8. GAIA

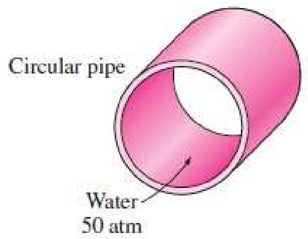
BARNEKO KONBEKZIO BEHARTUA

8.0 - HELBURUAK

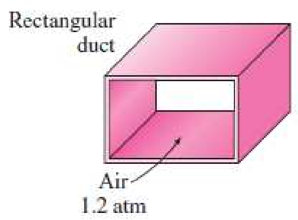
- Barne-fluxuaren **batez besteko abiadura** lortu, abiadura-profilean oinarrituta, eta **batez besteko tenperatura** lortu, tenperatura-profilean oinarrituta
- Barne-fluxuko **fluxu-eskualdeen ulermen** bisuala eduki, hala nola sarrera-eskualdearena eta erabat garatutako fluxu-eskualdearena, eta sarrera-luzera hidrodinamikoa eta termikoa kalkulatu
- Gainazal-tenperatura konstantea eta gainazaleko bero-fluxuaren baldintza konstanteak dituen hodi batean doan fluido bat **berotzeko eta hozteko prozesua aztertu**, eta **batez besteko tenperatura-diferentzia logaritmikoarekin** lan egin
- **Fluxu laminar** erabat garatuen abiadura-profilaren, presio-jaitsieraren, marruskadura-faktorearen eta Nusselten zenbakiaren **erlazio analitikoa lortu**
- **Fluxu turbulentu** erabat garatuen marruskadura-faktorea eta Nusselten zenbakia kalkulatu, erlazio enpirikoez baliatuta, eta presio-jaitsiera eta bero-transferentziaren abiadura kalkulatu

8.1 - SARRERA

Barne-fluxuan, fluidoa hodiaren barne-gainazalek inguratua dago



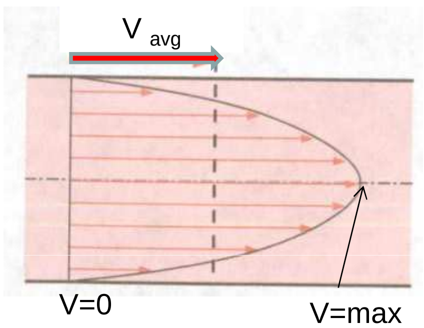
Fluidoa LIKIADOA denean



Fluidoa GASA denean

8.2 – BATEZ BESTEKO ABIADURA ETA TENPERATURA

BATEZ BESTEKO ABIADURA (V_{avg})



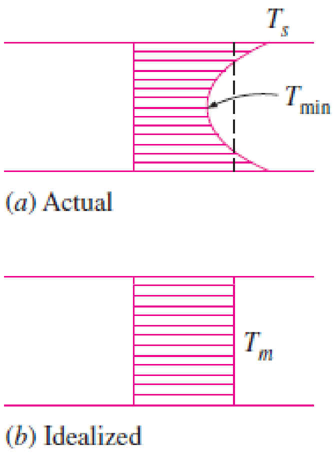
MASAREN kontserbazio-printzipioa

$$\dot{m} = \rho V_{avg} A_c = \int_{Ac} \rho u(r) dA_c$$

$$V_{avg} = \frac{2}{R^2} \int_0^R u(r) r dr$$

$u(r)$: abiadura - profila

BATEZ BESTEKO TENPERATURA (T_m)



