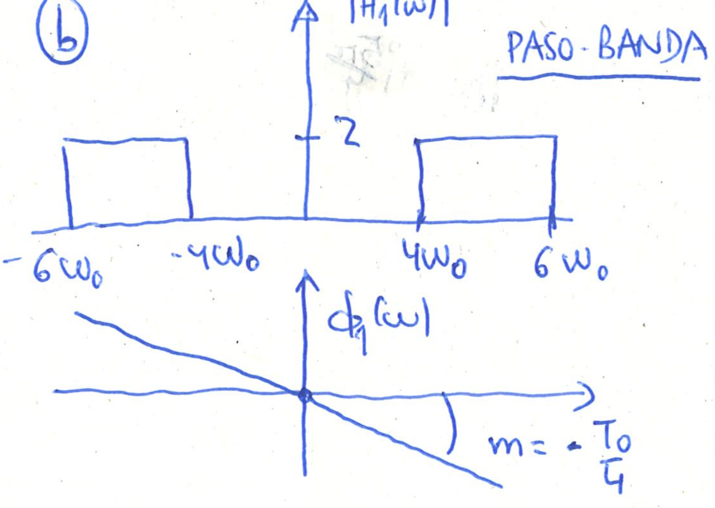
Que dibujado

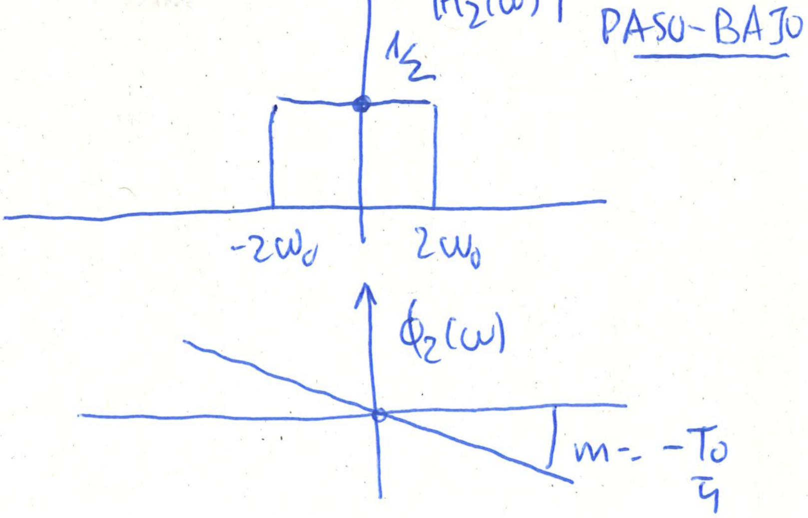


misma amplitud
fase cambiada de signo

$X(\omega) = X^*(-\omega)$ (real, simetria hermítica)

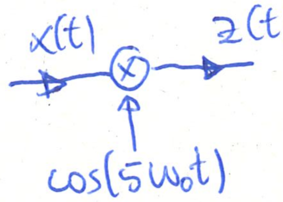
ⓑ





Además los dos filtros son reales $H(\omega) = H^*(-\omega)$!!

