6. GAIA

KONBEKZIOAREN OINARRIAK

Universidad del País Vasco Euskal Herriko Unibertsitatea

Makina eta Motor Termikoak Saila
Departamento de Máquinas y Motores Térmicos

6.0 - HELBURUAK

- **Konbekzioa**ren mekanismo fisikoak eta sailkapena ulertu.
- Gainazalen gaineko fluxuen **abiaduraren mugalde-geruza** eta **mugalde-geruza termikoa** irudikatu.
- Reynoldsen, Prandtlen eta Nusselten **zenbaki dimentsiogabeen** ezagutza praktikoa izan.
- Fluxu **laminarrak** eta **turbulentuak** bereizi, eta fluxu turbulentuen momentu- eta bero-transferentziako mekanismoak ulertu
- Konbekzioa deskribatzen duten **ekuazio diferentzialak garatu**, masa-, momentu- eta energia-balantzeetan oinarrituta
- Konbekzio-ekuazioak **dimentsiogabetu**, eta marruskaduraren eta bero-transferentziaren koefizienteen forma **funtzionalak** lortu.
- Momentu- eta bero-transferentziaren arteko **analogiak** erabili, eta bero-transferentziaren koefizientea kalkulatu, marruskadura-koefizientetik abiatuta

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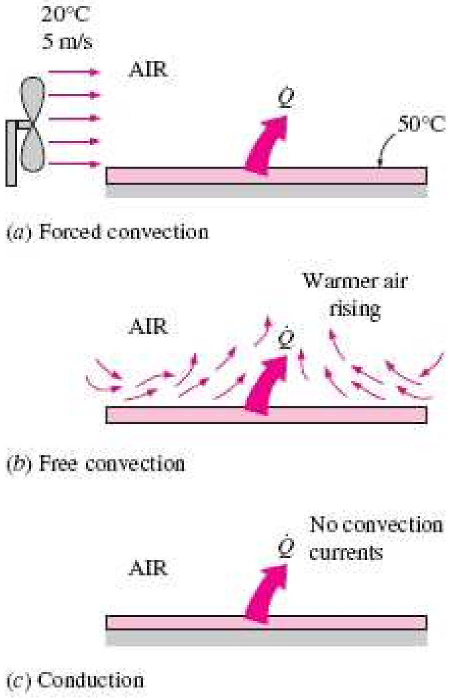
6.1 – KONBEKZIAOREN MEKANISMO FISIKOA



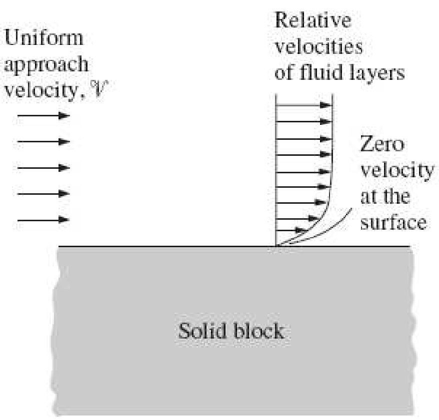
Konbekzio bero-transferentzia hurrengo parametroen araberakoa da:

- Biskositate dinamikoa μ_{fluid}
- Eroankortasun termikoa k_{fluid}
- Dentsitatea ρ_{fluid}
- Bero espezifikoa $c_{p,fluid}$
- Jariakinaren abiadura V_{fluid}
- Konfigurazio geometrikoa
- Solidoaren zimurtasuna
- Jariakinaren fluxu mota (laminarra edo turbulentua)

Newtonen hozte legea

$$\dot{Q} = h \cdot A \cdot (T_s - T_\infty) \quad \text{[W]}$$

6.1 – KONBEKZIAOREN MEKANISMO FISIKOA



Irristadurarik ezaren baldintza

Mugalde geruza

$$\dot{q}_{cond} = -k_{fluid} \cdot \left.\frac{\partial T}{\partial y}\right|_{y=0} \qquad [\text{W/m}^2]$$

