SEINALEEN PROZESAKETA: AZKEN AZTERKETA
(Lehen partziala)

Azterketak 3 ariketa ditu. Ariketa bakoitzak 10 puntu balio ditu, eta 1. Ariketako galdera guztiek pisu berdina dute.

1. ARIKETA (10 puntu, 40 minutu)

1. Aztertu hurrengo sistema lineala den, denboran ez-aldakorra, kausata edota egonkorra:

   $$y(t) = x(t/2)$$

   Ondoko sitemaren pultsu erantzuna, h[n], kalkulatu eta aztertu kausaltasuna eta egonkortasuna:

   $$y[n] = \sum_{k=2}^{\infty} x[n-k]2^{k-2}$$

2. Kalkulatu eta irudikatu $x(t) = \prod\left(\frac{t-1}{2}\right) * \mathrm{e}^{-\mathrm{t}}\mathrm{u}(\mathrm{t})$ seinalea

3. Izan bedi ondoko pultsu erantzuna duen sistema:

   $$h[n] = \begin{cases} (1/2)^n & 0 \le n \le 4 \\ 0 & resto \end{cases}$$

   Kalkulatu y[n] erantzuna n=0:6 tartean, sarrerako seinalea hau izanik: x[n]={$\underline{1}$,0,1,0,1,0,1,0,1,0,1,0,...}

2. ARIKETA (10 puntu, 40 minutu)

Izan bedi irudiko sistema, non hiru sistema seriean konektatu diren. Sistemen pultsu-erantzunak ondokoak dira: $h_1(t) = \prod(t)$, $h_2(t) = 2\prod(t)$ eta $h_3(t) = \delta(t-1)$.

$$\longrightarrow \boxed{h_1(t)} \longrightarrow \boxed{h_2(t)} \longrightarrow \boxed{h_3(t)} \longrightarrow$$

a. Aztertu hiru sistemak egonkorrak edota kausalak diren. (2 p)

b. Kalkulatu sistemaren $h(t)$ pultsu erantzuna, hau da $y(t) = x(t) * h(t)$.
Aztertu sistema egonkorra edota kausala den. (3 p)

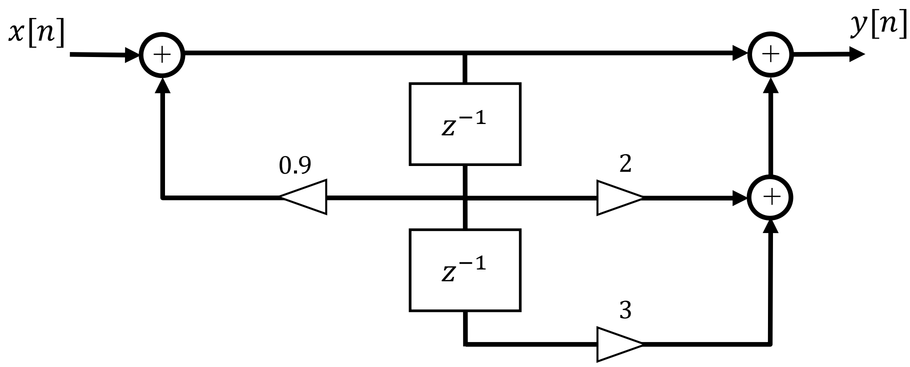
c. Irudikatu $y(t)$ irteera-seinalea, sarrera-seinalea $x(t) = \delta(t) - \delta(t-1)$ bada. (2.5 p)

d. Irudikatu $y(t)$ irteera-seinalea, sarrera-seinalea $x(t) = \sum_{k=-\infty}^{\infty} \delta(t-2k)$ bada. (2.5 p)

3. ARIKETA (10 puntu, 40 minutu)

Ariketa honek atal bi ditu, 3.1 eta 3.2, beraien artean loturik ez daudenak.

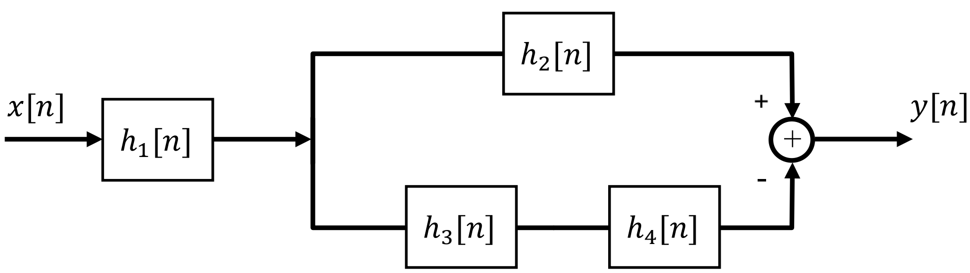
3.1. Izan bedi ondoko irudiko sistema:



a. Kalkulatu y[n] irteera sarrerako x[n] seinalearen lotzen duen diferentzia finituko ekuazioa. Adierazi zein motako sistema den eta bere ordena. (2.5 p)

b. Kalkulatu h[n] pultso erantzuna n guztietarako. (2.5 p)

3.2. Izan bedi irudiko eskema:



a. Adierazi multzo osoaren pultso erantzuna, $h_T[n]$, x[n] eta y[n] lotzen duena, $h_1[n]$, $h_2[n]$, $h_3[n]$ eta $h_4[n]$ pultso erantzunen funtzio bezala. (2 p)

b. Kalkulatu $h_T[n]$ hau kontuan izanik:

$h_1[n]=\{\underline{0.5}, 0.25, 0.5\}$

$h_2[n] = h_3[n] = (n+1)\, u[n]$

$h_4[n] = \delta\,[n-2]$ (3 p)

(1) $y(t) = x(t/2)$

(a) • linealidad:

$$y_1(t) = x_1(t/2) \qquad y_2(t) = x_2(t/2)$$

Si tenemos $x_T(t) = a x_1(t) + b x_2(t)$ $\quad y(t) = a x_1(t/2) + b x_2(t/2) =$

$= a y_1(t) + b y_2(t)$ lineal!!

• Invarianza temporal.

$$y(t-t_0) = x\left(\frac{t-t_0}{2}\right) = x(t/2 - t_0/2)$$

Si a la entrada ponemos $x(t-t_0)$

$$y(t) = x(t'-t_0)\Big|_{t'=t/2} = x(t/2 - t_0) \neq y(t-t_0)$$ no es tiempo invariante.

• Causal.

$$y(-2) = x\left(\frac{-2}{2}\right) = x(-1)$$ es decir necesito un valor posterior (futuro)

no es causal !!

• Estable

Si $|x(t)| < A \quad \forall t \rightsquigarrow \quad |x(t/2)| < A \quad \forall t \Rightarrow$

$|y(t)| = |x(t/2)| < A$ acotado.

Es estable.

