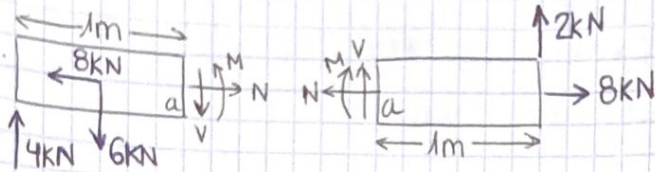


2. GAIA: Barne-indarrak. Habeak eta kableak

RESIS

Barne indarrak A sekzioan

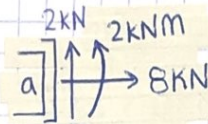


Eskuineko zatian oreka aztertu:

$$\begin{aligned} \sum F_x = 0 & \quad -N + 8 = 0 & \quad \boxed{N = 8 \text{ kN}} \\ \sum F_y = 0 & \quad +V + 2 = 0 & \quad \boxed{V = -2 \text{ kN}} \\ \sum M = 0 & \quad -M + 2 \cdot 1 = 0 & \quad \boxed{M = 2 \text{ kNm}} \end{aligned}$$

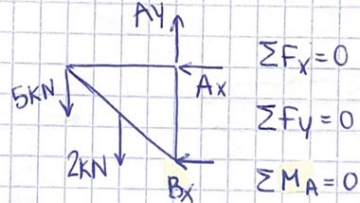
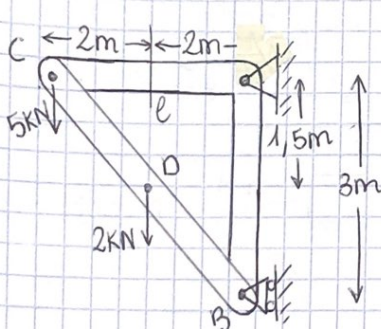
↑ sekzioan

Emaitza:



Kalkulatu tramatuaren elementu bakoitzean eragiten dituzten indarrak.

Kalkulatu e sekzioan kanpo-indarren ondorioz agertuko diren barne-indarrak.



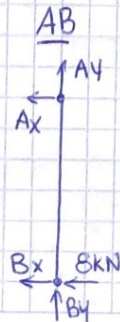
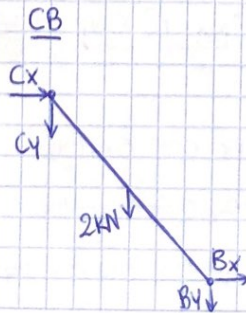
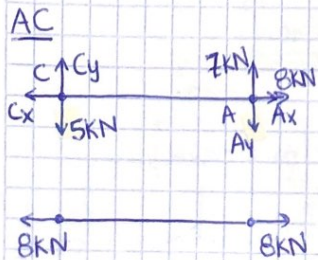
$$\begin{aligned} \sum F_x = 0 \\ \sum F_y = 0 \\ \sum M_A = 0 \end{aligned}$$

$$A_y - 5 - 2 = 0 \rightarrow \boxed{A_y = 7 \text{ kN}}$$

$$4 \cdot 5 + 2 \cdot 2 - B_x \cdot 3 = 0 \rightarrow 20 + 4 - 3B_x = 0 \rightarrow$$

$$\rightarrow 3B_x = 24 \rightarrow \boxed{B_x = 8 \text{ kN}}$$

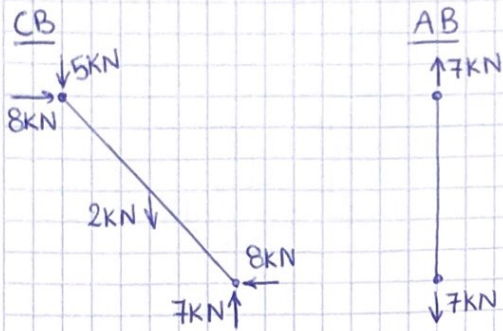
$$A_x = -B_x \rightarrow \boxed{A_x = -8 \text{ kN}}$$



Propietatea: Pieza bat bi indarren pean badago, biek larrokaturata, modulu = eta Kontrako noranzkoa izango dute.

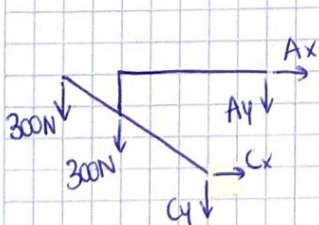
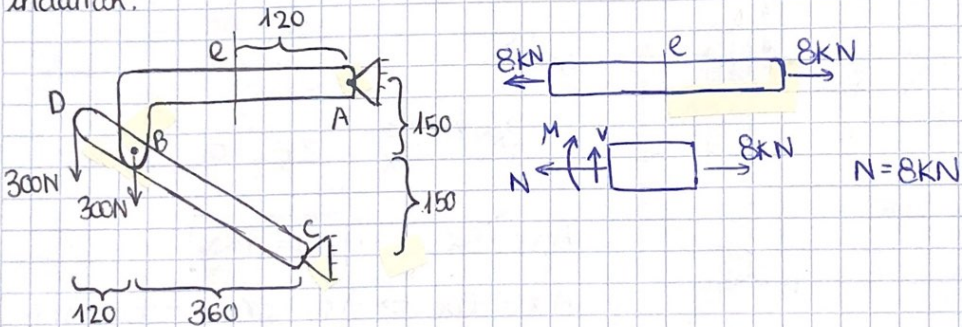
$$A_x = 0; B_x = -8\text{KN}; C_x = 8\text{KN}; A_y = 7\text{KN}; C_y = 5\text{KN}$$

$$\sum F_y = 0 \rightarrow -5 - 2 - B_y = 0 \rightarrow B_y = -7\text{KN}$$



X Kalkulatu tramatuaren elementu bakoitzean eragiten dituzten indarrak.

