Ingeniaritza Goi Eskola Teknikoa
Escuela Técnica Superior de Ingeniería
Bilbao

Euskal Herriko Unibertsitatea
Universidad del País Vasco

1. deitura / 1er apellido		Titulazioa / Titulación
2. deitura / 2º apellido		Ikasgaia / Asignatura
Izena / Nombre		Data / Fecha
Ikasturtea / Curso	Taldea / Grupo	Kalifikazioa / Calificación

EJERCICIO 1

1) $X Y^{-1} = B^t A X^{-1}$

$Y = A^{-1} B \cdot X \Rightarrow Y^{-1} = X^{-1} B^{-1} A$

$X \cdot X^{-1} B^{-1} A = B^t A X^{-1}$

$B^{-1} A = B^t A X^{-1}$

$A^{-1} B = X A^{-1} B \Rightarrow A^{-1} B B^{-1} = X A^{-1} B B^{-1} \Rightarrow A^{-1} = X A^{-1}$

$A^{-1} A = X A^{-1} A \Rightarrow \underline{\underline{X = I}}$

$\boxed{Y = A^{-1} B \cdot}$

EJERCICIO 3

i) $U = \left\{ \begin{bmatrix} \alpha - 2\beta + \gamma & \alpha - \gamma \\ \alpha - \gamma & \beta - \gamma \end{bmatrix} \right\}$ Base de U: Span $\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} -2 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ -1 & -1 \end{bmatrix}$

$$\begin{bmatrix} 1 & -2 & 1 \\ 1 & 0 & -1 \\ 1 & 0 & -1 \\ 0 & 1 & 1 \end{bmatrix} \begin{array}{l} F_2 - F_1 \\ F_3 - F_1 \end{array} \begin{bmatrix} 1 & -2 & 1 \\ 0 & 2 & -2 \\ 0 & 2 & -2 \\ 0 & 1 & -1 \end{bmatrix} \quad R_S = 2$$

$$B_U = \left\{ \underbrace{\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}}_{U_1}, \underbrace{\begin{bmatrix} -2 & 0 \\ 0 & 1 \end{bmatrix}}_{U_2} \right\}$$

a)

$$U^{\perp}: v = \begin{bmatrix} x & y \\ z & t \end{bmatrix} \;/\; \langle v \cdot u \rangle = 0, \quad \langle v \cdot u_2 \rangle = 0$$

$$\begin{cases} (x\ y\ z\ t)\begin{pmatrix}1\\1\\1\\0\end{pmatrix} = 0 & x+y+z=0 \\ (x\ y\ z\ t)\begin{pmatrix}-2\\0\\0\\1\end{pmatrix} = 0 & -2x+t=0 \end{cases} \qquad \begin{matrix} z=-x-y \\ t=2x \end{matrix}$$

$$v = \begin{bmatrix} x & y \\ -x-y & 2x \end{bmatrix} \qquad B_{U^{\perp}} = \left\{ \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \right\}$$

---

c) $U + U^{\perp}$

$$\begin{bmatrix} 1 & -2 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & -1 & -1 \\ 0 & 1 & 2 & 0 \end{bmatrix} \begin{matrix} F_2 - F_1 \\ F_3 - F_1 \end{matrix} \begin{bmatrix} 1 & -2 & 1 & 0 \\ 0 & 2 & -1 & 1 \\ 0 & 2 & -2 & -1 \\ 0 & 1 & 2 & 0 \end{bmatrix} \begin{matrix} F_3 - F_2 \\ F_4 - \frac{F_2}{2} \end{matrix}$$

$$\begin{bmatrix} 1 & -2 & 1 & 0 \\ 0 & 2 & -1 & 1 \\ 0 & 0 & -1 & -2 \\ 0 & 0 & \frac{5}{2} & -\frac{1}{2} \end{bmatrix} \quad R_S = 4 \qquad \underline{\mathrm{Dim}\ U + U^{\perp} = 4} \Rightarrow$$

$$U + U^{\perp} \equiv \mathbb{R}^4$$

---

b)

$$\mathrm{Dim}\ U + U^{\perp} = \mathrm{Dim}\ U + \mathrm{Dim}\ U^{\perp} - \mathrm{Dim}\ U \cap U^{\perp}$$

$$4 \qquad\qquad 2 \qquad\qquad 2 \qquad\qquad 0$$

$$\mathrm{Dim}\ U \cap U^{\perp} = 0 \Rightarrow U \cap U^{\perp} = \{\emptyset\}$$

---

d) Son suplementarios.