ⓑ $y[n] = \sum_{k=2}^{\infty} x[n-k]\, 2^{k-2}$ h[n] por $x[n] = \delta[n]$

$$h[n] = \underbrace{\sum_{k=2}^{\infty} \delta[n-k]}_{\delta[n-k]=1 \;\; n=k} 2^{k-2} = 1 \cdot 2^{n-2} \cdot \overbrace{u[n-2]}^{n \geq 2} = 2^{n-2} u[n-2]$$

$\delta[n-k] = 1$ $n=k$ cero en el resto de valores de k!!

Suma es en k.

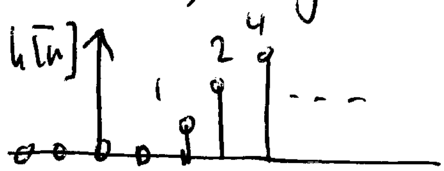
Como la suma empieza en $k=2$ $n \geq 2$ para que no sea siempre $\emptyset$.

Otra forma extender el sumatorio: 2^1 en $n=3$ 2^2 en $n=4$

$$h[n] = \sum_{k=2}^{\infty} \delta[n-k] 2^{k-2} = \underbrace{\delta[n-2]}_{n=2} 2^{\emptyset} + \underbrace{\delta[n-3]}_{n=3} 2^1 + \underbrace{\delta[n-4]}_{n=4} 2^2 + \dots$$

$$= 2^{n-2} \cdot u[n-2]$$

Causalidad y estabilidad, dibujamos h[n]



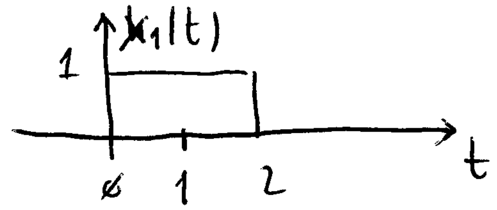
(a) Causal $h[n] = \emptyset$ $n < \emptyset$

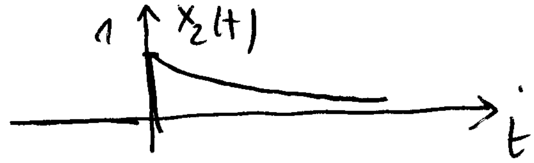
(b) Inestable

$$\sum_{n=-\infty}^{\infty} |h[n]| = \underbrace{2^{\emptyset} + 2^1 + 2^2 + \dots}_{\text{geométrica con } r>1} = \infty!!$$

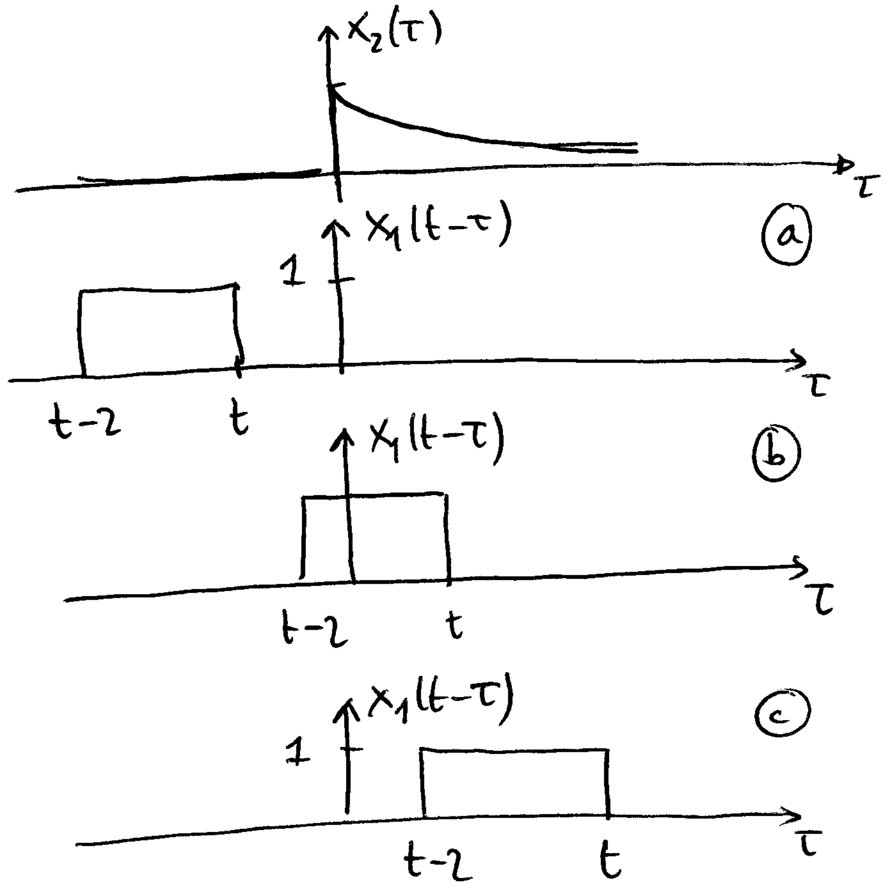
(2) $x(t) = \underbrace{\Pi\left(\frac{t-1}{2}\right)}_{x_1(t)} * \underbrace{e^{-t}u(t)}_{x_2(t)}$

$$= \int_{-\infty}^{\infty} x_1(t-\tau)\cdot x_2(\tau)\,d\tau$$





Hacemos la convolución:



(a) $t<0$ no se solapan

$x(t)=0$

(b) $t>0$ y $t-2<0$

$$\int_{-\infty}^{\infty} x_1(t-\tau)\,x_2(\tau)\,d\tau =$$

$$= \int_0^t 1\cdot e^{-\tau}\,d\tau = \left.\frac{e^{-\tau}}{-1}\right]_0^t =$$

$$= -e^{-t} + 1 = 1-e^{-t}$$

(c) $t-2>0$

$$x(t) = \int_{t-2}^{t} e^{-\tau}\,d\tau = \left.\frac{e^{-\tau}}{-1}\right]_{t-2}^{t} =$$

$$= -e^{-t} + e^{-(t-2)} =$$

$$= e^{-t}(e^2-1)$$

$$x(t) = \begin{cases} 0 & t \leq 0 \\ 1-e^{-t} & 0<t\leq 2 \\ e^{-t}(e^2-1) & t>2 \end{cases}$$

(3) $h[n] = \begin{cases} (1/2)^n & 0 \leq n \leq 4 \\ 0 & \text{resto} \end{cases}$

$x[n] = \{\underline{1}, 0, 1, 0, 1, 0, 1, \dots\} = \delta[n] + \delta[n-2] + \delta[n-4] + \dots = \sum_{k=0}^{\infty} \delta[n-2k]$