Kalkulatu e sekzioan kanpo-indarren ondorioz agertuko diren barne-indarrak.



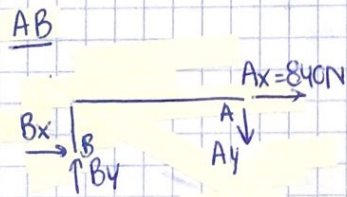
$$\sum F_x = 0 \rightarrow A_x + C_x = 0$$

$$\sum F_y = 0 \rightarrow -300 - 300 - C_y - A_y = 0$$

$$\sum M_C = 0 \rightarrow 300 \cdot 480 + 300 \cdot 360 - A_x \cdot 300 = 0 \rightarrow$$

$$\rightarrow \boxed{A_x = +840\text{N}} \rightarrow \boxed{C_x = -840\text{N}}$$

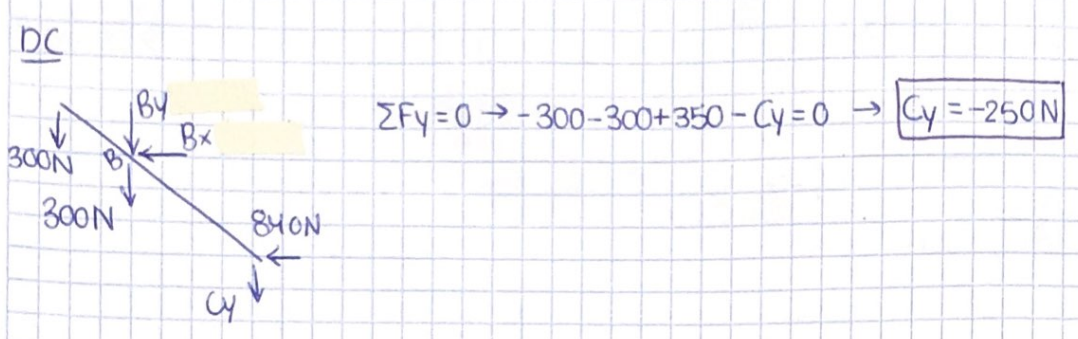
(?)



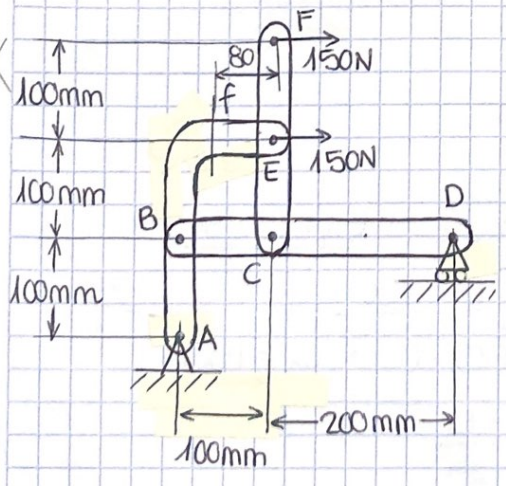
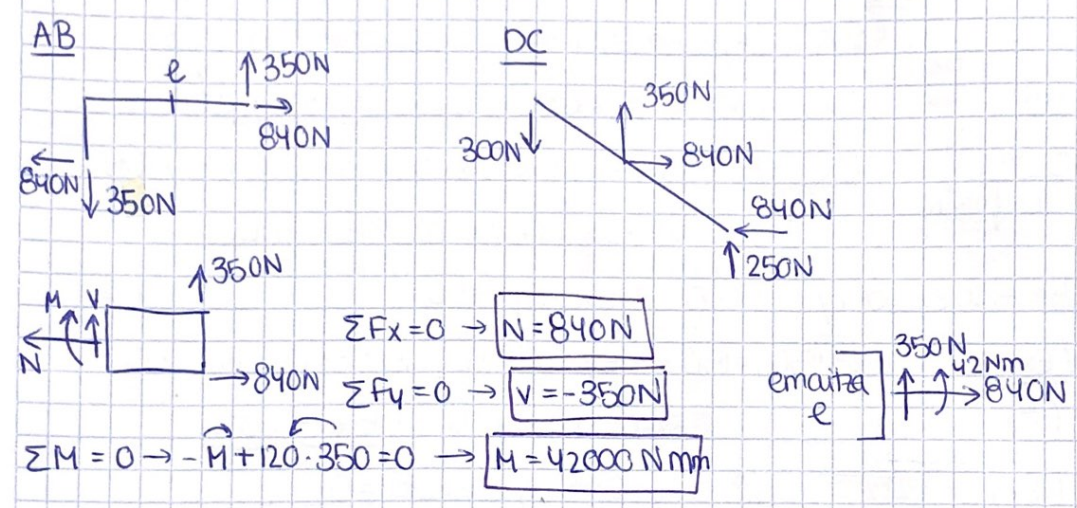
$$\sum F_x = 0 \rightarrow B_x + 840 = 0 \rightarrow \boxed{B_x = -840\text{N}}$$

$$\sum F_y = 0 \rightarrow B_y - A_y = 0 \rightarrow \boxed{B_y = 350}$$

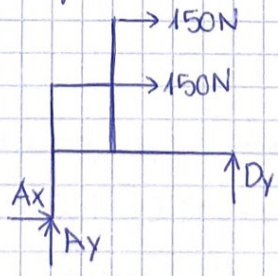
$$\sum M_B = 0 \rightarrow -840 \cdot 150 - A_y \cdot 360 = 0 \rightarrow \boxed{A_y = -350\text{N}}$$



Emaitza:



1) Egitura askatu \rightarrow erreakzioak



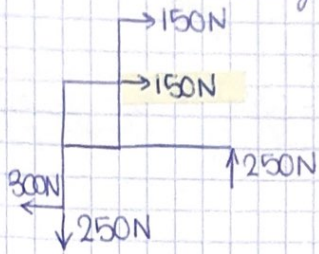
$\Sigma F_x = 0 \rightarrow A_x + 150 + 150 = 0 \rightarrow A_x = -300\text{N}$

$\Sigma F_y = 0 \rightarrow A_y + D_y = 0$

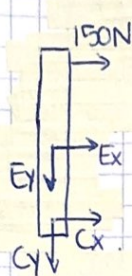
$\Sigma M_A = 0 \rightarrow -200 \cdot 150 - 300 \cdot 150 + 300 D_y = 0 \rightarrow D_y = 250\text{N}$

$A_y = -D_y = -250\text{N} = A_y$

Solido askearen diagrama



CEF

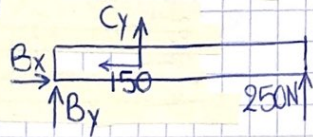


$$\sum F_x = 0 \rightarrow E_x + C_x + 150 = 0 \rightarrow E_x = -150 - 150 = -300 \text{ N} = E_x$$

$$\sum F_y = 0 \rightarrow -E_y - C_y = 0 \quad (*)$$

$$\sum M_E = 0 \rightarrow -100 \cdot 150 + 100 C_x = 0 \rightarrow C_x = 150 \text{ N}$$

BCD



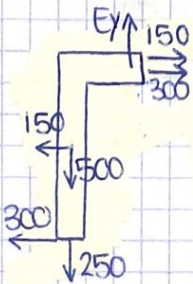
$$\sum F_x = 0 \rightarrow B_x - 150 = 0 \rightarrow B_x = 150 \text{ N}$$

$$\sum F_y = 0 \rightarrow B_y + C_y + 250 = 0$$

$$\sum M_B = 0 \rightarrow 300 \cdot 250 + 100 C_y = 0 \rightarrow C_y = -750 \text{ N}$$

$$B_y = -250 - C_y = 750 - 250 = 500 \text{ N} = B_y$$

ABE

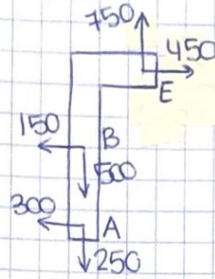


$$\sum F_y = 0 \rightarrow E_y - 500 - 250 = 0 \rightarrow E_y = 750 \text{ N}$$

$$(*) C_y = -E_y = -750 \text{ N} = C_y$$

Emaitzak:

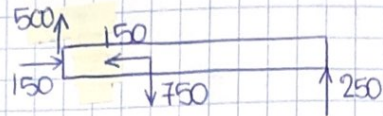
* ABE



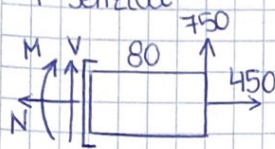
* CEF



* BCD



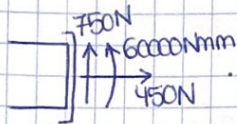
* f sekzioa



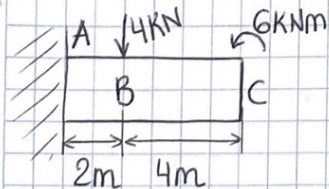
$$\sum F_x = 0 \rightarrow -N + 450 = 0 \rightarrow \underline{N = 450 \text{ N}}$$

$$\sum F_y = 0 \rightarrow V + 750 = 0 \rightarrow \underline{V = -750 \text{ N}}$$

$$\sum M = 0 \rightarrow -M + 750 \cdot 80 = 0 \rightarrow \underline{M = 60000 \text{ Nmm}}$$



X Marraztu inudiko habearen VM diagrama



1) Egitura askatu \rightarrow emeakizicak

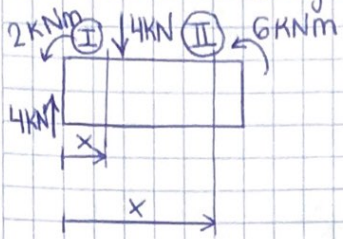


$$\sum F_x = 0 \rightarrow \underline{A_x = 0}$$

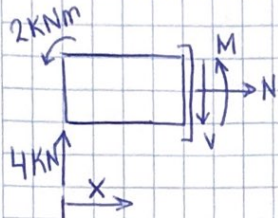
$$\sum F_y = 0 \rightarrow A_y - 4 = 0 \rightarrow \underline{A_y = 4 \text{ kN}}$$

$$\sum M_A = 0 \rightarrow \overset{\curvearrowright}{M_A} + \overset{\curvearrowleft}{6} - 2 \cdot \overset{\curvearrowright}{4} = 0 \rightarrow \underline{M_A = 2 \text{ kNm}}$$

Solido askearen diagrama:



I. eremua:

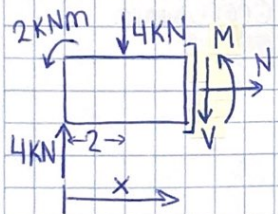


$$\sum F_x = 0 \rightarrow N = 0 \text{ kN}$$

$$\sum F_y = 0 \rightarrow 4 - V = 0 \rightarrow V = 4 \text{ kN}$$

$$\sum M = 0 \rightarrow \overset{\curvearrowleft}{2} + \overset{\curvearrowleft}{M} - \overset{\curvearrowright}{4 \cdot x} = 0 \rightarrow M = 4x - 2 \text{ (zuzen bat)}$$

