TRATAMIENTO DE SEÑALES: EXAMEN FINAL (Primer Parcial)

La puntuación total del examen es de 30 puntos divididos en:
Problema 1: 10 puntos. Todas las cuestiones tienen el mismo peso.
Problema 2: 10 puntos.
Problema 3: 10 puntos.
El tiempo estimado para resolver el examen es de hora y media.

PROBLEMA 1 (10 puntos, 30 minutos)

1. Dibujar y expresar analíticamente la parte par y la parte impar de la señal $x(t) = \Pi\left(\frac{t}{3}\right) + \Lambda(t-3)$.

2. Sea $x[n] = 3e^{j\frac{23\pi}{5}n} + 2e^{j\frac{63\pi}{10}n}$. Se pide:
   a. Determinar la frecuencia discreta de cada una de las exponenciales que forman x[n].
   b. Determinar el número de oscilaciones por periodo de cada una de las exponenciales que forman x[n].
   c. Determinar el periodo fundamental de x[n]. Razonar la respuesta.

3. Sea un sistema LTI discreto del que se conoce que para una entrada $x_1[n] = \{\underline{1}, 2, 1\}$, la respuesta del sistema es $y_1[n] = \{\underline{1}, 1\}$. Sea ahora la secuencia $x_2[n] = [\underline{0}, 2, 4, 2, -3, -6, -3]$. Se pide:
   a. Expresar $x_2[n]$ en función de $x_1[n]$.
   b. Obtener la respuesta del sistema ante la entrada $x_2[n]$.

PROBLEMA 2 (10 puntos, 30 minutos)

Sea el sistema:

$$y(t) = \int_{-\infty}^{t} x(\tau)d\tau - \int_{-\infty}^{t} x(\tau - 3)d\tau$$

a. Estudiar la linealidad, invarianza, y causalidad del sistema. (2 ptos)

b. Obtener la respuesta impulsional en función del pulso rectangular básico y estudiar a partir de ella la causalidad y estabilidad del sistema. (2 ptos)

c. Determinar la respuesta para una entrada $x(t) = \Pi\left(\frac{t-\frac{5}{2}}{3}\right)$ (2 ptos)

d. Determinar la respuesta para una entrada $x(t) = e^{-a(t-2)} \cdot u(t-2) \quad a > 0$ (4 ptos)

Nota: $\Lambda\left(\frac{t}{T}\right) = \frac{1}{T} \cdot \prod\left(\frac{t}{T}\right) * \prod\left(\frac{t}{T}\right)$

PROBLEMA 3 (10 puntos, 30 minutos)

Sea el sistema descrito por la siguiente ecuación en diferencias (las condiciones iniciales son nulas):

$$y[n] - \alpha^2 y[n-2] = x[n]$$

a. Especificar el tipo y orden del sistema. (1 pto)

b. Filtrar la siguiente señal $x[n] = \{\underline{-1}, 2,1,1,3\}$. (2 ptos)

c. Calcular $h[n]$, la respuesta impulsional del sistema. (4 ptos)

d. Indicar si el sistema es causal y determinar la condición que debe cumplir la constante α real para que el sistema sea estable. (3 ptos)

SEINALEEN PROZESAKETA: AZKEN AZTERKETA (Lehen partziala)

Azterketak 3 ariketa ditu. Ariketa bakoitzak 10 puntu balio ditu, eta 1. ariketako galdera guztiek pisu berdina dute. Ordu eta 30 minutu dituzue.

1. ARIKETA (10 puntu, 30 minutu)

1. Adierazi analitikoki eta irudikatu honako seinalearen zati bakoitia eta zati bikoitia:

   $x(t) = \Pi\left(\frac{t}{3}\right) + \Lambda(t-3)$

2. Izan bedi honako seinalea: $x[n] = 3e^{j\frac{23\pi}{5}n} + 2e^{j\frac{63\pi}{10}n}$.

   a. Adierazi x[n] seinalea osatzen duten esponentzial bakoitzaren maiztasun diskretua.
   b. Adierazi x[n] seinalea osatzen duten esponentzial bakoitzak zenbat oszilazio dituen periodoko.
   c. Adierazi x[n]ren oinarrizko periodoa. Arrazoitu erantzuna.

3. Izan bedi LTI sistema diskretua $x_1[n] = \{\underline{1}, 2{,}1\}$ sarrera-seinalearentzat $y_1[n] = \{\underline{1}, 1\}$ irteera-seinalea ematen duena. Sarrera-seinalea, $x_2[n] = [\underline{0}, 2{,}4{,}2, -3, -6, -3]$ izanez gero, honako hauek eskatzen dira:

   a. Adierazi $x_2[n]$ seinalea $x_1[n]$ren menpe.
   b. Kalkulatu sistemaren erantzuna $x_2[n]$ sarrera-seinalearentzat.

2. ARIKETA (10 puntu, 30 minutu)

Izan bedi honako sistema hau:

$$y(t) = \int_{-\infty}^{t} x(\tau)d\tau - \int_{-\infty}^{t} x(\tau - 3)d\tau$$

a. Aztertu sistemaren linealitatea, aldakortasuna denboran eta kausalitatea. (2 puntu)

b. Sistemaren pultsu erantzuna lortu, h(t), oinarrizko pultsu laukizuzenaren arabera. Aztertu sistemaren kausalitatea eta egonkortarsuna h(t)ren arabera. (2 puntu)

c. Lortu sistemaren erantzuna $x(t) = \Pi\left(\frac{t-\frac{5}{2}}{3}\right)$ seinalentzat. (2 puntu)

d. Lortu sistemaren erantzuna honako seinalearentzat:
$x(t) = e^{-a(t-2)} \cdot u(t-2) \quad a > 0$ (4 puntu)

Oharra: $\Lambda\left(\frac{t}{T}\right) = \frac{1}{T} \cdot \prod\left(\frac{t}{T}\right) * \prod\left(\frac{t}{T}\right)$

3. ARIKETA (10 puntu, 30 minutu)

Izan bedi hasierako baldintzak nuluak dituen eta honako diferentzia-ekuazioa betetzen duen sistema:

$$y[n] - \alpha^2 y[n-2] = x[n]$$

a. Adierazi sistemaren mota eta ordena. (1 puntu)

b. Iragazi honako seinalea: $x[n] = \{\underline{-1}, 2,1,1,3\}$. (2 puntu)

c. Kalkulatu $h[n]$, sistemaren pultsu-erantzuna. (4 puntu)

d. Kausala al da sistema ? Adierazi zer baldintza bete behar duen α koefiziente errealak sistema egonkorra izateko. (3 puntu)

Telecommunications engineering degree
2013-2014

Ingeniaritza Goi Eskola Teknikoa
Escuela Técnica Superior de Ingeniería
Bilbao

SIGNAL PROCESSING: FINAL EXAM
(First mid-term)

The exam scores a total of 30 points divided as follows:

Problem 1	: 10 points. All questions have equal weight.
Problem 2	: 10 points.
Problem 3	: 10 points.

The estimated time to complete the exam are 90 minutes.

PROBLEM 1 (10 points, 30 minutes)

1. Sketch and analitically express the even and odd parts of:
$$x(t) = \Pi\left(\frac{t}{3}\right) + \Lambda(t-3)$$

2. Consider the following signal: $x[n] = 3e^{j\frac{23\pi}{5}n} + 2e^{j\frac{63\pi}{10}n}$.
   a. Determine the discrete frequency of each of the exponentials.
   b. Determine the number of oscillations per period of each of the exponentials.
   c. Determine the fundamental period of $x[n]$. Reason your answer.

3. Consider an LTI system with output $y_1[n] = \{\underline{1}, 1\}$ for the input $x_1[n] = \{\underline{1}, 2, 1\}$. Consider now the following input signal: $x_2[n] = [\underline{0}, 2, 4, 2, -3, -6, -3]$.
   a. Express $x_2[n]$ in terms of $x_1[n]$.
   b. Compute the output of the system for $x_2[n]$.

PROBLEM 2 (10 points, 30 minutes)

Consider the following system:

$$y(t) = \int_{-\infty}^{t} x(\tau)d\tau - \int_{-\infty}^{t} x(\tau - 3)d\tau$$

a. Is the system linear, time-invariant and/or causal. Reason your answers. **(2 points)**

b. Compute the impulse response in terms of the rectangular pulse and study the causality and stability of the system. **(2 points)**

c. Compute the response for the following input: $x(t) = \Pi\left(\frac{t-\frac{5}{2}}{3}\right)$ **(2 points)**

d. Compute the response for the following input:
$x(t) = e^{-a(t-2)} \cdot u(t-2) \quad a > 0$ **(4 points)**

NOTE: $\Lambda\left(\frac{t}{T}\right) = \frac{1}{T} \cdot \prod\left(\frac{t}{T}\right) * \prod\left(\frac{t}{T}\right)$

PROBLEM 3 (10 points, 30 minutes)

Let a system be described by the following difference equation (zero initial conditions):

$$y[n] - \alpha^2 y[n-2] = x[n]$$

a. What are the type and order of the system. **(1 point)**

b. Filter the signal $x[n] = \{\underline{-1}, 2,1,1,3\}$. **(2 points)**

c. Compute $h[n]$, the impulse response of the system. **(4 points)**

d. Determine if the system is causal and the values of the real constant α for a stable system. **(3 points)**

Grado en Ingeniería Técnica de Telecomunicación
Telekomunikazioaren Ingeniaritza Teknikoko Gradua
2013-2014

Ingeniaritza Goi Eskola Teknikoa
Escuela Técnica Superior de Ingeniería
Bilbao

TRATAMIENTO DE SEÑALES: EXAMEN FINAL (Segundo Parcial)

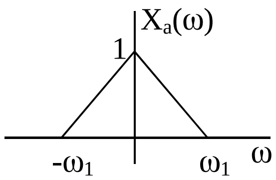
La puntuación total del examen es de 30 puntos divididos en:

Problema 1: 10 puntos. Todas las cuestiones tienen el mismo peso.
Problema 2: 10 puntos.
Problema 3: 10 puntos.