$y[n] = h[n] * x[n] = h[n] * \sum_{k=0}^{\infty} \delta[n-2k] = h[n-2k]$

en nuestro caso solo hay que calcular $n = 0, \dots, 6$

nos vale con hacer $y[n] = h[n] + h[n-2] + h[n-4] + h[n-6]$

$y[0] = h[0] = 1$

$y[1] = h[1] = \frac{1}{2}$

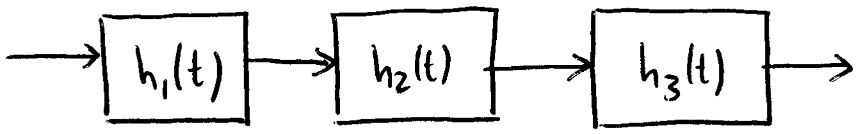
$y[2] = h[2] + h[0] = \frac{1}{4} + 1 = \frac{5}{4}$

$y[3] = h[3] + h[1] = \frac{1}{8} + \frac{1}{2} = \frac{5}{8}$

$y[4] = h[4] + h[2] + h[0] = \frac{1}{16} + \frac{1}{4} + 1 = \frac{21}{16}$

$y[5] = h[3] + h[1] = \frac{1}{8} + \frac{1}{2} = 5/8$

$y[6] = h[4] + h[2] + h[0] = \frac{21}{16}$

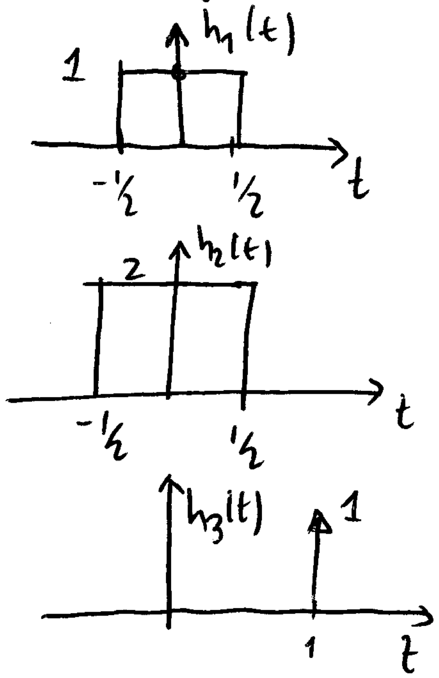
② 

$h_1(t) = \Pi(t)$

$h_2(t) = 2\Pi(t)$

$h_3(t) = \delta(t-1)$

ⓐ Dibujamos las $h_i(t)$



- Causal si $h_i(t) = 0 \quad \forall t \leq 0$

En nuestro caso

$h_1(t) \neq 0 \quad -\frac{1}{2} < t < 0$ no causal

$h_2(t) \neq 0 \quad -\frac{1}{2} < t \leq 0$ no causal

$h_3(t) = 0 \quad \forall t < 0$ causal

- Estable si $\int_{-\infty}^{\infty} |h_i(t)|\, dt < \infty$

En nuestro caso:

$\int_{-\infty}^{\infty} |h_1(t)| \cdot dt = 1 \cdot 1 = 1 < \infty$ (área bajo $h(t)$)

$\int_{-\infty}^{\infty} |h_2(t)|\, dt = 2 \cdot 1 = 2 < \infty$

$\int_{-\infty}^{\infty} |h_3(t)|\, dt = \int_{-\infty}^{\infty} \delta(t-1)\, dt = 1$

Todos los sistemas son estables.

ⓑ $y(t) = h_T(t) * x(t)$ en nuestro caso tenemos 3 sistemas en cascada:

$$h_T(t) = h_1(t) * h_2(t) * h_3(t) = \Pi(t) * 2\Pi(t) * \delta(t-1)$$

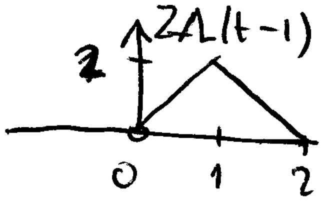
como la convolución es lineal y conmutativa:

$$h_T(t) = 2 \cdot (\underbrace{\Pi(t) * \Pi(t)}) * \delta(t-1) = 2 \cdot \Lambda(t) * \overset{\text{identidad}}{\delta(t-1)} = 2\Lambda(t-1)$$

desplazamiento y elem. identidad

sabemos que: $\Pi\left(\frac{t}{T}\right) * \Pi\left(\frac{t}{T}\right) = T \cdot \Lambda\left(\frac{t}{T}\right) \quad (T = 1)$

Dibujamos $h_T(t)$



$h_T(t) = \emptyset \quad \forall t < \emptyset \Rightarrow$ Causal

$$\int_{-\infty}^{\infty} |h_T(t)|\, dt = 2 \cdot 2 \cdot \frac{1}{2} = 2 < \infty$$

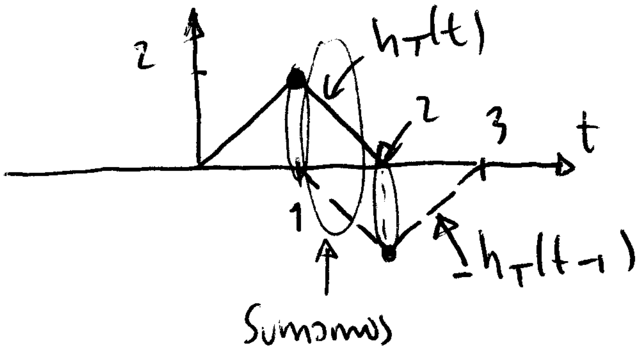
área bajo la curva.

Estable!!

ⓒ $x(t) = \delta(t) - \delta(t-1)$

asociativa, elem identidad, desplaz.

$$y(t) = h_T(t) * x(t) = h_T(t) * [\delta(t) - \delta(t-1)] = h_T(t) - h_T(t-1)$$



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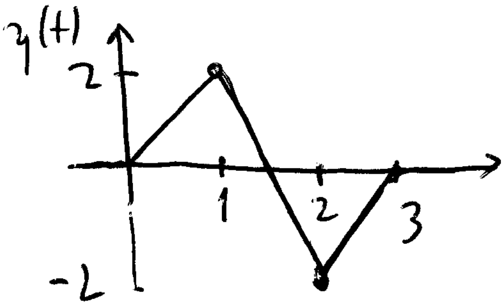


ⓓ $x(t) = \sum_{k=-\infty}^{\infty} \delta(t-2k)$

asociat.