ENERGIAREN kontserbazio-printzipioa

$$\dot{E}_{fluid} = \dot{m} c_p T_m = \int_{A_c} \rho C_p T(r) u(r) dA_c$$

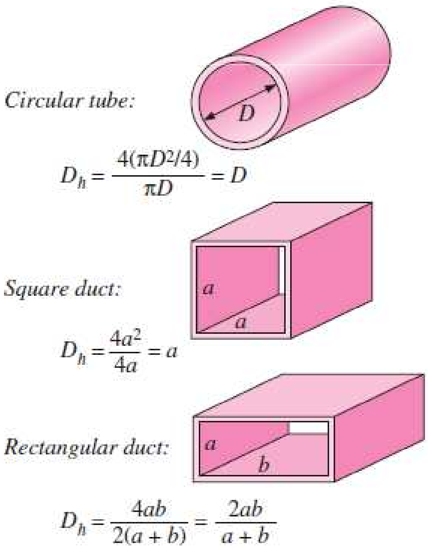
$$T_m = \frac{2}{V_{avg} R^2} \int_0^R T(r) u(r) r dr$$

FLUXU LAMINARRAK ETA TURBULENTUAK HODIETAN

$$\mathrm{Re} = \frac{\rho V_{avg} D}{\mu} = \frac{V_{avg} D}{\upsilon}$$

$$\upsilon = \mu / \rho$$ Biskositate zinematikoa

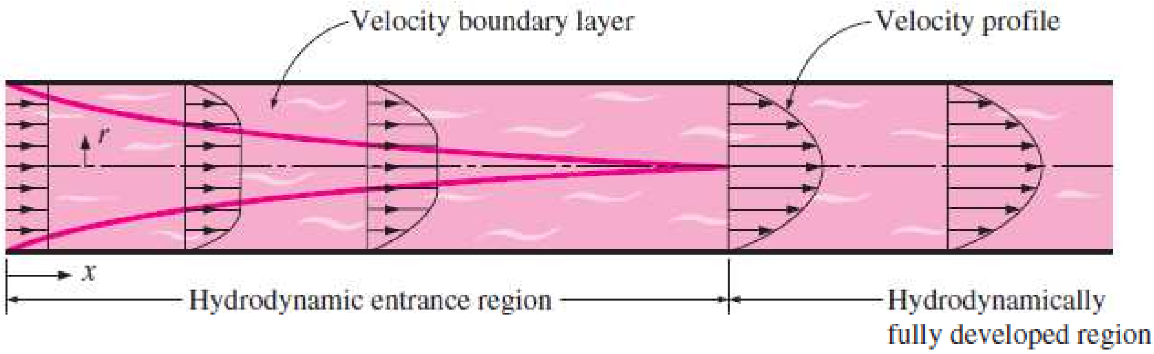
- **Re < 2.300 fluxu laminarra**: fluido oso biskosoak edo abiadura bajuak
- **Re > 10.000 fluxu turbulentoa**



Diametro Hidraulikoa

$$D_h = \frac{4A_c}{p}$$

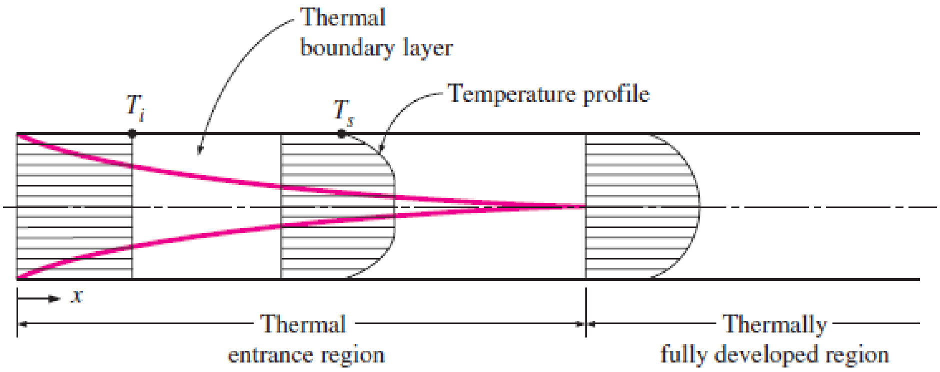
HODI BATEKO MUGALDE-GERUZA ABIADURAREN GARAPENA



Hidrodinamikoki erabat garatutako eskualdea

$$\frac{\partial u(r,x)}{\partial x} = 0 \rightarrow \mathrm{u} = \mathrm{u(r)}$$

