UPV EHU

Telekomunikazioaren Ingeniaritza Teknikoko Gradua

2014-2015

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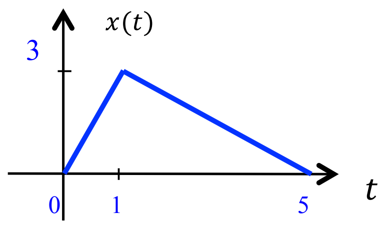
SEINALEEN PROZESAKETA: LEHEN PARTZIALA

Azterketak 3 ariketa ditu. Ariketa bakoitzak 10 puntu balio ditu, eta 1. ariketako galdera guztiek pisu berdina dute. Bi ordu dituzue.

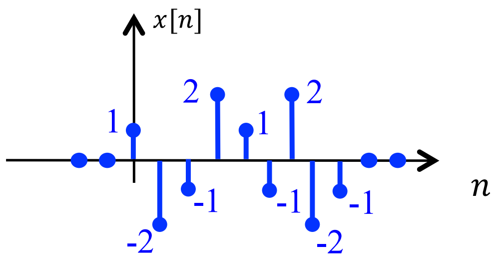
1.ariketa (10 puntu, 40 minutu)

1. GALDERA

a) Irudiko x(t) seinalearentzat irudikatu x(2t+1).



b) Analitikoki eta grafikoki adierazi y[n] sekuentzia, irudiko x[n] sekuentzia 2 faktorearekin detxematuz lortzen dena.



2. GALDERA

a) Lortu analitikokio eta adierazi grafikoki honako seinalea: $x(t)= \Pi\left(\frac{t-3}{2}\right) * \Pi\left(\frac{t+2}{2}\right)$

b) Kalkulatu $y[n] = x_1[n] * x_2[n]$ non: $x_1[n]=\{1, \underline{2}, -1, 3\}$ eta $x_2[n]=\{\underline{1}, 0, 1\}$ baitiren.

3. GALDERA

Kausalitatea eta egonkortasuna aztertu honako pultsu-erantzunak dituzten sistementzat:

a) $h(t) = e^t u(-1-t)$

b) $h[n] = \left(\frac{1}{2}\right)^n u[-n]$

2. ariketa (10 puntu, 40 minutu)

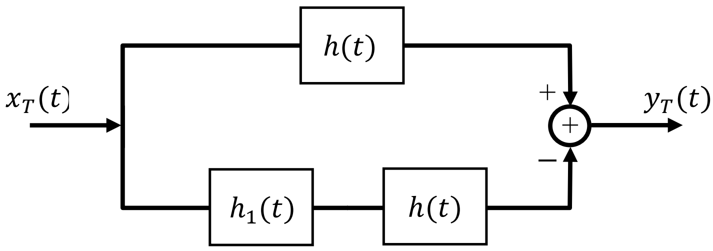
Izan bedi honako erlazioaz definitzen den sistema, non x(t) eta y(t) sarrera- eta irteera-seinaleak baitiren:

$$y(t) = \int_{-\infty}^{t} x(\tau)d\tau$$

a) Aztertu sistema lineala, aldakorra, kausala edota egonkorra den. **(3 puntu)**

b) Kalkulatu sistemaren pultsu erantzuna, $h(t)$, eta horren arabera aztertu kausala edota egonkorra den. **(2 puntu)**

c) Haurreko sistema, h(t), irudiko eskeman erabiltzen da. Kalkulatu $y_T(t)$ eta $x_T(t)$ erlazionatzen dituen sistema osoaren pultsu erantzuna, $h_T(t)$, kontuan hartuz: $h_1(t) = \delta(t-2)$. Aztertu $h_T(t)$ sistema kausala edota egonkorra den. **(3 puntu)**



d) Kalkulatu irteera-seinalea, $y_T(t)$, sarrera-seinalea honakoa bada: $x_T(t) = u(1-t)$ **(1 puntu)**

3. ariketa (10 puntu, 40 minutu)

Izan bedi honako erlazioaz definitzen den sistema, non x[n] eta y[n] sarrera- eta irteera-seinaleak baitiren:

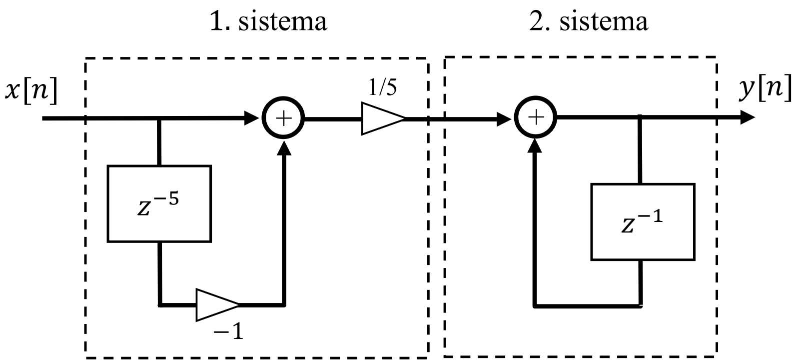
$$y[n] = \frac{1}{5}\sum_{k=0}^{4} x[n-k]$$

a) Adierazi sistemaren ordena eta mota. **(1 puntu)**

b) Lortu sistemaren pultsu-erantzuna. **(1 puntu)**

c) Iragazi honako seinalea: $x[n] = \{\underline{1}, 1,1,1,1,1,1,1,1,1\}$. **(1 puntu)**

d) Irudiko sistemarentzat kalkulatu x[n] eta y[n] erlazionatzen dituen diferentzia-ekuazioa. Seinaleak adierazi irudian. **(1 puntu)**



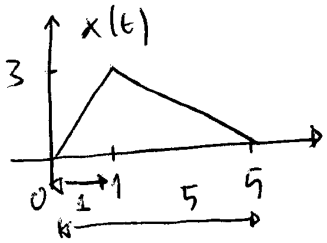
OHARRA: z^{-5} bost lagineko atzerapena adierazten du.

e) Sistemaren diferentzia-ekuazioatik abiatuz, kalkulatu sistema osoaren pultsu-erantzuna, h[n]. **(1 puntu)**

f) Lortu 1.sistemaren eta 2.sistemaren diferentzia-ekuazioak eta pultsu-erantzunak ($h_1[n]$ eta $h_2[n]$). Aztertu sistema bakoitzaren eta sistema osoaren egonkortasuna. **(3 puntu)**

g) Berriro kalkulatu sistema osoaren pultsu erantzuna, h[n], aurreko atalean lortutako $h_1[n]$ eta $h_2[n]$ erabiliz. Arrazoitu egiten dituzun eragiketak. **(2 puntu)**

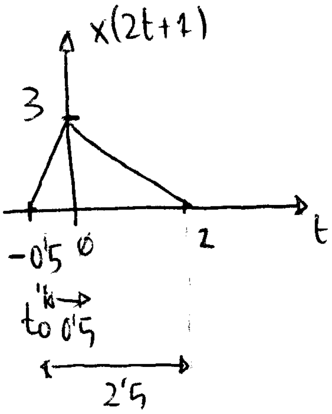
(1) (a)



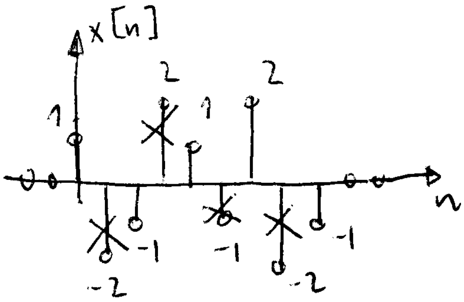
$x(2t+1)$

$\hookrightarrow$ Time compressed by a factor of 2

$2t_0+1=\emptyset \qquad t_0=-\frac{1}{2} \Rightarrow$ Time shift.