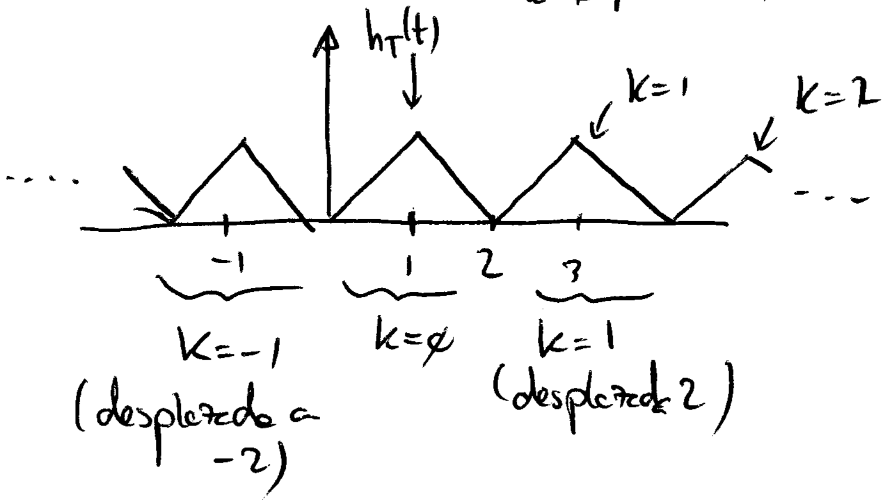
$$y(t) = h_T(t) * \sum_{k=-\infty}^{\infty} \delta(t-2k) =$$

elem ident + desplaz.

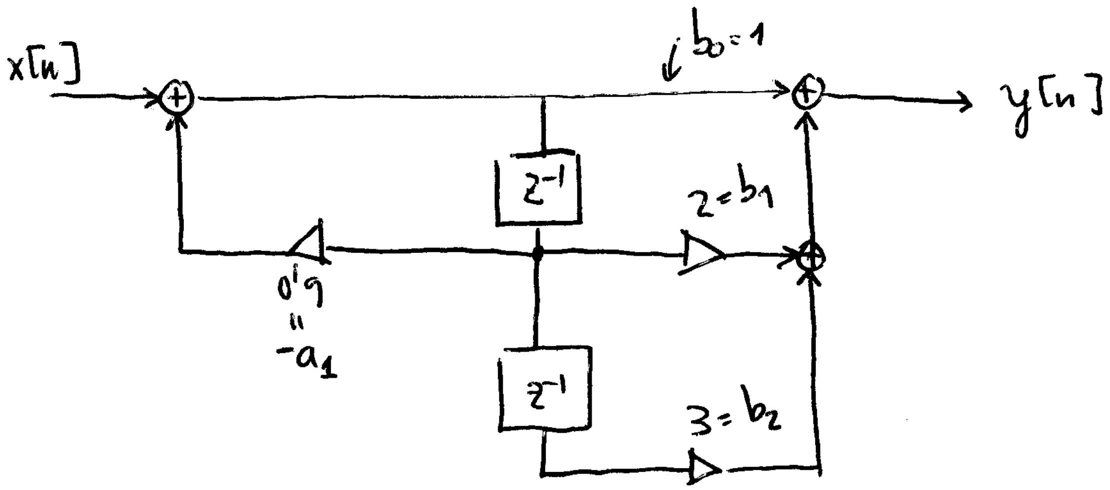
$$= \sum_{k=-\infty}^{\infty} h_T(t) * \delta(t-2k) = \sum_{k=-\infty}^{\infty} h_T(t-2k) =$$

~~elem~~

$$= \ldots + \underbrace{h_T(t+2)}_{k=-1} + \underbrace{h_T(t)}_{k=\emptyset} + \underbrace{h_T(t-2)}_{k=1} + \ldots$$



3.1.



ⓐ Sistema en forma directa II y por lo tanto podemos identificar los coeficientes a_k y b_k:

$$y[n] = \underset{1}{b_0}\, x[n] + \underset{2}{b_1}\, x[n-1] + \underset{3}{b_2}\, x[n-2] - \overset{-0'9}{a_1}\, y[n-1]$$

$$\boxed{y[n] = x[n] + 2x[n-1] + 3x[n-2] + 0'9\, y[n-1]}$$

Sistema IIR (recursivo $a_1 \neq \emptyset$) y de orden 2 (mayor retardo $x[n-2]$).

ⓑ $h[n]$ es la salida cuando $x[n] = \delta[n]$ con cond. iniciales nulas.

$$h[n] = \delta[n] + 2\delta[n-1] + 3\delta[n-2] + 0'9\, h[n-1] \quad , \quad h[-1] = \emptyset$$

$$h[\emptyset] = \delta[0] + \overbrace{2\delta[-1] + 3\delta[-2] + 0'9\, h[-1]}^{\emptyset} = 1$$

$$h[1] = \cancel{\delta[1]} + 2\delta[0] + 3\delta[-1] + 0'9\, h[0] = 2 + 0'9 \cdot 1 = 2'9$$

$$h[2] = \emptyset + 2\cdot\emptyset + 3\cdot\delta[0] + 0'9\, h[1] = 3 + 0'9 \cdot (2'9) = 5'61$$

$$h[3] = \emptyset + 2\cdot\emptyset + 3\cdot\emptyset + 0'9 \cdot h[2] = 0'9 \cdot 5'61$$

$$h[4] = 0'9\, h[3]$$

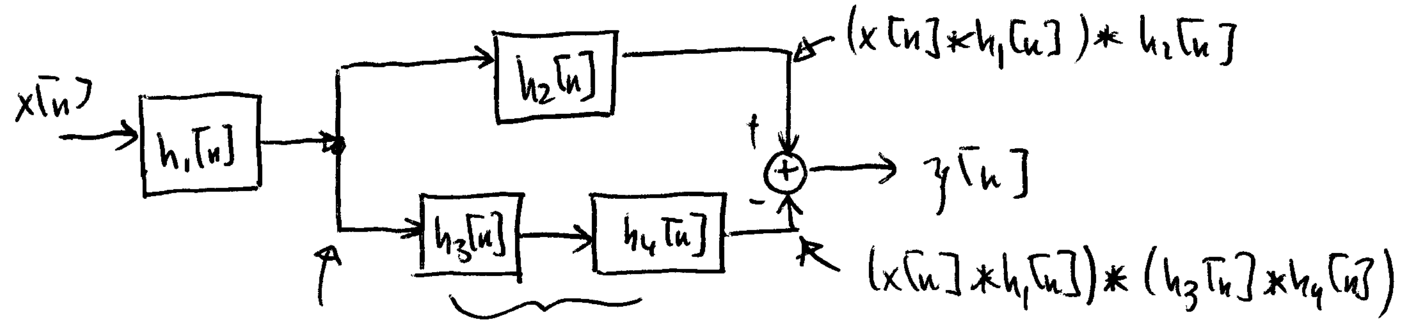
$$\vdots$$

$$h[n] = 0'9\, h[n-1]$$

$$\vdots$$

$$\boxed{h[n] = \delta[n] + 2'9\,\delta[n-1] + \underbrace{(0'9)^{n-2}\, 5'61\, u[n-2]}_{\text{empieza en } n=2}}$$