EJERCICIO 4

4) $f(2e_4) = -e_2 - e_4 \Rightarrow 2f(e_4) = -e_2 - e_4 \Rightarrow \boxed{f(e_4) = \frac{1}{2}[-e_2 - e_4]}$

$f(e_1 - e_3) = 3e_1 - 3e_3$

$f(e_4 - e_2) \in \ker f \Rightarrow f(e_4) - f(e_2) = 0 \Rightarrow \boxed{f(e_4) = f(e_2)}$

$f(e_1 + 2e_2) = e_1 - e_2 - 2e_3 - e_4$

$f(e_1) + 2f(e_2) = e_1 - e_2 - 2e_3 - e_4$

$f(e_1) - \cancel{e_2} - \cancel{e_4} = e_1 - \cancel{e_2} - 2e_3 - \cancel{e_4} \Rightarrow \boxed{f(e_1) = e_1 - 2e_3}$

$f(e_1) - f(e_3) = 3e_1 - 3e_3 \Rightarrow f(e_3) = f(e_1) + 3e_3 - 3e_1 = e_1 - 2e_3 + 3e_3 - 3e_1$

$\boxed{f(e_3) = -2e_1 + e_3}$

a)

$$A = \begin{bmatrix} 1 & 0 & -2 & 0 \\ 0 & -\frac{1}{2} & 0 & -\frac{1}{2} \\ -2 & 0 & 1 & 0 \\ 0 & -\frac{1}{2} & 0 & -\frac{1}{2} \end{bmatrix}$$

Ker f:

b)

$$\begin{bmatrix} 1 & 0 & -2 & 0 \\ 0 & -\frac{1}{2} & 0 & -\frac{1}{2} \\ -2 & 0 & 1 & 0 \\ 0 & -\frac{1}{2} & 0 & -\frac{1}{2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\left.\begin{array}{l} x_1 - 2x_3 = 0 \\ x_2 + x_4 = 0 \\ -2x_1 + x_3 = 0 \end{array}\right\} \begin{array}{l} x_4 = -x_2 \\ x_1 = x_3 = 0 \end{array}$$

$\begin{pmatrix} 0 \\ x_2 \\ 0 \\ -x_2 \end{pmatrix} \quad B_{\ker f} = \left\{ \begin{pmatrix} 0 \\ 1 \\ 0 \\ -1 \end{pmatrix} \right\} = \underline{\underline{e_2 - e_4}}$

Ingeniaritza Goi Eskola Teknikoa
Escuela Técnica Superior de Ingeniería
Bilbao

Euskal Herriko Unibertsitatea
Universidad del País Vasco

1. deitura / 1er apellido		Titulazioa / Titulación
2. deitura / 2º apellido		Ikasgaia / Asignatura
Izena / Nombre		Data / Fecha
Ikasturtea / Curso	Taldea / Grupo	Kalifikazioa / Calificación

Inf. $\dim E = \dim \mathrm{Im} f + \boxed{\dim \mathrm{Inf}}$

c) $4 = 1 + \boxed{3}$

$Rg\, A = 3$. $\quad B_{\mathrm{Inf}} = \left\{ \begin{pmatrix} 1 \\ 0 \\ -2 \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ -\frac{1}{2} \\ 0 \\ -\frac{1}{2} \end{pmatrix} \begin{pmatrix} -2 \\ 0 \\ 1 \\ 0 \end{pmatrix} \right\}$