$$\dot{q}_{conv} = h \cdot (T_s - T_\infty) \qquad [\text{W/m}^2]$$

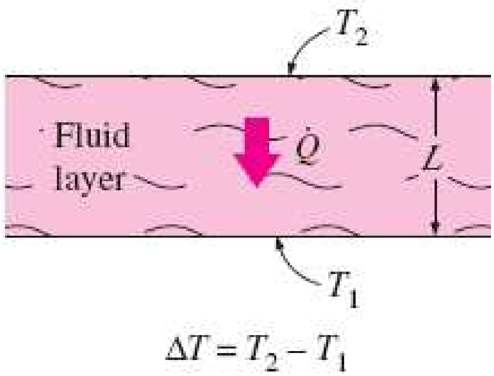
$$h = \frac{-k_{fluid} \cdot (\partial T / \partial y)_{y=0}}{T_s - T_\infty} \qquad [\text{W/m}^2 \cdot {}^{\text{o}}\text{C}]$$

Bataz besteko konbekzio koefizientea – Konbekzio koefiziente lokala

6.1 – KONBEKZIAOREN MEKANISMO FISIKOA

NUSSELTEN ZENBAKIA

$$Nu = \frac{h \cdot L_c}{k}$$



$$\left.\begin{aligned} \dot{q}_{conv} &= h \cdot \Delta T \\ \dot{q}_{cond} &= k \cdot \frac{\Delta T}{L} \end{aligned}\right\} \Rightarrow \frac{\dot{q}_{conv}}{\dot{q}_{cond}} = \frac{h \cdot \Delta T}{k \cdot \Delta T / L} = \frac{h \cdot L}{k} = Nu$$

6.2 – FLUIDO-FLUXUEN SAILKAPENA

Fluxu likatsua vs ez-likatsua.

Barne-fluxua vs kanpo-fluxua.

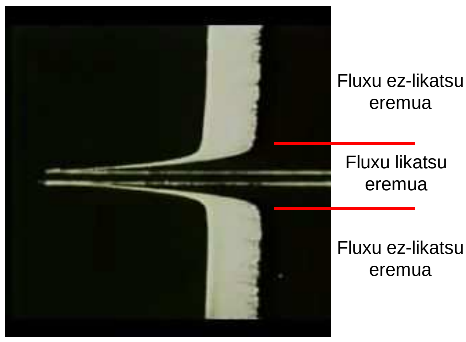
Fluxu konprimagarria vs konprimaezina.

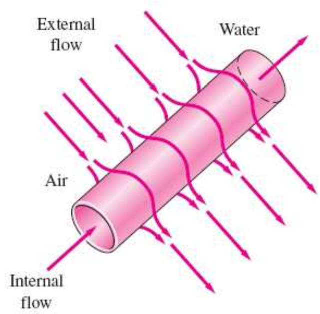
Fluxu laminarra vs turbulentua.

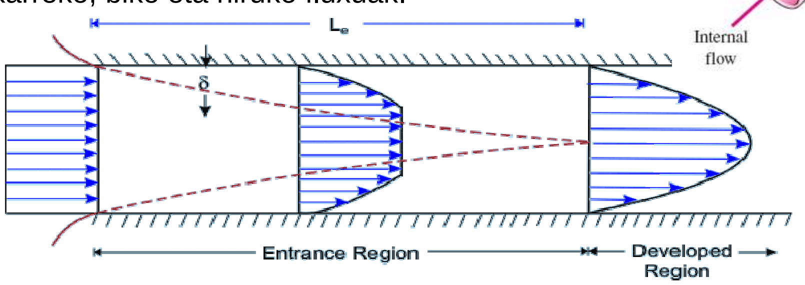
Fluxu naturala vs behartua.

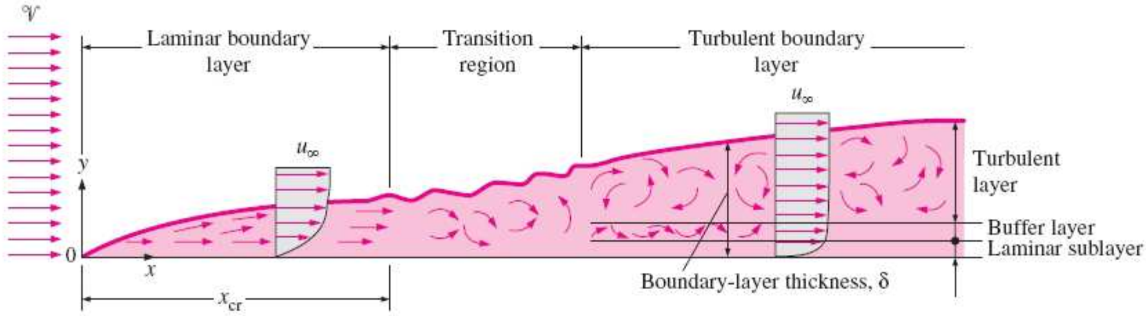
Fluxu geldikorra vs ez-geldikorra (iragankorra).