HODI BATEKO MUGALDE-GERUZA TERMIKOAREN GARAPENA

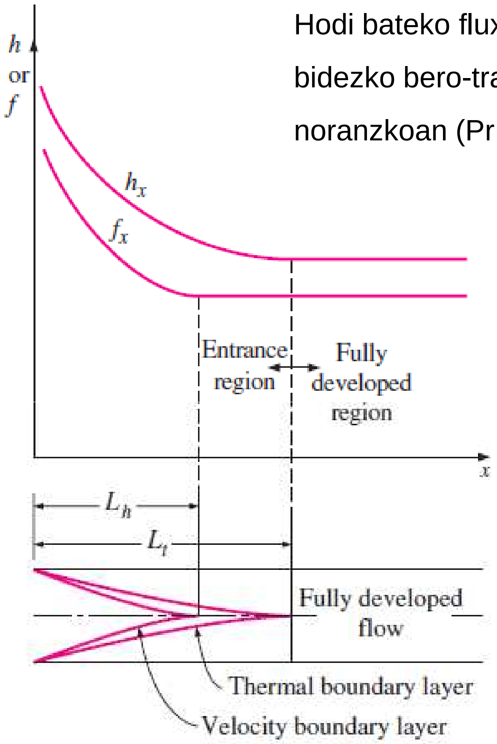


Termikoki erabat garatutako eskualdea

$$\frac{\partial}{\partial x}\left[\frac{T_s(x)-T(r,x)}{T_s(x)-T_m(x)}\right]=0$$

Gainazaleko bero-fluxua

$$\dot{q}_s = h_x(T_s - T_m) = k\left.\frac{\partial T}{\partial r}\right|_{r=R}$$



Hodi bateko fluxuaren marruskadura-faktorearen eta konbekzio bidezko bero-transferentziaren koefizientearen aldaketa fluxuaren noranzkoan (Pr > 1)