II. eremua:



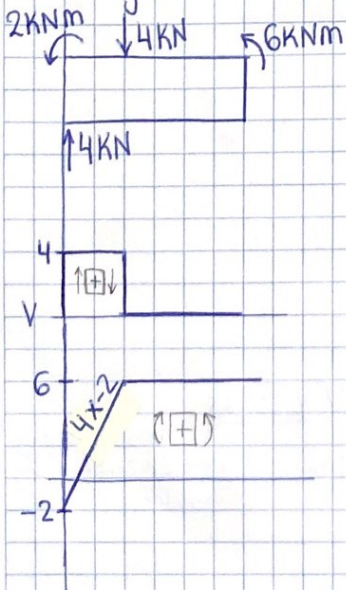
$$\sum F_x = 0 \rightarrow N = 0 \text{ kN}$$

$$\sum F_y = 0 \rightarrow V + 4 - 4 = 0 \rightarrow V = 0 \text{ kN}$$

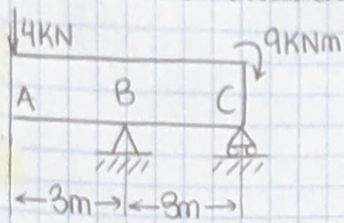
$$\sum M = 0 \rightarrow \overset{\curvearrowleft}{2} + \overset{\curvearrowleft}{M} - \overset{\curvearrowright}{4 \cdot x} + \overset{\curvearrowleft}{4(x-2)} = 0 \rightarrow 2 + M - 4x + 4x - 8 = 0 \rightarrow$$

$$\rightarrow M = 6 \text{ kNm}$$

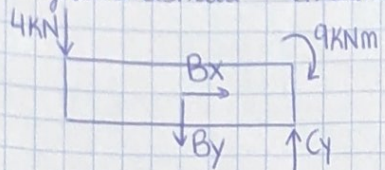
VM diagrama:



Marraztu irudiko habearen VM diagrama



1) Egitura askatu \rightarrow erreakzioak



$$\sum F_x = 0 \rightarrow \underline{B_x = 0}$$

$$\sum F_y = 0 \rightarrow -B_y + C_y - 4 = 0$$

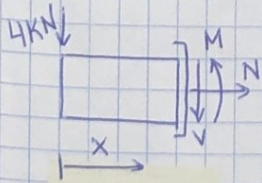
$$\sum M_B = 0 \rightarrow -9 + 3C_y + 3 \cdot 4 = 0 \rightarrow \underline{C_y = -1 \text{ kN}}$$

$$B_y = -1 - 4 = \underline{-5 \text{ kN}}$$

Solido askarearen diagrama:



I. eremua:

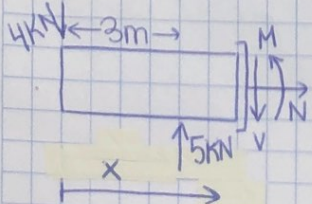


$$\sum F_x = 0 \rightarrow \underline{N = 0}$$

$$\sum F_y = 0 \rightarrow \underline{V = -4 \text{ kN}}$$

$$\sum M = 0 \rightarrow \overset{\curvearrowright}{M} + 4x = 0 \rightarrow \underline{M = -4x} \text{ (zuzena)}$$

II. eremua:



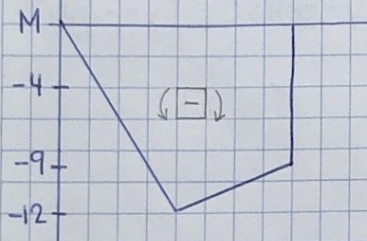
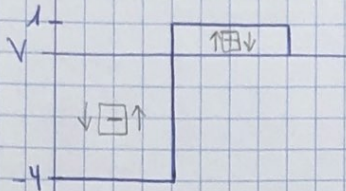
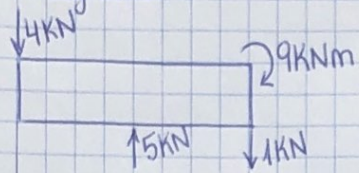
$$\sum F_x = 0 \rightarrow \underline{N = 0}$$

$$\sum F_y = 0 \rightarrow -V - 4 + 5 = 0 \rightarrow \underline{V = 1 \text{ kN}}$$

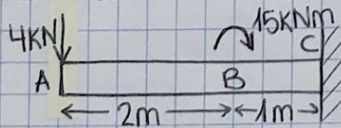
$$\sum M = 0 \rightarrow \overset{\curvearrowright}{M} + 4x - 5(x-3) = 0 \rightarrow M + 4x - 5x + 15 = 0 \rightarrow$$

$$\hookrightarrow \underline{M = x - 15} \text{ (zuzena)}$$

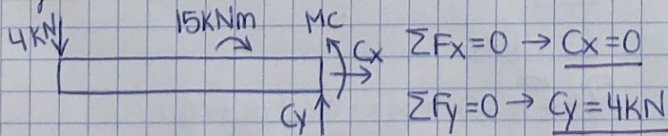
VM diagrama:



Marraztu irudiko habearen VM diagrama eta identifikatu sekzio kritikoa



Egitura askatu \rightarrow erreakzioak



$$\sum F_x = 0 \rightarrow C_x = 0$$

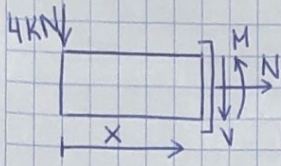
$$\sum F_y = 0 \rightarrow C_y = 4 \text{ kN}$$

$$\sum M_c = 0 \rightarrow \overset{\curvearrowright}{M_c} - 15 + 4 \cdot 3 = 0 \rightarrow \underline{M_c = 3 \text{ kNm}}$$

Solido askearen diagrama:



I. eremua:

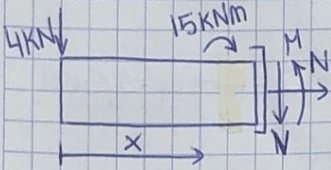


$$\sum F_x = 0 \rightarrow N = 0$$

$$\sum F_y = 0 \rightarrow V = -4 \text{ kN}$$

$$\sum M = 0 \rightarrow M + 4x = 0 \rightarrow M = -4x \text{ (zuzena)}$$

II. eremua:

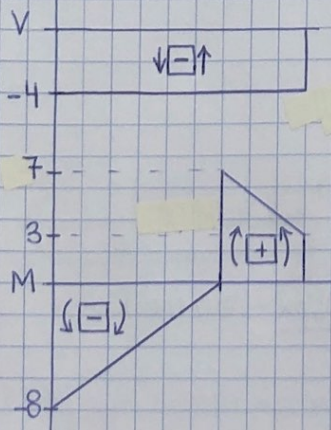
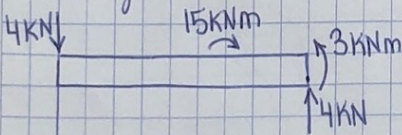


$$\sum F_x = 0 \rightarrow N = 0$$

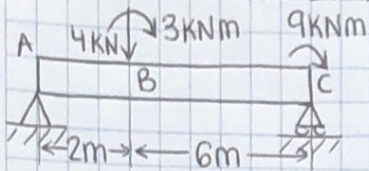
$$\sum F_y = 0 \rightarrow V = -4 \text{ kN}$$

$$\sum M = 0 \rightarrow M - 15 + 4x = 0 \rightarrow M = 15 - 4x \text{ (zuzena)}$$

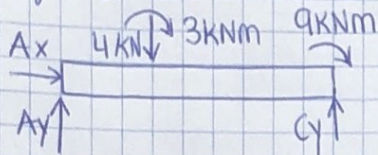
VM diagrama:



X Marraztu inuriko habearen VM diagrama eta identifikatu sekzio kritikoa



Egitura askatu \rightarrow erreakzioak



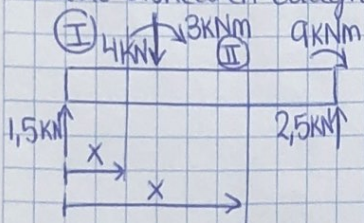
$$\sum F_x = 0 \rightarrow Ax = 0$$

$$\sum F_y = 0 \rightarrow Ay + Cy - 4 = 0$$

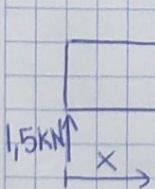
$$\sum M_A = 0 \rightarrow -3 - 9 - 4 \cdot 2 + 8 \cdot Cy = 0 \rightarrow Cy = 2,5 \text{ kN}$$

$$Ay = 4 - Cy = 4 - 2,5 = 1,5 \text{ kN} = Ay$$

Solido askeuren diagrama:



I. eremua:

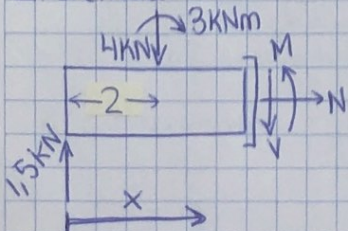


$$\sum F_x = 0 \rightarrow N = 0$$

$$\sum F_y = 0 \rightarrow V = 1,5 \text{ kN}$$

$$\sum M = 0 \rightarrow M - 1,5x = 0 \rightarrow M = 1,5x \text{ (zuzena)}$$

II. eremua:



$$\sum F_x = 0 \rightarrow N = 0$$

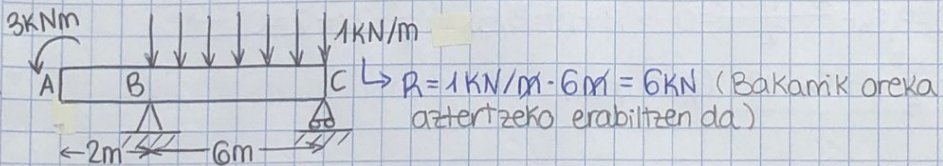
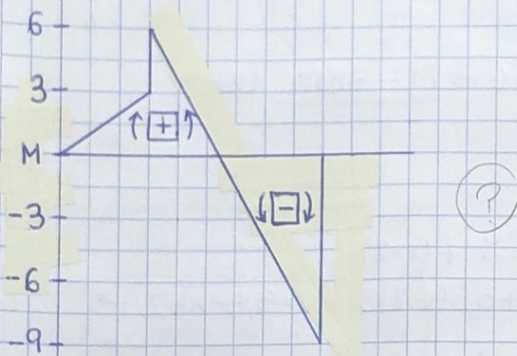
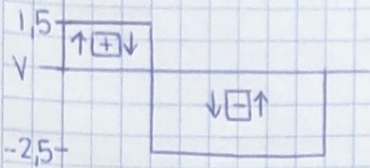
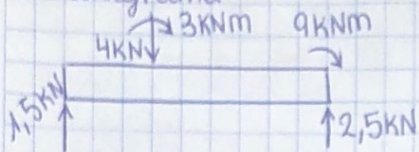
$$\sum F_y = 0 \rightarrow -V - 4 + 1,5 = 0 \rightarrow V = -2,5 \text{ kN}$$

$$\sum M = 0 \rightarrow M - 3 - 1,5x + 4(x - 2) = 0 \rightarrow$$

$$\rightarrow M - 3 - 1,5x + 4x - 8 = 0 \rightarrow M - 11 + 2,5x = 0 \rightarrow$$

$$\rightarrow M = 11 - 2,5x \text{ (zuzena)}$$

VM diagrama:



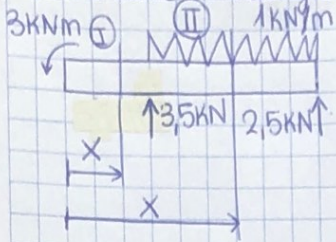
1) Egitura askatu \rightarrow erreakzioak

$$\begin{aligned} \sum F_x = 0 &\rightarrow B_x = 0 \\ \sum F_y = 0 &\rightarrow B_y + C_y - 6 = 0 \end{aligned}$$

$$\sum M_c = 0 \rightarrow 3 - 6 \cdot B_y + 6 \cdot 3 = 0 \rightarrow B_y = 3,5 \text{ kN}$$

$$C_y = 6 - B_y = 6 - 3,5 = 2,5 \text{ kN} = C_y$$

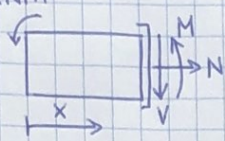
Solido askearen diagrama



⊗ Sekzioaren eta R-ren arteko distantzia

I. eremua: AB $x \in (0, 2)$

3 kNm



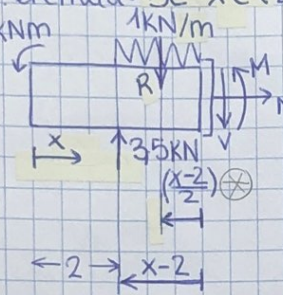
$$\sum F_x = 0 \rightarrow N = 0$$

$$\sum F_y = 0 \rightarrow V = 0$$

$$\sum M = 0 \rightarrow M + 3 = 0 \rightarrow M = -3 \text{ kNm (kte)}$$

II. eremua: BC $x \in (2, 8)$

3 kNm



$$\sum F_x = 0 \rightarrow N = 0 \quad | \cdot (x-2)$$

$$\sum F_y = 0 \rightarrow -V + 3,5 - R = 0 \rightarrow V = 3,5 - x + 2 \rightarrow$$

$$\hookrightarrow V = 5,5 - x \text{ (zutena)} \quad x-2$$

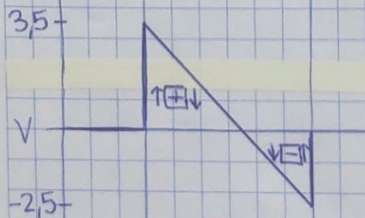
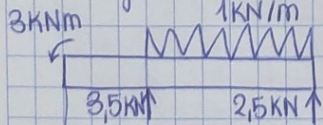
$$\sum M = 0 \rightarrow M + 3 - 3,5(x-2) + R \left(\frac{x-2}{2} \right) = 0 \rightarrow$$

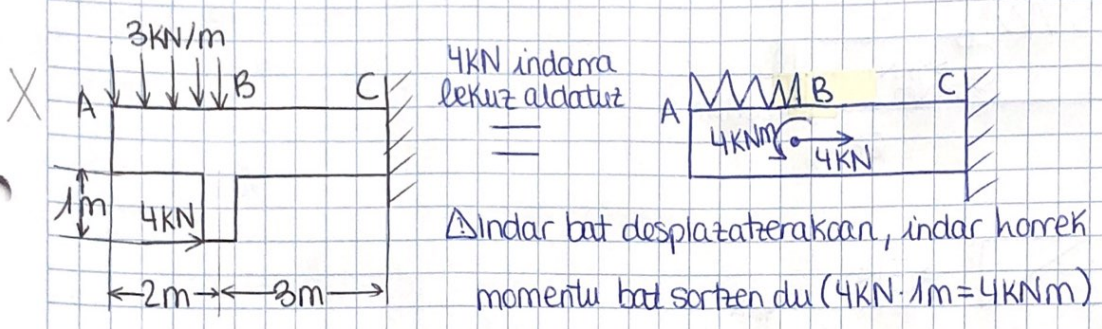
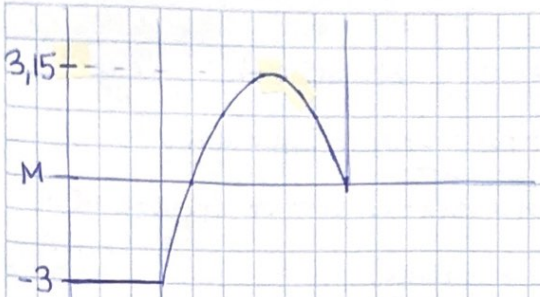
$$\hookrightarrow M + 3 - 3,5x + 7 + (x-2) \left(\frac{x-2}{2} \right) = 0 \rightarrow$$

$$\hookrightarrow M + 10 - 3,5x + \frac{x^2 - 4x + 4}{2} = 0 \rightarrow M + 10 - 3,5x + \frac{x^2}{2} - 2x + 2 = 0 \rightarrow$$

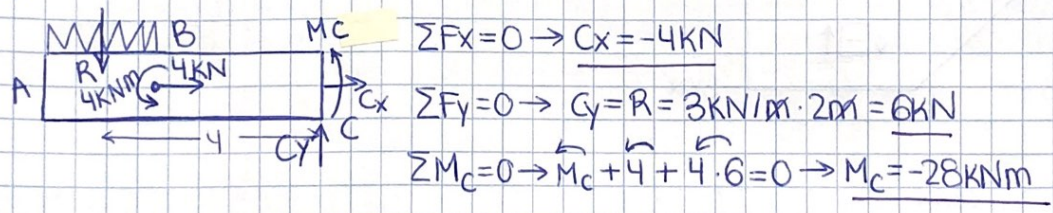
$$\hookrightarrow M = -\frac{x^2}{2} + 5,5x - 12 \text{ (parabola)}$$

