TRATAMIENTO DE SEÑALES: PRIMER PARCIAL

La puntuación total del examen es de 30 puntos divididos en:

Problema 1: 10 puntos. Todas las cuestiones tienen el mismo peso.

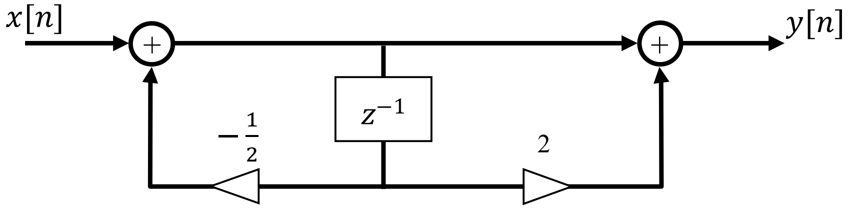
Problema 2: 10 puntos.

Problema 3: 10 puntos.

El tiempo estimado para resolver el examen es de dos horas.

PROBLEMA 1 (10 puntos, 40 minutos)

1. Un sistema LTI tiene la siguiente respuesta impulsional: $h(t) = e^{-t}u(t)$. Calcula la respuesta $y(t)$ si la entrada es: $x(t) = e^{-2t}u(t) - \delta(t)$.
2. Para las siguientes señales se pide:
   a. Dibuja la señal y calcula su energía: $x(t) = e^{-\alpha|t|}$, con $\alpha > 0$.
   b. Dibuja la señal y calcula su potencia media: $x[n] = A$
3. Se dispone del sistema cuya implementación en forma directa II es la de la figura. Filtra la siguiente señal de entrada mediante dicho sistema: $x[n] = \{\underline{2}, 1, 0, 2\}$ (toma $y[-1] = 0$).



PROBLEMA 2 (10 puntos, 40 minutos)

Sea el sistema descrito por la siguiente ecuación en diferencias para el que se consideran condiciones iniciales nulas:

$$y[n] - \alpha^2 y[n-2] = x[n], \text{ con } \alpha \text{ constante y real}$$

Se pide:

a. Indicar tipo de sistema y orden. **(1 p)**

b. Representar la implementación del sistema mediante un diagrama de bloques. **(1 p)**

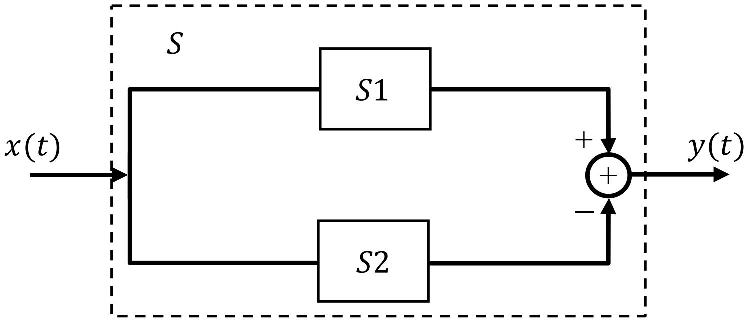
c. Filtrar la señal $x[n] = \{-\underline{1}, 2, 1, 0, 3, -1\}$ **(3 p)**

d. Calcular la respuesta impulsional del sistema. **(3 p)**

e. Determinar si el sistema es causal y obtener la condición que debe cumplir α para que el sistema sea estable. **(2 p)**

PROBLEMA 3 (10 puntos, 40 minutos)

Sea el sistema de la figura:



Con:

S1, $y_1(t) = \int_{-\infty}^{t} x(\tau + 2)d\tau$

S2, $y_2(t) = \int_{-\infty}^{t} x(\tau - 1)d\tau$

Se pide:

a. Estudiar la linealidad e invarianza de S1 y S2. **(1 p)**

b. Calcular y dibujar la respuesta impulsional de S1, S2 y del sistema completo S. **(2 p)**

c. Estudiar la causalidad y estabilidad del S1, S2 y S a partir de su respuesta impulsional. **(1 p)**

d. Calcular la salida para una entrada $x(t) = \prod\left(\frac{t}{3}\right)$. Nota: $\prod\left(\frac{t}{T}\right) * \prod\left(\frac{t}{T}\right) = T \wedge \left(\frac{t}{T}\right)$. **(2 p)**

Los sistemas S1 y S2 se conectan ahora en serie. Se pide:

e. Calcular la respuesta impulsional del sistema resultante, empezar calculando $u(t) * u(t)$. **(2 p)**

f. Determinar si el sistema es causal y estable. **(1 p)**

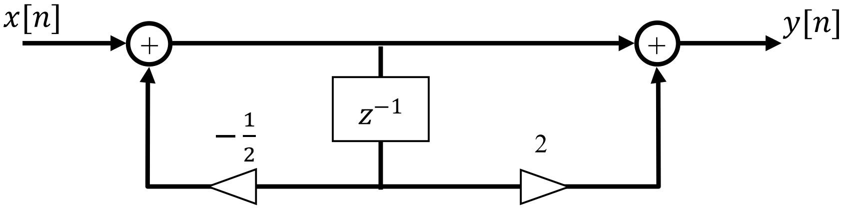
g. Calcular y dibujar la salida del sistema para una entrada $x(t) = \delta(t-1) - \delta(t-4)$ **(1 p)**

SEINALEEN PROZESATZEA: LEHEN PARTZIALA

Azterketak 3 ariketa ditu. Ariketa bakoitzak 10 puntu balio ditu, eta 1. ariketako galdera guztiek pisu berdina dute. Bi ordu dituzue.

1. ARIKETA (10 puntu, 40 minutu)

1. Izan bedi ondoko pultsu-erantzuna duen LTI sistema: $h(t) = e^{-t}u(t)$. Kalkulatu erentzuna $y(t)$ sarrera hau denean: $x(t) = e^{-2t}u(t) - \delta(t)$.
2. Hurrengo seinaleetarako honakoak erantzun:
   a. Irudikatu seinalea, eta ondoren kalkulatu bere energia: $x(t) = e^{-\alpha|t|}$, con $\alpha > 0$.
   b. Irudikatu seinalea, eta ondoren kalkulatu bere potentzia: $x[n] = A$
3. Sistema baten II. forma zuzeneko inplementazioa irudian adierazitakoa da. Iragazi hurrengo seinalea sistema hori erabilita: $x[n] = \{\underline{2}, 1, 0, 2\}$ (hartu $y[-1] = 0$).



2. ARIKETA (10 puntu, 40 minutu)

Izan bedi hurrengo diferentzia ekuazioa duen sistema, zeinek hasierako baldintzak nuluak dituen:

$$y[n]-\alpha^2 y[n-2]=x[n], \text{ non } \alpha \text{ konstante erreala den}$$

Honakoak eskatzen dira:

a. Sistemaren mota eta maila. **(1 p)**

b. Irudikatu sistemaren inplementazioa bloke-diagrama baten bidez. **(1 p)**

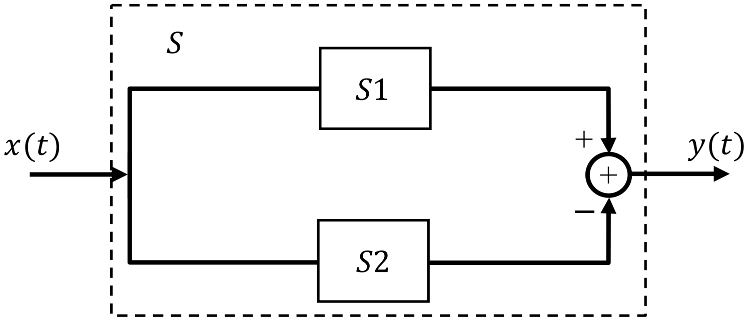
c. Iragazi hurrengo seinalea $x[n]=\{-\underline{1},2,1,0,3,-1\}$ **(3 p)**

d. Kalkulatu sistemaren pultsu-erantzuna. **(3 p)**

e. Aztertu sistema kausala den, eta lortu α konstanteak bete behar duen baldintza sistema egonkorra izan dadin. **(2 p)**

3. ARIKETA (10 puntu, 40 minutu)

Izan bedi irudiko sistema:



Non:

S1, $y_1(t)=\int_{-\infty}^{t} x(\tau+2)d\tau$

S2, $y_2(t)=\int_{-\infty}^{t} x(\tau-1)d\tau$

Honakoak eskatzen dira:

a. Aztertu S1 eta S2 sistemak linealak edota denboran aldakorrak diren. **(1 p)**

b. Kalkulatu eta irudikatu S1, S2 eta S sistema osoaren pultsu-erantzunak. **(2 p)**

c. Aztertu S1, S2 eta S kausalak edota egonkorrak diren pultsu-erantzunak erabiliz. **(1 p)**

d. Kalkulatu irteera, sarrera hau bada: $x(t)=\prod\left(\frac{t}{3}\right)$. Oharra: $\prod\left(\frac{t}{T}\right)*\prod\left(\frac{t}{T}\right)=T\wedge\left(\frac{t}{T}\right)$. **(2 p)**

Orain S1 eta S2 sistemak seriean konektatzen dira. Honakoak eskatzen dira:

e. Sistema osoaren pultsu-erantzuna. (Hasi $u(t)*u(t)$ kalkulatuz). **(2 p)**

f. Aztertu sistema kausala edota egonkorra den. **(1 p)**

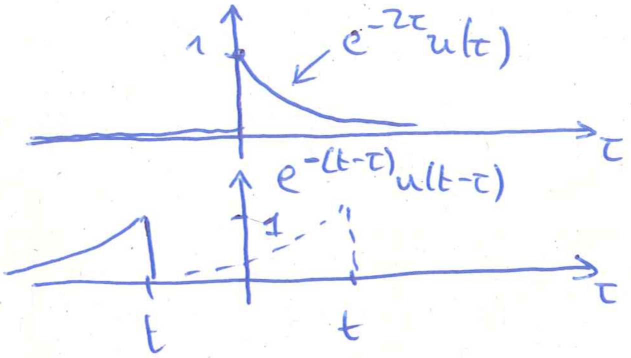
g. Kalkulatu eta irudikatu irteera, sarrera hau denean: $x(t)=\delta(t-1)-\delta(t-4)$. **(1 p)**

① Sistema LTI:

$$x(t)\rightarrow \boxed{h(t)} \rightarrow y(t)=h(t)*x(t)=\underbrace{e^{-t}\cdot u(t)}_{h(t)} * \underbrace{[e^{-2t}u(t)-\delta(t)]}_{x(t)} \overset{\text{distributiv.}}{=}$$

$$= e^{-t}u(t)*e^{-2t}u(t) - \underbrace{e^{-t}\cdot u(t)*\delta(t)}_{\text{elem. identidad}} = e^{-t}u(t)*e^{-2t}u(t)-e^{-t}u(t)$$

Calculemos $y_1(t)=e^{-t}u(t)*e^{-2t}u(t) = (e^{-t}-e^{-2t})u(t)$



$t<0$ $\quad y_1(t)=0$ sin solape

$$t>0 \quad y_1(t)=\int_{\underbrace{0}_{\text{solape}}}^{t} e^{-2\tau}\cdot e^{-(t-\tau)}\cdot d\tau$$

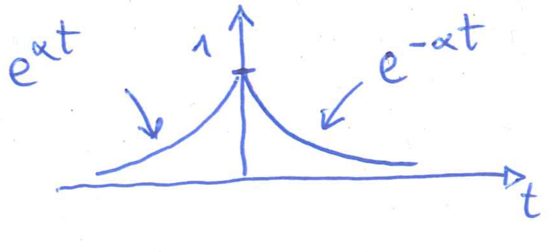
$$=\int_0^t e^{-\tau}\cdot e^{-t}\,d\tau=e^{-t}\cdot\frac{e^{-\tau}}{-1}\Big]_0^t=$$

$$=e^{-t}(1-e^{-t})$$

Por lo tanto $y_1(t)=(-e^{-2t}+e^{-t})u(t)$

$$y(t)=-e^{-2t}u(t)+e^{-t}\cancel{u(t)}-e^{-t}\cancel{u(t)}=\boxed{-e^{-2t}u(t)=y(t)}$$

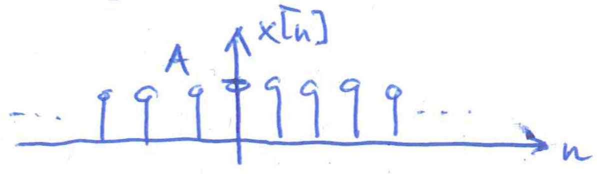
② ⓐ $x(t)=e^{-\alpha|t|}$ $\quad \alpha>0$ $\quad x(t)=\begin{cases}e^{-\alpha t} & t\geq 0\\ e^{\alpha t} & t<0\end{cases}$



$$E_x=\int_{-\infty}^{\infty}x^2(t)\,dt \underset{\substack{\uparrow\\ \text{simétrica}}}{=} 2\int_0^{\infty}(e^{-\alpha t})^2 dt$$

$$E_x=2\int_0^{\infty}e^{-2\alpha t}dt=\cancel{2}\,\frac{e^{2\alpha t}}{-\cancel{2}\alpha}\Big]_0^{\infty}=\boxed{\frac{1}{\alpha}=E_x}$$

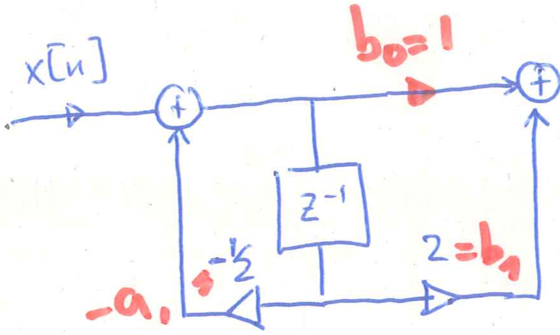
ⓑ $x[n]=A \quad \forall n$



$$P_x=\lim_{N\to\infty}\frac{1}{2N+1}\underbrace{\sum_{n=-N}^{N}}_{\substack{\text{se suma el mismo valor}\\ \text{2N+1 veces}}}\overset{\overset{A^2}{''}}{x^2[n]}$$

$$P_x = \lim_{N\to\infty} \frac{1}{\cancel{2N}+1} \cdot (\cancel{2N}+1)\cdot A^2 = \boxed{A^2 = P_x}$$

③



$b_0=1$
$b_1=2$
$a_1=\frac{1}{2}$
} identificando términos

Forma directa II por lo tanto.

$$y[n] = \underset{1}{b_0}\, x[n] + \underset{2}{b_1}\, x[n-1] - \underset{1/2}{a_1}\, y[n-1] = x[n] + 2x[n-1] - \frac{1}{2} y[n-1]$$

$$\boxed{y[n] = x[n] + 2x[n-1] - \frac{1}{2} y[n-1]}$$

$x[n] = \{\underline{2}, 1, 0, 2\}$

$y[-1] = 0$

$$y[0] = x[0] + 2\cancel{x[-1]}^{0} - \frac{1}{2} \cancel{y[-1]}^{0} = 2$$

$$y[1] = x[1] + 2x[0] - \frac{1}{2} y[0] = 1 + 2\cdot 2 - \frac{1}{2}\cdot 2 = 4$$

$$y[2] = x[2] + 2x[1] - \frac{1}{2} y[1] = 0 + 2\cdot 1 - \frac{1}{2}\cdot 4 = 0$$

$$y[3] = x[3] + 2x[2] - \frac{1}{2} y[2] = 2 + 2\cdot 0 - \frac{1}{2}\cdot 0 = 2$$

$$\boxed{y[n] = \{\underline{2}, 4, 0, 2\}}$$

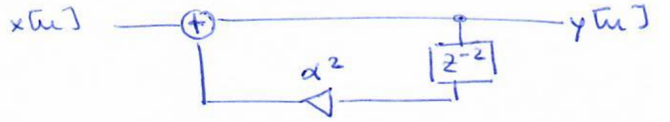
2. ARIKETA

$y[n] - \alpha^2 y[n-2] = x[n]$ non $\alpha = kte$ erreala

a) $y[n] = x[n] + \alpha^2 y[n-2]$

Sistema realimentado, $y[n-2]$, por lo que es un sistema IIR, de orden 2, que es el máximo retardo que aplica.

b)



c) Iragazi: $x[n] = \{-\underline{1}, 2, 1, 0, 3, -1\}$

Hasierako baldintzak nuluak: $y[-1] = y[-2] = \emptyset$

$y[0] = x[0] + \alpha^2 \cancel{y[-2]} = -1$

$y[1] = x[1] + \alpha^2 \cancel{y[-1]} = 2$

$y[2] = x[2] + \alpha^2 y[0] = 1 + \alpha^2(-1) = 1 - \alpha^2$

$y[3] = x[3] + \alpha^2 y[1] = 0 + \alpha^2 [2] = 2\alpha^2$

$y[4] = x[4] + \alpha^2 y[2] = 3 + \alpha^2 (1-\alpha^2) = 3 + \alpha^2 - \alpha^4$

$y[5] = x[5] + \alpha^2 y[3] = -1 + \alpha^2 \cdot 2\alpha^2 = -1 + 2\alpha^4$

Se obtien 6 muestras en la salida: $y[n] = \{-1, 2, 1-\alpha^2, 2\alpha^2, 3+\alpha^2-\alpha^4, -1+2\alpha^4\}$

d) $h[n] = \delta[n] + \alpha^2 h[n-2]$ non $x[n] = \delta[n] \Rightarrow y[n] = h[n]$