d) $|A-\lambda I| = \begin{vmatrix} 1-\lambda & 0 & -2 & 0 \\ 0 & -\frac{1}{2}-\lambda & 0 & -\frac{1}{2} \\ -2 & 0 & 1-\lambda & 0 \\ 0 & -\frac{1}{2} & 0 & -\frac{1}{2}-\lambda \end{vmatrix} = (1-\lambda) \begin{vmatrix} -\frac{1}{2}-\lambda & 0 & -\frac{1}{2} \\ 0 & 1-\lambda & 0 \\ -\frac{1}{2} & 0 & -\frac{1}{2}-\lambda \end{vmatrix} + (-2) \begin{vmatrix} 0 & -2 & 0 \\ -\frac{1}{2}-\lambda & 0 & -\frac{1}{2} \\ -\frac{1}{2} & 0 & -\frac{1}{2}- \end{vmatrix}$

$= (1-\lambda)(1-\lambda) \begin{vmatrix} -\frac{1}{2}-\lambda & -\frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2}-\lambda \end{vmatrix} + (-2)(2) \begin{vmatrix} -\frac{1}{2}-\lambda & -\frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2}-\lambda \end{vmatrix} =$

$= (\lambda^2 + 1 - 2\lambda - 4) \begin{vmatrix} -\frac{1}{2}-\lambda & -\frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2}-\lambda \end{vmatrix} = (\lambda^2 - 2\lambda - 3)\left(\left(-\frac{1}{2}-\lambda\right)^2 - \frac{1}{4}\right) =$

$= (\lambda^2 - 2\lambda - 3)\left(\lambda^2 + \frac{1}{4} + \lambda - \frac{1}{4}\right) = (\lambda^2 - 2\lambda - 3)(\lambda^2 + \lambda) = (\lambda^2 - 2\lambda - 3)(\lambda + 1)\lambda =$

$= (\lambda+1)^2 \lambda (\lambda - 3) = p(\lambda)$

$\lambda = \dfrac{2 \pm \sqrt{4 + 12}}{2} = \dfrac{2 \pm 4}{2} = \begin{cases} 3 \\ -1 \end{cases}$

$\lambda = 0 \quad m = 1 \quad \mu = 1$

$\lambda = 3 \quad m = 1 \quad \mu = 1$

$\lambda = -1 \quad m = 2 \quad \mu = ??$

$V(\lambda=-1)$

$$(A-\lambda I)\,x=[0] \Rightarrow (A+I)\,x=[0]$$

$$A+I=\begin{bmatrix} 2 & 0 & -2 & 0 \\ 0 & \frac{1}{2} & 0 & -\frac{1}{2} \\ -2 & 0 & 2 & 0 \\ 0 & -\frac{1}{2} & 0 & \frac{1}{2} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$x_1-x_3=0 \qquad x_1=x_3$

$x_2-x_4=0 \qquad x_2=x_4$

$$B_{V(\lambda=-1)}=\left\{\begin{pmatrix}1\\0\\1\\0\end{pmatrix}\begin{pmatrix}0\\1\\0\\1\end{pmatrix}\right\}$$

$$V(\lambda=0)=\ker f \Rightarrow \quad B_{V(\lambda=0)}=\left\{\begin{pmatrix}0\\1\\0\\-1\end{pmatrix}\right\}$$

$V(\lambda=3)$: $(A-3I)\,x=[0]$:

$$\begin{bmatrix} -2 & 0 & -2 & 0 \\ 0 & -\frac{7}{2} & 0 & -\frac{1}{2} \\ -2 & 0 & -2 & 0 \\ 0 & -\frac{1}{2} & 0 & -\frac{7}{2} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \Rightarrow x_2=x_4=0$$

$x_3=-x_1$

$$\begin{pmatrix}x_1\\0\\-x_1\\0\end{pmatrix} \qquad B_{V(\lambda=3)}=\left\{\begin{pmatrix}1\\0\\-1\\0\end{pmatrix}\right\}$$

DATO DEL PROBLEMA.

$$f(e_1-e_3)=3(e_1-e_3)$$

EJERCICIO 5

5) $\begin{bmatrix} a & 1 & 2 \\ 0 & b & 3 \\ 0 & 0 & 4 \end{bmatrix}$ $\quad a=4 \quad b\neq 4$

$\lambda = 4 \quad m=2$

$\lambda = b \quad m=1$

$V(\lambda=4)$

$(A-4I)x=[0]$

$$\begin{bmatrix} 0 & 1 & 2 \\ 0 & b-4 & 3 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$Rg\ (A-4I)$.