L_h luzera hidrodinamikoa

L_t luzera termikoa

SARRERA-LUZERAK EXPERIMENTALAK

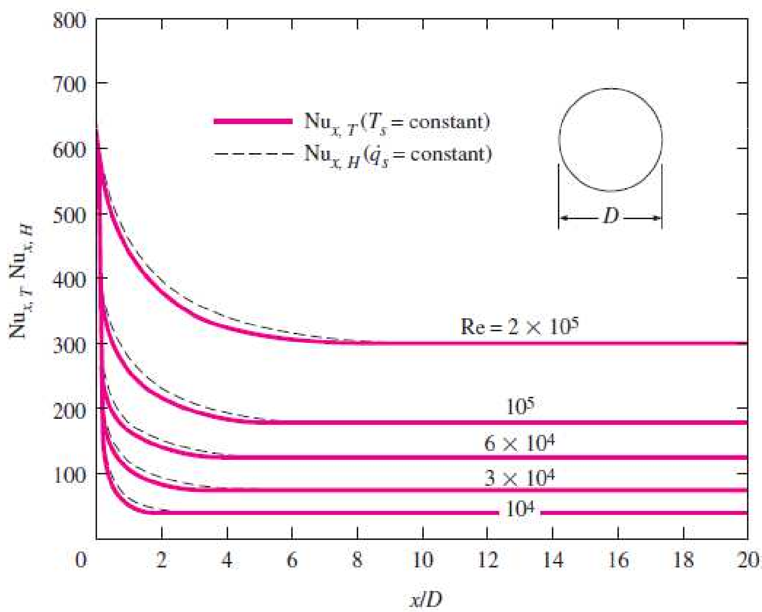
Fluxu Laminarra

$$L_{h,laminar} \approx 0{,}05\, Re D$$

$$L_{t,laminar} \approx 0{,}05\, Re\, Pr\, D = Pr\, L_{h,laminar}$$

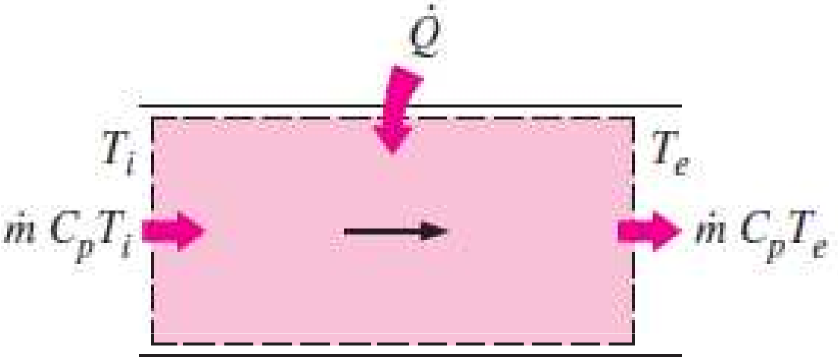
Fluxu turbulentua

$$L_{h,turbulent} \approx L_{t,turbulent} \approx 10D$$



Fluxu turbulentua duen hodi batean zeharreko Nusselten zenbaki lokalaren aldaketa

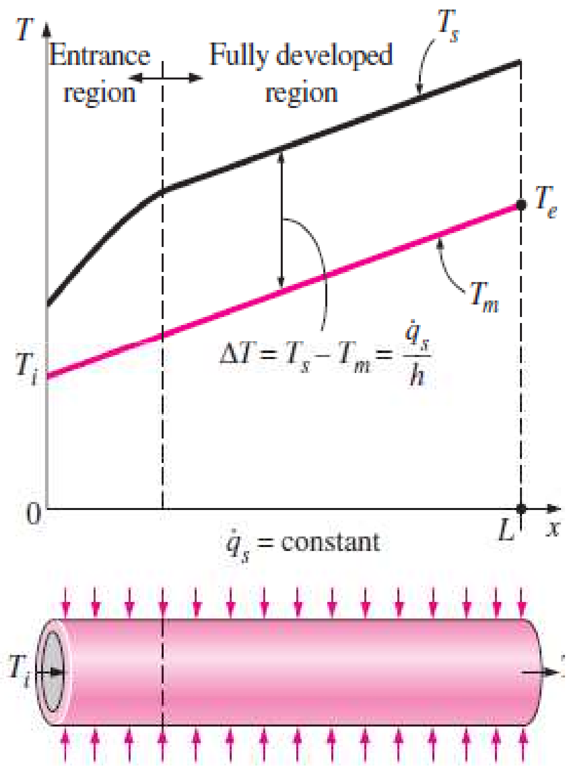
Hodi bateko fluido baten fluxu geldikorraren energia-kontserbazioaren ekuazioa



$$\dot{Q} = \dot{m}c_p(T_e - T_i) \quad (\mathrm{W})$$

$$\dot{q}_s = h_x(T_s - T_m) \quad (\mathrm{W/m^2})$$

GAINAZALEKO BERO-FLUXU KONSTANTEA (q_s= konstantea)



$$\boxed{\dot{Q} = \dot{q}_s A_s = \dot{m}c_p(T_e - T_i) \quad (\mathrm{W})} \quad T_e = T_i + \frac{\dot{q}_s A_s}{\dot{m}c_p}$$

$$\dot{q}_s = h\cdot(T_s - T_m) \rightarrow T_s = T_m + \frac{\dot{q}_s}{h}$$

$$h = kte \quad \rightarrow \quad T_s - T_m = kte$$

$$\dot{m}c_p dT_m = \dot{q}_s(pdx) \rightarrow \frac{dT_m}{dx} = \frac{\dot{q}_s p}{\dot{m}c_p} = kte$$

$$\frac{\partial T}{\partial x} = \frac{dT_s}{dx} = \frac{dT_m}{dx} = \frac{\dot{q}_s p}{\dot{m}c_p} = kte$$

