ELASTIKOTASUNA ETA MATERIALEN ERRESISTENTZIA

Azterketa Finala (16/01/15)

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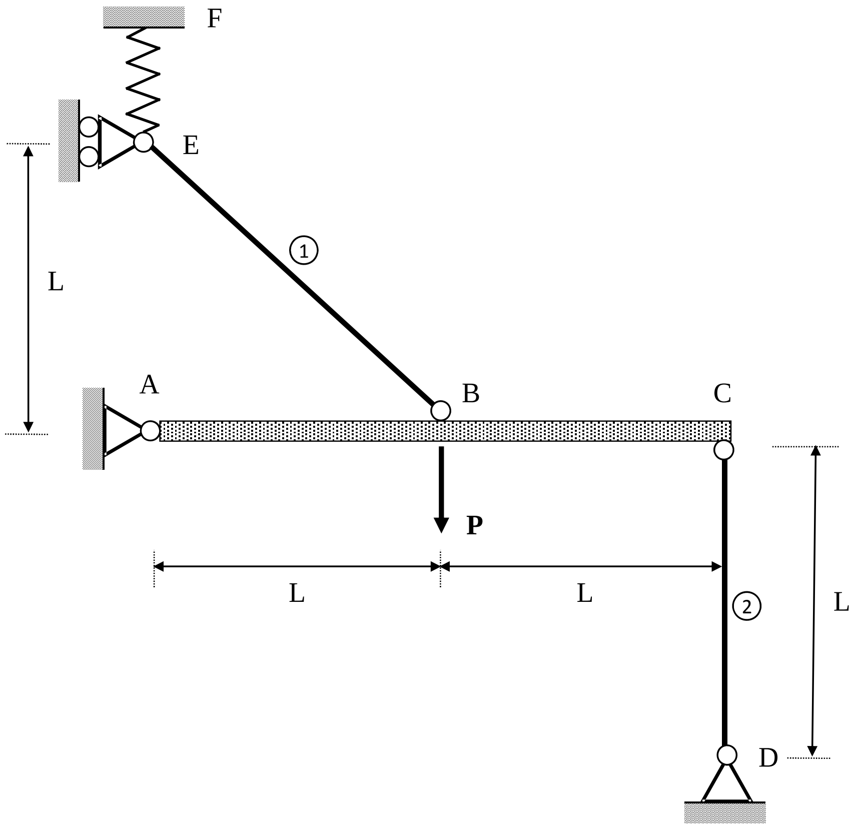
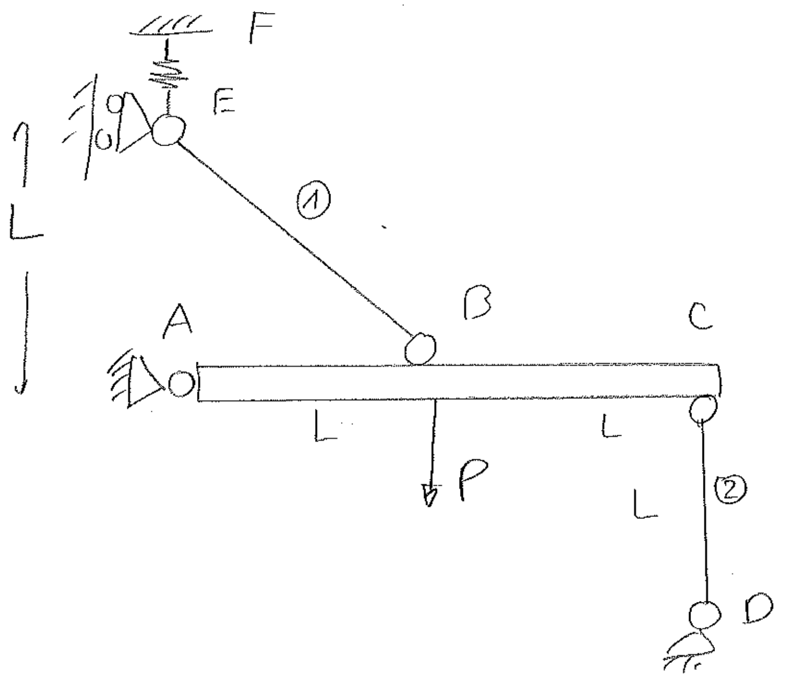
Irudiko egituran **ABC** barra deformaezina da, **A** euskarri finkoan artikulatua aurkitzen da eta muturretan artikulatutako **EB** eta **CD** tiranteen bidez eusten da, neurriak irudian adierazitakoak direlarik, non **L = 1 m** baita. Tiranteen sekzioa berdina da, **A = 10 cm^2**-koa, eta **E = 210 GPa**-eko elastikotasuna modulua duen altzairuarekin daude eginak. **D** euskarria, artikulatua eta finkoa da, **E** euskarria, ostera, mugikorra da eta norabide bertikalean **k = 2000 N/mm**-ko zurruntasuna duen malguki bati lotua aurkitzen da, **F**-n finkatutakoa. **B** sekzioan **P = 150 kN** balio duen karga bertikal bat aplikatzen bada, kalkulatu **ABC** barraren biraketa eta **E**-ren desplazamendu bertikala.