$h[0] = \delta[0] = 1$

$h[1] = 0$

$h[2] = \alpha^2 h[0] = \alpha^2$

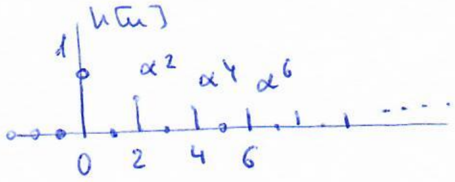
$h[3] = \alpha^2 h[1] = 0$

$h[4] = \alpha^2 h[2] = \alpha^4$

$h[5] = 0$

$h[6] = \alpha^6$

$\vdots \qquad \vdots$



$$h[n] = \sum_{K=\emptyset}^{\infty} \alpha^{2K} \delta[n-2K]$$

e) El sistema es causal porque $h[n] = \emptyset \ \forall n < \emptyset$

Para que sea estable: $\sum_n |h[n]| < \infty$

$$\sum_n |h[n]| = \sum_{k=0}^{\infty} \alpha^{2k} = 1 + \alpha^2 + \alpha^4 + \alpha^6 + \ldots \underset{\uparrow}{=} \frac{1}{1-\alpha^2} \quad \text{con} \quad \begin{matrix} |\alpha^2| \leq 1 \\ |\alpha| < 1 \end{matrix}$$

Serie geométrica
razon $= \alpha^2$, converge si $|z| < 1$

Problema 3

a. Linealidad

$$x(t)=ax_1(t)+bx_2(t) \longrightarrow y(t)=ay_1(t)+by_2(t)$$

con $y_1(t)=S\{x_1(t)\}$
$y_2(t)=S\{x_2(t)\}$

- S1: $y(t)=\int_{-\infty}^{t} x(\tau+2)\,d\tau$

Si fuera lineal, $y(t) \underset{\substack{\uparrow \\ x(t)=ax_1(t)+bx_2(t)}}{=} a\int_{-\infty}^{t} x_1(\tau+2)\,d\tau + b\int_{-\infty}^{t} x_2(\tau+2)\,d\tau \quad (1)$

La respuesta del sistema S1 es:

$$y'(t)=\int_{-\infty}^{t}\left[ax_1(\tau+2)+bx_2(\tau+2)\right]d\tau \underset{\substack{\uparrow \\ \text{Integral,} \\ \text{operador lineal}}}{=} a\int_{-\infty}^{t} x_1(\tau+2)\,d\tau + b\int_{-\infty}^{t} x_2(\tau+2)\,d\tau \quad (2)$$

Ambas expresiones (1) y (2) coinciden $\Rightarrow$ $\boxed{\text{S1 es lineal}}$

- S2: $y(t)=\int_{-\infty}^{t} x(\tau-1)\,d\tau$ → Siguiendo el mismo razonamiento $\Rightarrow$

$\Rightarrow$ $\boxed{\text{S2 es lineal}}$

Invarianza

$x(t) \longrightarrow y(t)$
$x(t-t_0) \longrightarrow y(t-t_0)$

- S1: Si fuera invariante, la respuesta sería:

$$y(t-t_0)=y(t)\Big|_{t=t-t_0} = \int_{-\infty}^{t-t_0} x(\tau+2)\,d\tau \quad (1)$$

La respuesta del sistema S1 es:

$$y'(t)=\int_{-\infty}^{t} x(\underbrace{\tau-t_0}_{\tau'}+2)\,d\tau \underset{\downarrow}{=} \int_{-\infty}^{t-t_0} x(\tau'+2)\,d\tau' \quad (2)$$

$\tau'=\tau-t_0$ $\quad$ $\tau\to-\infty$ $\quad$ $\tau'\to-\infty$
$d\tau'=d\tau$ $\quad$ $\tau=t$ $\quad$ $\tau'=t-t_0$

Coinciden (1) y (2)
$\Downarrow$
$\boxed{\text{S1 es INVARIANTE}}$

a. Invarianza (continuación)

- S2 $y(t-t_0) = \int_{-\infty}^{t-t_0} x(\tau-1)\,d\tau$. De forma similar:

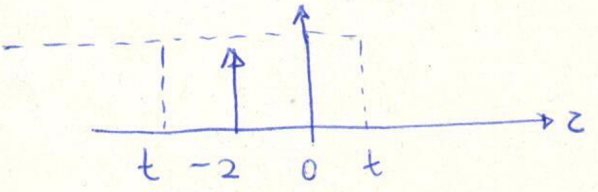
$$y'(t) = \int_{-\infty}^{t} x(\underbrace{\tau-t_0-1}_{\tau'})\,d\tau \underset{\downarrow}{=} \int_{-\infty}^{t-t_0} x(\tau'-1)\,d\tau'$$

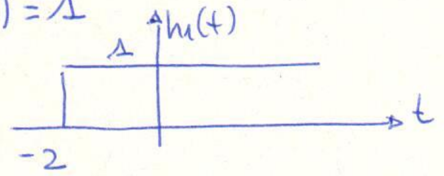
Cambio de variable $\tau' = \tau - t_0$

$\boxed{\text{S2 es INVARIANTE}}$

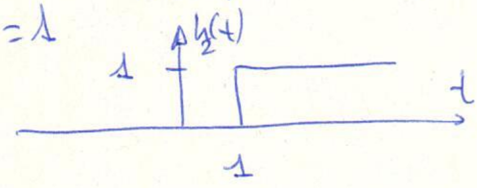
Por tanto, $\boxed{\text{S1 y S2 sistemas SLI}}$

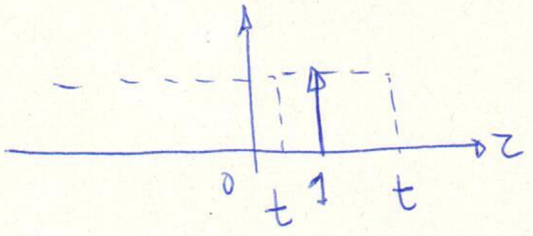
b. • S1: $h_1(t) = \int_{-\infty}^{t} \delta(\tau+2)\,d\tau = \begin{cases} t<-2 & h_1(t)=0 \\ t\geq 2 & h_1(t)=1 \end{cases}$ $h_1(t) = u(t+2)$



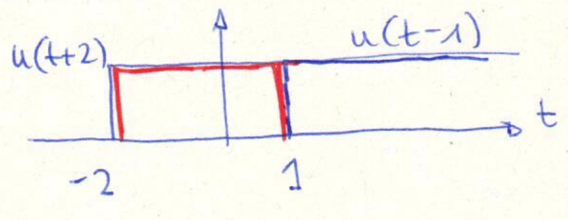


• S2: $h_2(t) = \int_{-\infty}^{t} \delta(\tau-1)\,d\tau = \begin{cases} t<1 & h_2(t)=0 \\ t\geq 1 & h_2(t)=1 \end{cases}$ $h_2(t) = u(t-1)$





$$S : h(t) = h_1(t) - h_2(t) = u(t+2) - u(t-1) = \Pi\left(\frac{t+1/2}{3}\right)$$



c. Causalidad $\Rightarrow h(t) = 0 \;\; \forall t < 0$

A partir de las representaciones gráficas:

S1	no causal
S2	causal
S	no causal

Estabilidad $\Rightarrow \int_{-\infty}^{\infty} |h(t)|\,dt < \infty$

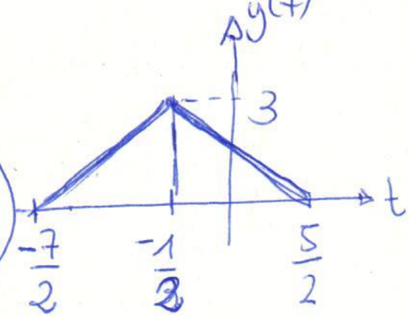
- S1 $\Rightarrow \int_{-\infty}^{\infty} u(t+2)\,dt = \int_{-2}^{\infty} 1 \cdot dt = t\Big]_{-2}^{\infty} \rightsquigarrow \infty$ INESTABLE
- S2 $\Rightarrow \int_{-\infty}^{\infty} u(t-1)\,dt = \int_{1}^{\infty} 1 \cdot dt = t\Big]_{1}^{\infty} \rightsquigarrow \infty$ INESTABLE
- S $\Rightarrow \int_{-\infty}^{\infty} \Pi\left(\frac{t+1/2}{3}\right) dt = \int_{-2}^{1} 1 \cdot dt = 3$ ESTABLE

d. $x(t)=\Pi\left(\frac{t}{3}\right)$ $\quad y(t)=\Pi\left(\frac{t}{3}\right)*\Pi\left(\frac{t+1/2}{3}\right)$

$y'(t)=\Pi\left(\frac{t}{3}\right)*\Pi\left(\frac{t}{3}\right)$; $y(t)=y'(t+1/2)$

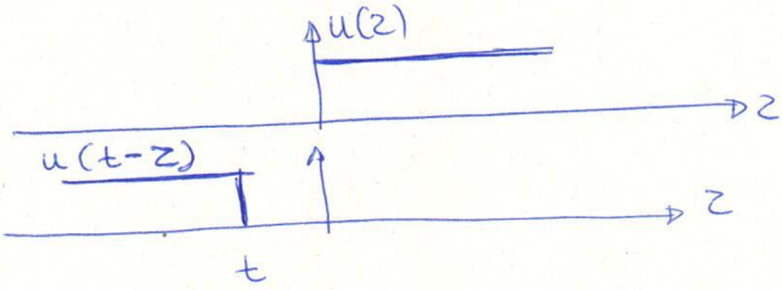
Aplicando $\Pi\left(\frac{t}{T}\right)*\Pi\left(\frac{t}{T}\right)=T\Lambda\left(\frac{t}{T}\right)$

$y'(t)=3\Lambda\left(\frac{t}{3}\right)$; $\underline{\underline{y(t)=3\Lambda\left(\frac{t+1/2}{3}\right)}}$



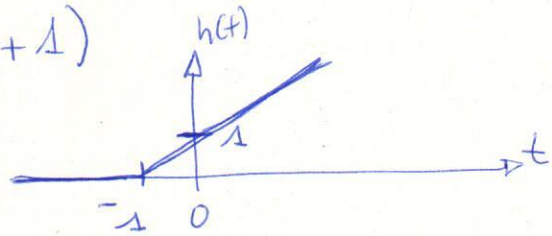
e. S1 y S2 conectados en serie: $h(t)=h_1(t)*h_2(t)=$
$=u(t+2)*u(t-1)$

$z(t)=u(t)*u(t) \Rightarrow h(t)=z(t+1)$



$$z(t)=\int_{-\infty}^{\infty}u(\tau)\cdot u(t-\tau)\,d\tau=\left|\begin{array}{ll} t<0 & z(t)=0 \\ t\geq 0 & z(t)=\int_0^t 1\,d\tau=t \end{array}\right|=t\cdot u(t)$$

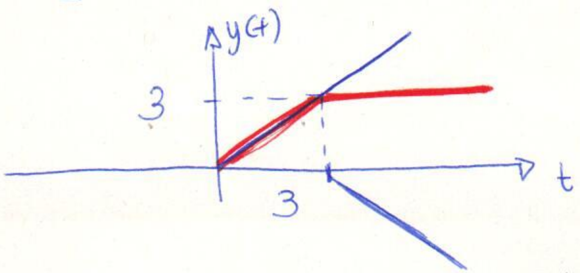
$h(t)=z(t+1)=(t+1)\,u(t+1)$



f. Causal y estable?

- No causal $h(t)\neq 0\ \forall t<0$
- INESTABLE $\int_{-\infty}^{\infty}|h(t)|\,dt=\int_{-1}^{\infty}(t+1)\,dt \longrightarrow \infty$

g. $x(t)=\delta(t-1)-\delta(t-4) \Rightarrow \Big[y(t)=x(t)*h(t)=$
$=\left[\delta(t-1)-\delta(t-4)\right]*(t+1)u(t+1)=t\cdot u(t)-(t-3)u(t-3)\Big]$



SIGNAL PROCESSING: FIRST MID-TERM

The exam scores a total of 30 points divided as follows:

Problem 1: 10 points. All questions have equal weight.

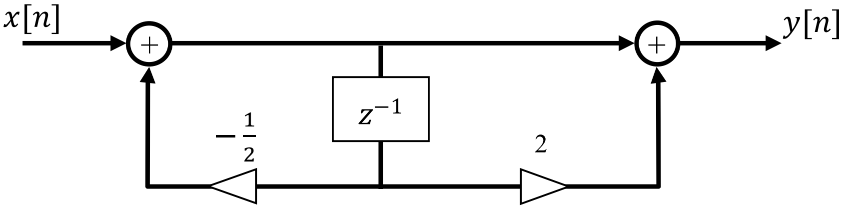
Problem 2: 10 points.

Problem 3: 10 points.

The estimated time to complete the exam is 2 hours.

PROBLEM 1 (10 points, 40 minutes)

1. A LTI system has the following impulse-response: $h(t) = e^{-t}u(t)$. Compute the response $y(t)$ if the input signal is: $x(t) = e^{-2t}u(t) - \delta(t)$.
2. For the following signals:
    a. Sketch the signal, then compute its energy: $x(t) = e^{-\alpha|t|}$, con $\alpha > 0$.
    b. Sketch the signal, then compute its power: $x[n] = A$
3. The figure shows the implementation of a system in direct form II. Filter the following signal using the system: $x[n] = \{\underline{2}, 1, 0, 2\}$ (take $y[-1] = 0$).



PROBLEM 2 (10 points, 40 minutes)

Consider the system described by the following difference equation, in which the initial conditions are zero:

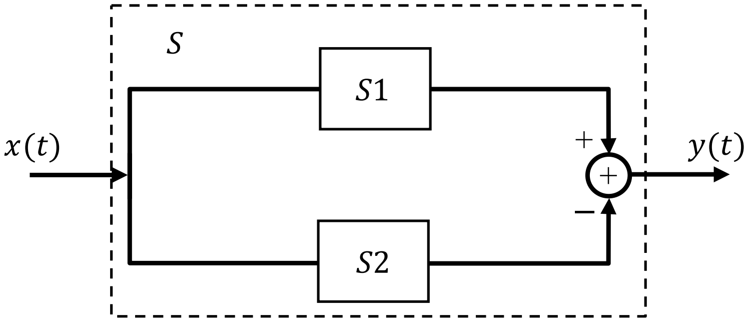
$$y[n] - \alpha^2 y[n-2] = x[n], \text{ where } \alpha \text{ is a real constant}$$

Answer the following questions:

a. Type and order of the system. **(1 p)**
b. Sketch the implementation of the system using a block diagram. **(1 p)**
c. Filter the following signal $x[n] = \{-\underline{1}, 2, 1, 0, 3, -1\}$ **(3 p)**
d. Compute the impulse-response of the system. **(3 p)**
e. Determine whether the system is causal and obtain the condition in α for the system to be stable. **(2 p)**

PROBLEM 3 (10 points, 40 minutes)

Consider the following system:



Where:

S1, $y_1(t) = \int_{-\infty}^{t} x(\tau + 2)d\tau$

S2, $y_2(t) = \int_{-\infty}^{t} x(\tau - 1)d\tau$

Answer the following questions:

a. Study the linearity and time-invariance of S1 and S2. **(1 p)**
b. Compute and sketch the impulse-response of S1, S2 and the complete system S. **(2 p)**
c. Study the causality and stability of S1, S2 and S using their impulse-responses. **(1 p)**
d. Compute the output if the input is $x(t) = \prod\left(\frac{t}{3}\right)$. Note: $\prod\left(\frac{t}{T}\right) * \prod\left(\frac{t}{T}\right) = T \wedge \left(\frac{t}{T}\right)$. **(2 p)**

Systems S1 and S2 are now connected in series. Answer the following questions:

e. Compute the impulse-response of the resulting system, start by computing $u(t) * u(t)$. **(2 p)**
f. Determine whether the system is causal and/or stable. **(1 p)**
g. Compute and sketch the output for the following input: $x(t) = \delta(t-1) - \delta(t-4)$. **(1 p)**

TRATAMIENTO DE SEÑALES
Convocatoria ordinaria. Primer parcial

La puntuación total del examen es de 30 puntos divididos en:

Problema 1: 10 puntos. Todas las cuestiones tienen el mismo peso.

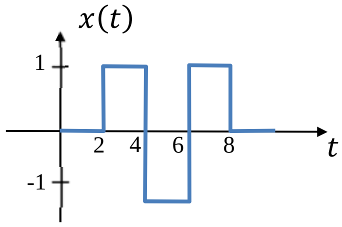
Problema 2: 10 puntos.

Problema 3: 10 puntos.

El tiempo estimado para resolver el examen es de dos horas.

PROBLEMA 1 (10 puntos, 30 minutos)

1. Sea la señal $x(t)$ de la figura. Se pide:



   a. Expresión analítica de x(t) en función del pulso rectangular $\prod(t)$.
   b. Se sabe que dicha señal es la entrada de un sistema LTI cuya respuesta impulsional es $h(t) = \prod\left(\frac{t-1}{2}\right)$. Obtener la expresión analítica de la respuesta $y(t)$ y representarla gráficamente. NOTA: $\Lambda\left(\frac{t}{T}\right) = \frac{1}{T}\prod\left(\frac{t}{T}\right) * \prod\left(\frac{t}{T}\right)$

2. Analizar si los siguientes sistemas son causales y/o estables:
   a. Sistema con la siguiente relación entrada-salida

$$y(t) = x(2t-1)$$

   b. Sistema LTI con la siguiente respuesta impulsional

$$h(t) = 3 \cdot \Lambda\left(-\frac{t}{2} + 1\right)$$

3.

4. Sea un sistema LTI discreto tipo FIR del que se conoce que para una entrada $x_1[n] = \{\underline{1}, 3, 5\}$, la respuesta del sistema es $y_1[n] = \{\underline{0}, 0, 0, 2, 6, 10\}$. Sea ahora la secuencia $x_2[n] = [\underline{0}, -1, -3, -4, 3, 5]$. Se pide:
   a. Respuesta impulsional del sistema
   b. Expresar $x_2[n]$ en función de $x_1[n]$.
   c. Obtener la respuesta del sistema ante la entrada $x_2[n]$.