Dimentsio bakarreko, biko eta hiruko fluxuak.







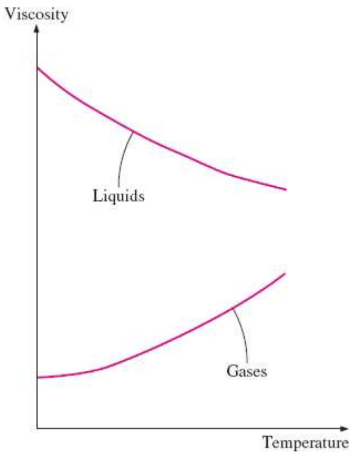


GAINAZALEKO EBAKIDURA-TENTSIOA $\tau = \mu \left.\frac{\partial u}{\partial y}\right|_{y=0}$ $[\mathrm{N/m^2}]$ Jariakin newtondarrak

Biskositate dinamikoa: μ [kg/m·s] [poise] 1 poise = 0,1 kg /m·s

Biskositate zinematikoa: $\nu = \frac{\mu}{\rho}$ [m^2/s] [stoke] 1 stoke = 10^{-4} m^2/s

GAINAZALEKO EBAKIDURA-TENTSIOA



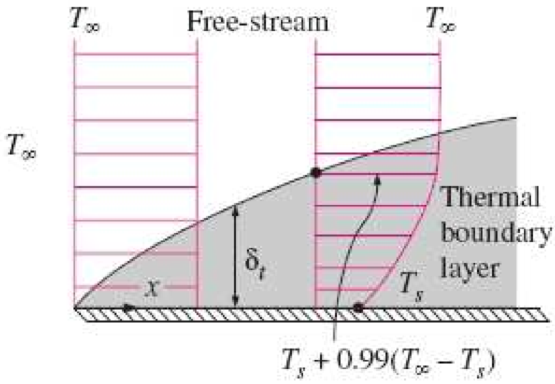
$$\tau_s = C_f \frac{\rho \cdot V^2}{2}$$

Marrusadura-koefizientea

Dynamic viscosities of some fluids at 1 atm and 20°C (unless otherwise stated)

Fluid	Dynamic viscosity μ, kg/m · s
Glycerin:	
−20°C	134.0
0°C	12.1
20°C	1.49
40°C	0.27
Engine oil:	
SAE 10W	0.10
SAE 10W30	0.17
SAE 30	0.29
SAE 50	0.86
Mercury	0.0015
Ethyl alcohol	0.0012
Water:	
0°C	0.0018
20°C	0.0010
100°C (liquid)	0.0003
100°C (vapor)	0.000013
Blood, 37°C	0.0004
Gasoline	0.00029
Ammonia	0.00022
Air	0.000018
Hydrogen, 0°C	0.000009

6.4 – MUGALDE-GERUZA TERMIKOA



PRANDTLEN ZENBAKIA

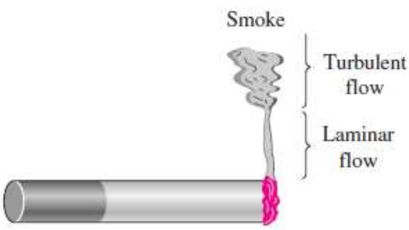
$$\Pr = \frac{\nu}{\alpha} = \frac{\mu \cdot c_p}{k}$$

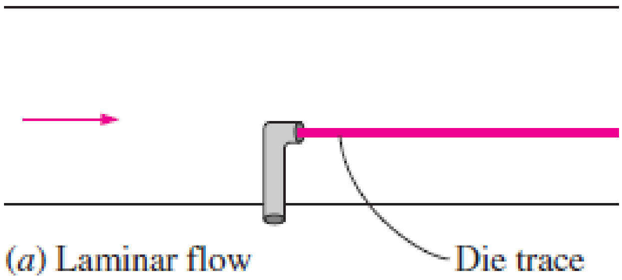
Typical ranges of Prandtl numbers for common fluids