$L = 1\,m$

$A = 10 \cdot 10^{-4}\,m^2$

$E = 200 \cdot 10^{9}\,Pa$

$k = 2 \cdot 10^{6}\,N/m$

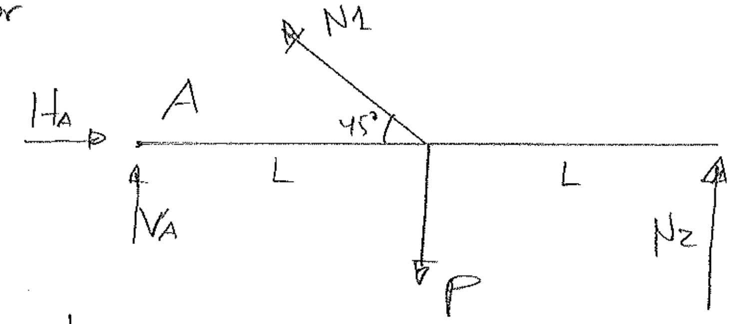
$P = 150 \cdot 10^{3}\,N$

$\frac{EA}{kL} = 100$

Hipótesis ① tr

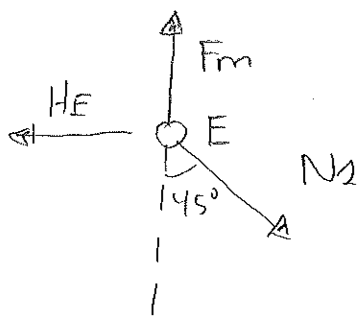
② compr

1) Equilibrio



$\Sigma M_A = 0$

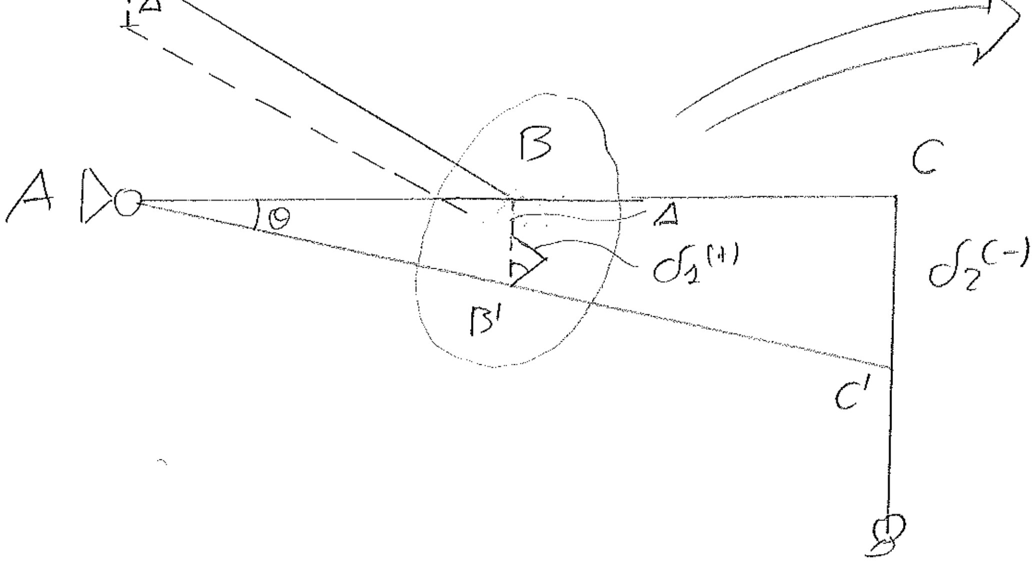
$$\boxed{N_1 \frac{\sqrt{2}}{2} + 2N_2 = P} \quad (1)$$