Si: $b-4 = \frac{3}{2} \longrightarrow \underline{\underline{Rg=1}}$

$b \neq \frac{11}{2} \longrightarrow$ NO DIAG.

$b = \frac{11}{2} \longrightarrow$ DIAG.

$V(\lambda=4)$ CUANDO $b=\frac{11}{2}$

$$\begin{bmatrix} 0 & 1 & 2 \\ 0 & \frac{3}{2} & 3 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix};$$

$x_2 + 2x_3 = 0 \qquad x_2 = -2x_3$

$$\begin{pmatrix} x_1 \\ -2x_3 \\ x_3 \end{pmatrix} = x_1\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + x_3\begin{pmatrix} 0 \\ -2 \\ 1 \end{pmatrix}$$

$$B_{V(\lambda=4)} = \left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}\begin{pmatrix} 0 \\ -2 \\ 1 \end{pmatrix} \right\}$$

$V(\lambda=\frac{3}{2})$.

$$\begin{bmatrix} 4-\frac{3}{2} & 1 & 2 \\ 0 & 0 & 3 \\ 0 & 0 & 4-\frac{3}{2} \end{bmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow x_3 = 0$$

$x_2 = \left(\frac{3}{2}-4\right)x_1 = -\frac{5}{2}x_1$

$$\begin{pmatrix} x_1 \\ -\frac{5}{2}x_1 \\ 0 \end{pmatrix} = x_1\begin{pmatrix} 1 \\ -\frac{5}{2} \\ 0 \end{pmatrix}$$

$$B_{V(\lambda=\frac{3}{2})} = \left\{ \begin{pmatrix} 1 \\ -\frac{5}{2} \\ 0 \end{pmatrix} \right\}$$

$$P = \begin{bmatrix} 1 & 0 & 1 \\ 0 & -2 & -\frac{5}{2} \\ 0 & 1 & 0 \end{bmatrix} \qquad D = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & \frac{3}{2} \end{bmatrix}$$

$b=4 \quad \lambda=4 \quad m=2$

$a\neq 4 \quad \lambda=a \quad m=1$

$$V(\lambda=4):\begin{bmatrix} a-4 & 1 & 2 \\ 0 & 0 & 3 \\ 0 & 0 & 0\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix} \quad x_3=0 \quad (a-4)x_1+x_2=0 \quad x_2=(4-a)x_1$$

$\text{Dim } V(\lambda=4)=1 \Rightarrow \mu=1$ <u>NO DIAG.</u>

$$\begin{bmatrix} a & 1 & 2 \\ 0 & b & 3 \\ 0 & 0 & 4\end{bmatrix} \quad \left.\begin{matrix}\lambda_1=a & m=1 & \mu=1 \\ \lambda_2=b & m=1 & \mu=1 \\ \lambda_3=4 & m=1 & \mu=1\end{matrix}\right\} \text{DIAG.}$$

$$V(\lambda=a).\begin{bmatrix} 0 & 1 & 2 \\ 0 & (b-a) & 3 \\ 0 & 0 & (4-a)\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix} \quad \begin{matrix}x_2=0\\x_3=0\end{matrix} \quad B_{V(\lambda=a)}=\left\{\begin{pmatrix}1\\0\\0\end{pmatrix}\right\}$$

$$V(\lambda=b)\begin{bmatrix} a-b & 1 & 2 \\ 0 & 0 & 3 \\ 0 & 0 & 4-b\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix} \quad \begin{matrix}x_2=(b-a)x_1\\x_3=0\end{matrix} \quad \begin{pmatrix}x_1\\(b-a)x_1\\0\end{pmatrix} \quad B_{V(\lambda=b)}=\left\{\begin{pmatrix}1\\b-a\\0\end{pmatrix}\right\}$$