ⓒ $z(t) = x(t) \cdot \cos(5\omega_0 t)$

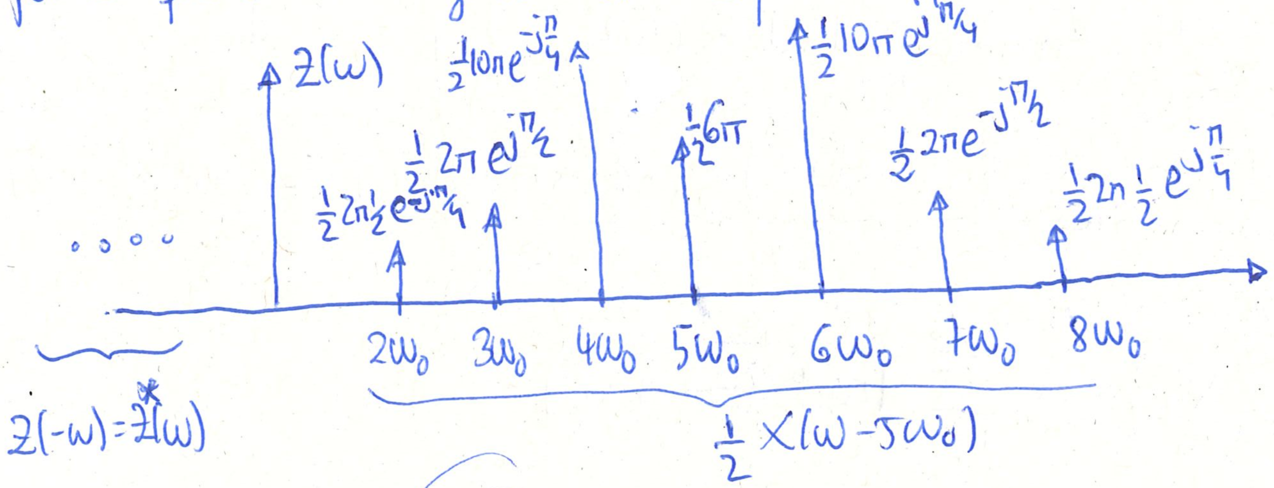


por la propiedad de modulación:

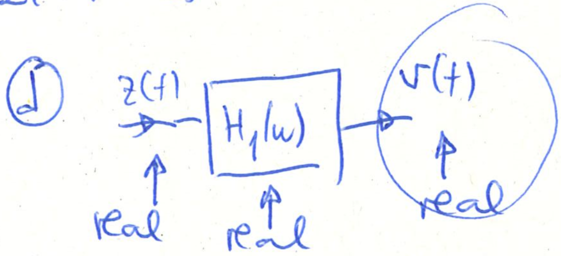
$$\boxed{Z(\omega) = \frac{1}{2}\left[X(\omega - 5\omega_0) + X(\omega + 5\omega_0)\right]}$$

Además si $x(t)$ es real $\rightarrow$ $z(t) = \underbrace{x(t)}_{\text{real}} \cdot \underbrace{\cos(5\omega_0 t)}_{\text{real}}$ es real

por lo que solo dibujamos su espectro por $\omega \geq 0$



ⓓ



$V(\omega) = H_1(\omega) \cdot Z(\omega)$

$|H_1(\omega)| = 2$ $\rightarrow$ amplitudes

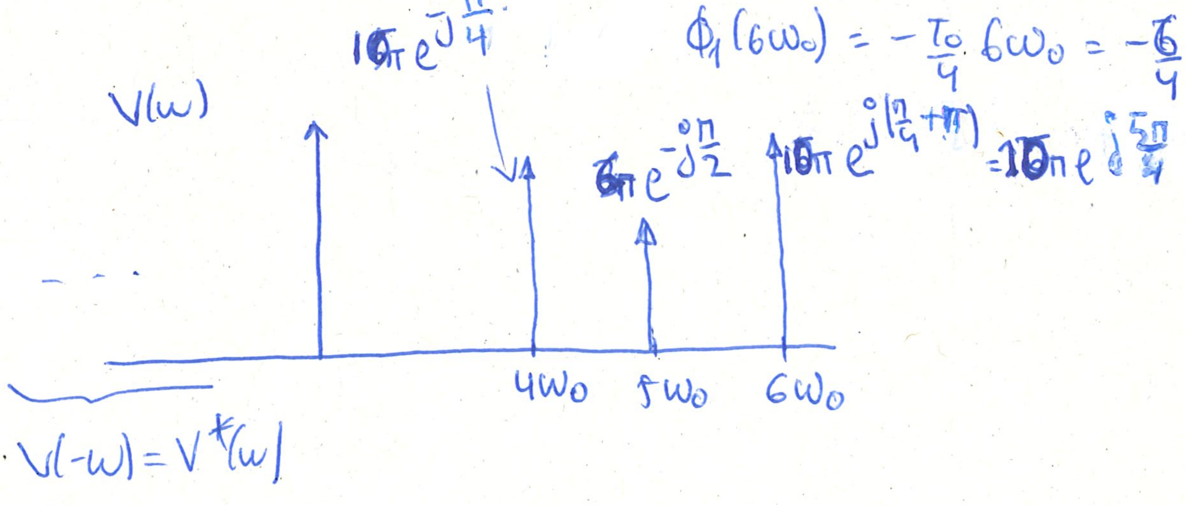
Como solo pasan las frecuencias entre $4\omega_0$ y $6\omega_0$