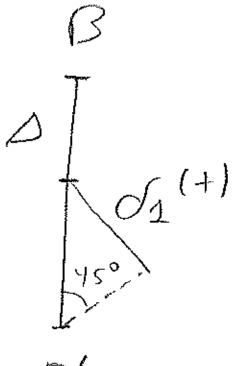


$$\boxed{F_m = N_1 \frac{\sqrt{2}}{2}} = K \cdot \Delta$$

$$\Delta = \frac{N_1}{K} \frac{\sqrt{2}}{2}$$

2) Compatibilidad de deformaciones





$$CC' = 2BB' \quad \begin{cases} CC' = \delta_2^{(-)} \\ BB' = \Delta + \dfrac{\delta_1^{(+)}}{\text{sen}\,45^\circ} \end{cases}$$

$$\delta_2^{(-)} = 2\left(\Delta + \delta_1^{(+)}\sqrt{2}\right)$$

$$\boxed{\delta_2^{(-)} = 2\sqrt{2}\,\delta_1^{(+)} + 2\Delta}$$

3) Ley de comportamiento

$$\delta_1^{(+)} = \frac{N_1 \cdot L\sqrt{2}}{EA}\,; \quad \delta_2^{(-)} = \frac{N_2 L}{EA}\,; \quad \Delta = \frac{N_1}{k}\,\sqrt{2}/2$$

Sustituyendo:

$$\boxed{\frac{N_2 L}{EA} = \frac{4 N_1 L}{EA} + \sqrt{2}\,N_1/k} \quad (2)$$

Resolviendo (1) y (2):

de (1): $\left(N_2 = P/2 - N_1\sqrt{2}/4\right)$

$$\frac{\left(P/2 - N_1\sqrt{2}/4\right)L}{EA} = \frac{4N_1 L}{EA} + \sqrt{2}\,N_1/k$$

$$N_1 = P\,\frac{\sqrt{2}/8}{\sqrt{2} + 1/8 + \frac{EA}{2kL}} = \boxed{0{,}514\,\text{kN}}$$

$$N_2 = P/2 - N_1\sqrt{2}/4 = \boxed{74{,}82\,\text{kN}}$$

$$\Theta = \frac{\delta_2^{(-)}}{2L} = \frac{N_2 \cancel{L}}{2EA\,\cancel{L}} = \boxed{1{,}871 \cdot 10^{-4}\,\text{rad}}$$

$$\delta_E(\downarrow) = \Delta = \frac{N_1}{k}\,\sqrt{2}/2 = \boxed{0{,}182\,\text{mm}}$$

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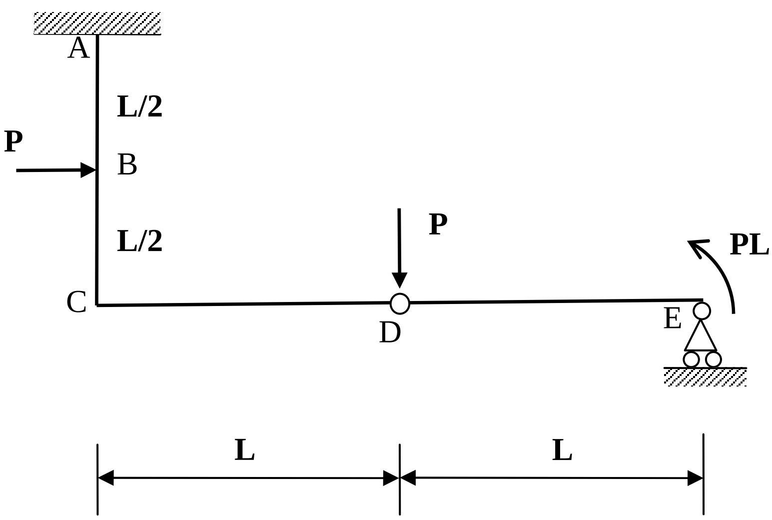
Irudiko egituran A landapen bat da, C sekzioan 90º-ko lotura zurrun bat dago, D-n errotula bat kokatu da eta E muturreko euskarria artikulatua eta mugikorra da. Egitura honetan irudian adierazitako indar eta momentuak aplikatzen dira. Sekzio zuzena lauki-luzea da, **6 x 10 cm**-koa, non alde luzeena marrazkiaren planoarekiko paraleloa baita, eta erabilitako materiala, altzairua, elastikotasun modulua **E=200 GPa** eta isurpen tentsioa **σ_f=300 MPa** direlarik. Zera eskatzen da:

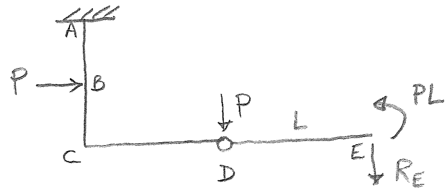
1º) Ardatzeko indarren, indar ebakitzaileen eta momentu makurtzaileen diagramak, P eta L-ren funtziopean, eta deformatuaren marrazkia gutxi gora-behera.
2º) Egituraren segurtasun koefizientea isurpenaren aurrean.
3º) D errotularen desplazamendua.

Makurdurak bakarrik sortutako tentsio eta deformazioak hartu kontutan.

<u>Datuak</u>: **P= 3 kN**, **L= 2 m**.

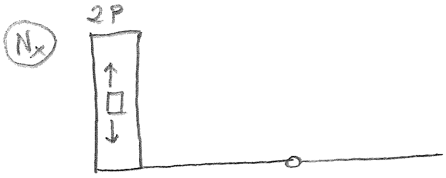


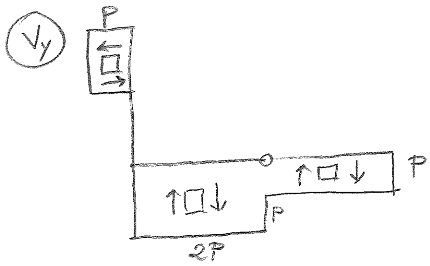
1º)

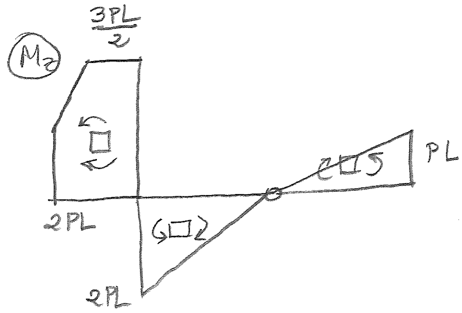


$M_{z_D}=0 \rightarrow R_E \cdot L = P \cdot L \rightarrow R_E = P$

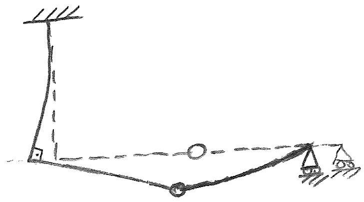
Diagramas:







Deformada:



2º) Sección más desfavorable: C

$$\sigma_{máx} = \frac{2PL \cdot y_{máx}}{I_z} = \frac{2PL \cdot 5\,cm}{500\,cm^4} = 120\,MPa \rightarrow \boxed{n = \frac{\sigma_e}{\sigma_{máx}} = \frac{300}{120} = 2'5}$$

$$I_z = \frac{1}{12}\, 6\, 10^3 = 500\,cm^4$$

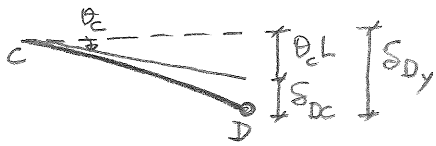
3º)

$$\boxed{(\leftarrow)\ \delta_{D_x}} = \delta_{CA} = \frac{1}{EI}\left(2PL^2\frac{L}{2} - \frac{1}{2}\frac{PL}{2}\frac{L}{2}\frac{5L}{6}\right) = \frac{43}{48}\frac{PL^3}{EI} = \boxed{2'15\,cm}$$

$$\boxed{(\downarrow)\ \delta_{D_y}} = \theta_C \cdot L + \delta_{DC} = \left(\frac{15}{8} + \frac{2}{3}\right)\frac{PL^3}{EI} = \frac{61}{24}\frac{PL^3}{EI} = \boxed{6'1\,cm}$$

$$\theta_C = \theta_{AC} = \frac{1}{EI}\left(2PL^2 - \frac{PL^2}{8}\right) = \frac{15}{8}\frac{PL^2}{EI}$$

$$\delta_{DC} = \frac{1}{EI}\left(\frac{1}{2}2PL^2\frac{2}{3}L\right) = \frac{2}{3}\frac{PL^3}{EI}$$