$$V(\lambda=4)\begin{bmatrix} a-4 & 1 & 2 \\ 0 & b-4 & 3 \\ 0 & 0 & 0\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix} \quad \begin{matrix}(a-4)x_1+x_2+2x_3=0 \\ (b-4)x_2-3x_3=0\end{matrix} \quad \begin{matrix} x_1=\dfrac{2x_3-\frac{3x_3}{b-4}}{a-4} \\ \Rightarrow x_2=\dfrac{3x_3}{b-4}\end{matrix}$$

$$x_1=\frac{(2b-11)x_3}{a-4}$$

$$\begin{pmatrix}\frac{2b-11}{a-4}\\ \frac{3}{b-4}\\ 1\end{pmatrix}=B(V\lambda=4)$$

$$D\begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & 4\end{bmatrix} \qquad P=\begin{bmatrix} 1 & 1 & \frac{2b-11}{a-4} \\ 0 & b-a & \frac{3}{b-4} \\ 0 & 0 & 1\end{bmatrix}$$

Ingeniaritza Goi Eskola Teknikoa
Escuela Técnica Superior de Ingeniería
Bilbao

Euskal Herriko Unibertsitatea
Universidad del País Vasco

1. deitura / 1er apellido	Titulazioa / Titulación
2. deitura / 2º apellido	Ikasgaia / Asignatura
Izena / Nombre	Data / Fecha
Ikasturtea / Curso — Taldea / Grupo	Kalifikazioa / Calificación

EJERCICIO 6

6)
$$\begin{vmatrix} 1-\lambda & a & 1 & 0 & 0 \\ 0 & 2-\lambda & 0 & -1 & 0 \\ 0 & 0 & 2-\lambda & 3 & 0 \\ 0 & 0 & 0 & 2-\lambda & 0 \\ 0 & 3 & 1 & 1 & 2-\lambda \end{vmatrix} = \cancel{(1-\lambda)}(2-\lambda)\begin{vmatrix} 1-\lambda & a & 1 & 0 \\ 0 & 2-\lambda & 0 & 0 \\ 0 & 0 & 2-\lambda & 0 \\ 0 & 3 & 1 & 2-\lambda \end{vmatrix} =$$

$$(2-\lambda)(2-\lambda)\begin{vmatrix} 1-\lambda & a & 1 \\ 0 & 2-\lambda & 0 \\ 0 & 0 & 2-\lambda \end{vmatrix} = (2-\lambda)^4(1-\lambda) \quad ; \quad \boxed{\begin{matrix} \lambda = 1 & m = 1 \\ \lambda = 2 & m = 4 \end{matrix}}$$

$V(\lambda=1)$

$$\begin{bmatrix} 0 & a & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 3 & 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad \begin{matrix} x_2 = 0 \\ x_3 = 0 \\ x_4 = 0 \\ x_5 = 0 \end{matrix} \quad B_{V(\lambda=1)} = \left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \right\}$$

$V(\lambda=2)$

$$\begin{bmatrix} -1 & a & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 3 & 1 & 1 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$x_4 = 0$

$3x_2 + x_3 = 0 \qquad x_3 = -3x_2$

$x_1 = ax_2 + x_3 = a\,x_2 - 3x_2 = (a-3)x$

$$\begin{pmatrix} (a-3)\,x_2 \\ x_2 \\ -3x_2 \\ 0 \\ x_5 \end{pmatrix} \Rightarrow \quad B_{V(\lambda=2)} = \left\{ \begin{pmatrix} a-3 \\ 1 \\ -3 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \right\}$$