$\phi_1(\omega) = -\frac{T_0}{4}\omega$

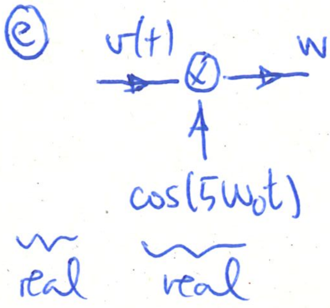
$\phi_1(4\omega_0) = -\frac{T_0}{4} \cdot 4\omega_0 = -2\pi \equiv 0$

$\phi_1(5\omega_0) = -\frac{T_0}{4} 5\omega_0 = -\frac{5}{4} 2\pi \equiv -\frac{\pi}{2}$

$\phi_1(6\omega_0) = -\frac{T_0}{4} 6\omega_0 = -\frac{6}{4} \cdot 2\pi \equiv \pi$

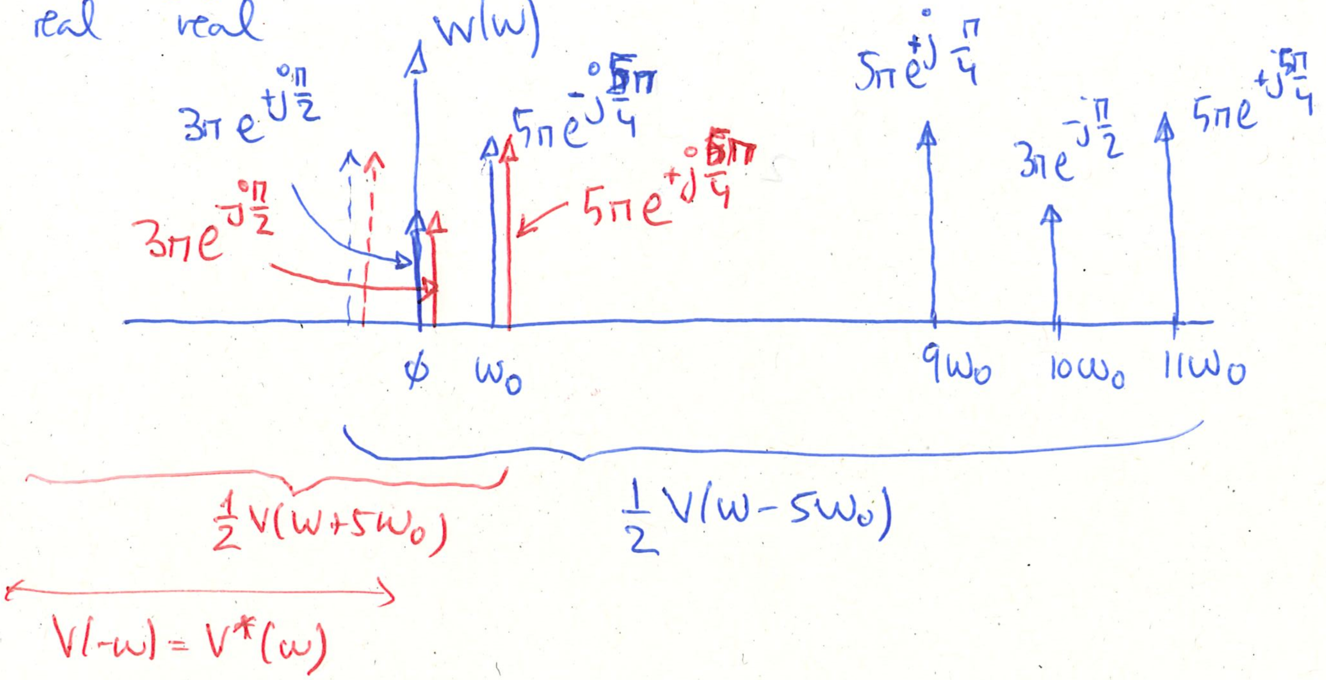


ⓔ

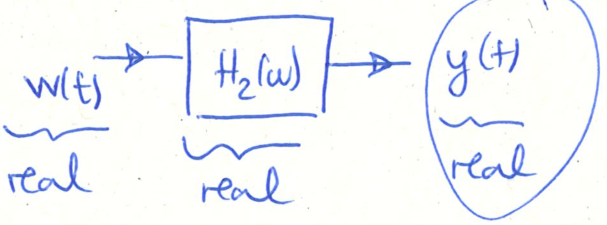


$\underbrace{w(t)}_{real} = v(t)\cdot\cos(5\omega_0 t)$

$$W(\omega) = \frac{1}{2}\left[V(\omega-5\omega_0) + V(\omega+5\omega_0)\right]$$



ⓕ En la última etapa



$Y(\omega) = H_2(\omega)\cdot W(\omega)$

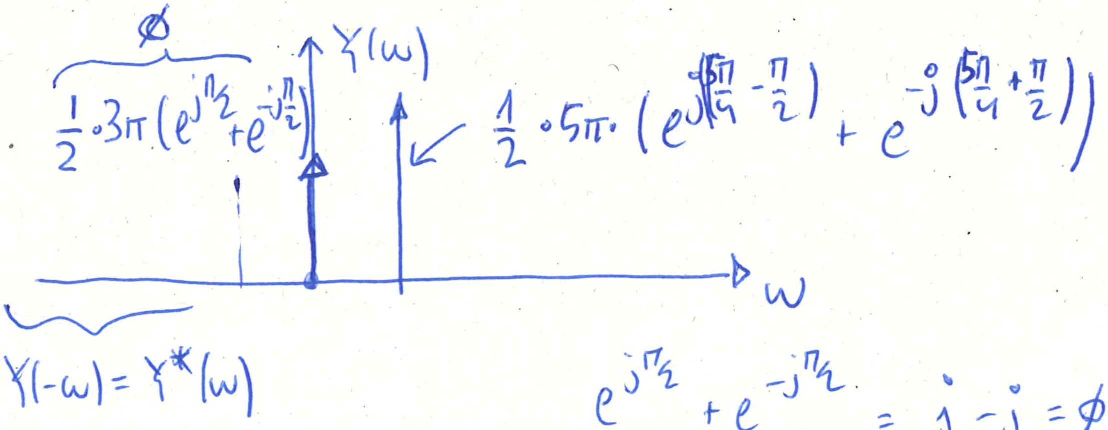