Fluid	Pr
Liquid metals	0.004–0.030
Gases	0.7–1.0
Water	1.7–13.7
Light organic fluids	5–50
Oils	50–100,000
Glycerin	2000–100,000

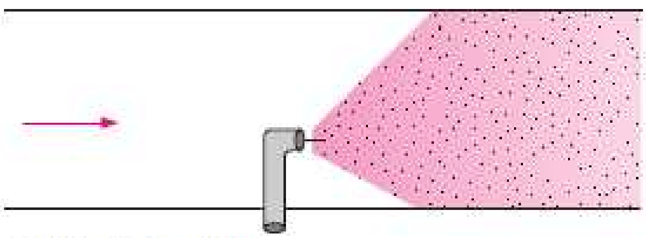
$$\Pr = \frac{\text{Momentuaren difusibitate molekularra}}{\text{Beroaren difusibitate molekularra}}$$

6.5 – FLUXU LAMINARRAK ETA TURBULENTOAK





(*a*) Laminar flow



(*b*) Turbulent flow

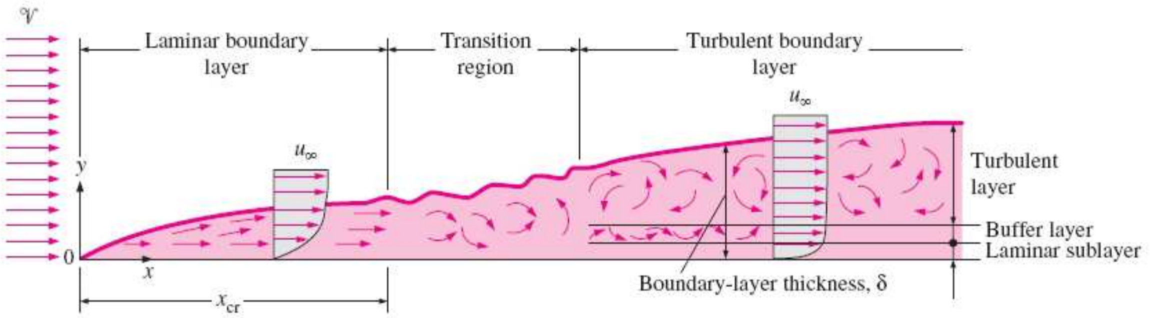
6.5 – FLUXU LAMINARRAK ETA TURBULENTOAK

REYNOLDSEN ZENBAKIA

$$\mathrm{Re} = \frac{V \cdot L_c}{\nu} = \frac{\rho \cdot V \cdot L_c}{\mu}$$

$$\mathrm{Re} = \frac{\text{Inertzi indarrak}}{\text{Biskositate - indarrak}}$$

- Re baxuak → Fluxu laminarra
- Re altuak → Fluxu turbulentua

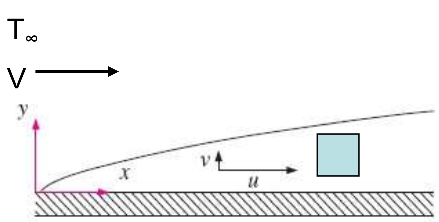


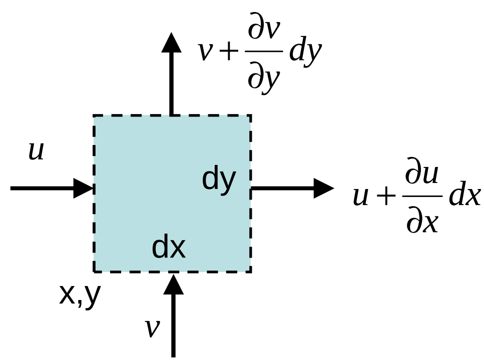
Reynolds kritikoa Xafla laua: $\mathrm{Re}_{cr} = V \cdot x_{cr}/\nu = 5 \cdot 10^5$

6.7 – KONBEKZIO-EKUAZIO DIFERENTZIALEN GARAPENA

<u>Hipotesiak</u>:
- Fluxu geldikorra bi dimentsiotan
- Jariakin Newtondarra
- Propietateak konstante (ρ,ν, k, …)