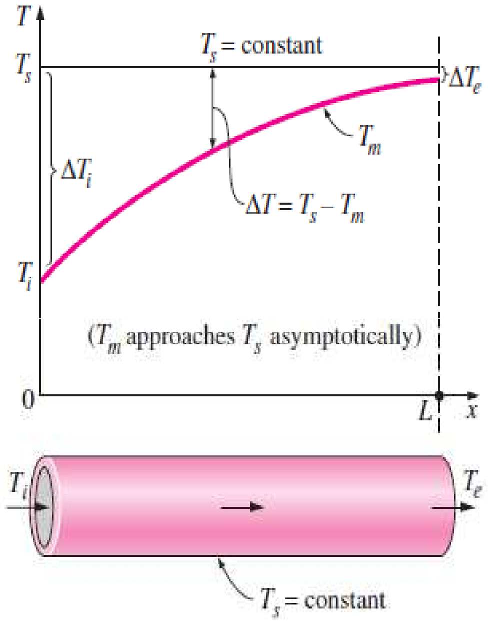
Tutu zirkularra:

$$\frac{\partial T}{\partial x} = \frac{dT_s}{dx} = \frac{dT_m}{dx} = \frac{2\dot{q}_s}{\rho V_{avg} c_p R} = kte$$

$$p = 2\pi R \ \text{ eta } \ \dot{m} = \rho V_{avg} A_c$$

8.4 – ANALISI TERMIKO OROKORRA

GAINAZAL-TENPERATURA KONSTANTEA (T_s= konstantea)



$$\dot{Q} = hA_s\Delta T_{avg} = hA_s(T_s - T_m)_{avg} \quad (W)$$

$$\Delta T_{avg} \approx \Delta T_{am} = \frac{\Delta T_i + \Delta T_e}{2} = \frac{(T_s - T_i) + (T_s - T_e)}{2}$$

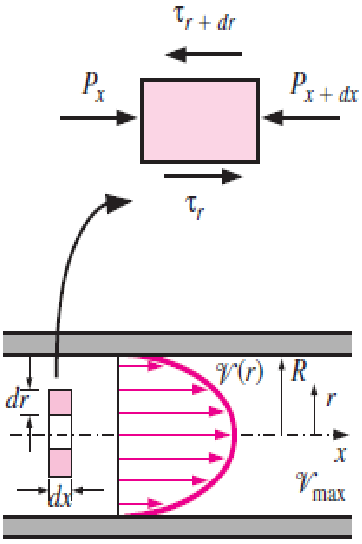
Energia Balantzea:

$$\dot{m}c_p dT_m = h(T_s - T_m)dA_s$$

$$T_e = T_s - (T_s - T_i)\exp(-hA_s / \dot{m}c_p)$$

$$\Delta T_{\ln} = \frac{(T_s - T_e) - (T_s - T_i)}{\ln[(T_s - T_e)/(T_s - T_i)]} = \frac{\Delta T_e - \Delta T_i}{\ln(\Delta T_e / \Delta T_i)}$$

8.5 – FLUXU LAMINARRA HODIETAN



Indar-balantzeak:

$$(2\pi r dr P)_x - (2\pi r dr P)_{x+dx} + (2\pi r dx \tau)_r - (2\pi r dx \tau)_{r+dr} = 0$$

Lim dr, dx→ 0 , ordezkapena $\tau = -\mu(du/dr)$

$$\frac{dP}{dx} = -\frac{2\tau_w}{R} = kte$$ berrantolatuz eta bi aldiz integratuz

$$u(r) = \frac{1}{4\mu}\left(\frac{dP}{dx}\right) + C_1 \ln r + C_2$$

$$u(r) = -\frac{R^2}{4\mu}\left(\frac{dP}{dx}\right)\left(1 - \frac{r^2}{R^2}\right)$$