$V_2(\lambda=2) = (A-2I)^2 =$

$$\begin{bmatrix} -1 & a & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 3 & 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} -1 & a & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 3 & 1 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & -a & -1 & -a+3 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$x_1 - a x_2 - x_3 \ (-a+3) x_4 = 0$$

$$x_1 = a x_2 + x_3 + (a-3) x_4$$

$$\begin{pmatrix} a x_2 + x_3 + (a-3) x_4 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} \Rightarrow B_{V_2(\lambda=2)} = \left\{ \overset{u_4}{\begin{pmatrix} a \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}} \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} a-3 \\ 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \right\}$$

$$u_4 = \begin{pmatrix} a \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}; \quad u_3 = (A-\lambda I) u_4 = \begin{bmatrix} -1 & a & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 3 & 1 & 1 & 0 \end{bmatrix} \begin{pmatrix} a \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 3 \end{pmatrix}$$

$$u_2 = \begin{pmatrix} a-3 \\ 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} \quad u_1 = (A-\lambda I) u_2 = \begin{bmatrix} -1 & a & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 3 & 1 & 1 & 0 \end{bmatrix} \begin{pmatrix} a-3 \\ 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} (3-a) \\ -1 \\ 3 \\ 0 \\ 1 \end{pmatrix}$$

$$P = \begin{bmatrix} 3-a & a-3 & 0 & a & 1 \\ -1 & 0 & 0 & 1 & 0 \\ 3 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 3 & 0 & 0 \end{bmatrix} \qquad J = \begin{bmatrix} 2 & 1 & & & \\ 0 & 2 & & & \\ & & 2 & 1 & \\ & & 0 & 2 & \\ & & & & 1 \end{bmatrix}$$

EJERCICIO 7

7) $\mathbb{R}^3 \times \mathbb{R}^3 \longrightarrow \mathbb{R}$

$$\langle x, y\rangle = 3x_1y_1 + x_1y_3 + x_3y_1 + 2x_2y_2 + 3x_3y_3$$

$$\langle x\, x\rangle \geq 0 = 3x_1^2 + x_1x_3 + x_3x_1 + 2x_2^2 + 3x_3^2 = 3x_1^2 + 2x_1x_3 + 2x_2^2 + 3x_3^2 =$$

$$= 2x_1^2 + 2x_2^2 + 2x_3^2 + (x_1 + x_3)^2 \geq 0 \quad \forall x \in \mathbb{R}^3.$$

$$\langle y, x\rangle = 3y_1x_1 + y_1x_3 + y_3x_1 + 2y_2x_2 + 3y_3x_3 = \langle x, y\rangle$$

$$\langle x, \alpha y + \beta z\rangle = 3x_1(\alpha y_1 + \beta z_1) + x_1(\alpha y_3 + \beta z_3) + x_3(\alpha y_1 + \beta z_1) + 2x_2(\alpha y_2 + \beta z_2) +$$

$$+ 3x_3(\alpha y_3 + \beta z_3) =$$

$$= \alpha(3x_1y_1 + x_1y_3 + x_3y_1 + 2x_2y_2 + 3x_3y_3) +$$

$$\beta(3x_1z_1 + x_1z_3 + x_3z_1 + 2x_2z_2 + 3x_3z_3) = \alpha\langle x\, y\rangle + \beta\langle x\, z\rangle$$

$B = \{e_1, e_2, e_3\}$.

B^*: $\quad u_1 = e_1 = \begin{pmatrix}1\\0\\0\end{pmatrix}$

$$u_2 = e_2 + \alpha u_1 = \begin{pmatrix}0\\1\\0\end{pmatrix} + \begin{pmatrix}\alpha\\0\\0\end{pmatrix} = \begin{pmatrix}\alpha\\1\\0\end{pmatrix} \qquad u_1 \cdot u_2 = 0$$

$$\langle u_1 \cdot u_2\rangle = 3(1)(\alpha) + 1\cdot 0 + 0\cdot\alpha + 2\,0\cdot 1 + 3\cdot 0\cdot 0 = 0 \Rightarrow \alpha = 0 \quad \underline{\underline{u_2 = \begin{pmatrix}0\\1\\0\end{pmatrix}}}$$

