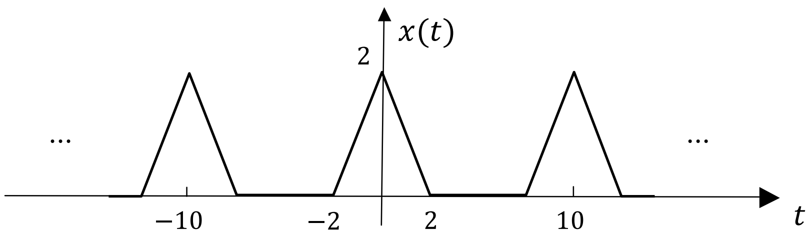
SEINALEEN PROZESAKETA: EZOHIZKO DEIALDIA

Azterketa bukatzeko ordu eta 30 minutu dituzue. Azterketak 3 ariketa ditu. Ariketa bakoitzak 10 puntu balio ditu, eta 1. ariketako galdera guztiek pisu berdina dute.

1. ARIKETA (10 puntu, 40 minutu)

1. Irudiko seinalea periodikoaren, a_k, Fourieren koefizienteen adierazpide analitiko itxia lortu lortu k-ren menpe.



2. Izan bedi honako maiztasun-erantzuna duen iragazkia:

$$\mathrm{H}(\Omega) = A \cdot e^{j\Omega} + 4 + 2 \cdot e^{-j\Omega}$$

   i) $x[n] = (-1)^n$ sarrera-seinalearentzat irteera-seinalea $y[n] = 0$ bada, kalkulatu A konstantearen balioa

   ii) Hartu aurreko ataleko A balioa eta gutxi gorabehera irudikatu maiztasun-erantzunaren modulua $0 \leq \Omega \leq \pi$ tartean. Zer motatako iragazkia da ?

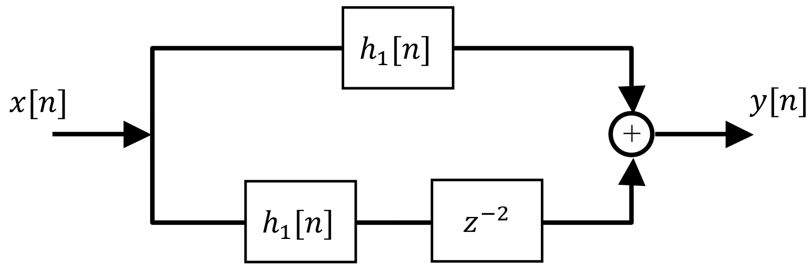
3. Izan bedi $x(t) = A_1 \cos(2\pi\, 100\, t) + A_2 \cos(2\pi\, 300\, t) + A_3 \cos(2\pi\, 1100\, t)$ seinale jarraitua A/D bihurgailu idealaren sarrera, f_{s1}=800Hz laginketa maiztasunarekin eta aliasing-aurkako iragazkirik digitalizatzen dena. Seinale lagindua goi-paseko iragazki ideal batetik pasatzen da, honako maiztasun-erantzuna duena:

$$H(\Omega) = 1 - \Pi\left(\frac{\Omega}{\pi}\right)$$

   Behin iragazita, seinale digitala, D/A bihurgailu ideal batetik pasatzen da f_{s2}=1kHz laginketa maiztasunarekin. Kalkulatu D/A bihurgailuaren irteeran lortzen den seinalearen adierazpide analitikoa.