PROBLEMA 2 (10 puntos, 30 minutos)

a) Determina si las siguientes señales son periódicas, y en caso de serlo calcula el periodo fundamental: (5 ptos.)

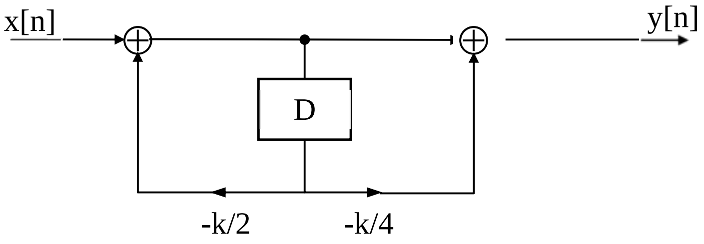
- $x(t) = 5\cos(\sqrt{3}\,t - \frac{\pi}{4})$
- $x(t) = 3\sin(\frac{\pi}{4}t) - 5\cos(\frac{\pi}{3}t - \frac{\pi}{2})$
- $x(t) = \cos(\pi t) + \sin(5t)$
- $x[n] = \cos(\frac{3\pi}{4}n + \frac{\pi}{4})$
- $x[n] = \sin(\frac{\pi}{2}n) - 3\cos(\frac{\pi}{3}n + \frac{\pi}{4})$

b) Demostrar que para una señal continua periódica de periodo fundamental T_0, la potencia media puede expresarse como: $P_x = \frac{1}{T_0}\int_{T_0}|x(t)|^2 dt$ (3 ptos.)

c) Usa el resultado anterior para calcular la potencia media de: $x(t) = 5\cos(\sqrt{3}\,t - \frac{\pi}{4})$ (2 ptos.)

PROBLEMA 3 (10 puntos, 30 minutos)

Se trata de analizar el siguiente esquema presentado en la forma directa II:



Siendo D, un elemento retardador de 1 muestra.

a) Encontrar la ecuación en diferencias que relaciona y[n] con x[n]. Indicar tipo y orden del sistema. (3 ptos.)

b) Calcular la respuesta impulsional del sistema, h[n]. (3 ptos.)

c) ¿Qué condición ha de satisfacer k para que el sistema sea estable? (2 ptos.)

d) Filtrar la señal $x[n]=\{\underline{-1},2,1,3\}$ con k=1 para obtener y[n]. (2 ptos.)

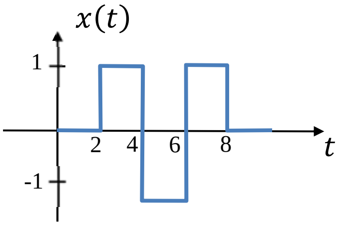
Telekomunikazio Teknologiaren Ingeniaritzako Gradua
2017-2018

SEINALEEN PROZESATZEA
Ohiko deialdia. Lehen partziala

Azterketak 3 ariketa ditu. Ariketa bakoitzak 10 puntu balio ditu, eta 1. ariketako galdera guztiek pisu berdina dute. Bi ordu dituzue.

1. ARIKETA (10 puntu, 30 minutu)

1. Izan bedi irudiko $x(t)$ seinalea. Honakoak eskatzen dira:



   a. $\mathrm{x(t)}$ seinalearen adierazpen analitikoa $\prod(t)$ pultsu laukizuzenaren menpe

   b. Aurreko seinalea $\mathrm{h(t)} = \prod\left(\frac{\mathrm{t}-1}{2}\right)$ pultsu-erantzuna duen LTI sistema baten sarrera-seinalea da. Lortu $\mathrm{y(t)}$ irteera-seinalearen adierazpide analitikoa eta irudikatu.

   Oharra: $\Lambda\left(\frac{t}{T}\right) = \frac{1}{T}\prod\left(\frac{t}{T}\right) * \prod\left(\frac{t}{T}\right)$

2. Aztertu honako sistemak kausalak edota egonkorrak diren:

   a. Honako sarrera-irteera erlazioa duen sistema:

$$y(t) = x(2t-1)$$

   b. Honako pultsu erantzuna duen LTI sistema:

$$h(t) = 3 \cdot \Lambda\left(-\frac{t}{2} + 1\right)$$

3. Izan bedi LTI sistema diskretua FIR motakoa, $x_1[n] = \{\underline{1}, 3, 5\}$ sarrera-seinaleari $y_1[n] = \{\underline{0}, 0, 0, 2, 6, 10\}$ erantzuna ematen duena. Izan bedi baita $x_2[n] = [\underline{0}, -1, -3, -4, 3, 5]$ sekuentzia. Honakoak eskatzen dira:

   a. Sistemaren pultsu-erantzuna.

   b. Adierazi $x_2[n]$ seinalea $x_1[n]$ seinalearen menpe.

   c. Lortu sistemaren erantzuna $x_2[n]$ seinaleari.

2. ARIKETA (10 puntu, 30 minutu)

a) Adieraz ezazu ondoko seinaleak periodikoak diren, eta periodikoak direnean kalkula ezazu oinarrizko periodoa: (5 puntu)

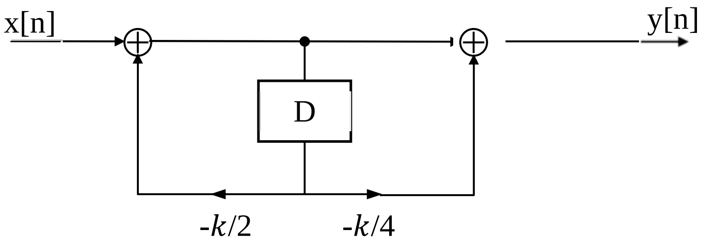
- $x(t) = 5\cos\left(\sqrt{3}\,t - \frac{\pi}{4}\right)$
- $x(t) = 3\sin\left(\frac{\pi}{4}t\right) - 5\cos\left(\frac{\pi}{3}t - \frac{\pi}{2}\right)$
- $x(t) = \cos(\pi t) + \sin(5t)$
- $x[n] = \cos(\frac{3\pi}{4}n + \frac{\pi}{4})$
- $x[n] = \sin\left(\frac{\pi}{2}n\right) - 3\cos(\frac{\pi}{3}n + \frac{\pi}{4})$

b) Froga ezazu T_0 periododun seinale jarraitu periodiko baten batezbesteko potentzia honela adieraz daitekeela: $P_x = \frac{1}{T_0}\int_{T_0}|x(t)|^2 dt$ (3 puntu)

c) Aurreko emaitza erabiliz ondoko seinalearen batezbesteko potentzia kalkulatu:
$x(t) = 5\cos\left(\sqrt{3}\,t - \frac{\pi}{4}\right)$ (2 puntu)

3. ARIKETA (10 puntu, 30 minutu)

Izan bedi irudiko sistema II. era zuzeneko egituran, non D lagin bateko atzeragailua den.



a) Lortu sistemaren diferentzia ekuazioa, y[n] eta x[n] seinaleak erlazionatzen dituena. Adierazi sistemaren mota eta maila. (3 puntu)

b) Lortu sistemaren pultsu-erantzuna, h[n]. (3 puntu)

c) Zer bete behar du k aldagaiak sistema egonkorra izateko? (2 puntu)

d) Iragazi x[n]={$\underline{-1}$,2,1,3} seinalea k=1 hartuz eta lortu y[n]. (2 puntu)

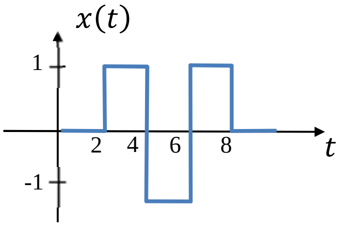
Telecommunications Engineering Degree
2017-2018

SIGNAL PROCESSING: Final exam
First mid-term

The estimated time to solve the exam are 1.5 hours.
The 3 short questions in problem 1 have all the same value.

PROBLEM 1 (10 points, 30 minutes)

1. Consider $x(t)$ the signal of the figure.



   a. Analytic expression of $x(t)$ in terms of the rectangular pulse, $\prod(t)$.

   b. We know $x(t)$ is the input signal to an LTI system with impulse response $h(t) = \prod\left(\frac{t-1}{2}\right)$. Obtain the analytic expression for the output $y(t)$ and sketch it graphically. NOTE: $\wedge\left(\frac{t}{T}\right) = \frac{1}{T}\prod\left(\frac{t}{T}\right) * \prod\left(\frac{t}{T}\right)$

2. Analyse whether the following systems are causal and/or stable:

   a. A system with the following input-output relation:

$$y(t) = x(2t-1)$$

   b. An LTI system with the following impulse response:

$$h(t) = 3 \cdot \Lambda\left(-\frac{t}{2}+1\right)$$

3. Consider an LTI FIR system, we know that for input $x_1[n] = \{\underline{1}, 3, 5\}$, the output is $y_1[n] = \{\underline{0}, 0, 0, 2, 6, 10\}$. Consider now $x_2[n] = [\underline{0}, -1, -3, -4, 3, 5]$.

   a. The system's impulse response.

   b. Express $x_2[n]$ in terms of $x_1[n]$.

   c. Obtain the output for input $x_2[n]$.

PROBLEM 2 (10 points, 30 minutes)

a) Determine which of the following signals are periodic, and if so compute their fundamental period: (5 points)

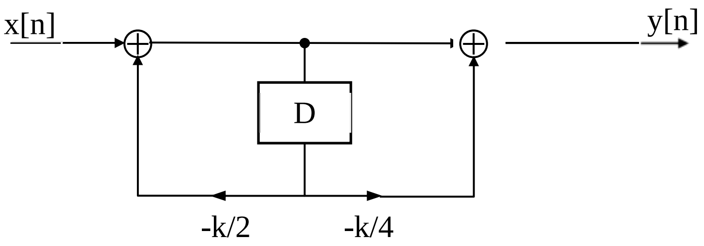
- $x(t) = 5\cos\left(\sqrt{3}\,t - \frac{\pi}{4}\right)$
- $x(t) = 3\sin\left(\frac{\pi}{4}t\right) - 5\cos\left(\frac{\pi}{3}t - \frac{\pi}{2}\right)$
- $x(t) = \cos(\pi t) + \sin(5t)$
- $x[n] = \cos\left(\frac{3\pi}{4}n + \frac{\pi}{4}\right)$
- $x[n] = \sin\left(\frac{\pi}{2}n\right) - 3\cos\left(\frac{\pi}{3}n + \frac{\pi}{4}\right)$

b) Prove that for a continuous periodic signal of fundamental period T_0, the average power can be expressed in the following way: $P_x = \frac{1}{T_0}\int_{T_0}|x(t)|^2 dt$ (3 points)

c) Use the previous result to compute the average power of: $x(t) = 5\cos\left(\sqrt{3}\,t - \frac{\pi}{4}\right)$ (2 points)

PROBLEM 3 (10 points, 30 minutes)

We will analyse the system of the figure which is in a direct form II implementation:



Where D is a one sample delay unit.

a) Find the difference-equation that relates y[n] with x[n]. What are the type and order of the system? (3 points)

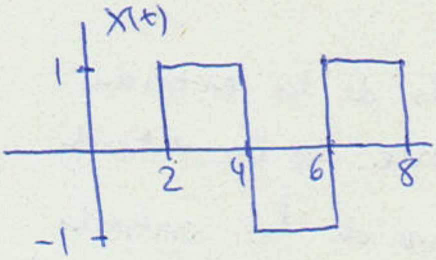
b) Compute the impulse response of the system, h[n]. (3 points)

c) What condition must k meet for the system to be stable? (2 points)

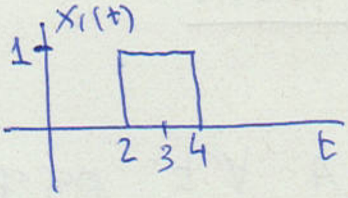
d) Filter the signal x[n]={$\underline{-1}$,2,1,3} with k=1 to obtain y[n]. (2 points)

PROBLEMA 1 PRIMER PARCIAL

CUESTION 1



a) $x(t) = x_1(t) + x_2(t) + x_3(t)$



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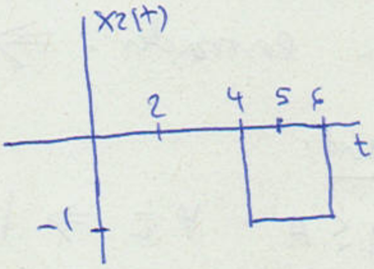

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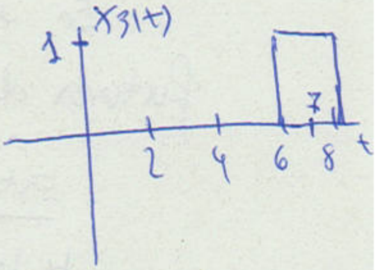
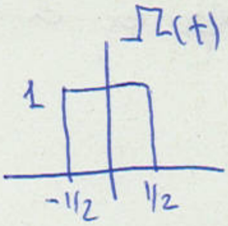




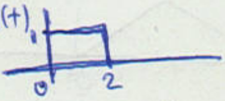
$$x_1(t) = \Pi\left(\frac{t-3}{2}\right)$$

$$x_2(t) = -\Pi\left(\frac{t-5}{2}\right)$$

$$x_3(t) = \Pi\left(\frac{t-7}{2}\right)$$

$$\boxed{x(t) = \Pi\left(\frac{t-3}{2}\right) - \Pi\left(\frac{t-5}{2}\right) + \Pi\left(\frac{t-7}{2}\right)}$$

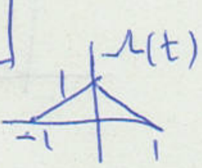
b) $h(t) = \Pi\left(\frac{t-1}{2}\right)$

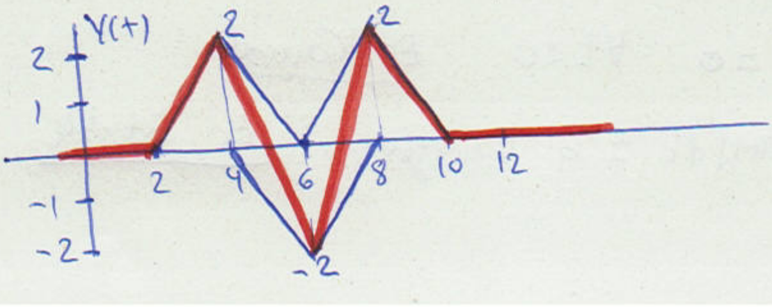


$$y(t) = x(t) * h(t) = \left[\Pi\left(\frac{t-3}{2}\right) - \Pi\left(\frac{t-5}{2}\right) + \Pi\left(\frac{t-7}{2}\right)\right] * \Pi\left(\frac{t-1}{2}\right)$$

$$y(t) = \Pi\left(\frac{t-3}{2}\right) * \Pi\left(\frac{t-1}{2}\right) - \Pi\left(\frac{t-5}{2}\right) * \Pi\left(\frac{t-1}{2}\right) + \Pi\left(\frac{t-7}{2}\right) * \Pi\left(\frac{t-1}{2}\right)$$

$$\boxed{y(t) = 2\,\Lambda\left(\frac{t-4}{2}\right) - 2\,\Pi\left(\frac{t-6}{2}\right) + 2\,\Pi\left(\frac{t-8}{2}\right)}$$





CUESTION 2

a) $y(t) = x(2t-1)$

Causalidad.

$t=0 \quad y(0) = x(-1)$ pasado de la entrada

$t=1 \quad y(1) = x(1)$ presente de la entrada.

$t=2 \quad y(2) = x(3)$ futuro de la entrada.

En algunos instantes la respuesta depende de valores futuros de la entrada. $\Rightarrow$ **No Causal**

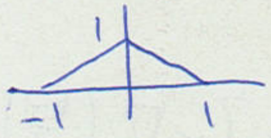
Estabilidad

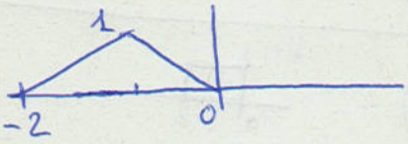
$|x(t)| \leq A \quad \forall t \Rightarrow |y(t)| \leq A \quad \forall t$ porque el sistema comprime y retarda la señal y no afecta a la amplitud. **Es estable**

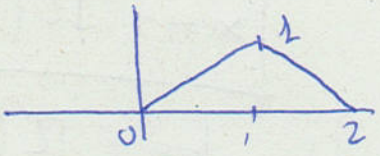
b) sistema LTI con $h(t) = 3\Lambda\left(-\frac{t}{2}+1\right)$

adelanto de 1s.   inversión.