3.2



$x[n]*h_1[n]$ $\qquad$ $\underbrace{h_3[n]*h_4[n]}$, conexión serie

Por lo tanto el total es:

$$y[n] = (x[n]*h_1[n])*h_2[n] - (x[n]*h_1[n])*(h_3[n]*h_4[n])$$

$$= x[n]*(h_1[n]*h_2[n]) - x[n]*(h_1[n]*h_3[n]*h_4[n])$$

asociativa y conmutativa de la convolución

$$= x[n]*\Big[\underbrace{h_1[n]*(h_2[n]-h_3[n]*h_4[n])}_{h_T[n]}\Big]$$

$$\boxed{h_T[n] = h_1[n]*(h_2[n]-h_3[n]*h_4[n])}$$

(b) $h_1[n] = \{\underline{0.5}, 0.25, 0.5\}$ $\qquad$ $h_2[n] = h_3[n] = (n+1)\cdot u[n]$ $\qquad$ $h_4[n] = \delta[n-2]$

$$h_2[n] - \underbrace{h_3[n]*h_4[n]}_{h_3[n]*\delta[n-2] = h_3[n-2]} = (n+1)u[n] - (n-1)u[n-2] = h_A[n]$$

$$h_1[n] = 0'5\,\delta[n] + 0'25\,\delta[n-1] + 0'5\,\delta[n-2]$$

$$h_1[n]*h_A[n] = 0'5\,h_A[n] + 0'25\,h_A[n-1] + 0'5\,h_A[n-2] =$$

$$= 0'5\left((n+1)u[n] - (n-1)\cancel{u[n-2]}\right) + 0'25\left(n\,u[n-1] - (n-2)u[n-3]\right) +$$

$$0'5\left(\cancel{(n-1)u[n-2]} - (n-3)\,u[n-4]\right)$$

$$= \underbrace{0'5\cdot\delta[n]}_{n=0} + (\underbrace{0'5\cdot 2 + 0'25}_{n=1})\delta[n-1] + (\underbrace{0'5\cdot 3 + 0'25\cdot 2}_{n=2})\delta[n-2] + (\underbrace{0'5\cdot 4 + 0'25\cdot 3}_{n=3} - 0'25)\delta[n-3]$$

$$+\left(\underbrace{0'5(n+1)+0'25n-0'25(n-2)}_{n\geq 4}-0'5(n-3)\right)\cdot u[n-4]$$

$$=0'5\delta[n]+1'25\delta[n-1]+2\delta[n-2]+2'5\delta[n-3]+(\cancel{0}\cdot n+2'5)u[n-4]$$

$$=\boxed{\{0'\underline{5},1'25,2,2'5\}+2'5u[n-4]=y[n]}$$

SEINALEEN PROZESAKETA: AZKEN AZTERKETA (Bigarren partziala)

Azterketa ukatzeko ordu eta 30 minutu dituzue. Azterketak 3 ariketa ditu. Ariketa bakoitzak 10 puntu balio ditu, eta 1. Ariketako galdera guztiek pisu berdina dute.

1. ARIKETA (10 puntu, 40 minutu)

1. $x(t) = 3e^{j(2\pi 100\, t+\frac{\pi}{3})}$ seinalea fs=200 l/s maiztasunarekin lagintzen da x[n] sekuentzia lortzeko. Sekuentzia hau LTI sistema batekin prozesatzen da, maiztasun erantzun hau duena: $H(\Omega) = \frac{1+2\, e^{-j2\Omega}+\, 5\, e^{-j4\Omega}}{1+\, e^{-j4\Omega}}$

   Kalkulatu sistemaren erantzuna, y[n].

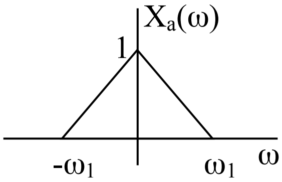
2. Adierazi ondoko sekuentziak periodikoak diren. Horrela bada kalkulatu Fourierren seriean garatzeko koefizienteak, a_k, k=0 eta N_o-1 artean, N_o sekuentziaren oinarrizko periodoa delarik.

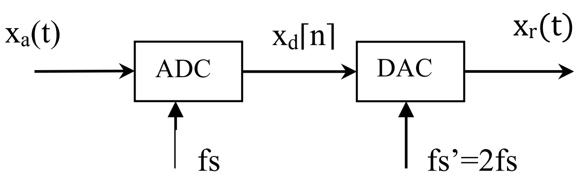
   a. $x[n] = 3 + 5\cos\left(\frac{5}{4}n\right) + 2\cos(\frac{3\pi}{5}n)$

   b. $x[n] = 2 + 5\cos\left(\frac{2\pi}{3}n + \frac{\pi}{3}\right) + 3\cos\,(\frac{7\pi}{2}n)$

3. $x_a(t)$ seinalea, irudian agertzen den espektroa duena ω_1=2π1000 rad/s balioarekin, fs=1600 l/s maiztasunarekin lagintzen da $x_d[n]$ sekuentzia lortzeko. Bihurgailu digital analogiko ideial batekin, fs'=2fs laginketa maiztasuna duena, $x_r(t)$ berreskuratzen da, irudian agertzen den bezala.

   a. Lortu grafikoki $x_d[n]$ sekuentziaren espektroa, $X_d(\Omega)$.
   b. Lortu grafikoki $x_r(t)$ seinalearen espektroa, $X_r(\omega)$.
   c. $x_a(t)$ seinale jarraituaren zein informazio banda mantentzen da aldatu gabe $x_r(t)$ seinalean?





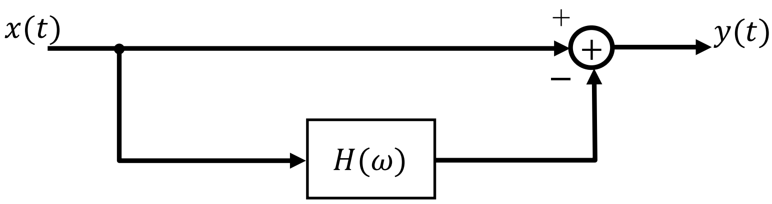
2. ARIKETA (10 puntu, 20 minutu)

Izan bitez ondoko maiztasun erantzuna duten bi iragazki ideal:

$$H_1(\omega) = \begin{cases} 1, & |\omega| \leq \omega_1 \\ 0, & |\omega| > \omega_1 \end{cases} \qquad H_2(\omega) = \begin{cases} 1, & |\omega| > \omega_2 \\ 0, & |\omega| \leq \omega_2 \end{cases}$$

a) Irudikatu $H_1(\omega)$ eta $H_2(\omega)$ (modulo eta fase) eta adierazi ze iragazki mota diren. (1p)

b) Aurreko iragazkiak irudiko eskeman erabili dira. Irudikatu (modulo eta fase) multzoaren maiztasun erantzuna $H(\omega)=H_1(\omega)$ denean eta $H(\omega)=H_2(\omega)$ denean. Adierazi kasu bakoitzean lortutako iragazki mota.. (4 p)



c) Banda kentzeko iragazki ideala lortu nahi dugu $H_1(\omega)$ eta $H_2(\omega)$ erabiliz, kendutako banda 2kHz < f <5 kHz delarik. Irudikatu eta justifikatu iragazkien konexioaren eskema, eta zehaztu ω_1 eta ω_2 balioak. (2.5 p)

d) Banda paseko iragazki ideala lortu nahi dugu $H_1(\omega)$ eta $H_2(\omega)$ erabiliz, paseko-banda 4MHz < f < 6 MHz delarik. Irudikatu eta justifikatu iragazkien konexioaren eskema, eta zehaztu ω_1 eta ω_2 balioak. (2.5 p)