El tiempo estimado para resolver el examen es de hora y 40 minutos.

PROBLEMA 1 (10 puntos, 30 minutos)

1. La señal $x_a(t)$, cuyo espectro se da en la figura, se muestrea con fs=1600 m/s, para obtener la secuencia $x_d[n]$.
   a. Calcular gráficamente el espectro de $x_d[n]$, $X_d(\Omega)$.
   b. ¿Qué información de la señal continua se mantiene inalterada en $x_d[n]$?
   c. ¿Cuál sería esta información si antes del muestreo se hubiera filtrado antialiasing?



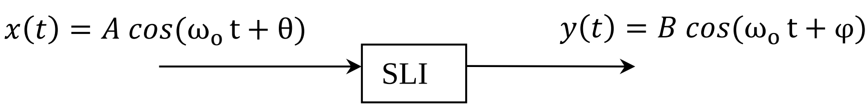
ω_1=2π1200 rad/s

2. La señal x(t)=3cos(2π20t+π/3) se muestrea con fs= 80 m/s para obtener la secuencia x[n]. Esta secuencia se procesa con un SLI promediador de 4 muestras cuya ecuación en diferencias es:

$$y[n] = \frac{1}{4}\sum_{k=0}^{3} x[n-k]$$

   Calcular analíticamente la respuesta del sistema y[n].

3. Demostrar que para un sistema SLI real, con respuesta frecuencial H(ω), la entrada x(t) da la salida y(t) de la figura. Obtén B, φ en función de A, θ y H(ω).



PROBLEMA 2 (10 puntos, 45 minutos)

Sea el sistema de la figura constituido por dos sistemas en cascada

$$x[n] \longrightarrow \boxed{h_1[n]} \longrightarrow \boxed{H_2(\Omega)} \longrightarrow y[n]$$

Considerando los siguientes sistemas:

$h_1[n] = A\left(\frac{1}{2}\right)^n u[n]$ y $H_2(\Omega) = \frac{3}{1-\frac{1}{8}\cdot e^{-j\Omega}}$

a) Calcular la respuesta frecuencial del sistema completo, H(Ω), y la ecuación en diferencias que relaciona y[n] con x[n]. (2 ptos)

b) Obtener el valor de A si para la señal de entrada $x[n] = \frac{1}{2}(-1)^n$ se obtiene la señal de salida $y[n] = \frac{2}{3}(-1)^n$. (2 ptos)

c) Sustituir el primer sistema por un sistema LTI tal que la respuesta total satisface que: $y[n] = x[n]$. Calcular y representar la nueva respuesta impulsional $h_1[n]$. (1.5 ptos)

Se sustituye el primer sistema por uno cuya respuesta impulsional es:

$$h_1[n] = \frac{\sin(\Omega_c(n-4))}{\pi(n-4)} \quad \text{con} \quad \Omega_c = \pi/4$$

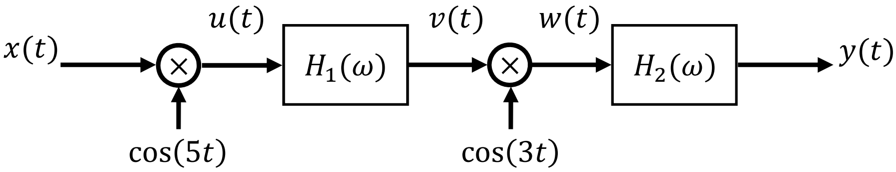
d) Calcular la respuesta frecuencial total del sistema, H(Ω). (0.5 ptos)

e) Si el sistema se excita con la señal $x[n] = \sum_{k=-\infty}^{k=\infty} \delta[n - kN_o]$ con N_o=10, calcular y representar el espectro X(Ω). (2 ptos)

f) Obtener la respuesta del sistema y[n] y representar su espectro Y(Ω). (2 ptos)

PROBLEMA 3 (10 puntos, 30 minutos)

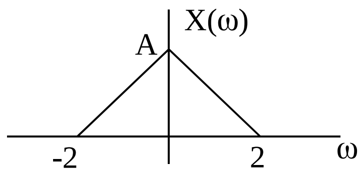
Sea el sistema de la figura, en el que las respuestas frecuenciales de los sistemas 1 y 2 son las indicadas.



$$H_1(\omega) = \begin{cases} 2, & 3 \le |\omega| \le 5 \\ 0, & \text{resto} \end{cases} \qquad H_2(\omega) = \begin{cases} 2, & |\omega| \le 3 \\ 0, & |\omega| > 3 \end{cases}$$

a. Dibuja $H_1(\omega)$ y $H_2(\omega)$ (módulo y fase) e indica el tipo de filtro que son. (2 ptos)

b. Dibuja el espectro de y(t) si el espectro de la señal de entrada es el de la figura. (4 ptos)



c. Calcula la señal de salida si la entrada es:

$$x(t) = \sin(t) + \cos(4t) \qquad \text{(4 ptos)}$$

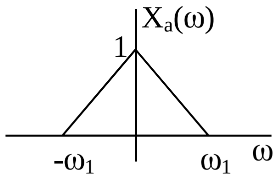
SEINALEEN PROZESAKETA: AZKEN AZTERKETA (Bigarren partziala)

Azterketak 3 ariketa ditu. Ariketa bakoitzak 10 puntu balio ditu, eta 1. ariketako galdera guztiek pisu berdina dute. Azterketa bukatzeko ordu eta 45 minutu dituzue.

1. ARIKETA (10 puntu, 30 minutu)

1. Izan bedi $x_a(t)$ seinalea, irudiko espektroa duena, fs=1600 l/s maiztasunarekin lagintzen dena $x_d[n]$ lortzeko.
    a. Kalkulatu grafikoki $x_d[n]$ren espektroa, $X_d(\Omega)$.
    b. Seinale jarraituaren zein informazio mantentzen da aldaketarik gabe (distortsiorik gabe)?
    c. Aurreko galderari erantzun antialiasing iragazki bat erabiltzen bada laginketaren aurretik.

$\omega_1=2\pi1200$ rad/s

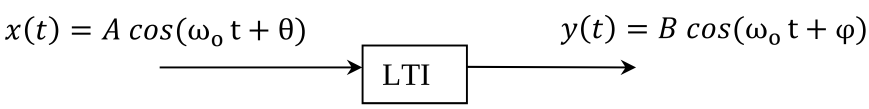


2. $x(t)=3\cos(2\pi20t+\pi/3)$ seinalea fs= 80 l/s maiztasunarekin lagintzen da x[n] sekuentzia lortzeko. Sekuentzia LTI batekin prozesatzen da, 4 laginen batezbestekoa egiten duena, honako diferentzia-ekuazioarekin:

$$y[n] = \frac{1}{4}\sum_{k=0}^{3} x[n-k]$$

Kalkulatu analitikoki sistemaren erantzuna, y[n].

3. Frogatu LTI sistema erreal batetan, $H(\omega)$ maiztasun-erantzuna duena, irudiko x(t) sarrera-seinaleak y(t) irteera-seinalea ematen duela. Lortu B eta φ-ren balioak A, θ eta $H(\omega)$ren menpe.