$\Lambda(t) \xrightarrow{t \to t+1} \Lambda(t+1) \xrightarrow{t \to -t} \Lambda(-t+1)$

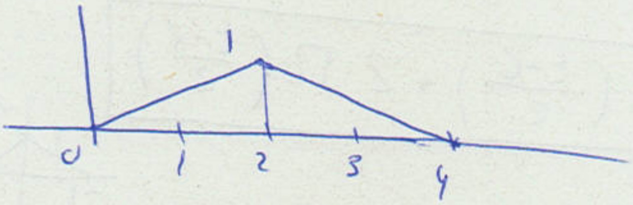


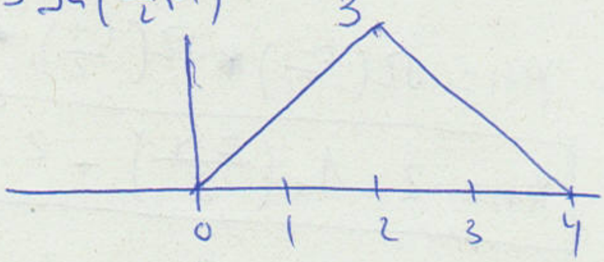




expansion   ×3

$\xrightarrow{t \to \frac{t}{2}} \Lambda\left(-\frac{t}{2}+1\right) \xrightarrow{} 3\Lambda\left(-\frac{t}{2}+1\right)$



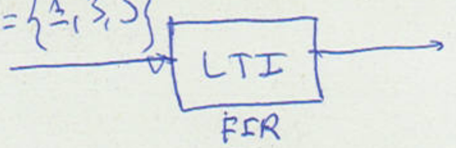


Causalidad $h(t) = 0 \quad \forall t<0$ **Es Causal**

Estabilidad $\int_{-\infty}^{\infty} |h(t)|\,dt = 6$ Convergente. **Es Estable**

CUESTION 3

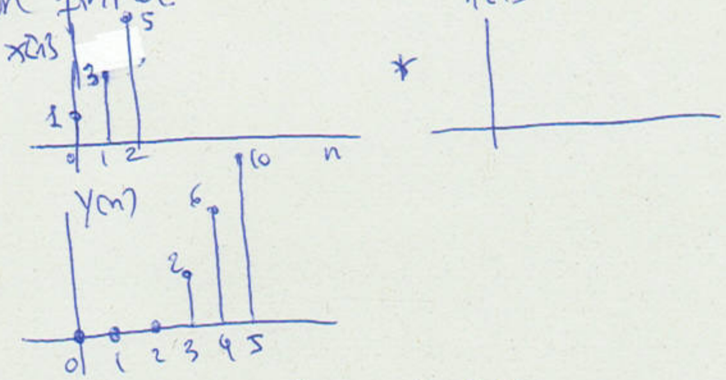
$x[n] = \{\underline{1}, 3, 5\}$ $\qquad$ $y[n] = \{\underline{0}, 0, 0, 2, 6, 10\}$



a)

FIR $\Rightarrow$ $h[n]$ de duración finita

$y[n] = x[n] * h[n]$



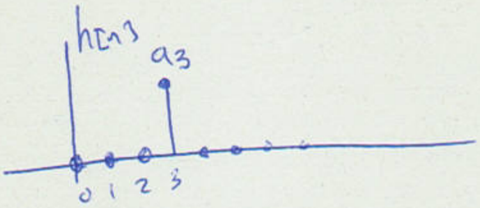
La convolución comienza en la suma de los comienzos

$3 = 0 + n_{h_0}$ $\qquad$ $n_{h_0} = 3$

La convolución termina en la suma de las terminaciones

$5 = 2 + n_{h_f}$ $\qquad$ $n_{h_f} = 3$

$h[n] = a_3\,\delta[n-3].$



sistema que retarda 3 muestras y amplifica por a_3 que a simple vista se ve que $a_3 = 2$

$$\boxed{h[n] = 2\,\delta[n-3]}$$

b) $x_2[n] = \{\underline{0}, -1, -3, -4, 3, 5\} = \{\underline{0}, -1, -3, -5, 0, 0\} + \{\underline{0}, 0, 0, 1, 3, 5\}$

$$\boxed{x_2[n] = -x_1[n-1] + x_1[n-3]}$$

c) Si la respuesta de $x_1[n]$ es $y[n] = \{\underline{0}, 0, 0, 2, 6, 10\}$.
La respuesta a $x_2[n]$ por ser el sistema LTI será $y_2[n] = -y[n-1] + y[n-3]$

$y_2[n] = \{\underset{0}{\underline{0}}, \underset{1}{0}, \underset{2}{0}, \underset{3}{0}, \underset{4}{-2}, \underset{5}{-6}, \underset{6}{-10}\} + \{\underset{0}{\underline{0}}, \underset{1}{0}, \underset{2}{0}, \underset{3}{0}, \underset{4}{0}, \underset{5}{0}, \underset{6}{2}, \underset{7}{6}, \underset{8}{10}\}$

$y_2[n] = \{\underset{0}{\underline{0}}, \underset{1}{0}, \underset{2}{0}, \underset{3}{0}, \underset{4}{-2}, \underset{5}{-6}\ \underset{6}{-8}\ \underset{7}{6}\ \underset{8}{10}\}$

otra forma:

$y_2[n] = 2x_2[n-3] = x_2[n] * 2\,\delta[n-3]$

2018-VI-8

2.

a) ⊛ $x(t) = 5\cos\left(\sqrt{3}t - \frac{\pi}{4}\right)$ cosinu jarraitua beti da periodikoa

$\cos\theta = \cos(\theta + 2k\pi)$
$\uparrow$ k osoa

$\exists T \,/\cdot\; x(t) = x(t+T)$

$$\underbrace{5\cos\left(\sqrt{3}t - \frac{\pi}{4}\right)}_{\theta} = 5\cos\left(\sqrt{3}(t+T) - \frac{\pi}{4}\right) = 5\cos\left(\underbrace{\sqrt{3}t - \frac{\pi}{4}}_{\theta} + \underbrace{\sqrt{3}T}_{2k\pi}\right)$$

$\sqrt{3}T = 2k\pi$

$T = \frac{2k\pi}{\sqrt{3}}$

$\boxed{T_0 = \frac{2\pi}{\sqrt{3}}}$

⊛ $x(t) = 3\sin\frac{\pi}{4}t - 5\cos\left(\frac{\pi}{3}t - \frac{\pi}{2}\right)$ osagai biak periodo berarekin izan behar dira periodiko:

$\frac{\pi}{4}T_1 = 2k_1\pi$

$T_1 = 8k_1$

$\frac{\pi}{3}T_2 = 2k_2\pi$

$T_2 = 6k_2 \rightarrow$ $T_1 = T_2$, $8k_1 = 6k_2$ $\frac{k_1}{k_2} = \frac{6}{8} = \frac{3}{4}$ $\nearrow T_1 = 8\cdot 3$, $\searrow T_2 = 6\cdot 4$

hau da: $T_0 = $ m.k.t$(8,6) = 24$

$T_1 = T_2 = \boxed{24 = T_0}$ periodikoa

⊛ $x(t) = \cos\pi t + \sin 5t$

$T_1 = 2$ $T_2 = \frac{2\pi}{5}$

m.k.t.$(2, \frac{2\pi}{5})$ $\nexists$

$2k_1 = \frac{2k_2\pi}{5}$

$k_1/k_2 = \pi/5$ $\leftarrow$ k_1 eta k_2 osoak izanik ezinezkoa.

$x(t)$ EZ da periodikoa

⊛ $x[n] = \cos\left(\frac{3\pi}{4}n + \frac{\pi}{4}\right)$

$x[n] \overset{?}{=} x[n+N]$ N osoa izanik

$\frac{3\pi}{4}N = 2k\pi \rightarrow N = \frac{8k}{3} \xrightarrow{k=3} \boxed{N_0 = 8}$ periodikoa

⊛ $x[n] = \sin\frac{\pi}{2}n - 3\cos\left(\frac{\pi}{3}n + \frac{\pi}{4}\right)$

$N_1 = \frac{2k\pi}{\pi/2} = 4k$ $N_2 = \frac{2k\pi}{\pi/3} = 6k$

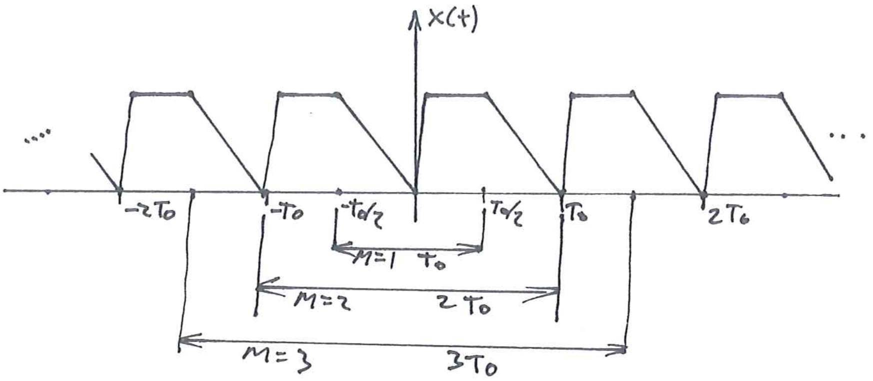
m.k.t.$(4,6) = \boxed{12 = N_0}$ periodikoa

2.

b)

$$P_x = \lim_{T\to\infty} \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} |x(t)|^2 dt \underset{\substack{\uparrow \\ \left\{\begin{array}{l} T = MT_0 \\ T\to\infty \Rightarrow M\to\infty \end{array}\right\}}}{=} \lim_{M\to\infty} \frac{1}{MT_0}\int_{-\frac{MT_0}{2}}^{\frac{MT_0}{2}} |x(t)|^2 dt =$$

M periodoko tarteko integrala M aldiz periodo bateko integrala da. Honen tartearen hasiera unea aldatzeak ez du emaitza aldatzen.



La integral en un intervalo de M periodos vale M veces lo que la integral en un periodo. En esta cambiar el punto de inicio del intervalo no cambia el resultado.

The integral in an interval of M periods is M times the integral of a single period. In this integral (of interval T_0) changing the initial point does not change its value.

$$= \lim_{M\to\infty} \frac{1}{MT_0} \cdot M \int_{T_0} |x(t)|^2 dt = \frac{1}{T_0}\int_{T_0} |x(t)|^2 dt$$

q.e.d.

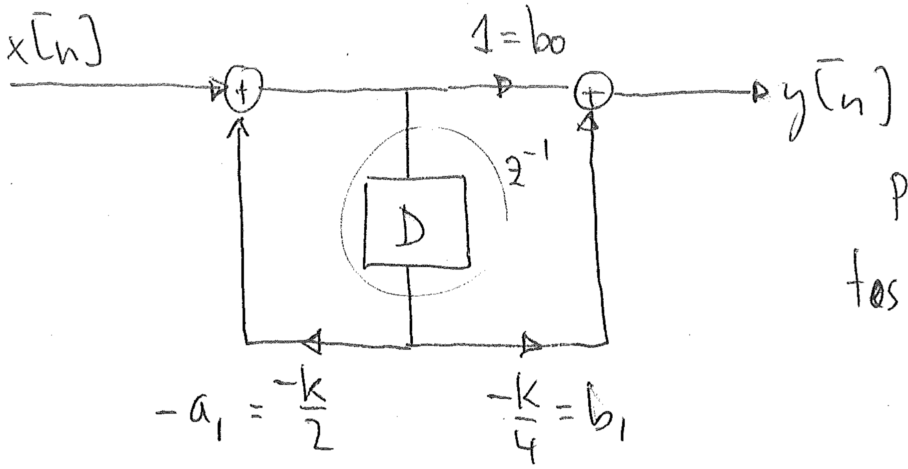
c) $x(t) = 5\cos\left(\sqrt{3}t - \frac{\pi}{4}\right)$

$T_0 = \frac{2\pi}{\sqrt{3}}$

$$P_x = \frac{1}{T_0}\int_0^{T_0} 5^2 \cos^2\left(\sqrt{3}t - \frac{\pi}{4}\right) dt =$$

$$= \frac{25\sqrt{3}}{2\pi}\int_0^{\frac{2\pi}{\sqrt{3}}} \left(\frac{1}{2} + \underbrace{\cos\left(2\sqrt{3} - 2\frac{\pi}{4}\right)}_{\hookrightarrow 0 \text{ periodo biko area}}\right) dt = \frac{25\sqrt{3}}{2\pi}\left[\frac{t}{2}\right]_0^{2\pi/\sqrt{3}} = \boxed{\frac{25}{2}}$$

PROBLEMA 3



En forma directa II

Podemos identificar los coeficientes en el esquema.

(a) Por lo tanto $y[n] = b_0 x[n] + b_1 x[n-1] - a_1 y[n-1]$

$$\boxed{y[n] = 1 \cdot x[n] - \frac{k}{4} x[n-1] - \frac{k}{2} y[n-1]} \quad (a)$$

recursivo

Si $k \neq 0$ el sistema es recursivo $y[n-1]$: IIR

el mayor retardo (ORDEN) : 1

(b) Se trata de un sistema causal en (a) $y[n]$ solo depende de valores presentes o pasados; $h[n] = 0 \quad n < 0$

Para obtener $h[n] \Rightarrow x[n] = \delta[n]$

$$h[n] = \delta[n] - \frac{k}{4}\delta[n-1] - \frac{k}{2} h[n-1]$$

$$h[0] = 1 - 0 - 0 = 1$$

$$h[1] = 0 - \frac{k}{4} - \frac{k}{2}(1) = -\frac{k}{4} \cdot 3 = \frac{-3}{2} \cdot \left(\frac{k}{2}\right)$$

$$h[2] = 0 - 0 - \frac{k}{2}\left(\frac{-3}{4}k\right) = +\frac{3}{2}\left(\frac{k}{2}\right)^2$$

$$h[3] = 0 - 0 - \frac{k}{2}\left(\frac{3}{2}\left(\frac{k}{2}\right)^2\right) = -\frac{3}{2}\left(\frac{k}{2}\right)^3$$

$\vdots$

$\vdots$

$$h[m] = (-1)^m \frac{3}{2}\left(\frac{k}{2}\right)^m$$

negativo impar

positivo par

Para todos los valores:

$$h[n] = \underset{\overset{\uparrow}{n=0}}{\delta[n]} + \underbrace{\frac{3}{2}(-1)^n \cdot \left(\frac{k}{2}\right)^n}_{\text{empezando } n=1} \cdot u[n-1]$$

ⓒ Estabilidad $\displaystyle\sum_{n=-\infty}^{\infty} |h[n]| < \infty$

$$\sum_{n=-\infty}^{\infty} |h[n]| = \sum_{n=0}^{\infty} |h[n]| = \underset{\underset{n=0}{\shortparallel}}{1} + \frac{3}{2} \underbrace{\sum_{n=1}^{\infty}}_{} \left|\frac{k}{2}\right|^n$$

serie geométrica ⇒ sea sumable

$\left|\frac{k}{2}\right| < 1$ $\boxed{|k| < 2}$

ⓓ $k=1$ $y[n] = x[n] - \frac{1}{4}x[n-1] - \frac{1}{2}y[n-1]$

$x[n] = \{\underline{-1}, 2, 1, 3\}$ ($y[n] = 0$ $n<0$)

$$y[0] = -1 - \frac{1}{4}\cdot 0 - \frac{1}{2}\cdot 0 = -1$$

$$y[1] = 2 - \frac{1}{4}(-1) - \frac{1}{2}(-1) = 2 + \frac{3}{4} = \frac{11}{4}$$

$$y[2] = 1 - \frac{1}{4}(2) - \frac{1}{2}\cdot\left(\frac{11}{4}\right) = 1 - \frac{15}{8} = \frac{-7}{8}$$

$$y[3] = 3 - \frac{1}{4}(1) - \frac{1}{2}\left(\frac{-7}{8}\right) = 3 + \frac{3}{16} = \frac{51}{16}$$

$x[n] = \{\underline{-1}, \frac{11}{4}, \frac{-7}{8}, \frac{51}{16}, \dots\}$ entran 4 muestras, salen 4 muestras.

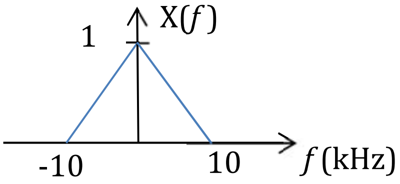
TRATAMIENTO DE SEÑALES: Convocatoria ordinaria
Segundo Parcial

El tiempo estimado para resolver el examen es de una hora y 30 minutos.
Las 3 cuestiones del problema 1 tienen el mismo valor.

PROBLEMA 1 (10 puntos, 30 minutos)

1. La señal $\mathrm{x(t)} = 5\cos(2\pi 400t) + 8\cos(2\pi 600t)$ se ha muestreado con f_s = 1000 Hz y después de realizar una conversión digital-analógica ideal se ha obtenido la señal $\mathrm{y(t)} = B_1 \cos(2\pi 400t)$. Obtener el valor de B_1 de forma razonada si:
   a. En el muestreo se ha utilizado un filtro antialiasing.
   b. En el muestreo no se ha utilizado filtro antialiasing.

2. Sea la señal x(t) cuyo espectro es:

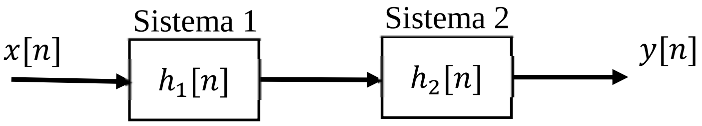


   Se desea muestrear la señal x(t) preservando su contenido espectral en la banda ±5kHz. No se dispone de filtro antialiasing. Se pide:
   a. Obtener la frecuencia de muestreo mínima necesaria.
   b. Para la frecuencia de muestreo del apartado anterior, dibujar el espectro X(Ω) de la secuencia obtenida entre −π y π, anotando los valores de frecuencia y amplitud significativos.

3. La señal $\mathrm{x(t)} = 2\cos(2\pi 30t)$ se muestrea con f_s = 120 Hz. La secuencia x[n] resultante se filtra con un sistema LTI promediador cuya ecuación en diferencias es $\mathrm{y}[n] = \frac{1}{4}\sum_{k=0}^{3} x[n-k]$. Calcular, trabajando en el dominio de la frecuencia, la salida del sistema y[n].