3. ARIKETA (10 puntu, 30 minutu)

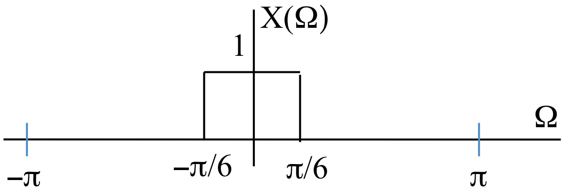
Izan bedi

$$p[n] = \sum_{k=-\infty}^{k=\infty} \delta[n - k\,3]$$

a) Irudikatu p[n]. Kalkulatu Fourierren seriean garatzeko koefizienteak, a_k, k=0 eta N_o-1 artean, N_o bere oinarrizko periodoa delarik. (2 p)

b) Kalkulatu analitikoki P(Ω) eta iriudikatu periodo bat 0 eta 2π artean. (1 p)

Izan bedi x[n] sekuentzia irudiko espektroa duena:



c) s[n]=x[n] p[n] seinalea osatzen da. Irudikatu s[n]. (1.5 p)

d) Kalkulatu analitikoki S(Ω) eta irudikatu 0 eta 2π artean. (2 p)

e) Izan bedi y[n]=s[3n] seinalea. Irudikatu y[n]. (1.5 p)

f) Fourierren transformatuaren definizioan oinarriturik, Y(Ω) eta S(Ω) zehazten duten batukariak garatu. Kalkulatu Y(Ω) S(Ω)-ren funtzio bezala. Irudikatu Y(Ω) 0 eta 2π artean. (2 p)

CUESTIONES

1 $x(t) = 3\,e^{j(2\pi 100 t + \pi/3)}$ $\quad f_s = 200 \quad$ $H(\Omega) = \dfrac{1 + 2e^{-j2\Omega} + 5e^{-j4\Omega}}{1 + e^{-j4\Omega}}$

$$x[n] = x(t)\Big|_{t = nT_s} = 3\,e^{j(2\pi \frac{100}{200} n + \pi/3)} = 3\,e^{j(\pi n + \pi/3)}$$

$x[n]$ exponencial compleja con $\Omega_0 = \pi \rightarrow y[n] = 3\,|H(\Omega_0)|\,e^{j(\Omega_0 n + \pi/3 + \phi(\Omega_0))}$

$$H(\pi) = \frac{1 + 2e^{-j2\pi} + 5e^{-j4\pi}}{1 + e^{-j4\pi}} = \frac{1+2+5}{1+1} = 4 \begin{cases} |H(\Omega_0)| = 4 \rightarrow y[n] = 12\,e^{j(\pi n + \pi/3)} \\ \phi(\Omega_0) = 0 \end{cases}$$

2 **a** $x[n] = 3 + \underbrace{5\cos(\tfrac{5}{4}n)}\, + 2\cos(\tfrac{3\pi}{5}n)$

$\Omega_1 = \frac{5}{4} = 2\pi f_1 \rightarrow f_1 = \frac{5}{8\pi} \rightarrow$ No racional $\Rightarrow 5\cos(\frac{5}{4}n)$ no periódica $\Rightarrow x[n]$ no periódica

b $x[n] = 2 + \underbrace{5\cos(\tfrac{2\pi}{3}n + \tfrac{\pi}{3})} + \underbrace{3\cos(\tfrac{7\pi}{2}n)}$

$\Omega_1 = \frac{2\pi}{3} = 2\pi f_1 \rightarrow f_1 = \frac{1}{3}$

$N_1 = k\frac{1}{f_1} = k\cdot 3 \xrightarrow{k=1} N_1 = 3$

$3\cos\left((\frac{7\pi}{2} - 4\pi)\cdot n\right) = 3\cos(-\frac{\pi}{2}n) = 3\cos(\frac{\pi}{2}n)$

$\Omega_2 = \frac{\pi}{2} = 2\pi f_2 \rightarrow f_2 = \frac{1}{4}$

$N_2 = k\frac{1}{f_2} = k\cdot 4 \xrightarrow{k=1} N_2 = 4$

$x[n]$ periódica de periodo $N_0 = m.c.m.(3,4) = 12 \Rightarrow \Omega_0 = \frac{2\pi}{12}$ $\begin{cases} \Omega_1 = \frac{2\pi}{3} = k_1\cdot\Omega_0 \rightarrow k_1 = 4 \\ \Omega_2 = \frac{\pi}{2} = k_2\cdot\Omega_0 \rightarrow k_2 = 3 \end{cases}$

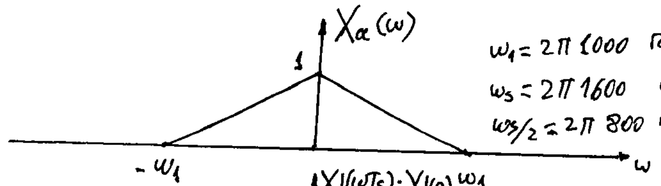
$$x[n] = 2 + \frac{5}{2}e^{j\pi/3}e^{j4\Omega_0 n} + \frac{5}{2}e^{-j\pi/3}e^{-j4\Omega_0 n} + \frac{3}{2}e^{j3\Omega_0 n} + \frac{3}{2}e^{-j3\Omega_0 n}$$

$\Rightarrow a_0 = 2\ ;\ a_4 = \frac{5}{2}e^{j\pi/3}\ ;\ a_{-4} = a_{-4+12} = a_8 = \frac{5}{2}e^{-j\pi/3}\ ;\ a_3 = \frac{3}{2}\ ;\ a_{-3} = a_{-3+12} = a_9 = 3/2$

$a_1 = a_2 = a_5 = a_6 = a_7 = a_{10} = a_{11} = 0$

3

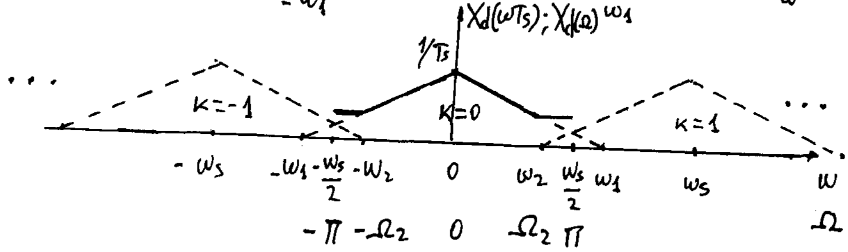
a $X_d(\Omega)\Big|_{\Omega = \omega\cdot T_s} = X_d(\omega T_s) = \frac{1}{T_s}\sum_k X_a(\omega - k\omega_s)$