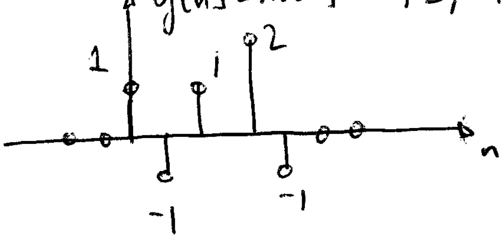


(b)



$y[n]=x[2n]$ we remove 1 every two samples (Decimate by a factor of 2)

$y[n]=x[2n] = \{ \underline{1}, -1, 1, 2, -1 \}$



(2) $x(t) = \Pi\left(\frac{t-3}{2}\right) * \Pi\left(\frac{t+2}{2}\right)$

We know that $\Pi\left(\frac{t}{T}\right) * \Pi\left(\frac{t}{T}\right) = T\Lambda\left(\frac{t}{T}\right)$

$T=2$: $\Pi\left(\frac{t}{2}\right) * \Pi\left(\frac{t}{2}\right) = 2\Lambda\left(\frac{t}{2}\right)$

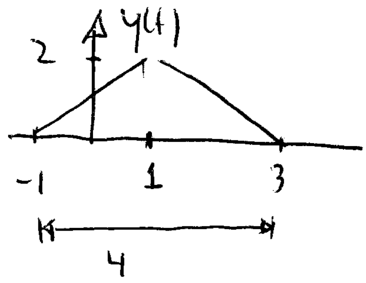
And we know (time-shifts in convolution)

$x(t-t_1) * h(t-t_2) = y(t-(t_1+t_2))$

in this case $t_1=3$, $t_2=-2$

$\Pi\left(\frac{t-3}{2}\right) * \Pi\left(\frac{t+2}{2}\right) = 2\cdot\Lambda\left(\frac{t-(3-2)}{2}\right)$

$y(t) = 2\Lambda\left(\frac{t-1}{2}\right)$



(b) $y[n] = x_1[n] * x_2[n]$ $\begin{cases} x_1[n] = \{\underset{\uparrow}{1}, \underline{2}, -1, 3\} \\ x_2[n] = \{\underline{1}, 0, 1\} \end{cases}$

$x_1[n] = \delta[n+1] + 2\delta[n] - \delta[n-1] + 3\delta[n-2]$
$x_2[n] = \delta[n] + \delta[n-2]$
$x_1[n] * x_2[n] =$

distributiva

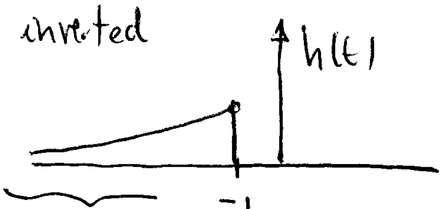
$= \{\delta[n+1] + 2\delta[n] - \delta[n-1] + 3\delta[n-2]\} * \{\delta[n] + \delta[n-2]\} =$

$\delta[n-k_1] * \delta[n-k_2] = \delta[n-k_1-k_2]$
delb es el elem. unidad $*$ y la propiedad desplaz.

$= \delta[n+1] + 2\delta[n] - \cancel{\delta[n-1]} + 3\delta[n-2] +$
$+ \delta[\cancel{n-1}] + 2\delta[n-2] - \delta[n-3] + 3\delta[n-4]$
$= \delta[n+1] + 2\delta[n] + 5\delta[n-2] - \delta[n-3] + 3\delta[n-4]$
$= \boxed{\{1, \underline{2}, 0, 5, -1, 3\} = y[n]}$

(3) Conditions for causality $\begin{cases} h(t) = 0 & t < 0 \\ h[n] = 0 & n < 0 \end{cases}$, stability $\begin{cases} \int_{-\infty}^{\infty} |h(t)| dt < \infty \\ \sum_{n=-\infty}^{\infty} |h[n]| < \infty \end{cases}$

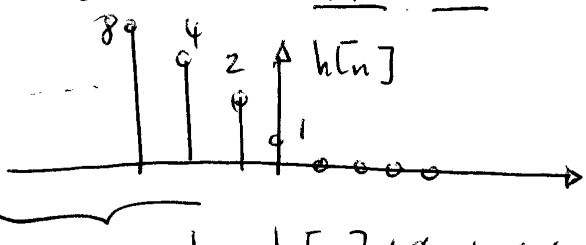
(a) $h(t) = e^t u(\underline{-1-t})$ → $t = -1$ and time inverted



non-causal $h(t) \neq 0$ $t < 0$

$\int_{-\infty}^{\infty} |h(t)| dt = \int_{-\infty}^{-1} e^t dt = \frac{e^t}{1}\Big]_{-\infty}^{-1} = e^{-1} - e^{-\infty} = e^{-1} < \infty \Rightarrow$ STABLE

(b) $h[n] = \left(\frac{1}{2}\right)^n u[\underline{-n}]$ → Time inverted



non causal $h[n] \neq 0$ $n < 0$

$\sum_{n=-\infty}^{\infty} |h[n]| = \sum_{n=-\infty}^{0} \left(\frac{1}{2}\right)^n =$

$= 1 + 2 + 2^2 + 2^3 \dots = \sum_{n=0}^{\infty} 2^n = \infty$ geometric series of $r=2$ (Non convergent)
UNSTABLE.