PROBLEMA 2 (10 puntos, 30 minutos)

Sea el sistema de la figura formado por la conexión serie de dos subsistemas:



Se conoce la respuesta frecuencial del sistema completo:

$$H(\Omega) = \frac{A}{1 - \frac{1}{3} \cdot e^{-j\Omega}} \frac{1 + e^{-j\Omega}}{1 - \frac{1}{2} \cdot e^{-j\Omega}}$$

a) Calcula el valor de A sabiendo que si la entrada es x[n]=1, la salida del sistema es y[n]=12. **(2 puntos)**

b) La ecuación en diferencias del Sistema 1 es:

$$y[n] = Ax[n] + \frac{1}{3} y[n-1]$$

Calcula

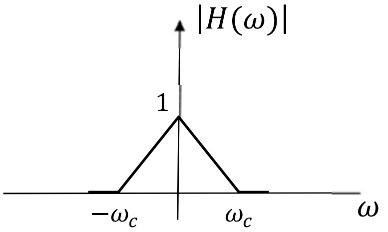
1. La respuesta impulsional del segundo sistema $h_2[n]$. **(2 puntos)**
2. La ecuación en diferencias del segundo sistema, e indica justificadamente el tipo y orden del sistema. **(2 puntos)**

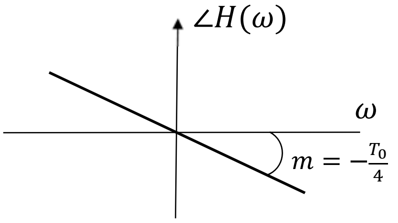
c) Calcula la salida del sistema completo si la entrada al sistema es: **(4 puntos)**

$$x[n] = 4 + 5\cos\left(\frac{\pi}{2} n + \frac{\pi}{4}\right) + 10\cos\left(\pi n + \frac{\pi}{3}\right)$$

PROBLEMA 3 (10 puntos, 30 minutos)

Se dispone de un filtro como el de la figura:





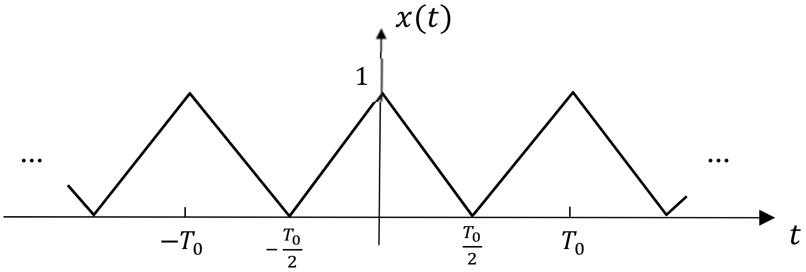
a) Indicar justificadamente el tipo de filtro, y si la la respuesta impulsional del filtro, $h(t)$, es real o compleja. **(1 punto)**

b) Sabemos que cuando la entrada el filtro es una señal periódica real, $x(t)$, la salida es de la forma indicada (a la salida solo tenemos 2 armónicos):

$$x(t) = A_0 + \sum_{k=1}^{\infty} A_k \cos(k\omega_0 t + \theta_k) \quad \Rightarrow \quad y(t) = B_0 + \sum_{k=1}^{2} B_k \cos(k\omega_0 t + \phi_k)$$

Calcula el máximo valor de ω_c para que dicha relación se cumpla. **(2 puntos)**

Considera la señal periódica, $x(t)$, mostrada en la figura:



Se pide:

c) Calcular los coeficientes en desarrollo en serie de Fourier de $x(t)$.

Nota: $\Lambda\left(\frac{t}{T}\right) = \frac{1}{T} \cdot \prod\left(\frac{t}{T}\right) * \prod\left(\frac{t}{T}\right)$ **(2 puntos)**

d) Obtener y representa el espectro de la señal periódica $x(t)$. **(2 puntos)**

e) La señal $x(t)$ es ahora la entrada al filtro del apartado *a*, con la ω_c calculada en *b*. Calcular la salida $y(t)$ en forma de suma de cosenos. **(3 puntos)**

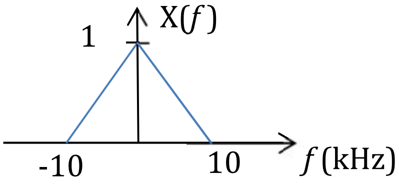
SEINALEEN PROZESATZEA: Ohiko deialdia
Bigarren partziala

Azterketak 3 ariketa ditu. Ariketa bakoitzak 10 puntu balio ditu, eta 1. ariketako galdera guztiek pisu berdina dute. Ordu eta erdi duzue.

1. ARIKETA (10 puntu, 30 minutu)

1. Izan bedi $\mathrm{x(t)} = 5\cos(2\pi 400t) + 8\cos(2\pi 600t)$ seinalea, f_s = 1000 Hz laginketa-maiztasunarekin lagindua, eta digital-analogiko bihurketa egin ondoren $\mathrm{y(t)} = B_1 \cos(2\pi 400t)$ seinalea sortzen duena. Lortu B_1 eta arrazoitu hurrengo kasuetan:
    - a. Laginketarako antialiasing iragazkia erabiltzen bada.
    - b. Laginketarako antialiasing iragazkia erabiltzen ez bada.

2. Izan bedi irudiko espektroa duen x(t) seinalea:

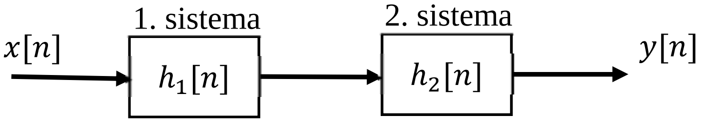


x(t) lagindu nahi dugu baina ±5kHz bandako maiztasun-osagaiak mantenduz. Antialiasing iragazkirik ez da erabiltzen. Honakoei erantzun:
    - a. Lortu erabili behar den laginketa-maiztasun txikiena.
    - b. Aurreko laginketa-maiztasuna erabiliz lortutako seinale digitalaren espektro, X(Ω), irudikatu (−π, π) tartean, eta adierazi maiztasun eta anplitude balio esanguratsuak.

3. Izan bedi $\mathrm{x(t)} = 2\cos(2\pi 30t)$ seinalea, f_s = 120 Hz laginketa-maiztasunarekin lagindu dena. Lortutako x[n] sekuentzia, $\mathrm{y}[n] = \frac{1}{4}\sum_{k=0}^{3} x[n-k]$ diferentzia ekuazioa duen batezbestekoa egiteko LTI sistema bat erabiliz iragazi da. Kalkulatu y[n] irteera-sekuentzia maiztasunaren eremuan lan eginez.

2. ARIKETA (10 puntu, 30 minutu)

Izan bedi irudiko sistema, bi azpisistema serie konektatuz lortu dena:



Sistema osoaren maiztasun-erantzuna hau da:

$$H(\Omega) = \frac{A}{1-\frac{1}{3}\cdot e^{-j\Omega}} \frac{1+e^{-j\Omega}}{1-\frac{1}{2}\cdot e^{-j\Omega}}$$

a) Kalkulatu A, jakinda sarrera x[n]=1 denean irteera y[n]=12 dela. **(2 puntu)**

b) 1 sistemaren diferentzia-ekuazioa hau da:

$$y[n] = Ax[n] + \frac{1}{3}y[n-1]$$

Kalkulatu:

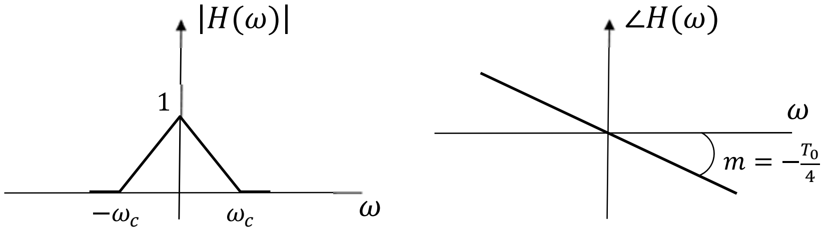
1. Bigarren sistemaren pultsu-erantzuna, $h_2[n]$. **(2 puntu)**
2. Bigarren sistemaren diferentzia-ekuazioa, adierazi modu arrazoituan sistema mota eta sistemaren maila zein diren. **(2 puntu)**

c) Kalkulatu sistema osoaren irteera, y[n], sarrera seinalea hau denean: **(4 puntu)**

$$x[n] = 4 + 5\cos\left(\frac{\pi}{2}n + \frac{\pi}{4}\right) + 10\cos\left(\pi n + \frac{\pi}{3}\right)$$

3. ARIKETA (10 puntu, 30 minutu)

Honako maiztasun-erantzuna duen sistema dugu:



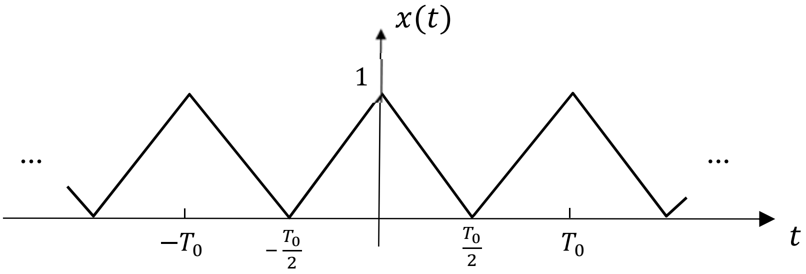
a) Adierazi eta arrazoitu, zer iragazki mota den eta pultsu-erantzuna, $h(t)$, erreala edo konplexua den. **(1 puntu)**

b) Iragazkiaren sarrera $x(t)$ seinale periodiko erreala denean, irteera seinaleak honako itxura hartzen du (bi harmoniko ditu soilik):

$$x(t) = A_0 + \sum_{k=1}^{\infty} A_k \cos(k\omega_0 t + \theta_k) \qquad \Rightarrow \qquad y(t) = B_0 + \sum_{k=1}^{2} B_k \cos(k\omega_0 t + \phi_k)$$

Kalkulatu ω_c balio maximoa adierazitako baldintza bete dadin. **(2 puntu)**

Demagun irudiko, $x(t)$, seinale periodikoa dugula:



Hurrengo galderei erantzun:

c) Kalkulatu $x(t)$ seinalearen Fourier serie garapeneko koefizienteak.

Oharra: $\Lambda\left(\frac{t}{T}\right) = \frac{1}{T} \cdot \prod\left(\frac{t}{T}\right) * \prod\left(\frac{t}{T}\right)$ **(2 puntu)**

d) Lortu eta irudikatu $x(t)$ seinale periodikoaren espektroa. **(2 puntu)**

e) $x(t)$ seinalea a ataleko iragazkiaren sarrera-seinalea da, iragazkiak b atalean kalkulatutako ω_c duela. Kalkulatu $y(t)$ irteera kosinuen batura gisa. **(3 puntu)**

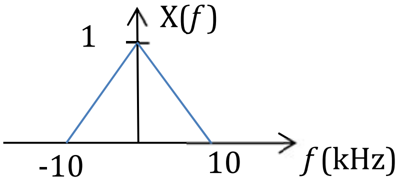
SIGNAL PROCESSING: Final exam
Second mid-term

The estimated time to solve the exam are three hours.
The 3 short questions in problem 1 have all the same value.

PROBLEM 1 (10 points, 30 minutes)

1. The signal $x(t) = 5\cos(2\pi 400t) + 8\cos(2\pi 600t)$ has been sampled with f_s = 1000Hz, and after its digital to analog conversion the resulting signal is $y(t) = B_1\cos(2\pi 400t)$. Obtain B_1 in a justified way in the following cases:
   a. In the sampling process an antialiasing filter was used.
   b. In the sampling process no antialiasing filter was used.

2. Consider the signal x(t) with the following spectrum:

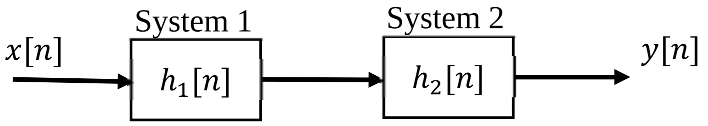


   We would like to sample x(t) but preserving its spectral content in the ±5kHz band. We do not have an antialiasing filter. Answer the following:
   a. Obtain the minimum sampling frequency needed.
   b. Sketch the spectrum X(Ω) of the resulting digital signal in (−π, π) range, indicating the significant frequency and amplitude values.

3. The signal $x(t) = 2\cos(2\pi 30t)$ is sampled with f_s = 120Hz sampling frequency. The resulting signal x[n] is filtered using an LTI average with the following difference equation $y[n] = \frac{1}{4}\sum_{k=0}^{3} x[n-k]$. Compute the output signal y[n] working in the frequency domain.

PROBLEM 2 (10 points, 30 minutes)

Consider the system of the figure composed of two systems connected in series:



The frequency response of the complete system is:

$$H(\Omega) = \frac{A}{1-\frac{1}{3}\cdot e^{-j\Omega}} \frac{1+e^{-j\Omega}}{1-\frac{1}{2}\cdot e^{-j\Omega}}$$

a) Compute A knowing that if the input is x[n]=1, the output is y[n]=12. **(2 points)**

b) The difference equation of Sytem 1 is:

$$y[n] = Ax[n] + \frac{1}{3}y[n-1]$$

Compute:

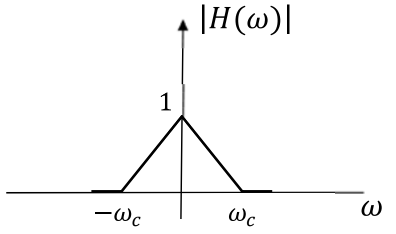
1. The impulse-response of the second system, $h_2[n]$. **(2 points)**
2. The difference-equation of the second system, indicating its type and order in a reasoned way. **(2 points)**

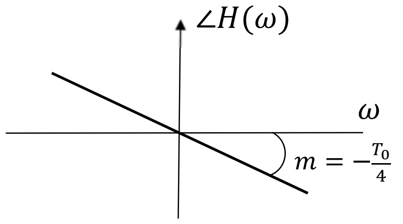
c) Compute the output of the complete system if the input is: **(4 points)**

$$x[n] = 4 + 5\cos\left(\frac{\pi}{2}n + \frac{\pi}{4}\right) + 10\cos\left(\pi n + \frac{\pi}{3}\right)$$

PROBLEM 3 (10 points, 30 minutes)

Consider the following filter:





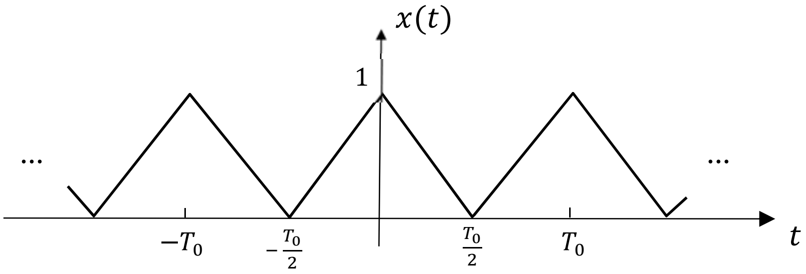
a) Indicate in a reasoned way the type of filter and whether the filter's impulse-response, $h(t)$, is real or complex. **(1 point)**

b) We know that when the input signal to the filter is a real periodic signal, $x(t)$, its output has the following form (it has only 2 harmonics):

$$x(t) = A_0 + \sum_{k=1}^{\infty} A_k \cos(k\omega_0 t + \theta_k) \qquad \Rightarrow \qquad y(t) = B_0 + \sum_{k=1}^{2} B_k \cos(k\omega_0 t + \phi_k)$$

Compute the maximum value of ω_c for that relation to hold. **(2 points)**

Consider the periodic signal, $x(t)$, shown in the figure:



Answer the following:

c) Compute the Fourier series coefficients of $x(t)$.