2. ARIKETA (10 puntu, 20 minutu)

Izan bedi irudiko sistema.



a. Kalkulatu sistemaren $h[n]$ pultsu-erantzuna, hau da, $y[n] = x[n] * h[n]$, $h_1[n]$ pultsu-erantzunaren menpe. (2.5 p)

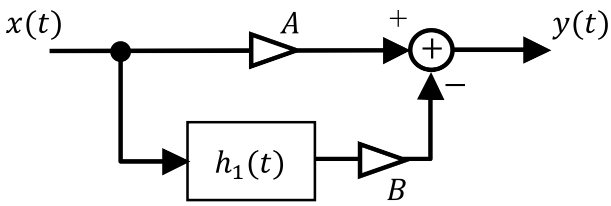
b. Irudikatu $h[n]$, $h_1[n] = \{\underline{1}, -1\}$ bada. Idatzi $y[n]$ eta $x[n]$ erlazionatzen dituen diferentzia-ekuazioa eta adierazi zein sistema mota eta zein mailakoa den. (2.5 p)

c. Kalkulatu $y[n]$ konboluzio bidez, $x[n] = \{\underline{1}, 1\}$ bada. (2.5 p)

d. Kalkulatu $y[n]$, $x[n] = u[n]$ bada. (2.5 p)

3. ARIKETA (10 puntu, 30 minutu)

Izan bedi irudiko sistema erreala:

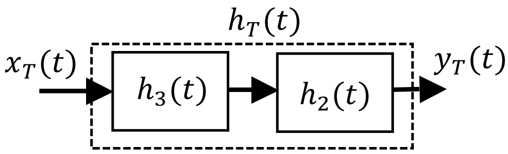


$$h_1(t) = \frac{\mathrm{sen}(\omega_{c1} t)}{\pi t}$$

a) Kalkulatu sistema osoaren pultsu-erantzuna, $h_2(t)$, eta maiztasun-erantzuna, $H_2(\omega)$, eskemako A, B eta ω_{c1} parametroen menpe. (2 p)
Oharra: $y(t) = h_2(t) * x(t)$.

b) Sarrera-seinalea $x(t) = 5 + 4\cos(100t)$ denean, irteera-seinalea $\mathrm{y}(t) = 20\cos(100t)$ bada, kalkulatu A eta B parametroak, eta adierazi ω_{c1} balioak bete beharreko baldintza. (3 p)

Aurreko sistemarekin jauzian behe-paseko iragazkia, $h_3(t)$, lotzen da irudian ageri den eran.



c) Kalkulatu ω_{c1} eta $h_3(t)$, sistema osoaren maiztasun-erantzuna honakoa izateko: $\mathrm{H}_T(\omega) = \Pi\left(\frac{\omega-200}{200}\right)$, $\omega \geq 0$. (3 p)

d) Izan bedi $x_T(t)$ seinale erreala eta periodikoa, T_0 periodoa duena, c ataleko sistemaren sarrera-seinalea. Kalkulatu zein T_0 balio tartearentzat irteeran sarrera-seinalearen oinarrizko osagaia izango dugun, $y_T(t) = A_1\cos(\omega_0 t + \theta_1)$. (2 p)

Problema 1

Q1. $x(t) = \sum_{l=-\infty}^{\infty} x_b(t - T_0 l)$ $\qquad T_0 = 10 \qquad \omega_0 = 2\pi/10$

$$x_b(t) = 2\Lambda\left(\tfrac{t}{2}\right) = \Pi\left(\tfrac{t}{2}\right) * \Pi\left(\tfrac{t}{2}\right)$$

$$X_b(\omega) = \left(\mathcal{F}\left\{\Pi\left(\tfrac{t}{2}\right)\right\}\right)^2 = \left(2\,\frac{\sin\omega}{\omega}\right)^2$$

$$\left[a_K = \frac{1}{T_0} X_b(\omega)\Big|_{\omega = K\omega_0} = \frac{4}{10}\,\frac{\sin^2 K\omega_0}{(K\omega_0)^2} = \boxed{10\,\frac{\sin^2 K\frac{2\pi}{10}}{K^2\pi^2}}\right.$$