JARRAITUTASUN-EKUAZIOA



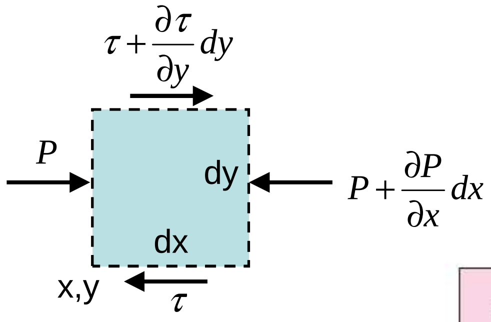


(Masa-emaria kontrol-bolumenaren barrurantz) = (Masa-emaria kontrol-bolumenaren kanporantz)

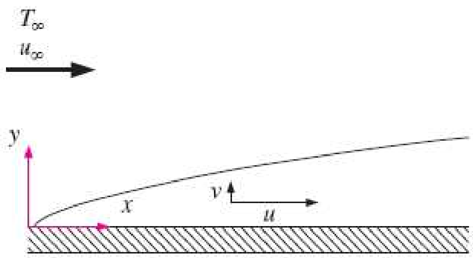
$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

MOMENTU-EKUAZIOAK

(Masa) (Azelerazioa, zehaztutako noranzkoan) = (Noranzko horretan eragiten duen indar garbia (gorputzekoa eta gainazalekoa))



$$\rho \left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = \mu \frac{\partial^2 u}{\partial y^2} - \frac{\partial P}{\partial x}$$



1) Velocity components:
$v \ll u$
2) Velocity grandients:
$\frac{\partial v}{\partial x} \approx 0, \frac{\partial v}{\partial y} \approx 0$
$\frac{\partial u}{\partial x} \ll \frac{\partial u}{\partial y}$
3) Temperature gradients:
$\frac{\partial T}{\partial x} \ll \frac{\partial T}{\partial y}$

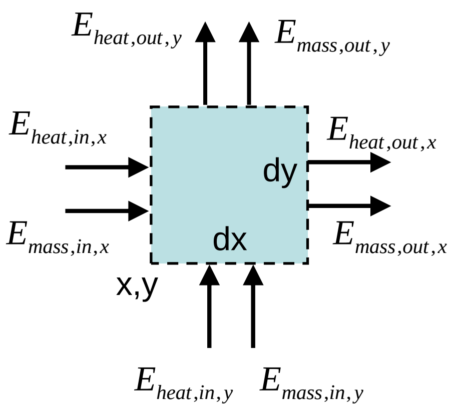
$$\frac{\partial P}{\partial y} = 0$$

ENERGIA-KONTSERBAZIOAREN EKUAZIOA

$$E_{in} - E_{out} = \Delta E_{sist} \quad \xrightarrow{\text{Egoera egonkorrean:}} \quad \dot{E}_{in} - \dot{E}_{out} = 0$$

$$\left(\dot{E}_{in} - \dot{E}_{out}\right)_{\text{by heat}} + \left(\dot{E}_{in} - \dot{E}_{out}\right)_{\text{by work}} + \left(\dot{E}_{in} - \dot{E}_{out}\right)_{\text{by mass}} = 0$$

$$e_{stream} = h + e_c + e_p = h + \frac{1}{2}V^2 + g \cdot z \approx h = c_p \cdot T$$



ENERGIA-KONTSERBAZIOAREN EKUAZIOA

$$\left(\dot{E}_{in}-\dot{E}_{out}\right)_{\text{by mass}}=-\rho\cdot c_p\cdot\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)dx\cdot dy$$

$$\left(\dot{E}_{in}-\dot{E}_{out}\right)_{\text{by heat}}=k\cdot\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)dx\cdot dy$$

$$\left(\dot{E}_{in}-\dot{E}_{out}\right)_{\text{by work}}=\mu\cdot\Phi\cdot dx\cdot dy=\mu\cdot\left(\frac{\partial u}{\partial y}\right)^2\cdot dx\cdot dy$$

Abiadura baxuetan mesprezagarria

$$\dot{E}_{in}-\dot{E}_{out}=0 \Rightarrow \rho\cdot c_p\cdot\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=k\cdot\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)$$