2. ARIKETA (10 puntu, 45 minutu)

Izan bedi irudiko sistema, jauzian dauden bi sistemez osatua.

$$x[n] \longrightarrow \boxed{h_1[n]} \longrightarrow \boxed{H_2(\Omega)} \longrightarrow y[n]$$

Honako bi sistemak hartuz hurrengo galderak erantzun:

$$h_1[n] = A\left(\tfrac{1}{2}\right)^n u[n] \quad \text{eta} \quad H_2(\Omega) = \frac{3}{1-\frac{1}{8}\cdot e^{-j\Omega}}$$

a) Kalkulatu sistema osoaren maiztasun-erantzuna, H(Ω). Kalkulatu y[n] eta x[n] erlazionatzen duen diferentzia-ekuazioa. (2 puntu)

b) Lortu A balioa $x[n] = \frac{1}{2}(-1)^n$ sarrera-seinalearen erantzuna $y[n] = \frac{2}{3}(-1)^n$ bada. (2 puntu)

c) Jauzian dagoen lehenengo sistema beste LTI sistema gatengatik ordezkatu, honako hau betearazten duena: $y[n] = x[n]$. Kalkula eta irudikatu $h_1[n]$ berria. (1.5 puntu)

Jauzian dagoen lehenengo sistema honako pultsu-erantzuna duen batengatik ordezkatu:

$$h_1[n] = \frac{\sin(\Omega_c(n-4))}{\pi(n-4)} \quad \text{non} \quad \Omega_c=\pi/4$$

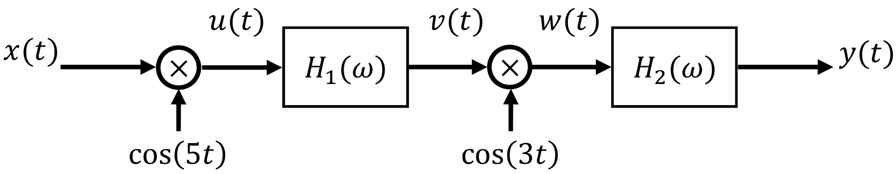
d) Kalkulatu sistema osoaren maiztasun-erantzuna, H(Ω). (0.5 puntu)

e) Sistemaren sarrera-seinalea $x[n] = \sum_{k=-\infty}^{k=\infty} \delta[n-kN_o]$ da, non N_o=10 baita. Kalkulatu eta irudikatu X(Ω) espektroa. (2 puntu)

f) Lortu sistemaren y[n] erantzuna, eta irudikatu bere espektroa, Y(Ω). (2 puntu)

3.ARIKETA (10 puntu, 30 minutu)

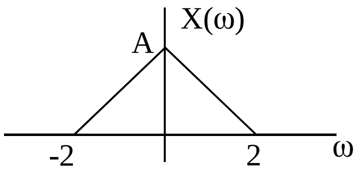
Izan bedi irudiko sistema, non 1. eta 2. sistemen maiztasun-erantzunak jarraian azaltzen baitira.



$$H_1(\omega) = \begin{cases} 2, & 3 \le |\omega| \le 5 \\ 0, & \text{resto} \end{cases} \qquad H_2(\omega) = \begin{cases} 2, & |\omega| \le 3 \\ 0, & |\omega| > 3 \end{cases}$$

a. Irudikatu $H_1(\omega)$ eta $H_2(\omega)$ (modulua eta fasea), eta adierazi zer iragazi mota diren. (2 puntu)

b. Irudikatu y(t)ren espektroa sarrera-seinalearen espektroa irudikoa bada. (4 puntu)



c. Kalkulatu irteera-seinalea honako sarrera-seinalearentzat:

$$x(t) = \sin(t) + \cos(4t)$$ (4 puntu)

SIGNAL PROCESSING: FINAL EXAM
(Second mid-term)

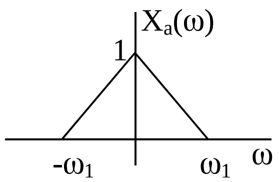
The exam scores a total of 30 points divided as follows:

Problem 1 : 10 points. All questions have equal weight.
Problem 2 : 10 points.
Problem 3 : 10 points.

The estimated time to complete the exam are 1h45min.

PROBLEM 1 (10 puntos, 30 minutos)

1. The signal $x_a(t)$ has the spectrum of the figure and is sampled without antialiasing filter and f_s=1600Hz to obtain the sequence $x_d[n]$.
    a. Graphically compute the spectrum of $x_d[n]$: $X_d(\Omega)$.
    b. What information of the continuous signal is unaltered in $x_d[n]$?
    c. What inforamtion of the continuos signal would be unaltered had an antialising filter been used?



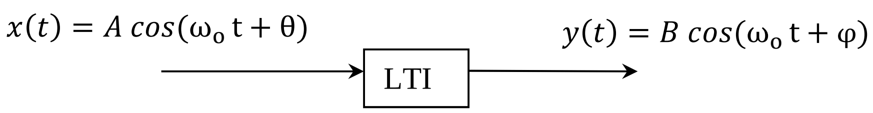
$\omega_1 = 2\pi 1200$ rad/s

2. The signal $x(t)=3\cos(2\pi 20t+\pi/3)$ is sampled with f_s= 80Hz to obtain the sequence x[n]. The sequence is processed with a 4 sample running average filter with the following difference equation:

$$y[n] = \frac{1}{4}\sum_{k=0}^{3} x[n-k]$$

Analitically compute the output of the system y[n].

3. Consider the real LTI system the figure whose frequency response is $H(\omega)$. Obtain the values of B and φ in terms of A, θ and the frequency response of the system.



PROBLEM 2 (10 points, 45 minutes)

The system of the figure is the cascade conexion of two systems:

$$x[n] \longrightarrow \boxed{h_1[n]} \longrightarrow \boxed{H_2(\Omega)} \longrightarrow y[n]$$

Consider the following systems:

$h_1[n] = A\left(\frac{1}{2}\right)^n u[n]$ and $H_2(\Omega) = \frac{3}{1-\frac{1}{8}\cdot e^{-j\Omega}}$

a) Compute the frequency response of the complete system, H(Ω), and the difference equation relating y[n] and x[n]. **(2 points)**

b) Obtain the value of A knowing that for the input $x[n] = \frac{1}{2}(-1)^n$ the output is $y[n] = \frac{2}{3}(-1)^n$. **(2 points)**

c) Substitute the first system by an LTI system so that the response is: $y[n] = x[n]$. Compute and sketch the new impulse response $h_1[n]$. **(1.5 points)**

The first system is substituted by:

$$h_1[n] = \frac{\sin(\Omega_c(n-4))}{\pi(n-4)} \quad \text{with} \quad \Omega_c = \pi/4$$

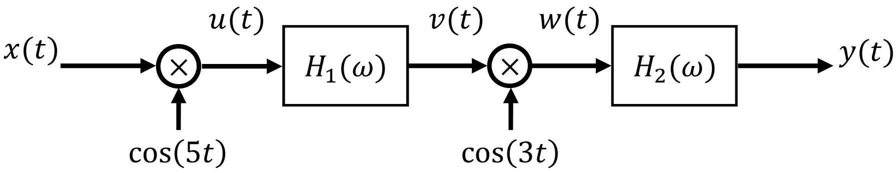
d) Compute the frequency response of the complete system, H(Ω). **(0.5 points)**

e) Consider the signal $x[n] = \sum_{k=-\infty}^{k=\infty} \delta[n-kN_o]$ with $N_o = 10$. Compute and sketch its spectrum $X(\Omega)$. **(2 points)**

f) Obtain the response $y[n]$ for the input $x[n]$ of the previous section and sketch its spectrum $Y(\Omega)$. **(2 points)**

PROBLEM 3 (10 points, 30 minutes)

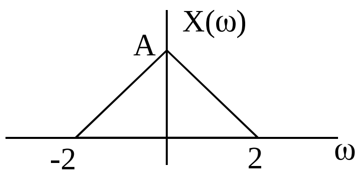
In the system of the figure the subsystems have the frequency responses indicated below.



$$H_1(\omega) = \begin{cases} 2, & 3 \le |\omega| \le 5 \\ 0, & \text{rest} \end{cases} \qquad H_2(\omega) = \begin{cases} 2, & |\omega| \le 3 \\ 0, & |\omega| > 3 \end{cases}$$

a. Plot $H_1(\omega)$ and $H_2(\omega)$ (modulus and phase) and indicate the type of filter. **(2 points)**

b. Plot the spectrum of y(t) for an input signal with the spectrum shown in the figure. **(4 points)**



c. Compute the output signal for: **(4 points)**

$$x(t) = \sin(t) + \cos(4t)$$

Seinaleen prozesatzea

Azken azterketako 1. partzialaren ebazpena 2014-V-I6

1. Ariketa. Galderak

1. Adierazi analitikoki eta irudikatu honako seinalearen zati bakoitia eta zati bikoitia: $x(t)=\Pi\left(\frac{t}{3}\right)+\Lambda(t-3)$

La parte par debe cumplir $x_e(t)=x_e(-t)$

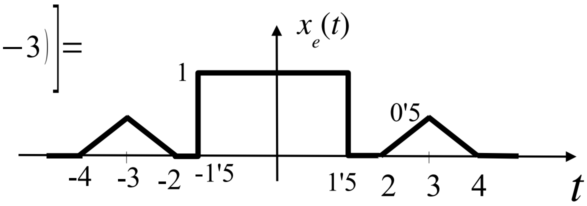
eta zati bakoitiak hau bete behar du $x_o(t)=-x_o(-t)$