Tambien se puede hacer con la formula general pero es muy trabajoso

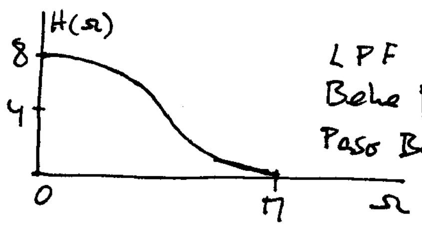
$$a_K = \frac{1}{T_0}\int_{T_0} x(t)\,e^{-jK\omega_0 t}\,dt = \frac{1}{10}\int_{-2}^{0}(t+2)\,e^{-jK\omega_0 t}\,dt + \frac{1}{10}\int_{0}^{2}(2-t)\,e^{-jK\omega_0 t}\,dt = \ldots \left\{\int t e^{at} = \frac{te^{at}}{a} - \frac{e^{at}}{a^2}\right\} \ldots$$

Q2. $x[n] = (-1)^n = e^{j\pi n} \xrightarrow{\mathcal{F}} X(\Omega) = 2\pi\delta(\Omega - \pi) \qquad -\pi < \Omega \leq \pi$

i)
$$\left.\begin{array}{l} Y(\Omega) = X(\Omega)\cdot H(\Omega) = 2\pi\delta(\Omega-\pi)\cdot H(\Omega) = 2\pi\,\delta(\Omega-\pi)\cdot H(\pi) \\ y[n] = 0 \xrightarrow{\mathcal{F}} Y(\Omega) = 0 \end{array}\right\} \Rightarrow H(\pi) = 0$$

$$H(\pi) = A\,e^{j\pi} + 4 + 2e^{-j\pi} = -A + 4 - 2 \quad \Longrightarrow A = 2$$

ii)
$$H(\Omega) = 2e^{j\Omega} + 4 + 2e^{-j\Omega} = 4 + 4\cos\Omega$$



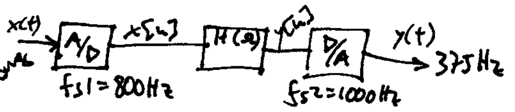
LPF
Behe paseko iragazkia
Paso Bajo

i) atala dentbora eremuan ere erraz egin daiteke

$h[n] = A\,\delta[n+1] + 4\,\delta[n] + 2\,\delta[n-1]$ $\qquad y[n] = x[n] * h[n] = A\,x[n+1] + 4x[n] + 2\,x[n-1]$

$y[n] = 0 \Rightarrow y[0] = 0 = A\,x[1] + 4\,x[0] + 2\,x[-1] = -A + 4 - 2 \quad \Rightarrow A = 2$

Q3. 100 Hz, 300, 1100 $> \frac{fs_1}{2}$ aliasing



$\boxed{A/D}$ $t = nT_{s1} = n/f_{s1}$

$$x[n] = A_1\cos 2\pi\cdot 100\frac{n}{800} + A_2\cos 2\pi\,\frac{300n}{800} + A_3\cos 2\pi\,\frac{1100n}{800} =$$

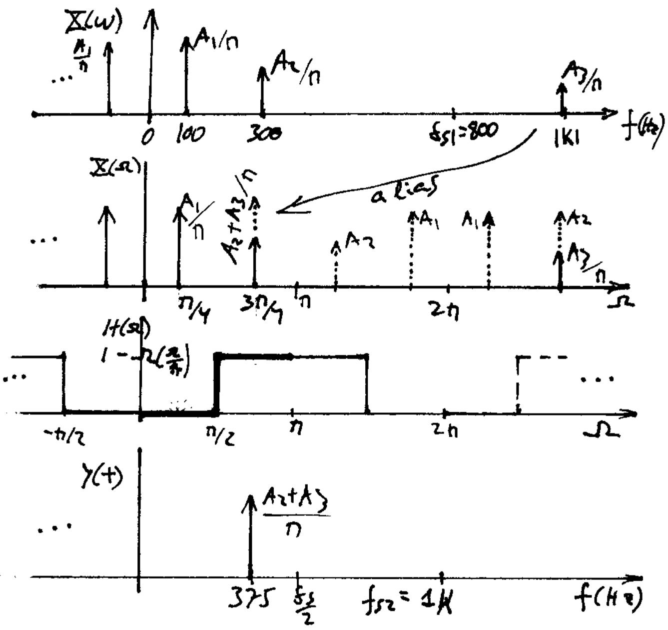
$$= A_1\cos\frac{\pi}{4}n + (A_2 + A_3)\cos\frac{3\pi}{4}n$$