Mugalde geruzaren barnean oso txikia

Aldagai guztiak dimentsiogabetuz:

$$x^*=\frac{x}{L};\quad y^*=\frac{y}{L};\quad u^*=\frac{u}{V};\quad v^*=\frac{v}{V};\quad P^*=\frac{P}{\rho V^2};\quad T^*=\frac{T-T_s}{T_\infty-T_s};$$

Jarraitutasuna:

$$\frac{\partial u^*}{\partial x^*}+\frac{\partial v^*}{\partial y^*}=0$$

Momentua:

$$u^*\frac{\partial u^*}{\partial x^*}+v^*\frac{\partial u^*}{\partial y^*}=\frac{1}{\mathrm{Re}_L}\frac{\partial^2 u^*}{\partial y^{*2}}-\frac{\partial P^*}{\partial x^*}$$

Energia:

$$u^*\frac{\partial T^*}{\partial x^*}+v^*\frac{\partial T^*}{\partial y^*}=\frac{1}{\mathrm{Re}_L\cdot\mathrm{Pr}}\left(\frac{\partial^2 T^*}{\partial y^{*2}}\right)$$

$$u^*(0,y^*)=1;\quad u^*(x^*,0)=0;\quad u^*(x^*,\infty)=1;\quad v^*(x^*,0)=0;$$

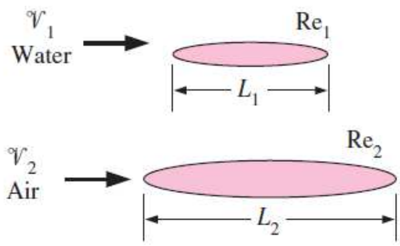
$$T^*(0,y^*)=1;\quad T^*(x^*,0)=0;\quad T^*(x^*,\infty)=1;$$

6.9 – KONBEKZIO-EKUAZIO DIMENTSIOGABEAK ETA ANTZEKOTASUNA

“Bi fenomeno fisiko *antzekoak* dira, baldin eta deskribatzen dituzten ekuazio diferentzialen eta mugalde-baldintzen forma dimentsiogabe berak badituzte.”

L, V, T_∞, T_S, ν, α ➡ Re_L, Pr

“Geometria jakin batean, antzekotasun-parametroen balio **bera** duten problemek **ebazpen berdin-berdinak** dituzte.”



Baldin $\mathrm{Re}_1 = \mathrm{Re}_2$ orduan $C_{f1} = C_{f2}$

6.10 – MARRUSKADURA- ETA KONBEKZIO-KOEFIZIENTEEN FORMA FUNTZIONALAK

3 ekuazio dimentsiogabeak:
- 3 funtzio ezezagun: $u^*,\ v^*$ y T^*
- 2 aldagai independente: $x^*,\ y^*$
- 2 parametro: Re_L y Pr

Lehenengo bi ekuazioetatik: $u^* = f_1(x^*, y^*, \mathrm{Re}_L)$

Ebakidura-tentsioa:
$$\tau_s = \mu \left.\frac{\partial u}{\partial y}\right|_{y=0} = \frac{\mu \cdot V}{L} \left.\frac{\partial u^*}{\partial y^*}\right|_{y^*=0} = \frac{\mu \cdot V}{L} f_2(x^*, \mathrm{Re}_L)$$

Marruskadura-koefizientea:

$$C_{f,x} = \frac{\tau_s}{\rho \cdot V^2/2} = \frac{\mu \cdot V / L}{\rho \cdot V^2/2} f_2(x^*, \mathrm{Re}_L) = f_3(x^*, \mathrm{Re}_L)$$

6.10 – MARRUSKADURA- ETA KONBEKZIO-KOEFIZIENTEEN FORMA FUNTZIONALAK

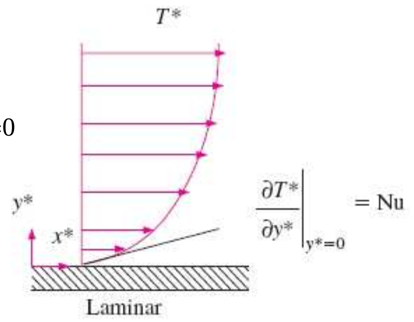
Azkenengo ekuaziotik.: $$T^* = g_1(x^*, y^*, \mathrm{Re}_L, \mathrm{Pr})$$