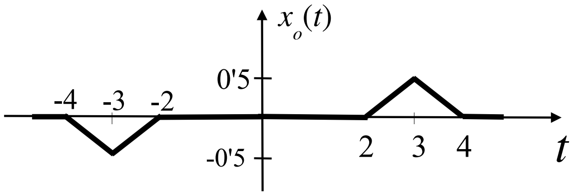
$$x_e(t)=\frac{x(t)+x(-t)}{2}=\frac{1}{2}\left[\Pi\left(\frac{t}{3}\right)+\Lambda(t-3)+\Pi\left(\frac{-t}{3}\right)+\Lambda(-t-3)\right]=$$

eta Π eta Λ bikoitiak direnez

$$\boxed{=\Pi\left(\frac{t}{3}\right)+\frac{\Lambda(t-3)+\Lambda(t+3)}{2}}$$

$$\boxed{x_o(t)=\frac{x(t)-x(-t)}{2}=\frac{\Lambda(t-3)-\Lambda(t+3)}{2}}$$





2. Izan bedi honako seinalea: $x[n]=3e^{j\frac{23\pi}{5}n}+2e^{j\frac{63\pi}{10}n}$
   a. Adierazi x[n] seinalea osatzen duten esponentzial bakoitzaren maiztasun diskretua.
   b. Adierazi x[n] seinalea osatzen duten esponentzial bakoitzak zenbat oszilazio dituen periodoko.
   c. Adierazi x[n]ren oinarrizko periodoa. Arrazoitu erantzuna.

a. $\Omega = 2\pi\cdot k/N = 2\pi f_d$ $\quad x_1[n]=3e^{j\frac{23\pi}{5}n}=3e^{j\frac{20\pi}{5}n}\cdot e^{j\frac{3\pi}{5}n}=3e^{j\frac{3\pi}{5}n}$ $\quad \Omega_1 = 3\pi/5 = 2\pi\cdot 3/10$ --> $\boldsymbol{f_{d1} = 3/10}$

$x_2[n]=2e^{j\frac{63\pi}{10}n}=2e^{j\frac{30\cdot 2\pi}{10}n}\cdot e^{j\frac{3\pi}{10}n}=2e^{j\frac{3\pi}{10}n}$ $\quad \Omega_2 = 3\pi/10 = 2\pi\cdot 3/20$ --> $\boldsymbol{f_{d2} = 3/20}$

b. $N_1 = 10$ $\quad$ $\mathbf{k_1 = 3}$ oszilazio periodoko

$N_2 = 20$ $\quad$ $\mathbf{k_2 = 3}$ oszilazio periodoko

c. $\mathbf{N = 20}$ lehen osagaiaren maiztasuna bigarrenaren bikoitza da. Oinarrizko osagaia bietatik geldoena da, $x_2[n]$, periodoa N_2=20 duena, eta $x_1[n]$ bigarren armonikoa da.

Tenemos la componente fundamental $x_2[n]$ y el segundo armónico, $x_1[n]$ de frecuencia doble (periodo mitad).

3. Izan bedi LTI sistema diskretua $x_1[n]=\{\underline{1},2,1\}$ sarrera-seinalearentzat $y_1[n]=\{\underline{1},1\}$ irteera-seinalea ematen duena. Sarrera-seinalea, $x_2[n]=\{\underline{0},2,4,2,-3,-6,-3\}$ izanez gero, honako hauek eskatzen dira:
   a. Adierazi $x_2[n]$ seinalea $x_1[n]$ ren menpe.
   b. Kalkulatu sistemaren erantzuna $x_2[n]$ sarrera-seinalearentzat.

a. Zuzenean ikusten da $x_2[n] = 2x_1[n-1]-3x_1[n-4]$

b. Sistema LTI denez $y_2[n] = 2y_1[n-1]-3y_1[n-4] = \mathbf{\{\underline{0},2,2,0,-3,-3,0\}}$

PROBLEMA 2

$$y(t) = \int_{-\infty}^{t} x(\tau)\,d\tau + \int_{-\infty}^{t} x(\tau-3)\,d\tau$$

a) Linealidad

homogeneidad.

$$x(t) \longrightarrow y(t) = \int_{-\infty}^{t} x(\tau)\,d\tau - \int_{-\infty}^{t} x(\tau-3)\,d\tau$$

$$a\,x(t) \longrightarrow y_1(t) = \int_{-\infty}^{t} a\,x(\tau)\,d\tau - \int_{-\infty}^{t} a\,x(\tau-3)\,d\tau$$

$$= a\left[\int_{-\infty}^{t} x(\tau)\,d\tau + \int_{-\infty}^{t} x(\tau-3)\,d\tau\right] = a\,y(t)$$

Se cumple homogeneidad

Superposición

$$x_1(t) \longrightarrow y_1(t) = \int_{-\infty}^{t} x_1(\tau)\,d\tau + \int_{-\infty}^{t} x_1(\tau-3)\,d\tau$$

$$x_2(t) \longrightarrow y_2(t) = \int_{-\infty}^{t} x_2(\tau)\,d\tau - \int_{-\infty}^{t} x_2(\tau-3)\,d\tau$$

$$x_1(t)+x_2(t) \longrightarrow y(t) = \int_{-\infty}^{t} (x_1(\tau)+x_2(\tau))\,d\tau - \int_{-\infty}^{t} (x_1(\tau-3)+x_2(\tau-3))\,d\tau$$

$$y(t) = \underbrace{\int_{-\infty}^{t} x_1(\tau) - \int_{-\infty}^{t} x_1(\tau-3)\,d\tau}_{y_1(t)} + \underbrace{\int_{-\infty}^{t} x_2(\tau)\,d\tau - \int_{-\infty}^{t} x_2(\tau-3)\,d\tau}_{y_2(t)}$$

$$y(t) = y_1(t) + y_2(t)$$

Se cumple superposición $\boxed{\text{Es lineal}}$

Invarianza

$$x(t) \longrightarrow y(t) = \int_{-\infty}^{t} x(\tau)\,d\tau - \int_{-\infty}^{t} x(\tau-3)\,d\tau.$$

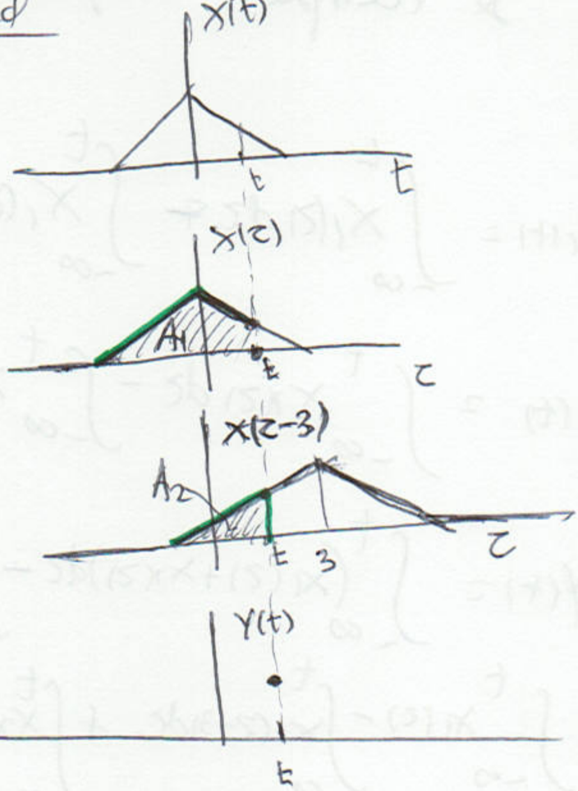
$$x(t-t_0) \longrightarrow y_1(t) = \int_{-\infty}^{t} x(\tau-t_0)\,d\tau - \int_{-\infty}^{t} x(\tau-t_0-3)\,d\tau. \quad \left\{\begin{array}{l} \tau - t_0 = \tau' \\ \tau \to -\infty \quad \tau' \to -\infty \\ \tau = t \quad \tau' = t - t_0 \end{array}\right.$$

$$y(t-t_0) = \int_{-\infty}^{t-t_0} x(\tau)\,d\tau + \int_{-\infty}^{t-t_0} x(\tau-3)\,d\tau$$

$$= \quad y_1(t) = \int_{-\infty}^{t-t_0} x(\tau)\,d\tau - \int_{-\infty}^{t-t_0} x(\tau-3)\,d\tau.$$

Es tiempo invariante

Causalidad

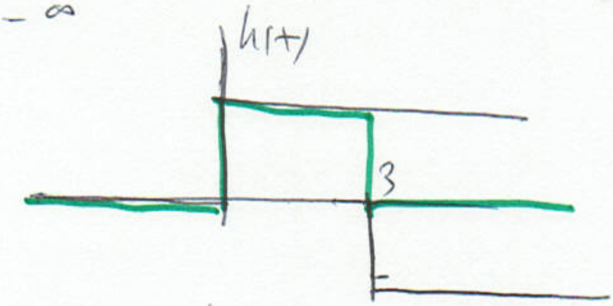


$$y(t) = \underbrace{\int_{-\infty}^{t} x(\tau)\,d\tau}_{A_1} - \underbrace{\int_{-\infty}^{t} x(\tau-3)\,d\tau}_{A_2}$$