PROBLEMA 3

$$y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau$$

a) • $\boxed{\text{LINEAL?}}$ $S\{a x_1(t) + b x_2(t)\} = a S\{x_1(t)\} + b S\{x_2(t)\} = a y_1(t) + b y_2(t)$

$$\left.\begin{array}{l} x_1(t) \rightarrow y_1(t) = \int_{-\infty}^{t} x_1(\tau)\, d\tau \\ x_2(t) \rightarrow y_2(t) = \int_{-\infty}^{t} x_2(\tau)\, d\tau \end{array}\right\} \quad a y_1(t) + b y_2(t) = a \int_{-\infty}^{t} x_1(\tau)\, d\tau + b \int_{-\infty}^{t} x_2(\tau)\, d\tau \quad ①$$

$$a x_1(t) + b x_2(t) \rightsquigarrow y'(t) = \int_{-\infty}^{t} [a x_1(\tau) + b x_2(\tau)]\, d\tau = \int_{-\infty}^{t} a x_1(\tau)\, d\tau + \int_{-\infty}^{t} b x_2(\tau)\, d\tau = a \int_{-\infty}^{t} x_1(\tau)\, d\tau +$$

$$+ b \int_{-\infty}^{t} x_2(\tau)\, d\tau \quad ②$$

① = ② ⇒ LINEAL

• $\boxed{\text{INVARIANTE?}}$ $S\{x(t - t_0)\} = y(t - t_0)$

$$\left.\begin{array}{l} x(t - t_0) \rightarrow y'(t) = \int_{-\infty}^{t} x(\tau - t_0)\, d\tau \quad ① \\ y(t - t_0) = \int_{-\infty}^{t - t_0} x(\tau)\, d\tau \quad ② \end{array}\right\}$$

Aplicando en ① el cambio de variable

$\tau' = \tau - t_0$; $d\tau' = d\tau$

$\tau \rightsquigarrow -\infty \Rightarrow \tau' \rightsquigarrow -\infty$

$\tau = t \Rightarrow \tau' = t - t_0$

queda: $y'(t) = \int_{-\infty}^{t - t_0} x'(\tau)\, d\tau'$

① = ② ⇒ INVARIANTE

• $\boxed{\text{CAUSAL?}}$ $y(t)$ sólo depende de valores de la entrada en el instante actual y en instantes pasados

$y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau$; $y(t)$ depende de valores de $x(t)$ desde $-\infty$ a t, es decir, el sistema es CAUSAL

• $\boxed{\text{ESTABLE?}}$ Ante entrada $x(t)$ acotada, $y(t)$ siempre acotada

ej: $x(t) = u(t)$ (acotado) $\rightarrow y(t) = \int_{-\infty}^{t} u(\tau)\, d\tau = \int_{0}^{t} 1\, d\tau = t \cdot u(t)$ (NO ACOTADA)

El sistema es INESTABLE

b) $x(t) = \delta(t) \Rightarrow y(t) = h(t) = \int_{-\infty}^{t} \delta(\tau)\, d\tau = u(t)$

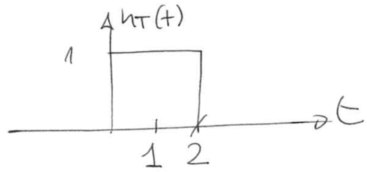
• CAUSAL? ⇒ $h(t) = 0 \quad \forall t < 0 \longrightarrow$ SÍ $\boxed{\text{CAUSAL}}$

• ESTABLE? ⇒ $\int_{-\infty}^{\infty} |h(t)|\, dt < \infty \Rightarrow \int_{-\infty}^{\infty} u(t)\, dt = t\Big]_{-\infty}^{\infty}$ ⇒ NO CONVERGE