Konbekzio koefizientea:

$$y = 0 \Rightarrow \dot{q}_{cond} = \dot{q}_{conv} \Rightarrow -k_{fluid} \cdot \left.\frac{\partial T}{\partial y}\right|_{y=0} = h \cdot (T_s - T_\infty)$$

$$h = -\frac{k \left.\frac{\partial T}{\partial y}\right|_{y=0}}{T_s - T_\infty} = -\frac{k \frac{T_\infty - T_s}{L} \cdot \left.\frac{\partial T^*}{\partial y^*}\right|_{y^*=0}}{T_s - T_\infty} = \frac{k}{L} \left.\frac{\partial T^*}{\partial y^*}\right|_{y^*=0}$$

$$Nu_x = \frac{h \cdot L}{k} = \left.\frac{\partial T^*}{\partial y^*}\right|_{y^*=0} = g_2(x^*, \mathrm{Re}_L, \mathrm{Pr})$$



0 eta 1 artean, X-ekiko integratuz

$$C_f = f_4(\mathrm{Re}_L) \qquad Nu = g_3(\mathrm{Re}_L, \mathrm{Pr})$$

Sarritan:

$$Nu = C \cdot \mathrm{Re}_L^m \cdot \mathrm{Pr}^n$$

6.11 – MOMENTU- ETA BERO TRANSFERENTZIAREN ARTEKO ANALOGIAK

Helburua: C_f eta Nu lortu ➡
- Reynoldsen analogia
- Chilton-Colburnen analogia

Baldin $\mathrm{Pr} = 1$ eta $\frac{\partial P^*}{\partial x^*} = 0$

Momentua: $$u^* \frac{\partial u^*}{\partial x^*} + v^* \frac{\partial u^*}{\partial y^*} = \frac{1}{\mathrm{Re}_L} \frac{\partial^2 u^*}{\partial y^{*2}}$$

Energia: $$u^* \frac{\partial T^*}{\partial x^*} + v^* \frac{\partial T^*}{\partial y^*} = \frac{1}{\mathrm{Re}_L} \frac{\partial^2 T^*}{\partial y^{*2}}$$

Mugalde baldintzak: $$u^*(0, y^*) = 1; \quad u^*(x^*, 0) = 0; \quad u^*(x^*, \infty) = 1;$$
$$T^*(0, y^*) = 1; \quad T^*(x^*, 0) = 0; \quad T^*(x^*, \infty) = 1;$$

u* eta T* berdinak behar dute izan

$$\left.\frac{\partial u^*}{\partial y^*}\right|_{y^*=0} = \left.\frac{\partial T^*}{\partial y^*}\right|_{y^*=0}$$

6.11 – MOMENTU- ETA BERO TRANSFERENTZIAREN ARTEKO ANALOGIAK

$$C_{f,x}\frac{\mathrm{Re}_L}{2} = Nu_x \qquad (\mathrm{Pr}=1)$$

Reynoldsen analogia

$$\frac{C_{f,x}}{2} = St_x \qquad (\mathrm{Pr}=1)$$

$$St = \frac{h}{\rho \cdot c_p \cdot V} = \frac{\mathrm{Nu}}{\mathrm{Re_L}\cdot\mathrm{Pr}}$$

Stantonen zenbakia

Prandtlen zenbakia zuzenduz

Reynoldsen analogia eraldatua edo Chilton-Colburnen analogia

$$C_{f,x}\frac{\mathrm{Re}_L}{2} = Nu_x \cdot \mathrm{Pr}^{1/3}$$

$$\frac{C_{f,x}}{2} = \frac{h_x}{\rho \cdot c_p \cdot V}\cdot \mathrm{Pr}^{2/3} \equiv j_H$$

Colburnen j faktorea

Baliogarria: 0,6 < Pr < 60 tartean

Fluxu turbulentua presio-gradienteak badaude ere

Fluxu laminarra baldin eta $\partial \mathrm{P}^* / \partial \mathrm{x}^* = 0$

6.12 – IRAKATSIKO EZ DIREN ATALAK

6.6 atala – Fluxu turbulentuen bero- eta momentu-transferentzia

6.8 atala – Xafla lau baten konbekzio-ekuazioen ebazpenak