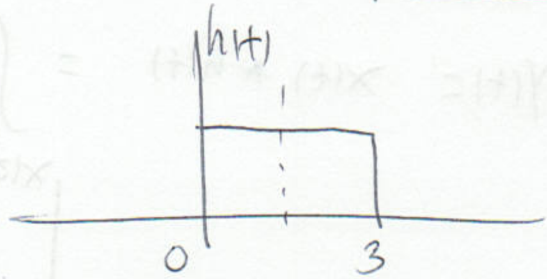
$y(t)$ depende de valores presentes o pasados de la entrada $x(t)$. Por tanto el sistema es causal.

b) $h(t)=\int_{-\infty}^{t}\delta(\tau)\,d\tau-\int_{-\infty}^{t}\delta(\tau-3)\,d\tau$

$h(t)=u(t)-u(t-3)$



$$\boxed{h(t)=\Omega\left(\frac{t-3/2}{3}\right)}$$



Es causal porque $\boxed{h(t)=0\ \forall\ t<0}$

Estabilidad $\int_{-\infty}^{\infty}|h(t)|\,dt = 3 < \infty$ Es estable

c) $x(t)=\Omega\left(\frac{t-5/2}{3}\right)$

$$y(t)=x(t)*h(t)=\Omega\left(\frac{t-5/2}{3}\right)*\Omega\left(\frac{t-3/2}{3}\right)$$

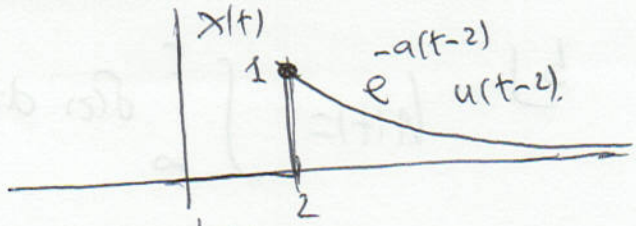
$$\Lambda\left(\frac{t}{T}\right)=\frac{1}{T}\,\Omega\left(\frac{t}{T}\right)*\Omega\left(\frac{t}{T}\right)$$

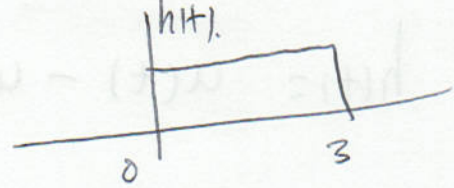
$$\Omega\left(\frac{t}{3}\right)*\Omega\left(\frac{t}{3}\right)=3\Lambda\left(\frac{t}{3}\right)$$

Propiedad de desplazamiento

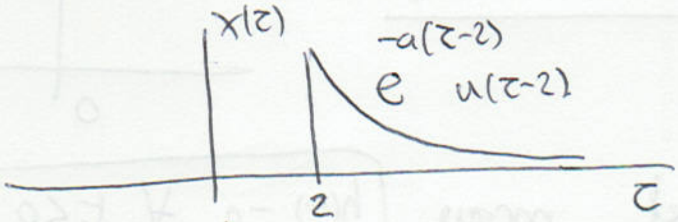
$$\Omega\left(\frac{t-5/2}{3}\right)*\Omega\left(\frac{t-3/2}{3}\right)=\boxed{3\Lambda\left(\frac{t-4}{3}\right)}$$

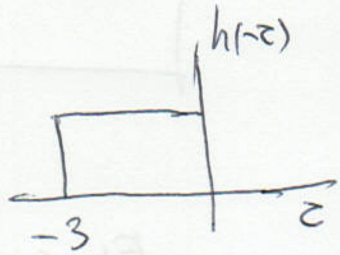
d) $x(t) = e^{-a(t-2)} u(t-2) \quad a>0$

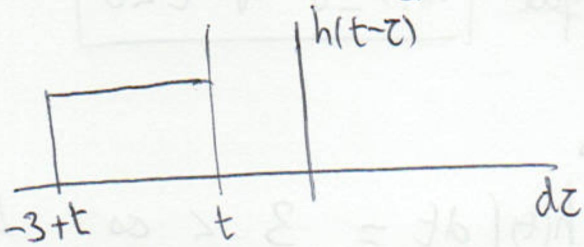




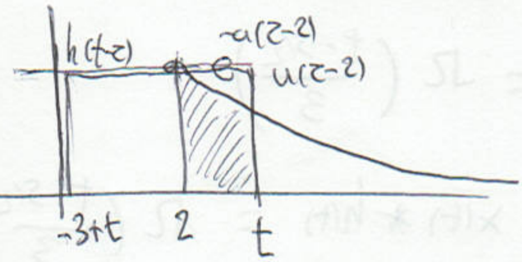
$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$







$-\infty < t < 2 \qquad y(t) = 0 \longrightarrow$ No hay solapamiento



$2 \leq t < 5$ Esta entrando

$-3+t = 2 \qquad \underline{\underline{t=5}}$

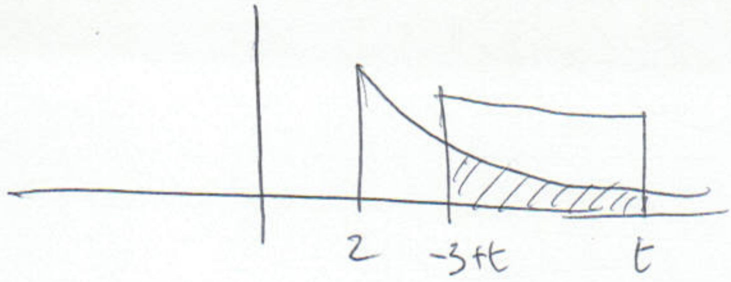
$$y(t) = \int_2^t e^{-a(\tau-2)}\, d\tau$$

$$y(t) = e^{2a} \int_2^t e^{-a\tau}\, d\tau$$

$$y(t) = e^{2a} \left(\frac{-1}{a}\right) e^{-a\tau}\Big]_2^t = e^{2a}\left[\frac{-1}{a}\left(e^{-at} - e^{-2a}\right)\right.$$

$$y(t) = \frac{-1}{a}\left(e^{-a(t-2)} - 1\right) = \frac{1 - e^{-a(t-2)}}{a}$$

$5 \leq t < \infty$



$$y(t) = \int_{-3+t}^{t} e^{-a(t-2)} \, d\tau = e^{2a}\left(\frac{-1}{a}\right) e^{-a\tau}\Big]_{-3+t}^{t}$$

$$y(t) = e^{2a}\left(\frac{-1}{a}\right)\left[e^{-at} - e^{-a(-3+t)}\right]$$

$$y(t) = \frac{e^{5a} \cdot e^{-at} - e^{2a} e^{-at}}{a} = e^{-at}\left[\frac{e^{5a} - e^{2a}}{a}\right]$$

$$\boxed{y(t) = \begin{cases} 0 & -\infty < t < 2 \\ \dfrac{1 - e^{-a(t-2)}}{a} & 2 \leq t < 5 \\ e^{-at}\left(\dfrac{e^{5a} - e^{2a}}{a}\right) & 5 < t < \infty \end{cases}}$$

PROBLEMA 3

$$y[n] = \alpha^2 y[n-2] - x[n]$$

a) Es un IIR porque tiene parte recursiva (término $y[n-2]$) y es de orden 2 porque es el máximo retardo de la entrada o la salida.

b) $x[n] = \{\underline{-1}, 2, 1, 1, 3\}$
$n = 0, 1, 2, 3, 4$

$y[0] = x[0] + \alpha^2 \cancel{y[-2]}^{\,0}$ C.I nulas $= (-1)$

$y[1] = x[1] + \alpha^2 \cancel{y[-1]}^{\,0}$ C.I nulas $= 2$

$y[2] = x[2] + \alpha^2 y[0] = 1 + \alpha^2(-1) = 1 - \alpha^2$

$y[3] = x[3] + \alpha^2 y[1] = 1 + \alpha^2 \cdot 2 = 1 + 2\alpha^2$

$y[4] = x[4] + \alpha^2 y[2] = 3 + \alpha^2(1-\alpha^2) = 3 + \alpha^2 - \alpha^4$

$$\boxed{y[n] = \{\underline{-1}, 2, 1-\alpha^2, 1+2\alpha^2, 3+\alpha^2-\alpha^4, \ldots\}}$$

c) $h[n]$ $\quad x[n] = \delta[n]$ $\quad h[n] = \delta[n] + \alpha^2 h[n-2]$

$h[0] = \delta[0] + \alpha^2 \cancel{h[-2]}^{\,0}$ C.I nulas $= 1$

$h[1] = \cancel{\delta[1]}^{\,0} + \alpha^2 \cancel{h[-1]}^{\,0}$ C.I nulas $= 0$

$h[2] = \cancel{\delta[2]}^{\,0} + \alpha^2 h[0] = \; = +\alpha^2$

$h[3] = \cancel{\delta[3]}^{\,0} + \alpha^2 h[1] = \; = 0$

$h[4] = \cancel{\delta[4]}^{\,0} + \alpha^2 h[2] = \; = \alpha^4$

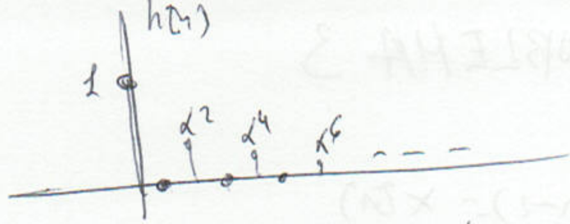
$h[5] = \cancel{\delta[5]}^{\,0} + \alpha^2 h[3] = \; = 0$

$h[6] = \cancel{\delta[6]}^{\,0} + \alpha^2 h[4] = \; = +\alpha^6$

$$\boxed{h[n] = \sum_{k=0}^{\infty} \alpha^{2k} \delta[n-2k]}$$

$h[n] =$

d)



Es causal porque $h[n]=0 \quad \forall\, n<0$.