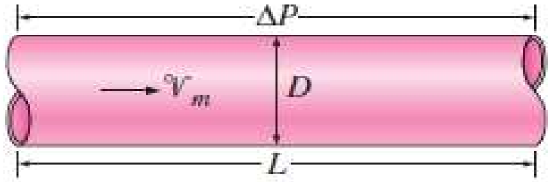
$$V_{avg} = \frac{2}{R^2}\int_0^R u(r) r dr = -\frac{R^2}{8\mu}\left(\frac{dP}{dx}\right)$$

$$u(r) = 2V_{avg}\left(1 - \frac{r^2}{R^2}\right)$$

non r=0 → $u_{max} = 2V_{avg}$

PRESIO-GALERA (ΔP)

$$\frac{dP}{dx} = \frac{P_2 - P_1}{L}$$



Fluxu Laminarra →

$$\Delta P = P_1 - P_2 = \frac{8\mu L V_{avg}}{R^2} = \frac{32\mu L V_{avg}}{D^2}$$

Presio Galera →

$$\Delta P_L = f \frac{L}{D} \frac{\rho V^2{}_{avg}}{2}$$

f= Darcy-ren marruskadure faktorea →

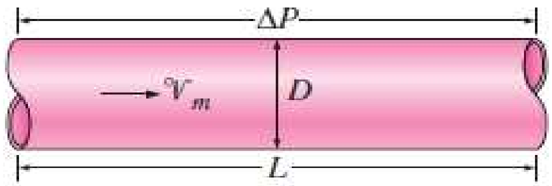
$$f = \frac{8\tau_w}{\rho V^2{}_{avg}}$$

Hodi zirkularra, Laminarra →

$$f = \frac{64\mu}{\rho D V_{avg}} = \frac{64}{Re}$$

KARGA-GALERA (h_L)

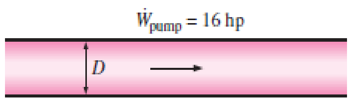
$$\Delta P = \rho g h$$



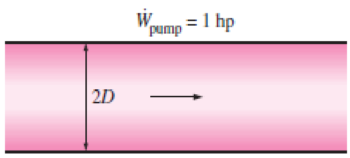
Karga Galera →

$$h_L = \frac{\Delta P_L}{\rho g} = f \frac{L}{D} \frac{V^2{}_{avg}}{2g}$$

PONPATZE-POTENTZIA



$$W_{pump,L} = \dot{V}\Delta P_L = \dot{V}\rho g h_L = \dot{m} g h_L$$

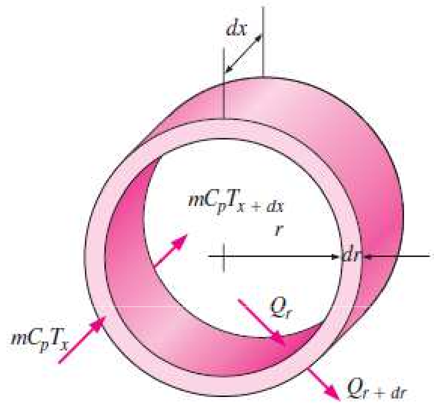


$$V_{avg} = \frac{(P_1 - P_2)R^2}{8\mu L} = \frac{\Delta P D^2}{32\mu L}$$

Poiseuille-ren Legea →

$$\dot{V} = V_{avg} A_c = \frac{(P_1 - P_2)R^2}{8\mu L}\pi R^2 = \frac{\Delta P \pi D^4}{128\mu L}$$

TENPERATURA-PROFILA ETA NUSSELTEN ZENBAKIA



Fluxu laminarra geldikorra, u=u(r)

Energia balantzea

$$\dot{m}c_pT_x - \dot{m}c_pT_{x+dx} + \dot{Q}_r - \dot{Q}_{r+dr} = 0$$

$$u\frac{\partial T}{\partial x} = -\frac{1}{2\rho c_p \pi r \mathrm{dx}}\frac{\partial \dot{Q}}{\partial r}$$

