

①  $A = \begin{bmatrix} a & 0 & 0 & 0 \\ 1 & 2 & 0 & -1 \\ 1 & 0 & 2 & -1 \\ -1+a & 0 & 0 & 1 \end{bmatrix}$

$|A - \lambda I| = (a - \lambda)(2 - \lambda)^2(1 - \lambda) = 0$

$$\begin{cases} \lambda_1 = a & (m_1 = 1) \\ \lambda_2 = 2 & (m_2 = 2) \\ \lambda_3 = 1 & (m_3 = 1) \end{cases}$$

$a = 2$

$$\begin{cases} \lambda_1 = 2 & (m_1 = 3) \\ \lambda_2 = 1 & (m_2 = 1) \end{cases}$$

$V(\lambda = 2) \rightarrow \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & -1 \\ 1 & 0 & 0 & -1 \\ 1 & 0 & 0 & -1 \end{bmatrix}$

$h = 1 \rightarrow \dim = 4 - 1 = 3 = m \rightarrow$  diagonalizable

$\rightarrow \begin{cases} x_1 = x_4 \end{cases}$   
 $\rightarrow B_{V_{\lambda=2}} = \{(1, 0, 0, 1), (0, 1, 0, 0), (0, 0, 1, 0)\}$

$V(\lambda = 1) \rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & -1 \\ 1 & 0 & 1 & -1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$

$\rightarrow \begin{cases} x_1 = 0 \\ x_2 = x_4 \\ x_3 = x_4 \end{cases}$   
 $B_{V_{\lambda=1}} = \{(0, 1, 1, 1)\}$

$h = 3 \rightarrow \dim = 1$

$D = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$       $P = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix}$

$a = 1$

$$\begin{cases} \lambda_1 = 2 & (m_1 = 2) \\ \lambda_2 = 1 & (m_2 = 2) \end{cases}$$

$V(\lambda = 2) \rightarrow \begin{bmatrix} -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & -1 \\ 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & -1 \end{bmatrix}$

$h = 2 \rightarrow \dim = 4 - 2 = 2 = m_1$

$V(\lambda = 1) \rightarrow \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & -1 \\ 1 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$h = 2 \rightarrow \dim = 2 = m_2$

diagonalizable

$\rightarrow \begin{cases} x_1 = 0 \\ x_1 = x_4 \\ x_4 = 0 \end{cases}$   
 $B_{V_{\lambda=2}} = \{(0, 1, 0, 0), (0, 0, 1, 0)\}$

$\rightarrow \begin{cases} x_1 + x_2 - x_4 = 0 \\ x_1 + x_3 - x_4 = 0 \end{cases} \begin{cases} x_2 = x_3 \\ x_4 = x_1 + x_2 \end{cases}$

$D = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$       $P = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$

$B_{V_{\lambda=1}} = \{(1, 0, 0, 1), (0, 1, 1, 1)\}$



$$a \neq 2, 1$$

$$V(\lambda=2) \rightarrow \begin{bmatrix} a-2 & 0 & 0 & 0 \\ 1 & 0 & 0 & -1 \\ 1 & 0 & 0 & -1 \\ -1+a & 0 & 0 & -1 \end{bmatrix}$$

$$\rightarrow \begin{cases} x_1=0 \\ x_4=0 \end{cases} B_{V_{\lambda=2}} = \{(0,1,0,0), (0,0,1,0)\}$$

$h=2 \rightarrow \dim=2=n \rightarrow$  diagonalizable

$$V_{\lambda=a} \rightarrow \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 2-a & 0 & -1 \\ 1 & 0 & 2-a & -1 \\ -1+a & 0 & 0 & 1-a \end{bmatrix}$$

$$\rightarrow \begin{cases} x_1+(2-a)x_2=x_4 \\ x_1+(2-a)x_3=x_4 \\ (-1+a)x_1=(-1+a)x_4 \end{cases} \begin{cases} x_2=0 \\ x_3=0 \\ x_1=x_4 \end{cases}$$

$$B_{V_{\lambda=a}} = \{(1,0,0,1)\}$$

$h=3 \rightarrow \dim=1$

$$V_{\lambda=1} \rightarrow \begin{bmatrix} a-1 & 0 & 0 & 0 \\ 1 & 1 & 0 & -1 \\ 1 & 0 & 1 & -1 \\ -1+a & 0 & 0 & 0 \end{bmatrix}$$

$$\rightarrow \begin{cases} x_1=0 \\ x_2=x_4 \\ x_3=x_4 \end{cases}$$

$$B_{V_{\lambda=1}} = \{(0,1,1,1)\}$$

$h=3 \rightarrow \dim=1$

$$D = \begin{bmatrix} a & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$P = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix}$$

(2)

$$\begin{bmatrix} -1 & 1 & -2 & -a & | & -6 \\ -2 & 1 & 1 & 5 & | & 2(1-a) \\ 2 & 1 & -2 & -2a & | & a \\ 3 & 0 & -1 & -4 & | & 8 \end{bmatrix}$$

$$\xrightarrow{\begin{matrix} E_2 - 2E_1 \\ E_3 + 2E_1 \\ E_4 + 3E_1 \end{matrix}} \begin{bmatrix} -1 & 1 & -2 & -a & | & -6 \\ 0 & -1 & 5 & 5+2a & | & 14-2a \\ 0 & 3 & -6 & -4a & | & a-12 \\ 0 & 3 & -7 & -4-3a & | & -10 \end{bmatrix}$$

$$\xrightarrow{\begin{matrix} E_3 + 3E_2 \\ E_4 + 3E_2 \end{matrix}}$$

$$\rightarrow \begin{bmatrix} -1 & 1 & -2 & -a & | & -6 \\ 0 & -1 & 5 & 5+2a & | & 14-2a \\ 0 & 0 & 9 & 15+2a & | & 30-5a \\ 0 & 0 & 8 & 11+3a & | & 32-6a \end{bmatrix}$$

$$\xrightarrow{E_4 - \frac{8}{9}E_3} \begin{bmatrix} -1 & 1 & -2 & -a & | & -6 \\ 0 & -1 & 5 & 5+2a & | & 14-2a \\ 0 & 0 & 9 & 15+2a & | & 30-5a \\ 0 & 0 & 0 & -\frac{7}{3} + \frac{11}{9}a & | & \frac{16}{3} - \frac{8}{9}a \end{bmatrix}$$

$$\frac{11}{9}a = \frac{7}{3} \rightarrow a = \frac{21}{11}$$

$$\begin{bmatrix} x & x & x & x & | & x \\ 0 & x & x & x & | & x \\ 0 & 0 & x & x & | & x \\ 0 & 0 & 0 & 0 & | & x \end{bmatrix}$$

$$h(A) = 3 < 4 = h(A_2)$$

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$$a \neq \frac{21}{11}$$

$$\begin{bmatrix} x & x & x & x & | & x \\ 0 & x & x & x & | & x \\ 0 & 0 & x & x & | & x \\ 0 & 0 & 0 & x & | & x \end{bmatrix}$$

$$h(A) = 4 = h(A_2)$$

ezet. kop.

BATERAGARRI ZEHATUA

$$\frac{16}{3} = \frac{8}{9}a \rightarrow a = 6$$