$H(\Omega)$ elimina $\Omega < \frac{\pi}{2} \longrightarrow y[n] = (A_2 + A_3)\cos\frac{3\pi}{4}n$

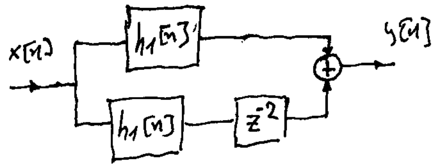
$\boxed{D/A}$ $n = t\cdot f_{s2}$

$$y(t) = (A_2 + A_3)\cos\frac{3\pi}{4}1000t = \underline{(A_2 + A_3)\cos 2\pi\cdot 375t}$$

La señal ha sufrido aliasing, filtrado y escalado



PROBLEMA 2

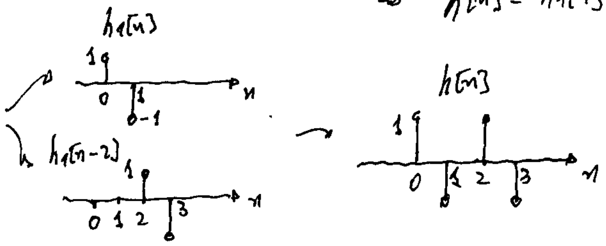


a) $y[n] = x[n] * h_1[n] + x[n] * h_1[n] * \delta[n-2] \Rightarrow$

$\Rightarrow y[n] = x[n] * h_1[n] + x[n] * h_1[n-2] = x[n] * (h_1[n] + h_1[n-2]) \Rightarrow$

$\Rightarrow h[n] = h_1[n] + h_1[n-2]$

b) $h_1[n] = \{\underline{1}, -1\}$

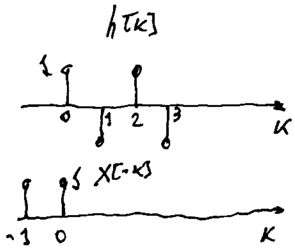


$h[n] = \{\underline{1}, -1, 1, -1\}$

$h[n] = \{\underline{1}, -1, 1, -1\} = \delta[n] - \delta[n-1] + \delta[n-2] - \delta[n-3] \Rightarrow y[n] = x[n] * h[n] = x[n] - x[n-1] + x[n-2] - x[n-3]$ ec. dif.

Sistema no recurrente (FIR) $n=3$

c) $x[n] = \{\underline{1}, 1\}$ $\quad y[n] = x[n] * h[n] = \sum_k h[k] \cdot x[n-k]$



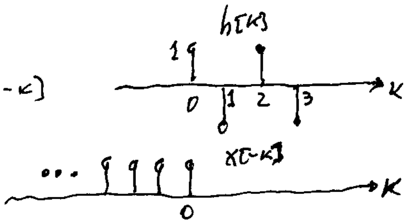
$\Rightarrow y[n] = \{\underline{1}, 0, 0, 0, -1\}$

También: $x[n] = \delta[n] + \delta[n-1]$
$x[n-1] = \delta[n-1] + \delta[n-2]$
$x[n-2] = \delta[n-2] + \delta[n-3]$
$x[n-3] = \delta[n-3] + \delta[n-4]$

aplicando la ec. en dif.: $y[n] = x[n] - x[n-1] + x[n-2] - x[n-3] \Rightarrow$

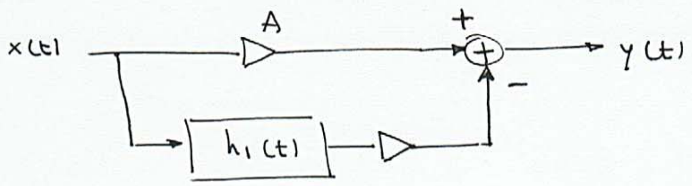
$\Rightarrow y[n] = \delta[n] - \delta[n-4] = \{\underline{1}, 0, 0, 0, -1\}$

d) $x[n] = u[n]$ $\quad y[n] = x[n] * h[n] = \sum_k h[k]\, x[n-k]$



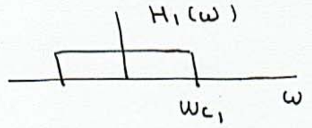
$\Rightarrow y[k] = \{\underline{1}, 0, 1, 0, 0 \dots\} = \{\underline{1}, 0, 1\}$