Note: $\Lambda\left(\frac{t}{T}\right) = \frac{1}{T} \cdot \prod\left(\frac{t}{T}\right) * \prod\left(\frac{t}{T}\right)$ **(2 points)**

d) Obtain and sketch the spectrum of $x(t)$. **(2 points)**

e) The signal $x(t)$ is now the input to the filter of section a, with the ω_c computed in section b. Compute the output $y(t)$ as a sum of cosines. **(3 points)**

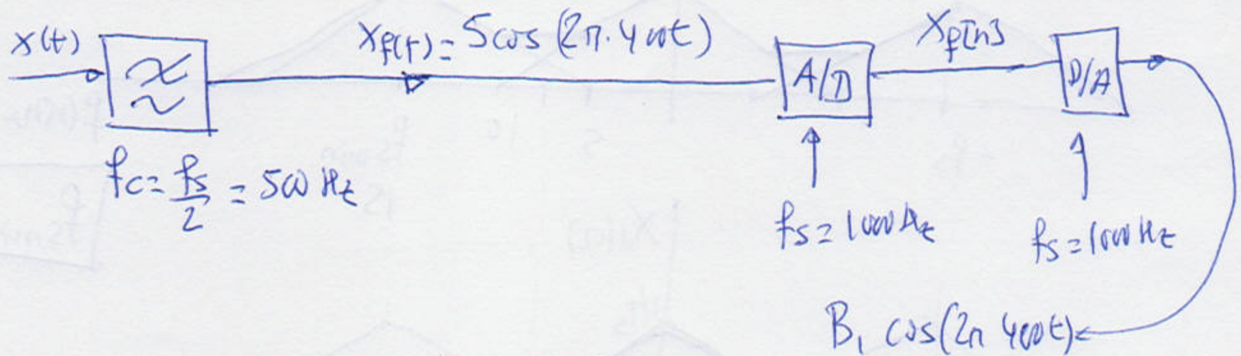
PROBLEMA 1 SEGUNDO PARCIAL

CUESTION 1

$$x(t) = 5\cos(2\pi 400 t) + 8\cos(2\pi 600\, t)$$

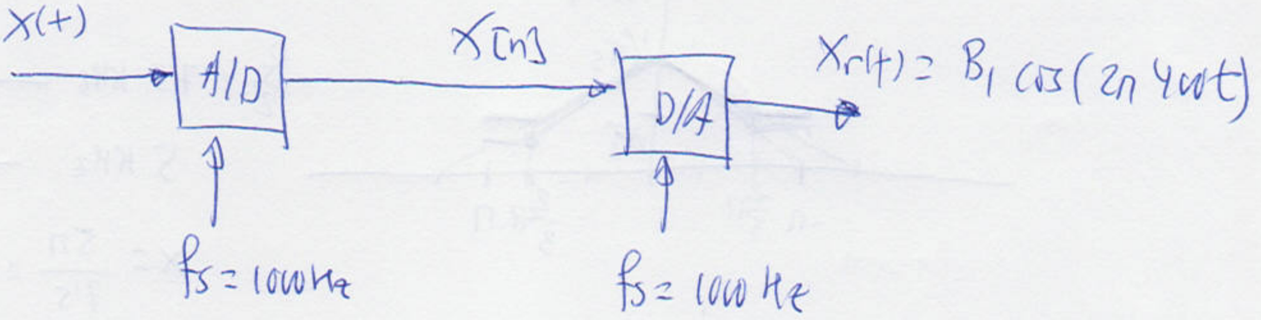
$f_s = 1000\ Hz$

a)



$$\boxed{B_1 = 5}$$

b)



$$x[n] = x(t)\Big|_{t=\frac{n}{f_s}} = 5\cos\left(2\pi \cdot 400 \frac{n}{1000}\right) + 8\cos\left(2\pi \cdot 600 \frac{n}{1000}\right)$$

$$x[n] = 5\cos\left(2\pi \frac{2}{5} n\right) + 8\cos\left(2\pi \frac{3}{5} n\right)$$

$f_{d_1} = \frac{2}{5} < \frac{1}{2}$

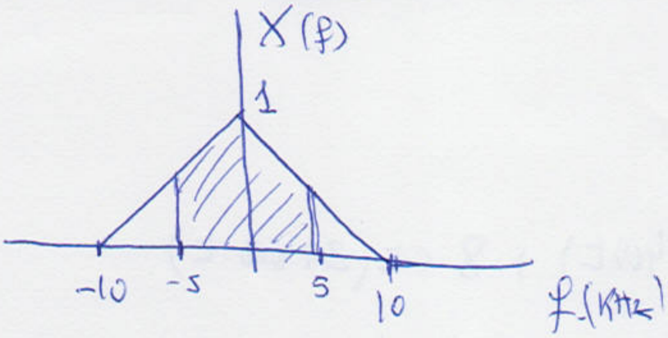
$$\boxed{B_1 = 13}$$

$f_{d_2} = \frac{3}{5} > \frac{1}{2} \quad f'_{d_2} = \frac{3}{5} - 1 = \frac{-2}{5}$

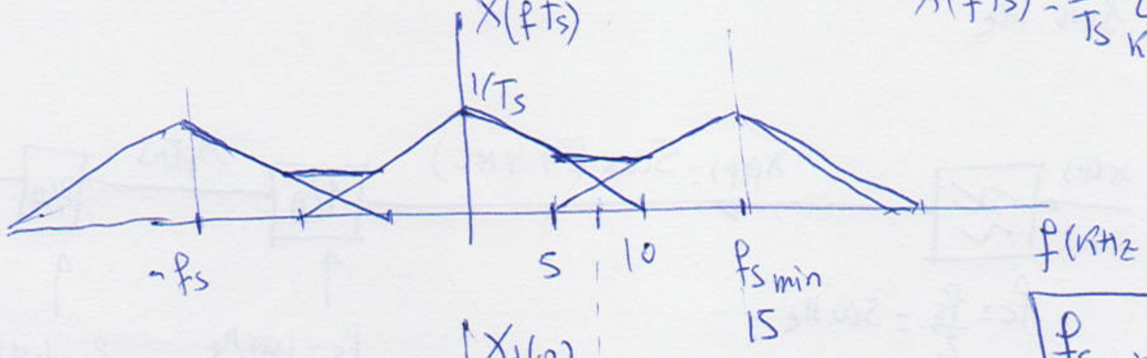
$$x[n] = 5\cos\left(2\pi \frac{2}{5} n\right) + 8\cos\left(2\pi \left(-\frac{3}{5}\right) n\right) = 5\cos\left(2\pi \frac{2}{5} n\right) + 8\cos\left(2\pi \frac{2}{5} n\right)$$

$x[n] = 13\cos\left(2\pi \frac{2}{5} n\right) \qquad x_r(t) = x[n]\Big|_{n = t \cdot f_s} \qquad x_r(t) = 13\cos\left(2\pi \frac{2}{5} \cdot 1000\, t\right)$

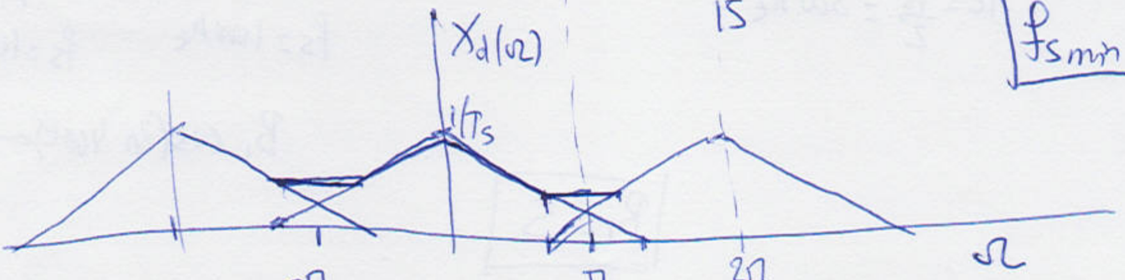
CUESTION 2

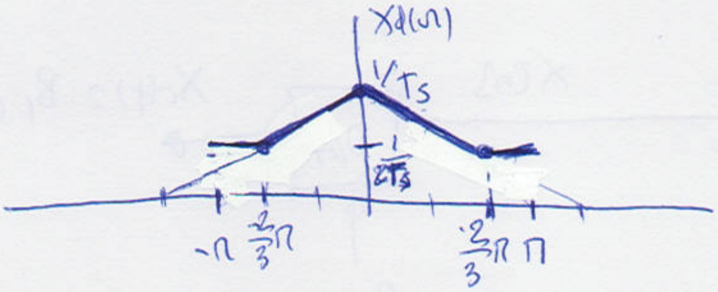


$$X(fTs) = \frac{1}{Ts}\sum_{K=-\infty}^{\infty} X(f - Kf_s)$$



$$\boxed{f_{s\,min} = 15 \text{ KHz}}$$



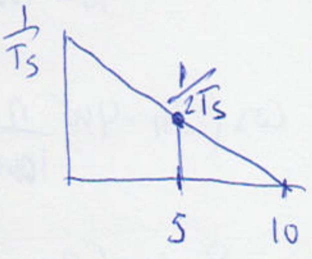


$\frac{f_s}{2} = 7'5$ KHz $\longrightarrow \pi$

5 KHz $\longrightarrow x$

$$x = \frac{5\pi}{7'5} = \frac{5\pi}{\frac{15}{2}} = \frac{10}{15}\pi$$

$x = \frac{2}{3}\pi$

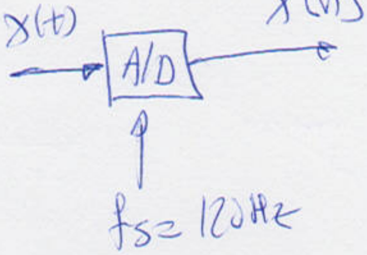


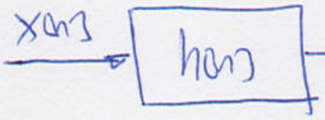
CUESTION 3

$x(t) = 2\cos(2\pi 30 t)$

$f_s = 120\ Hz$

$x[n] = x(t)\big|_{t=\frac{n}{f_s}} = 2\cos\left(2\pi 30 \frac{n}{120}\right) = 2\cos\left(2\pi \frac{1}{4} n\right)$

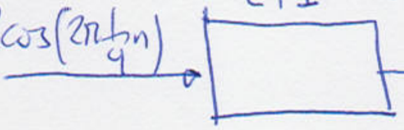




$y[n] = \frac{1}{4}\sum_{k=0}^{3} x[n-k]$

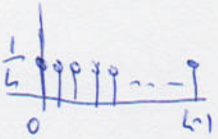
FIR $h[n] = \left\{\underline{\frac{1}{4}}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}\right\}$

LTI h[n] real



$x[n] = 2\cos\left(2\pi \frac{1}{4} n\right)$

$y[n] = 2\left|H(\Omega)\right|_{\Omega=\frac{\pi}{2}} \cos\left(\frac{\pi}{2} n + \measuredangle H\left(\frac{\pi}{2}\right)\right)$

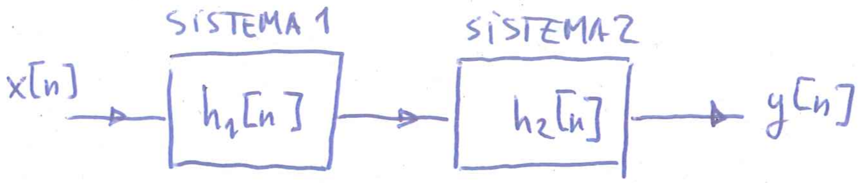


$H(\Omega) = TF\{h[n]\} = \frac{1}{L} \frac{\operatorname{sen} \Omega \frac{L}{2}}{\operatorname{sen} \frac{\Omega}{2}} e^{-j\frac{L-1}{2}\Omega}\Bigg|_{L=4} = \frac{1}{4} \frac{\operatorname{sen} 2\Omega}{\operatorname{sen} \frac{\Omega}{2}} e^{-j\frac{3}{2}\Omega}$

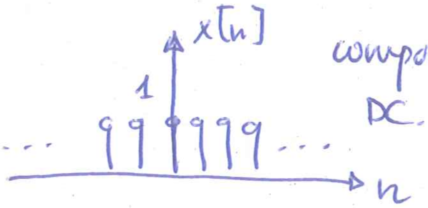
$H(\Omega)\big|_{\Omega=\frac{\pi}{2}} = \frac{1}{4} \frac{\operatorname{sen} \pi}{\operatorname{sen} \frac{\pi}{4}} e^{-j\frac{3\pi}{4}} = \frac{1}{4} \cdot 0 = 0$

$$\boxed{y[n] = 0 \quad \forall n}$$

PROBLEMA 2



$$H(\Omega) = \frac{A}{1-\frac{1}{3}e^{-j\Omega}} \cdot \frac{1+e^{-j\Omega}}{1-\frac{1}{2}e^{-j\Omega}}$$



componente DC.

ⓐ Si $x[n]=1 \Rightarrow y[n]=12$

Es decir que para la componente de $\Omega=0$ (DC, continuo)

$$H(\Omega)\Big|_{\Omega=0} = \frac{A}{1-\frac{1}{3}e^{j0}} \cdot \frac{1+1e^{-j0}}{1-\frac{1}{2}e^{-j0}} = \frac{A}{2/3}\;\frac{2}{1/2} = 6\cdot A$$

Es la ganancia a esa frecuencia $\Rightarrow \frac{Y(0)}{X(0)} = H(0) = 6\cdot A = \frac{12}{1}$

$$\boxed{A=2}$$

ⓑ Sistema 1 tiene

$$y[n] = A\cdot x[n] + \frac{1}{3}y[n-1]$$ es decir que:

$$Y(\Omega) = A\cdot X(\Omega) + \frac{1}{3}e^{-j\Omega}\,Y(\Omega)$$

$$Y(\Omega)\left[1-\frac{1}{3}e^{-j\Omega}\right] = AX(\Omega) \Rightarrow H_1(\Omega) = \frac{A}{1-\frac{1}{3}e^{-j\Omega}}$$

Como los dos sistemas están en serie:

$$h[n]=h_1[n]*h_2[n] \Rightarrow H(\Omega)=H_1(\Omega)\cdot H_2(\Omega)$$

$$\underbrace{\cancel{\frac{A}{1-\frac{1}{3}e^{-j\Omega}}}\cdot\frac{1+e^{-j\Omega}}{1-\frac{1}{2}e^{-j\Omega}}}_{H(\Omega)} = \underbrace{\cancel{\frac{A}{1-\frac{1}{3}e^{-j\Omega}}}}_{H_1(\Omega)}\cdot\underbrace{\frac{1+e^{-j\Omega}}{1-\frac{1}{2}e^{-j\Omega}}}_{H_2(\Omega)}$$

y si $H_2(\Omega) = \dfrac{\overset{b_0}{1} + \overset{b_1}{1}\, e^{-j\Omega}}{\underset{a_0}{1} - \underset{a_1}{\frac{1}{2}}\, e^{-j\Omega}}$ $\Rightarrow$ ec. en diferencias $\boxed{y[n] = \underset{b_0}{1}x[n] + \underset{b_1}{1}x[n-1] + \underset{-a_1}{\frac{1}{2}}y[n-1]}$

Sistema IIR (recursivo, $y[n-1]$)

Orden 1, mayor retardo

El sistema completo IIR orden 2 en el denominador $e^{-j2\Omega}$ (retardo 2)

$$h_2[n] = F^{-1}\{H_2(\Omega)\} = F^{-1}\left\{\frac{1+e^{-j\Omega}}{1-\frac{1}{2}e^{-j\Omega}}\right\}$$

Pares básicos: $F^{-1}\left\{\dfrac{1}{1-\frac{1}{2}e^{-j\Omega}}\right\} = \left(\frac{1}{2}\right)^n \cdot u[n] = h_{21}[n]$

Desplaz tiempo $F^{-1}\left\{\dfrac{e^{-j\Omega}}{1-\frac{1}{2}e^{-j\Omega}}\right\} = \left(\frac{1}{2}\right)^{n-1} u[n-1] = h_{22}[n]$

linealidad $h_2[n] = h_{21}[n] + h_{22}[n]$

$$\boxed{h_2[n] = \left(\frac{1}{2}\right)^n u[n] + \left(\frac{1}{2}\right)^{n-1} u[n-1]}$$

ⓒ Vemos que el sistema completo es real, todos los coef. de las ecuaciones en diferencias son reales. $\therefore H(\Omega) = H^*(-\Omega)$

Por lo tanto sabemos que si $x_1[n] = A_0 \cdot \cos(\Omega_0 n + \theta_0)$

$$\downarrow\; y_1[n] = A_0 \cdot |H(\Omega_0)| \cdot \cos(\Omega_0 n + \theta_0 + \phi(\Omega_0))$$

Aplicamos esa relación a cada una de las componentes de:

$$x[n] = \underset{\Omega=0}{4} + 5\cdot\cos\left(\underset{\Omega=\frac{\pi}{2}}{\frac{\pi}{2}n} + \frac{\pi}{4}\right) + 10\cdot\cos\left(\underset{\Omega=\pi}{\pi n} + \frac{\pi}{3}\right)$$

calculadora ↓

$H(\Omega)\big|_{\Omega=0} = 12$ (apartado a)

$H(\Omega)\big|_{\Omega=\pi/2} = \dfrac{2}{1-\frac{1}{3}(-j)} \cdot \dfrac{1+(-j)}{1-\frac{1}{2}(-j)} = -2'4j = 2'4\angle -\frac{\pi}{2}$

$H(\Omega)\big|_{\Omega=\pi} = \dfrac{2}{1+\frac{1}{3}} \cdot \dfrac{1-1}{1+\frac{1}{2}} = 0$ (se cancela)

$$y[n] = 4\cdot 12 + 5\cdot 2{,}4\cdot\cos\left(\frac{\pi}{2}n \overset{\frac{\pi}{4}-\frac{\pi}{2}}{-\frac{\pi}{4}}\right) + 0 = \boxed{48 + 12\cos\left(\frac{\pi}{2}n - \frac{\pi}{4}\right) = y[n]}$$

2. PARTZIALA 3. ARIKETA

a] Behe paseko iragazkia.