$$u_3 = e_3 + \alpha u_1 + \beta u_2 = \begin{pmatrix}0\\0\\1\end{pmatrix} + \begin{pmatrix}\alpha\\0\\0\end{pmatrix} + \begin{pmatrix}0\\\beta\\0\end{pmatrix} = \begin{pmatrix}\alpha\\\beta\\1\end{pmatrix}$$

$$\langle u_1 u_3\rangle = 0 = 3\cdot 1\cdot\alpha + 1\cdot 1 + 0\cdot\alpha + 2\,0\cdot\beta + 3\,0\cdot 1 = 0; \quad 3\alpha + 1 = 0 \Rightarrow \alpha = -\frac{1}{3}$$

$$\langle u_2 u_3\rangle = 0 = 3\,0\,\alpha + 0\,1 + 0\,\alpha + 2\cdot 1\cdot\beta + 3\cdot 0\cdot 1 = 0 \Rightarrow \beta = 0$$

$$u_3 = \begin{pmatrix}-\frac{1}{3}\\0\\1\end{pmatrix};$$

$\langle u_1 u_1\rangle = 3$

$\langle u_2 u_2\rangle = 2$

$$\langle u_3 u_3\rangle = 3\cdot\left(\frac{1}{3}\right)^2 + 2\cdot\frac{1}{3}\cdot 1 + 3 = \frac{1}{3} + \frac{2}{3} + 3 = 4$$

$$v_1 = \begin{pmatrix}\frac{1}{\sqrt{3}}\\0\\0\end{pmatrix} \qquad v_2 = \begin{pmatrix}0\\\frac{1}{\sqrt{2}}\\0\end{pmatrix} \qquad v_3 = \begin{pmatrix}\frac{1}{6}\\0\\\frac{1}{2}\end{pmatrix}$$

EJERCICIO 8

8

$$\left.\begin{array}{l} x+y+z=a-1 \\ 2x+y+az=a \\ x+ay+z=1 \end{array}\right\} \quad \left[\begin{array}{ccc|c} 1 & 1 & 1 & (a-1) \\ 2 & 1 & a & a \\ 1 & a & 1 & 1 \end{array}\right] \begin{array}{l} F_2-2F_1 \\ F_3-F_1 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & a-1 \\ 0 & -1 & a-2 & 2-a \\ 0 & a-1 & 0 & 2-a \end{array}\right]; \quad F_3+(a-1)F_2 \quad \left[\begin{array}{ccc|c} 1 & 1 & 1 & a-1 \\ 0 & -1 & a-2 & 2-a \\ 0 & 0 & (a-2)(a-1) & (2-a)\cdot a \end{array}\right];$$

$a=2 \rightarrow$ S. C. INDETERMINADO

$a=1 \rightarrow$ S. INCOMPATIBLE.

$a\neq 2$ y $a\neq 1$ S. C. DET.

$a=2$; $\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & -1 & 0 & 0 \end{array}\right] \quad \left.\begin{array}{l} x=1-z \\ y=0 \end{array}\right\}$

$a\neq 2$
$a\neq 1$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & a-1 \\ 0 & -1 & (a-2) & (2-a) \\ 0 & 0 & (a-2)(a-1) & (2-a)\cdot a \end{array}\right]; \quad z=\frac{(2-a)\cdot a}{(a-2)(a-1)}=\boxed{\frac{a}{(1-a)}=z}$$

$y=(2-a)+(2-a)\cdot z$; $-y=(2-a)(1+z)$; $y=(a-2)(1+z)$

$$y=(a-2)\left[\frac{(1-a)}{(1-a)}+\frac{a}{(1-a)}\right]=(a-2)\cdot\frac{1}{(1-a)}=\frac{(a-2)}{1-a}=0 \quad \boxed{y=\frac{a-2}{(1-a)}}$$

$$x=(a-1)-y-z=(a-1)-\frac{(a-2)}{(1-a)}-\frac{a}{(1-a)}=\frac{((a-1)(1-a))-2a+2}{(1-a)}=$$

$$x=\frac{[(a-1)(1-a)]+2(1-a)}{(1-a)}=(a-1)+2=\boxed{a+1=x} \quad \text{OK}$$

BIEN