$\boxed{\text{INESTABLE}}$

c) $h_T(t) = h(t) - h_1(t) * h(t) = h(t) - \delta(t-2) * h(t) = h(t) - h(t-2)$

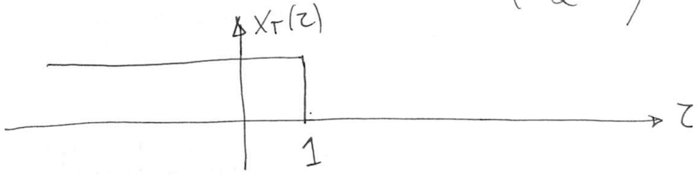
$h_T(t) = u(t) - u(t-2) = \Pi\left(\frac{t-1}{2}\right)$

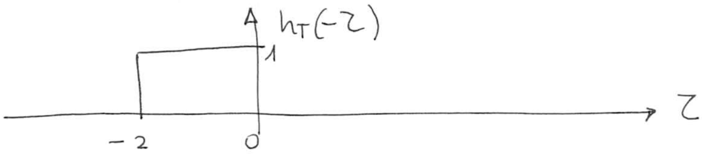


CAUSAL : $h_T(t) = 0 \quad \forall t < 0$

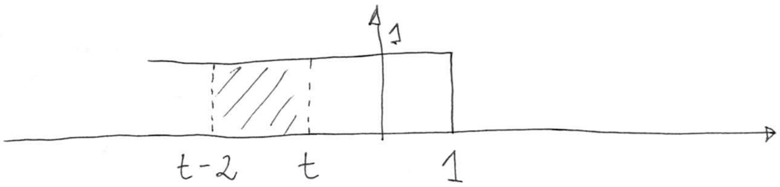
ESTABLE : $\int_{-\infty}^{\infty} |h_T(t)|\, dt = \int_0^2 1\, dt = 2$ /

d) $y_T(t) = x_T(t) * h_T(t) = u(1-t) * \Pi\left(\frac{t-1}{2}\right)$



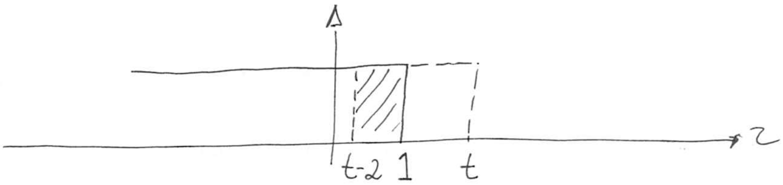


CASO 1



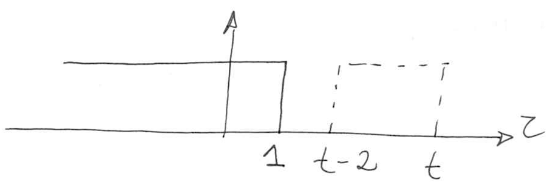
$t < 1 \rightarrow y_T(t) = \int_{t-2}^{t} 1\, d\tau = t - t + 2 = 2$ /

CASO 2

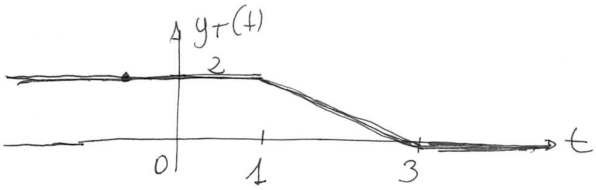


$1 < t < 3 \rightarrow y_T(t) = \int_{t-2}^{1} 1\, d\tau = 1 - t + 2 = 3 - t$ /

CASO 3 $\quad t > 3 \rightarrow y_T(t) = 0$

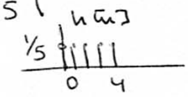


$$y_T(t) = \begin{cases} t < 1 & 2 \\ 1 \leq t \leq 3 & 3 - t \\ t > 3 & 0 \end{cases}$$



a) FIR, 4. maila

b) $h[n] = \frac{1}{5}\left(\delta[n] + \delta[n-1] + \delta[n-2] + \delta[n-3] + \delta[n-4]\right) = \frac{1}{5}\left(u[n] - u[n-5]\right)$



c) $x[n] = \{\underline{1}, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1\}$