$h(t)$ = real porque $H(\omega)$ presenta simetría hermítica / $|H(\omega)| = |H(-\omega)|$, $\measuredangle H(\omega) = -\measuredangle H(-\omega)$

b] Sólo pasan dos harmónicos de la señal, hasta la componente en $2\omega_0$. Para ello el filtro con frecuencia de corte ω_c ha de ser mayor que $2\omega_0$ y menor de $3\omega_0$ para que no pase el tercer armónico $\Rightarrow$

$\omega_{c_{max}} = 3\omega_0$.

c] $a_k = \frac{1}{T_0} X_b(\omega)\Big|_{\omega = k\omega_0}$ siendo $x_b(t) = \Lambda\left(\frac{t}{T_0/2}\right) = \frac{1}{T_0/2} \Pi\left(\frac{t}{T_0/2}\right) * \Pi\left(\frac{t}{T_0/2}\right.$

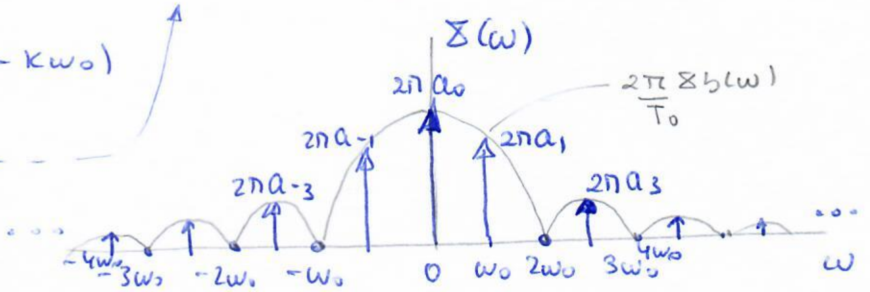
$$X_b(\omega) = \frac{1}{T_0/2} \cdot \left(2\frac{\text{sen}\ \omega T_0/4}{\omega} \right)\left(2\frac{\text{sen}\ \omega T_0/4}{\omega} \right)$$

$a_k = 2 \frac{\text{sen}^2(k\pi/2)}{\pi^2 k^2}$ ordezkatuz $\omega_0 = \frac{2\pi}{T_0}$

$a_0 = 1/2$, $a_{\pm 1} = \frac{2}{\pi^2}$, $a_{\pm 2} = 0$, $a_{\pm 3} = 2/9\pi^2$, $a_{\pm 4} = 0$, ...

d] $X(\omega) = 2\pi \cdot \sum_{k=-\infty}^{\infty} a_k\, \delta(\omega - k\omega_0)$

con $a_k = \frac{X_b(k\omega_0)}{T_0}$



e] $Y(\omega) = X(\omega) H(\omega) = 2\pi \sum_{k=-2}^{2} b_k\, \delta(\omega - k\omega_0)$ pq el filtro limita en $\omega_c = 3\omega_0$; pasan 2 armónicos

$b_k = a_k \cdot H(k\omega_0)$

$b_0 = a_0 \cdot = 1/2$

$b_1 = a_1 \cdot H(\omega_0) = a_1 \cdot 2/3\, e^{-j\frac{T_0}{4}\frac{2\pi}{T_0}} = \frac{4}{3\pi^2} e^{-j\pi/2}$

$b_{-1} = a_{-1}\, H(-\omega_0) = \frac{4}{3\pi^2} e^{j\pi/2}$

$b_2 = \cancel{a_2} \cdot H(2\omega_0) = \emptyset = b_{-2}$.

$$Y(\omega) = 2\pi \cdot a_0\, \delta(\omega) + 2\pi \cdot \frac{4}{3\pi^2} e^{-j\pi/2} \delta(\omega - \omega_0) + 2\pi \frac{4}{3\pi^2} e^{j\pi/2} \delta(\omega + \omega_0)$$

$\Big\downarrow TF^{-1}$

$$y(t) = \frac{1}{2} + \frac{8}{3\pi^2} \cos(\omega_0 t - \pi/2)$$

TRATAMIENTO DE SEÑALES
Convocatoria extraordinaria

La puntuación total del examen es de 30 puntos divididos en:

Problema 1: 10 puntos. Todas las cuestiones tienen el mismo peso.

Problema 2: 10 puntos.

Problema 3: 10 puntos.

El tiempo estimado para resolver el examen es de dos horas.

PROBLEMA 1 (10 puntos, 30 minutos)

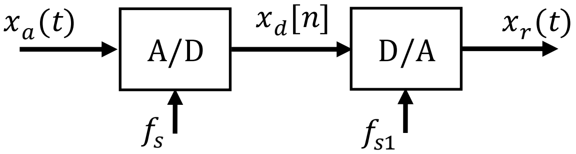
1. Obtener analíticamente y representar gráficamente la respuesta, $y(t)$, de un sistema de respuesta impulsional $h(t) = \Pi\left(\frac{t-2}{2}\right)$ a la señal de entrada $x(t) = e^{-t}\,u(t)$.

2. Representar $x[n]$, sabiendo que es una señal periódica de pulsación fundamental $\Omega_0 = \frac{3\pi}{5}$, que satisface la siguiente expresión:

$$x[n] = \sum_{k=-\infty}^{\infty} w[n - kN_0]$$

siendo N_0 su periodo fundamental, y $w[n]$ la señal cuya Transformada de Fourier es de la forma $W(\Omega) = \frac{sen(3\Omega/2)}{sen(\Omega/2)}$.

3. En el esquema de la figura se muestrea $x_a(t)$, sin filtro antialiasing, con $f_s = 2$kHz para obtener $x_d[n]$. Mediante un conversor digital analógico ideal trabajando con $f_{s1} = 4$kHz se obtiene $x_r(t)$ a partir de $x_d[n]$.



Para $x_a(t) = A_1\cos(2\pi f_1 t) + A_2\cos(2\pi f_2 t)$, con $f_1 = 500$Hz y $f_2 = 1500$Hz se pide:

a. Calcular $x_d[n]$.
b. Calcular $x_r(t)$.

PROBLEMA 2 (10 puntos, 30 minutos)

Sea la siguiente señal periódica: $p(t)=\sum_{k=-\infty}^{\infty}\delta(t-k\,T_0)$

a. Representar la señal e identificar su periodo. (1 pto)

b. Calcular sus coeficientes del desarrollo en serie de Fourier. (2 ptos)

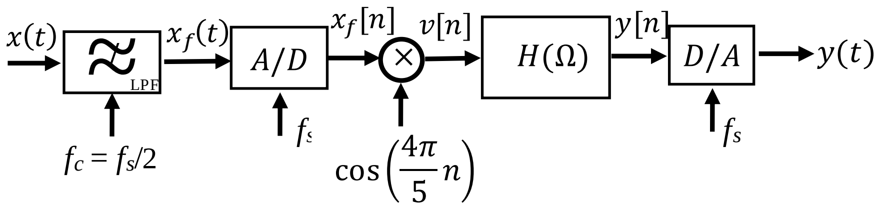
c. Obtener y representar gráficamente $P(\omega)$. (2 ptos)

d. Dada la señal $y(t)=x(t)\cdot p(t)$ y siendo $x(t)=\left(\frac{\sin\left(\frac{\omega_0 t}{3}\right)}{\pi t}\right)^2$, calcular y representar gráficamente $Y(\omega)$. (5 ptos)

Nota: $\Lambda\left(\frac{\omega}{\omega_c}\right)=\frac{1}{\omega_c}\cdot\Pi\left(\frac{\omega}{\omega_c}\right)*\Pi\left(\frac{\omega}{\omega_c}\right)$

PROBLEMA 3 (10 puntos, 30 minutos)

En el esquema de la figura la señal de entrada es: $x(t)=\sum_{k=-\infty}^{\infty}\Pi\left(\frac{t-k\,T_0}{T_0/_2}\right)$



a. Representar gráficamente x(t). (0.5 pto)

b. Calcular y representar gráficamente $X(\omega)$. (2.5 ptos)

c. Obtener $x_f(t)$ como suma de cosenos si $f_s=5/T_0$. (1.5 ptos)

d. Obtener la secuencia, $x_f[n]$, a la salida del conversor A/D y su espectro $X_f(\Omega)$. (2 ptos)

e. Obtener la secuencia, $v[n]$, tras el modulador y su espectro $V(\Omega)$. (1.5 ptos)

f. Obtener la señal de salida $y(t)$ si el sistema $H(\Omega)$ es un filtro paso alto ideal de pulsación de corte $\Omega_c=3\pi/5$. (2 ptos)

Universidad del País Vasco
Euskal Herriko Unibertsitatea
BILBOKO INGENIARITZA ESKOLA
ESCUELA DE INGENIERÍA DE BILBAO

Telekomunikazio Teknologiaren Ingeniaritzako Gradua
2017-2018

SEINALEEN PROZESATZEA
Ezohiko deialdia

Azterketak 3 ariketa ditu. Ariketa bakoitzak 10 puntu balio ditu, eta 1. ariketako galdera guztiek pisu berdina dute. Bi ordu dituzue.

1. ARIKETA (10 puntu, 30 minutu)

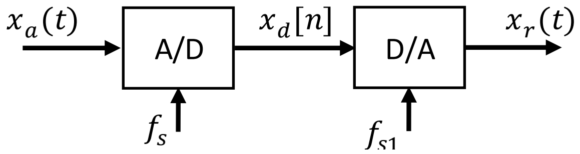
1. Lortu adierazpen analitikoa eta irudikatu $h(t) = \Pi\left(\frac{t-2}{2}\right)$ pultsu erantzuna duen sistemak ematen duen irteera-seinalea, $y(t)$, honako sarrera-seinalearentzat: $x(t) = e^{-t}\,u(t)$.

2. Irudikatu $x[n]$ seinale periodikoa, jakinda oinarrizko pultsazioa $\Omega_0 = \frac{3\pi}{5}$ duela eta honakoa betetzen duela:

$$x[n] = \sum_{k=-\infty}^{\infty} w[n - kN_0]$$

non N_0 bere oinarrizko periodoa den, eta $w[n]$ seinalearen Fourierren transformatua $W(\Omega) = \frac{sen(3\Omega/2)}{sen(\Omega/2)}$.

3. Irudiko $x_a(t)$ seinalea lagintzen da antialiasing iragazkirik gabe $f_s = 2\text{kHz}$ laginketa-maiztasunarekin. Lortzen den $x_d[n]$ seinalea D/A bihurgailu ideal baten bidez bihurtzen da, $f_{s1} = 4\text{kHz}$ laginketa-maiztasunarekin, $x_r(t)$ izateko.



Hartu $x_a(t) = A_1\cos(2\pi f_1 t) + A_2\cos(2\pi f_2 t)$, non $f_1 = 500\text{Hz}$ eta $f_2 = 1500\text{Hz}$ diren, eta honakoak egin:

   a. Kalkulatu $x_d[n]$.
   b. Kalkulatu $x_r(t)$.

2. ARIKETA (10 puntu, 30 minutu)

Izan bedi honako seinale periodikoa: $p(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT_0)$

a. Irudikatu $p(t)$ seinalea eta identifikatu bere oinarrizko periodoa. (1 p)

b. Fourierren seriezko garapeneko koefizienteak kalkulatu. (2 p)

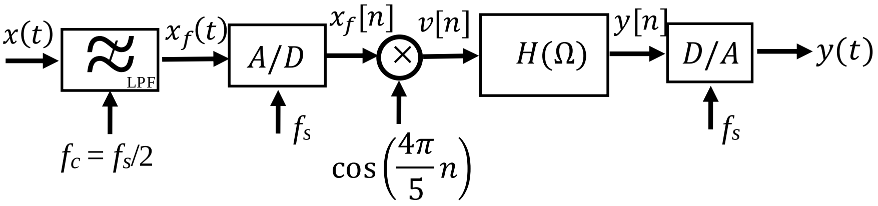
c. Lortu eta irudikatu $p(t)$ seinalearen Fourierren transformatua $P(\omega)$. (2 p)

d. Izan bedi $y(t) = x(t) \cdot p(t)$ non $x(t) = \left(\frac{\sin\left(\frac{\omega_0 t}{3}\right)}{\pi t}\right)^2$ den. Kalkulatu eta irudikatu y(t) seinalearen Fourierren transformatua, $Y(\omega)$. (5 p)

Oharra: $\Lambda\left(\frac{\omega}{\omega_c}\right) = \frac{1}{\omega_c} \cdot \prod\left(\frac{\omega}{\omega_c}\right) * \prod\left(\frac{\omega}{\omega_c}\right)$

3. ARIKETA (10 puntu, 30 minutu)

Izan bedi irudiko eskema, non sarrera-seinalea $x(t) = \sum_{k=-\infty}^{\infty} \prod\left(\frac{t-k\,T_0}{T_0/2}\right)$ den.



a. Irudikatu x(t) seinalea. (0.5 p)

b. Kalkulatu eta irudikatu $X(\omega)$. (2.5 p)

c. Adierazi $x_f(t)$ seinalea cosinuen batura bezala, $f_s=5/T_0$ hartuta. (1.5 p)

d. Lortu $x_f[n]$, A/D bihurgailuaren irteera-seinalea, eta bere espektroa $X_f(\Omega)$. (2 p)

e. Lortu $v[n]$, modulatzailearen irteera, eta bere espektroa $V(\Omega)$. (1.5 p)

f. Lortu irteera-seinalea, $y(t)$, $H(\Omega)$ goi-paseko iragazki ideala baldin bada mozketa-maiztasuna $\Omega_c=3\pi/5$ duena. (2 p)

Telecommunications Engineering Degree
2017-2018

SIGNAL PROCESSING
Extraordinary exam

The estimated time to solve the exam are 2 hours.
The 3 short questions in problem 1 have all the same value.

PROBLEM 1 (10 points, 30 minutes)

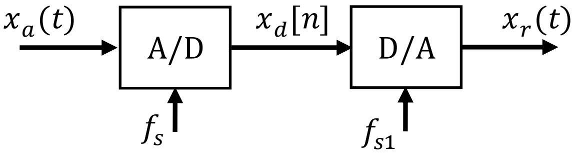
1. Analytically obtain and graphically sketch the output-signal, $y(t)$, of an LTI system with impulse response $h(t) = \Pi\left(\frac{t-2}{2}\right)$ and input-signal $x(t) = e^{-t}\,u(t)$.

2. Sketch $x[n]$, knowing that it is a periodic signal with fundamental angular frequency $\Omega_0 = \frac{3\pi}{5}$, and the following expression:

$$x[n] = \sum_{k=-\infty}^{\infty} w[n - kN_0]$$

   where N_0 is the fundamental period, and $w[n]$ a signal whose Fourier Transform is $W(\Omega) = \frac{sen(3\Omega/2)}{sen(\Omega/2)}$.

3. In the block diagram of the figure $x_a(t)$ is sampled without antialiasing filter and sampling frequency $f_s = 2kHz$, to obtain the signal $x_d[n]$. Using a digital to analog converter with $f_{s1} = 4kHz$ we obtain $x_r(t)$ from $x_d[n]$.



   For $x_a(t) = A_1\cos(2\pi f_1 t) + A_2\cos(2\pi f_2 t)$, where $f_1 = 500Hz$ and $f_2 = 1500Hz$, answer the following questions:
   a. Compute $x_d[n]$.
   b. Compute $x_r(t)$.

PROBLEM 2 (10 points, 30 minutes)

Consider the following periodic signal: $p(t)=\sum_{k=-\infty}^{\infty}\delta(t-k\,T_0)$

a. Sketch the signal and identify its period. (1 point)

b. Compute its Fourier series coefficients. (2 points)

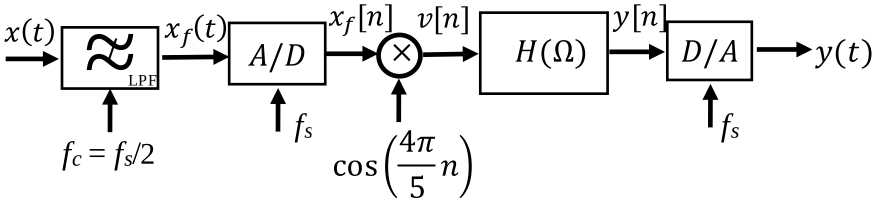
c. Obtain and represent its Fourier transform, $P(\omega)$. (2 points)

d. Given the signal $y(t)=x(t)\,\cdot\,p(t)$, where $x(t)=\left(\frac{\sin\left(\frac{\omega_0 t}{3}\right)}{\pi t}\right)^2$, compute and sketch its Fourier transform $Y(\omega)$. (5 points)

Nota: $\Lambda\left(\frac{\omega}{\omega_c}\right)=\frac{1}{\omega_c}\cdot\Pi\left(\frac{\omega}{\omega_c}\right)*\Pi\left(\frac{\omega}{\omega_c}\right)$

PROBLEM 3 (10 points, 30 minutes)

The input signal to the system of the figure is: $x(t)=\sum_{k=-\infty}^{\infty}\Pi\left(\frac{t-k\,T_0}{T_0/2}\right)$



a. Graphically sketch x(t). (0.5 points)

b. Compute and graphically sketch X(ω), its Fourier transform. (2.5 points)

c. Obtain x_f(t) as a sum of cosines if f_s=5/T_0. (1.5 points)

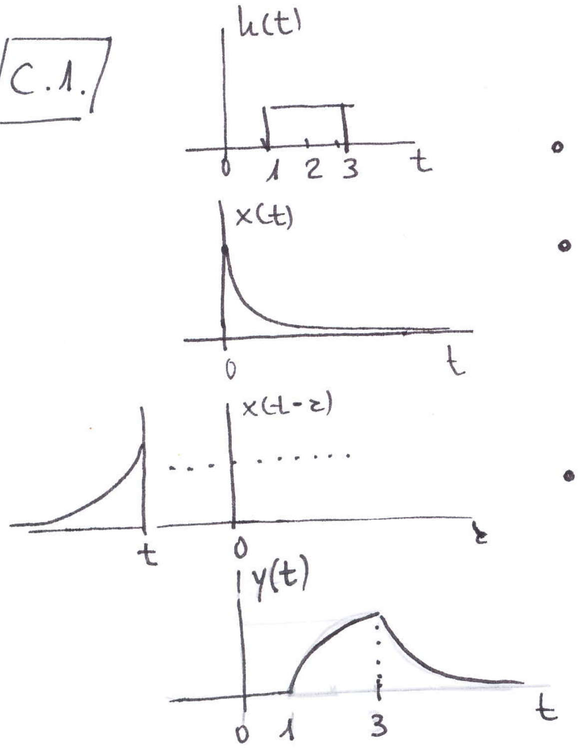
d. Obtain, x_f[n], at the output of the AD converter, and its spectrum X_f(Ω). (2 points)

e. Obtain the sequence after the modulator, v[n] and its spectrum V(Ω). (1.5 points)

f. Obtain the output signal y(t) if the system H(Ω) is an ideal high-pass filter with cutoff frequency Ω_c=3π/5. (2 points)

2017_18 TdS - Extra. CUESTIONES.

C.1.