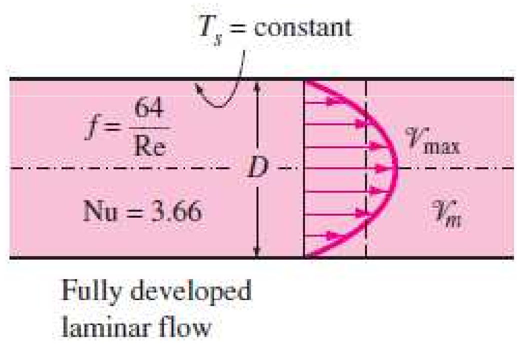
$$u\frac{\partial T}{\partial x} = \frac{\alpha}{r}\frac{\partial}{dr}\left(r\frac{\partial T}{\partial r}\right)$$

GAINAZALEKO BERO-FLUXU KONSTANTEA (q_s= kte)

$$\mathrm{Nu} = \frac{\mathrm{hD}}{\mathrm{k}} = 4{,}36$$

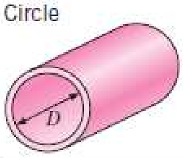
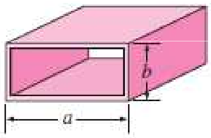
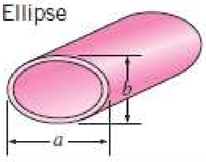
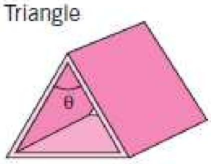
GAINAZAL-TENPERATURA KONSTANTEA (T_s= kte)

$$\mathrm{Nu} = \frac{\mathrm{hD}}{\mathrm{k}} = 3{,}66$$



FLUXU LAMINARRA HODI EZ-ZIRKULARRETAN

$$Nu = \frac{hD}{k} \Rightarrow$$

Tube Geometry	a/b or θ°	Nusselt Number T_s = Const.	Nusselt Number $\dot{q}_s$ = Const.	Friction Factor f
Circle 	—	3.66	4.36	64.00/Re
Rectangle 	a/b			
	1	2.98	3.61	56.92/Re
	2	3.39	4.12	62.20/Re
	3	3.96	4.79	68.36/Re
	4	4.44	5.33	72.92/Re
	6	5.14	6.05	78.80/Re
	8	5.60	6.49	82.32/Re
	∞	7.54	8.24	96.00/Re
Ellipse 	a/b			
	1	3.66	4.36	64.00/Re
	2	3.74	4.56	67.28/Re
	4	3.79	4.88	72.96/Re
	8	3.72	5.09	76.60/Re
	16	3.65	5.18	78.16/Re
Triangle 	θ			
	10°	1.61	2.45	50.80/Re
	30°	2.26	2.91	52.28/Re
	60°	2.47	3.11	53.32/Re
	90°	2.34	2.98	52.60/Re
	120°	2.00	2.68	50.96/Re

GARAPEN-FASEKO FLUXU LAMINARRA SARRERA-ESKUALDEAN

Hodi zirkularra, L, T_s=kte

$$Nu = 3{,}66 + \frac{0{,}0065(D/L)\,Re\,Pr}{1 + 0{,}04[(D/L)\,Re\,Pr]^{2/3}}$$

Hodi zirkularra, L, T_s =kte, $T_s >> T_{jariakin}$ denean

$$Nu = 1{,}86 + \left(\frac{Re\,Pr\,D}{L}\right)^{1/3}\left(\frac{\mu_b}{\mu_s}\right)^{0{,}14}$$

Plaka paralelo isotermoen arteko fluxua, L, Re≤ 2800

$$Nu = 7{,}54 + \frac{0{,}03(D_h/L)\,Re\,Pr}{1 + 0{,}016[(D_h/L)\,Re\,Pr]^{2/3}}$$