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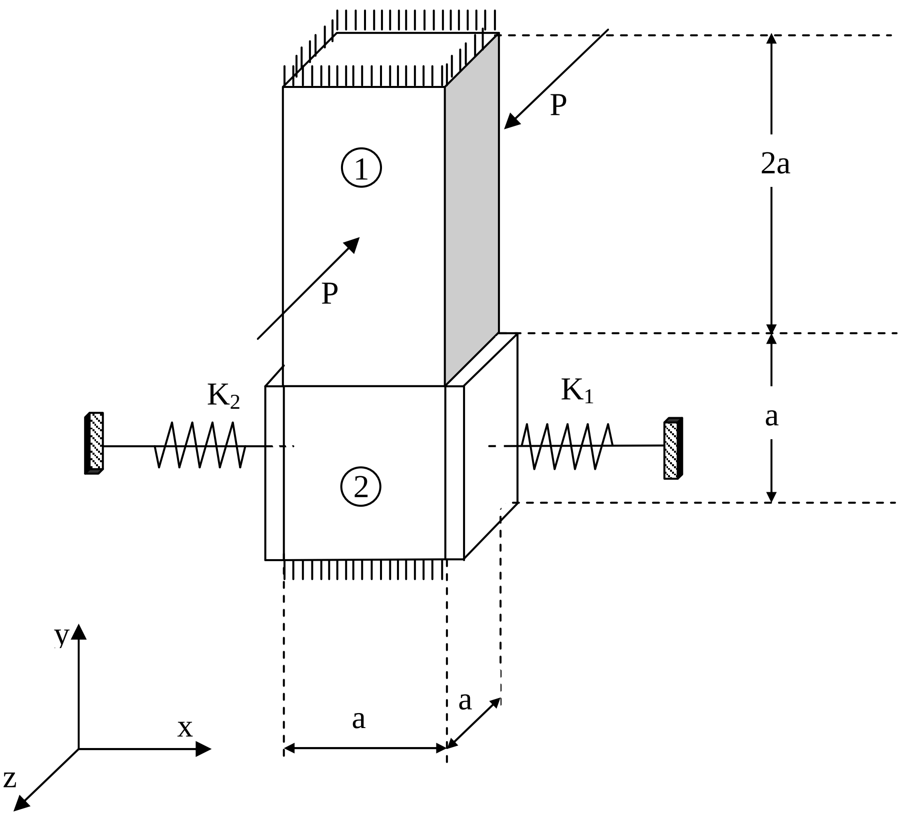
Irudian bi paralelepipedo elastiko (1) eta (2) errepresentatzen dira, elkarri atxikiak eta goian eta behean dauden bi horma zurrunen artean kokatuta. Irudiko ardatzen, (x,y,z), araberako paralelepípedoen dimentsioak **a x 2a x a** eta **a x a x a** dira hurrenez hurren, **a=10 mm** izanik. (1) paralelepipedoa z ardatzeko norabidea duen P konpresiozko karga baten eraginpean dago, ekiten duen aurpegian uniformeki banatuta dagoena. (2) paralelepipedoa, ordea, bi plaka zurrunetara lotuta dago eta hauek x ardatzeko norabidea duten bi malgukien bidez bi horma finkoetara konektatzen dira, irudian ikus daitekeen bezala. Malgukien zurruntasunak $\mathbf{K_1=10^6}$ **N/mm** eta $\mathbf{K_2=2.10^6}$ **N/mm** dira. Bi paralelepipedoen materialak **E=200 GPa** elastikotasun modulua, $\boldsymbol{\nu}$**=0,3** poisson-en koefizientea eta $\boldsymbol{\sigma_f}$**=240 MPa** isurpen tentsioa du.