Para que sea estable $\sum_{n=-\infty}^{\infty} |h[n]| < \infty$

$$\sum_{n=-\infty}^{\infty} |h[n]| = 1+\alpha^2+\alpha^4+\alpha^6+ \dots \dots \alpha^{\infty}$$

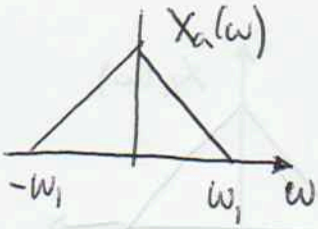
serie geométrica de razón $r=\alpha^2$

para que sea convergente

$|r|<1 \qquad \alpha^2<1$

$$\boxed{-1<\alpha<1}$$

1) a) El espectro de la señal analógica es:

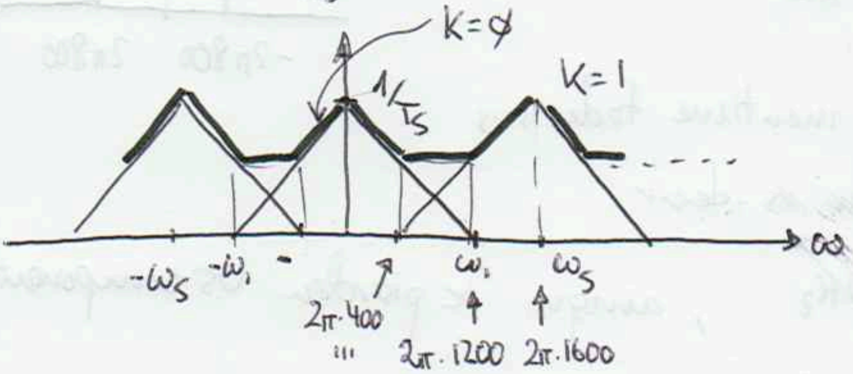


$\omega_1 = 2\pi \cdot 1200$ r/s

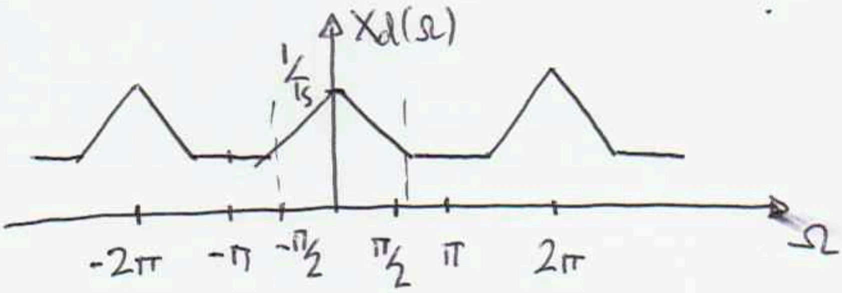
y su relación con el espectro discreto es (señal muestreada):

$$X_d(\Omega)\Big|_{\Omega=\omega T_s} = \frac{1}{T_s}\sum_{K=-\infty}^{\infty} X_a(\omega - K\omega_s)$$

Se ponen repeticiones cada $\omega_s = 2\pi \cdot f_s = 2\pi \cdot 1600$ r/s



$\Omega = \omega \cdot T_s = 2\pi \frac{f}{f_s}$



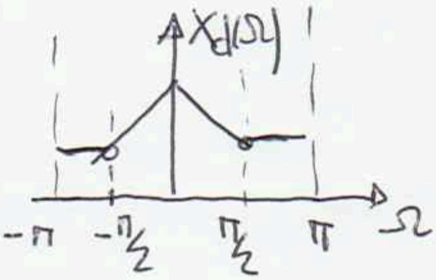
las frecuencias discretas son:

$\omega_s \to \Omega = 2\pi \frac{f_s}{f_s} = 2\pi$

$\omega_1 \to \Omega = 2\pi \frac{1200}{1600} = \frac{3\pi}{2}$

$\omega_2 \to \Omega = 2\pi \frac{(1600-1200)}{1600} = \frac{\pi}{2}$

b) Las componentes espectrales que se mantienen son:



$-\frac{\pi}{2} \leq \Omega \leq \frac{\pi}{2}$ ↓ $\omega = \Omega \cdot f_s$

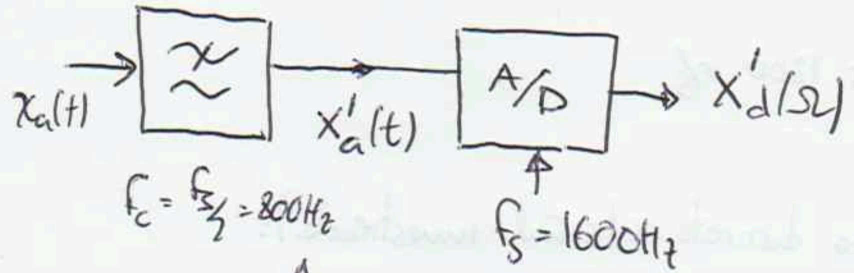
$-2\pi\, 400 \leq \omega \leq \frac{\pi}{2} \cdot 1600 = 2\pi \cdot 400$

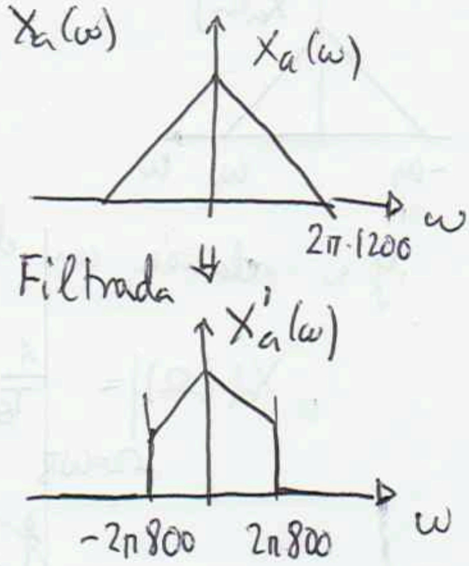
$-400\,Hz \leq f \leq 400\,Hz$ ↙ $f = \frac{\omega}{2\pi}$

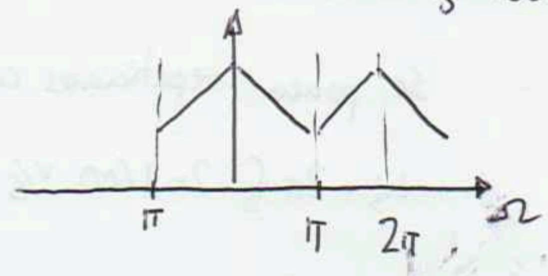
↳ distorsión (no son los de $X_a(\omega)$)

Sin distorsión (como en $X_a(\omega)$)

ⓒ Si aplicamos un filtro antialiasing (con $f_c = f_s/2$, pasobajo):





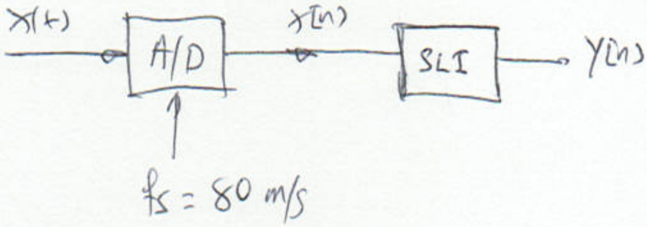


En este caso $X_d(\Omega)$ mantiene todas sus componentes sin distorsión es decir

$$-800Hz \leq f \leq 800Hz$$

, aunque se pierden las componentes de la señal original entre $800Hz$ y $1200Hz$.