$\omega_1 = 2\pi\,1000$ rad/s
$\omega_s = 2\pi\,1600$ "
$\omega_s/2 = 2\pi\,800$ "

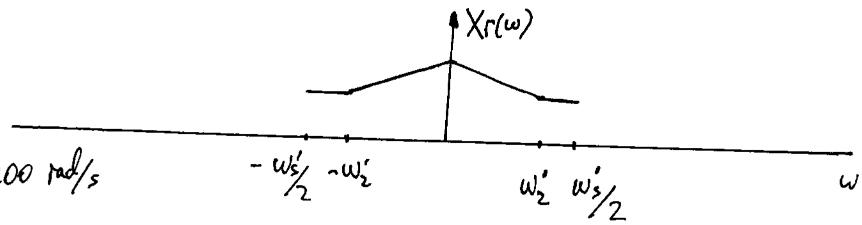
$\omega_2 = \omega_s - \omega_1 = 2\pi\,600$ rad/s

$\Omega_2 = \dfrac{\omega_2}{f_s} = \dfrac{2\pi\,600}{1600} = \dfrac{3\pi}{4}$ rad/m



b $\dfrac{\omega_s'}{2} = \dfrac{2\pi\,3200}{2} = 2\pi\,1600$ rad/s

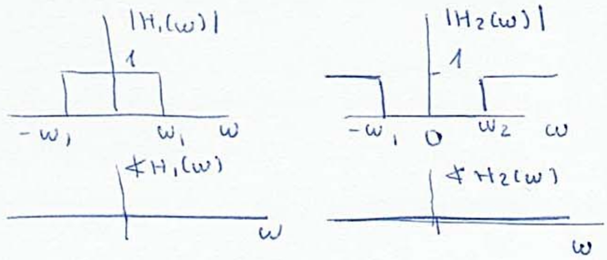
$\omega_2' = \Omega_2\cdot f_s' = \frac{3\pi}{4}\cdot 3200 = 2\pi\,1200$ rad/s



c En $x_r(t)$ se mantiene la información de la señal $x_a(t)$ entre $\pm\,\omega_2$, esto es, las componentes frecuenciales de $x_a(t)$ entre ± 600 Hz.

2.ARIKETA

a)



$H_1(\omega)$ = Behe-paseko iragazkia

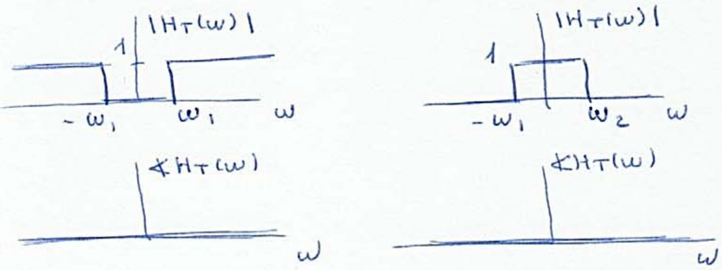
$H_2(\omega)$ = Goi-paseko "

b)

$Y(\omega) = X(\omega) - X(\omega)\cdot H(\omega)$

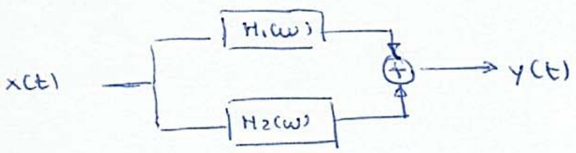
$H_T(\omega) = \frac{Y(\omega)}{X(\omega)} = 1 - H(\omega)$

$H(\omega) = H_1(\omega)$ $\qquad$ $H(\omega) = H_2(\omega)$



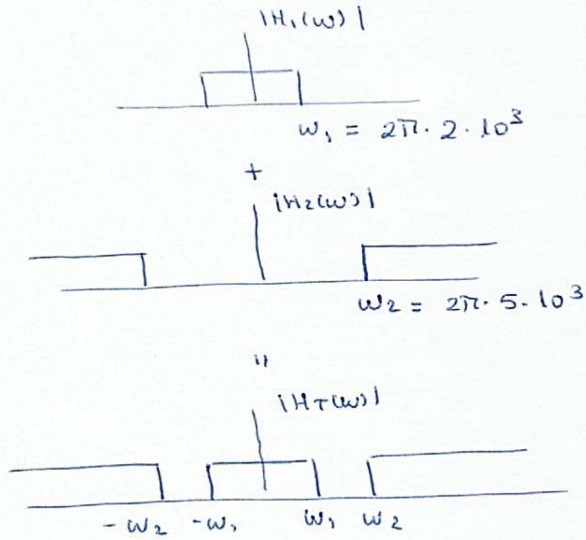
Goi-paseko iragazkia $\qquad$ Behe-paseko iragazkia

c) Paraleloan konektatuta



$Y(\omega) = X(\omega)\cdot[H_1(\omega) + H_2(\omega)]$

$H_T(\omega) = H_1(\omega) + H_2(\omega)$



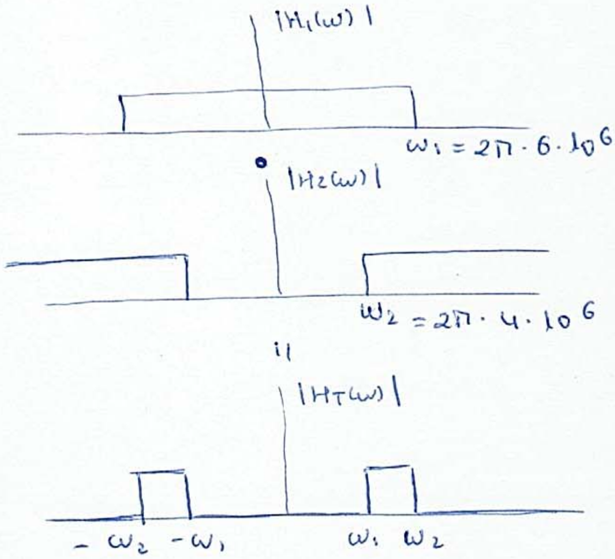
Banda-kentzeko iragazkia

d)

Seriean konektatuta.