Kalkulatu:

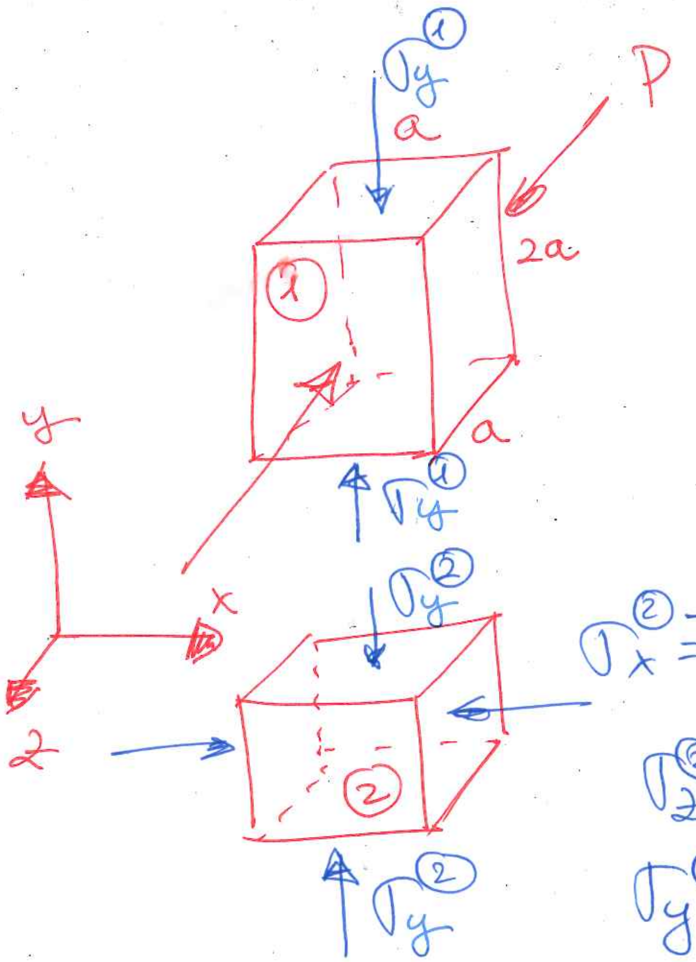
1º) P kargaren balio maximoa, Tresca-ren hutsegite erizpidearen arabera.
2º) Aurretik kalkulatutako P kargaren balio horretarako paralelepípedo bakoitzaren tentsio nagusiak.

Oharra: Kontaktuak marruskadurarik gabe ematen dira.



1

HIPOTESIS:
Tensiones a compresión
Alargamientos



$$\sigma_z^{(1)} = -P/2a^2$$

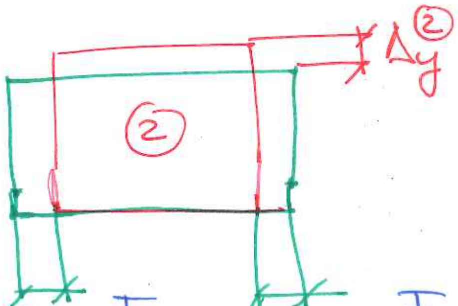
$$\sigma_x^{(1)} = 0$$

$$\sigma_y^{(1)} = -\sigma_y$$

$$\sigma_x^{(2)} = -\frac{F_m}{a^2} = -\sigma_x^{(2)}$$

$$\sigma_z^{(2)} = 0$$

$$\sigma_y^{(2)} = -\sigma_y$$



$$\Delta_x^{(2)'} = \frac{F_m}{k_2} \qquad \Delta_x^{(2)''} = \frac{F_m}{k_1}$$

$$\Delta_x^{(2)} = \Delta_x^{(2)'} + \Delta_x^{(2)''} = \left(\frac{1}{k_1} + \frac{1}{k_2}\right) F_m$$

EQUILIBRIO:

$$\sigma_y^{(1)} \cdot a \cdot a = \sigma_y^{(2)} \cdot a \cdot a \Rightarrow \sigma_y^{(1)} = \sigma_y^{(2)} = \sigma_y$$