VM diagrama

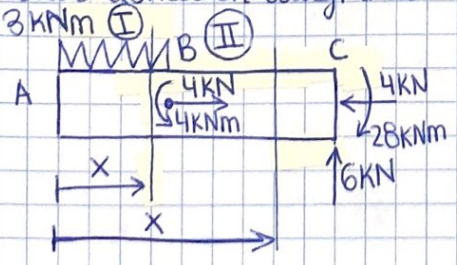




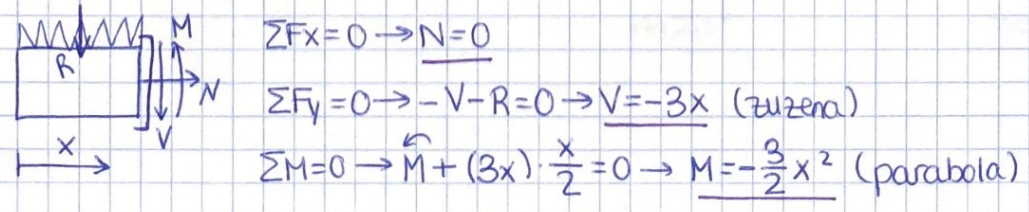
Egitura askatu → erreakzioa



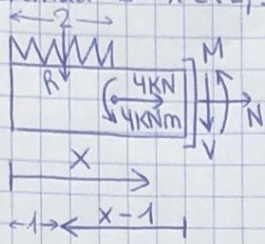
Solido askaren diagrama



I. eremua: AB $x \in (0, 2)$



II. eremua: BC $x \in (2,5)$



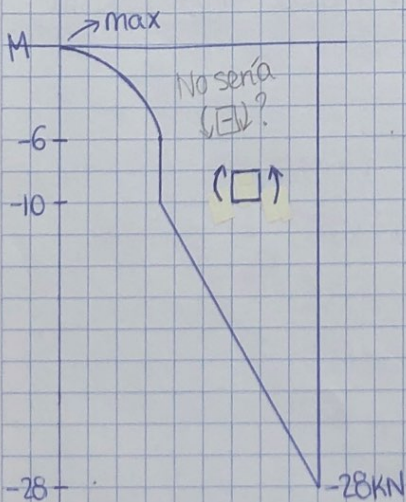
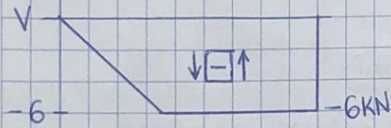
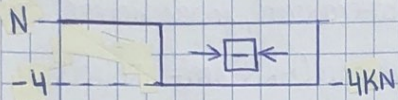
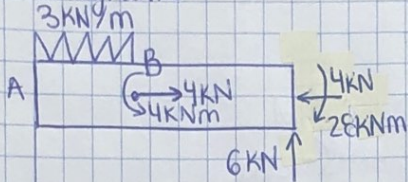
$$\sum F_x = 0 \rightarrow N = -4 \text{ kN (konpresioa)}$$

$$\sum F_y = 0 \rightarrow V = -R = -3 \text{ kN/m} \cdot 2 \text{ m} = -6 \text{ kN}$$

$$\sum M = 0 \rightarrow \bar{M} + \bar{4} + R(x-1) = 0 \rightarrow M + 4 + 6x - 6 = 0 \rightarrow$$

$$\rightarrow M = 2 - 6x \text{ (zutena)}$$

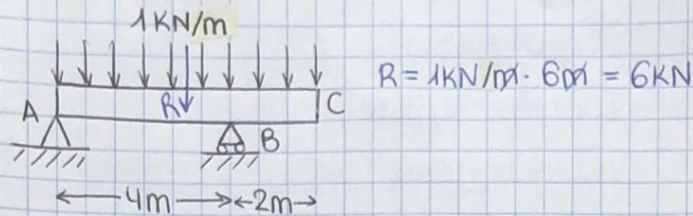
VM diagrama



$$M(x=2) = -10 \text{ kNm}$$

$$M(x=5) = -28 \text{ kNm}$$

Marraztu irudiko habearen barne-indarren VM diagrama. Identifikatu puntu kritikoa.



Egitura askatu \rightarrow erreakzicak

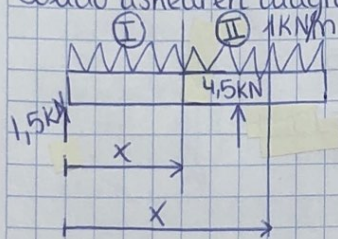
$$\sum F_x = 0 \rightarrow A_x = 0$$

$$\sum F_y = 0 \rightarrow A_y + B_y - 6 = 0$$

$$\sum M_A = 0 \rightarrow -6 \cdot 3 + 4 B_y = 0 \rightarrow B_y = 4,5 \text{ kN}$$

$$A_y = 6 - B_y = 6 - 4,5 = 1,5 \text{ kN} = A_y$$

Solido askearen diagrama:



I. eremua: AB $x \in (0,4)$

$$\sum F_x = 0 \rightarrow N = 0$$

$$\sum F_y = 0 \rightarrow 1,5 - R - V = 0 \rightarrow V = 1,5 - x \quad (\text{zutena})$$

$$\sum M = 0 \rightarrow M - 1,5x + R \cdot \frac{x}{2} = 0 \rightarrow M - 1,5x + x \cdot \frac{x}{2} = 0 \rightarrow$$

$$\hookrightarrow M = -\frac{x^2}{2} + 1,5x \quad (\text{parabola})$$

II. eremua: BC $x \in (4,6)$

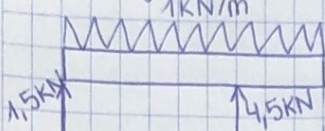
$$\sum F_x = 0 \rightarrow N = 0$$

$$\sum F_y = 0 \rightarrow -V - R + 1,5 + 4,5 = 0 \rightarrow V = 6 - x \quad (\text{zutena})$$

$$\sum M = 0 \rightarrow M - 1,5x - 4,5(x-4) + R \cdot \frac{x}{2} = 0 \rightarrow$$

$$\hookrightarrow M - 1,5x - 4,5x + 18 + x \cdot \frac{x}{2} = 0 \rightarrow M = -\frac{x^2}{2} + 6x - 18 \text{ (parabola)}$$