También: aplicando la ec. en dif.: $y[n] = \underbrace{u[n] - u[n-1]}_{\delta[n]} + \underbrace{u[n-2] - u[n-3]}_{\delta[n-2]} = \delta[n] + \delta[n-2] = \{\underline{1}, 0, 1\}$



a) $y(t) = A\,x(t) - B\,x(t) * h_1(t)$

$$\begin{cases} x(t) = \delta(t) \\ h_2(t) = A\,\delta(t) - B\,h_1(t) \end{cases} \quad \text{non } h_1(t) = \frac{\sin w_{c_1} t}{\pi t} \xrightarrow{FT}$$



$H_2(w) = FT\{h_2(t)\} = A - B\,H_1(w)$

$H_1(w) = \Pi\left(\frac{w}{2w_{c_1}}\right)$

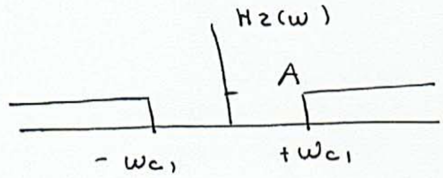
b) $x(t) = 5 + 4\cos 100t \longrightarrow y(t) = \emptyset + 20\cos 100t$

Sistemak osagai jarraia anulatzen du eta $w=100$ osagaia $\times 5$ egin, beraz:

$H_2(w=\emptyset) = \emptyset$

$H_2(w=100) = 5$

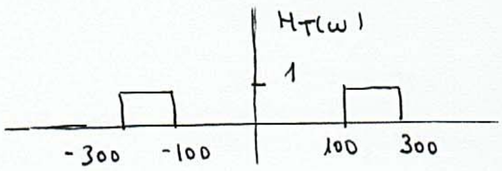
Beraz, goi-paseko iragazkia da:



$H_2(\emptyset) = A - B = \emptyset \quad \Rightarrow A = B$

$H_2(100) = A = 5$ baldin $w_{c_1} < 100$

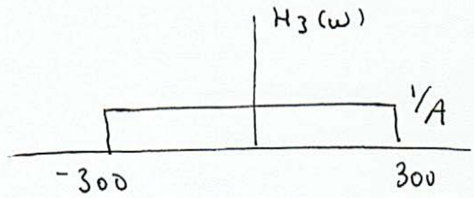
c) $H_T(w) = \Pi\left(\frac{w-200}{200}\right) \quad w \geq \emptyset$



$H_T(w) = H_2(w) \cdot H_3(w)$

Beraz, $H_3(w)$ behe-paseko iragazkia da:

eta $w_{c_1} = 100$



$H_3(w) = \frac{1}{A}\,\Pi\left(\frac{w}{2\cdot 300}\right) \longrightarrow h_3(t) = \frac{1}{A}\,\frac{\sin 300t}{\pi t}$

d) $x_T(t) = \sum_{k=1}^{\infty} 2|a_k| \cos(k w_0 t + \measuredangle a_k)$, non $k w_0$, k. armonikoa baita.

$y_T(t)$ seinaleak oinarrizko osagaia izateko, 100-300 bandan oinarrizko osagaia bakarrik pasa behar da, ez armonikorik.:

$w_0 : 150 \rightarrow 300$ (2. armonikoa pasa ez dadin)

$T_0 : \frac{2\pi}{300} \rightarrow \frac{2\pi}{150} \qquad \frac{2\pi}{300} < T_0 < \frac{2\pi}{150}$