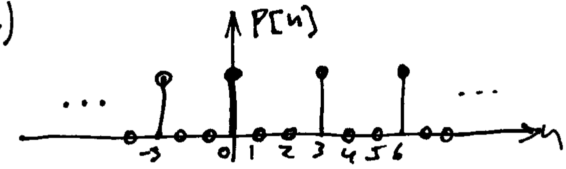
$Y(\omega) = X(\omega)\cdot H_1(\omega)\cdot H_2(\omega)$

$H_T(\omega) = H_1(\omega)\cdot H_2(\omega)$



Banda paseko iragazkia.

a)



$$a_K = \frac{1}{N_0}\sum_{n=0}^{N_0-1} x[n]\, e^{-jK\Omega_0 n} \qquad N_0 = 3 \qquad \Omega_0 = \frac{2\pi}{3}$$

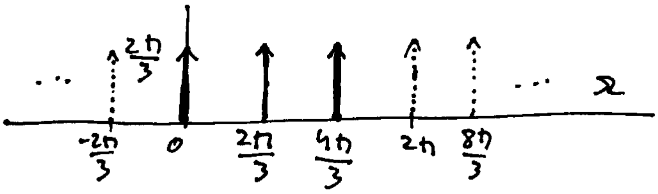
$$a_K = \frac{1}{3}\sum_{n=0}^{2} p[n]\, e^{-jK\frac{2\pi}{3}n} = \frac{1}{3}\left(1\cdot e^{-jK\frac{2\pi}{3}0} + 0\cdot e^{\dots 1} + 0\cdot e^{\dots 2}\right) = \frac{1}{3}$$

Tambien se puede hacer apoyándose en la señal base $P_b(\Omega)$ $\quad a_K = \frac{1}{N_0} P_b(\Omega)\Big|_{\Omega = K\Omega_0}$

$$a_0 = a_1 = a_2 = \frac{1}{3}$$

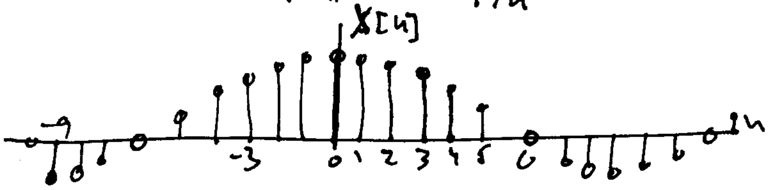
b) $$X(\Omega) = \sum_{K=-\infty}^{\infty} 2\pi a_K\, \delta(\Omega - K\Omega_0)$$

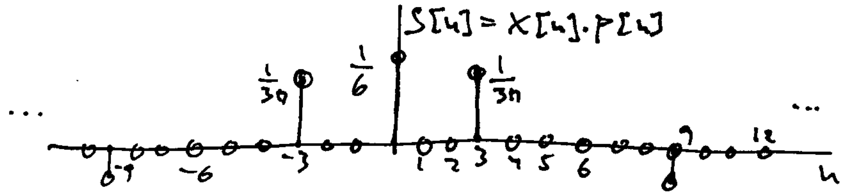
$$P(\Omega) = \sum_{K=-\infty}^{\infty} \frac{2\pi}{3}\, \delta\!\left(\Omega - K\frac{2\pi}{3}\right)$$



c) $$X(\Omega) = \Pi\left(\frac{\Omega}{2\pi/6}\right)_{\Omega \neq k\pi} \xrightarrow{\mathcal{F}^{-1}} x[n] = \frac{\sin \frac{\pi}{6} n}{\pi n}$$

$x[0] = \frac{1}{6}$ $\quad x[1] = \frac{\sin \pi/6}{6\pi}$ $\quad x[2] = \frac{\sin \pi/3}{2\pi}$ $\quad x[3] = \frac{\sin \pi/2}{3\pi} = \frac{1}{3\pi}$ $\quad \dots \quad x[6] = 0$



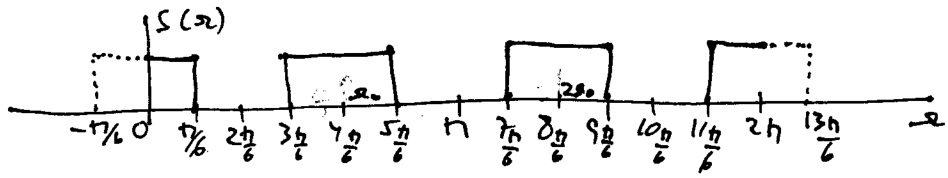


Sampling...

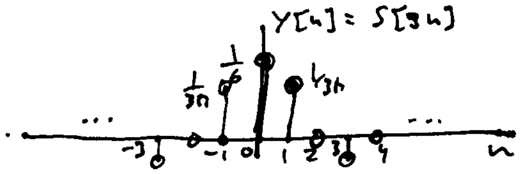
d) $$S(\Omega) = \frac{1}{2\pi} X(\Omega) \circledast P(\Omega) = \frac{1}{2\pi}\, \Pi\left(\frac{\Omega}{2\pi/6}\right) * \frac{2\pi}{3}\left[\delta(\Omega) + \delta\left(\Omega - \frac{2\pi}{3}\right) + \delta\left(\Omega - \frac{4\pi}{3}\right)\right] =$$

$$= \frac{1}{3}\left[\Pi\left(\frac{\Omega}{\pi/3}\right) + \Pi\left(\frac{\Omega - 2\pi/3}{\pi/3}\right) + \Pi\left(\frac{\Omega - 4\pi/3}{\pi/3}\right)\right]$$

... without info losing.



e)



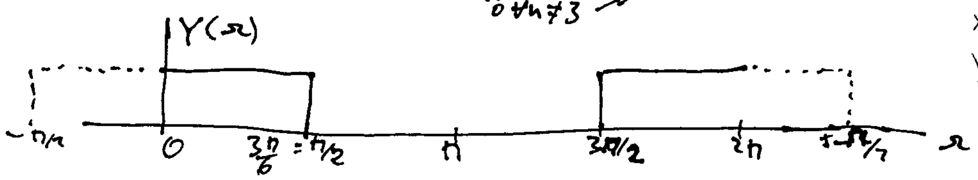
$$y[n] = S[3n] = x[3n]\cdot\underbrace{\sum_{K=-\infty}^{\infty}\delta[3n-3K]}_{1}$$

$$Y(\Omega) = \sum_{n=-\infty}^{\infty} x[3n]\, e^{-j\Omega n}$$

f) $$S(\Omega) = \sum_{n=-\infty}^{\infty} s[n]\, e^{-j\Omega n} = \sum_{n=-\infty}^{\infty} x[n]\cdot\underbrace{\sum_{K=-\infty}^{\infty}\delta[n-K3]}_{0\ \forall n \neq \dot{3}} e^{-j\Omega n} \underset{n=3n'}{=} \sum_{n'=-\infty}^{\infty} x[3n']\, e^{-j\Omega 3n'}$$

$$S(\Omega) = Y(3\Omega)$$

$$\boxed{Y(\Omega) = S\left(\frac{\Omega}{3}\right)}$$



Y[n] S[n] estutuada

Y(Ω) S(Ω) zabaldua da