INCOGNITAS: $\sigma_y, \sigma_x^{(2)}$

COMPATIBILIDAD: $\Delta_y^{(1)+} + \Delta_y^{(2)+} = 0$

$$\varepsilon_{yy}^{(1)+} \cdot 2a + \varepsilon_{yy}^{(2)+} a = 0$$

$$\boxed{2\varepsilon_{yy}^{(1)+} + \varepsilon_{yy}^{(2)+} = 0} \quad (1)$$

$$\Delta_x^{(2)+} = \Delta_x^{(2)'+} + \Delta_x^{(2)''+} = \left(\frac{1}{k_1} + \frac{1}{k_2}\right) F_m =$$

$$= \left(\frac{1}{k_1} + \frac{1}{k_2}\right) a^2 \cdot \sigma_x^{(2)}$$

$$\varepsilon_{xx}^{(2)+} \cdot a = a^2 \sigma_x^{(2)} \left(\frac{1}{k_1} + \frac{1}{k_2}\right)$$

$$\boxed{\varepsilon_{xx}^{(2)} = a \cdot \sigma_x^{(2)} \left(\frac{1}{k_1} + \frac{1}{k_2}\right)} \quad (2)$$

HIPOTESIS: Tensiones a compresión
Alargamientos.

LEY DE COMPORTAMIENTO:

$$\varepsilon_{xx}^{(2)} = \frac{-\sigma_x^{(2)} - \nu(-\sigma_y + 0)}{E} = \frac{-\sigma_x^{(2)} + \nu \sigma_y}{E}$$

$$\varepsilon_{yy}^{(1)} = \frac{-\sigma_y - \nu\left(0 - P/2a^2\right)}{E} = \frac{-\sigma_y + \nu\, P/2a^2}{E}$$

$$\varepsilon_{yy}^{(2)} = \frac{-\sigma_y - \nu\left(-\sigma_x^{(2)} + 0\right)}{E} = \frac{-\sigma_y + \nu \sigma_x^{(2)}}{E}$$

ECUACIONES

1 E desaparece

$$-2\sigma_y + \nu P/a^2 - \sigma_y + \nu \sigma_x^{(2)} = 0$$

$$\boxed{-3\sigma_y + \nu P/a^2 + \nu \sigma_x^{(2)} = 0}$$

2
$$\boxed{\frac{-\sigma_x^{(2)} + \nu \sigma_y}{E} = a\,\sigma_x^{(2)}\left(\frac{1}{k_1} + \frac{1}{k_2}\right)}$$

RESOLUCIÓN:

Datos: $a = 10\,mm$

$E = 200\ GPa = 200000\ MPa = 200000\ N/mm^2$

$\nu = 0'3$

$\sigma_f = 240\ MPa = 240\ N/mm^2$

$k_1 = 10^6\ N/mm$, $k_2 = 2.10^6\ N/mm$

1 $-3\sigma_y + 0'3\,\sigma_x^{(2)} = -0'3\,P/10^2$

2 $-\sigma_x^{(2)} + 0'3\,\sigma_y = 10.200000\left(\frac{1}{10^6} + \frac{1}{2.10^6}\right)\sigma_x^{(2)}$

$$-\sigma_x^{(2)} + 0'3\,\sigma_y = \frac{3.2}{2}\sigma_x^{(2)} = 3\,\sigma_x^{(2)}$$

$$0'3\,\sigma_y = (1+3)\,\sigma_x^{(2)} = 4\,\sigma_x^{(2)}$$

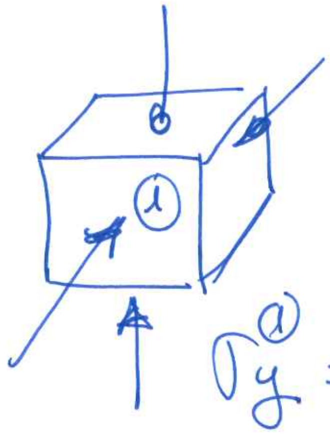
$$\boxed{3\,\sigma_y = 40\,\sigma_x^{(2)}}$$ Sustituimos en 1

$$-40\,\sigma_x^{(2)} + 0'3\,\sigma_x^{(2)} = -0'3\,P/10^2$$

$$-39'7\,\sigma_x^{(2)} = -0'3P/10^2 \Rightarrow \sigma_x^{(2)} = 7'556.10^{-5}\cdot P$$