CUESTION 2



$$x(t) = 3\cos(2\pi\, 20 t + \pi/3)$$

$$x[n] = x(t)\Big|_{t=\frac{n}{f_s}} = 3\cos\left(2\pi\, 20 \frac{n}{80} + \frac{\pi}{3}\right)$$

$$x[n] = 3\cos\left(2\pi \frac{1}{4} n + \frac{\pi}{3}\right) = 3\cos\left(\frac{\pi}{2} n + \frac{\pi}{3}\right)$$

$$Y(\Omega) = X(\Omega)\cdot H(\Omega)$$

$$y[n] = \frac{1}{4}\sum_{k=0}^{3} x[n-k]$$

$$Y(\Omega) = \frac{1}{4}\sum_{k=0}^{3} X(\Omega)\, e^{j\Omega k}$$

$$Y(\Omega) = X(\Omega)\cdot \frac{1}{4}\sum_{k=0}^{3} e^{-j\Omega k}$$

$$H(\Omega) = \frac{Y(\Omega)}{X(\Omega)} = \frac{1}{4}\sum_{k=0}^{3} e^{-j\Omega k}$$

$$H(\Omega) = \frac{1}{4}\left(1 + e^{-j\Omega} + e^{-j2\Omega} + e^{-j3\Omega}\right)$$

SLI real

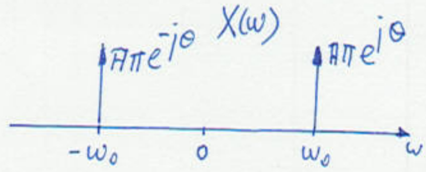
$$y[n] = B\cos\left(\frac{\pi}{2} n + \alpha\right)$$

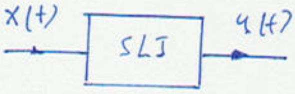
$$B = 3\, H\left(\frac{\pi}{2}\right) = 3\cdot\frac{1}{4}\left(1 + e^{-j\frac{\pi}{2}} + e^{-j\pi} + e^{-j\frac{3\pi}{2}}\right) = \frac{3}{4}\left(\cancel{1} + \cancel{(-j)} + \cancel{(-1)} + \cancel{j}\right) = 0$$

$$\boxed{y[n] = 0}$$

CUESTION 3

$$x(t) = A\cos(\omega_0 t+\theta) \xrightarrow{\mathcal{F}} X(\omega) = A\pi e^{j\theta}\delta(\omega-\omega_0) + A\pi e^{-j\theta}\delta(\omega+\omega_0)$$





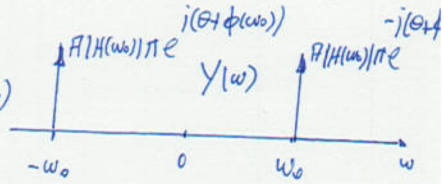
$$\to Y(\omega) = X(\omega)\cdot H(\omega) \to Y(\omega) = A\pi e^{j\theta}\underbrace{H(\omega)\,\delta(\omega-\omega_0)}_{H(\omega_0)\,\delta(\omega-\omega_0)} + A\pi e^{-j\theta}\underbrace{H(\omega)\,\delta(\omega+\omega_0)}_{H(-\omega_0)\,\delta(\omega+\omega_0)}$$

SLI real $\Rightarrow H(\omega) = H^*(-\omega) \Rightarrow$

$\to H(\omega_0) = |H(\omega_0)|\, e^{j\phi(\omega_0)}$

$\to H(-\omega_0) = H^*(\omega_0) = |H(\omega_0)|\, e^{-j\phi(\omega_0)}$

$$Y(\omega) = A|H(\omega_0)|\pi e^{j(\theta+\phi(\omega_0))}\delta(\omega-\omega_0) + A|H(\omega_0)|\pi e^{-j(\theta+\phi(\omega_0))}\delta(\omega+\omega_0)$$



$\downarrow \mathcal{F}^{-1}$

$$y(t) = \underbrace{A|H(\omega_0)|}_{B}\cos(\omega_0 t + \underbrace{\theta+\phi(\omega_0)}_{\varphi})$$

2014 ko ekaina.
2. partziala

2. ARIKETA

$$x[n] \longrightarrow \boxed{h_1[n]} \longrightarrow \boxed{H_2(\Omega)} \longrightarrow y[n]$$

a)

$$H_1(\Omega) = \frac{A}{1-\frac{1}{2}e^{-j\Omega}} \qquad H_2(\Omega) = \frac{3}{1-\frac{1}{8}e^{-j\Omega}}$$

$$H(\Omega) = H_1(\Omega)\cdot H_2(\Omega) = \frac{3A}{1-\frac{5}{8}e^{-j\Omega}+\frac{1}{16}e^{-j2\Omega}} = \frac{Y(\Omega)}{X(\Omega)}$$

$$y[n] - \frac{5}{8}y[n-1] + \frac{1}{16}y[n-2] = 3A\,x[n]$$

b)

$$x[n] = \frac{1}{2}e^{j\pi n}$$

$$y[n] = \frac{2}{3}e^{j\pi/2}$$

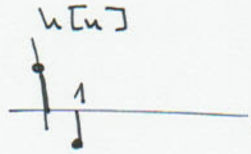
$$2/3 : \frac{1}{2} = \frac{4}{3} = H(\Omega)\big|_{\Omega=\pi}$$

$$H(\pi) = \frac{A}{1-\frac{1}{2}e^{-j\pi}} \cdot \frac{3}{1-\frac{1}{8}e^{-j\pi}} = \frac{4}{3} \implies A = \frac{3}{4}$$

c)

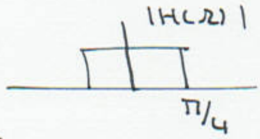
$$H_1(\Omega) = \frac{1}{H_2(\Omega)} = \frac{1}{3}\left(1-\frac{1}{8}e^{-j\Omega}\right)$$

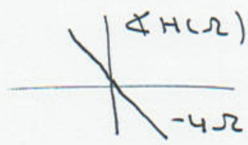
$$h_1[n] = \frac{1}{3}\delta[n] - \frac{1}{24}\delta[n-1]$$



d)

$$H_1(\Omega) = \Pi\left(\frac{\Omega}{\pi/2}\right)e^{-j4\Omega}$$

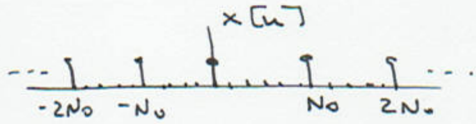




$$H(\Omega) = \Pi\left(\frac{\Omega}{\pi/2}\right)\frac{3}{1-\frac{1}{8}e^{-j\Omega}}\cdot e^{-j4\Omega}$$

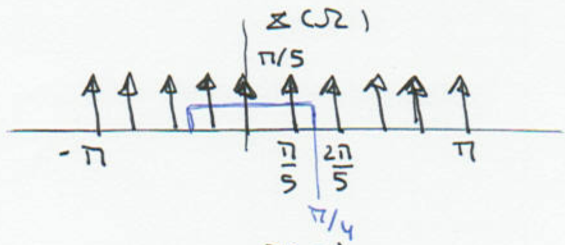
e)

$$x[n] = \sum_k \delta[n-kN_0]$$



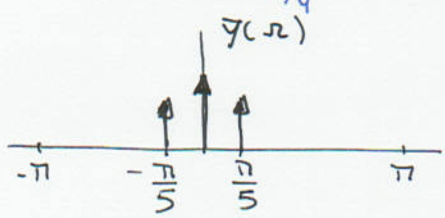
$$a_k = \frac{1}{N_0}\sum_{n=0}^{N_0-1} x[n] = \frac{1}{N_0} \quad \forall k, \quad N_0 = 10$$

$$X(\Omega) = 2\pi\sum_k a_k\,\delta\left(\Omega - \frac{2\pi k}{N_0}\right) = \frac{\pi}{5}\sum_k \delta\left(\Omega - \frac{k\pi}{5}\right)$$



f)

$$Y(\Omega) = 2\pi\cdot a_0 H(0)\,\delta(\Omega) +$$
$$+ 2\pi\, a_1 H(\pi/5)\,\delta(\Omega-\pi/5) +$$
$$+ 2\pi\, a_{-1} H(-\pi/5)\,\delta(\Omega+\pi/5)$$



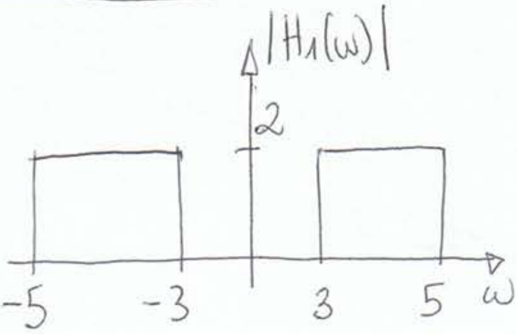
$$H(\Omega=0) = 3'42$$

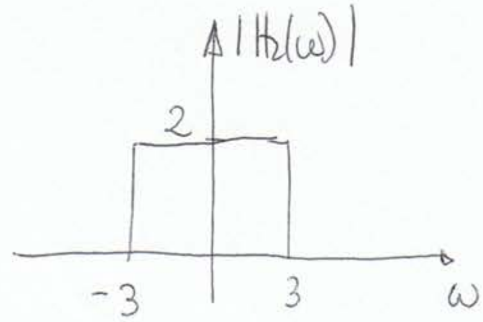
$$H(\Omega=\pm\pi/5) = 3'32\,e^{\mp j\,2'59}$$

$$y[n] = \frac{3'42}{10} + 2\cdot\frac{3'32}{10}\cdot\cos\left(\frac{\pi}{5}n - 2'59\right)$$