HODI LEUNA

Petukhov ➜ $f = (0{,}790 \ln Re - 1{,}64)^{-2} \qquad 3000 < Re < 5x10^6$

Chilton-Colburn ➜ $Nu = 0{,}125 \cdot f \cdot Re\, Pr^{1/3}$

Colburn ➜ $$Nu = 0{,}023 Re^{0,8} Pr^{1/3} \quad \begin{pmatrix} 0{,}7 \le Pr \le 160 \\ Re > 10\,000 \end{pmatrix}$$

Dittus-Boelter ➜ $$Nu = 0{,}023 Re^{0,8} Pr^{n} \quad \begin{pmatrix} n = 0{,}4 \text{ heating} \\ n = 0{,}3 \text{ cooling} \end{pmatrix}$$

Sieder-Tate ➜ $$Nu = 0{,}027 Re^{0,8} Pr^{1/3} \left(\frac{\mu}{\mu_s}\right)^{0,14} \quad \begin{pmatrix} 0{,}7 \le Pr \le 17\,600 \\ Re \ge 10\,000 \end{pmatrix}$$

Re > 10 000 Y HODI LEUNA ETA ZIMURRAK

Gnielinski ➜ $$Nu = \frac{(f/8)(Re - 1000)Pr}{1 + 12{,}7 (f/8)^{0,5} (Pr^{2/3} - 1)} \quad \begin{pmatrix} 0{,}5 \le Pr \le 2\,000 \\ 3x10^3 < Re < 5x10^6 \end{pmatrix}$$

GAINAZAL ZIMURRAK

Colebrook ➜ $$\frac{1}{\sqrt{f}} = -2{,}0 \log\left(\frac{\varepsilon/D}{3{,}7} + \frac{2{,}51}{Re\sqrt{f}}\right)$$

Moody ➜ $$\frac{1}{\sqrt{f}} = -1{,}8 \log\left(\frac{6{,}9}{Re} + \left(\frac{\varepsilon/D}{3{,}7}\right)^{1,11}\right)$$

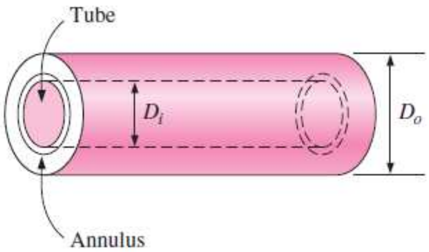
Equivalent roughness values for new commercial pipes*

Material	Roughness, ε ft	Roughness, ε mm
Glass, plastic	0 (smooth)	
Concrete	0.003–0.03	0.9–9
Wood stave	0.0016	0.5
Rubber, smoothed	0.000033	0.01
Copper or brass tubing	0.000005	0.0015
Cast iron	0.00085	0.26
Galvanized iron	0.0005	0.15
Wrought iron	0.00015	0.046
Stainless steel	0.000007	0.002
Commercial steel	0.00015	0.045

*The uncertainty in these values can be as much as ±60 percent.

8.6 – FLUXU TURBULENTUA HODIETAN

FLUXUA HODI ZENTROKIDEEN ARTEKO ERAZTUN-FORMAKO ESPAZIOAN



$$D_h = \frac{4A_c}{p} = D_o - D_i$$

$$Nu_i = \frac{h_i D_h}{k} \qquad Nu_o = \frac{h_o D_h}{k}$$

Erabat garatutako fluxu turbulentuetan ➡ **Gnielinski**

Gnielinski ekuazioaren obezpena:

$$\mathrm{Nu_i} = \mathrm{F_i}\,\mathrm{Nu_{Gnielinski}} \qquad F_i = 0{,}86\left(\frac{D_i}{D_o}\right)^{-0{,}16}$$ Kanpoko horma adiabatikoa

$$\mathrm{Nu_o} = \mathrm{F_o}\,\mathrm{Nu_{Gnielinski}} \qquad F_o = 0{,}86\left(\frac{D_i}{D_o}\right)^{-0{,}16}$$ Barneko horma adiabatikoa

8.7 – IRAKATSIKO EZ DIREN ATALAK