$y[n] = \frac{1}{5}\left(x[n] + x[n-1] + x[n-2] + x[n-3] + x[n-4]\right)$

$y[0] = \frac{1}{5}\{1 + 0 + 0 + 0 + 0\} = 1/5$

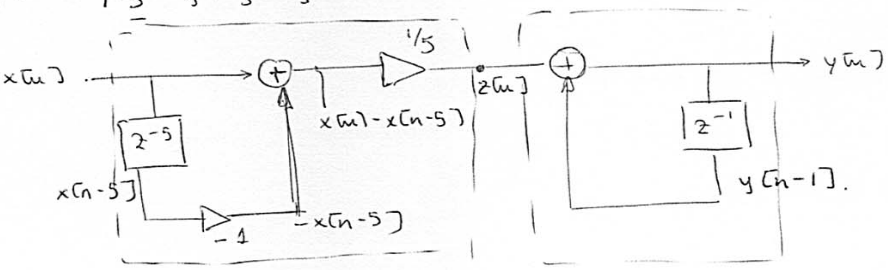
$y[1] = \frac{1}{5}\{1 + 1 + 0 + 0 + 0\} = 2/5$ $\quad y[2] = \frac{3}{5} \quad y[3] = \frac{4}{5} \quad y[4] = 1$

$\vdots$

$y[5] = y[6] = y[7] = y[8] = y[9] = 1$

$y[n] = \left\{\frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}, 1, 1, 1, 1, 1, 1\right\}$

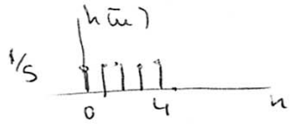
d)



$z[n] = \frac{1}{5}x[n] - 1/5\, x[n-5]$

$y[n] = z[n] + y[n-1] = \frac{1}{5}x[n] - \frac{1}{5}x[n-5] + y[n-1]$

e) $h[n] = \frac{1}{5}\delta[n] - \frac{1}{5}\delta[n-5] + h[n-1]$



$h[0] = 1/5 - 0 + 0 = 1/5$

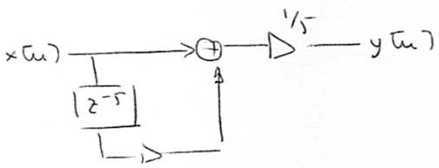
$h[1] = 0 - 0 + 1/5 = 1/5$

$h[2] = 0 - 0 + 1/5 = 1/5$

$h[3] = h[4] = 1/5$

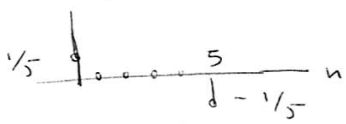
$h[n \geq 5] = \emptyset$

f)



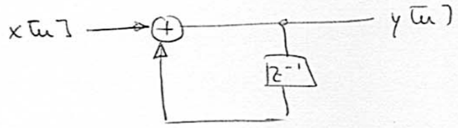
$y[n] = \frac{1}{5}x[n] - \frac{1}{5}x[n-5]$

$h_1[n] = \frac{1}{5}\delta[n] - \frac{1}{5}\delta[n-5]$



$\sum_n |h_1[n]| = \frac{1}{5} + \frac{1}{5} = \frac{2}{5} < \infty$

Sistema estable



$y[n] = x[n] + x[n-1]$

$h_2[n] = \delta[n] + h_2[n-1]$

$h_2[0] = 1$

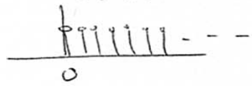
$h_2[1] = 1$

$h_2[2] = 1$

$\vdots$

$h_2[n \geq 0] = 1$

$h_2[n] = u[n]$



$\sum_n |h_2[n]| = \sum_n |u[n]| = \infty$

Sistema inestable

g) $h[n] = h_1[n] * h_2[n] = \left(\frac{1}{5}\delta[n] - \frac{1}{5}\delta[n-5]\right) * u[n]$

Prop. distributiva
+
Prop elemento neutro conv

Dos sistema en cascada

$h[n] = \frac{1}{5}u[n] - \frac{1}{5}u[n-5]$