PROBLEMA 3

a)

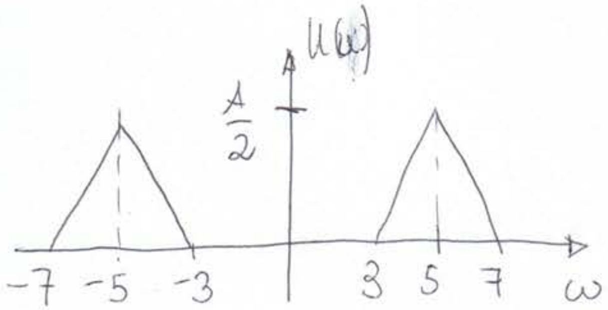




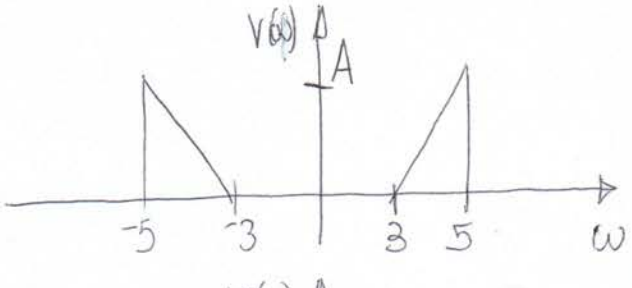
FILTRO IDEAL PASO BANDA

FILTRO IDEAL PASO BAJO

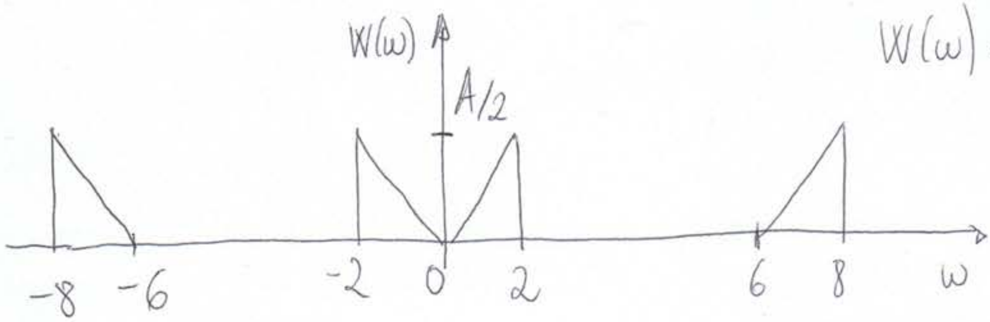
b)



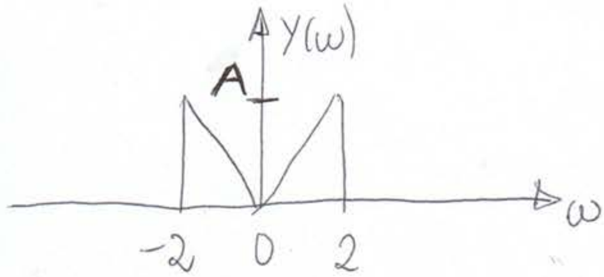
$$U(\omega)=\frac{1}{2}\left[X(\omega-5)+X(\omega+5)\right]$$



$$V(\omega)=U(\omega)\cdot H_1(\omega)$$

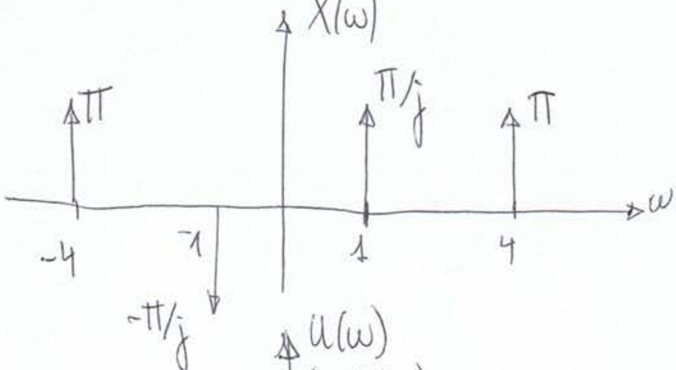


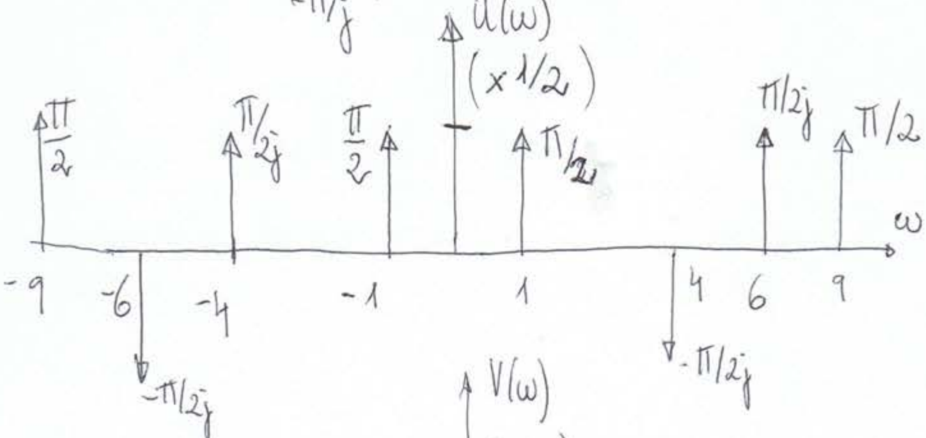
$$W(\omega)=\frac{1}{2}\left[V(\omega-3)+V(\omega+3)\right]$$

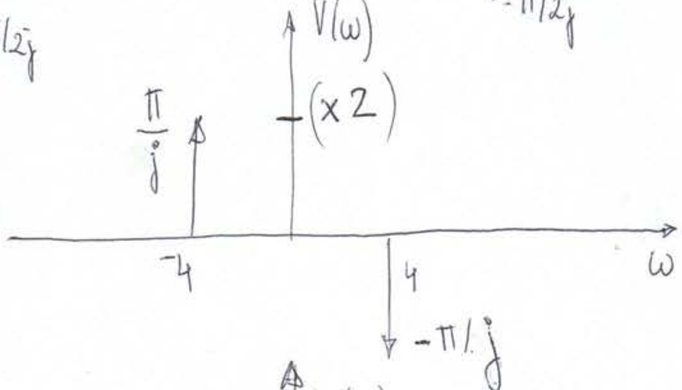


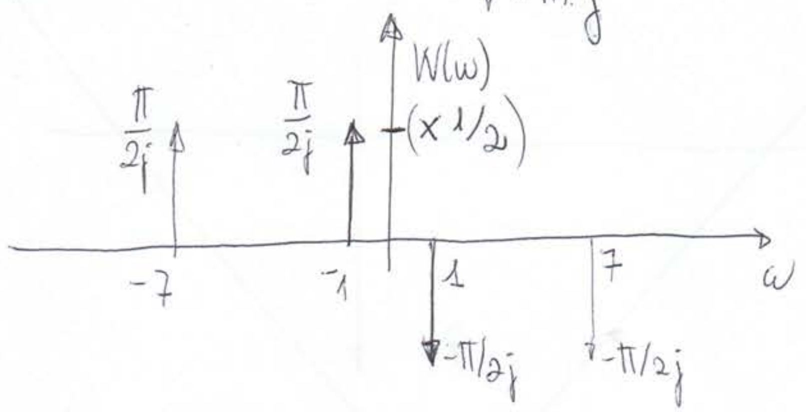
$$Y(\omega)=W(\omega)\cdot H_2(\omega)$$

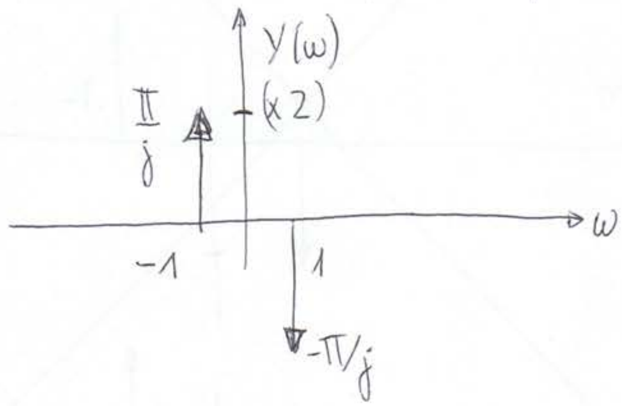
c)











$$Y(\omega) = -\frac{\pi}{j}\,\delta(\omega-1) + \frac{\pi}{j}\left(\delta(\omega+1)\right) \Rightarrow$$

$$Y(\omega) = -\left(\frac{\pi}{j}\,\delta(\omega-1) - \frac{\pi}{j}\,\delta(\omega+1)\right)$$

$$\Rightarrow \quad y(t) = -\,\mathrm{sen}(t)$$