$|H_2(\omega)| = \frac{1}{2}$

$\Phi_2(\omega) = -\frac{T_0}{4}\cdot\omega$

$\Phi_2(0) = 0$

$\Phi_2(\omega_0) = -\frac{T_0}{4}\omega_0 = -\frac{2\pi}{4} = -\frac{\pi}{2}$

Solo deja pasar $|\omega| \leq 2\omega_0$

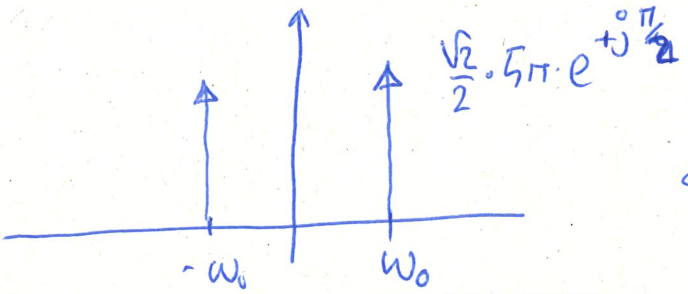


$$e^{j\pi/2} + e^{-j\pi/2} = j - j = 0$$

$$e^{+j\frac{3\pi}{4}} + \underbrace{e^{-j\frac{7\pi}{4}}}_{e^{j\pi/4}} = -\frac{\sqrt{2}}{2} + j\frac{\sqrt{2}}{2} + \left(+\frac{\sqrt{2}}{2} + j\frac{\sqrt{2}}{2}\right) = +j\sqrt{2}$$

$$= \sqrt{2}\, e^{+j\pi/2}$$

Por lo que:



← coseno!!

$$\boxed{y(t) = \frac{5\sqrt{2}}{2} \cdot \cos\left(w_0 t + \frac{\pi}{2}\right)}$$