$y(t) = x(t) * h(t)$

- $y(t) = \varnothing \quad \forall\, t < 1$
- $1 \leq t \leq 3 \quad y(t) = \int_1^t h(\tau)x(t-\tau)\,d\tau = \int_1^t 1 \cdot e^{-(t-\tau)}\,d\tau =$

$$= 1 - e^{-t+1}$$

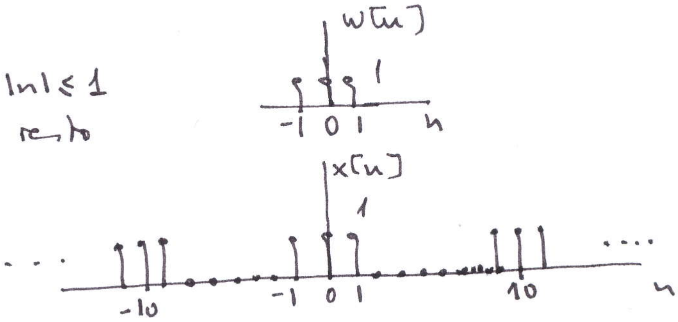
- $t \geq 3 \quad y(t) = \int_1^3 1 \cdot e^{-(t-\tau)}\,d\tau = e^{3-t} - e^{1-t}$

C.2

$\Omega_0 = \frac{3\pi}{5} \rightarrow N_0 = 10$

$w[n] = TF^{-1}\{W(\Omega)\} = \begin{cases} 1 & |n| \leq 1 \\ 0 & \text{resto} \end{cases}$

$x[n] = \sum_k w[n - kN_0]$



C.3

$x_a(t) = A_1 \cos 2\pi f_1 t + A_2 \cos 2\pi f_2 t$

$x_d[n] = x_a(t = nT_s) = A_1 \cdot \cos 2\pi \frac{500}{2000} n + A_2 \cos 2\pi \frac{1500}{2000} n =$

$T_s = \frac{1}{f_s} = \frac{1}{2000}$

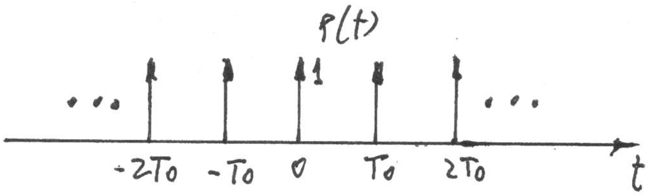
$= A_1 \cos \frac{\pi}{2} n + A_2 \cos \frac{2\pi \cdot 3n}{4} =$

$= A_1 \cos \frac{\pi}{2} n + A_2 \cos \frac{\pi}{2} n = (A_1 + A_2) \cos \frac{\pi}{2} n$

$x_r(t) = x_d[n = t f_s] = (A_1 + A_2) \cos \frac{\pi}{2} . 4000 t = (A_1 + A_2) \cos 2\pi \cdot 1000 t$

PROBLEMA 2

a) $p(t)=\sum_{k}\delta(t-kT_0)$



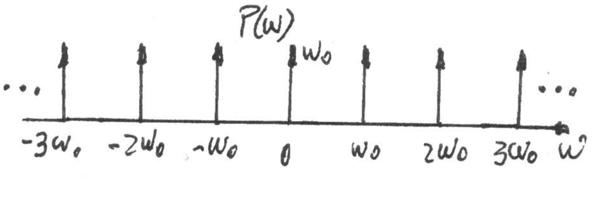
tren de deltas periódico de periodo T_0 y pulsación fundamental $\omega_0=\frac{2\pi}{T_0}$

b) a_k?

Dominio tiempo: $a_k=\frac{1}{T_0}\int_{T_0}p(t)e^{-jk\omega_0 t}dt=\frac{1}{T_0}\int_{0}^{T_0^-}p(t)e^{-jk\omega_0 t}dt=\frac{1}{T_0}\int_{0}^{T_0^-}\delta(t)e^{-jk\omega_0 t}dt=\frac{1}{T_0}\int_{0}^{T_0^-}\delta(t)dt=\frac{1}{T_0}$

Dominio frecuencia: $a_k=\frac{1}{T_0}X_b(\omega)\Big|_{\omega=k\omega_0}$; $x_b(t)=\delta(t)\Rightarrow X_b(\omega)=1\Rightarrow a_k=\frac{1}{T_0}\ \forall k$

c) $P(\omega)=\mathcal{F}\{p(t)\}=\sum_{k}2\pi a_k\delta(\omega-k\omega_0)=\sum_{k}\frac{2\pi}{T_0}\delta(\omega-k\omega_0)$

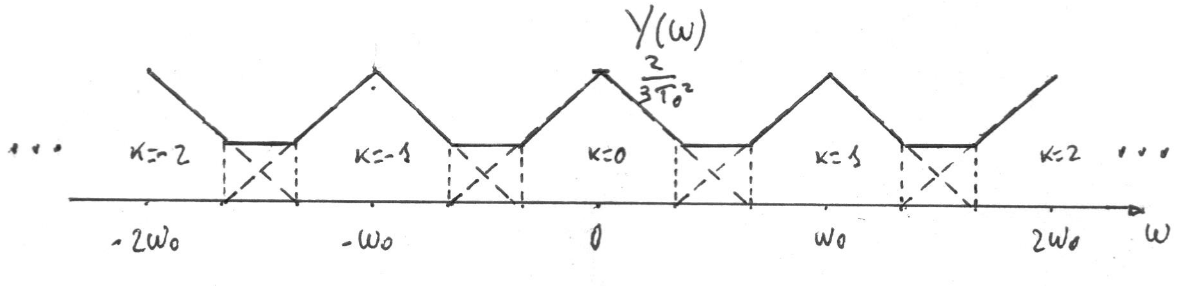


d) $y(t)=x(t)\cdot p(t)\xrightarrow{\mathcal{F}}Y(\omega)=\frac{1}{2\pi}X(\omega)*P(\omega)$

$$x(t)=\underbrace{\frac{\sin\left(\frac{\omega_0 t}{3}\right)}{\pi t}}_{x_1(t)}\cdot\underbrace{\frac{\sin\left(\frac{\omega_0 t}{3}\right)}{\pi t}}_{x_1(t)}\qquad X_1(\omega)=\mathcal{F}\{x_1(t)\}=\Pi\left(\frac{\omega}{2\frac{\omega_0}{3}}\right)$$

$$X(\omega)=\frac{1}{2\pi}X_1(\omega)*X_1(\omega)=\frac{1}{2\pi}\,2\frac{\omega_0}{3}\,\Lambda\left(\frac{\omega}{2\frac{\omega_0}{3}}\right)=\frac{2}{3T_0}\Lambda\left(\frac{\omega}{2\frac{\omega_0}{3}}\right)$$

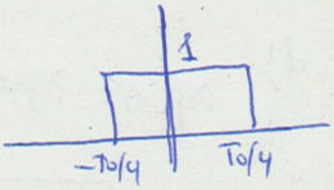
$$Y(\omega)=\frac{1}{2\pi}\cdot\frac{2}{3T_0}\Lambda\left(\frac{\omega}{2\frac{\omega_0}{3}}\right)*\sum_{k}\frac{2\pi}{T_0}\delta(\omega-k\omega_0)=\frac{2}{3T_0^2}\sum_{k}\Lambda\left(\frac{\omega-k\omega_0}{2\frac{\omega_0}{3}}\right)$$

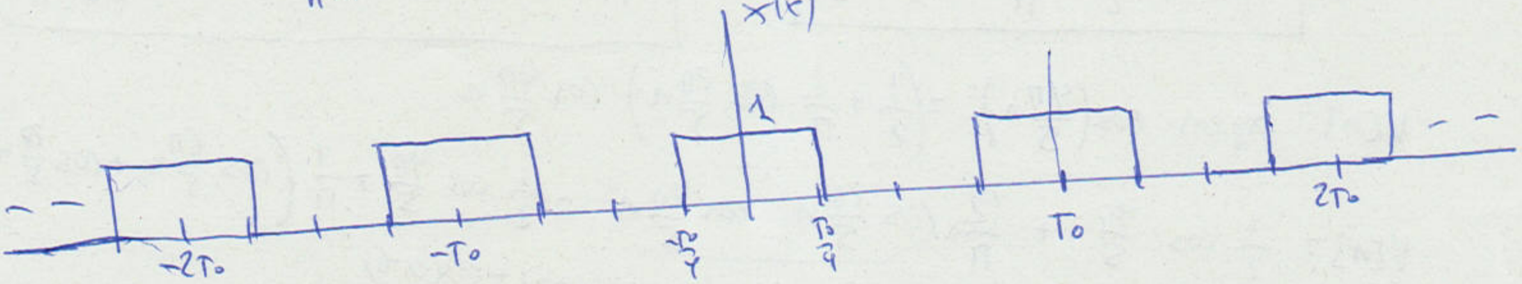


PROBLEMA 3

a) $x(t) = \sum_{k=-\infty}^{\infty} \Pi\left(\frac{t-kT_0}{T_0/2}\right)$

$\Pi\left(\frac{t}{T_0/2}\right)$



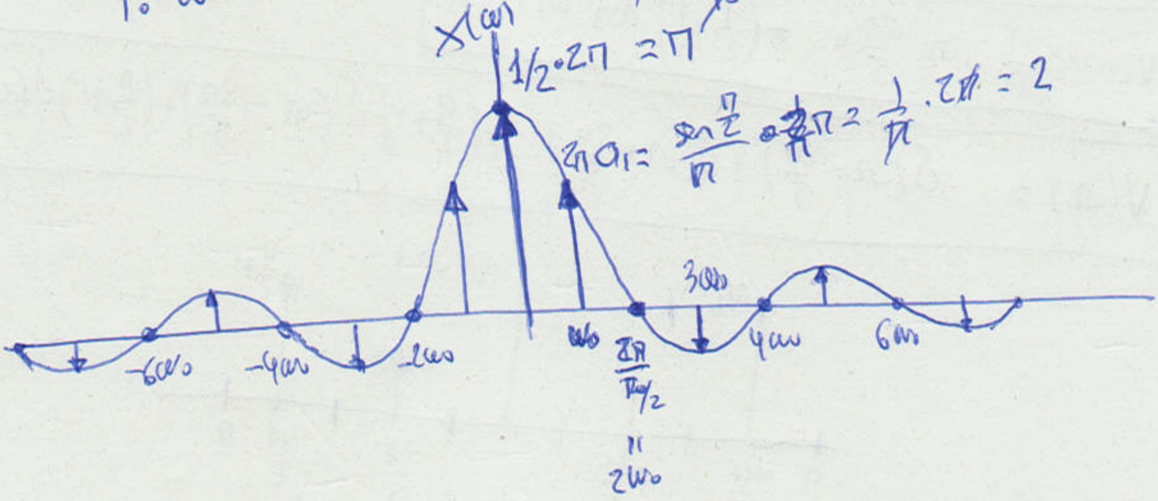


b) $X(\omega) = 2\pi \sum_{k=-\infty}^{\infty} a_k \delta(\omega - k\omega_0)$

$a_k = \left.\frac{X_b(\omega)}{T_0}\right|_{\omega=k\omega_0}$

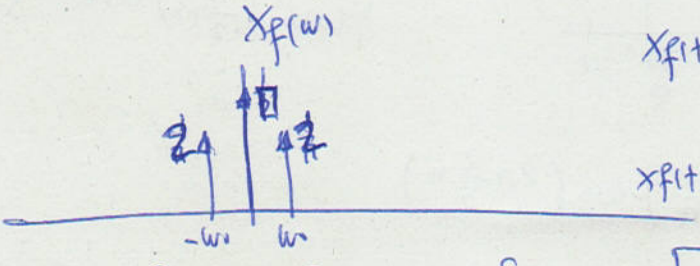
$x_b(t) = \Pi\left(\frac{t}{T_0/2}\right) \xrightarrow{F} X_b(\omega) = \frac{2\,\text{sen}\,\omega\frac{T_0}{4}}{\omega}$

$$a_k = \left.\frac{2\,\text{sen}\,\omega\frac{T_0}{4}}{T_0\,\omega}\right|_{\omega=k\omega_0} = \frac{\cancel{2}\,\text{sen}\left(k\frac{\cancel{2}\pi}{\cancel{T_0}}\frac{\cancel{T_0}}{\cancel{4}}\right)}{\cancel{T_0}\frac{\cancel{2}\pi}{\cancel{T_0}}k} = \frac{\text{sen}\,\frac{k\pi}{2}}{k\pi}$$



$2\pi a_1 = \frac{\text{sen}\,\frac{\pi}{2}}{\pi} \cdot 2\pi = \frac{1}{\pi} \cdot 2\cancel{\pi} = 2$

c) $f_s = \frac{5}{T_0} = 5 f_0$ $\quad f_c = \frac{5f_0}{2}$ $\quad \omega_c = 2'5\,\omega_0$



$x_f(t) = \sum_{k=-1}^{1} a_k e^{jk\omega_0 t}$

$x_f(t) = \frac{1}{\pi} e^{-j\omega_0 t} + \frac{1}{2} + \frac{1}{\pi} e^{j\omega_0 t}$

$X_f(\omega) = \pi\delta(\omega) + 2\,\delta(\omega-\omega_0) + 2\delta(\omega+\omega_0)$

$\boxed{x_f(t) = \frac{1}{2} + \frac{2}{\pi}\cos\omega_0 t}$

$f_s = 5f_0 = 5\frac{\omega_0}{2\pi}$

d) $x_f[n] = x_f(t)\Big|_{t=\frac{n}{f_s}} = \frac{1}{2} + \frac{2}{\pi}\cos\left(\cancel{\omega_0}\frac{n}{\frac{5\omega_0}{2\pi}}\right)$

$$\boxed{x_f[n] = \frac{1}{2} + \frac{2}{\pi}\cos\left(2\pi\frac{n}{5}\right)}$$

$$\boxed{X_f(\Omega) = \pi\delta(\Omega) + 2\,\delta\left(\Omega - \frac{2\pi}{5}\right) + 2\delta\left(\Omega + \frac{2\pi}{5}\right) \quad |\Omega| \le \pi}$$

$V[n] = x_f[n]\cdot\cos\left(\frac{4\pi}{5}n\right) = \left(\frac{1}{2} + \frac{2}{\pi}\cos\frac{2\pi}{5}n\right)\cos\frac{4\pi}{5}n$

$V[n] = \frac{1}{2}\cos\frac{4\pi}{5} + \frac{2}{\pi}\cos\frac{2\pi}{5}n\cos\frac{4\pi}{5}n = \frac{1}{2}\cos\frac{4\pi}{5} + \frac{1}{\pi}\left(\cos\frac{6\pi}{5}n + \cos\frac{2\pi}{5}n\right)$

$2\cos a\cos b = \cos(a+b) + \cos(a-b)$

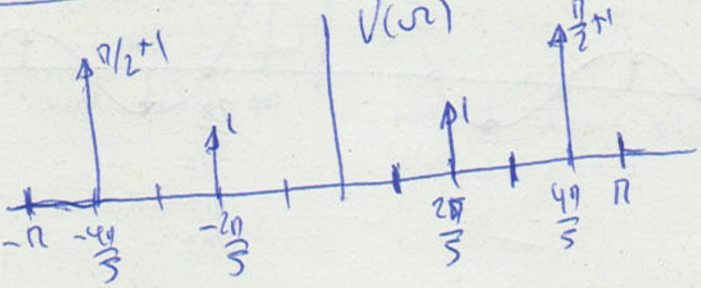
$V[n] = \frac{1}{\pi}\cos\frac{2\pi}{5}n + \frac{1}{2}\cos\frac{4\pi}{5}n + \frac{1}{\pi}\cos\frac{6\pi}{5}n$ $\quad 2\pi\frac{6}{10} = 2\pi\frac{3}{5}$

corregir. $f_d = \frac{3}{5} > \frac{1}{2} \Rightarrow \frac{3}{5} - 1 = \frac{-2}{5} \overset{\cos}{=} \frac{2}{5}$

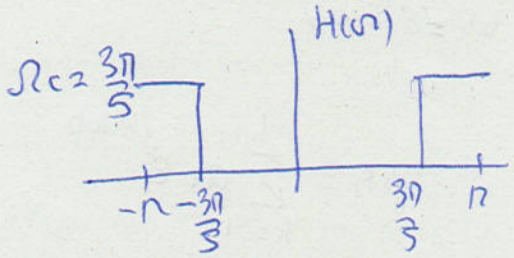
$V[n] = \frac{1}{\pi}\cos\frac{2\pi}{5}n + \frac{1}{2}\cos\frac{4\pi}{5}n + \frac{1}{\pi}\cos 2\pi\frac{2}{5}n$

$$\boxed{V[n] = \frac{1}{\pi}\cos\frac{2\pi}{5}n + \left(\frac{1}{2} + \frac{1}{\pi}\right)\cos 2\pi\cdot\frac{2}{5}n.}$$

$$\boxed{V(\Omega) = \delta\left(\Omega - \frac{2\pi}{5}\right) + \delta\left(\Omega + \frac{2\pi}{5}\right) + \left(\frac{\pi}{2}+1\right)\delta\left(\Omega - \frac{4\pi}{5}\right) + \left(\frac{\pi}{2}+1\right)\delta\left(\Omega + \frac{4\pi}{5}\right) \quad |\Omega| \le \pi}$$



e)



$Y(\Omega) = V(\Omega)\cdot H(\Omega)$

$Y(\Omega) = \left(\frac{\pi}{2}+1\right)\delta\left(\Omega - \frac{4\pi}{5}\right) + \left(\frac{\pi}{2}+1\right)\delta\left(\Omega + \frac{4\pi}{5}\right) \quad |\Omega| \le \pi$

$Y[n] = \left(\frac{1}{2} + \frac{1}{\pi}\right)\cos\left(2\pi\frac{2}{5}n\right)$

$Y(t) = Y[n]\Big|_{n = t f_s = t\cdot\frac{5\omega_0}{2\pi}} = \left(\frac{1}{2} + \frac{1}{\pi}\right)\cos\left(\cancel{2\pi}\frac{3}{\cancel{5}}\cdot\frac{\cancel{5}\omega_0}{\cancel{2\pi}}t\right)$

$$\boxed{Y(t) = \left(\frac{1}{2} + \frac{1}{\pi}\right)\cos\left(3\omega_0 t\right)}$$