$$\sigma_y = \frac{40}{3}\sigma_x^{(2)} = 1'007.10^{-3}P$$

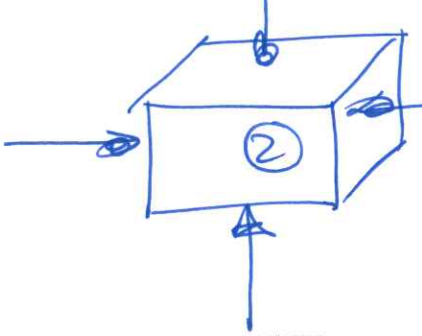
ESTADO DE TENSION EN CADA CUBO:



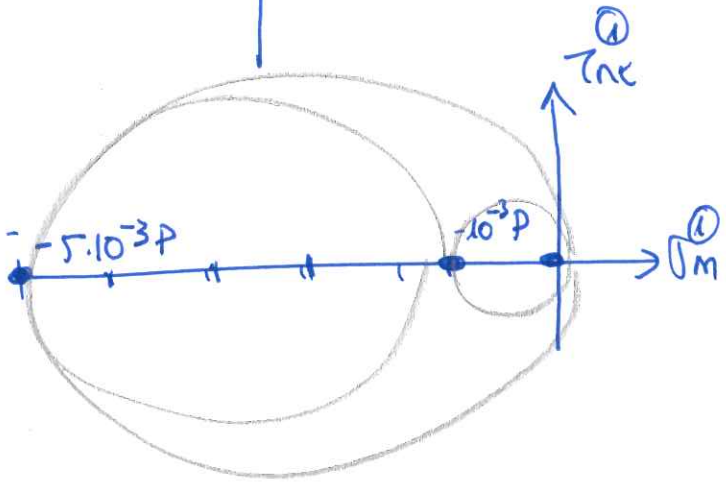
$$\sigma_z^{(1)} = \frac{-P}{2 \cdot 10^2} = -5 \cdot 10^{-3} P$$

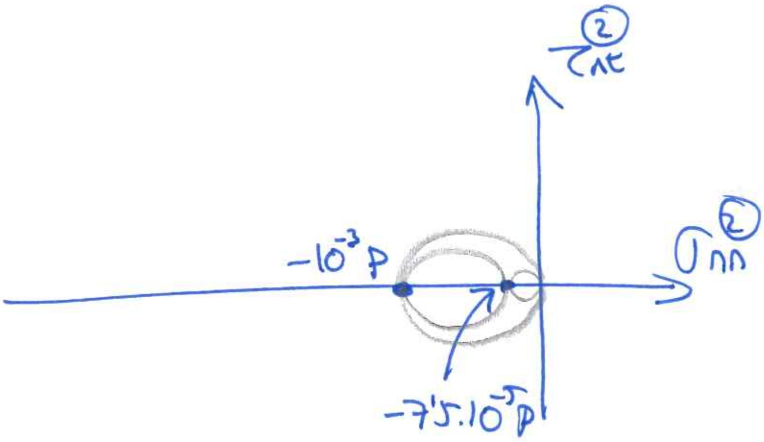
$$\sigma_y^{(1)} = -\sigma_y \cong -1'007 \cdot 10^{-3} P \simeq -10^{-3} P$$

$$-10^{-3} P = \sigma_y^{(2)}$$



$$\sigma_x^{(2)} = -7'556 \cdot 10^{-5} P$$





CRITERIO DE TRESCA: $R^{max} = \frac{\sigma_f}{2}$

$$\frac{+5 \cdot 10^{-3} P}{2} = \frac{\sigma_f}{2} \longrightarrow 5 \cdot 10^{-3} P = 240$$

$$\boxed{P = 48000 \text{ N}}$$

TENSIONES PARA ESA CARGA

$$\sigma_x^{(1)} = 0$$

$$\sigma_y^{(1)} = -1'007 \cdot 10^{-3} P = -48'36 \text{ MPa}$$

$$\sigma_z^{(1)} = -5 \cdot 10^{-3} \cdot P = -240 \text{ MPa}$$

$$\sigma_x^{(2)} = -3'626 \text{ MPa}$$

$$\sigma_y^{(2)} = -48'36 \text{ MPa}$$

$$\sigma_z^{(2)} = 0.$$