VM diagrama

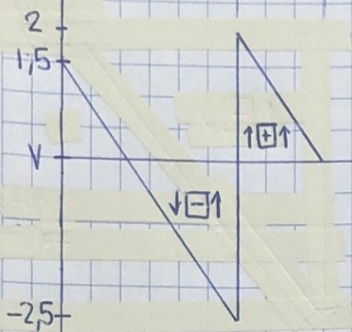


$$V(x=0) = 1,5 \text{ kN}$$

$$V(x=4) = -2,5 \text{ kN}$$

$$V(x=4) = 2 \text{ kN}$$

$$V(x=6) = 0 \text{ kN}$$

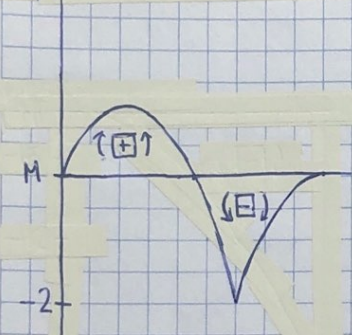


$$M(x=0) = 0 \text{ kNm}$$

$$M(x=4) = -2 \text{ kNm}$$

$$M(x=4) = -2 \text{ kNm}$$

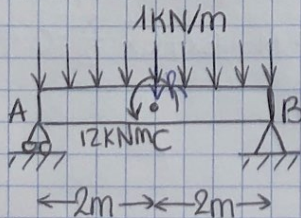
$$M(x=6) = 0 \text{ kNm}$$



$$\max_2 \begin{cases} x = \frac{-6}{2(-1/2)} = 6 \\ f(6) = 0 \end{cases}$$

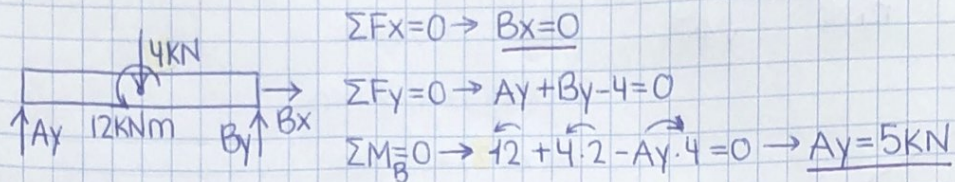
$$\max_1 \begin{cases} x = \frac{-1,5}{2(-1/2)} = 1,5 \\ f(1,5) = 1,125 \end{cases}$$

Marraztu haren VM diagrama



$$R = 1 \text{ kN/m} \cdot 4 \text{ m} = 4 \text{ kN}$$

Egitura askatu \rightarrow erreakzioak



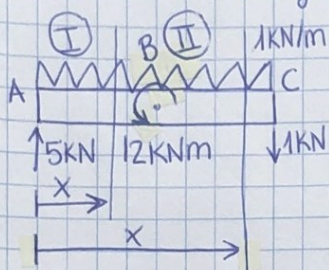
$$\sum F_x = 0 \rightarrow B_x = 0$$

$$\sum F_y = 0 \rightarrow A_y + B_y - 4 = 0$$

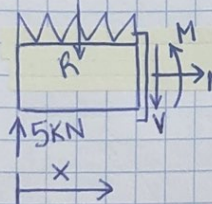
$$\sum M_B = 0 \rightarrow 12 + 4 \cdot 2 - A_y \cdot 4 = 0 \rightarrow A_y = 5 \text{ kN}$$

$$B_y = 4 - A_y = 4 - 5 = -1 \text{ kN} = B_y$$

Solido askearen diagrama:



I. eremua: AB $x \in (0, 2)$



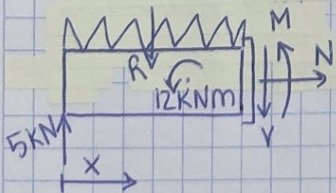
$$\sum F_x = 0 \rightarrow N = 0$$

$$\sum F_y = 0 \rightarrow 5 - R - V = 0 \rightarrow V = 5 - x \text{ (zuzena)}$$

$$\sum M = 0 \rightarrow M + R \cdot \frac{x}{2} - 5 \cdot x = 0 \rightarrow M = 5x - x \cdot \frac{x}{2} \rightarrow$$

$$\hookrightarrow M = -\frac{x^2}{2} + 5x \text{ (parabola)}$$

II. eremua: BC $x \in (2, 4)$



$$\sum F_x = 0 \rightarrow N = 0$$

$$\sum F_y = 0 \rightarrow 5 - R - V = 0 \rightarrow V = 5 - x \text{ (zuzena)}$$

$$\sum M = 0 \rightarrow M + R \cdot \frac{x}{2} + 12 - 5x = 0 \rightarrow$$

$$\hookrightarrow M = -\frac{x^2}{2} + 5x - 12 \text{ (parabola)}$$

VM diagrama



$$M(x=0) = 0 \text{ kNm}$$

$$M(x=1) = 4,5 \text{ kNm}$$

$$M(x=2) = 8 \text{ kNm}$$

$$M(x=2) = -4 \text{ kNm}$$

$$M(x=3) = -1,5 \text{ kNm}$$

$$M(x=4) = 0 \text{ kNm}$$

